Cyclotrons : specific techniques

Chapter 3

Acceleration and RF cavities

maximal energy

Turn separation δr

Bunch size Δr and energy dispersion ΔE

Injection

Axial injection

Radial injection

Extraction

Stripping

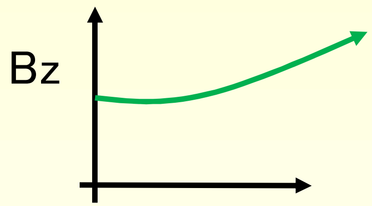
turn separation, precession, resonance

Cyclotron Summary

$$\omega_{rev} = 2\pi\, F_{rev} = \frac{qB}{m\,\gamma}$$

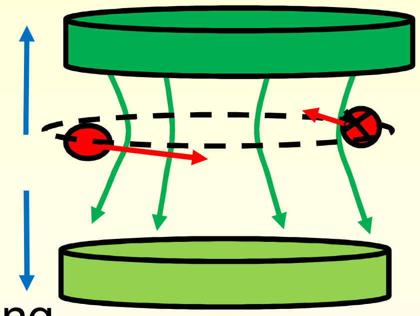
Longitudinal dynamics: particles synchronous with RF

Isochronous cyclotron = constant revolution frequency



Bz=Bz(R)

Radius

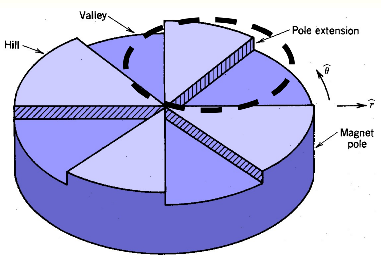


radial component

Br vertical defocusing

Transverse dynamics : vertical defocusing forces have to be compensated

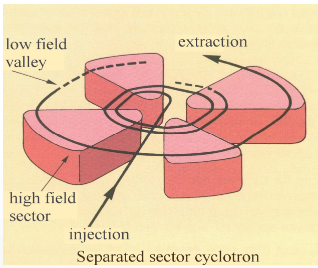
Azimutal(θ) **Field modulation = vertical focusing** Bz(R,θ) => Bθ



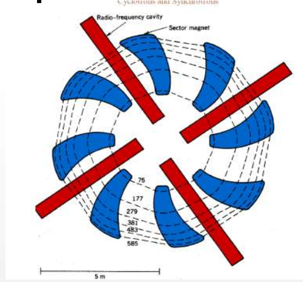
Straight sectors

Spiraled sectors

Separeted sectors



Separated and Spiraled sectors



Cyclotrons Tutorial 4

•**An cyclotron is supposed to accelerate ions with A nucleons and a charge state Q.**

•**Demonstrate that the maximal kinetic energy *E/A* of a cyclotron is**

$$E_K/A = K_b \, . \, (Q/A)^2$$

Nota : Give the *Kb* factor in a non relativistic approximation
using the extraction radius R,
the maximal average magnetic field B.

The mass of the ions is $m = Am_0$ & the charge of the ions is $q = Qe_0$

Cyclotrons Tutorial 4

- **An cyclotron accelerate ions with A nucleon and a charge state Q.**

$$E_K/A = K_b \,.\, (Q/A)^2 \quad ?$$

Answer : $E_K=(\gamma-1)mc^2 \sim \frac{1}{2}\, mV^2 = \frac{1}{2}\, m\,(R\omega)^2$

$$E_K = \frac{1}{2}\, m\,(R\, qB/m)^2 = \frac{1}{2}\, A\, m_0\, (R\, Q\, e_0\, B / A\, m_0)^2$$

$$E_K/A = \frac{1}{2}\, (e_0\, R\, B)^2 / m_0 \quad (Q/A)^2$$

$$E_K/A\ [MeV/A] = K_b \,.\, (Q/A)^2$$

$$K_b \sim (R_{extract.}\, B_{max})^2$$

Cyclotrons Tutorial 5

- A compact cyclotron have a Kb factor of 30 MeV

$$(E_K/A = Kb\ .\ (Q/A)^2)$$

What is the maximal kinetic energy
we could reach with such a cyclotron magnet (Kb=**30 MeV**)

a) With a proton beam
b) With a carbon beam (with Q= 6+)

The cyclotron magnet have <B>=1Tesla, what is
the revolution frequency ? (Frev =$\omega/2\pi$)
c) of a proton beam
d) of a carbon beam (with Q= 6+)

Can we work with the same RF cavity for the two beams ?
($\omega rf = h\ \omega = h\ qB/m\gamma$)

Max Energy for Superconducting Cyclotrons
not limited by (B x Rextraction)

We can demonstrate that isochronism imply $n(R) = (1-\gamma^2) < 0$

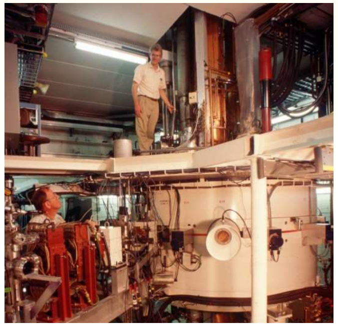
Stability : isochronous field condition *compensated* by Flutter

$$Q_z^2 = \langle n \rangle + \frac{N^2}{N^2-1} Fl.(1 + 2\tan^2(\xi)) + \ldots$$

At high energy field index n compensation not possible ($Qz^2<0$)

the max energy is not given by *Kb* ~48 (*B.R*extraction)^2

but $\mathbf{K_f}$ the so-called "*focusing* factor":



•Focusing limitation (stronger than B limitation)

$$\left[\frac{E}{A}\right]_{max} = Kf . \left\{\frac{Q}{A}\right\}^2 < Kb . \left\{\frac{Q}{A}\right\}^2$$

Kb~48 (B.Rextract)^2

Kf~ f(FLUTTER)

Acceleration in a cyclotron and orbit separation δr

•The final energy is independent of the accelerating potential $V = V_0 \cos \varphi$.

If V_0 varies, the number of turn varies. (but $B\rho_{final}$ =<B>.$R_{extraction}$)

• The energy gain per turn depends on the peak voltage V_0,

if the cyclotron is isochronous (for a particle φ =const) $\delta E = N_{gap}\, q\, V \cos(\varphi)$:

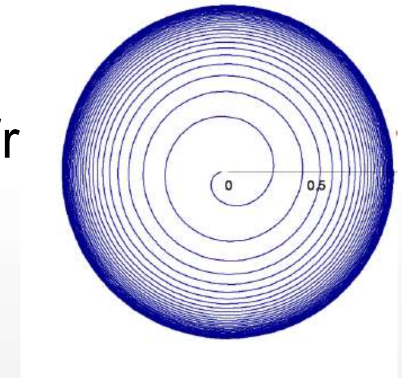
$$\frac{\delta r}{r} = \frac{\delta B\rho}{B\rho} = \frac{\delta p}{p} = \frac{\gamma}{\gamma+1}\frac{\delta E}{E} \approx \frac{qV_0 \cos\varphi}{2E} \propto \frac{1}{r^2}$$

$(\gamma \sim 1)$ $\quad E = \frac{1}{2} m v^2 = \frac{1}{2} m r^2 \omega^2$

• The radial separation δr between two turns varies as 1/r

$$\delta r \propto \frac{1}{r}$$

At large radius r, the different orbits are very close

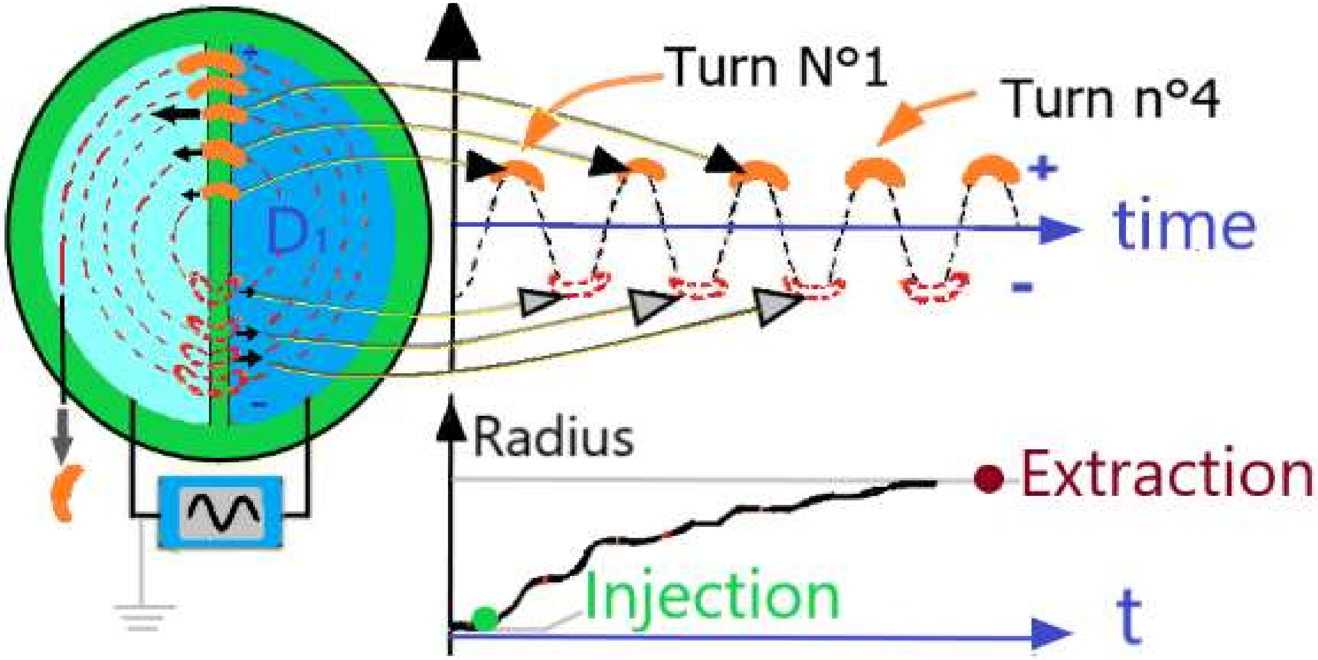


Acceleration in a cyclotron

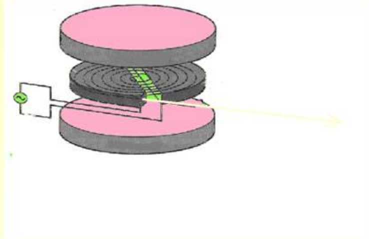
$$\delta E_{turn} = \delta E_{gap1} + \delta E_{gap2} = qV(\varphi_{gap1}) - qV(\varphi_{gap2})$$

$$\delta E_{turn} = N_{gap}\, q\, V_0 . \cos(\varphi_{gap})$$

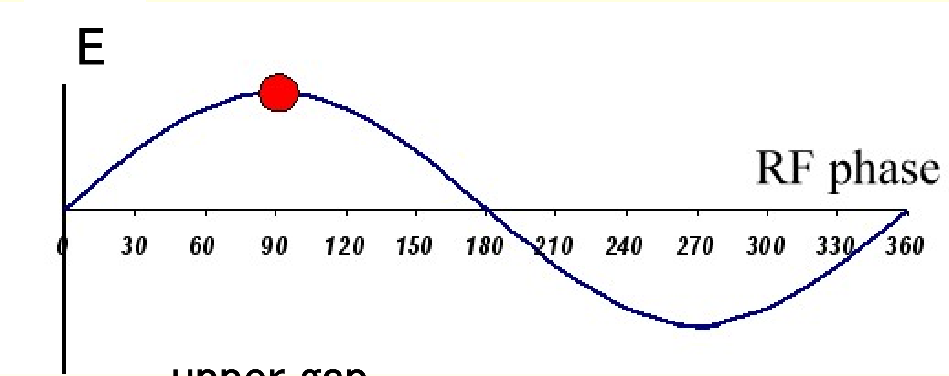
. $\omega_{rf} = \mathbf{H}\ \omega_{rev}$ Vdee $= V_0 . \cos(\mathbf{H}\ \omega_{rev}\ t)$



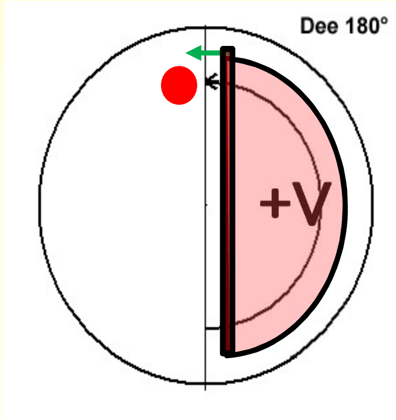
Harmonic number H=F$_{rf}$ /F$_{revolution}$



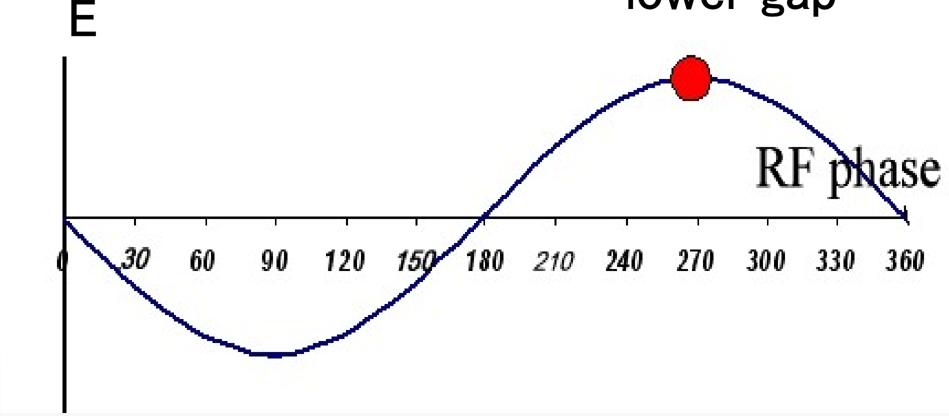
H = 1 : 1 bunch by turn $\omega_{rf} = \mathbf{H}\ \omega_{rev}$

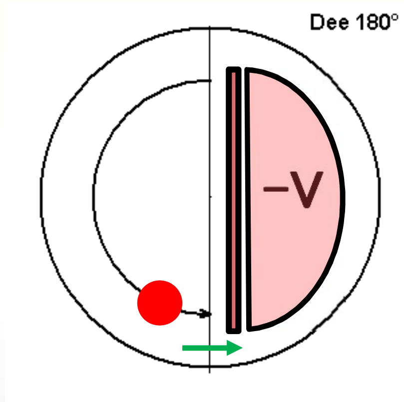


upper gap



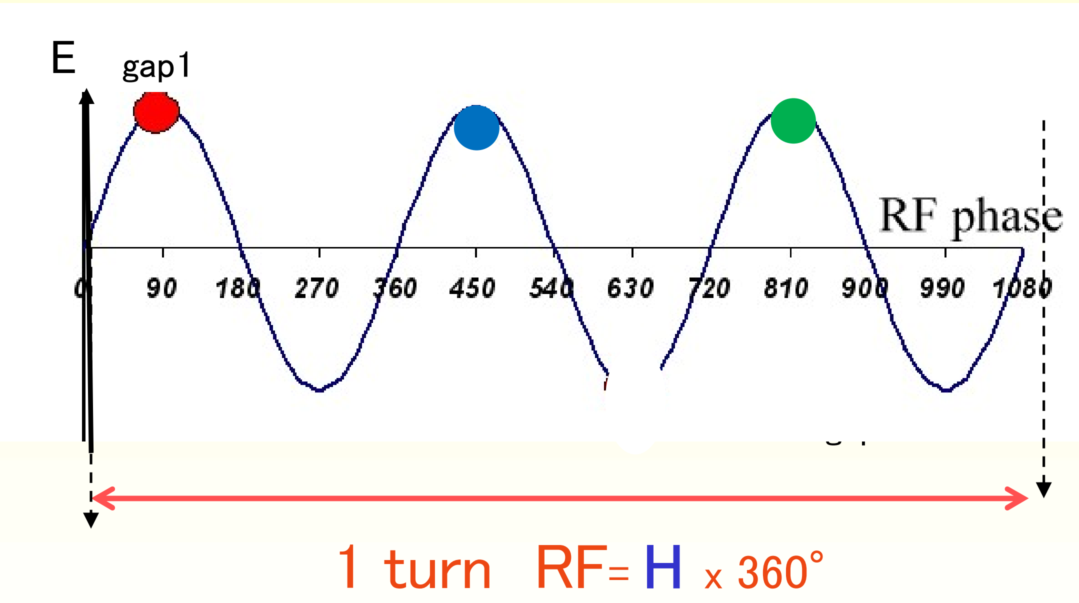
lower gap

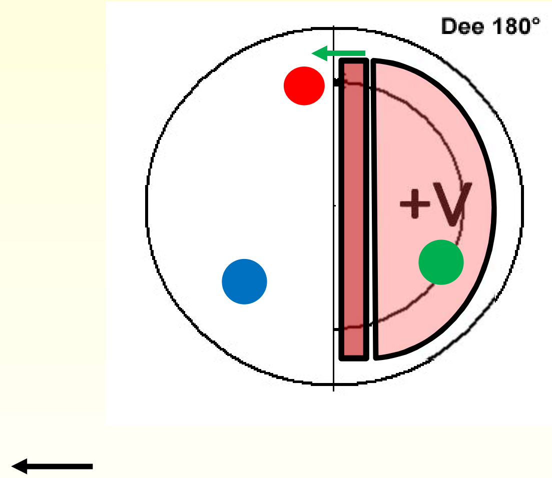




Harmonic number H =Frf / Frev

H = 3 : 3 bunches by turn $\omega_{rf} = \mathbf{H}\ \omega_{rev}$





Particle azimuth θ

$$\theta = \omega_{rev}\ t + constant$$

Rf phase

$$\varphi_{gap1} = H\,\omega_{rev}\ t + C$$

Acceleration in a cyclotron

and bunch size $\Delta r = f(\Delta t_{bunch})$

Two particles arriving at different time in accelerating gap will get a different energy :

$$E_1 = E_0 + q\,V_0\,.\cos(0) \quad \text{and} \quad E_2 = E_0 + q\,V_0\,.\cos(\omega_{rf}\,.\delta t)$$

The radial position of a particle evolves with energy : $R = B\rho_0 / B_z$

$$\Delta r / r = \Delta B\rho \,/\, B\rho = \Delta p \,/\, p = \gamma \,/\, (\gamma + 1)\;.\;\Delta E \,/\, E$$

A long bunch length $\Delta\phi = H\,\omega_{rev}\,\Delta t$: - induces a large energy dispersion ΔE

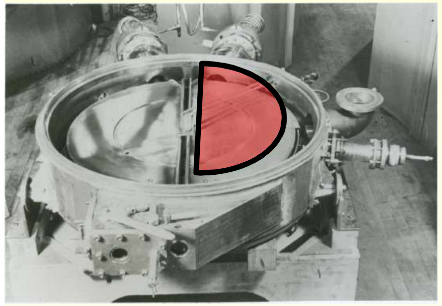
: - induces a large bunch size Δr

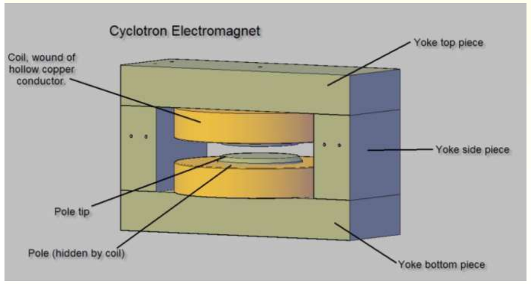
$$\Delta r / r \approx {}^{1}/_{2}\,.\Delta E / E \approx {}^{1}/_{2}\;\Delta\cos(\varphi) = {}^{1}/_{2}\,(\cos(0°) - \frac{\cos\Delta\varphi}{2}) \approx {}^{1}/_{4}\,\Delta\varphi^2$$

Use buncher to reduce $\Delta\phi$: it reduces energy dispersion and beam size (emittance)

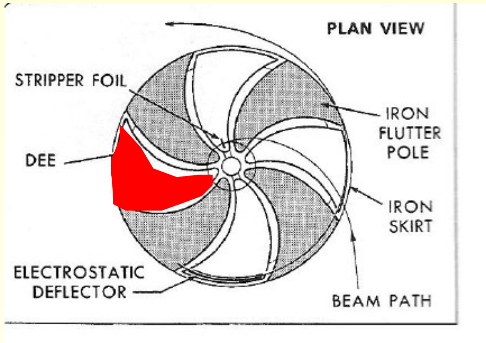
Acceleration RF Technology

Magnetic structure => RF cavity's shape





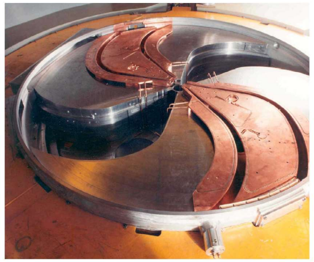
The classical "D" shape cyclo

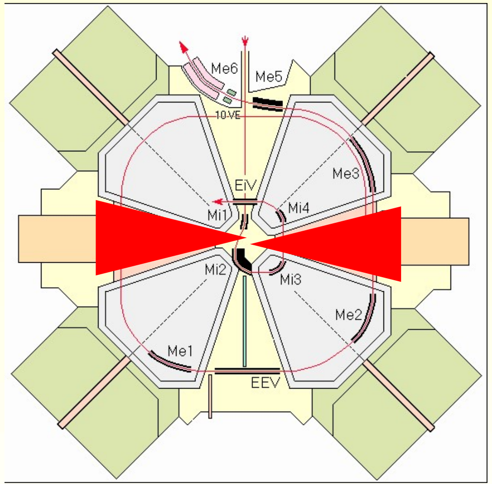


"Curved sector"

For spiral AVF

C235 poles and valleys



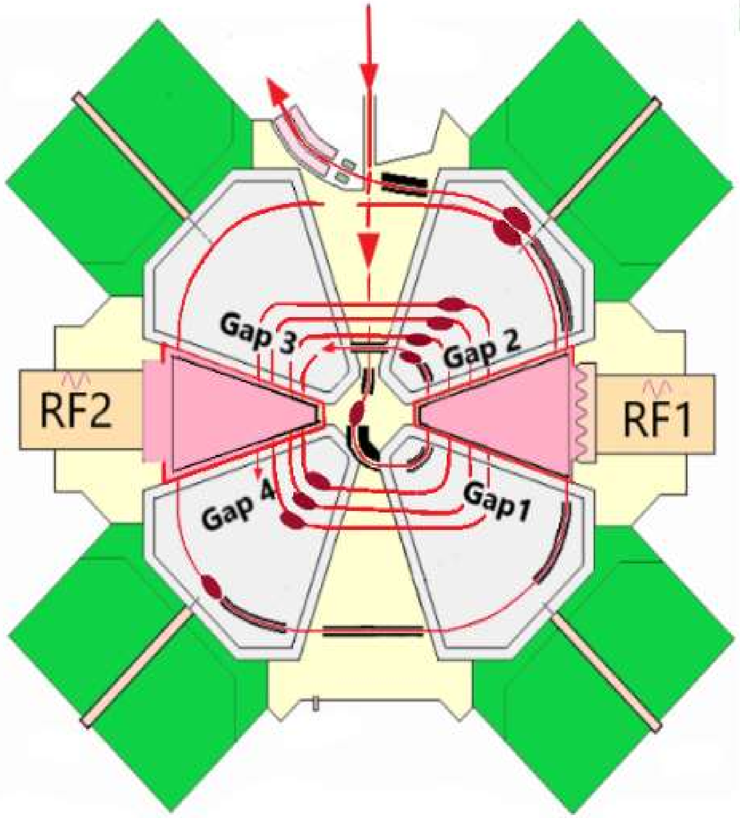


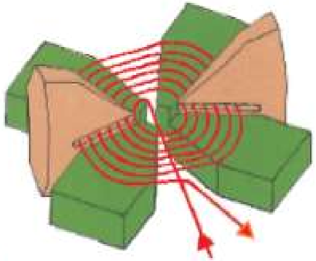
"Triangle" shape

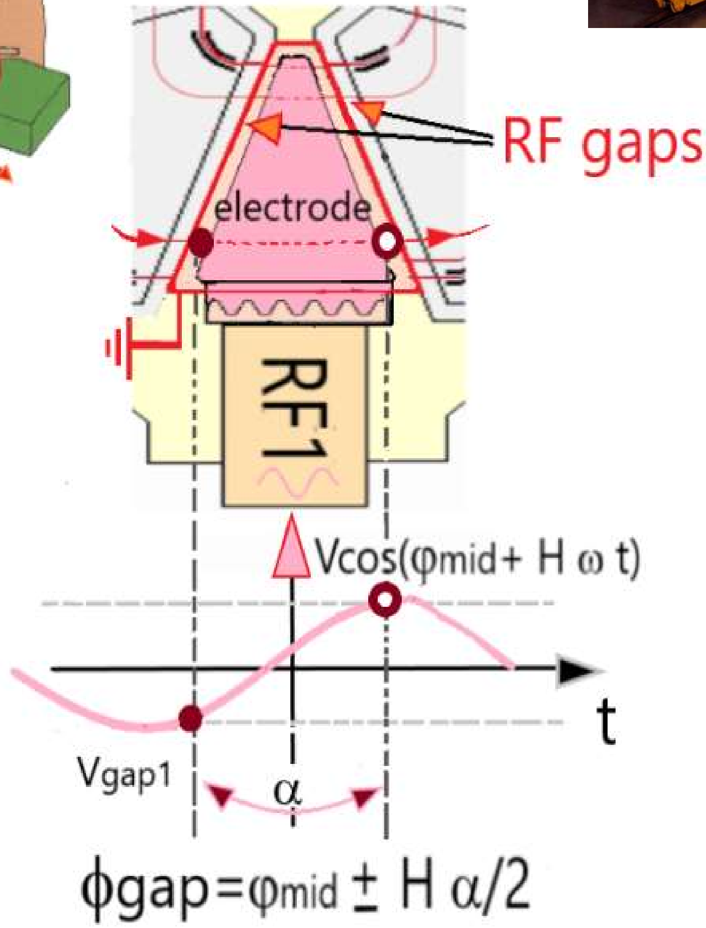
For separated sector

The choice of the pole shape and the number of sectors N have a great impact on the available space for RF systems. Dees have to fit into the gaps and/or valley sections

RF Cavities : example 1
for Separated Sectors Cyclotron







RF Cavities : example 1
for Separated Sectors Cyclo

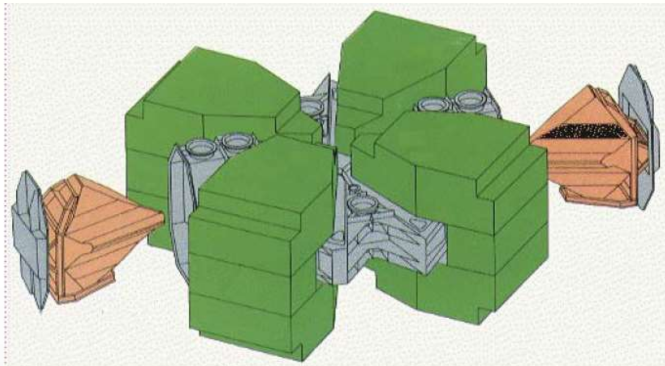
Energy gain in 2 gaps ~ $cos(\phi - h\alpha/2) + cos(\phi + h\alpha/2)$

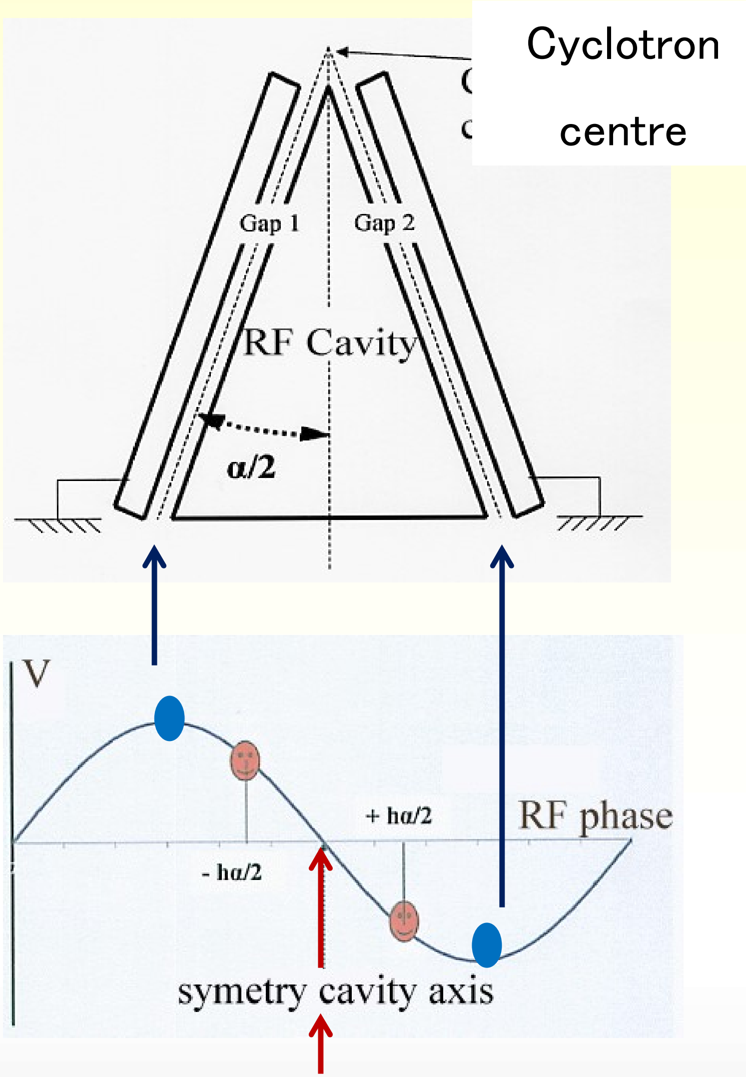
$$\delta E_{turn} = N_{gap}\, q\, V_0 . \sin\left({}^{H\alpha_{cav}}/_{2}\right) . \cos(\varphi_{mid})$$

- For a maximum energy gain(: ϕ=0)

δEturn optimum is

for H. αcav / 2 = 90 degree





ϕmiddle=0

Ion Sources for cyclotron

- Internal source (inside cyclotron)

–PIG : Penning or Philips Ionization Gauge i*on source*

for very light beam with low charge state H+,D+,He+

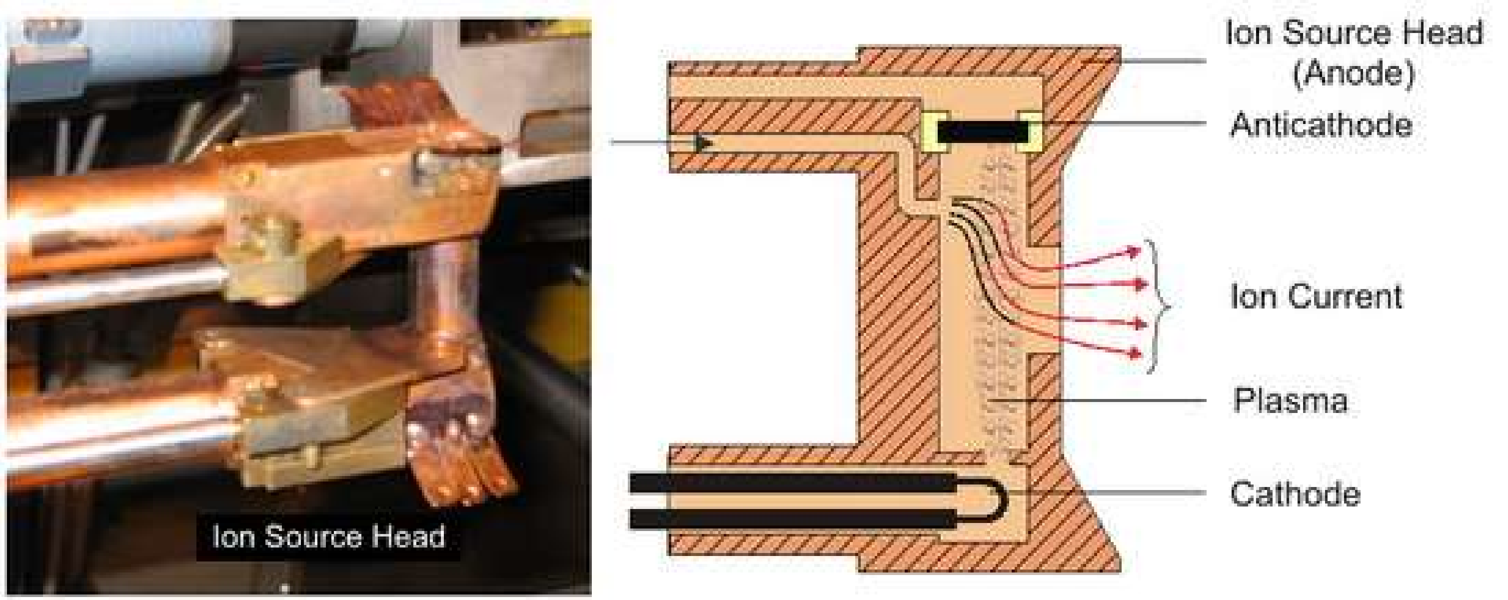
- External source (outside cyclotron with injection line)

– Multi–CUSP source ;: *for negativ ion H– or D–*

– ECRIS（Electron cyclotron resonance）

for high charge state He++ up to U^{35+}

Internal ion source: Cold cathode PIG Ion Source

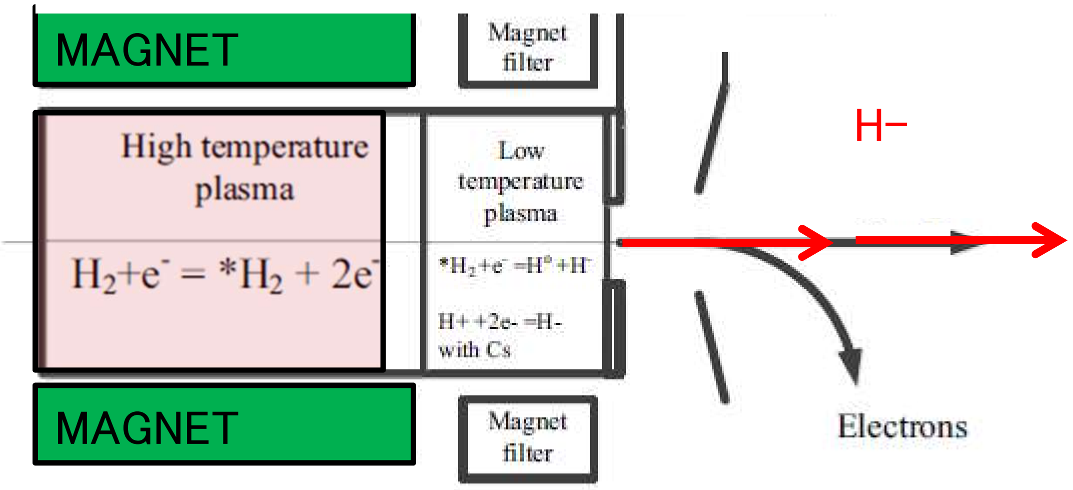


- Electron emission due to electrical potential on the cathodes
- Electron confinement due to the magnetic field along the anode axis
- Electrons produced by thermionic emission and ionic bombardment
    - Start-up: 3 kV to strike an arc
    - At the operating point : 100 V
- Cathodes heated by the plasma (100 V is enough to pull an outer e- off the gas atoms)

External ion source : Multi-CUSP source

for negativ ions : H-//D- with high current

Confinement + filtering + extraction



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e- $+H_2$ => e- + p +H-

- Larger Than the PIG source (Magnets)
- Better emittance
- Larger current (Magnet confinement+ Filter)

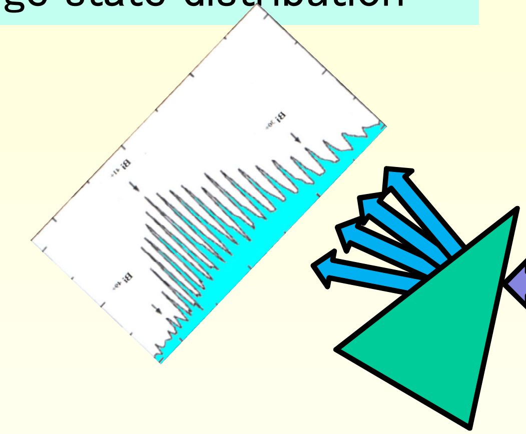
Larger Size ⇨ External Source

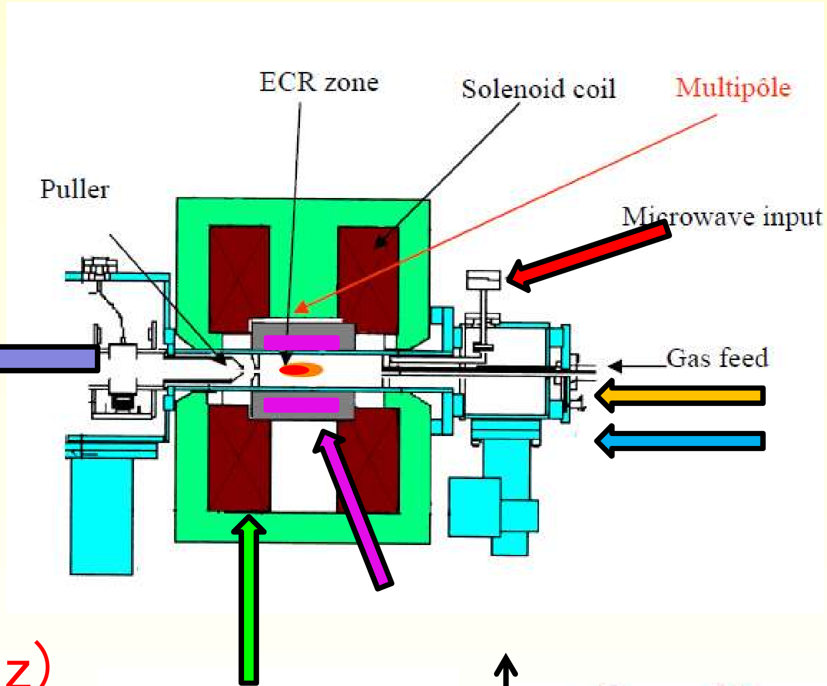
External ion source : ECR ion source

Heavy ions with high charge state

Pantechnic
Nanogan®

Charge state distribution

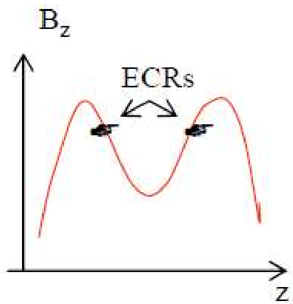


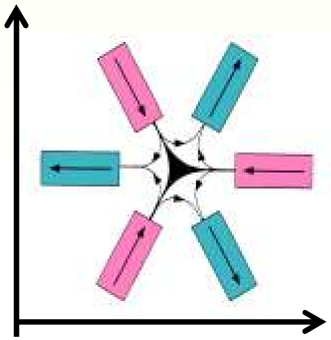


Gas (He,O,..) + RF(microWave 10-18Ghz)
Plasma (ions + electrons) :
+ ATOMS

electrons + ions impacts

Ionize any injected heavy atoms
(He,Li,······.U)



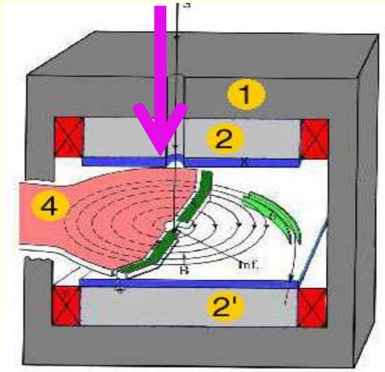


Bz + Br
3D plasma confinement

Beam injection in cyclotron

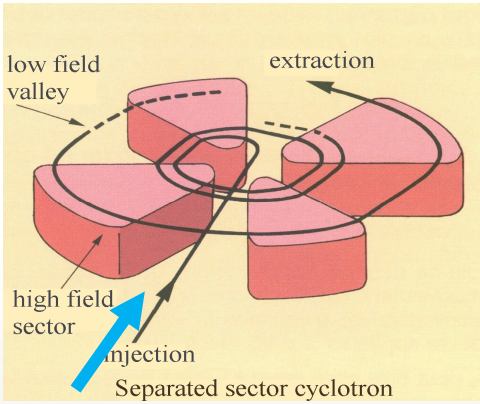
- Low energy cyclotron : axial injection

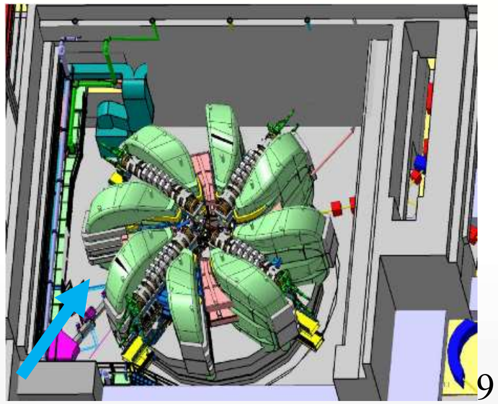
Injection from the top



- Higher energy cyclotron : radial injection

Injection In between sectors



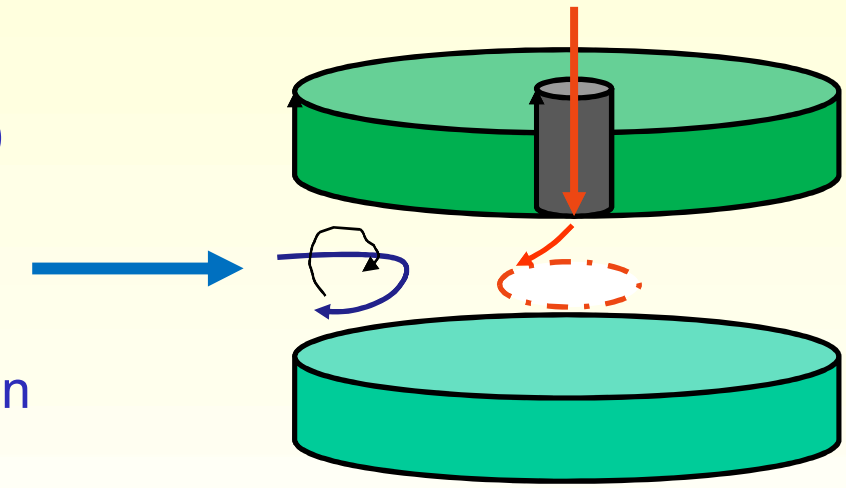


injection in compact cylotron

- Goal : Put the beam on the « good orbit »

Can we inject the beam
from the side ? (horizontal plan)

Not possible :
Magnetic force too strong !!
Particle with low rigidity at injection



Idea : inject Vertically : if $V_z // B_z$ $F = V_z \times B_z \sim 0$

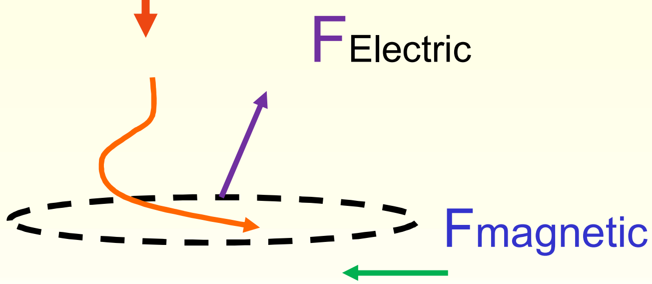
Beam Axial injection from the top of the cyclotron

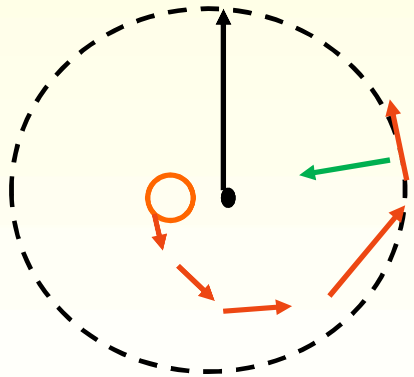
Axial injection with inflector

- Goal : Put the beam on the « good orbit » at the good phase

  with a very compact geometry

Generate a Vertical force with an electrostatic device





-Outside cyclotron

axial motion (vertical)

- Inside cyclotron (Magnetic force is radial)

radial motion (horizontal)

$R_m = B\rho \;/\; B_{center}$

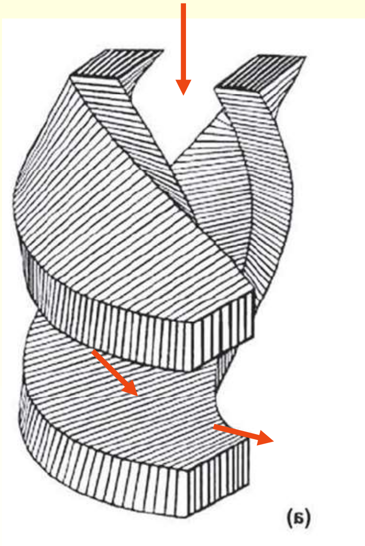
$R_E = mV^2/Q \;/\; E_{inflector}$

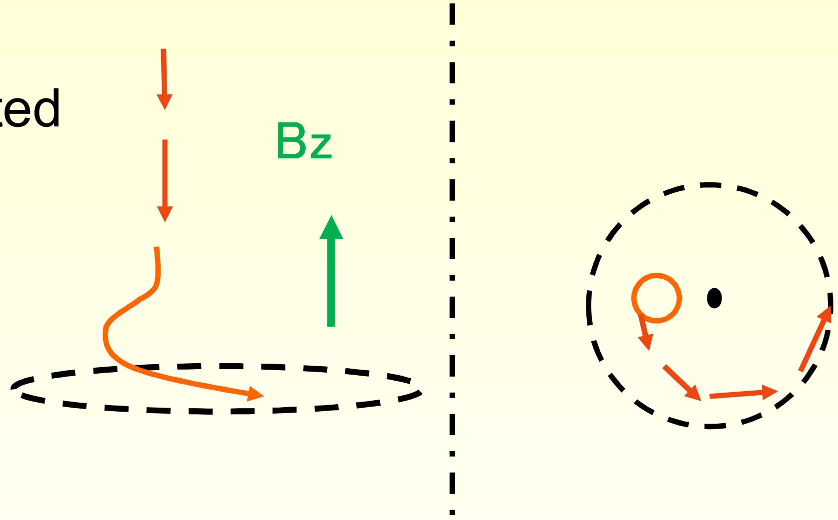
Axial injection : the Spiral inflector

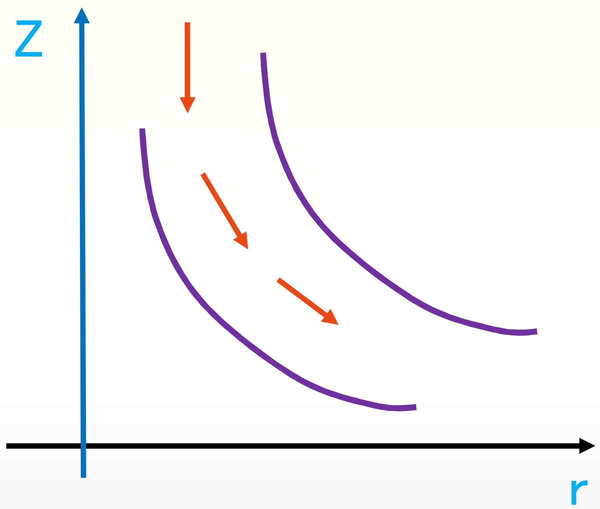
=Twisted electrodes with **E** ⊥ v

1. Spiral inflector (or helical channel)

   principle: 90° electrostatic deflector twisted



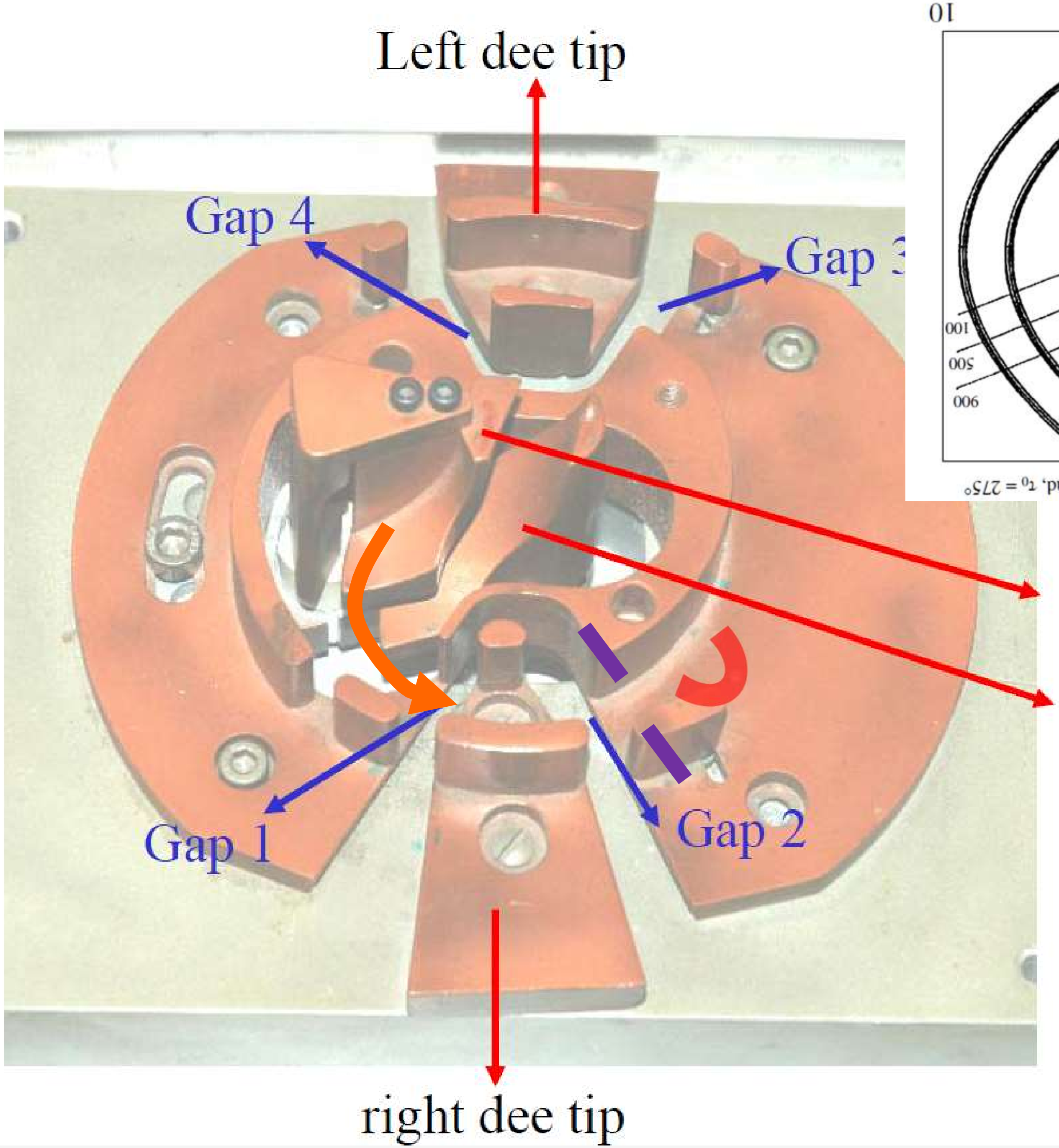


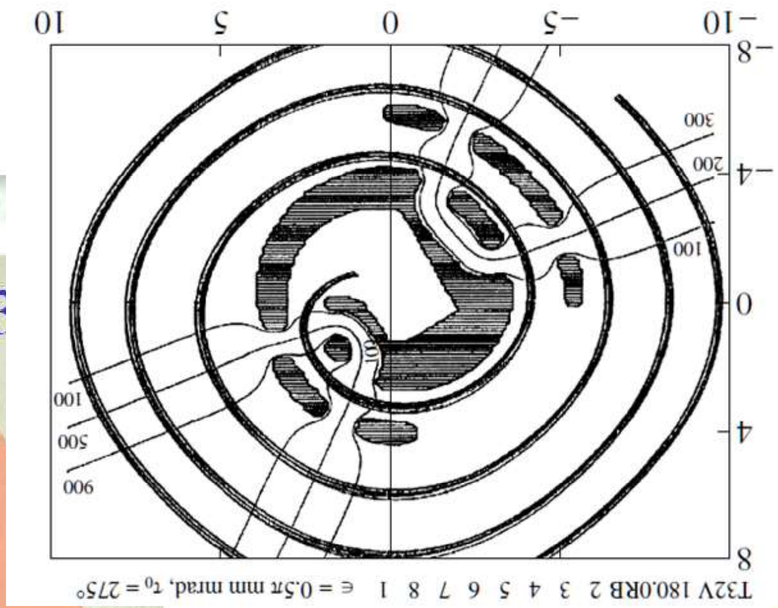


E always perpendicular to v (**E** ⊥ v)

B=B_z constant (cyclo center)

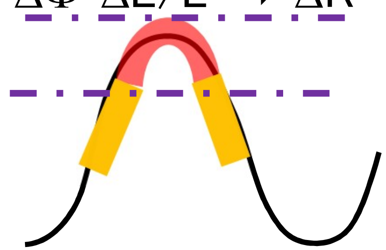
Complex geometry, very compact





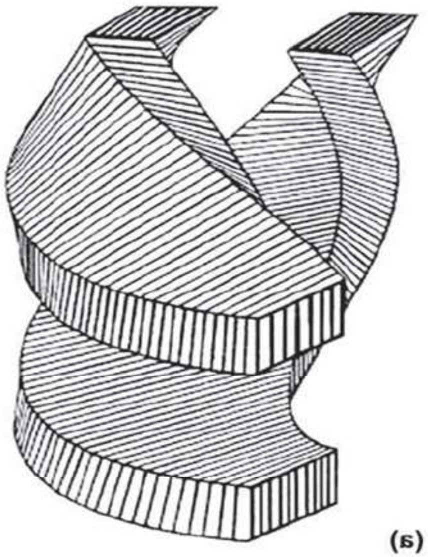
Phase selection
with RF

$\Delta\Phi \tilde{} \Delta E/E \Rightarrow \Delta R$



Axial injection : the Spiral inflector
=Twisted electrodes with $\mathbf{E} \perp v$

$$m\ddot{x} = qE_x - qv_yB_0,$$
$$m\ddot{y} = qE_y + qv_xB_0,$$
$$m\ddot{z} = qE_z.$$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Trajectory Equations are very funny :

Parametric equation of the trajectory $\theta = [0,\pi/2]$

$$x_c = \lambda(1 - \sin k\theta \sin\theta - \cos k\theta \cos\theta)$$
$$y_c = \lambda(\sin k\theta \cos\theta - \cos k\theta \sin\theta) \quad ,$$
$$z_c = A(\sin\theta - 1)$$

$$k = A/R_m + k'$$
$$\lambda = A/(k^2 - 1)$$

Two parameters : A the inflector Height
k' the tilt

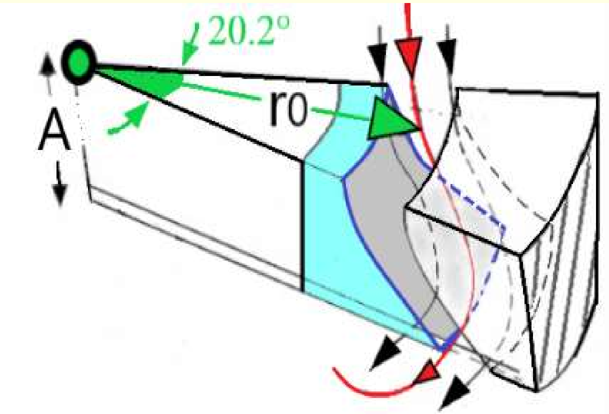
2 forces bend the beam
Electric radius $A = RE = mV^2/Q / Eo$
Magnetic radius $Rm = Br / Bo$

Axial injection 2: hyperboloid inflector

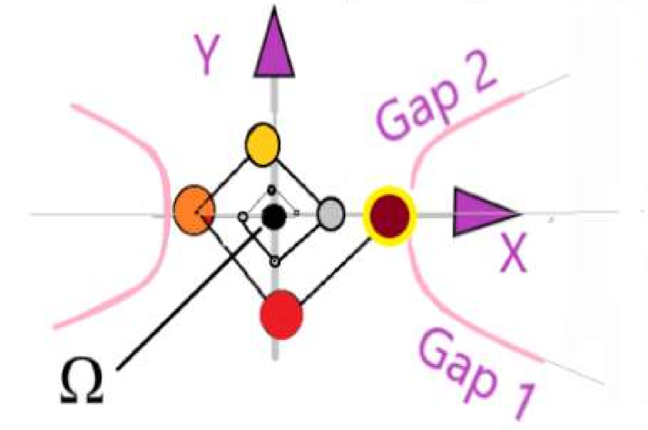
= Electrodes are surface of revolution

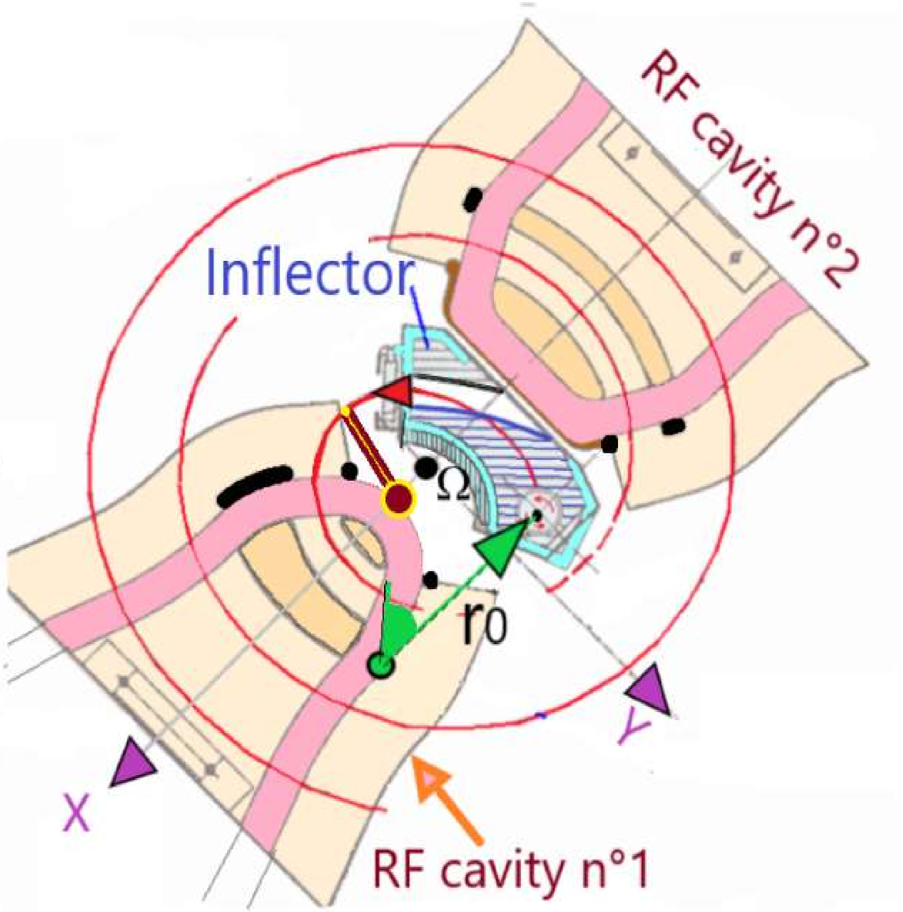
$$V = -K z^2/2 + K r^2/4$$

$$x = \frac{r_0}{2}\{-b\cos(akt) + a\cos(bkt)\},$$
$$y = \frac{r_0}{2}\{-b\sin(akt) + a\sin(bkt)\},$$
$$z = \frac{r_0}{2}\sin(kt),$$



Evolution of trajectory center



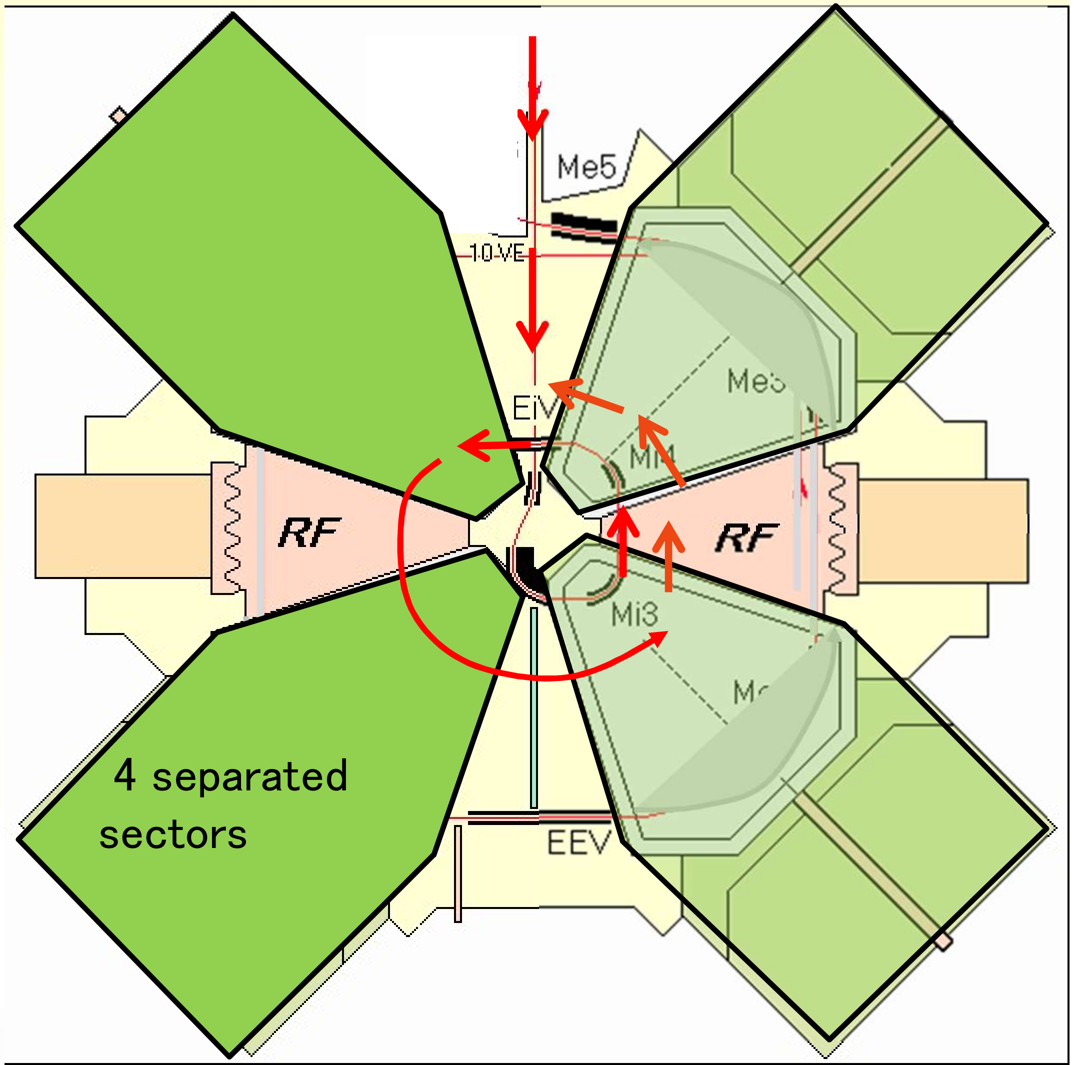


Radial injection in separated sector cyclotron

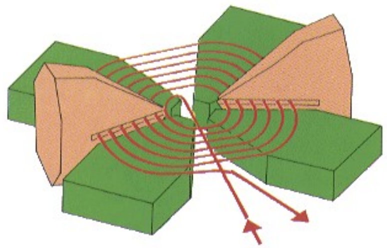
- More room to insert bending elements.

Injected beam is pre-accelerated (high Bρ)

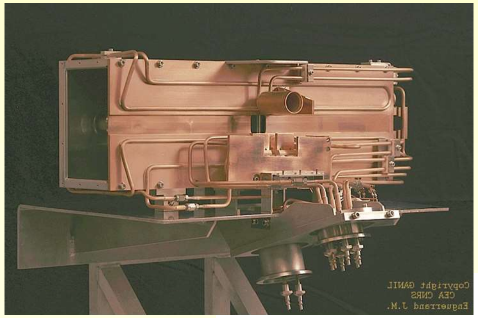
Beam injected between sector magnets

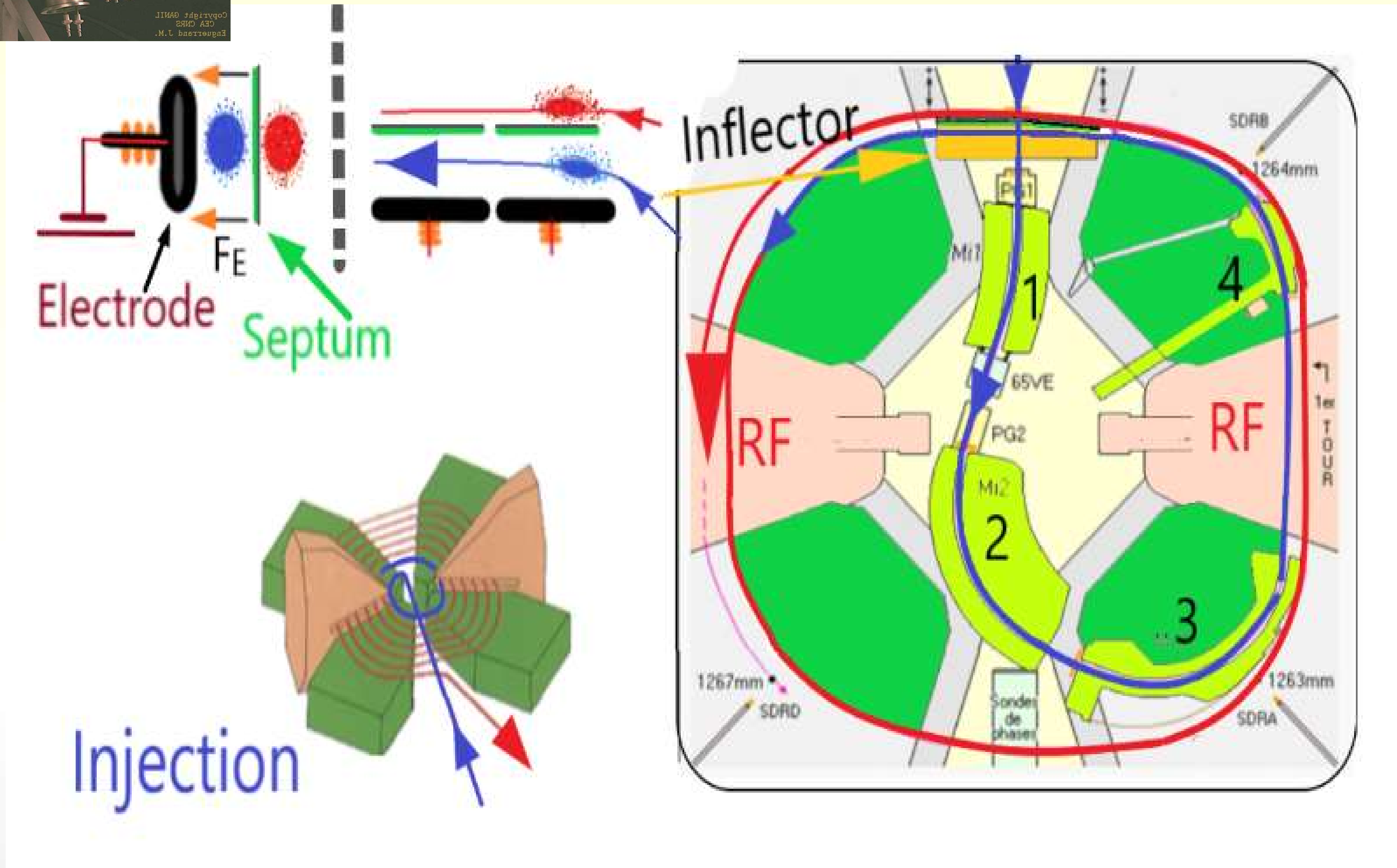


- The beam coming from the pre-injector enters the SSC horizontally.



Electrostatic Inflector for radial injection





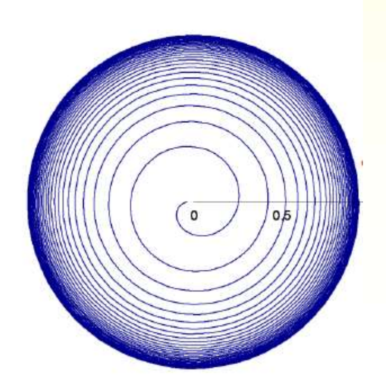
Cyclotron Extraction

Extraction by stripping negative ions

simple and low cost

Extraction using the radial separation between turn n°N & n°N+1

turn separation $\delta \mathbf{r}$ > bunch size Δr



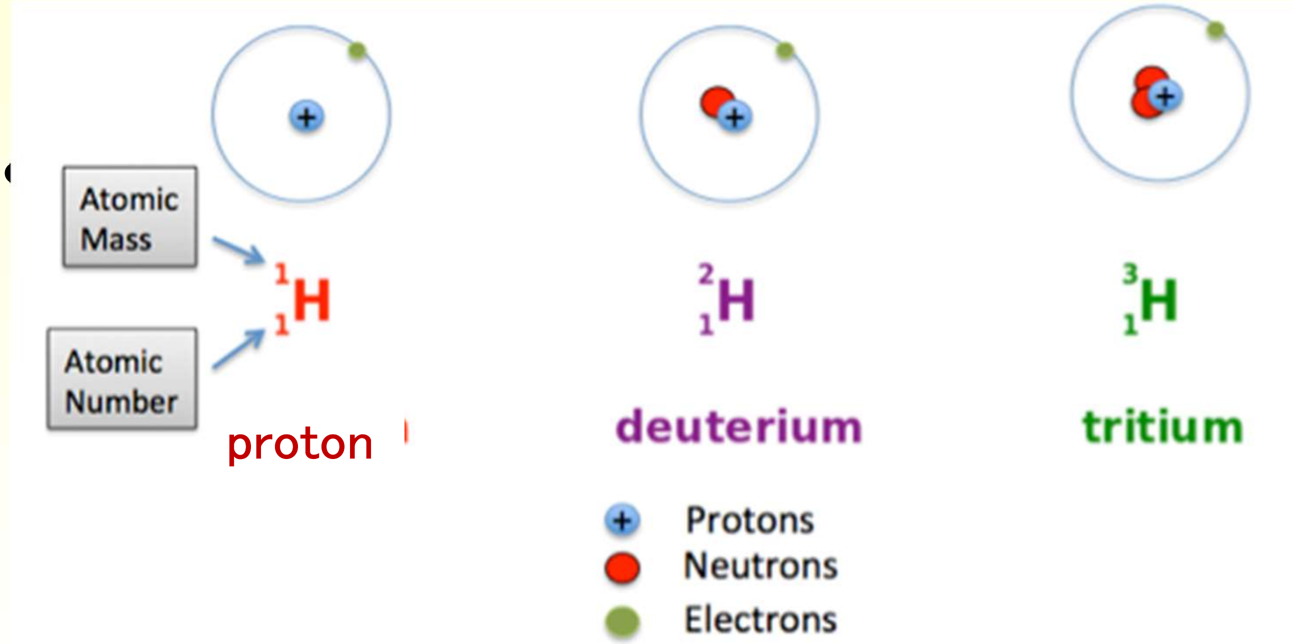
method 2.a : natural acceleration

method 2.b : precession

method 2.c : resonance

Negative Hydrogene isotopes for proton & deuteron beam

very convenient for stripping extraction



Hydrogene isotopes

PIG sources or multiscup sources for negative ions of H,D

H− (proton+ 2 e−)

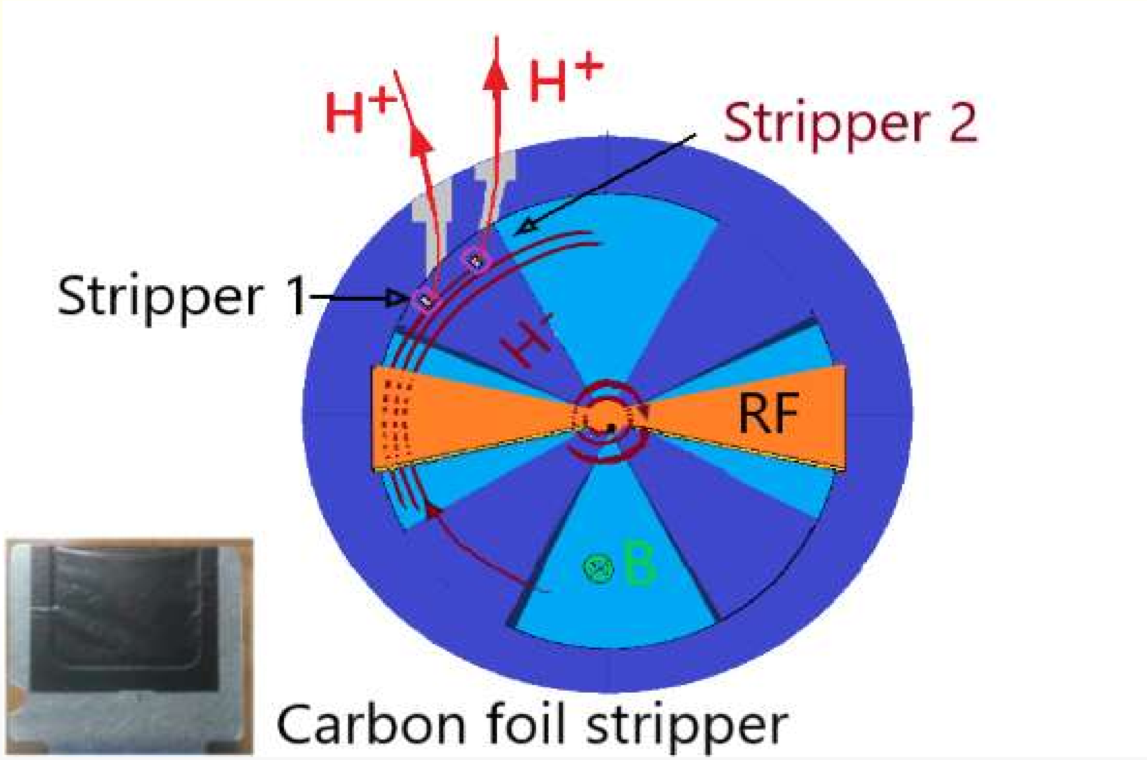
D− (deuteron+2 e−) D= good for several nuclear reactions (radio isotopes production)

Extraction by stripping negativ ions with a very thin carbon foil (stripper)

The magnetic force is inverted when H^- loses its 2 electrons

$F_r \sim - v.B_z \Rightarrow + v.B_z$

$Q = -1 \Rightarrow Q = +1$

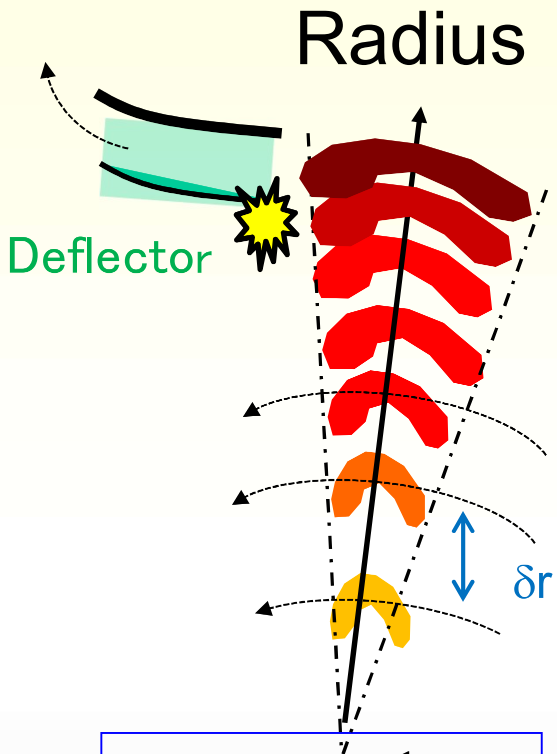


Beam extraction at the cyclotron exit

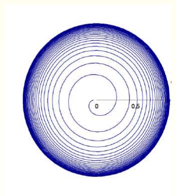
- if $\Delta r < \delta r$ (bunch size < turn separation)

Extraction with an electrostatic deflector

100% efficiency : singlr turn extrction



if $\Delta r > \delta r$ (bunch size> turn separation)



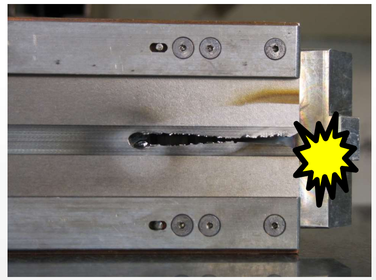
last turns are not separated

Beam losses in the extraction channel

Multi Turn extraction

Deflector sparking or damaged

$$\delta r \propto \frac{1}{Radius}$$



Extraction : 3 mechanisms possible

Goal : High extraction efficiency with well separated orbits

δr = Acceleration + Precession + increase oscillation by a field bump (resonance extraction)

a. Extraction by acceleration

b. Precession extraction : radial oscillations help to separate orbits

c. Resonant extraction : increase the precession by a field bump

If turn separation not enough then magnetic perturbations are used. Particles are forced to oscillate around their equilibrium orbit with a magnetic bump

a. Extraction by acceleration

Method 2.a : Extraction by acceleration in exit fringe field n<0)

Maximise The radial δR separation between 2 consecutive turns

- δE_{turn} : Energy gain per turn as high as possible (V_{RF})
- Accelerate the beam to fringing field (Bz decrease,n>0)

Demonstration : We have $B\rho = \langle B \rangle . \langle R \rangle$ and $B_z \sim R^{-n}$

$$\frac{\delta\langle R\rangle}{\langle R\rangle} = \delta \ln(R) = \delta \ln\left(\frac{B\rho}{\langle B\rangle}\right) = \frac{\delta B\rho}{B\rho} - \frac{\delta\langle B\rangle}{\langle B\rangle} = \frac{\delta p}{p} + n\frac{\delta\langle R\rangle}{\langle R\rangle}$$

So rearranging δR on two side : $\frac{\delta\langle R\rangle}{\langle R\rangle} = \frac{\delta p}{p} . \frac{1}{1-n} = \frac{1}{2} . \frac{\delta E_{turn}}{E_N} . \frac{1}{1-n}$

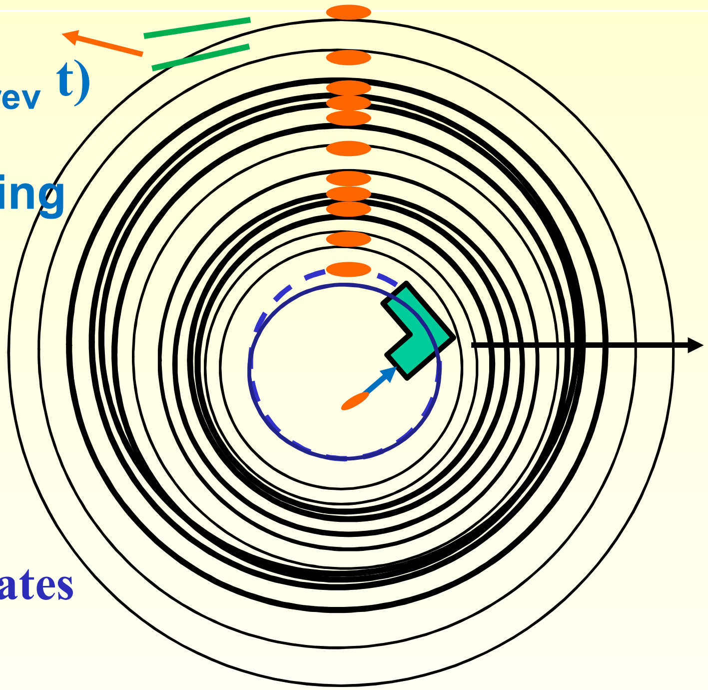
b. Extraction with precession

$r(t) = R_0(t) + x_0 \cos(Q_r \, \omega_{rev} \, t)$

X0 given by injection tuning

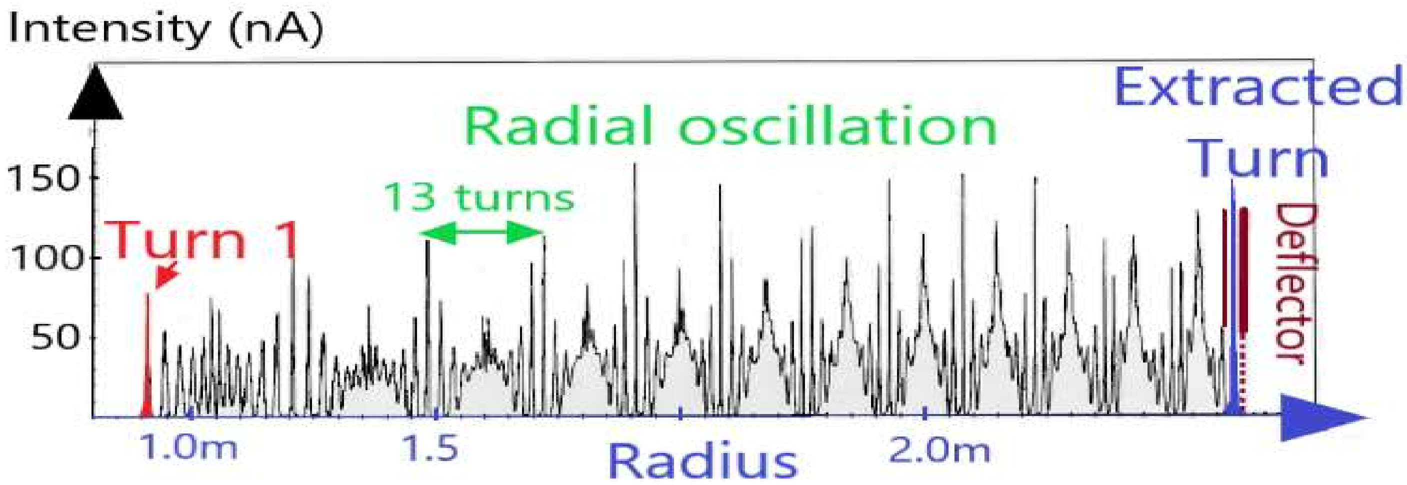
X0 =0 No precession
X0 ≠ 0 precession

Radial Distance between bunches oscillates



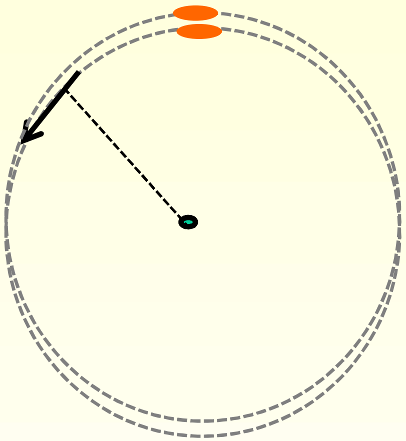
At certain radii bunches are close

At certain radii Bunches are well separated
- Good for extraction

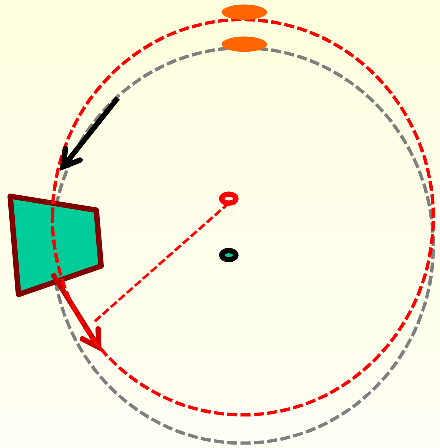


c. Resonant extraction with a magnetic bump

Step 1 : circular motion
+ small oscillations

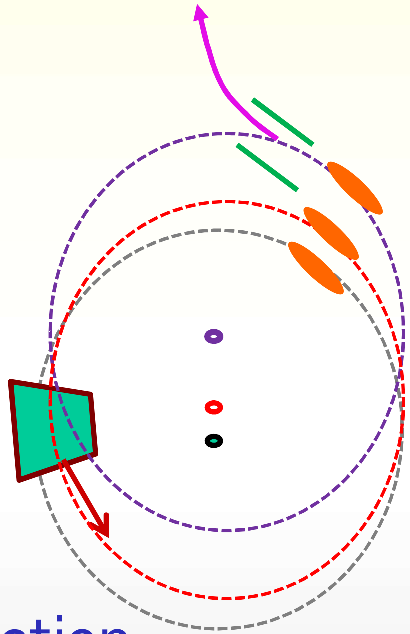


Step 2 : A dipolar magnetic bump
shift of the orbit center
you get larger deviation



$Q_r \sim 1$: 1oscillation per turn

Step 3 : Several turns
produce
Large amplitude oscillation

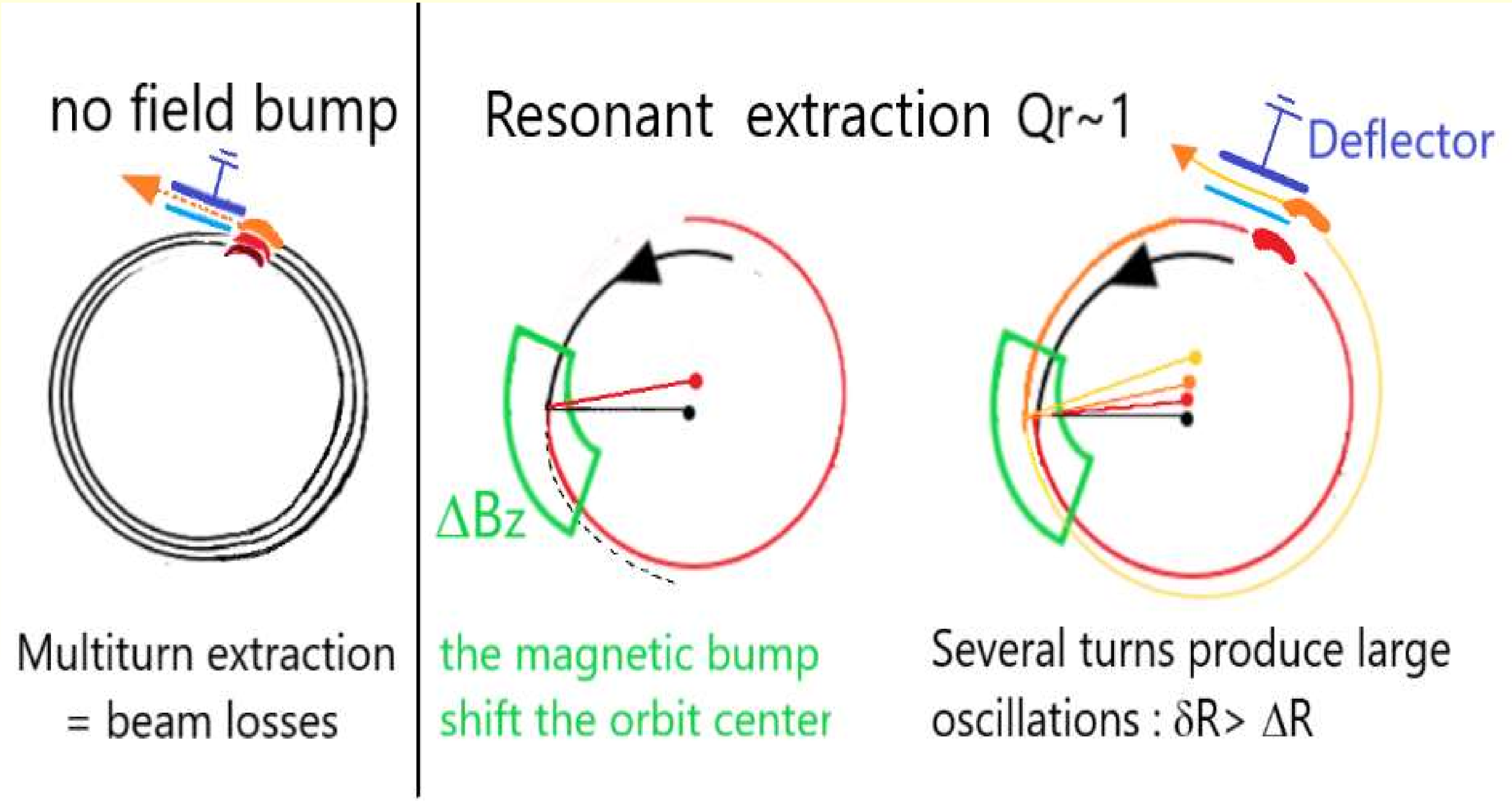


Larger &Larger &Larger

$$\delta r \approx \left[\frac{1}{2} \frac{\delta E_{RF}}{E} \right] + \Delta x_0 \sin(\nu_r . \omega t)$$

Large δR=easy extraction

C. Resonant extraction with a magnetic bump



c. Resonant extraction shown by equations

Radial Equation without magnetic Perturbation

$$\frac{d^2x}{dt^2} + \omega^2 . Qr^2\, x \; = 0$$

$$x(t) = x_0 \cos(Q_r\, \omega_r\, t)$$

Radial Equation with magnetic Perturbation = cos(P ω t) *

$$\left[\frac{d^2x}{d\theta^2} + \omega^2\, Q_r{}^2\, x\right] = A\cos(P\theta)$$

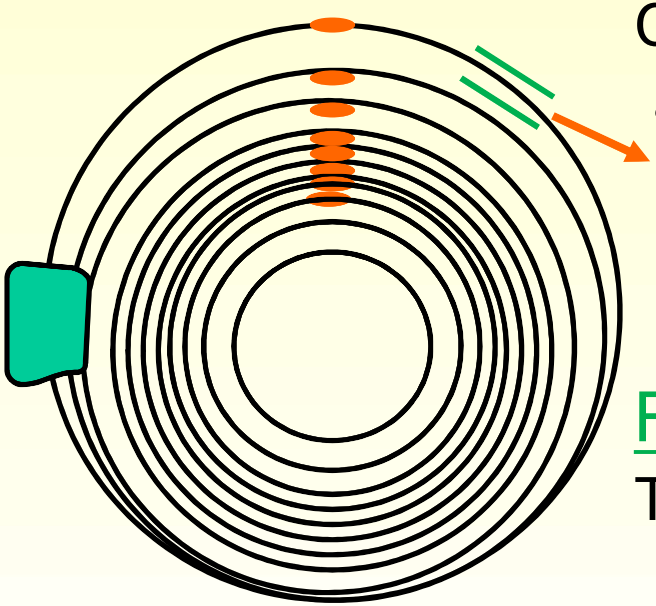
Driven oscillator excited at the « frequency » P

if the excitation is at the resonance frequency P = Qr
you get Large amplitude oscillations δr (easy extraction)

One field Bump correspond to harmonic P=1

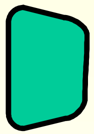
coil for fieldbump

C. Extraction with resonance excitation



Close to extraction radius,
a field bump Increase the bunch separation δr

<u>Field bump</u>



The excitation must correspond to
the natural radial oscillation Q_r

Very small excitation is sufficient
if resonance …..

if $\mathbf{Q_r}$ ~ 1 One field bump
if $\mathbf{Q_r}$ ~ 2 two field bump