Cyclotrons

Chapter 4 : Cyclotron Design

- Cyclotron summary
- How to design a cyclotron ?
- Simulation code
- Simulation algorithm
- Simulation
  (closed orbit, matched beam, backward tracking to injection
  forward tracking up to extraction

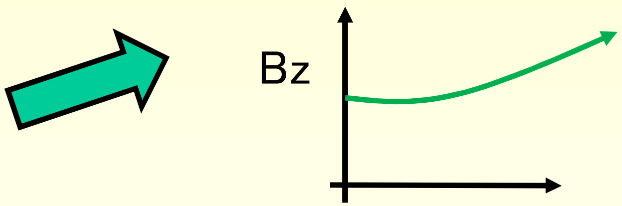
Design Strategy for K=10 MeV cyclo (Most diffused ion cyclotron)

Design Strategy for a research facility (E/A vs Intensity)

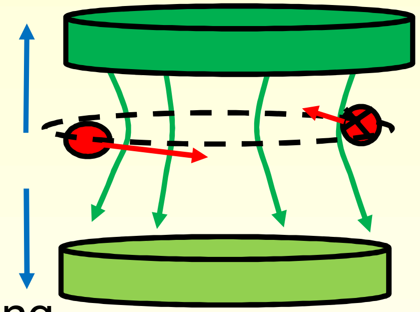
Cyclotron Summary $\omega_{rev} = 2\pi\, F_{rev} = \frac{qB}{m\,\gamma}$

Longitudinal dynamics: particles synchronous with RF

Isochronous cyclotron = constant revolution frequency



Bz=Bz(R) Radius

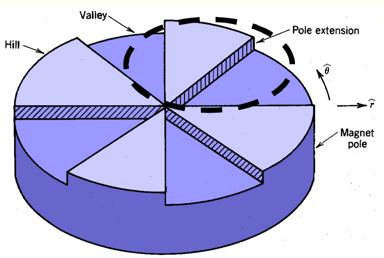


radial component

Br vertical defocusing

Transverse dynamics vertical defocusing forces have to be compensated

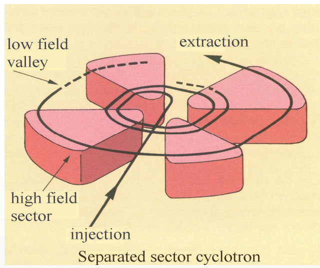
Azimutal(θ) Field modulation = vertical focusing Bz(R,θ) => Bθ



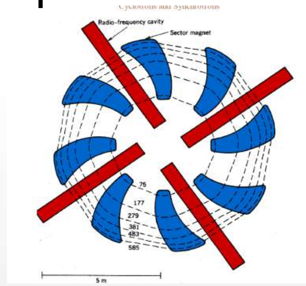
Straight sectors

Spiraled sectors

Separeted sectors



Separated and Spiraled sectors



Cyclotron Summary : with formulas

Isochronous cyclotron = constant revolution frequency

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m} = const$$

field index n <0

$$B_z = B_0 \, (R/R_0)^{-n}$$

$$n(R) = 1-\gamma^2$$

$$< R >= \frac{B\rho}{< B_z >} = \frac{\gamma\, m\mathrm{v}}{q < B_z >}$$

$$E/A = Kb\,.\,(Q/A)^2$$

Vertical stability in isochronous cyclotron $B_z = F(R,\theta)$

requires Azimuthal Field Modulation (N sectors)

$$\frac{d^2z}{dt^2} = -\omega^2 . Q_z^2 \,. z \qquad Q_z^2 =< \left(n + \frac{v_r}{\omega B_0} . \frac{dB_z}{Rd\theta}\right) > 0$$

$z(t) \sim z_{0.} \exp(-i\, Q_z\, \omega\, t)$: vertical tune Q_z; real for stability

$$Q_z^2 =< n > + \frac{N^2}{N^2-1} Fl.(1 + 2\tan^2(\xi)\,) + \ldots$$

Tutorial : Isochronous fied B(R) and field index n(R)

Compute the field index n(R) in cyclotron as a function of γ factor

$$n = -\frac{R}{B_{0z}}\frac{\partial B_z}{\partial R} \quad \Longleftrightarrow \quad \frac{dB}{B} = -n\frac{dR}{R}$$

$$B\rho = <B>.<R> = \frac{p}{q}$$

$$\frac{dp}{p} = \frac{dB}{B} + \frac{dR}{R} = (1-n)\frac{dR}{R}$$

Longitudinal dynamics lecture

$$\frac{dp}{p} = \gamma^2\frac{d\beta}{\beta} = \gamma^2\frac{d(\omega_{rev}.R)}{\omega_{rev}.R} = \gamma^2\frac{dR}{R}$$

$$1 - n = \gamma^2$$

$$n(R) = (1-\gamma^2)$$

« At high energy » isochronism requires n << 0

The Azimutal modulations are not sufficient.
It is a (Focusing) limit for high energy isochronous cyclotron

How to design a cyclotron : first parameters

- **1) Define the basic parameters of the cyclotron (B,R,F) :**

  Particle choice - ion : A/Q Energy => $B\rho$ γ

  - final Intensity ? I_f

  - Desired Accelerator transmission= I_f / I_0 (Duty cycle)

2) Magnet design

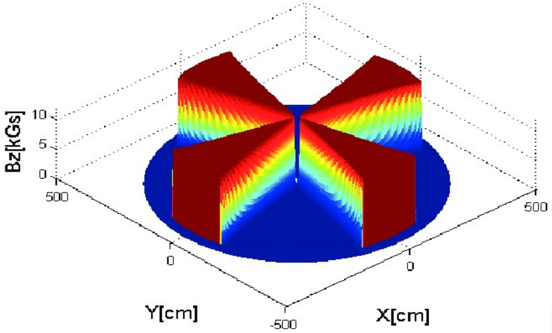
- magnet technology = <B>
- magnet size ; Rextraction= $B\rho$ /<B>
- magnet field index n= f(Energy) = 1- γ^2

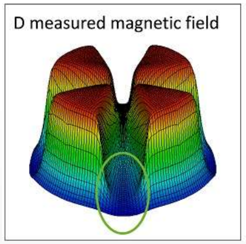
Vertical stability in theory :

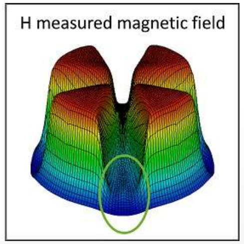
What Flutter *Fl* is required , spiral angle (ξ) for $Qz^2 > 0$

$$Q_z^2 = <n> + \frac{N^2}{N^2-1} Fl.(1 + 2\tan^2(\xi)) + \ldots$$

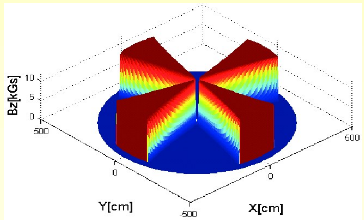
Compute a magnetic model Bz, Br, $B\theta$ = F(R, z,θ)







How to design a cyclotron : Beam dynamics



0) Get a magnetic map model B(r,θ,z)

1) Simulation Code (MadX ?, ….) : integration in θ

2) **Find the closed orbit** (1 particle) in the middle of the cyclotron

3) **Find a matched beam** in the cyclotron (multiparticles) **toward injection (backward tracking)**

4) **Forward tracking (multiparticles) toward extraction**
Extraction (multiparticles) : (deflector, precession, resonance)

Iterative process
Problems ? : restart with a new magnet model : iterative process

Simulation code:
Particle Tracking with a computer code

Simulation : tracking ions (M,Q,v_0)

Transport Matrix not precise enough

Multi-particle code in « realistic » magnetic field
In cylindrical coodinates

~~TRANSPORT (1rst order)
MAD (3rd order)
CPO~~

~~Cyclotron Magnets are too complex~~

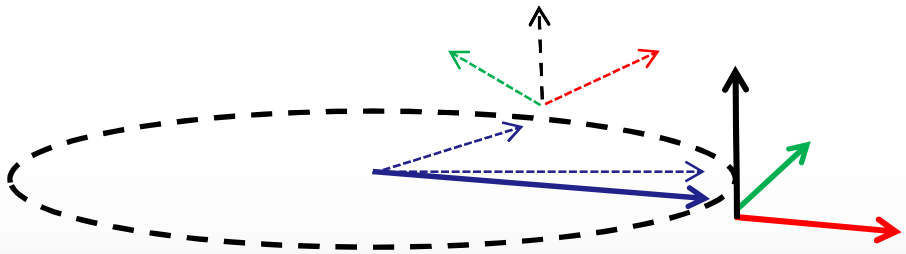
$$\mathbf{r} = r \cdot \mathbf{e}_r + z \cdot \mathbf{e}_z$$

Comoving Frame : $e_r = f(t)$

$$d\mathbf{e}_r = \mathbf{e}_\theta \cdot d\theta \qquad d\mathbf{e}_z = 0 \qquad d\mathbf{e}_\theta = -\mathbf{e}_r \cdot d\theta$$

$$\dot{\mathbf{r}} = \dot{r} \cdot \mathbf{e}_r + \dot{z} \cdot \mathbf{e}_z + r \cdot \dot{\mathbf{e}}_r + z \cdot \dot{\mathbf{e}}_z$$

$$\frac{d}{dt}\left[m\gamma \, \dot{\mathbf{r}} \right] = q \cdot (\mathbf{E} + \dot{\mathbf{r}} \times \mathbf{B})$$



Unit vectors are evolving in time !!!

Simulation code:
Particle tracking with a computer code

Tracking ions (M,Q,v0) In cylindrical coodinates

Let's track one particle   Start $\theta=\theta_0$ (At t=0)

$r = r0$ $pr = pr\_0$

$z = z0$ $pz = pz\_0$

$p\theta = p\theta\_0$

What is the particle position at $\theta=\theta_0+\Delta\theta$ (At $t=0+\Delta\theta$ [dt/dθ])

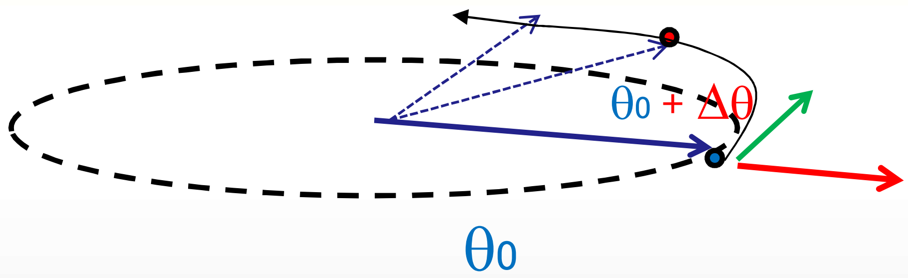
$r(\theta_0+\Delta\theta) = r0 + \Delta\theta$ [dr/dθ]   first order extrapolation= euler algorithm)

$z = z0 + \Delta\theta$ [dz/dθ]

$pr = pr\_0 + \Delta\theta$. [d *pr* /dθ]

$pz = pz\_0 + \Delta\theta$.[d *pz* /dθ]

$p\theta = p\theta\_0 + \Delta\theta$ [d pθ /dθ]



[d r /dθ] =

[d pr /dθ] = cylindrical equation of motion = f [B(r, θ z)]

Cyclotrons simulation: cylindrical equation

$$\frac{d\mathbf{p}}{dt} = q\cdot(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\dot{\theta} \\ B_r & B_z & B_\theta \end{vmatrix} =$$

$$= (\dot{z}\cdot B_\theta - r\dot{\theta}\cdot B_z)\cdot\mathbf{e}_r + (r\dot{\theta}\cdot B_x - \dot{r}\cdot B_\theta)\cdot\mathbf{e}_z + (\dot{r}\cdot B_z - \dot{z}\, B_r)\cdot\mathbf{e}_\theta$$

Evolution in time t is not convenient, evolution in θ is better !!!

$$\frac{d}{dt} = \frac{d\theta}{dt}\frac{d}{d\theta} = \dot{\theta}\frac{d}{d\theta}$$

$$\frac{d\mathbf{p}}{dt} = \dot{\theta}\frac{d\mathbf{p}}{d\theta} = q\cdot(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

$$\frac{d}{d\theta}\left[m\gamma\,\dot{r}\right] = \frac{d}{d\theta}[p_r] = m\gamma\, r\dot{\theta} + \frac{q}{\dot{\theta}}(\dot{z}\cdot B_\theta - r\dot{\theta}\cdot B_z)$$

$$\frac{d}{d\theta}\left[m\gamma\,\dot{z}\right] = \frac{d}{d\theta}[pz] = \frac{q}{\dot{\theta}}(r\dot{\theta}\cdot B_x - \dot{r}\cdot B_\theta)$$

$$\frac{d}{d\theta}\left[m\gamma\, r\dot{\theta}\right] = \frac{d}{d\theta}[p_\theta] = \frac{q}{\dot{\theta}}\ldots.$$

$$\frac{dr}{rd\theta} = \frac{\dot{r}}{r\dot{\theta}} = \frac{p_r}{p_\theta} \qquad \frac{dz}{rd\theta} = \frac{\dot{z}}{r\dot{\theta}} = \frac{p_z}{p_\theta}$$

Cyclotrons simulation : the simplest algorithm
Euler algorithm

Loop j=1, Nparticles

Initial position and momentum : $\theta = 0$ r, z pr, pz, pθ

Loop i=1,Nstep **// step in $\Delta\theta$**

FIELD MAP

$$B_r = BR(r,z,\theta) \qquad B_z = BZ(r,z,\theta) \qquad B_\theta = B\theta(r,z,\theta)$$

$$\theta = \theta_0 + \Delta\theta$$

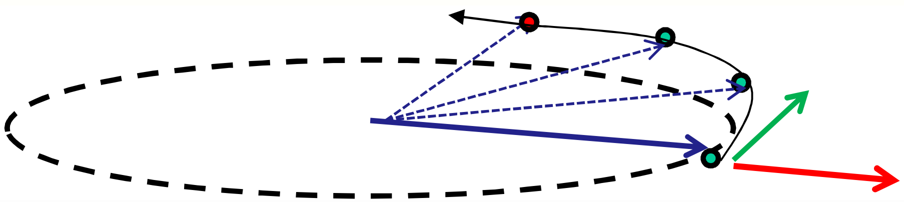
$$r(\theta = \theta + d\theta) = r + \frac{dr}{d\theta}\Delta\theta \qquad p_r(\theta) = pr_0 + \frac{dp_r}{d\theta}\Delta\theta$$

$$\frac{d}{d\theta}[pr] = m\gamma\, r\dot{\theta} + \frac{q}{\dot{\theta}}(\dot{z}\cdot B_\theta - r\dot{\theta}\cdot B_z)$$

$$z(\theta = \theta + d\theta) = z + \frac{dz}{d\theta}\Delta\theta \qquad p_z(\theta) = \ldots\ldots$$

Endloop I // **end $\Delta\theta$ loop**

Endloop J // Nparticle **loop**



Euler algorithm (second order accurate in $d\theta$)

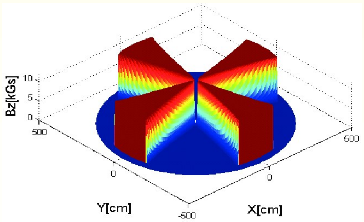
Cyclotrons simulation : the algorithm

- Numerical integration of the equations of motion

What algorithm ?

- Euler algorithm (second order accurate in $\Delta\theta$) is not the best !!
- RK4 (Runge Kutta order 4) is better (4th order accurate in $\Delta\theta$)
- RK5 : slightly better
- …..

See a Numerical analysis Lecture



- **Fied Interpolation of the magnetic map :**

Be very carefull

The field interpolation between the points of the field map B (ri, θi, zi) is the main source of error

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Beam dynamics in cyclotrons

4 Steps :

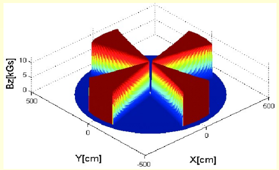
- Find the closed orbit (1 particle) in the middle of the cyclotron

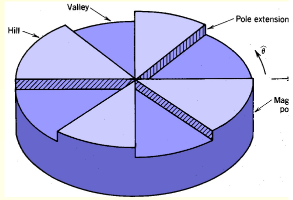
- **Find a matched beam** in the cyclotron (multiparticles)

- **Backward tracking toward injection (**decelerate beam toward injection)

- **Forward tracking toward extraction**
And define Extraction : (deflector, precession, resonance)

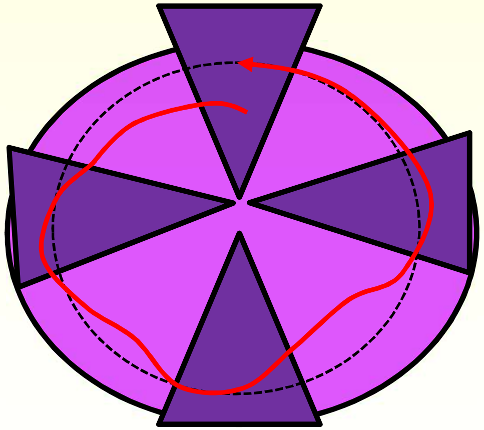
Simulation: Find the closed orbit at Ref. Radius



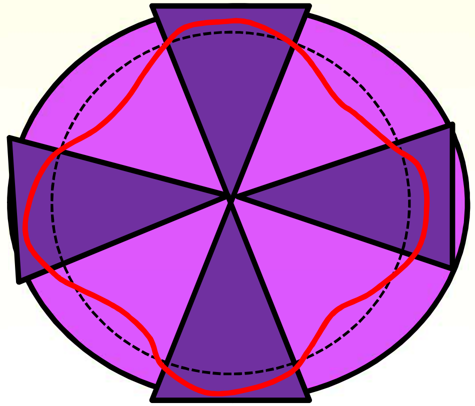


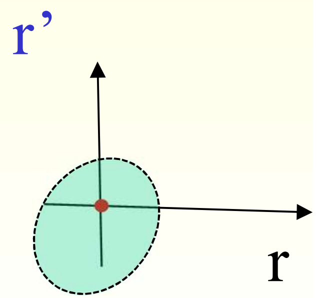
Dynamic of 1 particle in the middle of the cyclotron
Without acceleration in the field map

- Choose reference particle (1 particle) (M,Q, $B\rho_0$)
- Choose a field Level **$B(r,\theta)$ = k. FIELD MAP**



Send a particle in the code
Trajectory is not a closed orbit
Not a good starting point
Change the initial position



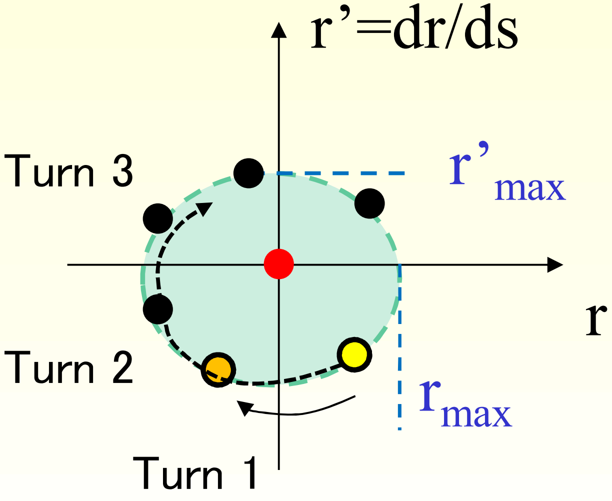


Trajectory is a closed orbit : OK

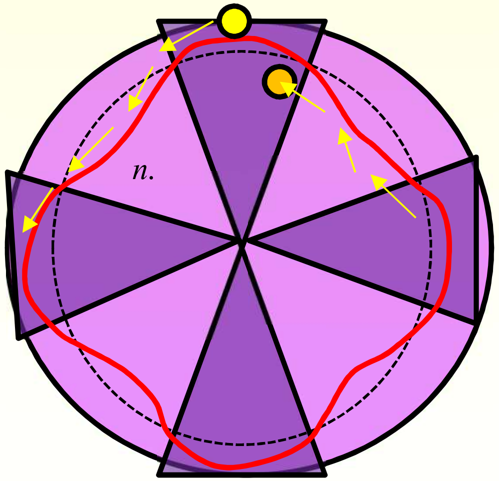
$$\langle R_{ref} \rangle = \langle B(r,\theta) \rangle / B\rho$$

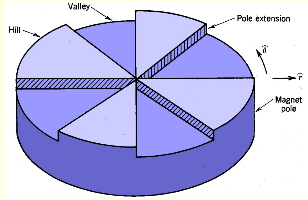
Simulation: Find a matched beam in the cyclotron at Rref

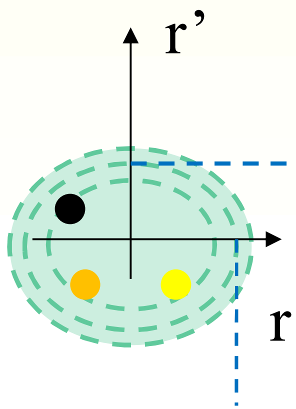
Around the reference trajectory, send a particle for many turns



Hill-Valley is a periodic lattice







A particle trajectory follows an ellipse

Beam matching =

Choose a beam ellipse with

$\Delta r' / \Delta r = r'_{max} / r_{max}$

$r(t) = r0 + r_{max} \cos(Q_r \omega_0 t)$

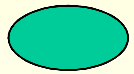
$r'(t) = r'_{max} \sin(Q_r \omega_0 t)$

This ellipse occupy the minimal size in the cyclotron

Mismatched beam recall

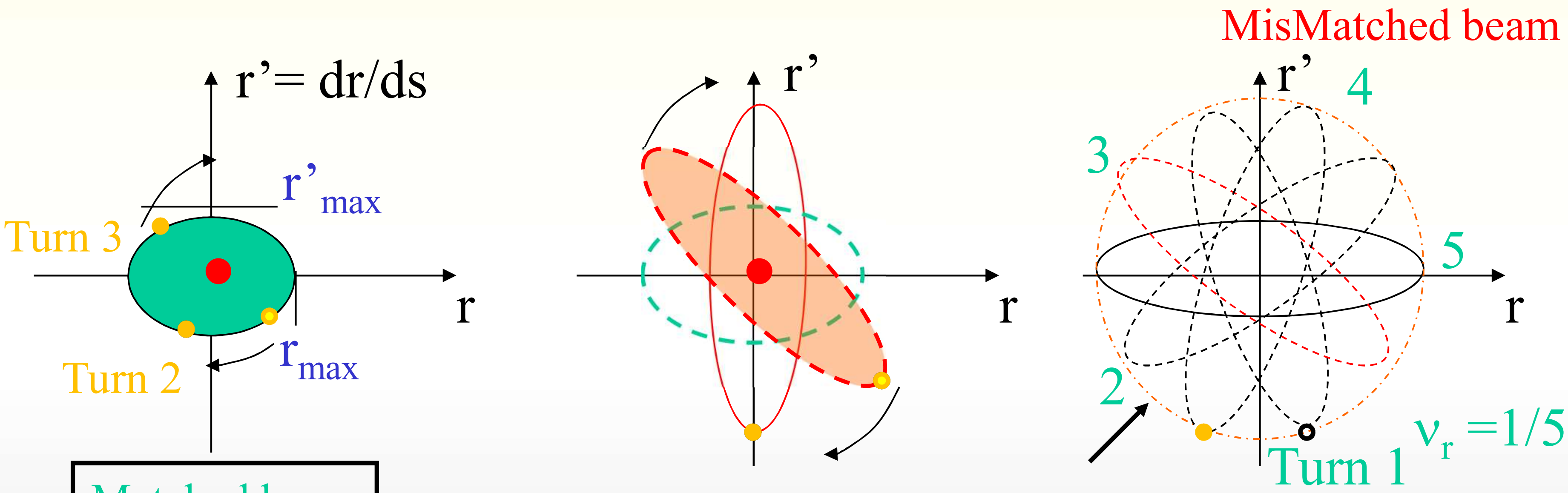
Because of each individual trajectory over N turn

$$\begin{cases} r(t) = r_0 + r_{max}\cos(Q_r\,\omega_0 t) & \text{(without acceleration)} \\ r'(t) = r'_{max}\sin(Q_r\,\omega_0 t) \end{cases}$$

it exist an optimal ellipse 

for a given beam Emittance : $\varepsilon = \pi\,\Delta r_{max}.\,\Delta r'_{max}$

Betatron oscillation with mismatched beam



Matched beam

Larger acceptance required for the cyclo : not good

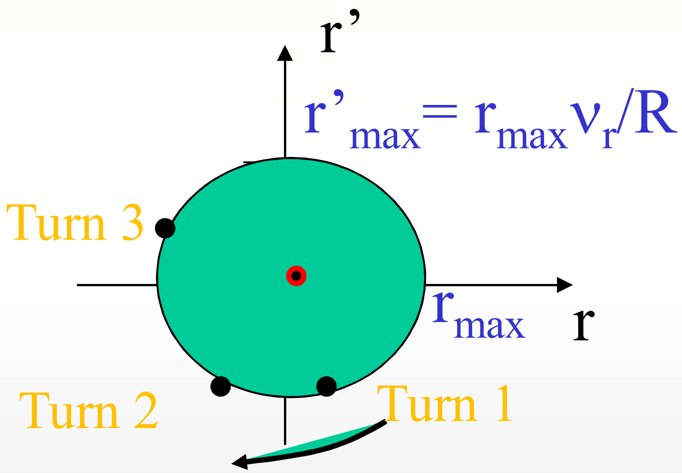
Matched beam recall

$$\begin{cases} r(t) = r_0 + r_{max} \cos(Q_r\, \omega\, t) \\ r'(t) = dr/ds = dr / R\omega dt = -(r_{max} Q_r / R) \sin(Q_r\, \omega\, t) \end{cases}$$

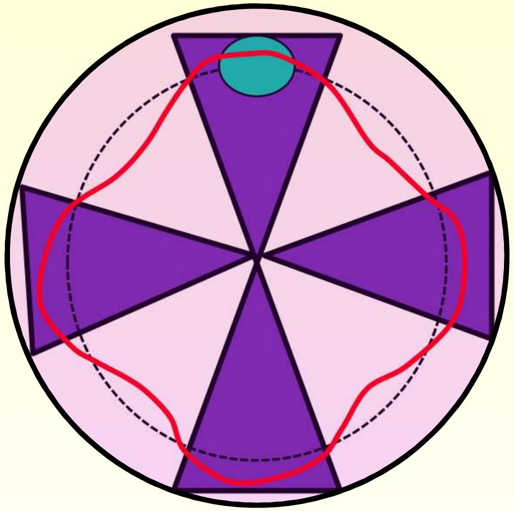
the Matched ellipse $|r'_{max}| = |r_{max} Q_r / R|$

⇨ Initial beam conditions depend of the tune (Q_r) of the cyclotron at the matching point.

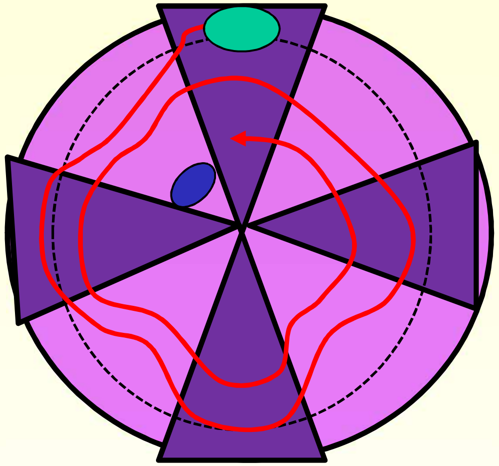
- ⇨ Betatron oscillation disappears
- ⇨ Matched beam
- ⇨ Minimum of acceptance



Simulation: backward tracking toward injection



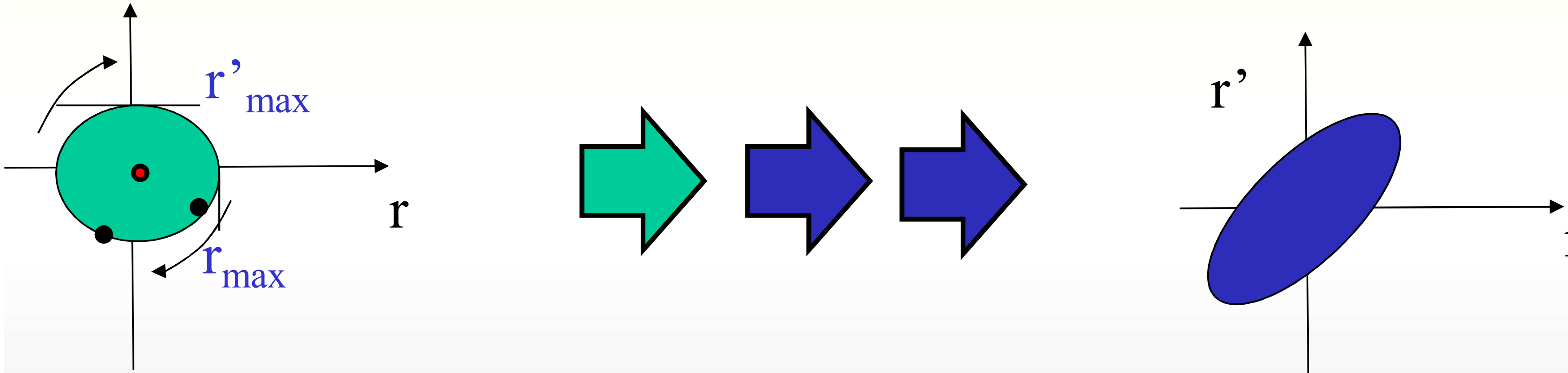
Central Trajectory is a closed orbit :OK



turn on RF : backward toward injection

Adjust Vrf, central field.....

Beam matched : OK

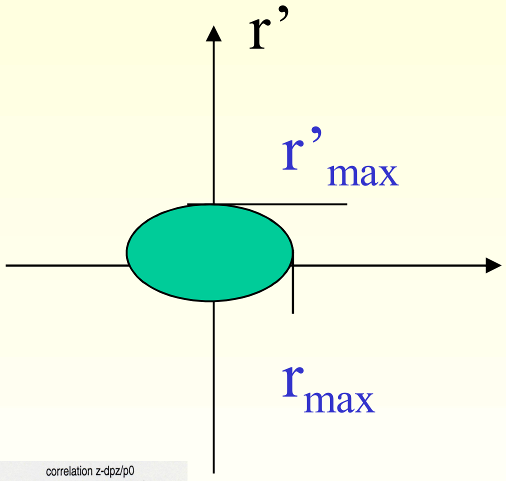


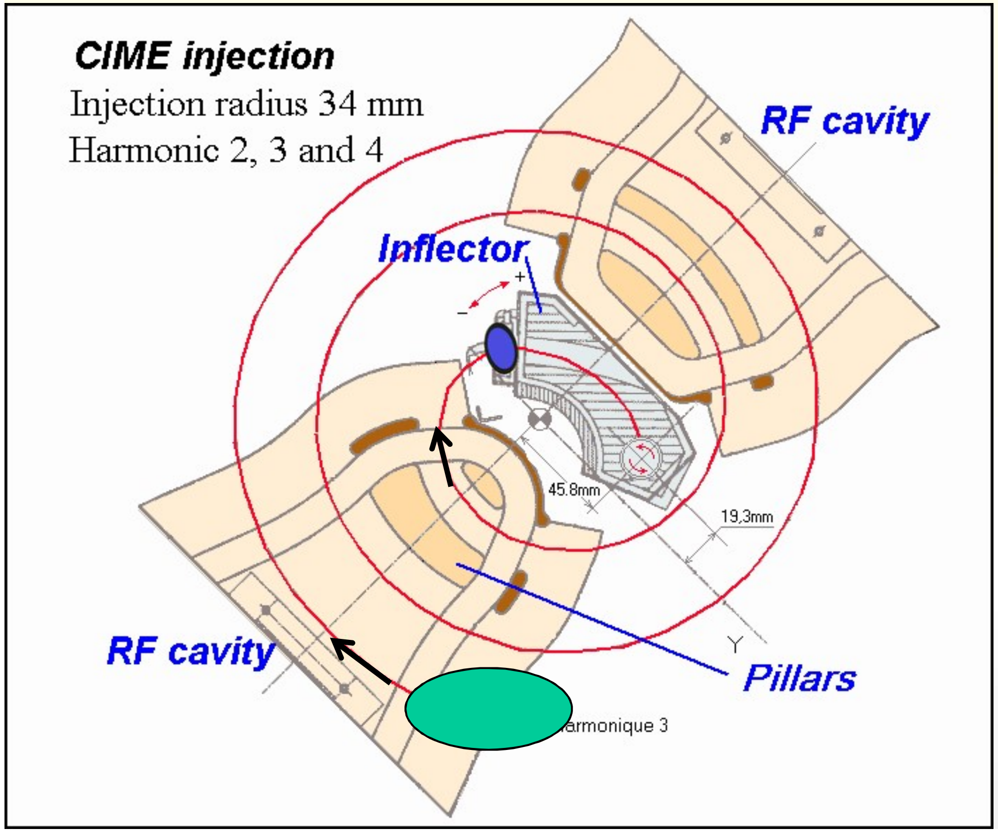
Corresponding optimal beam at injection radius

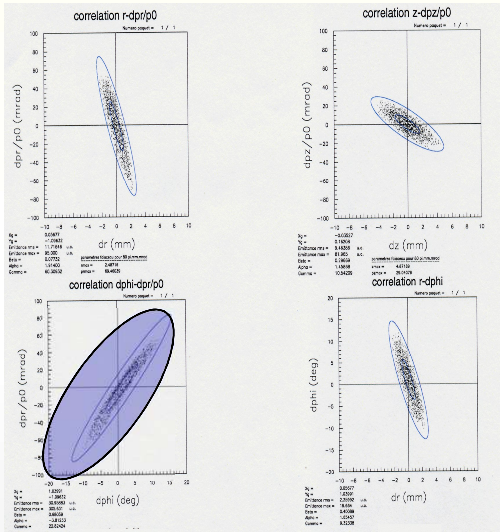
Simulation: backward tracking toward injection

Start with matched beam in the cyclotron (multiparticles) at large radius
Then Adjust Vrf, central field to reach injection Radius

Find the optimal beam at injection radius





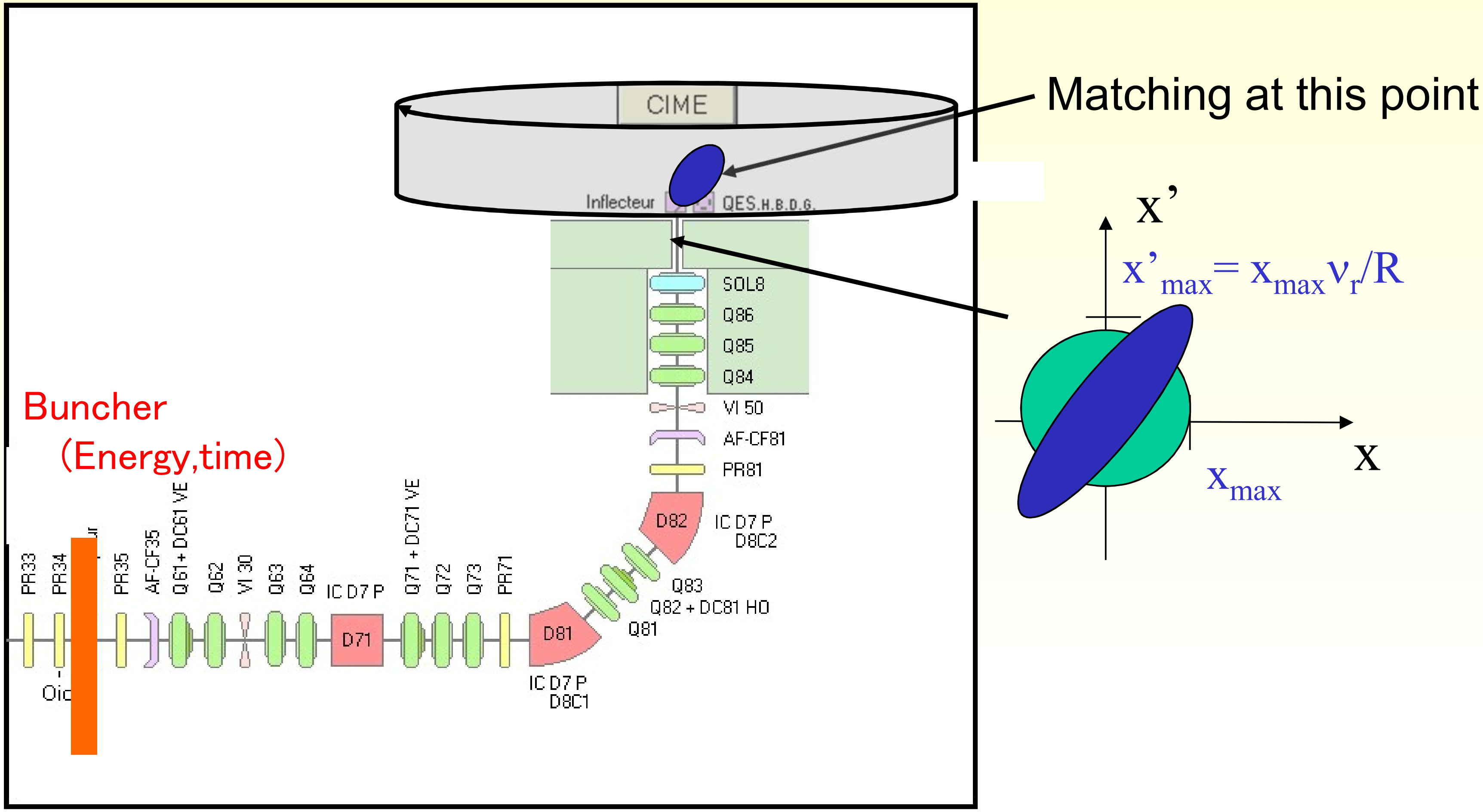


Beam Obtained With backward tracking

(r, r', z, z', E, ϕ)

Simulate the injection beam line

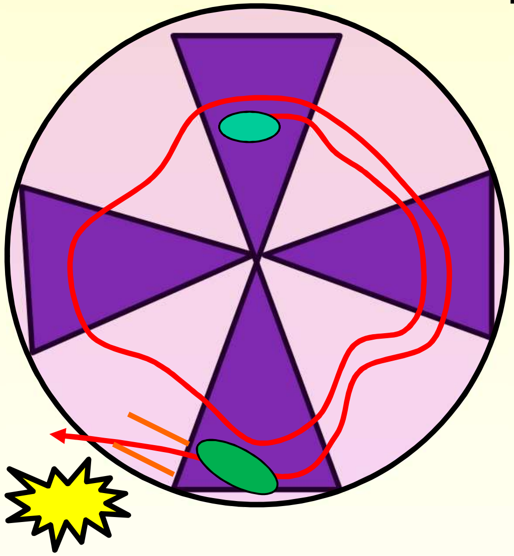
to get the perfect beam at injection



Classical transport line problems :

Adjust quads to get desired beam at injection (r,r') (z,z') (t,E)

Simulation: Forward tracking up to extraction



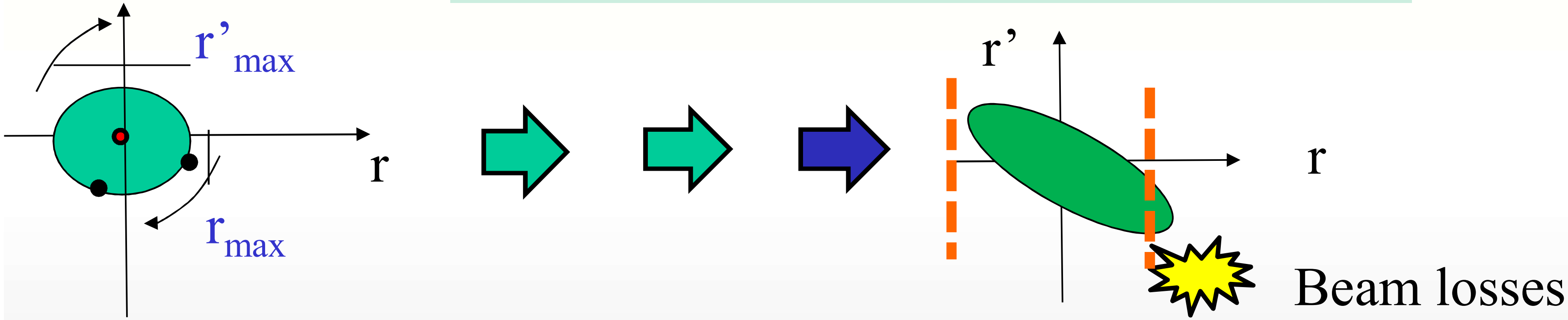
turn on RF : Forward toward extraction

tune the isochronism $\langle B(r)\rangle = \langle B\rangle \gamma(r)$

Extraction
- design the extraction (deflector +..)
- turn separation
  (RF +precession + magnetic bump)
- beam losses ?

Start with a Beam matched

beam at extraction radius : Watch the beam losses in the deflector



The cyclotron is simulated, Let's construct it !

Cyclotron Design strategies

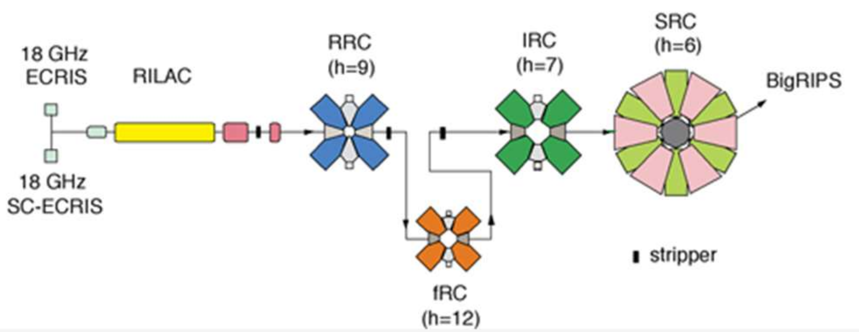
Radio-Isotopes production

cost & reliability

Nuclear physics& Research facility

performance , intensity,..

Mode (1): RILAC + RRC + (stripper2) + fRC + (stripper3) + IRC + SRC



Strategy for Radio-Isotopes production (medical applications)

10 MeV Protons / 5 MeV Deutons: @ low cost

$B\rho max$ =0.458 T.m = ⟨B⟩ Rextraction

Rextract = 0.34 m

⟨B⟩ = 1.35Tesla [hill =1.8 T // valley =0.5 T]

AVF with 4 straight sectors (sufficient z-focusing)

Ibeam ~ 0.1 –0.05 mA

Rf Dees : 2 (so 4 gaps)

2 possibilities for extraction

Extraction By stripping :
external target (18F, radiotracer)

No Extraction :
internal target (in yoke)

A « low energy » industrial cyclotron
Cyclone 10/ 5 : 2 particle kinds : ^{1}H & ^{2}D

Kb=10 MeV

Fixed energy ;

4 straight sectors 50°

fixed **Frf =42Mhz**

<B> =1.35 Tesla

Harmonic H=2(p) ,4 (D)

Internal source

Rextraction=0.33m

Bρmax=0.33x 1.35=0.45 T.m

$$E/A = Kb\ .\ (Q/A)^2$$

Eprotons=10 MeV protons= $^{1}H^{1+}$ A=1 Q=1

(E/A=Kb*1^2 =10MeV/A)

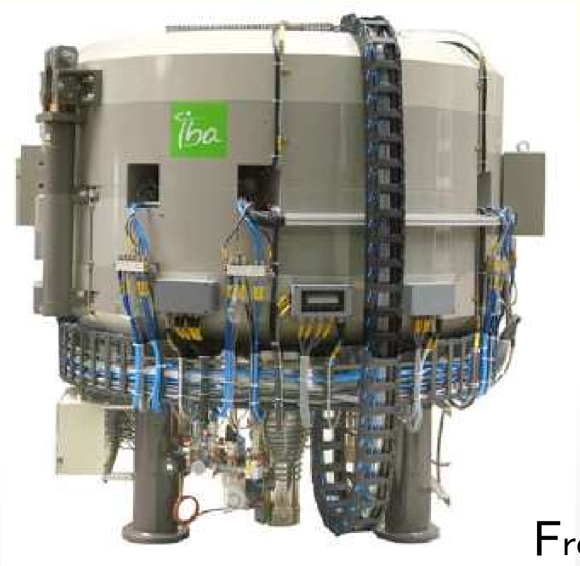
RF Harmonic =2 Frev=42 Mhz/ h= 21 Mhz

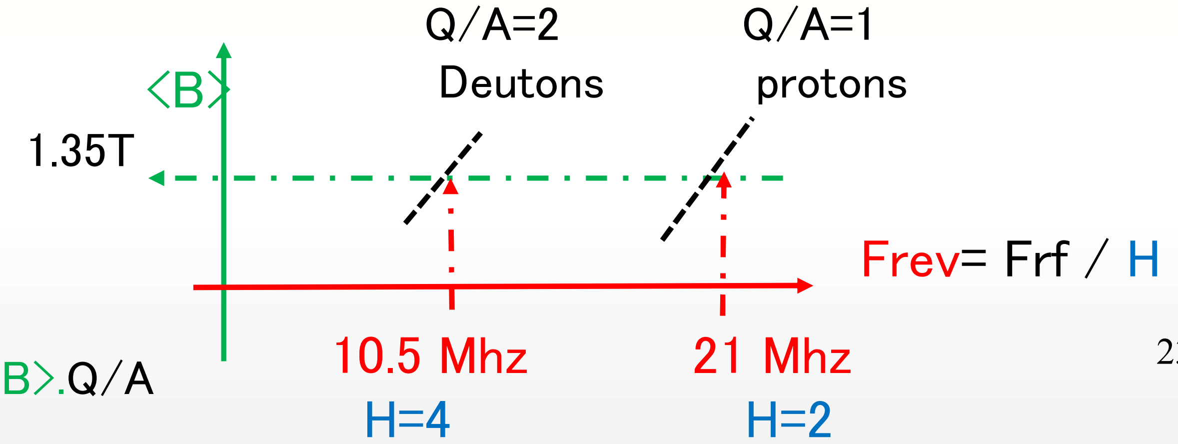
EDeutons=5 MeV Deutons= $^{2}H^{1+}$ A=2 Q=1

(E/A=Kb*0.5^2= 2.5 MeV/A)

RF Harmonic =4

$$F_{rev} \propto \frac{Q.B_{cyclo}}{A}$$





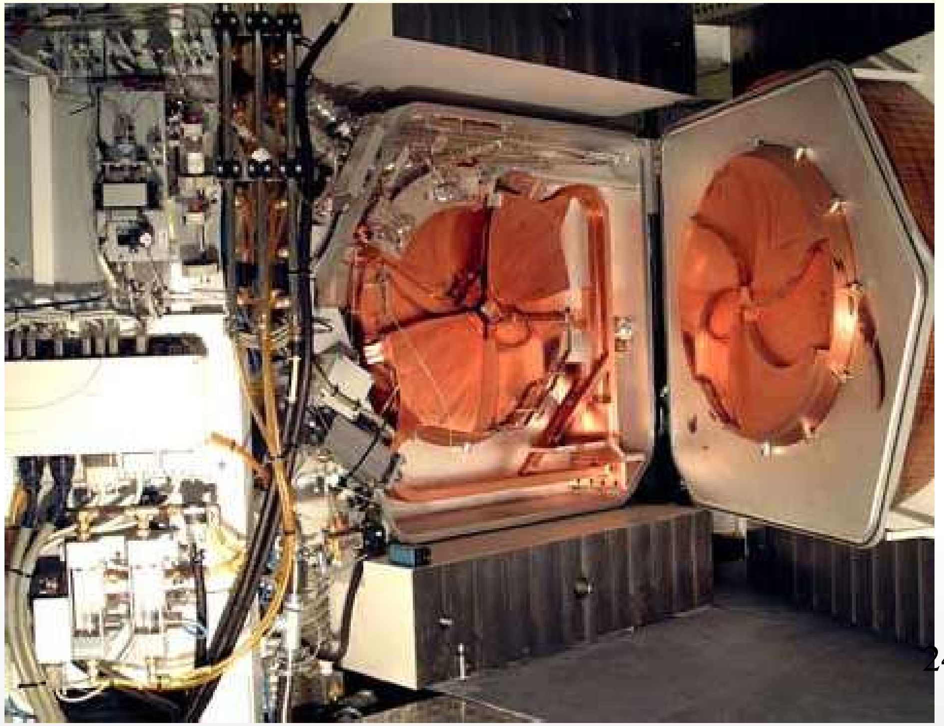
F_{rev}~ <B>.Q/A

Cyclone 10/5 MeV (IBA)

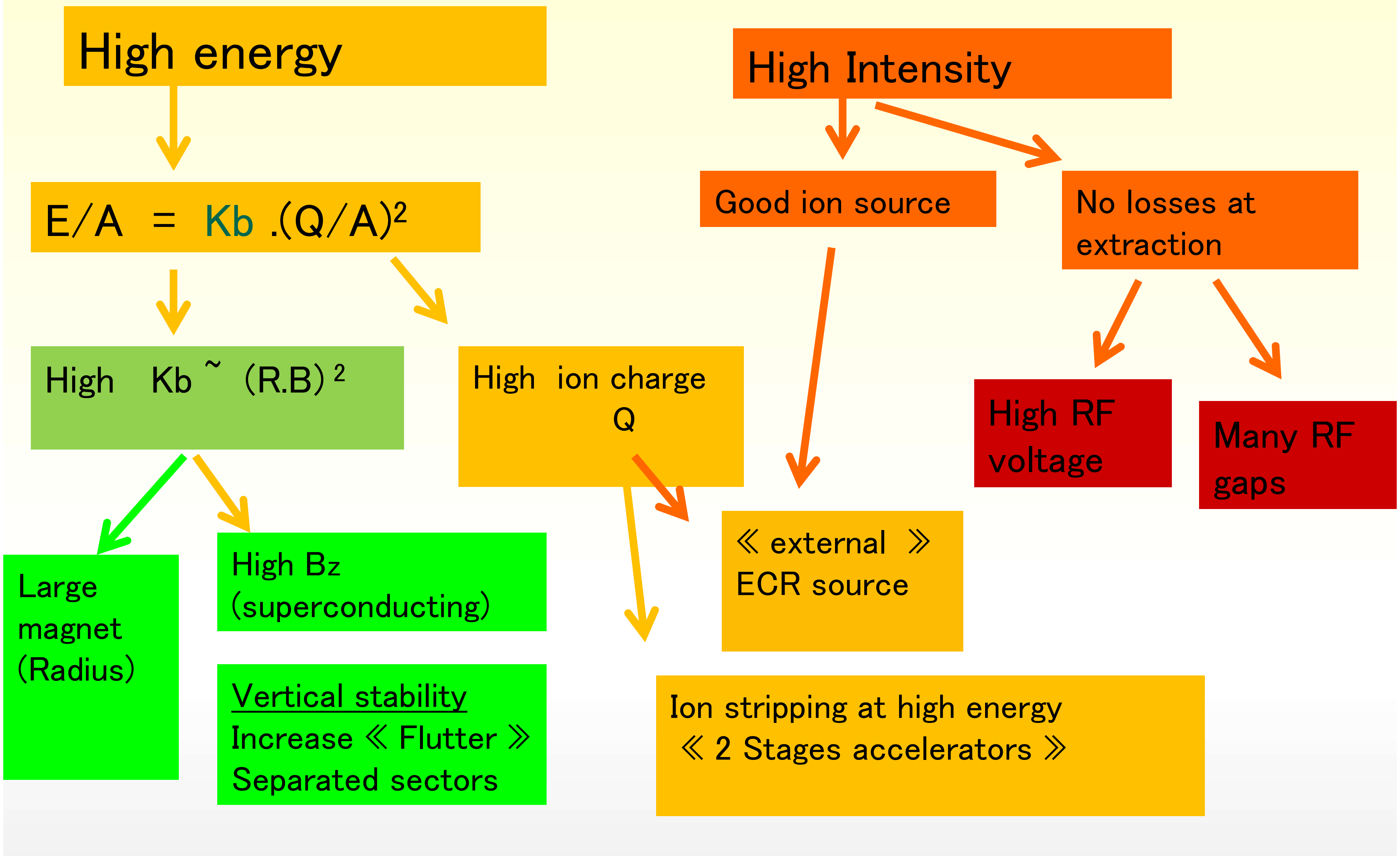
= 10 MeV proton (Kb=10MeV)

= 5 MeV Deuteron

cyclone 3D ("vertical implantation")

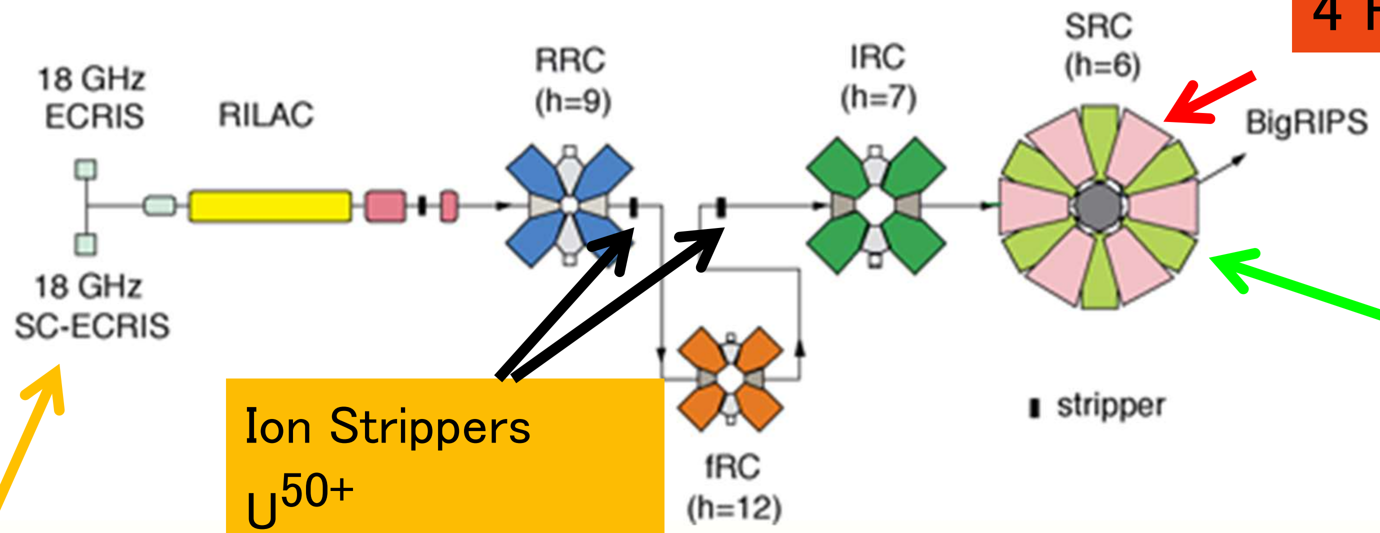


Strategy for a Cyclotron in a research facility



RIBF (Japan) : SRC (K=2600 MeV) –the bigest cyclo
Uranium beam $^{238}U^{88+}$ @345 MeV/A cw

Mode (1): RILAC + RRC + (stripper2) + fRC + (stripper3) + IRC + SRC



4 RF cavities

Radius=6m
<B>=3.8T

Ion Strippers
U^{50+}
U^{88+}

ECR source
U^{30+}

5 Stages accelerators

1 LINAC
+ 4 Cyclotrons

Coupling of 2 Cyclotrons : velocity matching

-Two cyclotrons can be used to reach higher energy:

Harmonic & Radius of the 2 cyclotrons have to be matched

The velocity of extraction Cyclo n°1 = velocity of injection Cyclo n°2

$$v_1 = R^1_{extraction} . \omega = R^1_{extraction} \frac{\omega^1_{rf}}{H_1} \qquad v_2 = R^2_{inject} . \omega = R^2_{inject} \frac{\omega^2_{rf}}{H_2}$$

$$R^1_{extraction} \frac{F^1_{rf}}{H_1} = R^2_{inject} \frac{F^2_{rf}}{H_2}$$

- **Ion stripping can be used, to increase the charge state Q before injection into the second cyclo**

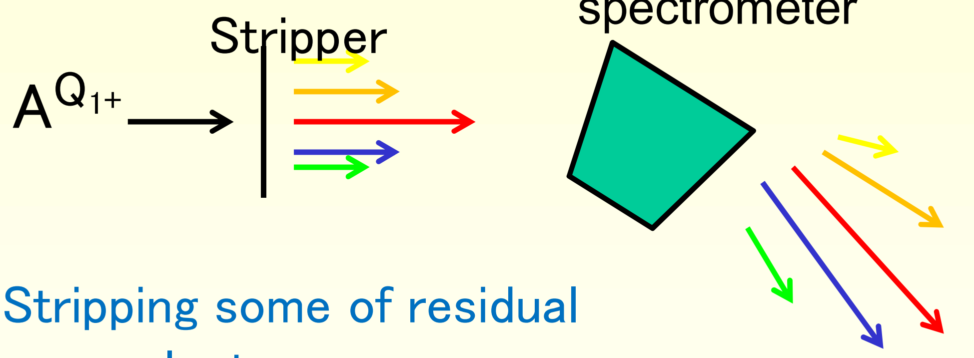
large Q ⇨ large *Emax*

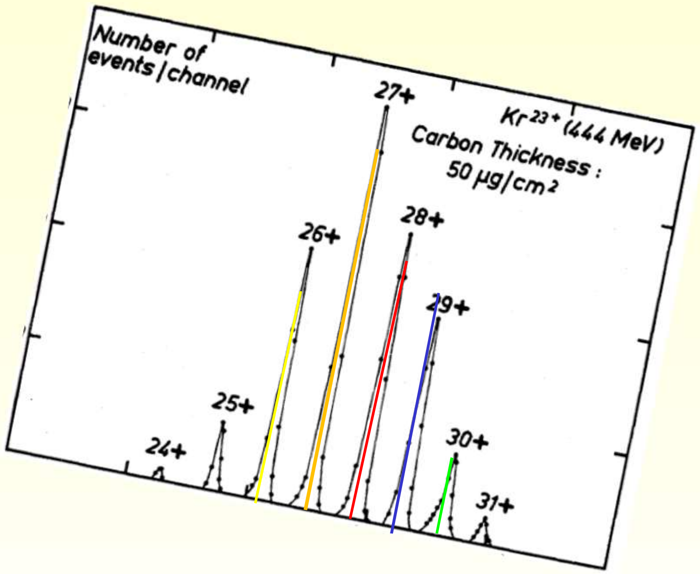
$$\left[\frac{E}{A}\right]_{\max} = K_b \left\{\frac{Q}{A}\right\}^2$$

Ion Stripping at high energy

Heavy ions are not fully stripped by ion sources :

Incoming Ions





Stripping some of residual electrons

final Ions A^{Q2+}

Carbon foil Stripper

$Q_2 > Q_1$

$B\rho_2 < B\rho_1$

$$B\rho = \frac{P}{q} = \frac{\gamma\, m.v}{q}$$

Ion Stripping help to increase the maximal energy of a given cyclotron….

$$[E/A]\max = Kb\left[\frac{Q}{A}\right]^2$$

Important facts for cyclotron:

1) Magnetic structure provide the vertical stability
(field index n compensated by sectors)

2) Simulations are done with realistic magnetic field
(not transport matrices)

3) The Beam matching at injection for better transverse acceptance

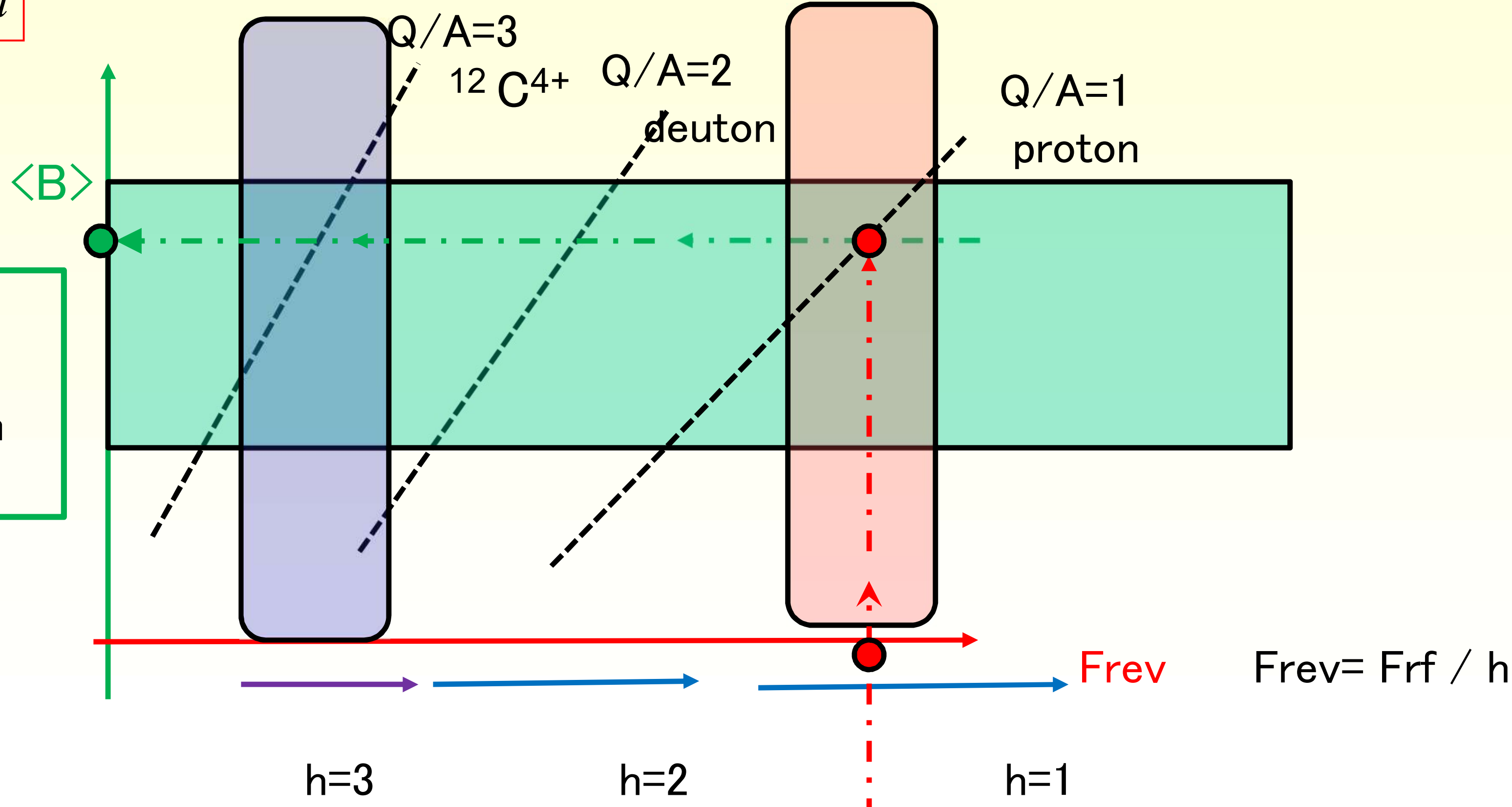
…..

Diagram for
The variable energy cyclotrons

$$F_{rev} \propto \frac{Q.B_{cyclo}}{A} \propto h \cdot F_{RF}$$

A= nucleon number
Q= charge number

$$\omega_{rev} = \frac{qB}{\gamma\, m}$$



2) Compute <B>
For a given (Q,A)

1) Select the energy (Frev)
Select the ions (Q,A)
Adjust <B>

Bρ # <B> Rextract

E/A (MeV/A) # K (Q/A)^2