Transverse non-linear manipulations

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Some known properties of beam dynamics

Let us consider some general statements

- Transverse emittances are an invariant of motion
- The transverse beam distribution is a Gaussian

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This seminar is devoted to show how **non-linear effects** can be used to manipulate **emittances** and **distributions**.

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- Exploit **nonlinear effects** in transverse motion:
  - Change phase space **topology** (new separatrices, islands)

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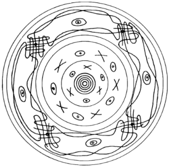
  - Change phase space **topology** (new separatrices, islands)
- Use **slow** variation of parameters
  - Change surfaces of phase-space regions
  - Perform particle **trapping & transport** in phase-space regions
- To **manipulate** the transverse distribution for:
  - Beam splitting
  - Sharing of transverse emittances
  - Cooling of annular beams

Theoretical frameworks

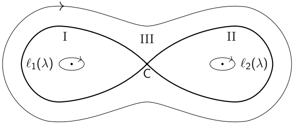
Hénon-like maps

$$\begin{pmatrix} x_{n+1} \\ p_{n+1} \end{pmatrix} = \mathrm{R}(\omega_0) \begin{pmatrix} x_n \\ p_n + f(x_n) \end{pmatrix}$$

Poincaré-Birkhoff



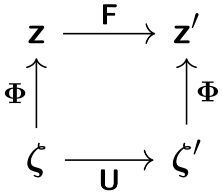
Separatrix-crossing theory



$$P_{\mathrm{III}\to i} = \frac{\mathrm{d}A_i/\mathrm{d}t}{\mathrm{d}A_{\mathrm{III}}/\mathrm{d}t}$$

$$J_f = A_i/2\pi$$

Normal Forms



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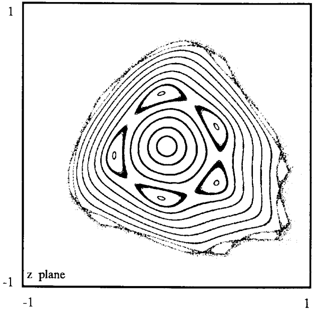
M. Hénon was an astronomer

- A simple, although general model.
- It represents the transverse motion using Courant-Snyder co-ordinates
  - FODO cell $\rightarrow$ rotation matrix
  - non-linear kick $\rightarrow f(x_n)$, f is typically a polynomial function
- It can be generalised to 4D.

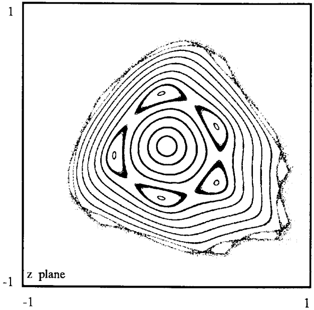


Phase-space portrait for $\omega_0 = 0.205/(2\pi)$

Poincaré-Birkhoff theorem

Loosely speaking, it states that non-linear maps feature chains of islands in phase space

- Each chain is made of:
  - Stable (or **elliptic**) fixed points at the centre of the islands.
  - Unstable (or **hyperbolic**) fixed points in between islands.
  - The unstable fixed points are connected by a **separatrix**.



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Normal Forms

It is a perturbative technique that aims at finding a change of co-ordinates so that in the new reference the symmetries of the system are explicit.

$$
\begin{array}{ccc}
\mathbf{z} & \xrightarrow{\mathbf{F}} & \mathbf{z}' \\
\Phi \uparrow & & \uparrow \Phi \\
\zeta & \xrightarrow[\mathbf{U}]{} & \zeta'
\end{array}
$$

which is equivalent to

$$\mathbf{F} \circ \Phi = \Phi \circ \mathbf{U}$$

note the resemblance with the similarity relationship for matrices

F is the original transfer map.
U is the Normal Form.
Ψ, Φ are the change of co-ordinates.
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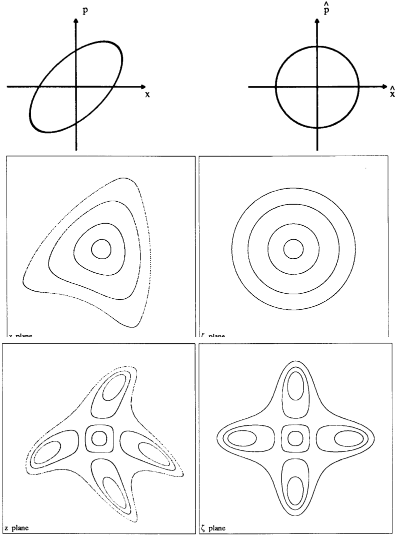
Link established between maps and Hamiltonians

Normal Forms



Courant-Snyder transformation → linear Normal Form

Left: map
Right: Non-resonant Normal Form

Left: map
Right: Resonant Normal Form

Separatrix-crossing theory

The following statements can be made rigorous from a mathematical point of view...

- In a **time-independent** Hamiltonian linear system, the action is an invariant.
- In a **time-independent** Hamiltonian non-linear system, the action is an approximation of the true invariant.
- In a **time-dependent** Hamiltonian, the action is an adiabatic invariant.
- **The issue is due to the presence of a separatrix in the phase space.**

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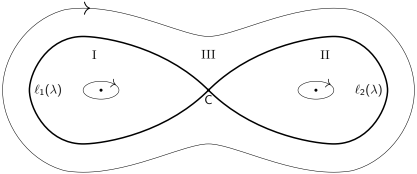
- In a **time-dependent** Hamiltonian system, the separatrix **can** be crossed.



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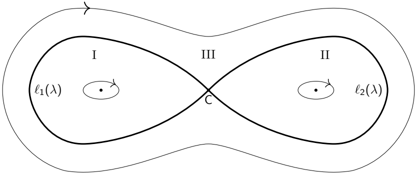

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All non-linear manipulations rely on all this!!!

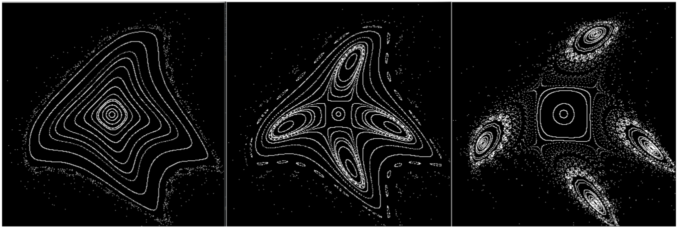
The origin of all this: Multi-Turn Extraction (MTE)

Hénon map:

$$\begin{pmatrix} x_{n+1} \\ p_{n+1} \end{pmatrix} = R(\omega_0) \begin{pmatrix} x_n \\ p_n + x_n^2 \end{pmatrix}$$

$\omega_0 \approx 2\pi r/s$: s islands.

- **split beam** in $s+1$ beamlets
- used for beam transfer from PS to SPS



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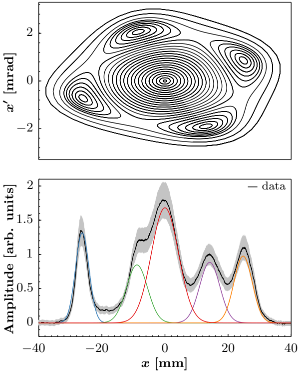

A measured profile of a split

beam

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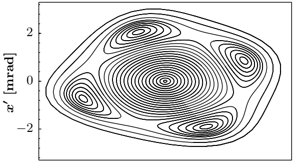

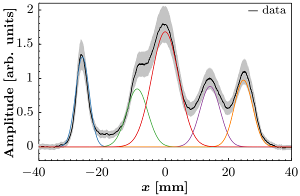

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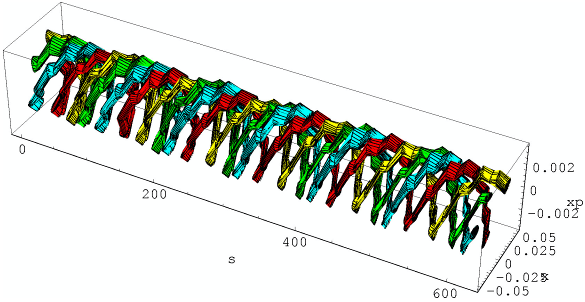


Split beam structure along the ring

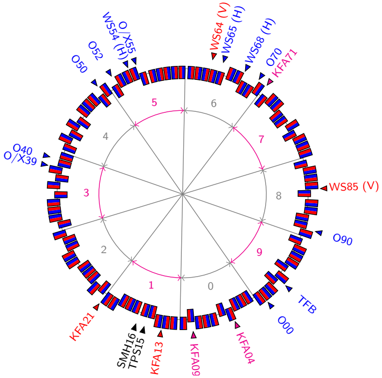
Animation from numerical simulations

Animation from beam measurements

From theory to practice: beam splitting in the CERN PS

The main ingredients of MTE

- **Sextupoles** and **octupoles** → stables islands
- **Quadrupoles** → vary the tune to cross 4th-order resonance



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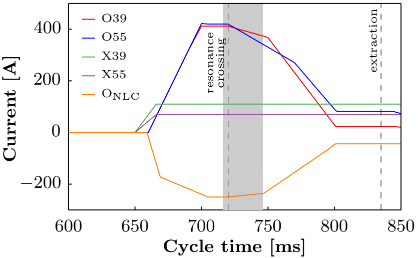
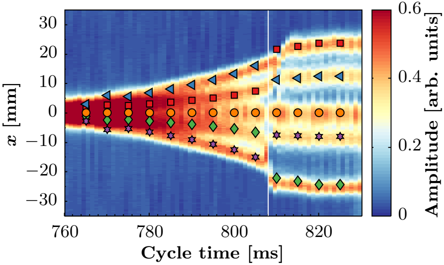


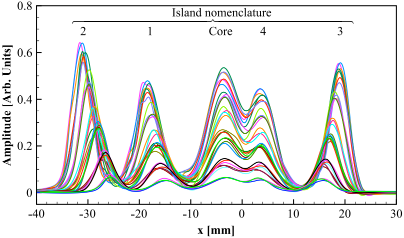
Evolution of sextupoles and octupoles strength



Waterfall measurement of the horizontal distribution

Digression: intensity-dependent effects in split beams

Beam splitting has been studied as single-particle process. Space-charge effects have been probed experimentally:



Measured distribution after splitting for different beam intensities: **beamlets position changes!**

- Keep magnetic settings constant
- Change total beam beam intensity
- Perform splitting
- Check properties of split beam

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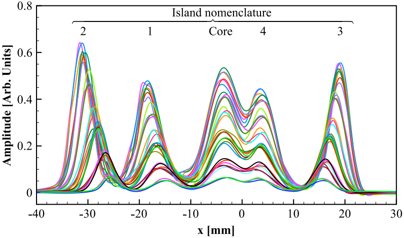

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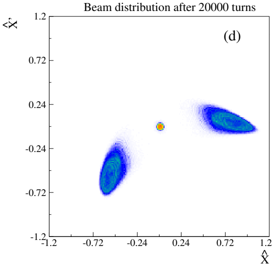
Numerical studies have been performed, but this field is still in its infancy!

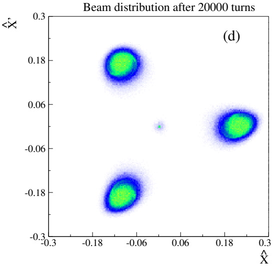
Extending MTE: other resonances

Any resonance can be used to perform beam splitting!

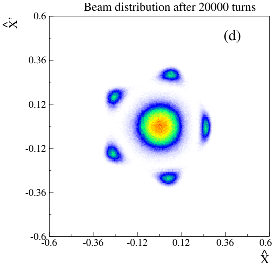
NB: the higher the order the smaller the islands. Hence, the larger is size and intensity of the core.



Split beam with **2nd-order** resonance



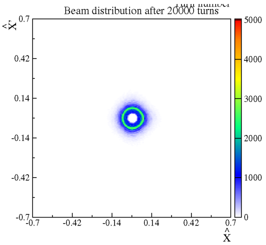
Split beam with **3rd-order** resonance



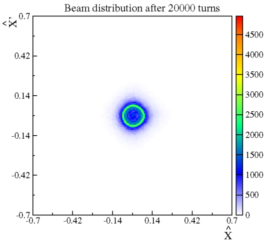
Split beam with **5th-order** resonance

Reversing MTE: Multi-Turn Injection (MTI)

The splitting process can be reversed: the beam is injected in the islands and the tune is used to merge them. **Two variants are possible: with or without beam injected in the core.**



Beamlets merged without core



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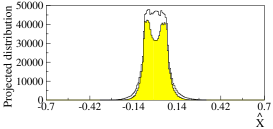

Projected beam profile after injection: with (white) and without (yellow) core

Extending MTE: an external exciter

- Splitting a beam and transporting beamlets works well in both simulations and real machines.
- Excellent agreement between theoretical predictions, numerical simulations, and measurements.
- This technique implies the possibility of accurate tune control and the possibility to set the tune close to a resonant value.
- **What if the tune value is imposed by other considerations, e.g. space-charge effect?**

Extending MTE: an external exciter

- **The solution consists in using an external exciter.**
- A so-called AC dipole generates an oscillating dipole field at a given frequency.
- Such a device is used to perform optics measurements. It allows kicking a bunch avoiding the filamentation of its transverse emittances.
- In this scheme, **the resonance condition is generated between the machine tune and the frequency of the AC element.**

Extending MTE: an external exciter

$$\begin{pmatrix} x_{n+1} \\ p_{n+1} \end{pmatrix} = R(\omega_0) \begin{pmatrix} x_n \\ p_n + x_n^2 + \varepsilon x_n^{\ell-1} \cos \omega n \end{pmatrix}$$

NB: it is assumed that a generic AC 2ℓ-pole can be built.

ω_0 fixed, ω varies, $\omega \approx m\omega_0 \to m$ islands.

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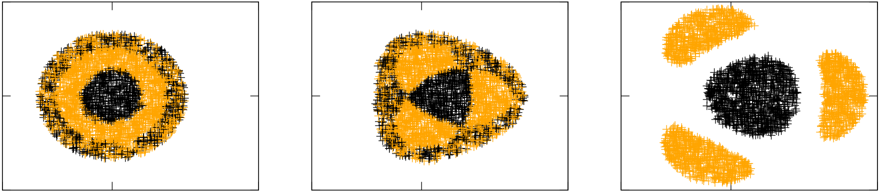

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- A detailed analysis has been carried out of both the map and the Hamiltonian models.
- General agreement between the two models.
- Trapping is fully explained via the time variation of islands' surface for maps and Hamiltonians.
- Scaling laws of key quantities on the models parameters have been established.

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This confirms the possibility of performing beam splitting without varying the accelerator's tune

Intermediate summary

- It should be clear that non-linear effects allow
  - Breaking the conservation of the emittance.
  - Generating multiple Gaussian beamlets.
- **This can be achieved in a controlled way.**
- There is another essential consequence
  - **Multiple closed orbits are present in the particle accelerator: one is the standard one, then there is the additional closed orbit linked with the fixed points.**
- This aspect is not discussed further, but is at the heart of a novel gamma-transition gymnastics.

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Let us move forward to 2 degrees of freedom...

Crossing a 2D resonance

- MTE and its extensions are all based on resonance crossing in a single plane, i.e. $\mathbf{m}\,\omega_{\mathbf{x}} \approx \mathbf{p}$.
- In reality, the transverse dynamics has two degrees of freedom, each with a tune. In this case, the resonance condition reads $\mathbf{m}\,\omega_{\mathbf{x}} + \mathbf{n}\,\omega_{\mathbf{y}} \approx \mathbf{p}$.

What could we achieve by crossing a 2D resonance?

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What could we achieve by crossing a 2D resonance? Sharing of transverse emittances

Sharing transverse emittances

The Hamiltonian model for this case is

$$\mathcal{H}(\phi_x,\, J_x,\, \phi_y,\, J_y) = \omega_x J_x + \omega_y J_y + \alpha_{xx} J_x^2 + 2\alpha_{xy} J_x J_y + \alpha_{yy} J_y^2 + \\ + G J_x^{m/2} J_y^{n/2} \cos(m\phi_x - n\phi_y)$$

$\alpha_{xx}, \alpha_{xy}, \alpha_{yy} \rightarrow$ **amplitude-detuning parameters.**
$m\,\omega_x - n\,\omega_y \approx 0 \rightarrow$ **quasi-resonant condition.**
The canonical transformation

$$\begin{aligned} J_x &= m J_1\,, & \phi_1 &= m\phi_x - n\phi_y\,, \\ J_y &= J_2 - n J_1\,, & \phi_2 &= \phi_y\,, \end{aligned}$$

casts the Hamiltonian into the form

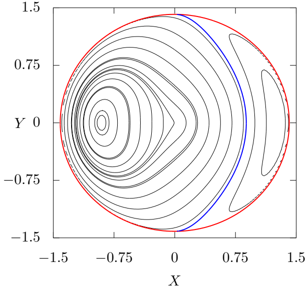
$$\mathcal{H}(\phi_1, J_1) = (\delta + \alpha_{12} J_2)\, J_1 + \alpha_{11} J_1^2 + G (m J_1)^{\frac{m}{2}} (J_2 - n J_1)^{\frac{n}{2}} \cos\phi_1 + \left[\omega_y J_2 + \alpha_{22} J_2^2\right]$$

$\delta = m\,\omega_x - n\,\omega_y \rightarrow$ **resonance-distance parameter**
$\alpha_{11}, \alpha_{12}, \alpha_{22} \rightarrow$ **functions of** $\alpha_{xx}, \alpha_{xy}, \alpha_{yy}$

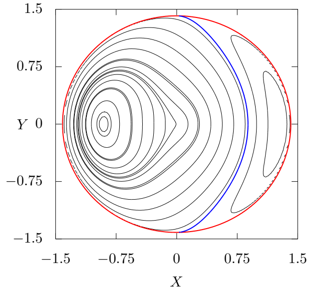
Sharing transverse emittances

- $J_y > 0 \implies J_1 < J_2/n \rightarrow$ **the dynamics is limited to an allowed circle.**
- $J_2 = nJ_x/m + J_y$ **is approximately conserved.**
- **The 4D system reduces to a 2D one dependent on a parameter (J_2).**
- The phase-space topology can be studied
  - **Existence and stability of fixed points.**
  - **Existence of separatrices.**
- Not easy to perform analytically. The topology depends on m, n.

Sharing transverse emittances



Sharing transverse emittances



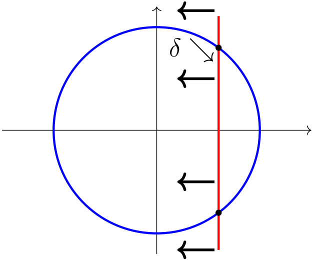
- Vary ω_x, ω_y $\rightarrow$ separatrix crossing
- For each particle, can we make $J_{y,f} = (m/n)J_{x,i} \implies \varepsilon_{x,f} = (m/n)\varepsilon_{y,i}$?

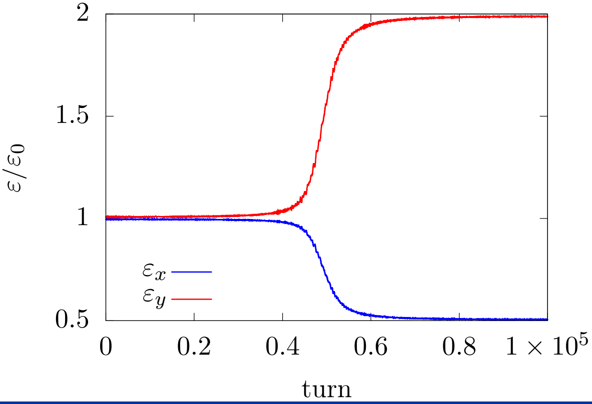
4D Hénon-like map

$$\begin{pmatrix} x' \\ p_x' \\ y' \\ p_y' \end{pmatrix} = R(\omega_x, \omega_y) \begin{pmatrix} x \\ p_x + \operatorname{Re} f(x,y) \\ y \\ p_y - \operatorname{Im} f(x,y) \end{pmatrix}$$

Sharing transverse emittances

$m = 1, n = 2$:



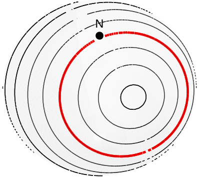


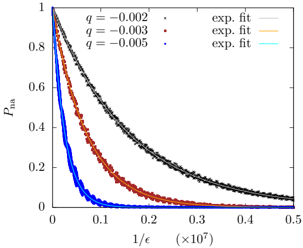
Sharing transverse emittances

- Studied **2D Hamiltonian and 4D map models**: exchange mechanism explained via separatrix crossing theory.
- **Resonances higher than 3**: presence of additional fixed points $\rightarrow$ more phase-space regions.
- **Improved adiabatic theory**: error on final J depends on **adiabaticity**:
  - Resonance $(1, 2)$ and higher: **power-law**.
  - Resonance $(1, 1)$ (**coupling resonance**): **exponential**.

Digression: sharing transverse emittances with linear coupling

- **Observed features linked to the analytical properties of the Hamiltonian.**
- Exponential behaviour of emittance exchange already observed: now fully explained by adiabatic theory.
- Relationship between coupling strength and adiabaticity established.





Intermediate summary

- It should be clear that non-linear effects allow
  - Manipulating transverse emittances.
  - The emittance sharing process depends on the properties of the 2D resonance used.
  - Hamiltonian and adiabatic theories provide a detailed explanation of the process.
- **The emittance sharing can be achieved in a controlled way.**

Intermediate summary

- It should be clear that non-linear effects allow
  - Manipulating transverse emittances.
  - The emittance sharing process depends on the properties of the 2D resonance used.
  - Hamiltonian and adiabatic theories provide a detailed explanation of the process.
- **The emittance sharing can be achieved in a controlled way.**

Let us move back to 1 degree of freedom and the use of the exciter...

Trapping, transporting and external exciters

- Non-linear effects can be used to create stable islands
- Slow variation of the tune can be used to cross a separatrix so that
  - **Trapping** inside islands can occur.
  - **Transport** of trapped particles inside the islands can be performed.

Let us assume to have **an annular beam distribution, generated by kicking a beam and let it filament, what could we do with that?**

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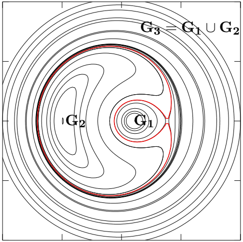
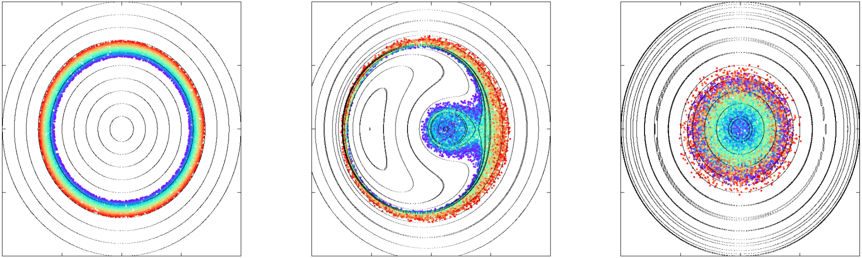
Emittance cooling!

Cooling an annular beam

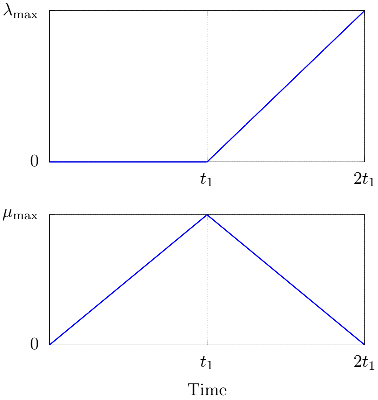
- AC dipole and nonlinearity:
  $\mathcal{H} = \omega_0 J + \Omega_2 J^2/2 + \varepsilon\sqrt{2J}\cos\phi\cos\omega t$
- Vary ω, ε as a function of time;
- Engineer areas to optimise trapping and transport;
- As a result: **up to 90% cooling**

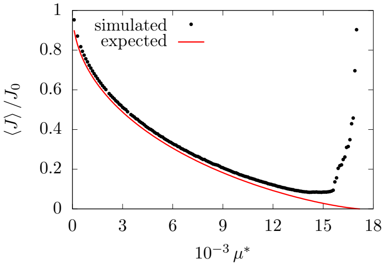
Animations from numerical simulations available here





Cooling an annular beam





An alternative cooling protocol has been devised

Conclusions and outlook

- Non-linear effects can be used efficiently to manipulate the transverse beam emittances and distributions.
- Hamiltonian and adiabatic theories are the ideal framework to develop this field.

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- Design experimental configurations to perform **beam tests** of some of these techniques (mainly at the CERN PS).

Conclusions and outlook

- Non-linear effects can be used efficiently to manipulate the transverse beam emittances and distributions.
- Hamiltonian and adiabatic theories are the ideal framework to develop this field.
- Design experimental configurations to perform **beam tests** of some of these techniques (mainly at the CERN PS).
- **More beam manipulations are expected to be studied in the near future.**

Thank you for your attention!!!

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