Beam dynamics for cyclotrons

Bertrand Jacquot

Cyclotrons JUAS 2023

Chapter 1 : theory

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- Cyclotrons JU

Chapter 1 : theory

 History & Principle

 isochronism

 Transverse dynamics (stability) Cyclotrons JUA

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Cyclotrons JUAS

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• AVF cyclotron

• Synchro-cyclotron

• FFAG Chapter 1 : theory

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Chapter 3 :specific problems • Transverse dynamics (stability)
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Chapter 3 :specific problems
• Acceleration , RF
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Chapter 3 :specific problems

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Chapter 4 : design
• Design strategy
• Tracking S 2023
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• Tracking

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Chapter 4 : design

• Design strategy

• Tracking

Chapter 5 :

-Theory vs reality (cost,

tunes, Isochronism,...) -Theory vs reality (cost,

tunes, Isochronism,...)

-Examples Medical cyclotron Research facility

Chapter 1 : part a

Cyclotron history

The Inventor, E. Lawrence, get the Nobel in Physics (1939) hapter 1 : part a

Cyclotron history

The Inventor, E. Lawrence, get the Nobel in Physics (1939)

(first nuclear reactions Without alpha source)

Brilliant idea (E. Lawrence, Berkeley,

1929) : RF accelerating field is te

**Cyclotron history

Solution Carry (September 1989)**

Refilliant idea (E. Lawrence, Berkeley,

1999) : RF accelerating field is technically

Implex and expensive. a
 Cyclotron history
 Example 30
 Example 30
 Cyclotron history
 Example 30
 Cyclotron history
 Cyclotron biggers
 Cyclotron accelerating field is technically
 Cyclotron Beam
 Cyclotron Solid is technic a
 Cyclotron history
 Cyclotron history
 Cyclotron history
 Complex and solution is a
 Complex and expensive.
 Complex and expensive.

So Let 's use **only 1 RF cavity**, but many

times **Cyclotron history

So Let 's use only 1 RF cavity, but many

A device is put into a magnetic field,

A device is put into a magnetic field,

A device is put into a magnetic field,

A device is put into a magnetic field, Cyclotron Filstory**
 E. Lawrence, get the Nobel in Physics (1939)

reactions Without alpha source)

 Calculate E. Lawrence, Berkeley,

1929) : RF accelerating field is technically

complex and expensive.

So Let 's curving the ion trajectories and only one **Electrons Without alpha source**)
 electrons Without alpha source)

 electrode is Unit alpha source (Berkeley, 1929) : RF accelerating field is technically complex and expensive.

So Let 's use only 1 RF cavity, but

times

From 1935-1980 : unique tool for Research facility

The metric of the control of the co

(nuclear physics)

1980-today : Radio Isotope Production for the hospitals

In 2021 : 1300 cyclotrons in Operation !

What is a cyclotron ? What is a cyclotron?
•RF accelerator for the ions :
• from proton A=1 to Uranium A=238
• Fnergy range for proton 1MeV 1GeV (*(v*, 1.2) What is a cyclotron ?
for the ions :
from proton A=1 to Uranium A=238
for proton 1MeV -1GeV (γ ~1-2) **What is a cyclotron ?**
• RF accelerator for the ions :
• from proton A=1 to Uranium A=238
• Energy range for proton 1MeV -1GeV (γ ~1-2)
• Circular machine with 100% duty cycle
• Weak focusing **What is a cyclotron ?**

• RF accelerator for the ions :

from proton A=1 to Uranium A=238

• Energy range for proton 1MeV -1GeV (γ ~1-2)

• Circular machine with 100% duty cycle

• Weak focusing

• Size Radius=30cm to **What is a cyclotron?**
• RF accelerator for the ions :
• from proton A=1 to Uranium A=238
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• Size Radius=30cm to R=6m

- What is a cycl
• RF accelerator for the ions :
• from proton A=1 to Ur
• Energy range for proton 1MeV -1Ge
• Circular machine with 100% duty cycle
• Weak focusing
• Size Radius=30cm to R=6m
• RF Frequency : 10 MHz -60 MHz **• RF** accelerator for the ions :

• FRF accelerator for the ions :

• from proton A=1 to Uranium A=238

• Energy range for proton 1MeV -1GeV (γ ~1-2)

• Circular machine with 100% duty cycle

• Weak focusing

• Size Ra
-
-
-
-

RF accelerator for the ions :

from proton A=1 to Uranium A=238

Energy range for proton 1MeV -1GeV (γ ~1-2)

Circular machine with 100% duty cycle

Weak focusing

Size Radius=30cm to R=6m

RF Frequency : 10 MHz -60 MHz
 e with 100% duty cycle

cm to R=6m

: 10 MHz -60 MHz

Nuclear physics

from fundamental to applied research

Medical applications :

Radio Isotopes production (for PET scan,....)

Cancer treatment

pact and Cost effective

Quality: Compact and Cost effective

4

Useful concepts for the cyclotrons

$$
B\rho = \frac{P}{q} = \frac{\gamma \, m.v}{q}
$$

$$
E_K = (\gamma - 1).mc^2
$$

Cyclotron coordinates

- r Radial = horizontal
- z Axial $=$ vertical
- $\theta \ll$ Azimuth » = cylindrical angle

MeV/A= kinetic energy unit in MeV per nucleon

Maxwell equations

 $\nabla \times B = 0$

Principle: spiraling trajectories

Accelerating Dee's

2 Copper boxes \neq potential one is at the ground potential

A compact cyclotron in reality

7

Trajectory in uniform B field

$$
\frac{d(\gamma m\vec{v})}{dt} = \vec{F}
$$

A ion with a charge q and a mass m circulating at a speed v_{θ} in a uniform induction field \bm{B} . = $(0, Bz, 0)$

The motion equation can be derived from the Newton's law in a cylindrical coordinate system (er,,ez, e θ):

1.1 Trajectory in uniform B field
$$
\frac{d(y m\vec{v})}{dt} = \vec{F}
$$
\n\nA ion with a charge **q** and a mass **m** circulating at a speed v_{θ} \nin a uniform induction field **B** = (0, Bz, 0)\n\nThe motion equation can be derived from the Newton's law in a cylindrical coordinate system (er, ez, e θ):\n\n
$$
\frac{d\vec{p}}{dt} = \vec{F} = q(\vec{v} \times \vec{B}) = -qv_{\theta}B_z \vec{e}_r
$$
\n
$$
\frac{dp}{dx} = my \frac{dv}{dt} = q(v_{\theta}.B_z).e_r
$$
\n
$$
\frac{dv}{dt} = \left(\frac{||v||^2}{R}\right).e_r
$$
\n
$$
mv \frac{v^2}{R} e_r = q(v_{\theta}.B_z).e_r
$$
\n
$$
\frac{dv}{dt} = \left(\frac{||v||^2}{R}\right).e_r
$$
\n
$$
mv \frac{v^2}{R} = q.vB_z
$$
\n
$$
\frac{d^2v}{dt} = \left(\frac{||v||^2}{R}\right).e_r
$$
\n
$$
R = \frac{\gamma mv}{qB_z}
$$

Trajectory in uniform Bz field

The acceleration in cyclotron

100 million of the Contract of

Cyclotrons Tutorials 0

- Cyclotrons Tutorials 0
• Demonstrate that the revolution frequency $(F_{rev} = \omega_{rev}/2\pi$)
of an ion in a perpendicular uniform field B_z is
 $\omega_{rev} = \alpha R_z / m_v$ **Cyclotrons Tutorials 0**
 OF an ion in a perpendicular uniform field B_z **is**
 OF an ion in a perpendicular uniform field B_z **is**
 OF $\exp(-\mathbf{q} \cdot \mathbf{r})$ $\exp(-\mathbf{q} \cdot \mathbf{r})$ $\exp(-\mathbf{q} \cdot \mathbf{r})$ $\begin{array}{l} \text{lotrons} \text{ Tutorials 0} \\\\ \text{revolution frequency} \quad (F_{rev} = \text{Orev}/2\pi \text{)} \\\\ \text{cular uniform field } B_z \text{ is} \\\\ \text{Orev} = \textbf{q} \ B_z / \textbf{m} \gamma \\\\ \text{rentz equation (demo 1)} \end{array}$ Cyclotrons Tutorials 0

Demonstrate that the revolution frequency $(F_{rev} = \omega_{rev}/2\pi$)

of an ion in a perpendicular uniform field Bz is
 $\omega_{rev} = q Bz / m\gamma$

1) With the Newton-Lorentz equation (demo 1)

2) With magnetic rigi Demonstrate that the revolution frequency $(F_{rev} = \text{Orev}/2\pi$)
of an ion in a perpendicular uniform field Bz is
 $\text{Orev} = q \ Bz / m\gamma$
1) With the Newton-Lorentz equation (demo 1)
2) With magnetic rigidity (demo 2)
Nota: Elect • Demonstrate that the revolution frequency $(F_{rev} = \text{Crev}/2\pi$)

of an ion in a perpendicular uniform field Bz is
 $\text{Crev} = q \ Bz / m\gamma$

1) With the Newton-Lorentz equation (demo 1)

2) With magnetic rigidity (demo 2)

Nota rate that the revolution frequency $\int (Frev - \omega r \text{eV})/2\pi r^2 dr$

in a perpendicular uniform field Bz is
 $\omega r \text{eV} = q Bz / m\gamma$

e Newton-Lorentz equation (demo 1)

agnetic rigidity (demo 2)

tric field is supposed to be Zero
-
-

orev = $\frac{1}{2}$ Bz / mγ

wton-Lorentz equation (demo 1)

lic rigidity (demo 2)

eld is supposed to be Zero

V|| = constant and

hence γ = $[1-v^2/c^2]$ $\frac{-1}{2}$ = constant $/c^2$] $^{-1/2}$ = constant

Cyclotrons Tutorials 0

Cyclotrons Tutorials 0
Demonstrate that the revolution frequency $F_{rev} = \omega_{rev}/2\pi$
of an ion in a perpendicular uniform field B_z is $\omega_{rev} = q B_z / m\gamma$ Cyclotrons Tutorials 0

Demonstrate that the revolution frequency $F_{rev} = \omega_{rev} / 2\pi$

of an ion in a perpendicular uniform field Bz is $\omega_{rev} = q Bz / m\gamma$

Demo N° 1: Newton equation Cyclotrons Tu

Demonstrate that the revolution frequency

of an ion in a perpendicular uniform

Demonstrate that the revolution frequency

Demonstrate (v x B)
 $\begin{vmatrix} e_r & e_z & e_{\theta} \end{vmatrix}$

Demo N° 1 : Newton equation

$$
\mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \mathbf{0} & \mathbf{0} & \mathbf{V}\theta \\ \mathbf{0} & B_z & \mathbf{0} \end{vmatrix}
$$

\n
$$
\mathbf{m} \gamma \ (\mathbf{d} \ \mathbf{v} / \ \mathbf{d} \mathbf{t}) = - \mathbf{q} \ \mathbf{v}\theta \ \mathbf{B} \mathbf{z} \ \mathbf{e} \mathbf{r}
$$

$$
m \gamma \left(-\nu^2 / R\right) er = - q \nu \theta Bz er
$$

 $R = q v Bz / m \gamma$ m qB R Frevolution πR 2 π γ 1 $\overline{2}$ v $=\frac{v}{2}$ =

Demo N° 2 : Bp formula

$$
\begin{array}{c|c}\n\cdot & \mathbf{e}_z & \mathbf{e}_{\theta} \\
\hline\n\mathbf{0} & \mathbf{0} & \mathbf{V}\theta\n\end{array}\n\qquad\n\begin{array}{c|c}\n\end{array}\n\qquad\nR = \frac{B\rho}{B_z} = \frac{\gamma \ m \mathbf{V}}{qB_z}
$$

Demo N° 2 : Bp formula
\n
$$
R = \frac{B\rho}{B_z} = \frac{\gamma m v}{qB_z}
$$
\n
$$
Frevolution = \frac{v}{2\pi R} = \frac{1}{2\pi} \frac{qB}{\gamma m}
$$
\n
$$
0
$$
\n
$$
0
$$
\n
$$
r = 2\pi
$$
\n
$$
Frev = qBz / m \gamma
$$

$$
0 = 2\pi \ Frev = q \ Bz / m \gamma
$$

Desynchronization of bunches in uniform field cyclotron Bz=B₀

$$
\omega_{rev} = \frac{qB_0}{m\gamma} = \frac{qB_0}{m} \sqrt{1 - \frac{v^2}{c^2}} = \frac{qB_0}{m} \sqrt{1 - \frac{R^2 \omega^2}{c^2}}.
$$

14 **Isochronism** $\omega_{\text{rev}} = \text{constant}$ $B_z(R) = B_0/(1 - R^2 \omega^2/c^2)^{1/2}$

Isochronism condition: ω_{rev} =constant

with $\omega_{\rm rf}$ = $\rm{H\omega_{rev}}$, the particle is synchronous with the RF wave.

In other words, the particle arrives always at the same RF phase in the middle of the accelerating gap.

Cyclotrons Tutorial 1

Cyclotrons Tutorial 1
•An isochronous cyclotron uses a RF cavity
at 60 MHz at the RF harmonic H=3 Cyclotrons Tutorial 1

velotron uses a RF cavity

at 60 MHz at the RF harmonic H=3

pe peeded to perform Cyclotrons Tutorial 1

An isochronous cyclotron uses a RF cavity

at 60 MHz at the RF harmonic H=3

a. Compute the time needed to perform

one turn Trev for the accelerated ions. Cyclotrons Tutorial 1

onous cyclotron uses a RF cavity

at 60 MHz at the RF harmonic H=3

te the time needed to perform

one turn Trev for the accelerated ions. b. Compute the average field Bz needed orthom and a solution and a solution at 60 MHz at the RF harmonic H=3

is time needed to perform

turn Trev for the accelerated ions.

e average field Bz needed

to accelerate a proton beam

(in a non relativistic approxim a RF cavity

it the RF harmonic H=3

perform

accelerated ions.

needed

ton beam

(in a non relativistic approximation)

Cyclotrons Tutorial 1

Cyclotrons Tutorial 1
•An isochronous cyclotron uses a RF cavity at 60 MHz at the RF harmonic h=3
•An isochronous cyclotron uses a RF cavity at 60 MHz at the RF harmonic h=3
• Compute the time needed to perform one turn fo Cyclotrons Tutorial 1
An isochronous cyclotron uses a RF cavity at 60 MHz at the RF harmonic h=3
a. Compute the time needed to perform one turn for the accelerated ions.
b. Compute the average field B needed to accelerate Cyclotrons Tutorial 1
An isochronous cyclotron uses a RF cavity at 60 MHz at the RF harmonic h=3
a. Compute the time needed to perform one turn for the accelerated ions.
b. Compute the average field B needed to accelerate **Controlled as a non-**
 Controlled as a non-
 Controlled as a non relativistic approximation
 Controlled as a non relativistic approximation
 Controlled as a non-
 Controlled as a non-
 Controlled as a non-
 Cyclotrons Tutorial 1

n isochronous cyclotron uses a RF cavity at 60 MHz at the RF harmonic h=3

Compute the time needed to perform one turn for the accelerated ions.

Compute the average field B needed to accelerate pro ^b ^w =qB/^g ^m ⁼ ^wrf / ^H and we have ^g close to 1 bochronous cyclotron uses a RF cavity at 60 MHz at the RF harmonic h=3
mpute the time needed to perform one turn for the accelerated ions.
mpute the average field B needed to accelerate proton
in a non relativistic approx

.

Solution freq=60/3 =20 MHz

in a non relativistic appr

nswer a Revolution freq=60/3 =20 MHz

b $\omega = qB/\gamma m = \omega rf/H$ and we

proton mass~ 1.6 10^{-27 kg} // proton ch

Frf=60 MHz= $\omega rf/2\pi$

Bz = m_p/q . $2\pi F rf/H$ = 10⁻⁸. 10⁶ $Bz = m_p/q$. $2\pi Frf/H = 10^{-8}$. 10⁶ 20.2 π . = 1.26 Tesla Revolution freq=60/3 = 20 MHz $\Delta T = 1/$ Frev = 50ns
 $\omega = qB/\gamma m = \omega_f / H$ and we have γ close to 1

ass~ 1.6 10^{-27 kg} // proton charge~ 1.6 10⁻¹⁹ C

Hz= $\omega_f / 2\pi$

/q. $2\pi F_f / H = 10^{-8}$. 10⁶ 20.2 $\pi = 1.26$ Tesla z $\Delta T = 1/Frev = 50ns$
we have γ close to 1
n charge~ 1.6 10⁻¹⁹ C
20 .2π .= 1.26 Tesla

Cyclotrons Tutorial 2

Example 19 CONTROV CONTROV CONTROVER CONTROVIDED

•Demonstrate than in a uniform circular motion , the radial acceleration is
 $a_r = |V^2/R|$. **Example 12 Series Cyclotrons Tutorial**
 Example:
 Acceleration is
 a_r= $|V^2/R|$. Cyclotrons Tutorial 2
than in a uniform circular motion
 $a_r = |V^2/R|.$ clotrons Tutorial 2

a uniform circular motion, the radial
 \angle R|. monstrate than in a uniform circular n

eleration is
 $a_r = |V^2/R|$.
 \therefore You can use parametric equations : etcos(αt)
 $Y(t) = R \sin(\omega t)$ monstrate than in a uniform circular

eleration is
 $a_r = |V^2 / R|$.

a: You can use parametric equations $\frac{R}{t}$
 $X(t) = R \cos(\omega t)$
 $Y(t) = R \sin(\omega t)$

an compute the velocity and the acceleration

$$
\mathbf{a}_{\mathrm{r}}^{\mathrm{}}=|\mathrm{V}^2/|\mathrm{R}|\,.
$$

Nota : You can use parametric equations
 $X(t) = R \cos(\omega t)$
 $Y(t) = R \sin(\omega t)$

Then compute the velocity and the acceleration.

Demonstrate that the acceleration is radial

Nota : $\omega t = \theta$ $\omega = d\theta/dt$

Cyclotron Tutorial 2

Cyclotron Tutorial

uniform circular motion
 $X(t) = R \cos(\omega t)$
 $Y(t) = R \sin(\omega t)$ $Cyclotron$ Tutorian
 $X(t) = R cos(\omega t)$
 $Y(t) = R sin(\omega t)$
 $v = W(t) cos(t) cos(\omega t)$
 $v = W(t) cos(t) cos(t) cos(t) cos(t)$

Answer

$$
|\mathbf{a}| = (ax^2 + ay^2)^{1/2} = \omega^2 R = v^2 / R
$$

 $a = -v^2/R$ $/R$

Answer
\n
$$
\mathbf{V_x} = -\omega \mathbf{R} \sin(\omega t) \qquad \mathbf{v_{\theta}} = |\mathbf{v}| = (Vx^2 + Vy^2)^{1/2} = \omega R
$$
\n
$$
\mathbf{V_y} = +\omega \mathbf{R} \cos(\omega t)
$$
\n
$$
|\mathbf{a}| = (ax^2 + ay^2)^{1/2} = \omega^2 R = v^2 / R
$$
\nrependicular to a (since $\mathbf{v} \cdot \mathbf{a} = 0$)
\n
$$
\mathbf{a} = -v^2 / R
$$
\n2adial vector **e**r and longitudinal vector **e**₀
\n
$$
\mathbf{e_r} = \begin{vmatrix} cos(\omega t) & det/dt & = \omega e\theta \\ sin(\omega t) & d^2 e\mathbf{r}/dt^2 & = -\omega^2 e\mathbf{r} = v^2 / R^2 e\mathbf{r} \end{vmatrix}
$$

Transverse dynamics in a cyclotron without acceleration

We will use cylindrical coordinates $(er, e\theta, ez)$

The reference trajectory is $r = R_0 e_r$ $R_0 = B_0/B_z$ What happen to a particle with $r = (R_0 + x_0)$ er + z_0 ez

In Radial plane (horizontal) :

 $r(t) = R_0 + x_0 \cos(Q_r \omega_{rev} t)$

Radial tune Q

In the Vertical (axial) plane :

$$
z(t) = z_0 \cos(Q_z \omega_{rev} t)
$$

Axial tune Q_z

Transverse dynamics with Bz (R) $B_{7}(R) = B_{0}/(1 - R^{2} \omega^{2}/c^{2})^{1/2}$ $\binom{2}{2}$ $\frac{1}{2}$ $\frac{2}{\pi}$ $\frac{1}{2}$

Cylindrical coordinates (e_r, e_θ, e_z) and define $x \& z$ a small orbit deviation with $Bz=Bz(r)$ (not constant)

Motion Equat. With $x \neq 0$ & $z \neq 0$?

$$
m\frac{d(\vec{v})}{dt} = m\frac{d^2(\mathbf{r})}{dt^2} = ?
$$

Vertical dynamics with $B_z(R)$ (No RF)

• Taylor expansion of the field B_z around the median plane:

Vertical dynamics with B_z(R) (M

• Taylor expansion of the field B_z around the median plane:

definition of $n(r)$ Bz ~B₀ (r/R₀)⁻ⁿ n=field index

uniform) definition of $n(r)$ $Bz \sim Bo(r/R_0)^{-n}$ n=field index Bz is not uniform)

$$
\begin{vmatrix} with & n = -\frac{R_0}{B_0} \frac{\partial B_z}{\partial r} = -\frac{R_0}{B_0} \frac{\partial B_z}{\partial x} \end{vmatrix}
$$

 $\textsf{SO} \ \ \textit{Bz} \ \textit{(Ro+x)} = \textit{B}_{0}(\textit{Ro}) + \ \textit{(dB/dx)} \ \textit{x} + ... = \textit{B}_{0} \ \textit{(1-nx/Ro)}$ How evolves an ion, in this non uniform Bz :

 $r(t) = (R + x(t))$ er + $z(t)$ ez

er

 $m \gamma$ d² (z ez)/dt² = **F_z ez** Vertical motion e θ \leftarrow \leftarrow \rightarrow

$$
\begin{aligned}\n\text{Fz vertical plan: } \quad \mathbf{ez} = \text{constant} \\
m\gamma \frac{d^2 z}{dt^2} &= F_z = q \left(v \times B \right)_z = -q \left(r \, B_\theta - r \, \theta \, B_r \right)\n\end{aligned}
$$

Vertical dynamics with B (r)

$$
m\gamma \frac{d^2z}{dt^2} = F_z = q \left(v \times B \right)_z = -q \left(r \cancel{B_\theta} - r \frac{\partial}{\partial B_r} \right)
$$

$$
\mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \mathbf{\dot{r}} & \mathbf{\dot{z}} & r\dot{\theta} \\ B_r & B_z & \mathbf{0} \end{vmatrix}
$$

Br=? B0 0 r B z B^r ^z z r B B n oz r For isochronism, we have n < 0 (therefore Q^z is imaginary) + Bz =B0 r –n

Motion equation

$$
\frac{d^2z}{dt^2} = -\omega^2 \cdot Q_z^2 \cdot z
$$

Harmonic oscillator ?

$$
z(t) = z_0 \cos(Q_z \omega_{rev} t)
$$

$$
Q_z^2 = n < 0
$$
 $Q_z = i |n| \dots$

X Watch the <u>vertical oscillations</u> !! $\frac{1}{2}$ Watch the <u>vertical oscillations</u> $\frac{1!}{2}$ $\frac{1}{2}$
Isochronism condition : $n < 0$: Bz(r)~r⁻ⁿ : increase with Radius
Vertical tune $Q_x^2 = n < 0$

Vertical tune $Q_z^2 = n < 0$

Isochronism condition will induce unstable oscillations

equation: the vel total oscillations

\ncondition:
$$
n < 0
$$
 : $Bz(r) \sim r^{-n}$: increase

\nVertical tune $Q_z^2 = n < 0$

\ncondition will induce unstable oscillates:

\n $z(t) = z_0 \cos(Q_z \omega_{rev} t)$

\n $z(t) \sim z_0 \exp(\pm |\mathbf{n}|^{1/2} \omega_{rev} t)$

\nUnstable oscillations in d

Unstable oscillations in Z

 $=$ exponential growth $=$ beam losses

Radial (horizontal) dynamics with $Bz(R)$ (No RF)

-
- **Radial (horizontal) dynamics with Bz(R)** (*N*

 Taylor expansion of the field B_z around the median plane:

 definition of $n(R)$ Bz (R)~B₀ (R /R₀)⁻ⁿ n= called field

 SQ Bz (Baty)= B (Batt (dB(dy) xt, = B (1 py • definition of $n(R)$ Bz $(R) \sim B_0 (R/R_0)^{-n}$ n= called field index dial (horizontal) dynamics with
lor expansion of the field B_z around the median p
finition of n(R) $Bz (R) \sim B_0 (R/R_0)^{-n}$
so $Bz (R_0+x) = B_0 (R_0) + (dB/dx) x + ... = B_0 (1-nx/R_0)$ SO $Bz (R_0+x)=B_0 (R_0)+$ (dB/dx) $x+...= B_0 (1-nx/R_0)$

 $\left(1-\frac{x}{R}\right)$ |er + 2 x .e₀ $\frac{1}{2} = -q \mathbf{v} \times \mathbf{B}$ $\begin{array}{cc} \bullet & \sqrt{2} & x \\ \hline \theta & \sqrt{2} & x \end{array}$ $\left(e\right)$ + 2 V' ($(r \cdot \vec{er})$ • v_{θ}^2 2 $2\overrightarrow{r}$ \overrightarrow{vr} $\overrightarrow{v_0}$ \overrightarrow{v} \overrightarrow{v} 2 $\ddot{}$ $\overline{}$ $\overline{}$ \perp \cdot L L $\overline{\mathsf{L}}$ $\overline{}$ $=\left(x-\frac{\sqrt{\theta}}{R}\left(1-\frac{x}{R}\right)\right)$ |er + 2 x $=(x-\frac{\nu\theta}{\rho})er+$ $\ddot{}$ $=-q \mathbf{v} \times$ R° \mathcal{X} R $x-\frac{\nu}{\rho}$ $er + 2xe$ r \mathcal{X} dt $d^2(r \cdot er)$ dt d^2r $m\gamma$ r θ $(1 - \frac{\lambda}{2})$ = - q **v**₀ B_{0z} (1 - n $\frac{\lambda}{2}$) \overline{V}_l $\overline{0}$ 2 R \mathcal{X} $q \mathbf{v}_a B_{0z} (1 - n)$ R \mathcal{X} R $\left| m\gamma \right| \left| x-\frac{\mathbf{v}\,\theta}{R}\left(1-\frac{x}{R}\right) \right| = -q\,\mathbf{v}_{\theta}B_{0z}\left(1-\frac{x}{R}\right)$ $\overline{}$ \int \setminus $\overline{}$ $\overline{}$ \setminus $\left(\right)$ $-\frac{\nu \theta}{2}$ (1 – $\bullet \bullet$ $\left| \gamma \right| x - \frac{\nu}{p} \left(1 - \frac{x}{p} \right) = -q \mathbf{V}_\theta$ •How evolves an ion, in this non uniform Bz : $r(t) = R_0+x(t)$ $d^2 er/dt^2 = -\omega^2 er = v^2/r^2 er$ er $der/dt = \omega e \theta$ radial motion (projection on er) $\left(1 - \frac{\lambda}{D}\right)$ 1 $(1 + \frac{\lambda}{\lambda})$ $1 \qquad \qquad 1$ \overline{R} \mathcal{X} R R \overline{x} r R $\approx \frac{1}{\sqrt{1-1}}$ $\ddot{}$ $=$ $r = R(1 + x/R)$

Radial dynamics with Bz(R)

$$
m\gamma \left(\frac{\mathbf{w}}{x} - \frac{\mathbf{v}_{\theta}^{2}}{R}\left(1 - \frac{x}{R}\right)\right) = -q B_{0z} \left(1 - n\frac{x}{R}\right). \mathbf{v}_{\theta}
$$

After simplification :

$$
\text{and} \qquad \omega_{rev} = \frac{q B_{0z}}{\gamma m} = \omega_{0} \approx \frac{\mathbf{v}_{\theta}}{R}
$$

$$
\frac{d^{2}x}{dt^{2}} = -\omega^{2} Q_{r}^{2} x \qquad Q_{r} = (1 - n)^{1/2}
$$

Horizontal stability condition (Qr real) : $\vert n < 1 \vert$

We have n < 0 $\,$ for isochronism : therefore $\mathrm{Q}_\mathrm{r}^{\,2}$ >0 $\,$

Horizontal stability is guarantee in isochronous cyclotron

Radial dynamics (Qr real) : Stable oscillations

Harmonic oscillator with the frequency $(Q_r \omega_{rev})$

$$
\frac{d^2x}{dt^2} = -\omega^2(1-n)x
$$

 $x(t) = x_0 \cos(Q_r \omega_{rev} t)$

Horizontal stability if n <1 : always satisfied Stable oscillations
 $Q_{r} \omega_{rev}$)

if n <1 : always satisfied
 $Qr^2 = 1 - n$ > 0

dition Bz should increase

n< 0 : isochronism condition Bz should increase Stability condition $(Qr^2 > 0)$ OK

$$
\mathbf{r} = \mathbf{R}_0 + \mathbf{x}_0 \cos(\mathbf{Q}_r \omega_{rev} \mathbf{t})
$$

Tunes : Q_r & Q_z

oscillations around reference trajectory

 $r(t) = R_0(t) + x_0 \cos(Q_r \omega_{rev} t)$

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Q_r: Number of radial oscillations per cyclotron turn in horizontal (radial) plan

 $r = R_0(t) + x_0 \cos(2\omega_0 t)$

 $Q_r^2 = 1 - n$ stable oscillations

Q_r : Number of radial oscillations per cyclotron turn in horizontal (radial) plan		
$Q_r^2 = 1 - n$	stable oscillations	$r = R_0(t) + x_0 \cos(\theta)$
$Z(t) = z_0 \cos(Q_z \omega_{rev} t) = z_0 \cos(Q_z \theta)$	$\sum_{v \neq x} M_z$	
$Q_z^2 = n < 0$ unstable oscillations $(Q_z = i Q_z)$	$\sum_{v \neq y} M_z$	
$Z(t) \sim z_0 \exp(\pm Q_z \theta)$	$\sum_{v \neq y} M_z$	

The vertical instability generated by $Bz(R)$

 $\langle F_z \rangle = q \langle v_r \cdot B_\theta \rangle$: additional Vertical focusing force

What is the azimuthal field Be

 $\langle F_z \rangle = q \langle v_r \cdot B_0 \rangle$ ~ - z : Vertical focusing force

Vertical stability $Bz(R)$ Defocus in $z + B\theta$ Focus in z

Azimuthally varying Field (AVF) Exemple : 30 MeV compact proton cyclotron 4 straight sectors

C30 poles and valleys

-2 RF cavities Inserted in the valleys = 4 accelerating gaps

4 Hills + 4 Valleys $B= B(R,\theta)$

Bz Field varies