



| The European Synchrotron

Injection and extraction

JUAS 2023

Nicola Carmignani, Thomas Perron, Laurent Farvacque

January 2023

Introduction

Injection

- On-axis single turn injection

- Multi-turn injection – without damping

- Multi-turn injection – with damping

- RF capture, swap out and injection perturbations

Extraction

- Fast extraction

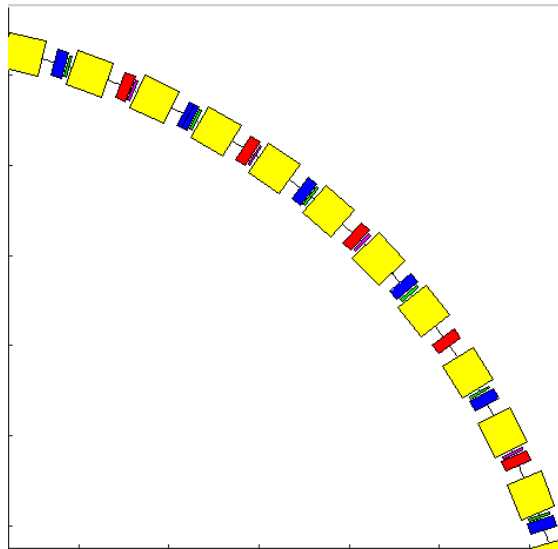
- Slow extraction

Magnetic elements for injection and extraction

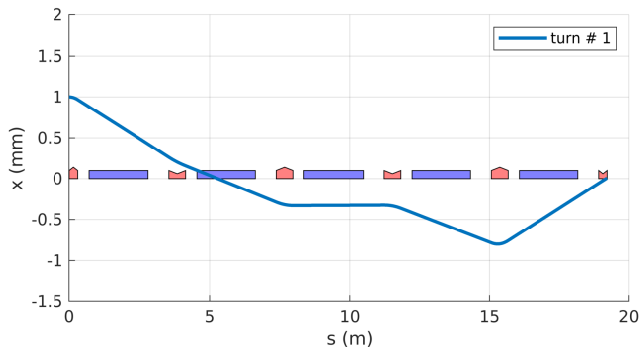
Examples / exercise

We use a simple accelerator lattice to remember what the phase space is.

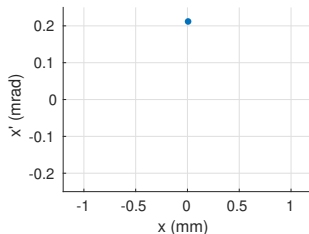
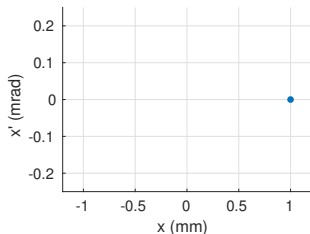
The **FODO** lattice is a series of focusing (**F**) and defocusing (**D**) quadrupoles with dipoles, sextupoles and drift spaces in between.



Phase space in an accelerator

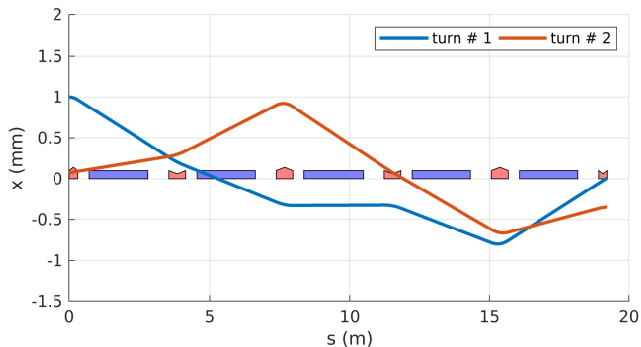


Tracking of a particle in a FODO arc.

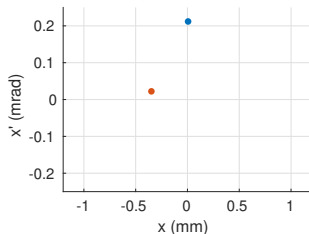
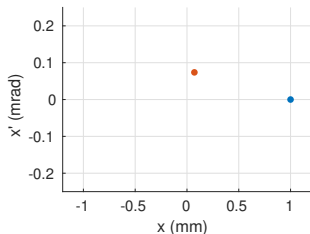


Phase space plots at a QF and a QD.

Phase space in an accelerator

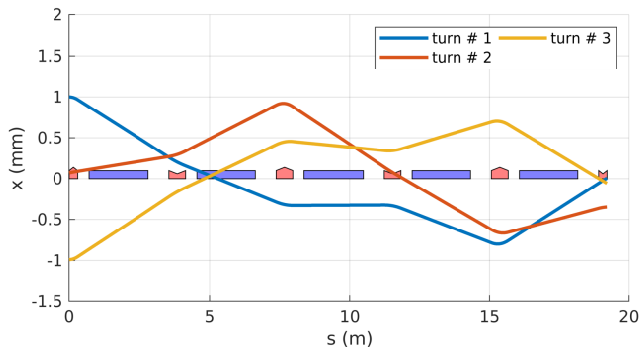


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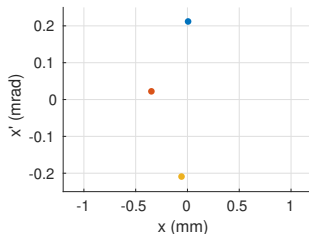
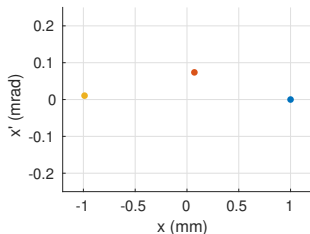


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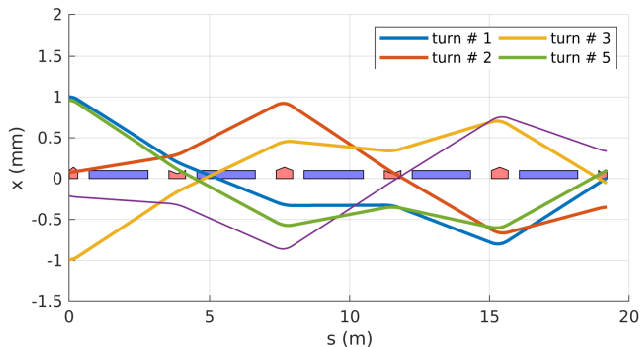


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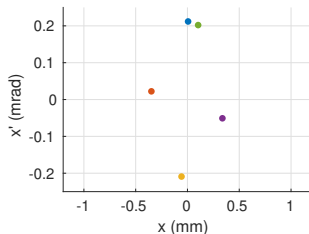
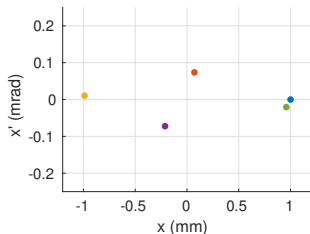


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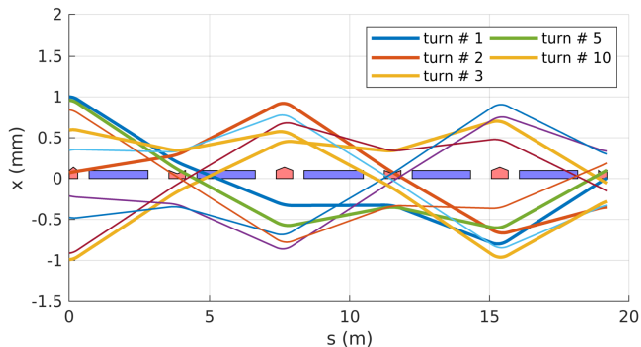


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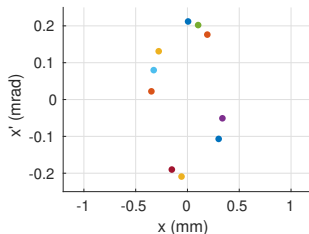
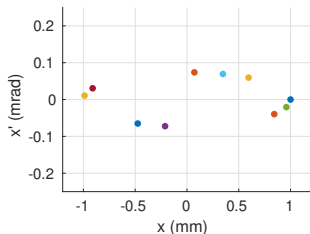


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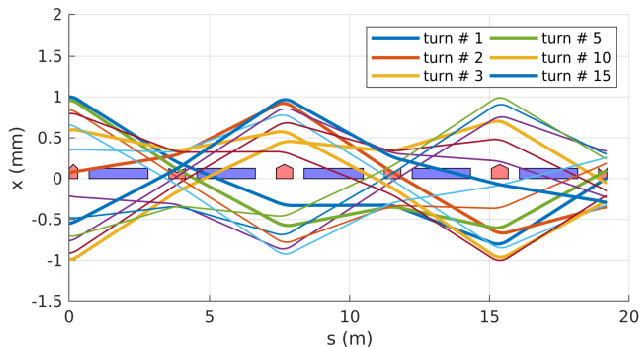


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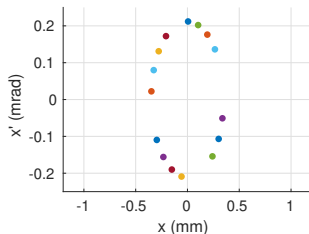
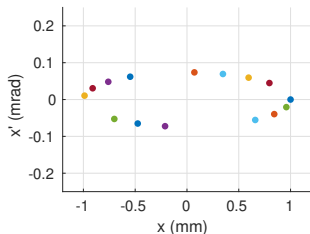


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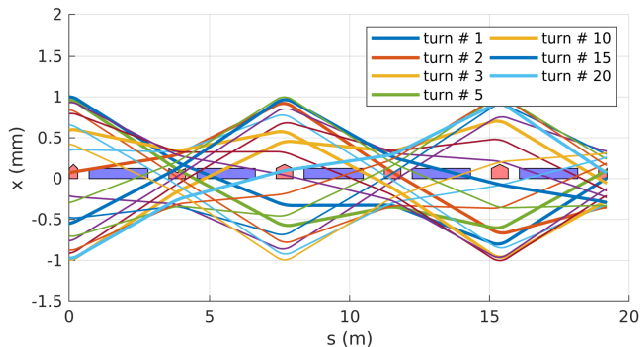


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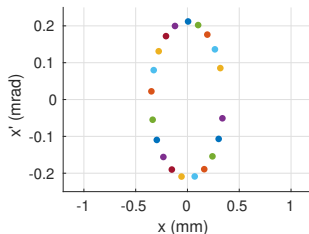
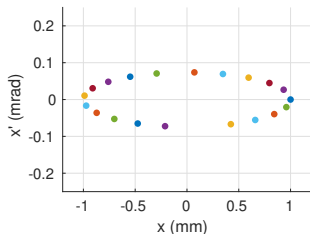


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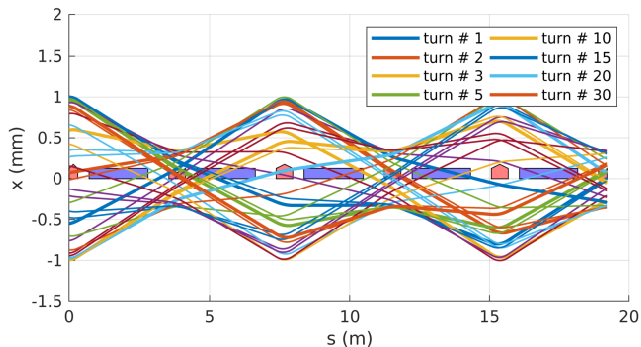


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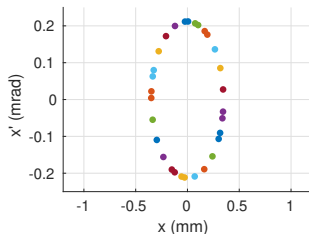
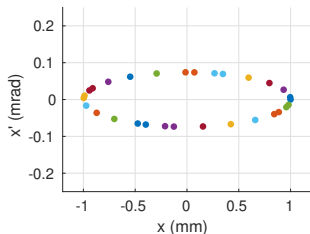


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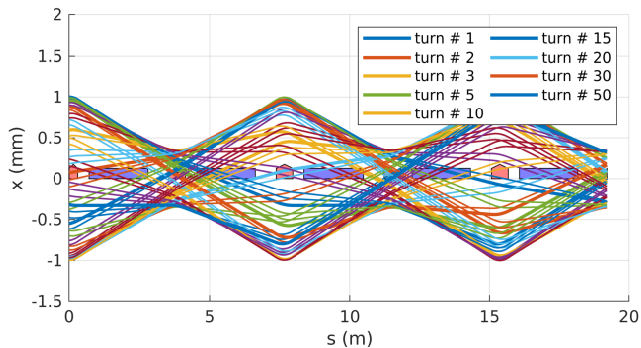


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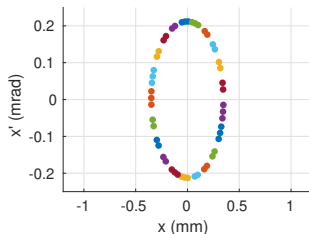
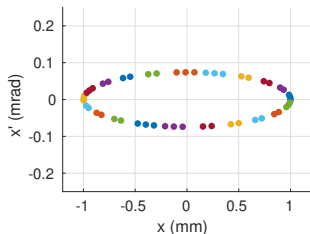


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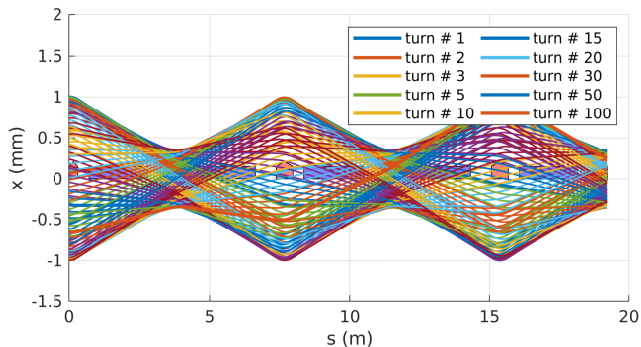


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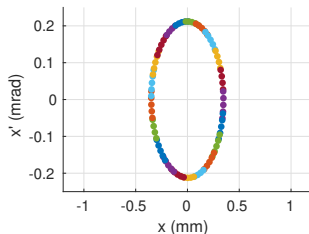
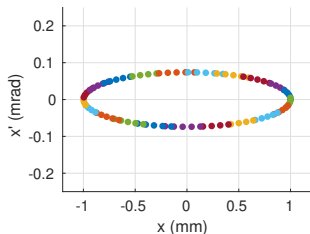


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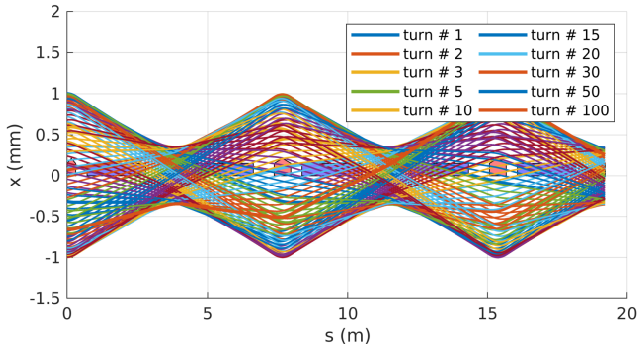


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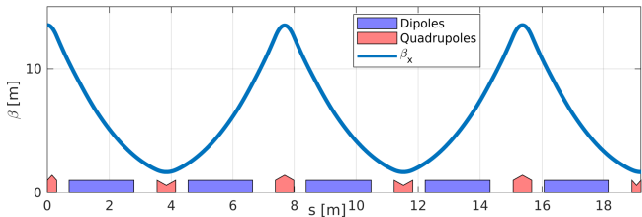


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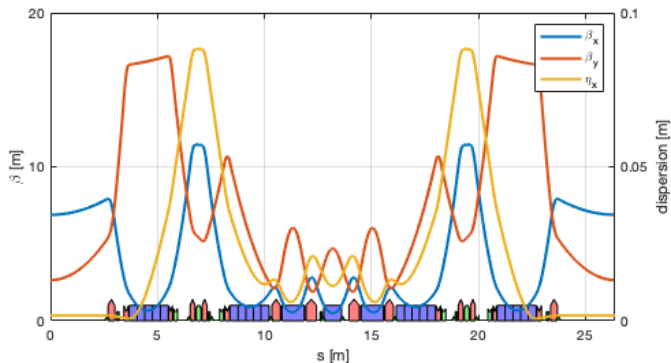
β functions



The envelope of the particle trajectories is proportional to the square root of β function.



Horizontal β function.



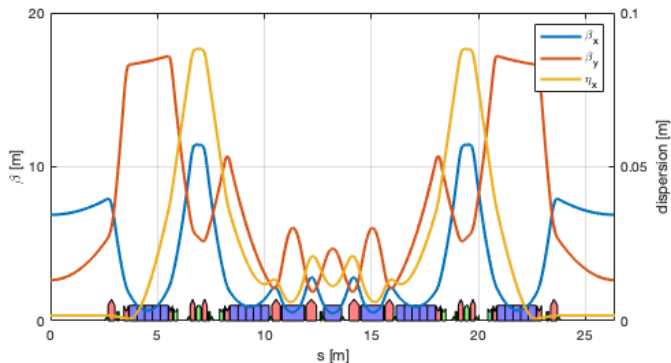
β functions of an ESRF storage ring cell.

From the β functions we can define the two other Twiss functions α and γ .

$$\alpha = -\frac{\beta'}{2} \quad \gamma = \frac{1 + \alpha^2}{\beta}$$

where $\beta' = \frac{d\beta}{ds}$

β functions



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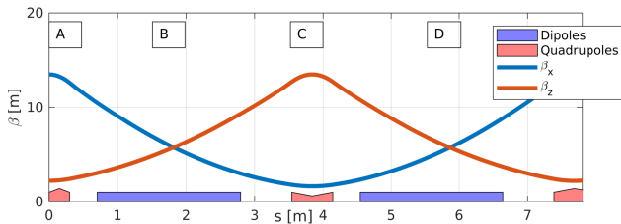
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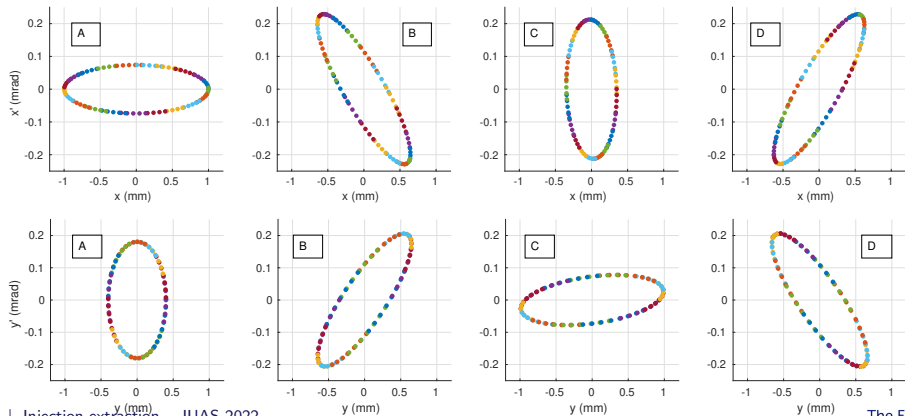
where $\beta' = \frac{d\beta}{ds}$

Be careful! β and γ here are not the relativistic β and γ !

β functions



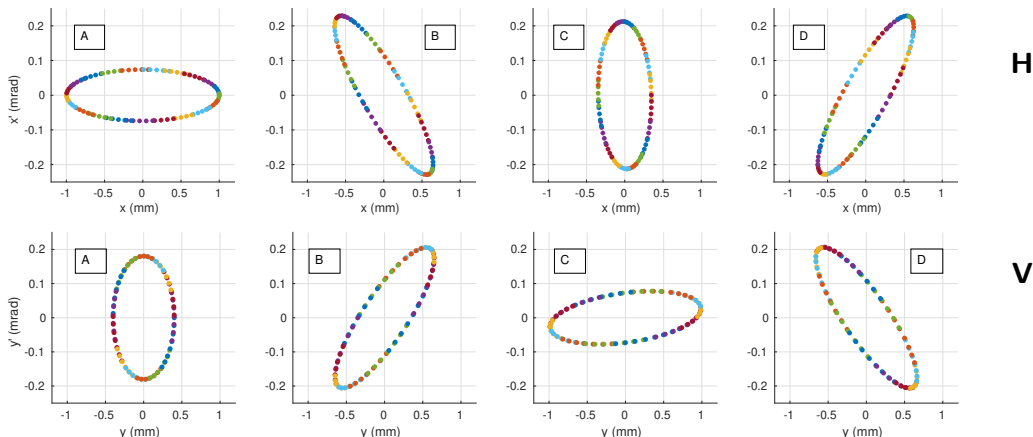
The shape of the ellipses changes along the ring, but the area is constant.



H

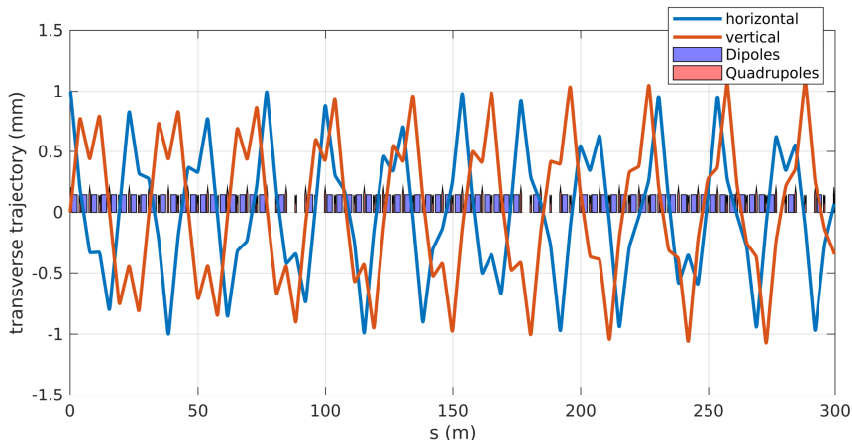
V

β functions



When there are many particles, each one rotates in a different ellipse and the area of the “average ellipse” is the emittance (ϵ). The emittance unit of measure is m·rad.

- Position **A**: large horizontal beam size and small vertical
- Position **C**: large vertical beam size and small horizontal
- Position **B**: $\alpha_x > 0$ and $\alpha_y < 0$



The tune (Q) is the number of transverse (betatron) oscillations that a particle performs in a turn of the machine.

Tunes of the ESRF booster:

$$Q_x = 11.75$$

$$Q_y = 9.65$$

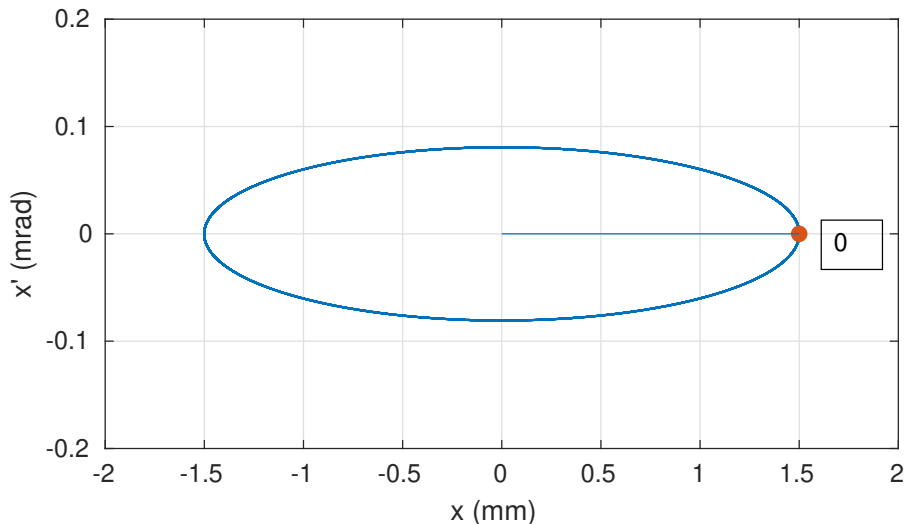
Tunes of the ESRF SR:

$$Q_x = 76.21$$

$$Q_y = 27.34$$

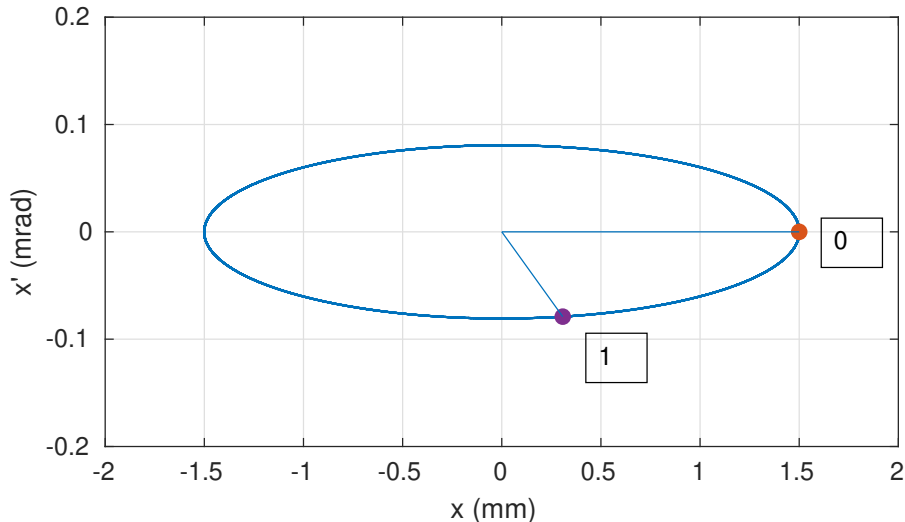
The fractional part of Q is the fraction of full turn in the phase space ellipse that a particle covers in one turn of the accelerator.

In this case, the fractional part of the tune is 0.21 (Q_x of the ESRF storage ring).



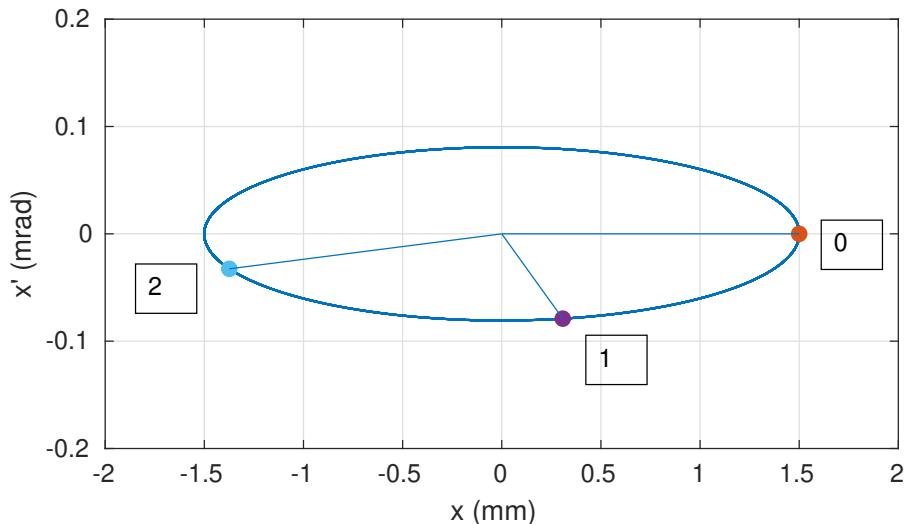
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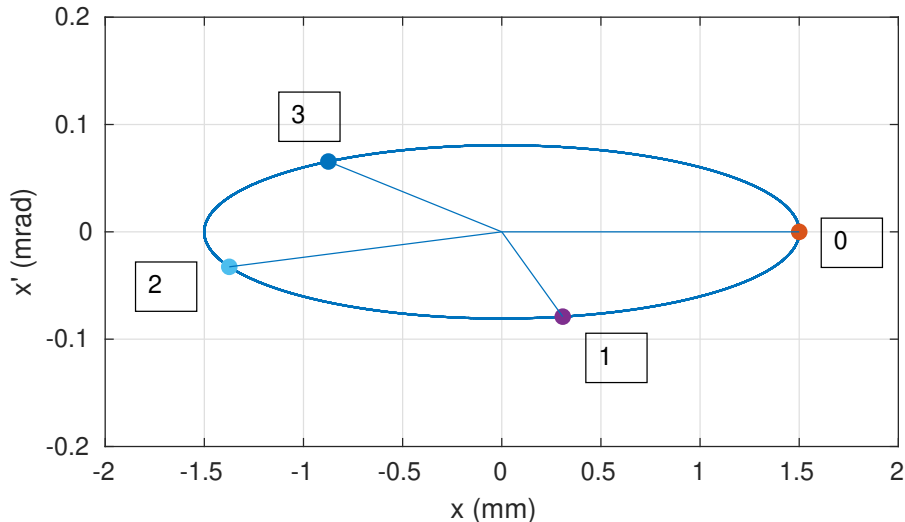
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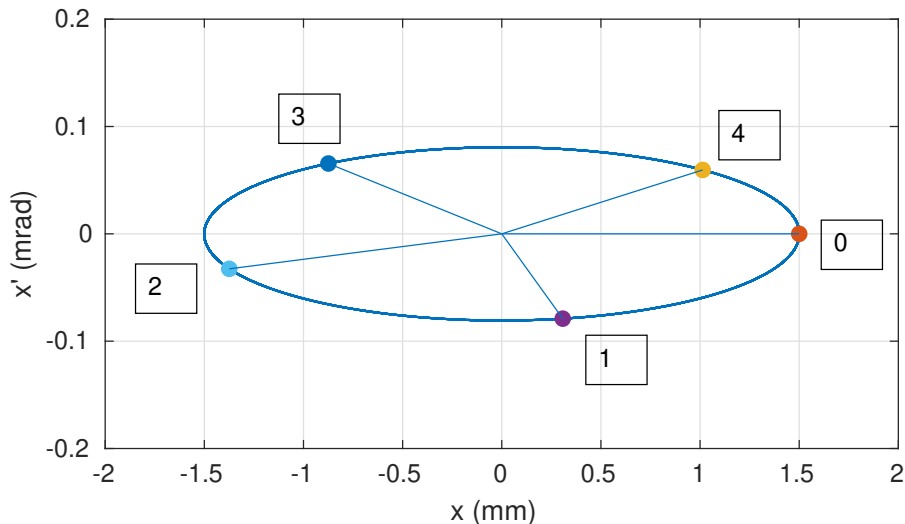
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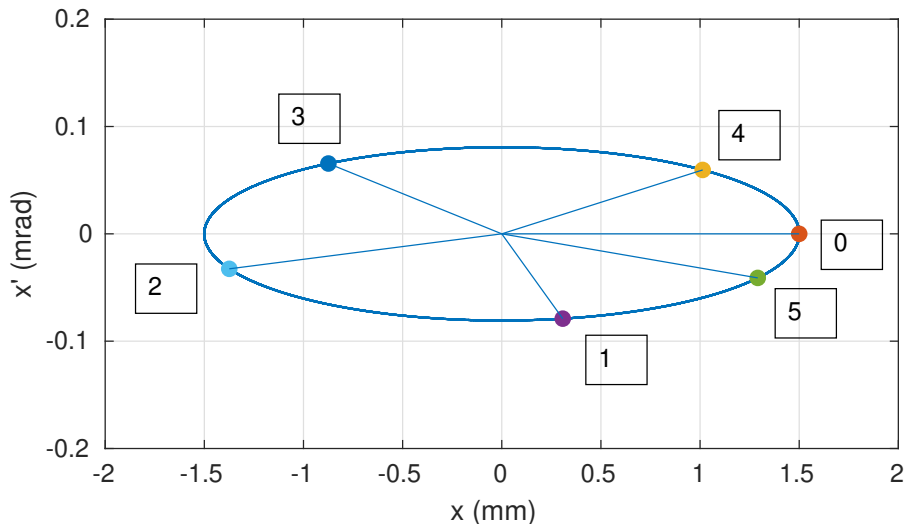
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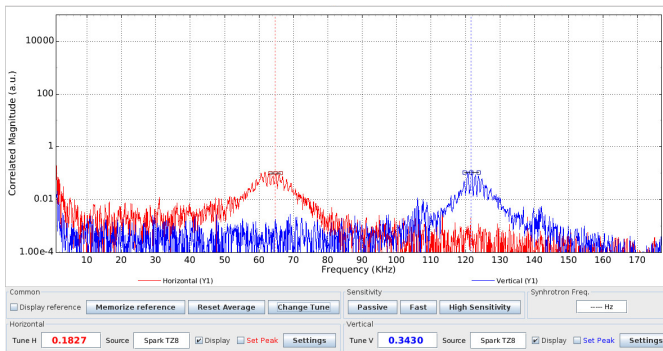


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We can perform an FFT analysis of the beam position measured at each turn to get the betatron tunes.



Betatron tunes of the ESRF storage ring:

$$Q_x = 76.18$$

$$Q_y = 27.34$$

The frequency obtained from the FFT is the fractional part of Q times the revolution frequency ($f_0 = 355$ kHz).

$$f_{Q_x} = .18 \cdot f_0 \simeq 64 \text{ kHz}$$

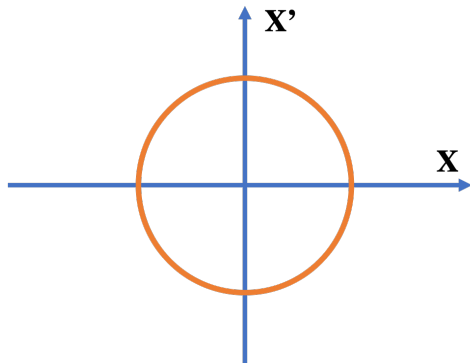
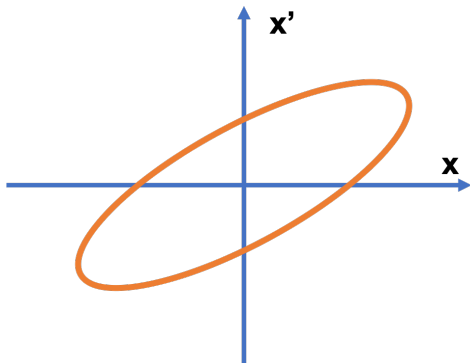
$$f_{Q_y} = .34 \cdot f_0 \simeq 121 \text{ kHz}$$

Normalized coordinates

Phase space ellipses can be transformed in circles with the Courant Snyder transformations:

$$\begin{cases} X = \frac{x}{\sqrt{\beta}} \\ X' = x' \sqrt{\beta} + \alpha \frac{x}{\sqrt{\beta}} \end{cases}$$

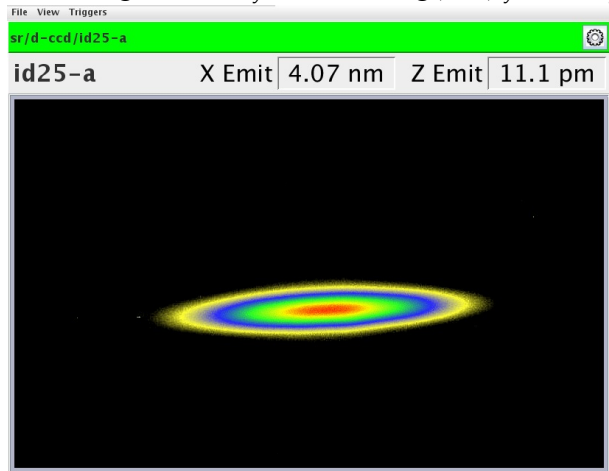
$$\begin{cases} x = X \sqrt{\beta} \\ x' = \frac{X'}{\sqrt{\beta}} - \alpha \frac{X}{\sqrt{\beta}} \end{cases}$$



Emittance and beam size

Beam sizes (σ_x and σ_y) depend on the β functions and dispersion and on the emittances (ε_x and ε_y) and energy spread (σ_δ) β functions and dispersion change along the ring, while emittance is constant.

Measuring σ_x and σ_y and knowing β_x , β_y , D_x , D_y , σ_δ we can extract the emittances.



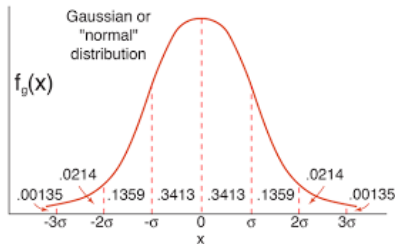
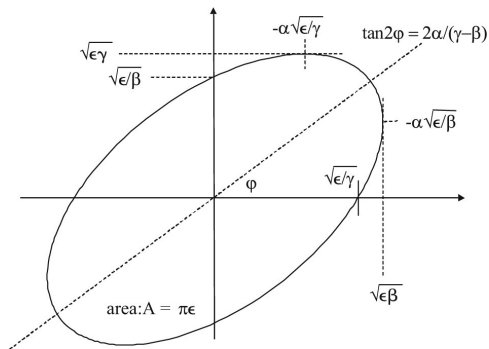
$$\sigma_x = \sqrt{\beta_x \varepsilon_x + D_x^2 \sigma_\delta^2}$$

$$\sigma_y = \sqrt{\beta_y \varepsilon_y + D_y^2 \sigma_\delta^2}$$

A particle beam is often reasonably well described by a two dimensional Gaussian distribution in phase space.

The curves of constant phase space density are then ellipses, with equation:

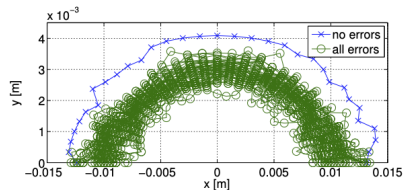
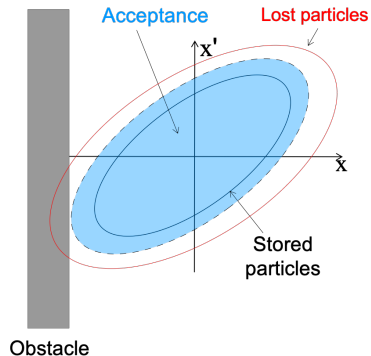
$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon$$



The acceptance is the phase space area defined by the largest possible oscillation amplitude: if a particle is outside the acceptance it will be lost.

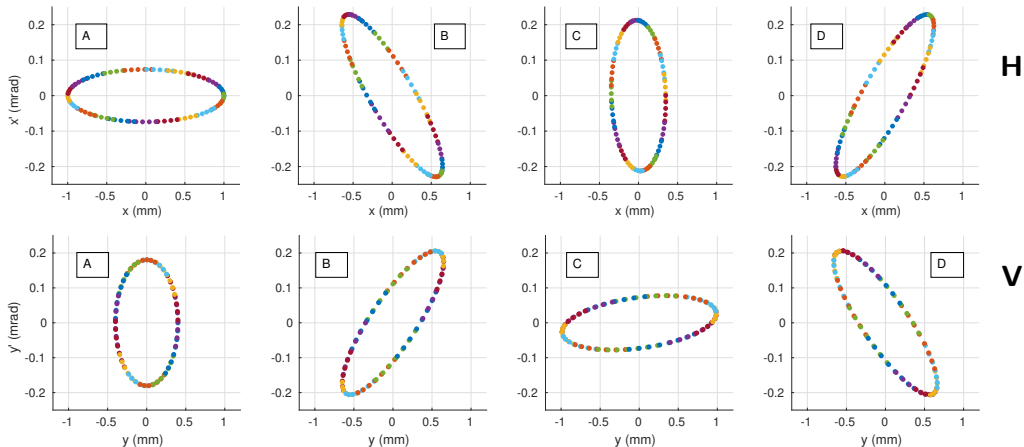
The acceptance can be limited by two factors:

- Physical apertures, defined by the most limiting aperture in the machine
- Dynamic aperture, defined by the nonlinear dynamics



Liouville theorem

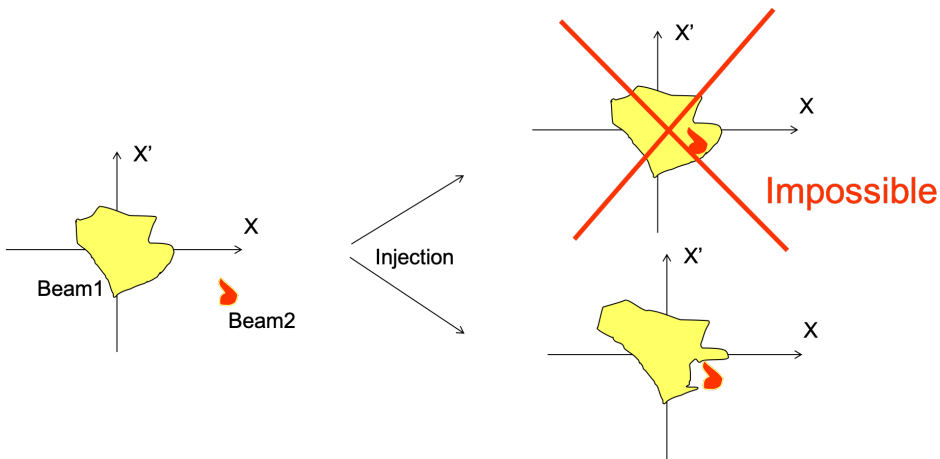
If there is no damping and no space charge effects, beam volume in 6D phase space is constant.



The area of the 2D phase space ellipses are constant along the ring.

Liouville theorem

If we want to inject a bunch on top of an already circulating bunch we can't have a superposition of the new bunch with the old bunch, because the total volume has to be unchanged.



No superposition is possible in the injection process.

Introduction

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- Multi-turn injection – without damping

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- Slow extraction

Magnetic elements for injection and extraction

Examples / exercise

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Off-axis injections are very different in case we have an electron/positron accelerator or an accelerator with heavy particles. The reason is that with electron there is the radiation damping.

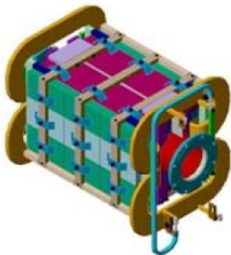
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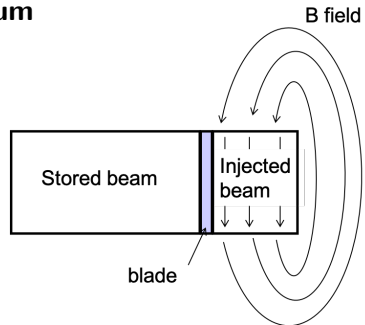
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In the majority of cases, injection is performed in the horizontal plane.

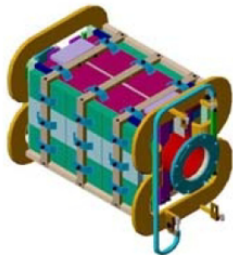
Kicker



Septum



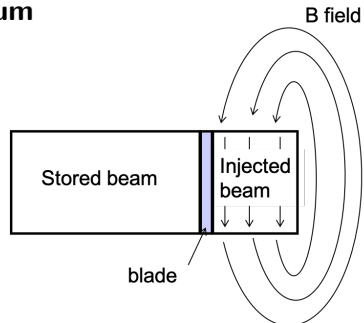
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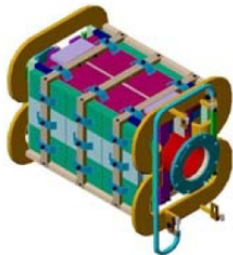
Fast dipoles with homogeneous field over the vacuum chamber cross section.

Pulsed magnets with fast raising and falling times required to control the timing of interaction with the beam.

Septum



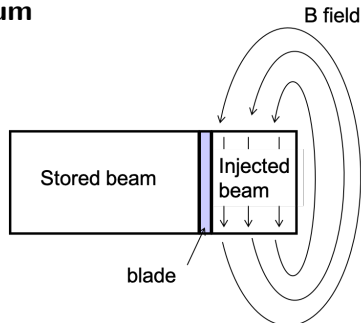
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Fast dipoles with homogeneous field over the vacuum chamber cross section.

Pulsed magnets with fast raising and falling times required to control the timing of interaction with the beam.

Septum



Dipole on one side of the blade, no field on the other side. The blade is screening the field but is an obstacle to the stored beam. Usually pulsed magnets to reduce average power.

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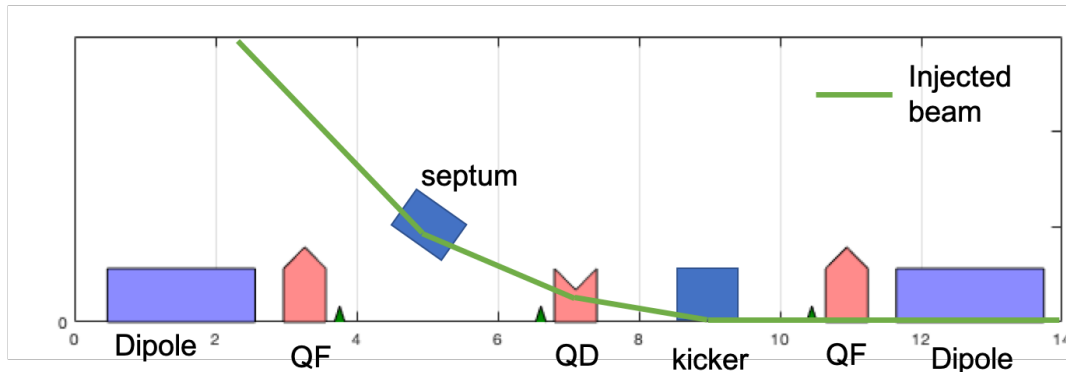
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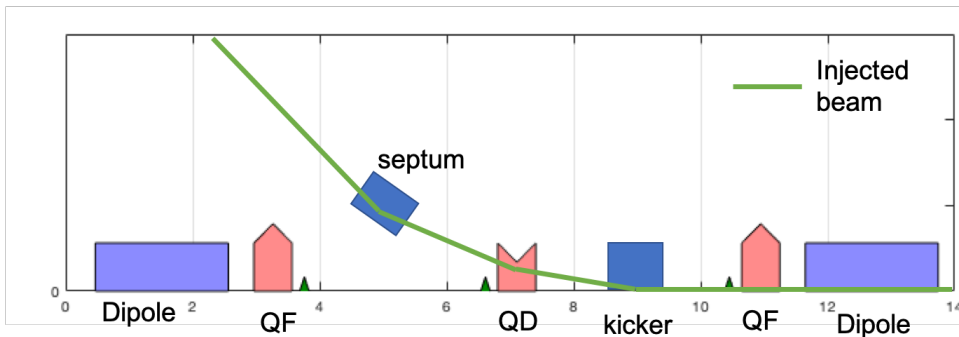
Magnetic elements for injection and extraction

Examples / exercise

On-axis single turn injection

- The beam is injected in the central orbit using electromagnetic elements (usually a septum magnet and a kicker)
- As the injected beam is put on the ring axis at the exit of the kicker, this is called “on-axis injection”.
- If some beam was still stored at the injection time, the kicker would kick it out.





The septum and the kicker should not act on the injected beam when it comes back after one turn, otherwise it would be thrown out of the acceptance of the accelerator. This requires that:

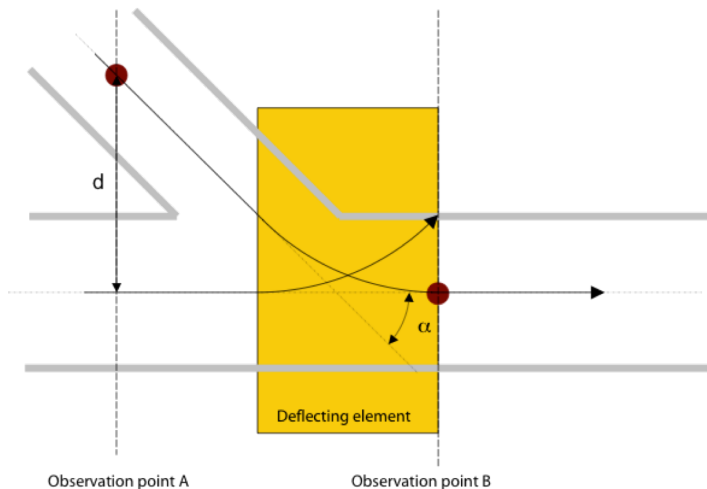
- The stray field of the septum unit is at an acceptable level
- The kicker field is reduced to zero in a time that is short compared to the revolution period.

On-axis injection, real space

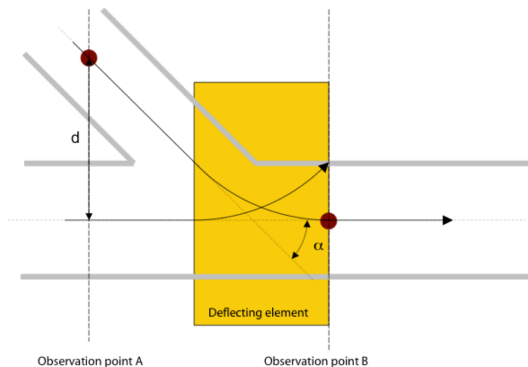
The yellow element is the kicker magnet.

The injected beam is deflected by an angle α .

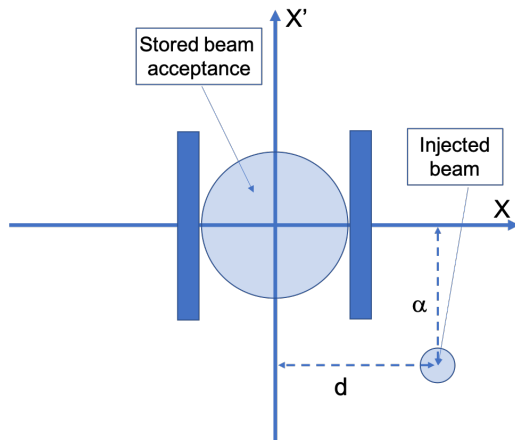
If some beam was stored at injection time, it would also be deflected by the same angle α and if would be sent to the beam pipe.



On-axis injection, phase space – normalised coordinates

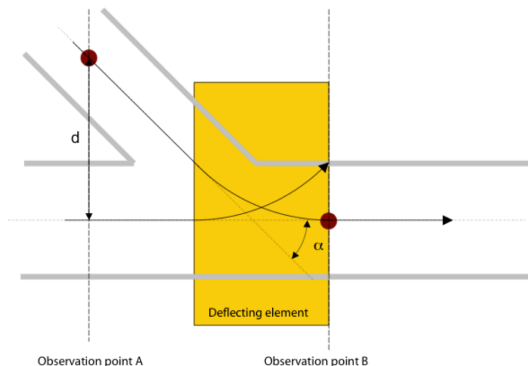


Real space

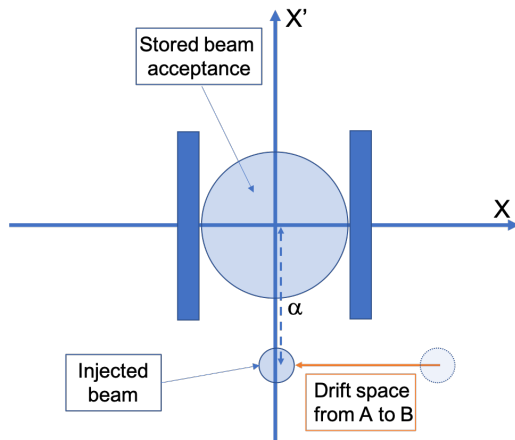


Phase space (in normalised coordinates)
Observation point A

On-axis injection, phase space – normalised coordinates

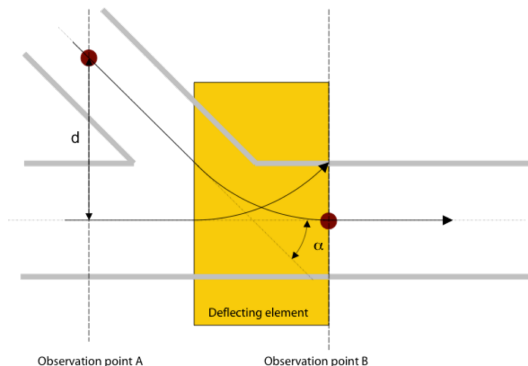


Real space

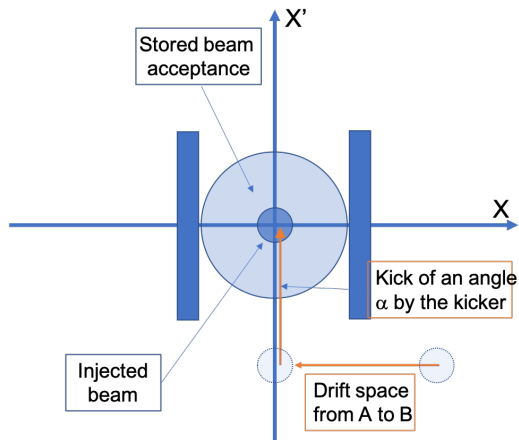


Phase space (in normalised coordinates)
At kicker, before kick

On-axis injection, phase space – normalised coordinates



Real space

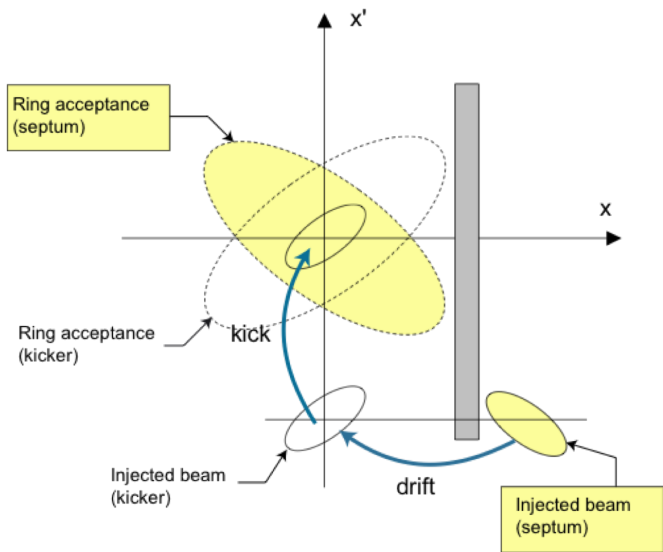


Phase space (in normalised coordinates)
Observation point B

Phase space in real coordinates

Drift septum to kicker:
injected beam moves in X ,
acceptance changes shape.

Kick:
injected beam moves in X' .



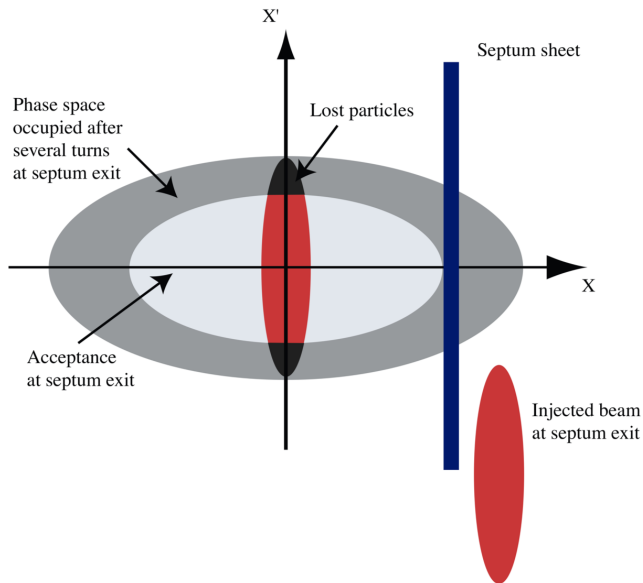
- A requirement for the injected beam is that it is matched at the entry point to the ring. This means that, at the exit of the septum unit, the betatron and dispersion functions

$$\beta_x, \alpha_x, \beta_y, \alpha_y, D_x, D'_x, D_y, D'_y$$

must be identical with the machine lattice parameters at that point.

- This is primordial when the emittance of the injected beam is comparable to the acceptance of the ring.
- This matching is performed using the dipoles and quadrupoles of the upstream transfer line.

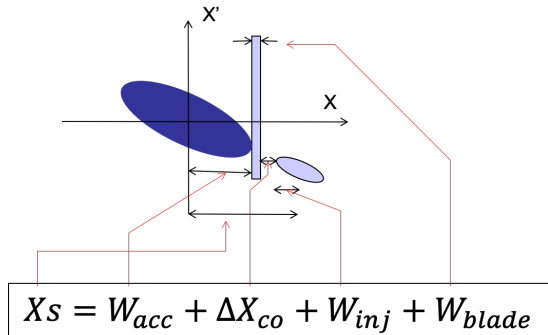
On-axis injection – example of bad matching



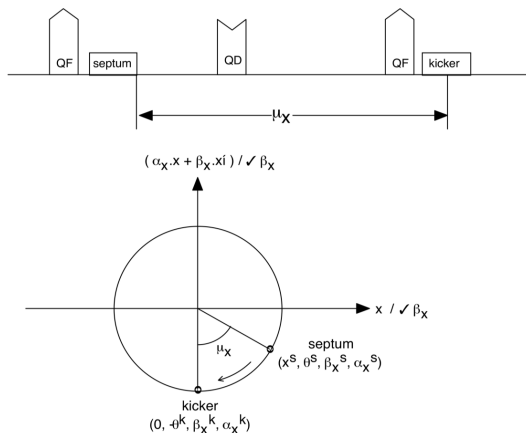
Even if the phase space area of the injected beam is less than the accelerator acceptance, if the beam is not matched we could lose a fraction of it.

Usually the injection septum is the main limitation of the horizontal acceptance (Minimum kick angle to be provided by the fast kicker => septum as close as possible to the machine axis). The center of the injected beam is at a position X_s :

- W_{acc} distance between machine axis and septum blade.
- ΔX_{co} margin for orbit errors or defects of the septum blade.
- W_{inj} half width of injected beam.
- W_{blade} septum blade width.



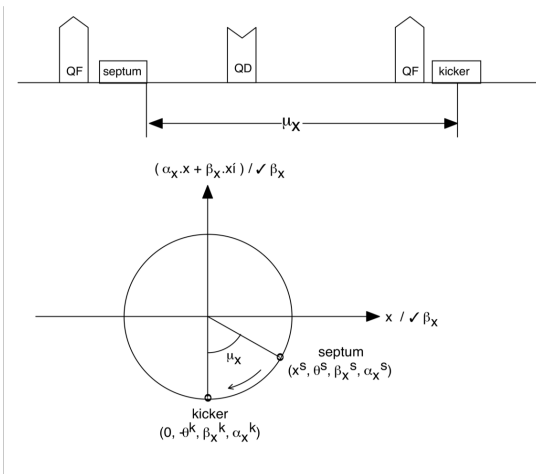
Injection parameters: exercise



In a standard on-axis injection scheme, we would like to know what is the needed kicker angle θ_k in order to optimise the parameters to minimise the kicker strength.

The distance between the injected beam and the accelerator axis at the septum position x_s is the one we derived in the previous slide.

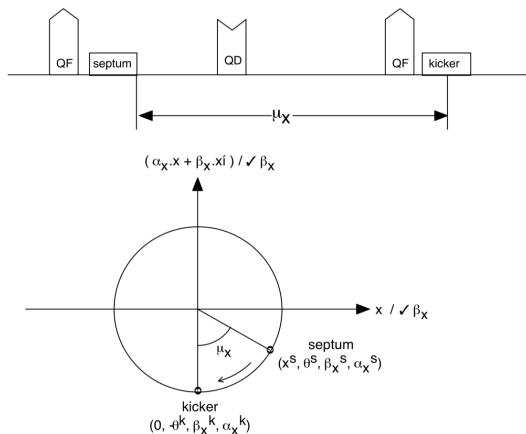
Injection parameters: exercise



In a standard on-axis injection scheme, we would like to know what is the needed kicker angle θ_k in order to optimise the parameters to minimise the kicker strength.

The distance between the injected beam and the accelerator axis at the septum position x_s is the one we derived in the previous slide.

Let's compute θ_k step-by-step.



μ is the betatron phase advance between the exit of the septum and the kicker. The transfer matrix from septum to kicker, in normalised coordinates, is a rotation matrix:

$$\begin{pmatrix} X_k \\ X'_k \end{pmatrix} = \begin{pmatrix} \cos \mu & \sin \mu \\ -\sin \mu & \cos \mu \end{pmatrix} \times \begin{pmatrix} X_s \\ X'_s \end{pmatrix}$$

x_s derived in previous slide

$X_k = 0$ because on-axis injection

We want to find x'_k , that is the required kicker strength, to minimise it. x'_s is the angle at septum exit.

$$\begin{pmatrix} X_k \\ X'_k \end{pmatrix} = \begin{pmatrix} \cos \mu & \sin \mu \\ -\sin \mu & \cos \mu \end{pmatrix} \times \begin{pmatrix} X_s \\ X'_s \end{pmatrix}$$

Transfer matrix septum to kicker

$$X_k = 0 = X_s \cos \mu + X'_s \sin \mu$$

$$X'_s = -X_s \frac{\cos \mu}{\sin \mu}$$

First line of transfer matrix

$$\begin{pmatrix} X_k \\ X'_k \end{pmatrix} = \begin{pmatrix} \cos \mu & \sin \mu \\ -\sin \mu & \cos \mu \end{pmatrix} \times \begin{pmatrix} X_s \\ X'_s \end{pmatrix}$$

Transfer matrix septum to kicker

$$X'_s = -X_s \frac{\cos \mu}{\sin \mu}$$

From first line

$$X'_k = -X_s \sin \mu + X'_s \cos \mu$$

$$X'_k = -X_s \sin \mu - X_s \frac{\cos^2 \mu}{\sin \mu}$$

Second line of transfer matrix

$$X'_k = -\frac{X_s}{\sin \mu}$$

Now we go back to normal coordinates using the Courant Snyder transformations:

$$\begin{cases} X = \frac{x}{\sqrt{\beta}} \\ X' = x' \sqrt{\beta} + \alpha \frac{x}{\sqrt{\beta}} \end{cases} \quad \begin{cases} x = X \sqrt{\beta} \\ x' = \frac{X'}{\sqrt{\beta}} - \alpha \frac{X}{\sqrt{\beta}} \end{cases}$$

The beam angle at the exit of the septum, from X'_s to x'_s :

$$X'_s = -X_s \frac{\cos \mu}{\sin \mu}$$
$$x'_s = -\frac{\left(\alpha_s + \frac{\cos \mu}{\sin \mu} \right) X_s}{\beta_s}$$

where α_s and β_s are the Twiss functions at the septum.

And the kick angle that the kicker must provide to the beam to bring it back to the ring orbit:

$$X'_k = -\frac{X_s}{\sin \mu}$$

$$\theta_k = -x'_k = \frac{x_s}{\sin \mu \sqrt{\beta_k \beta_s}}$$

where β_k and β_s are the β functions at the kicker and at the septum location.

$$\theta_k = \frac{x_s}{\sin \mu \sqrt{\beta_k \beta_s}}$$

To minimise the angle to be provided by the kicker we need to have:

- large β values at the septum and at the kicker locations;
- phase advance μ_x close to $\pi/2$.

This is the case in a standard FODO lattice where the septum and kicker are located in the vicinity of focusing quadrupoles at a distance of one cell.

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Examples / exercise

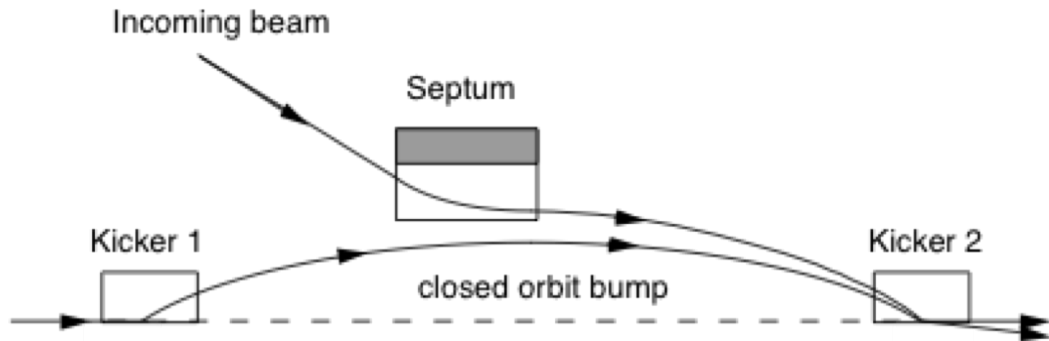
If the phase space of the accelerator is empty at the injection time, we can perform on-axis injection, but if there is already some beam we cannot!

With standard on-axis injection, we would throw out the circulating beam from the accelerator acceptance.

- Off-axis injection is needed if we have to inject in more than one turn or if there is already some beam in the accelerator, that is the typical case in a storage ring.
- Different techniques exist for off-axis injection in case of electrons/positrons and in case of proton or other heavy particles.

Off-axis injection

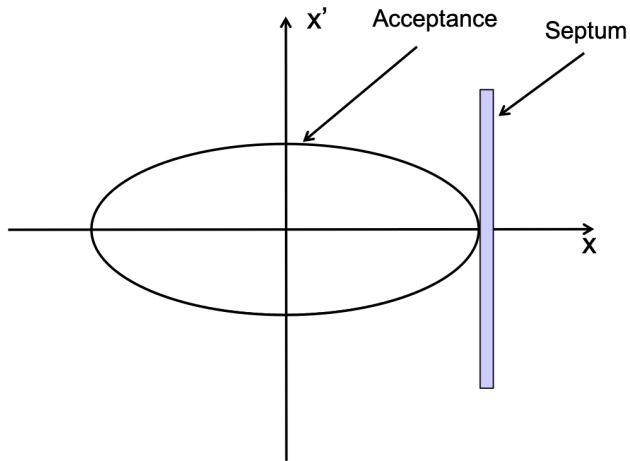
Conventional multi-turn injection uses an input septum unit associated with a programmed closed orbit bump in the vicinity of the septum. Such localised bump can be produced by two or more kicker magnets.



The role of the orbit bump is to shift the horizontal transverse acceptance of the ring towards the injected beam at the exit of the septum.

Off-axis injection – no damping

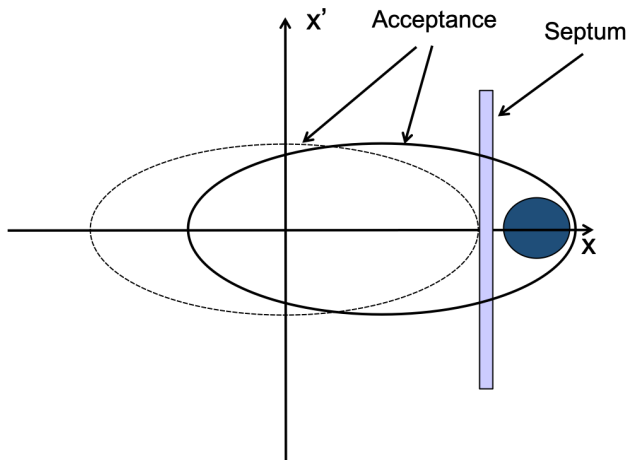
Let's study the limits for the accumulation process and first consider the dependance on the tune. In this following case, the beam is injected off-axis with a horizontal fractional tune of 0.25.



■ before injection

Off-axis injection – no damping

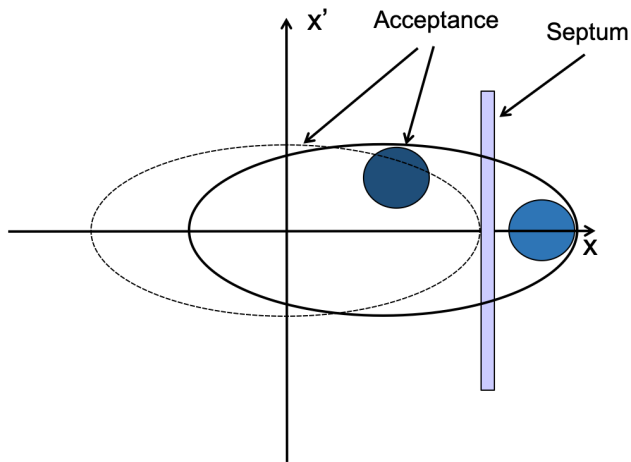
Let's study the limits for the accumulation process and first consider the dependence on the tune. In this following case, the beam is injected off-axis with a horizontal fractional tune of 0.25.



- before injection
- bump on: 1st bunch injected

Off-axis injection – no damping

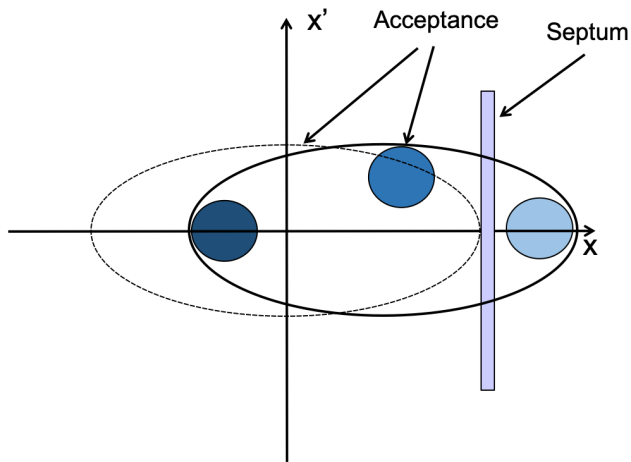
Let's study the limits for the accumulation process and first consider the dependance on the tune. In this following case, the beam is injected off-axis with a horizontal fractional tune of 0.25.



- before injection
- bump on: 1st bunch injected
- 2nd bunch injected

Off-axis injection – no damping

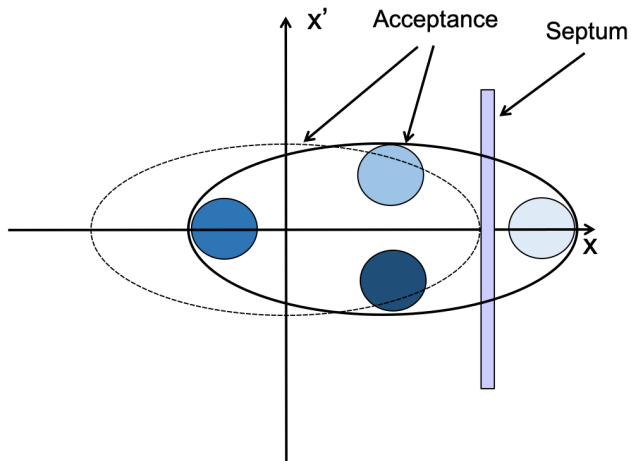
Let's study the limits for the accumulation process and first consider the dependance on the tune. In this following case, the beam is injected off-axis with a horizontal fractional tune of 0.25.



- before injection
- bump on: 1st bunch injected
- 2nd bunch injected
- 3rd bunch injected

Off-axis injection – no damping

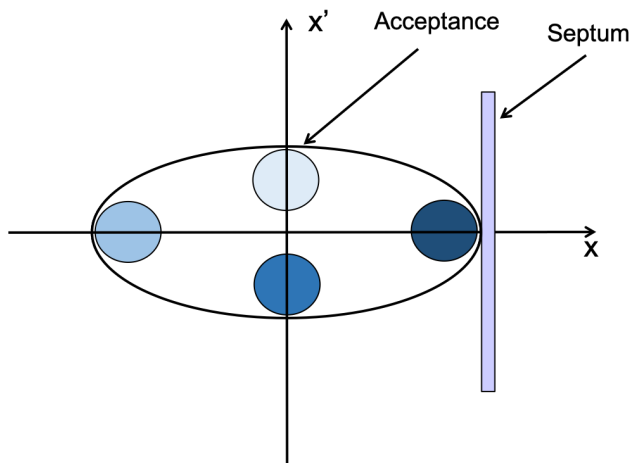
Let's study the limits for the accumulation process and first consider the dependance on the tune. In this following case, the beam is injected off-axis with a horizontal fractional tune of 0.25.



- before injection
- bump on: 1st bunch injected
- 2nd bunch injected
- 3rd bunch injected
- 4th bunch injected

Off-axis injection – no damping

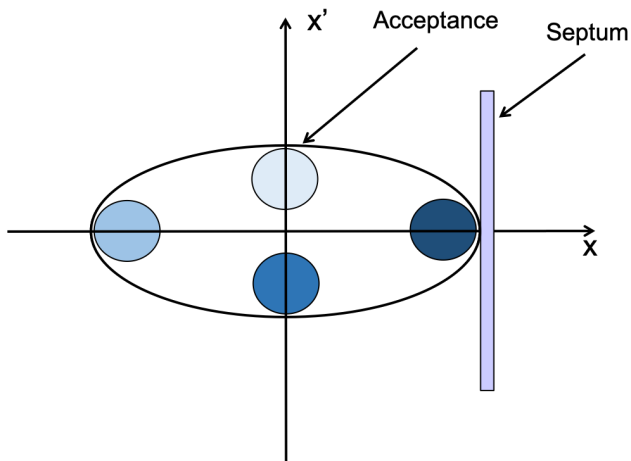
Let's study the limits for the accumulation process and first consider the dependance on the tune. In this following case, the beam is injected off-axis with a horizontal fractional tune of 0.25.



- before injection
- bump on: 1st bunch injected
- 2nd bunch injected
- 3rd bunch injected
- 4th bunch injected
- bump off

Off-axis injection – no damping

Let's study the limits for the accumulation process and first consider the dependence on the tune. In this following case, the beam is injected off-axis with a horizontal fractional tune of 0.25.



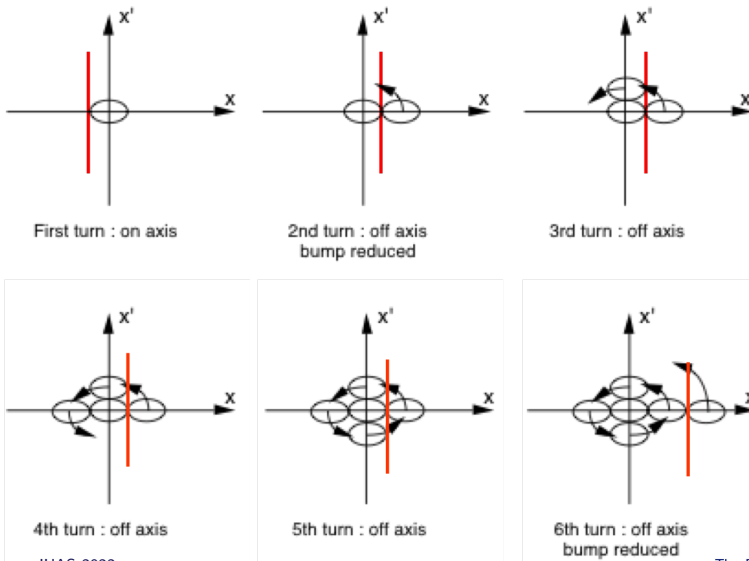
- before injection
- bump on: 1st bunch injected
- 2nd bunch injected
- 3rd bunch injected
- 4th bunch injected
- bump off

In this case, the kicker bump has to be switched off, at the latest, before the injected beam completes the fourth turn.

- In the case of the orbit bump making the closed orbit coincide with the center of the incoming beam in the septum magnet one would again find the case of an on-axis injection as considered before.
- The orbit bump can be made large enough to bring the first particles on axis, and then has to be reduced to enable stacking during the next four turns, and reduced again for the following four turns, until the full acceptance has been filled.

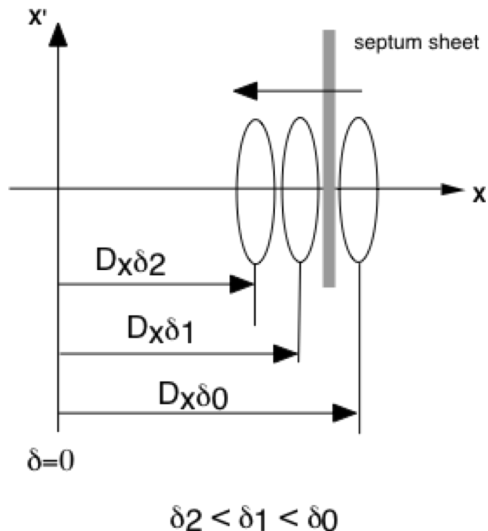
Off-axis injection – without damping

In this case we inject into the same bucket in 5 bunches. The first is a normal on-axis injection. This technique is also called phase space painting.



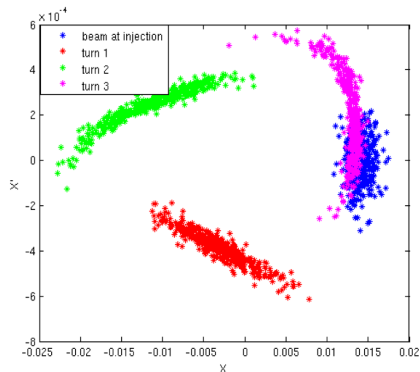
- There are others ways of preventing the stored beam from hitting the septum after one turn, such as using a large dispersion at the place of the septum and injecting the beam off-momentum.
- The injected beam will then start a betatron oscillation around the chromatic closed orbit. Then by either ramping the energy of the machine or decelerating the injected beam, the chromatic closed orbit can be moved further from the septum.

- Evolution of beam pulses which are injected at an energy δ_0 above the ring energy and then decelerated in a non dispersive section just after being injected.
- The advantage of such methods is a reduced horizontal emittance of the stored beam but with a larger energy spread.



There is some emittance dilution due to the transverse space charge effects and to tune shift with amplitude. They induce non-negligible tune shifts which result in the loss of some particles.

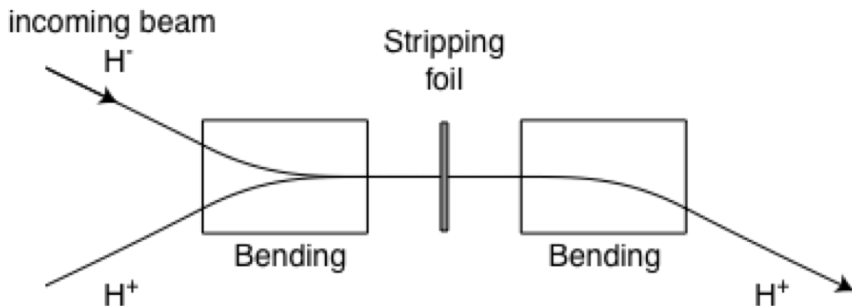
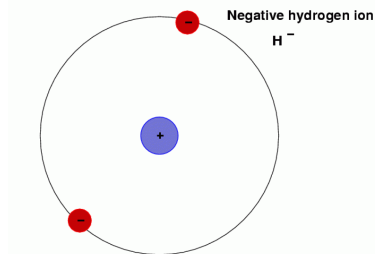
With the increasing intensity of the machines, the consequence of the beam losses has become increasingly significant and new injection schemes were investigated.

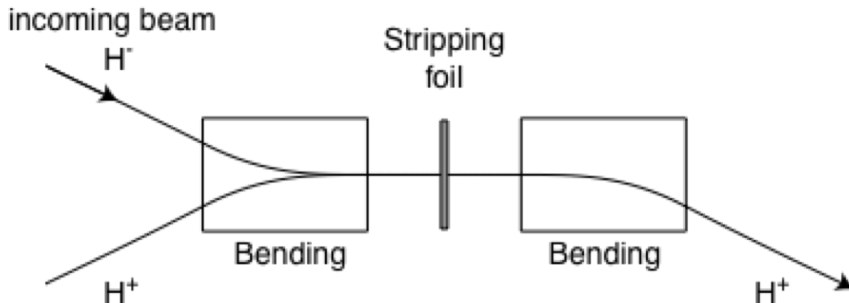


Example of phase space dilution due to tune shift in the ESRF storage ring.

Charge exchange injection

To overcome the saturation problem, and following the development of intense H^- ion sources, most of the high intensity proton rings now use the H^- charge exchange injection scheme.





The principle is to change the charge of the injected particle after it has passed the injector magnet. This is done by inserting a thin stripping foil which removes the two loosely bound electrons of the H^- .

The foil thickness is chosen to give a high stripping efficiency (98%) without introducing appreciable scattering and momentum spread (this depends on the injection energy).

- The advantage of this process is that there is no need to shift the transverse acceptance outside the physical aperture: Injection can be performed during a large number of turns with superposition in phase space.
- An orbit bump is still useful to avoid crossing the stripping foil after injection is finished.
- This method enables to fill the transverse acceptance with a uniform transverse density distribution to minimise the space charge forces.
- Such a uniform filling may be achieved by applying a “painting” of the acceptance: a vertical steering of the H⁻ in the injection line and a variation of the horizontal orbit bump in the vicinity of the stripping foil are applied simultaneously to fill the acceptance.

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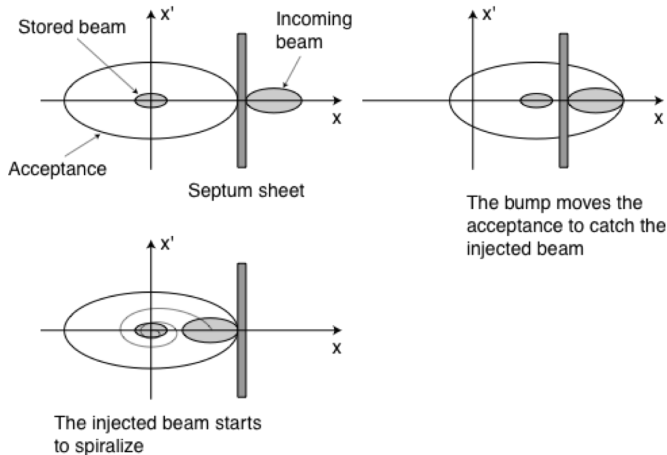
Magnetic elements for injection and extraction

Examples / exercise

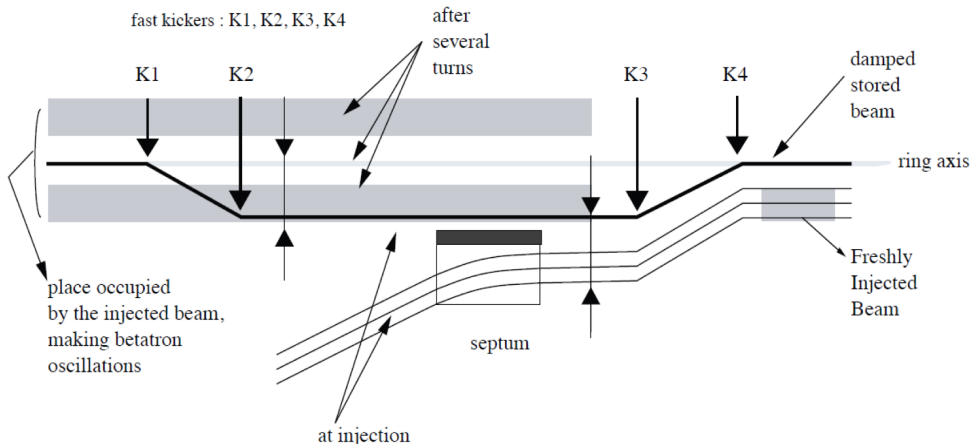
- Electrons (and positrons) are at a sufficient energy for space-charge forces to be insignificant.
- Electron circular machine are submitted to the Synchrotron radiation damping: after a few damping times the emittance is the equilibrium from radiation damping and quantum diffusion.
- A single bunch is first injected off-axis. Radiation damping effect in the ring then sends the beam in the center of the closed orbit. The injection process may then be repeated.
- The injection is not limiting the maximum intensity which can be stored on electron storage rings.

Off-axis injection – with damping

- A bump reduces the distance between stored and injected beam.
- After a few damping times, the injected electrons occupy the center of the density distribution and have freed the phase space at the outer areas of the acceptance.
- The sequence is repeated until we have a sufficient current.



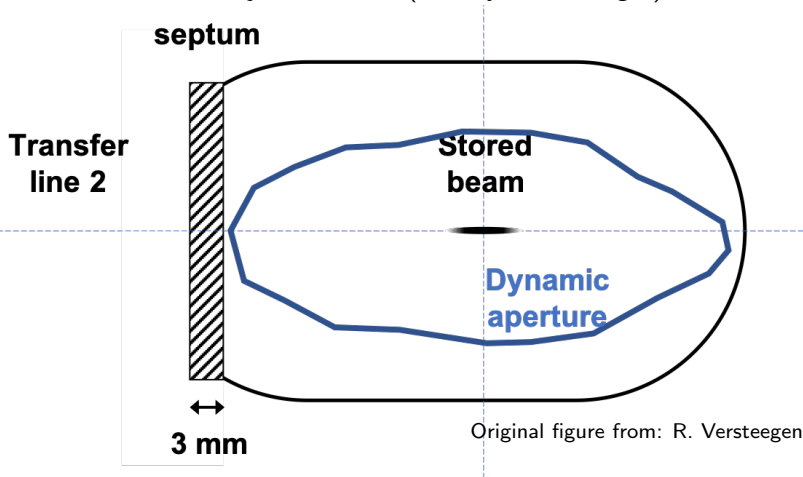
Off-axis injection – with damping



The beam after injection occupies the full chamber width up to the septum, then after a few damping times, it is reduced to the equilibrium size and the same process can be repeated again.

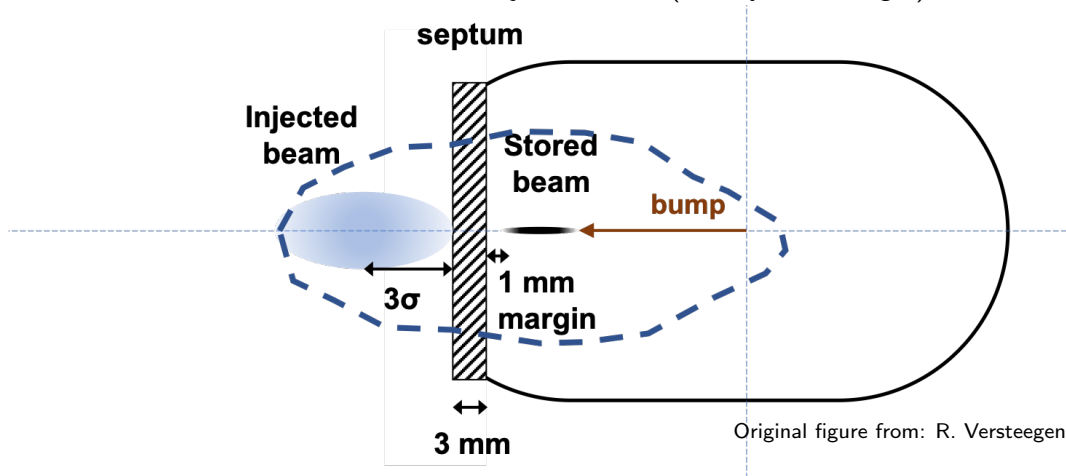
Off-axis injection – with damping

For off-axis injection, the dynamic aperture (DA) has to be sufficiently large. The DA needed for off-axis injection depends on the thickness of the septum blade and on the size of the stored beam and the injected beam (usually much larger).



Off-axis injection – with damping

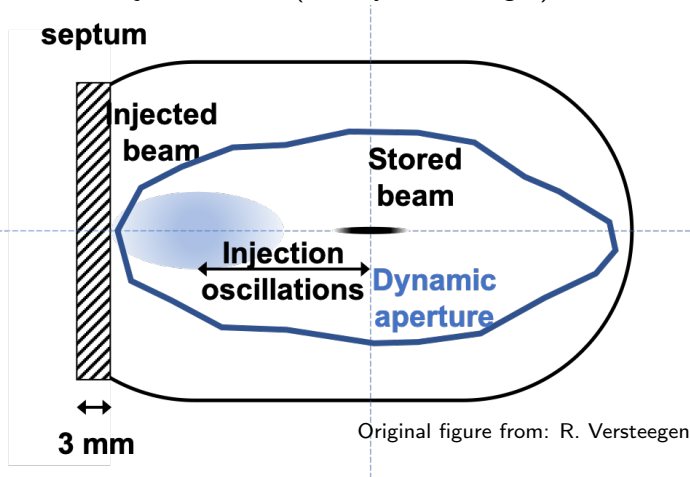
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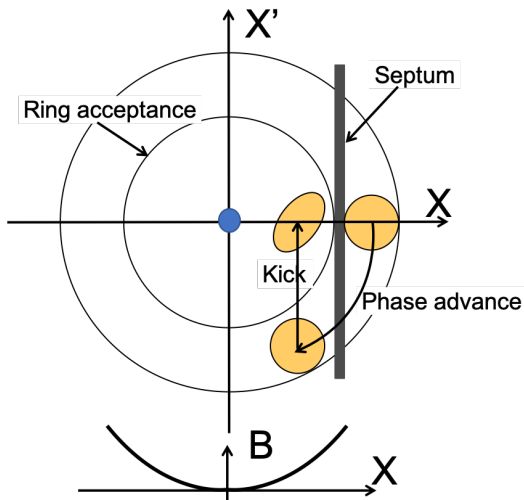


Original figure from: R. Versteegen

Off-axis injection – with damping

For off-axis injection, the dynamic aperture (DA) has to be sufficiently large. The DA needed for off-axis injection depends on the thickness of the septum blade and on the size of the stored beam and the injected beam (usually much larger).

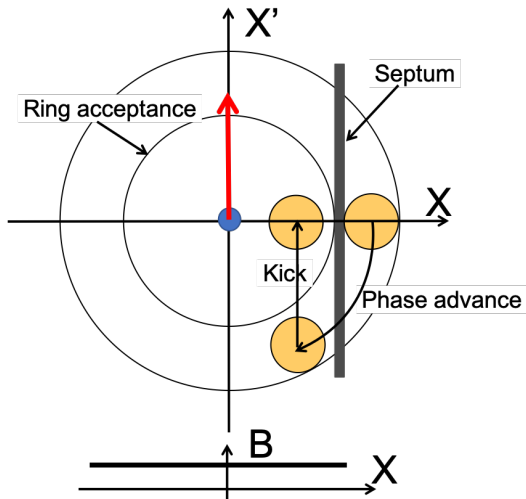




Y. Kobayashi and K. Harada, "Possibility of the beam injection using a single pulsed sextupole magnet in electron storage rings". EPAC 2006.

- the injected beam is at the septum outside the acceptance
- the beam drifts along the phase space ellipse
- the beam is kicked into the acceptance
- stored beam does not see any field

The magnetic field of the kicker must be zero at the center, in order to keep the stored beam unperturbed.

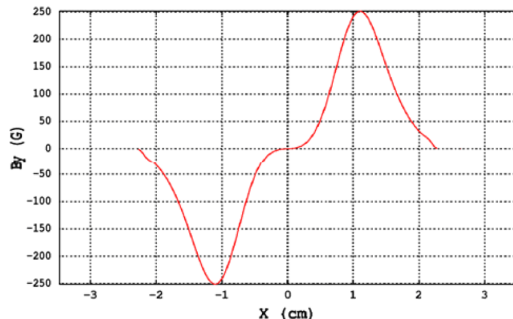


Y. Kobayashi and K. Harada, "Possibility of the beam injection using a single pulsed sextupole magnet in electron storage rings". EPAC 2006.

- the injected beam is at the septum outside the acceptance
- the beam drifts along the phase space ellipse
- the beam is kicked into the acceptance
- stored beam does not see any field

A normal kicker would send the stored beam outside of the acceptance.

The nonlinear kicker can also be a combination of different multipoles in order to have a flat field where the injected beam is.

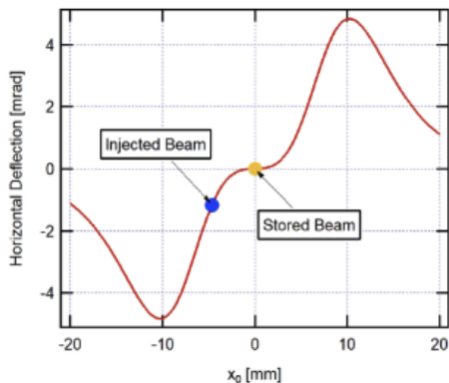


T. Atkinson, M. Dirsat, O. Dressler, P. Kuske, "Development of a non-linear kicker system to facilitate a new injection scheme for the Bessy II Storage Ring." IPAC 2011.

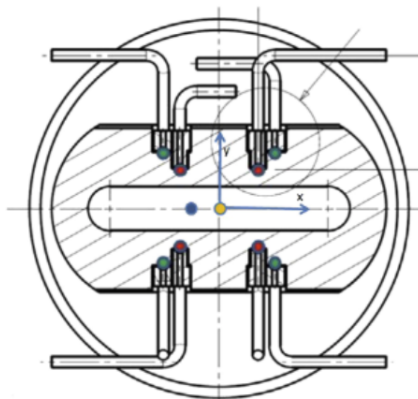
Successful tests at BESSY in 2013. Some machines injection schemes are based on this concept, mainly low energy machines.

Innovative injection schemes: nonlinear kicker

Similar solutions are considered for future storage rings, for example the upgrade of the Soleil synchrotron in Paris.



P. Alexandre, R. Ben El Fekih, A. Letrésor, S. Thorau, J. da Silva Castro, F. Bouvet, J. Breunlin, Å. Andersson, P. Fernandes Tavares, "Transparent top-up injection into a fourth-generation storage ring." Nuclear Inst. and Methods in Physics Research, A.



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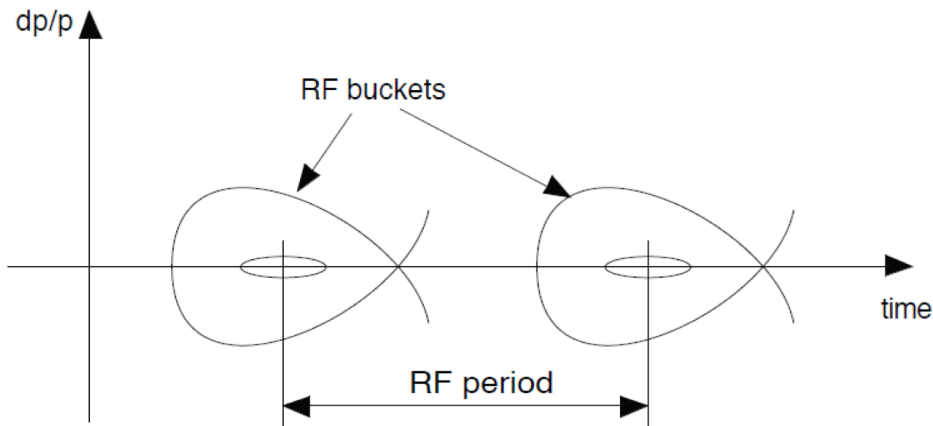
Examples / exercise

The injected beam has also to be trapped in the longitudinal acceptance of the accelerator. Several cases are to be considered depending on whether the injected beam is already bunched or not, and when the RF system of the ring is turned on:

- Bunched beam with RF on
- Coasting beam (unbunched) with RF on
- Injection without RF

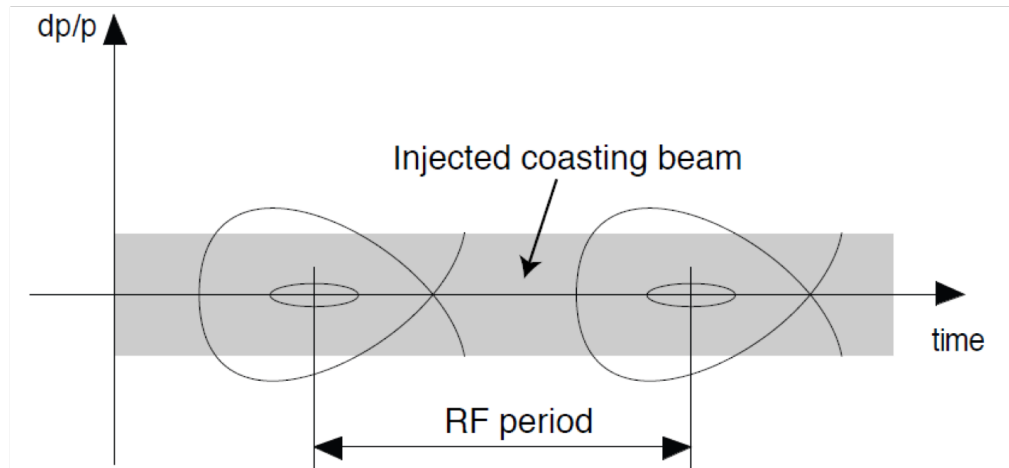
Bunched beam with RF on

The simplest case is when the frequency of the RF injector system is the same as the one of the ring. The phase between the two RF systems and the energy of the incoming beam have to be adjusted correctly so that the injected bunches fall just inside the ring RF buckets.



Coasting beam with RF on

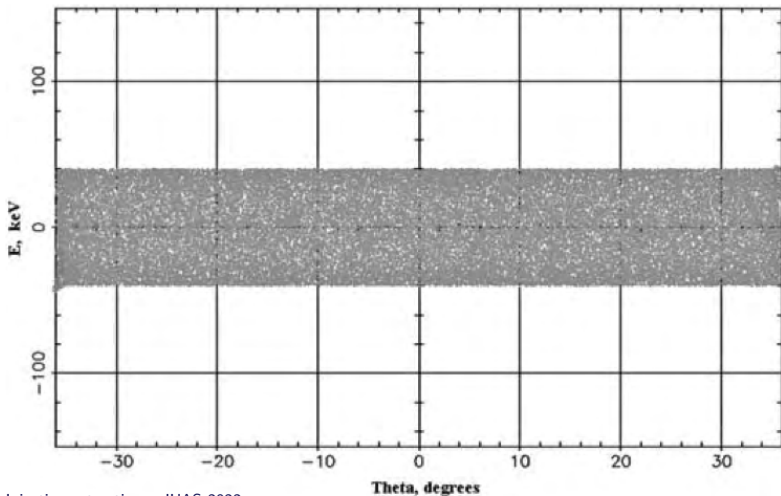
Only the particles that fall inside the RF buckets will be captured. The bucket size (RF voltage) has to be optimised in order to have the maximum phase acceptance while keeping the maximum energy deviation within the transverse energy acceptance of the machine.



Injection without RF on

Once the injection process is completed, the RF is turned on progressively in order to bunch the beam (adiabatic capture).

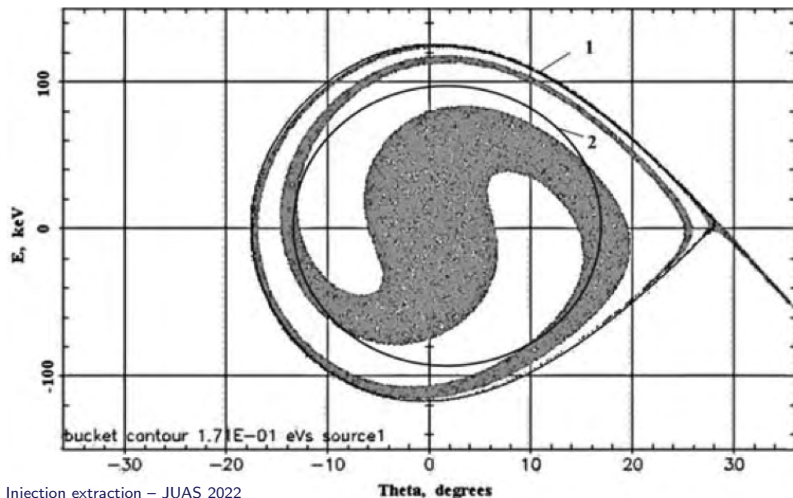
RF ramp has to be optimised to limit the losses.



Injection without RF on

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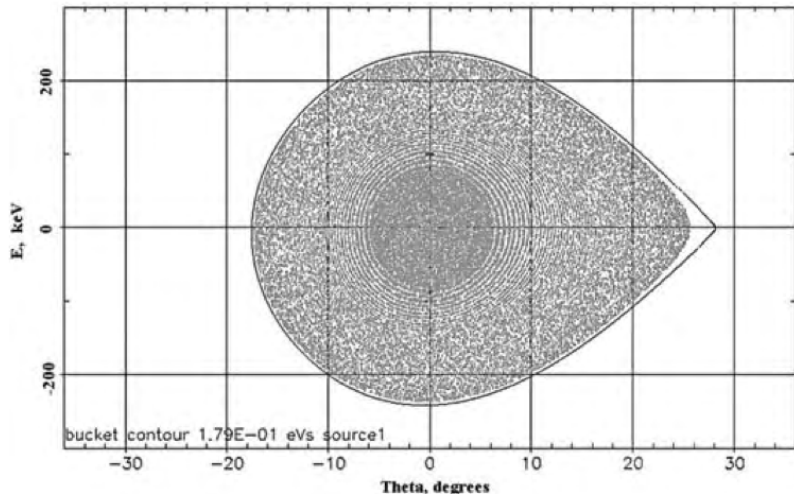
RF ramp has to be optimised to limit the losses.



Injection without RF on

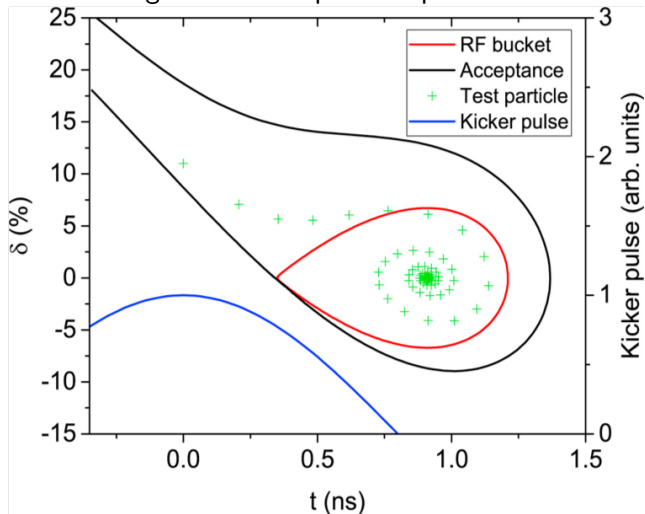
Once the injection process is completed, the RF is turned on progressively in order to bunch the beam (adiabatic capture).

RF ramp has to be optimised to limit the losses.



Longitudinal injection

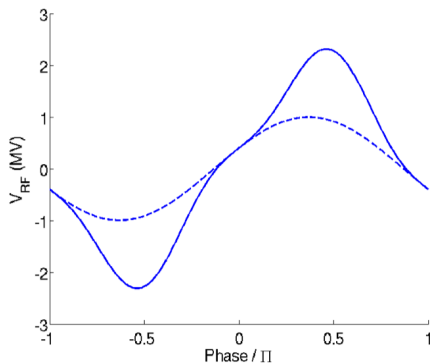
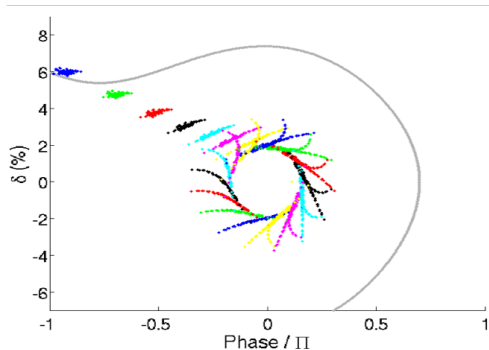
In electron machines, the shape of the longitudinal bucket is not symmetric on the energy direction. High energy particles emit more radiation than low energy ones, so there is a “golf club” shaped acceptance.



This allows to inject beam in the same bucket, on-axis, but off-phase and slightly off-energy.

The technique requires a very fast kicker ($\lesssim 1$ ns)

A variation on the same scheme is proposed at Soleil. A nonlinear longitudinal kicker that would increase the change of energy for the injected particles, keeping untouched the stored beam.

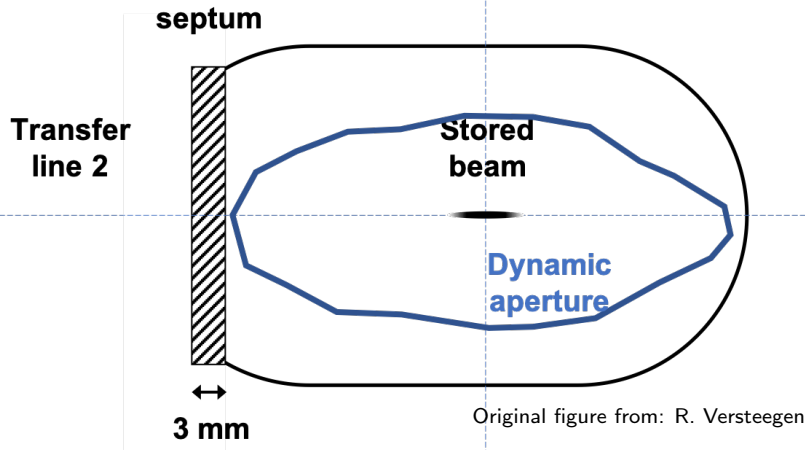


M.-A. Tordeux, Topical workshop on Injection, Berlin, August 28–30 2017

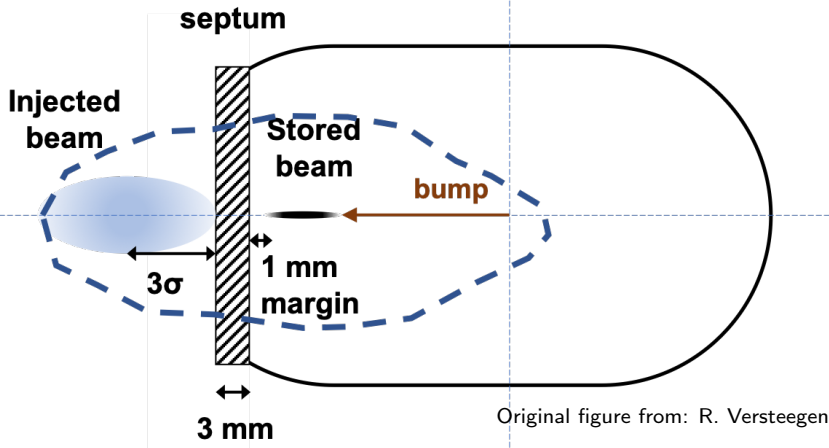
Several fourth generation synchrotron light source have a lattice with an extremely small emittance, but also with a very small transverse dynamic aperture.

In order to perform off-axis injection, at least a 4-5 mm of transverse dynamic aperture are needed.

The DA needed for off-axis injection depends on the thickness of the septum blade and on the size of the stored beam and the injected beam (usually much larger).

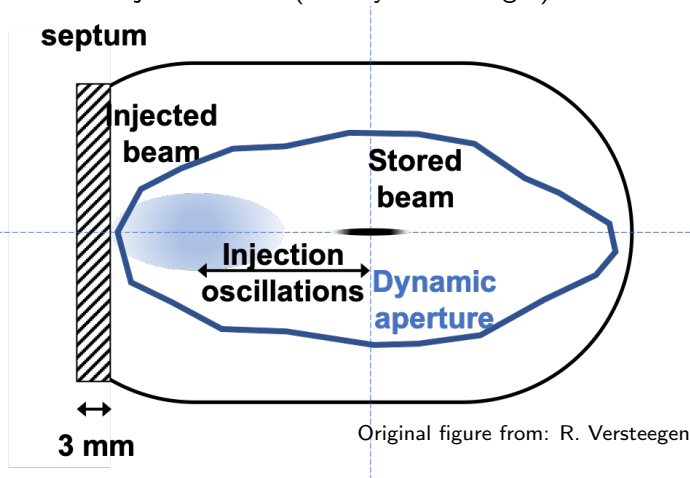


The DA needed for off-axis injection depends on the thickness of the septum blade and on the size of the stored beam and the injected beam (usually much larger).



Original figure from: R. Versteegen

The DA needed for off-axis injection depends on the thickness of the septum blade and on the size of the stored beam and the injected beam (usually much larger).



If the DA is less than 2-3 mm, off-axis injection is impossible.

A solution is to have an accumulator ring and inject on-axis a full bunch or a train of bunches from the accumulator to the SR.

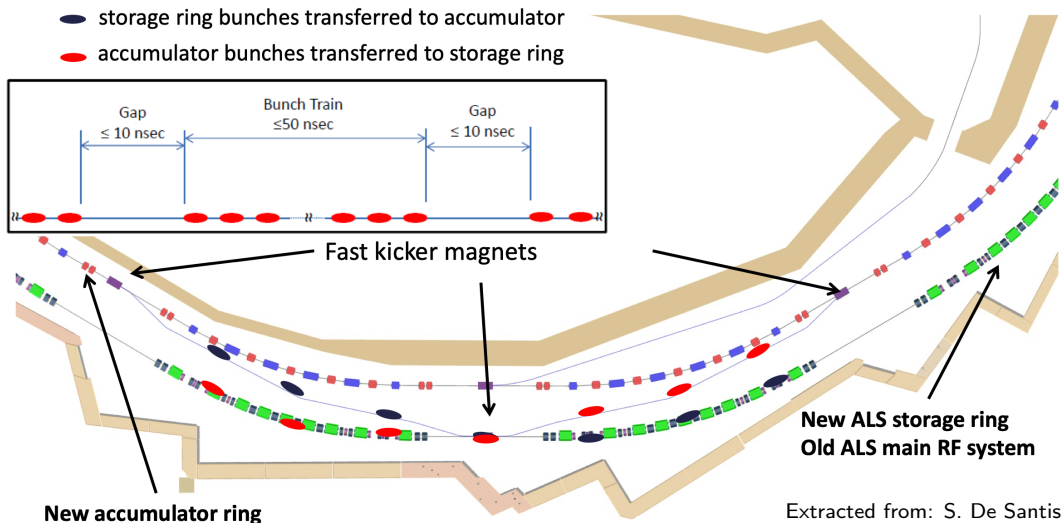
The kick used to inject on-axis would extract the stored bunch from the storage ring.

We can either decide to dump the previously stored bunch (APS-U choice) or to reinject it into the accumulator (ALS-U choice).

The bunches previously stored in the SR can be extracted from the SR and injected in the accumulator. This is ALS-U choice.

Swap-out injection

Scheme of the ALS-U injection.



Extracted from: S. De Santis
"On-Axis Swap-Out Injection At ALS-U"

Drawbacks of swap-out injections are:

- additional accumulator ring needed
- fast kickers needed
- the accumulator needs to be able to store high charge
- extraction line from SR needed
- uniform filling mode with short repetition rate are impossible

The advantage is that we can inject in a very small dynamic aperture and so we can push the design of the machine to a smaller emittance.

The last stage of a transfer line must enable the matching of the optical functions in the two transverse phase spaces: this means controlling at the exit of the septum unit the following betatron and dispersion functions

$$\beta_x, \alpha_x, \beta_y, \alpha_y, D_x, D'_x, D_y, D'_y$$

These are 8 constraints, which require 8 quadrupoles.

In most of the cases, the injector and the ring are in the same horizontal plane, which means that there are not vertical bending magnets and there is no vertical dispersion:

$$D_y = 0 \quad D'_y = 0$$

Then only 6 quadrupoles are required.

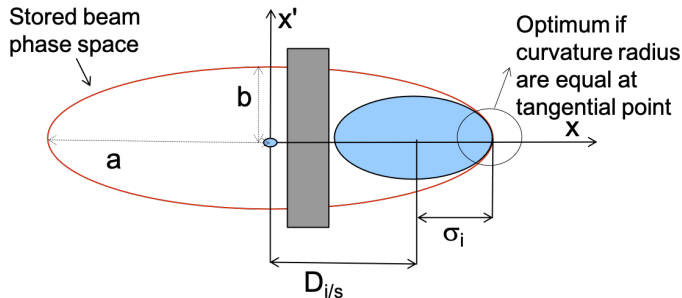
Magnets involved in injection/extraction provide a horizontal bending angle, this has to be taken into account for the matching of D_x and D'_x .

If there are several bending magnets along the transfer line it can be interesting to separate the roles of the quadrupoles which match the beta functions by placing them in a dispersion-free straight section.

The steering in the horizontal plane is generally made by fine tuning of the injection magnets, whereas two steerer magnets are required at the end of the line to adjust the vertical beam angle and position at the injection point.

Transfer line matching in off-axis injections

For off-axis injection, the phase space matching is more complicated.



Red ellipse

$$a = \sqrt{Ac\beta_s} = D_{i/s} + \sigma_i \quad b = \sqrt{\frac{Ac}{\beta_s}}$$

$$r_s = \frac{\sqrt{Ac}}{\beta_s^{3/2}} = \frac{a}{\beta_s^2} = \frac{D_{i/s} + \sigma_i}{\beta_s^2}$$

Curvature radius is:

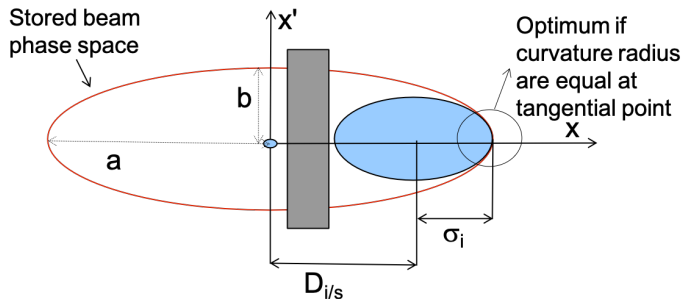
$$r = \frac{b^2}{a}$$

Blue ellipse

$$a = \sigma_i \quad b = \frac{\sigma_i}{\beta_i}$$

$$r_i = \frac{\sigma_i}{\beta_i^2}$$

Transfer line matching in off-axis injections



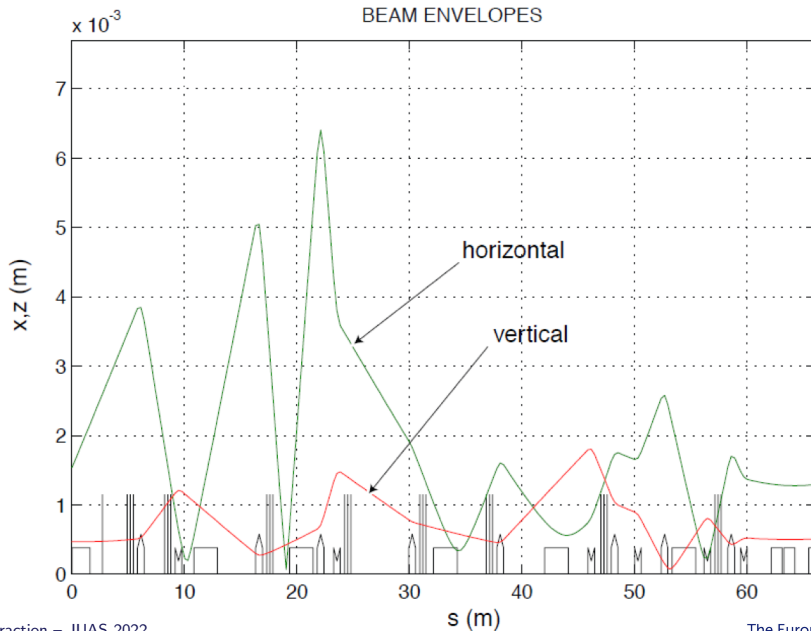
A. Streun, "SLS booster-to-ring transferline optics for optimum injection efficiency." PSI SLS-TME-TA-2002-0193. 2005.

Optimum beta function of injected beam (β_i) as a function of beta function of the stored beam (β_s).

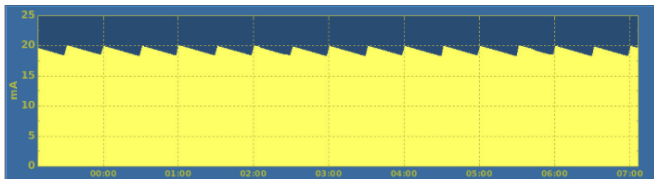
$$\beta_i^2 = \frac{\sigma_i}{D_{i/s} + \sigma_i} \beta_s^2$$

This calculation assumes a linear machine, but injection takes place at the edge of acceptance, where phase space is usually distorted by non-linearities. Final tuning of beta functions looking at injection efficiency is usually needed.

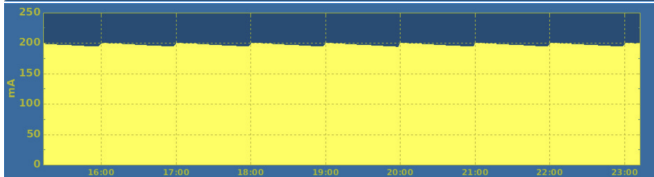
Transfer line matching in off-axis injections



In synchrotron light sources, the injections can be performed without interrupting the experiments in the beamlines. In this case we are in top-up injection mode.

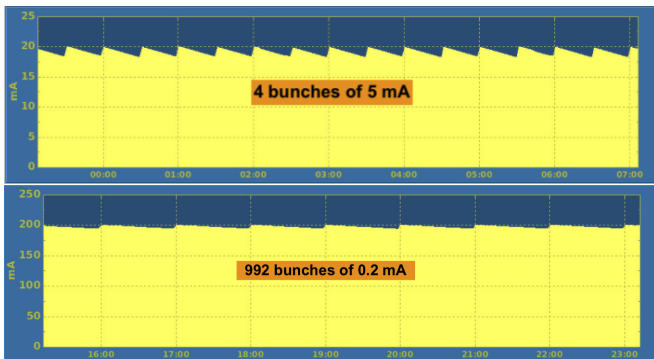


Injection every 30 minutes
with 5 h lifetime.
 $\Delta I/I \simeq 10\%$



Injection every hour with
30 h lifetime.
 $\Delta I/I \simeq 3\%$

In synchrotron light sources, the injections can be performed without interrupting the experiments in the beamlines. In this case we are in top-up injection mode.

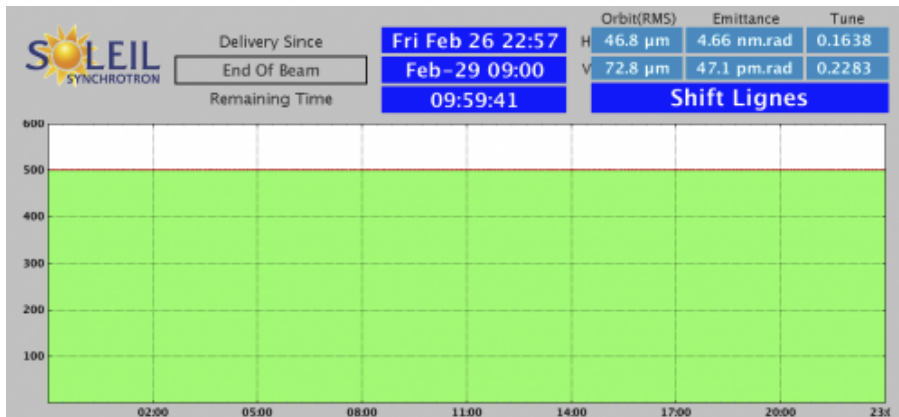


Injection every 30 minutes
with 5 h lifetime.
 $\Delta I/I \simeq 10\%$

Injection every hour with
30 h lifetime.
 $\Delta I/I \simeq 3\%$

Frequency of injections usually depends on the beam lifetime, that depends on the operation mode.

In other synchrotron light sources, the injections are much more frequent and the change of current is much smaller. This is the case of the French synchrotron light source Soleil.



At Soleil, the injections are performed every 4 minutes and the current variation is within $\pm 0.5\%$.

Most of synchrotron light sources are running in top-up mode.

There are advantages on this operation:

- higher average beam current and therefore x-ray brilliance
- lower variation of heat load on the beamline optics elements

And disadvantages:

- the injector have to be switched on and off frequently
- some beamlines have to stop the data acquisition during injections, because the injection bump introduced some small orbit perturbations

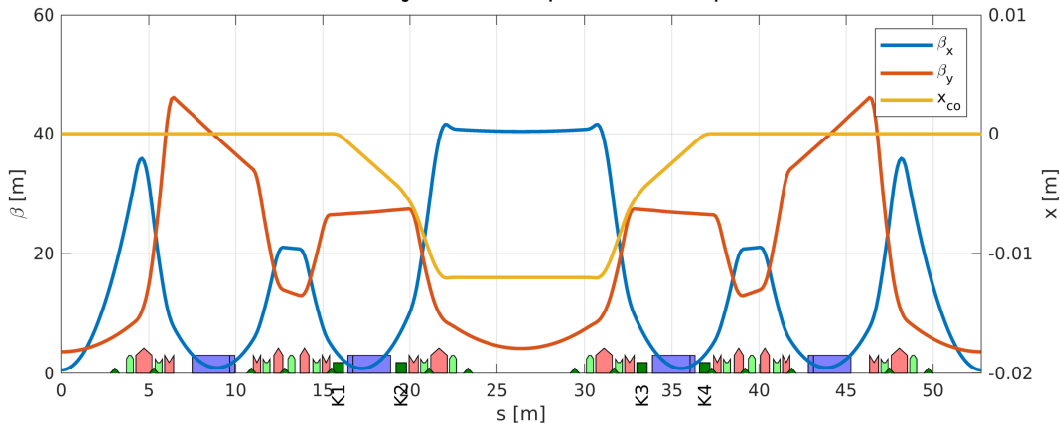
Injection perturbation is the reason why at ESRF the top-up frequency is limited.

One of the limitations for very frequent top-up injections is the injection perturbations.

This is a problem in synchrotron light sources, where a lot of experiments (about 40 at ESRF) are performed in parallel, 24 h per day, even during the injections.

Injection perturbations – old ESRF example

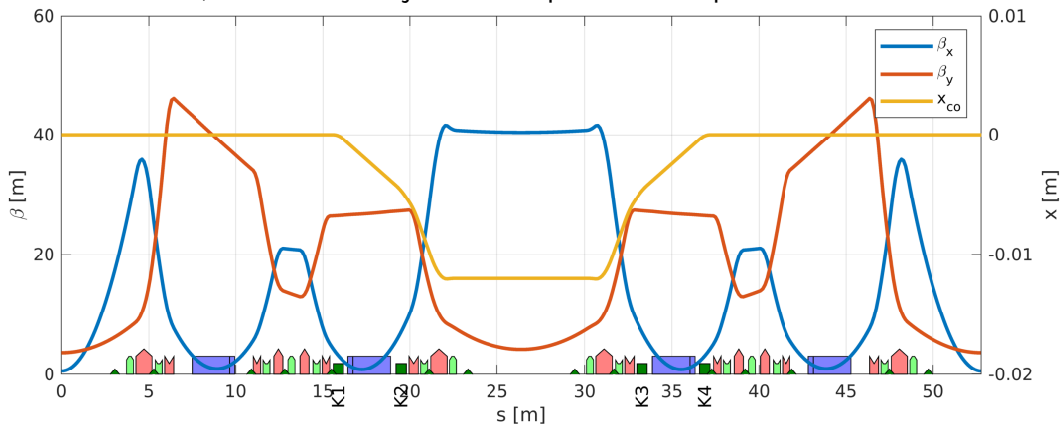
In the old ESRF, the 4-kickers injection bump had 4 sextupoles inside:



When the bump is ramping up or ramping down, the quadrupolar feed-down of the sextupoles are different and therefore the bump is not perfectly closed.

Injection perturbations – old ESRF example

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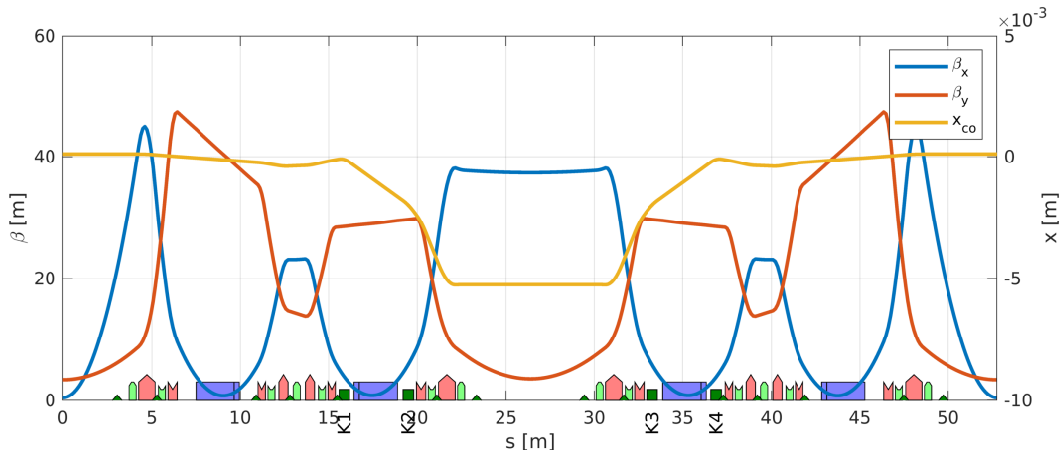


When the bump is ramping up or ramping down, the quadrupolar feed-down of the sextupoles are different and therefore the bump is not perfectly closed.

When the ramp is at half of the full amplitude, it is not closed.

Injection perturbations – old ESRF example

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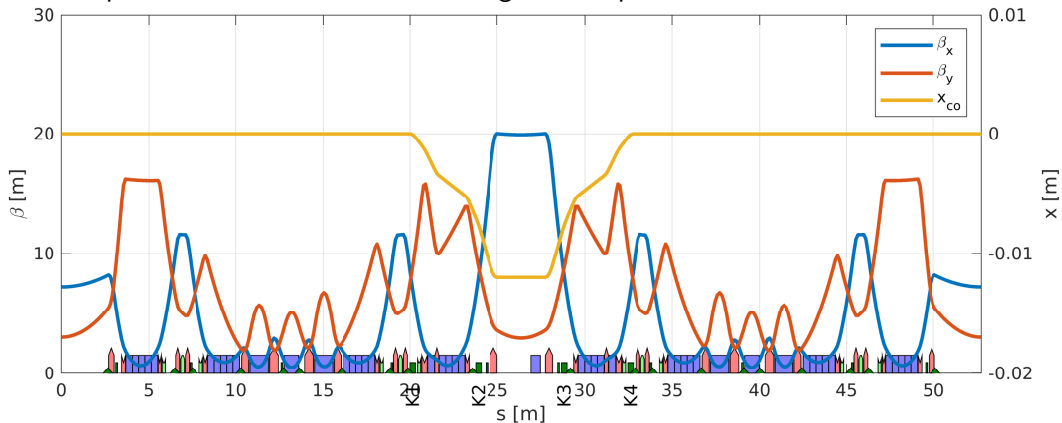
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When the ramp is at half of the full amplitude, it is not closed.

Injection perturbations – new ESRF example

In the upgrade of the ESRF, this design problem was fixed!

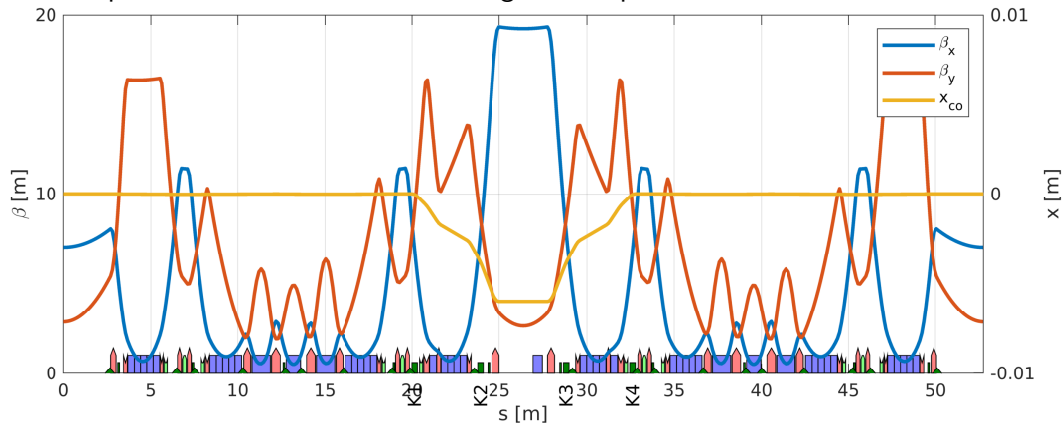
There are still two sextupoles inside the bump, but they are just at the beginning of the bump, so the nonclosure effect during the ramp is much smaller.



Injection perturbations – new ESRF example

In the upgrade of the ESRF, this design problem was fixed!

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- On-axis single turn injection

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Extraction

- Fast extraction

- Slow extraction

Magnetic elements for injection and extraction

Examples / exercise

Extraction is the mechanism used to remove beam from an accelerator. Different techniques are used to extract the beam from a circular machine:

- Fast extraction: all the beam is extracted in one turn from a circular accelerator to be injected in another machine.
- Slow-extraction: a small fraction of the stored beam is extracted at each turn, with the result of having a low intensity beam for a long time (ms or even seconds).

As the injection, extraction is generally performed in the horizontal plane.

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- The first stage of fast extraction consists in creating an orbit bump to slowly bring the beam as close as possible to the first extraction septum.
- Then a fast kicker magnet is powered such that the beam is deflected into the extraction channel where it receives sufficient angular deflection to leave the machine. If particles arrive at the kicker while it is ramping, they will be killed onto the septum blade. As a consequence, the rise time of the kicker must be as short as possible.
- The duration and switch-on/switch-off times of the kicker pulse depend on the mode of extraction. Typical values of rise and fall times are 40 to 50 ns.

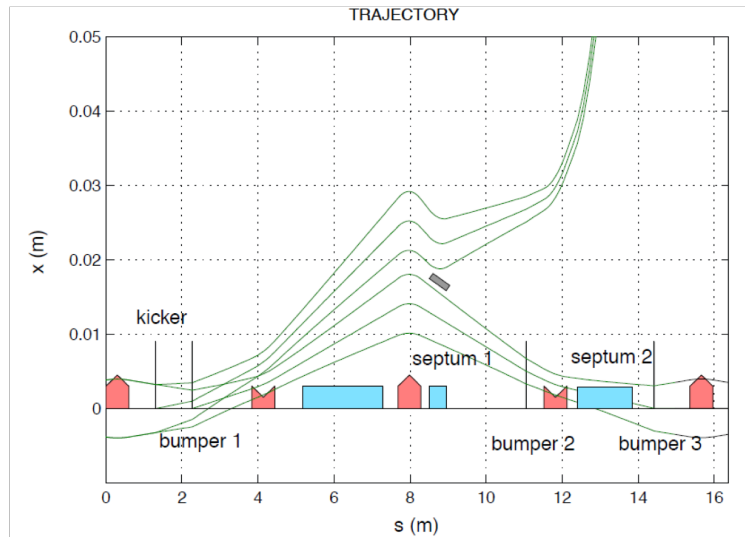
- For one-turn extraction, the rise time must be short compared to the revolution period of the ring (then only a small fraction of the particles are lost), the pulse duration is the revolution period and the switch-off time is arbitrary.
- For bunch-by-bunch extraction, the rise time and the fall time must be shorter than the time interval between two successive circulating bunches, and the pulse duration is the bunch repetition period (or less).

As for the on-axis injection, the kicker must deflect the beam by:

$$\theta_k = \frac{x_s}{\sin \mu_x \sqrt{\beta_x^k \beta_x^s}}$$

- β_x^k and β_x^s are the horizontal beta functions at the kicker and at the septum
- μ_x is the betatron phase advance from the kicker to the septum
- x_s is the required displacement at the septum entrance.

The initial orbit bump is used to reduce the value of x_s .



Schematic view of extraction region of the ESRF booster. There are three slow bumpers B1, B2, B3, two extraction septa SE1 and SE2 and one fast kicker KE.

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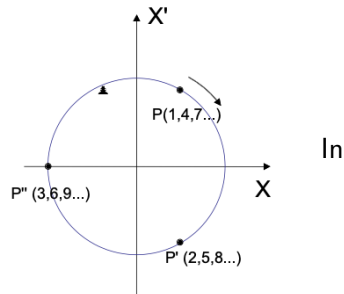
Examples / exercise

Slow extraction can be achieved by controlled excitation of a non-linear betatron resonance of the ring, often a third-integer resonance.

The efficiency of slow extraction depends on the thickness of the first extraction septum as compared with the growth of the resonant betatron amplitudes in the final few turns before extraction.

The slow extraction is used in hadrotherapy machines or in particle physics experiment where a uniform extracted beam is needed.

The trajectory of a particle which has a horizontal tune $Q_x = p/3$ is closed over 3 turns. Such a motion can be described by 3 fixed points P, P' and P'' in the normalised phase space diagram: after 3 turns the particle is back to the same angle and position.



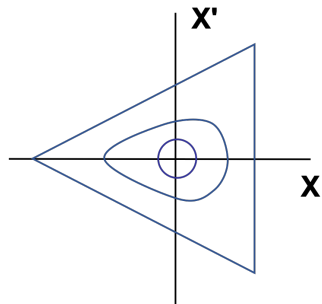
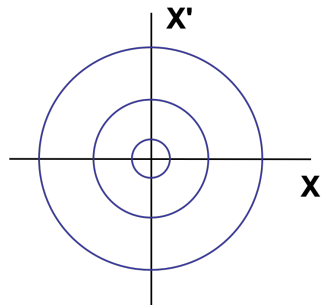
the presence of a sextupolar perturbation, the particle will get an angular kick proportional to the square of its amplitude, but will still describe a close trajectory between the 3 fixed points P, P' and P''.

A particle which has a horizontal tune $Q_x = p/3 + \epsilon$ (ϵ small) describes triangular trajectories in the normalised phase space diagram.

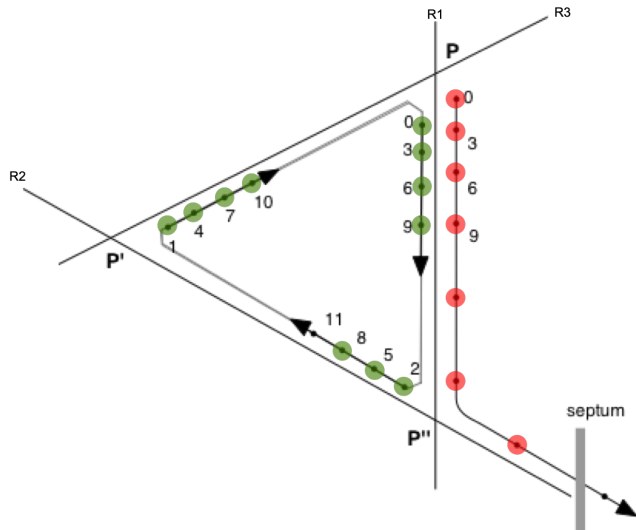
In a completely linear machine, the phase space is elliptical at small and large amplitudes. It is circular in normalized coordinates.

When there is a third order resonance, the phase space becomes triangular at large amplitudes and the particles are unstable outside the triangle.

The stable area is defined by three separatrices. The particles which start to move inside this triangle are stable and describe closed trajectories. The particles which are outside this triangle are unstable and describe open trajectories with a diverging motion to the outside.



Resonant extraction



The size of the stable triangle can be reduced by changing the sextupoles values or by approaching the tunes to the $1/3$ resonance. The tunes can be changed with the quadrupoles or also by changing the energy of the beam, in case of non-zero chromaticity.

Particles are brought closer to the resonance by varying their tune with:

- A quadrupole change.
- A finite chromaticity and a change of energy.

At the location of the extraction septum, within the last three turns the betatron amplitude of the particle must have grown enough to jump from one side of the septum to the other. This requires a very thin septum blade (electrostatic septum). This growth in amplitude of a particle depends on its momentum, its initial amplitude and the proximity of its tune to the resonant tune value.

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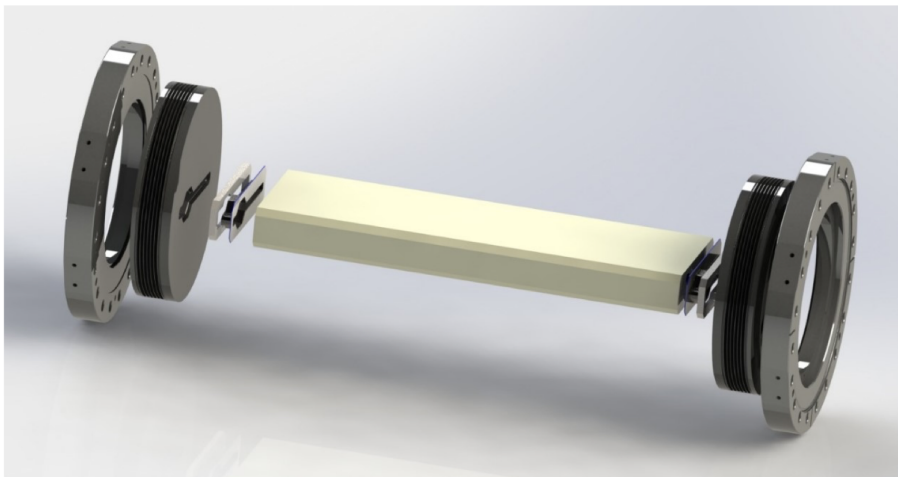
Examples / exercise

- Fast kicker magnets need to be switched on/off in times typically of 50 to 150 ns. This is only possible with magnets with extremely small inductance. Small inductance magnets are usually “air coil magnets”, which cannot produce a high magnetic field without going to impracticable currents
- The kickers are powered by pulse-forming networks (PFN) which are charged some time prior to the injection and rapidly discharged via fast switches (thyratrons).
- Ferrites are frequently used to contain the field, usually they cannot be tolerated inside the vacuum of the ring, and must be installed around a ceramic vessel. Such ceramic vessels have to be coated with a thin conducting layer deposited on the inner wall of the ceramic, in order to ensure the circulation of the beam image current.

Kicker magnets

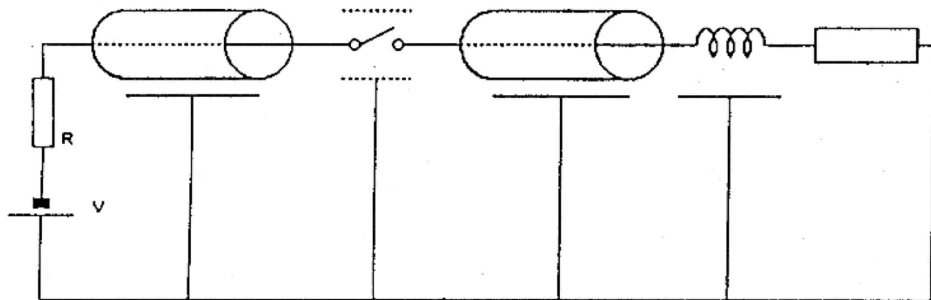
Kicker magnets are very fast and the vacuum chamber may have to be in ceramic, so that the eddy currents don't screen the magnetic field. The ceramic then has to be coated by a thin layer of metal.

Kicker ceramic chamber of ESRF EBS, coated with titanium.



Kicker magnets

Typical layout of a kicker power supply



DC-HV
Power
supply

HV-Coax
cable acting
as PFN

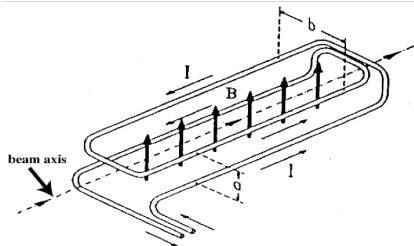
Thyratron
switch with
coaxial
mounting

Coax cable
to kicker
magnet

Kicker
magnet
constructed
as delay line

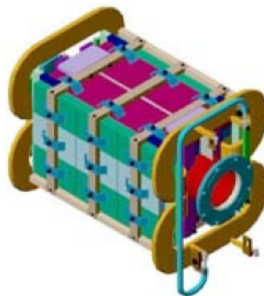
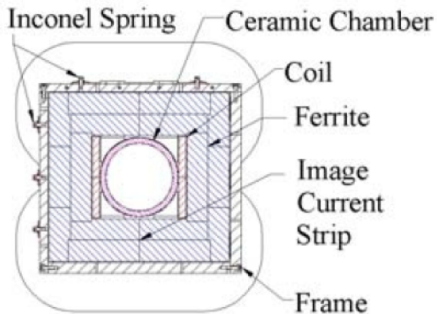
Z termination

Kicker magnets



Example of a kicker magnet coil

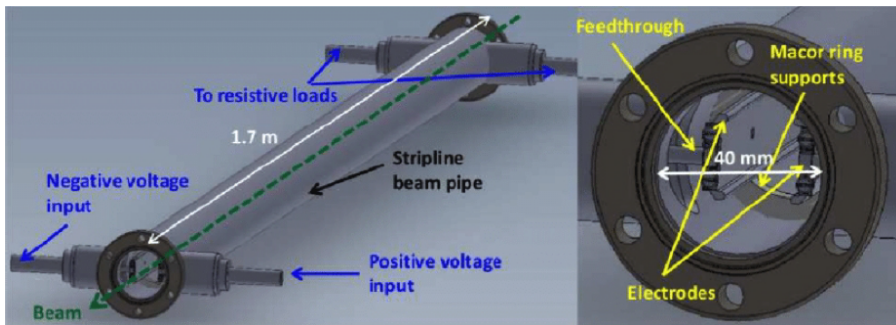
Spallation Neutron Source (SNS) injection kicker, courtesy C. Pai:



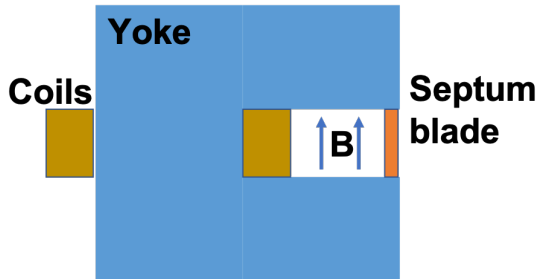
Stripline kicker

Two electrodes with 50Ω characteristic impedance, to avoid inductance and allow faster rise times. It can kick selected bunches.

It is less efficient than the magnetic devices and need to be combined to fast high voltage pulser.



from C. Belver-Aguilar thesis.



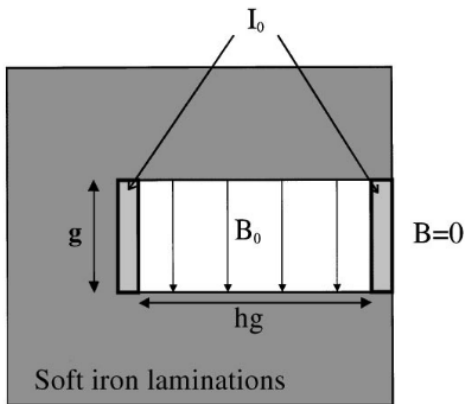
A septum magnet gives a uniform magnetic field inside a small region and nothing outside.

It is used to bend only the injected or extracted beam without perturbing the stored beam.

A septum can be pulsed or not. If it is not pulsed, can be a permanent magnet.

- A septum gives a much larger bending angle than a kicker and allows to bring the injected beam close to the stored beam, without interfering with the stored beam.
- The septum blade separates the injected/extracted beam from the stored beam. The blade can be a few mm thick.
- The septum blade can be part of the coils or an eddy current shield.

Septum magnets: active septa

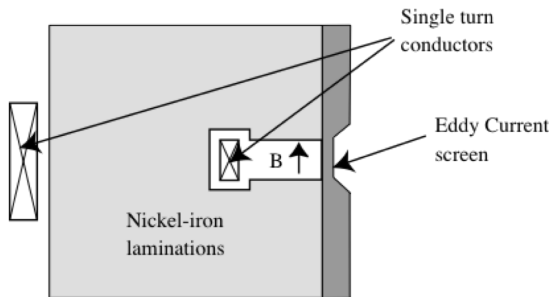


The blade is the return conductor, so there is no field outside.

For a large field, there is a lot of heating in the blade, so it can't be too small.

Can be DC or pulsed (usually ms).

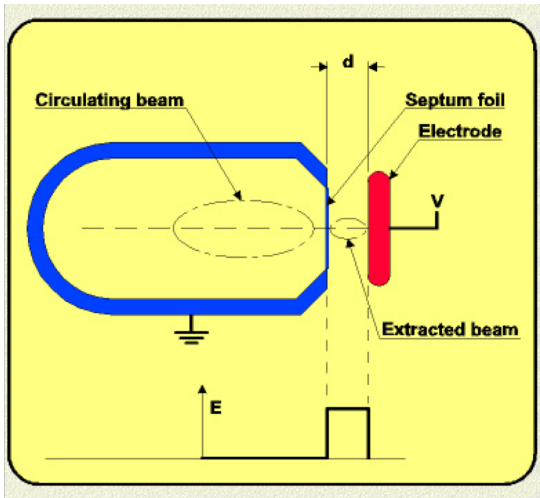
Septum magnets: passive septa



For thinner blades, we use very short pulses and the blade is a conductor screening the magnetic field with eddy currents.

The blade has to be larger than the skin depth in the material of an electromagnetic field at the frequency of the pulse.

The pulse can be few tens of μs .

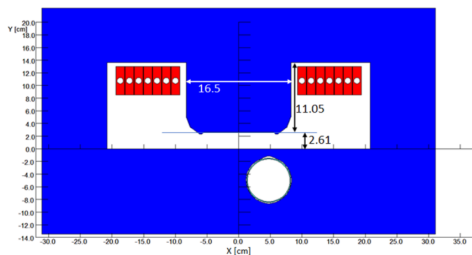


Electrostatic septa use electric field to bend the beam instead of magnetic field.

The blade can be very small.

The deflection is smaller.

Lambertson septum



From: N.Tsoupas, “A state of the art Lambertson septum magnet of the RHIC beam injection system”, NIM-A, 2020.

The circulating beam travels through the Yoke and does not see the field.

Deflection and offset are not in the same plane. In this example deflection is horizontal and offset vertical.

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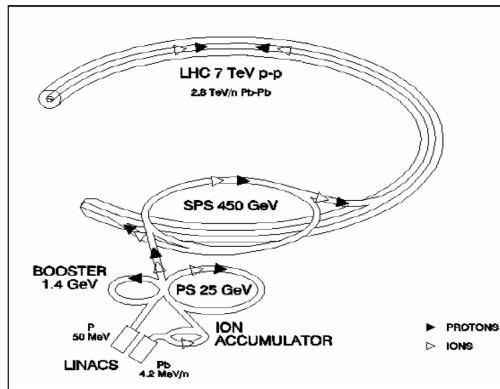
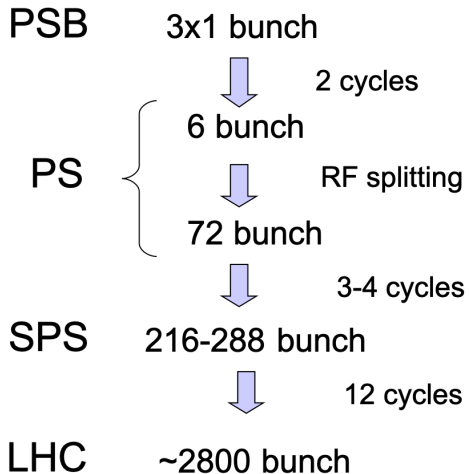
- Fast extraction

- Slow extraction

Magnetic elements for injection and extraction

Examples / exercise

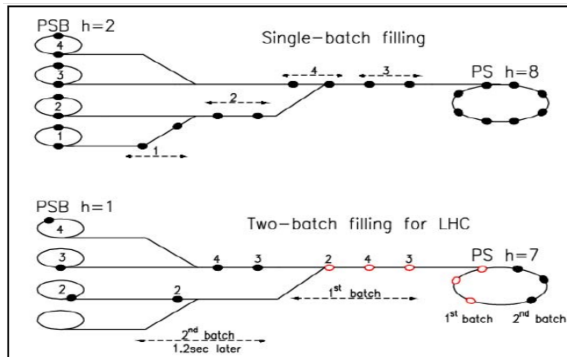
LHC injection chain



Injection efficiency of the full chain is about 85 %.

PSB has four accelerators on top of each others which can be synchronised to send the bunches in a single transfer line.

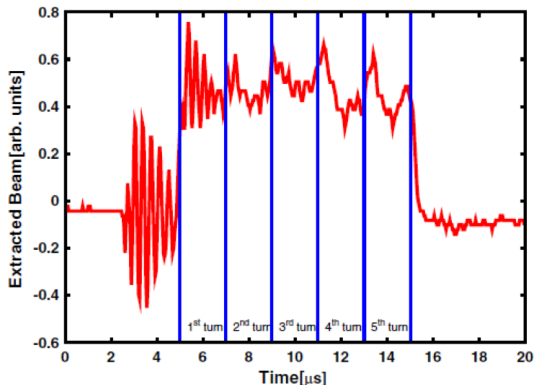
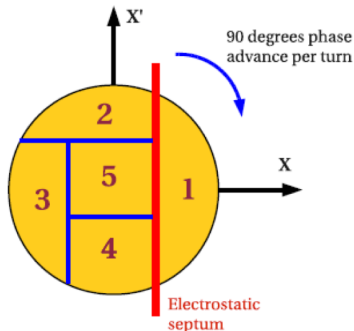
Two schemes possible, with one or two bunches in each PSB accelerator.



The 6 bunches injected in the PSB are split into 72 bunches with a technique called RF splitting. The 72 bunches are then injected into the SPS accelerator.

Example: PS extraction for fixed target experiments

For fixed target experiment, extraction from the PS at CERN is performed in 5 turns. Once the energy is reached, the fractional tune is $1/4$. Fast magnets approach the beam to the septum.



from: J. Barranco Garcia and S. Gilardoni,
Beam loss studies during the CERN PS CT Extraction

Example: ESRF injection chain

ESRF is a fourth generation 6 GeV synchrotron light source.

200 MeV linac



Single turn injection

6 GeV Booster synchrotron 300m
~335 bunches



Single turn extraction
+
Accumulation in the storage ring

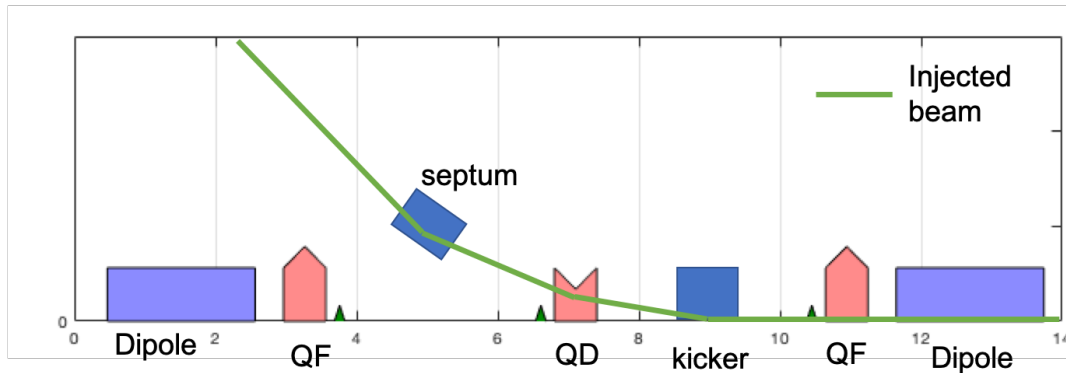
6 GeV storage ring 844m
992 bunches



Example: ESRF injection chain

The linac can produce a train of about 350 bunches (long pulse mode) or a few single bunches (short pulse mode).

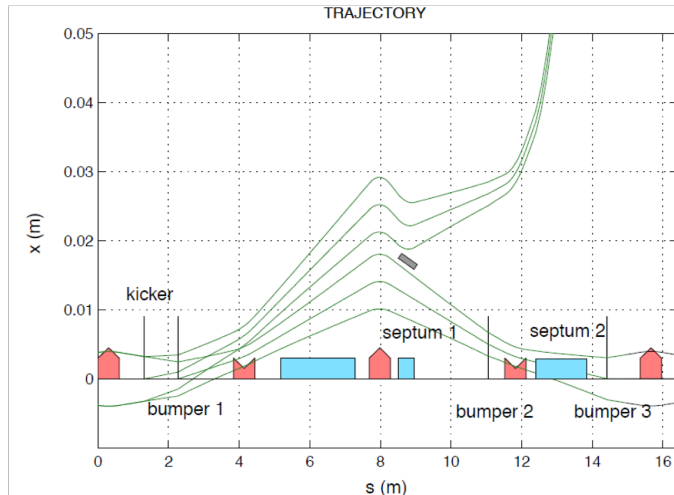
Linac to booster injection is a single turn on-axis injection with a pulsed septum and a kicker with a $1 \mu\text{s}$ pulse.



Example: ESRF injection chain

The beam is accelerated in the booster in 150 ms from 200 MeV to 6 GeV and then it is extracted in a single turn.

Three slow bumpers generate a closed bump of about 15 mm in 3-4 ms, then the extraction kicker kicks the beam to the other side of the blade of the extraction septa.



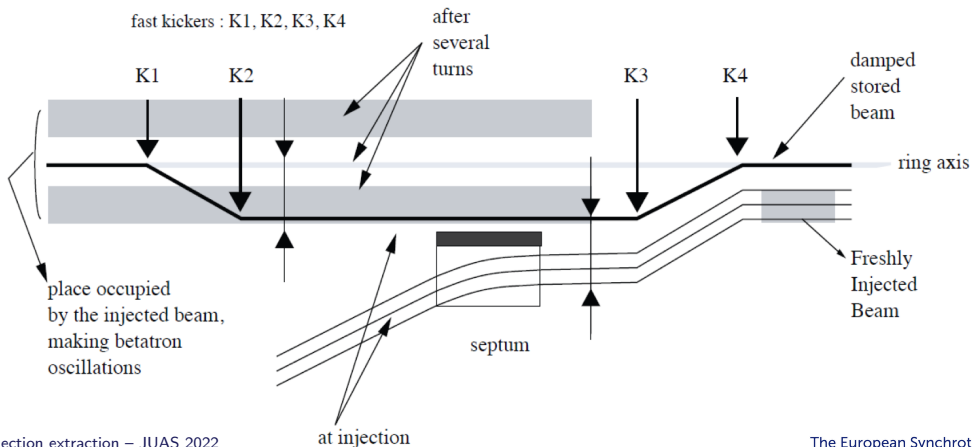
After extraction the beam enters in the transfer line 2.

Example: ESRF injection chain

From the transfer line 2, the beam is injected off-axis in the storage ring on top of the already circulating one.

The stored beam is bumped by 4 injection kickers K1, K2, K3 and K4. The closed bump is about 14 mm. The injected beam is bended by two septa.

Injection oscillations of 6 mm are damped in about 10-20 ms.



Example: injection in AdA

AdA is the first electron positron collider, built in 1961 in Frascati, close to Rome. AdA means *Anello di Accumulazione*, which is Storage Ring in Italian.

The beam energy was 200 MeV.

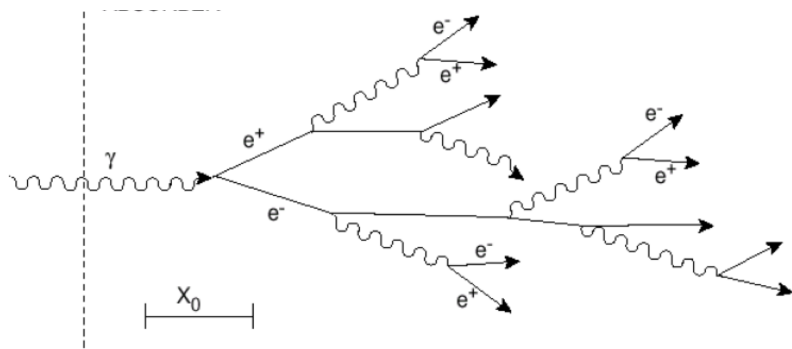
The beam of electrons and the beam of positrons were in the same vacuum chamber in the same accelerator, travelling in opposite directions.



Example: injection in AdA

The injection was achieved through the conversion of γ rays in a tantalum plate installed in the vacuum chamber. The γ rays were produced by bremsstrahlung from higher energy electrons beam.

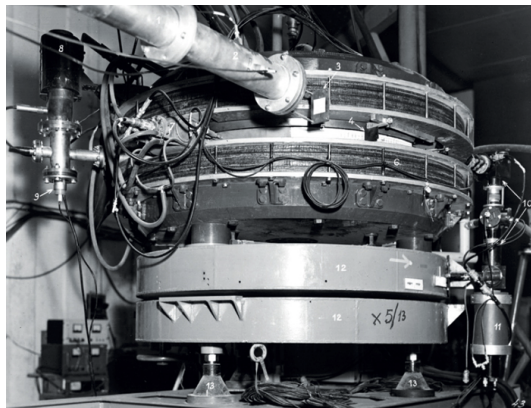
Many electrons and positrons were produced in the electromagnetic shower and only a very few were captured in AdA.



Example: injection in AdA

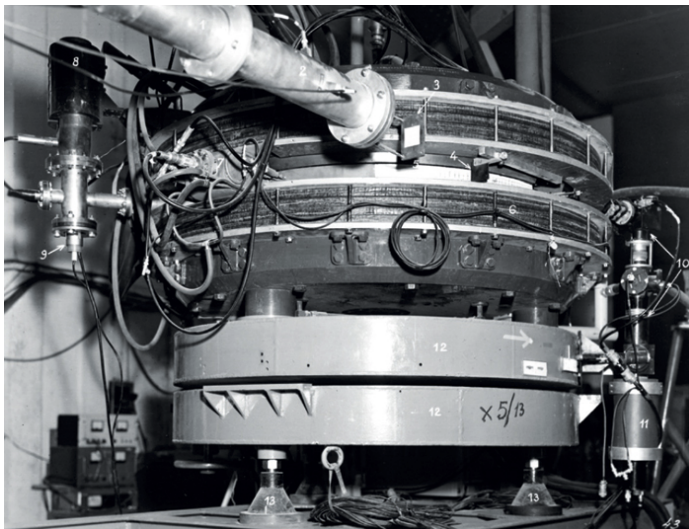
The γ rays were produced by bremsstrahlung of the electron beam of the electron synchrotron of the Frascati laboratories.

In order to increase the injection rate, in 1962 AdA was moved at the *Laboratoire de l'Accélérateur Linéaire* (LAL) in Orsay, close to Paris.



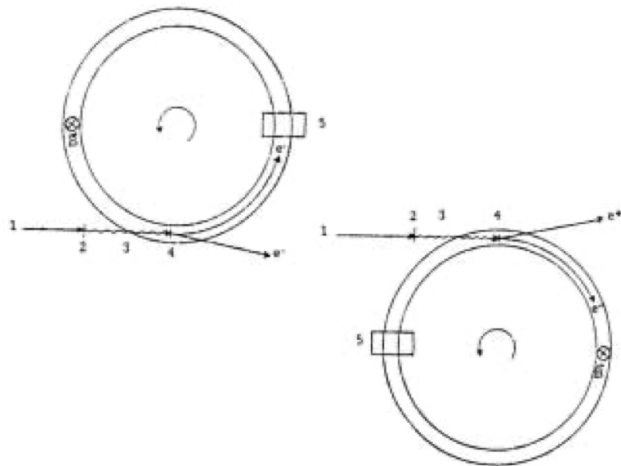
Example: injection in AdA

In order to inject both electrons and positrons using the same big linac had some problems: moving the linac to a different part of AdA was not a practical solution!



Example: injection in AdA

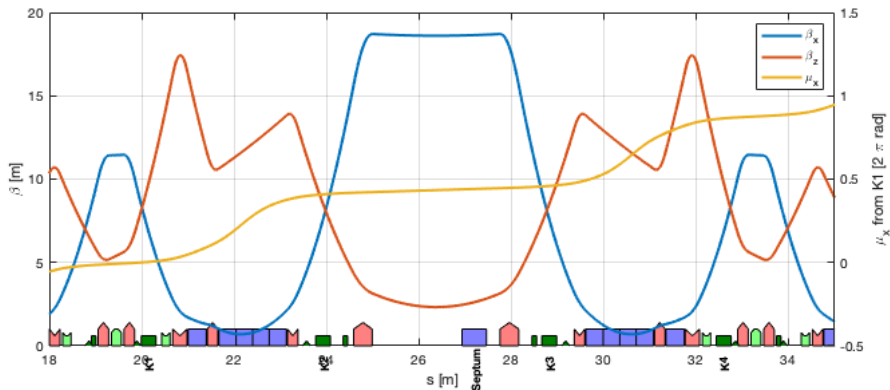
In order to inject both electrons and positrons using the same big linac had some problems: moving the linac to a different part of AdA was not a practical solution!
A more clever solution was to rotate the small AdA by 180° as shown in this drawing.



Original drawing from Jacques Haïssinski

Exercise: single kicker injection

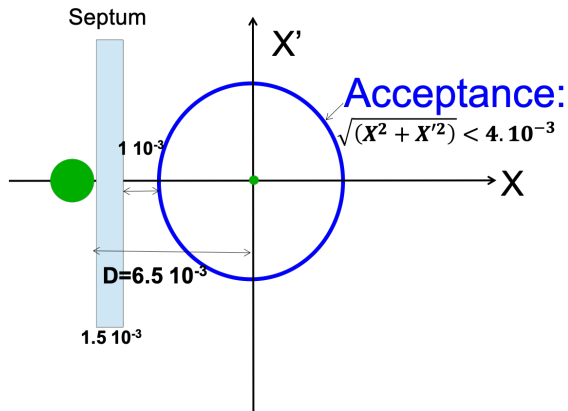
You work in a synchrotron light source where there is top-up injection with a four kickers bump scheme.



One day the kickers power supplies go in fire and you have to manage to inject with the only spare power supply available.
Is a single kicker injection possible?

Exercise: single kicker injection

Let's do all in normalised coordinates.



Acceptance:

$$\sqrt{X^2 + X'^2} = 4 \cdot 10^{-3}$$

Distance stored beam to injected beam:

$$D = 6.5 \cdot 10^{-3}$$

Half size of injected beam:

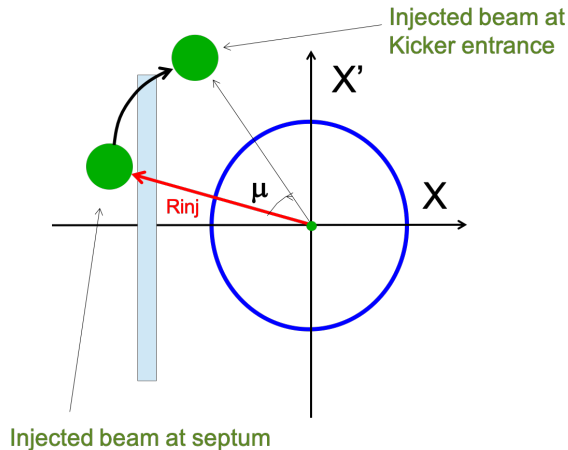
$$\sigma_X = 1 \cdot 10^{-3}$$

The size of the stored beam is almost zero compared to the acceptance, so we consider it as a single particle.

If we have only a single kicker can we still inject?

Exercise: single kicker injection

This solution is not optimal.



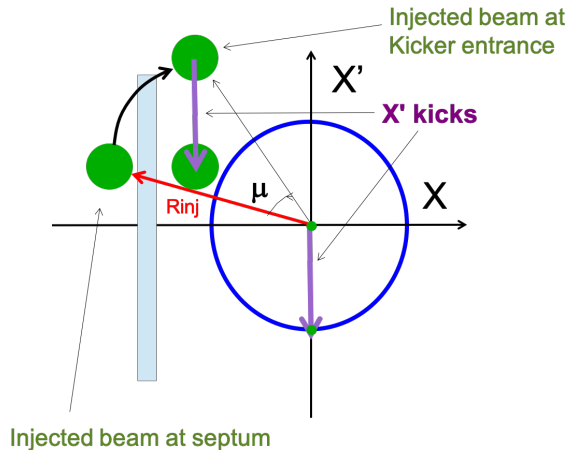
The kick in X' must be smaller than the acceptance, otherwise the stored beam is lost.

R_{inj} must be as small as possible, so X' of the injected beam must be 0.

μ_x must be close to $\pi/2$ rad.

Exercise: single kicker injection

This solution is not optimal.

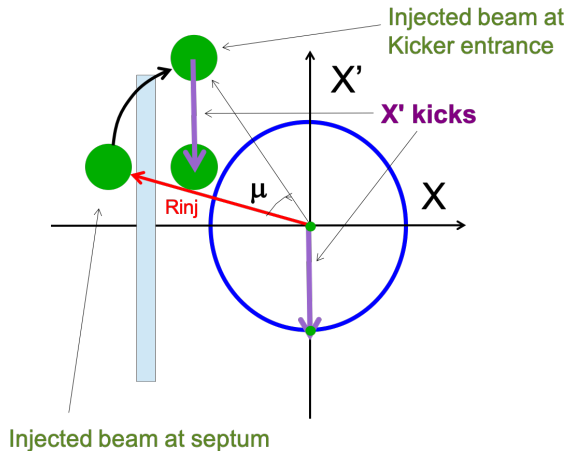


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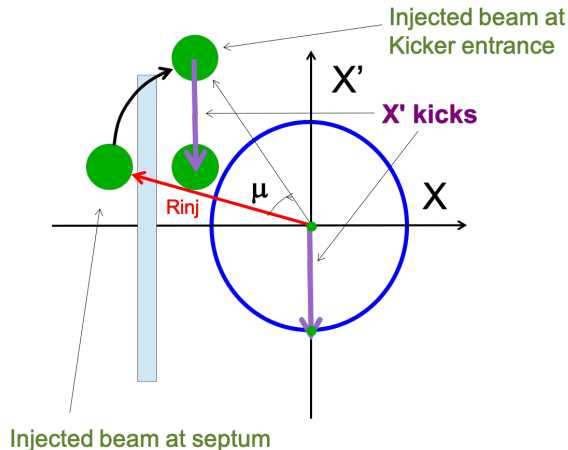
Exercise: single kicker injection



The questions are:

- Can we inject?
- Which of the 4 kickers should we use?
- What is the kicker strength needed?
- Can we have 100% injection efficiency?

Exercise: single kicker injection

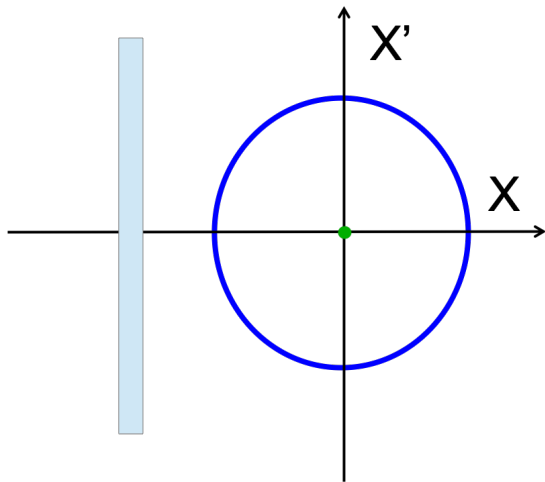


The questions are:

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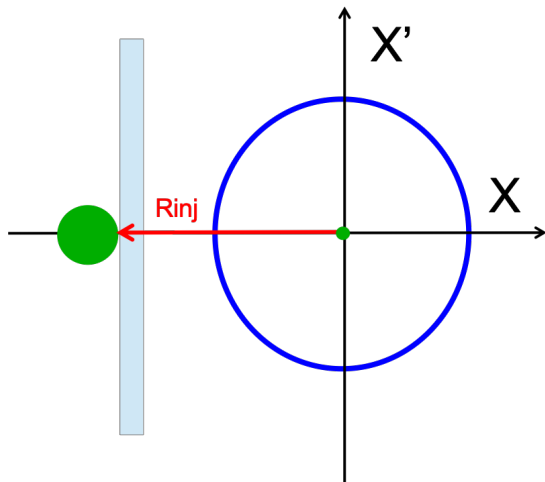
We will try to improve this set-up.

Exercise: single kicker injection



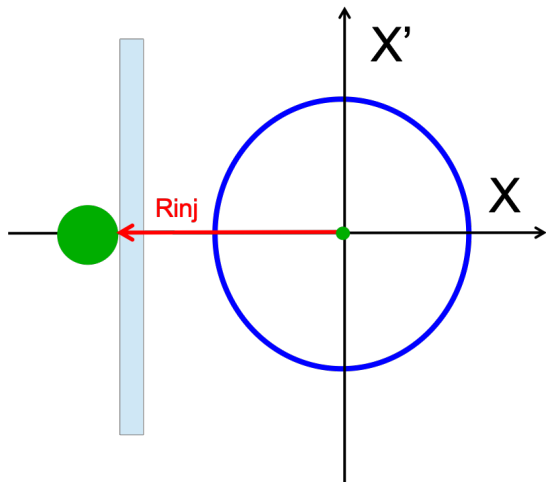
- R_{inj} should be minimised.

Exercise: single kicker injection



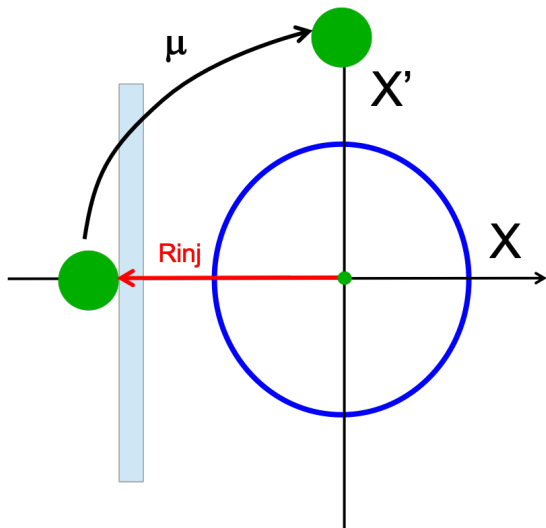
- R_{inj} should be minimised.
 - $X'_{inj} = 0$ at septum

Exercise: single kicker injection



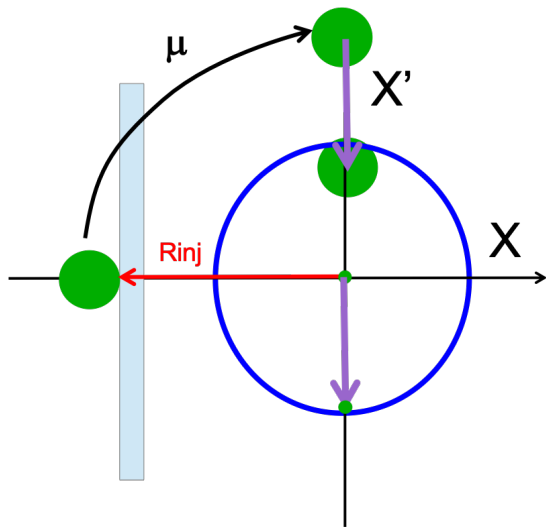
- R_{inj} should be minimised.
 - $X'_{inj} = 0$ at septum
- $X_{inj} \simeq 0$ at kicker

Exercise: single kicker injection



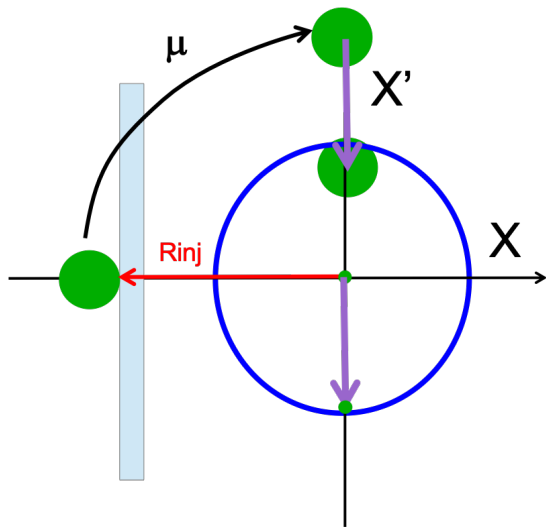
- R_{inj} should be minimised.
 - $X'_{inj} = 0$ at septum
- $X_{inj} \simeq 0$ at kicker
 - $\mu_x \simeq \pi/2$ rad.

Exercise: single kicker injection



- R_{inj} should be minimised.
 - $X'_{inj} = 0$ at septum
- $X_{inj} \simeq 0$ at kicker
 - $\mu_x \simeq \pi/2$ rad.
- Maximum kicker strength is the radius of the acceptance R_{acc}
 - $\theta_k = \Delta X'_{inj} \leq R_{acc}$

Exercise: single kicker injection

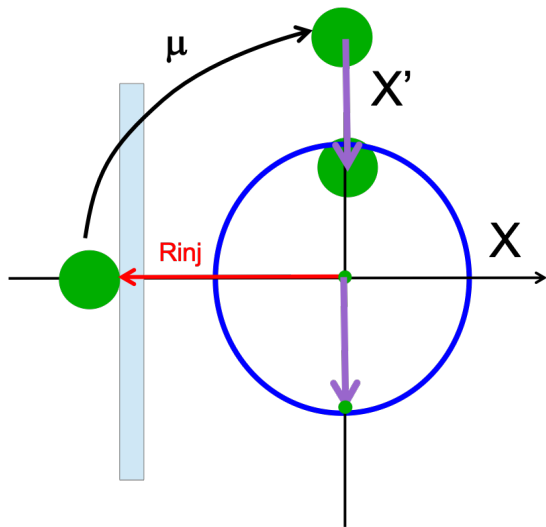


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For injection efficiency > 0 :

$$R_{inj} < 2 \times R_{acc}$$

Exercise: single kicker injection



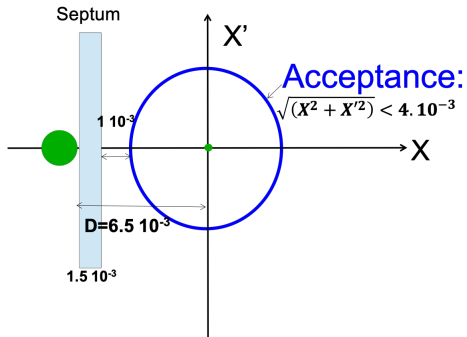
- R_{inj} should be minimised.
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 - $\theta_k = \Delta X'_{inj} \leq R_{acc}$

For injection efficiency > 0 :

For injection efficiency = 100%:

$$R_{inj} < 2 \times R_{acc}$$
$$R_{inj} + 2 \times \sigma_{inj} < 2 \times R_{acc}$$

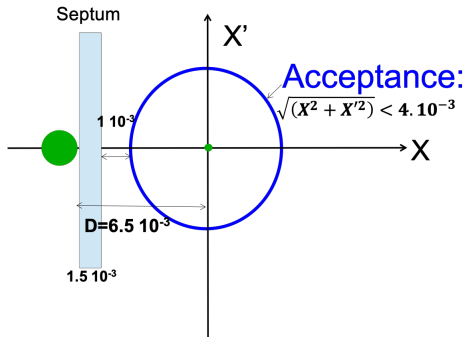
Now we can answer the questions.



The questions are:

- Can we inject with one kicker?

Now we can answer the questions.

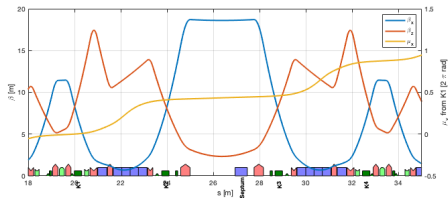


The questions are:

- Can we inject with one kicker?
 - Yes! $R_{inj} < 2R_{acc}$
 - $6.5 \text{ mm}^{-1/2} < 2 \cdot 4 \text{ mm}^{-1/2}$

Exercise: single kicker injection

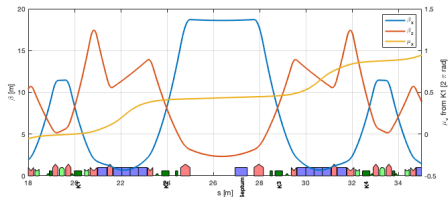
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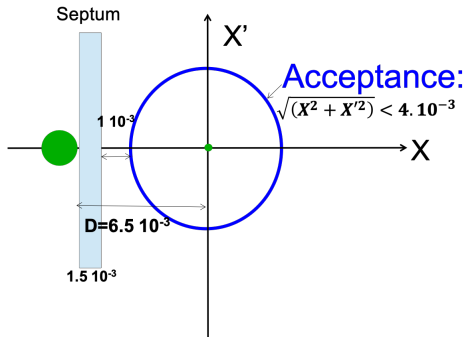
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- Which of the 4 kickers should we use?
 - K4 is not too far from $\mu_x = \pi/2$ from the septum!

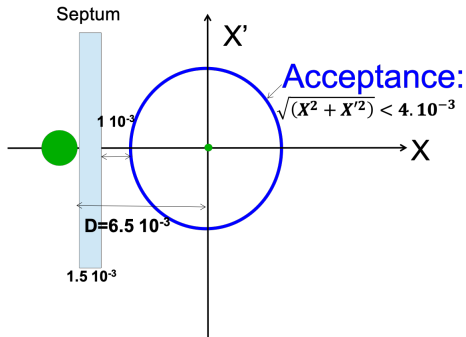
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- Which of the 4 kickers should we use?
 - K4 is not too far from $\mu_x = \pi/2$ from the septum!
- Can we have 100% injection efficiency?

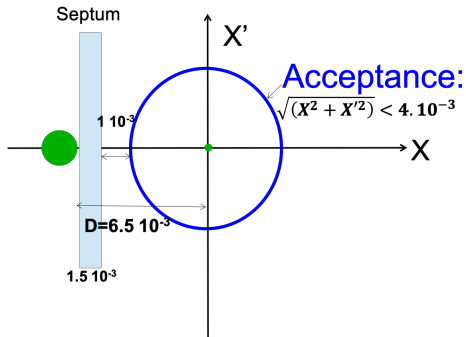
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The questions are:

- Can we inject with one kicker?
 - Yes! $R_{inj} < 2R_{acc}$
 - $6.5 \text{ mm}^{-1/2} < 2 \cdot 4 \text{ mm}^{-1/2}$
- Which of the 4 kickers should we use?
 - K4 is not too far from $\mu_x = \pi/2$ from the septum!
- Can we have 100% injection efficiency?
 - No! $2R_{acc} < R_{inj} + 2\sigma_{inj}$
$$8 \text{ mm}^{-1/2} < (6.5 + 2) \text{ mm}^{-1/2}$$

Now we can answer the questions.

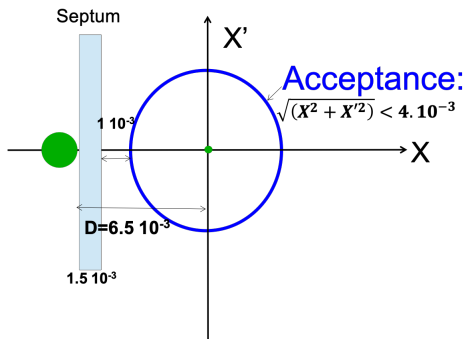


So we can inject, but not with 100% injection efficiency

The questions are:

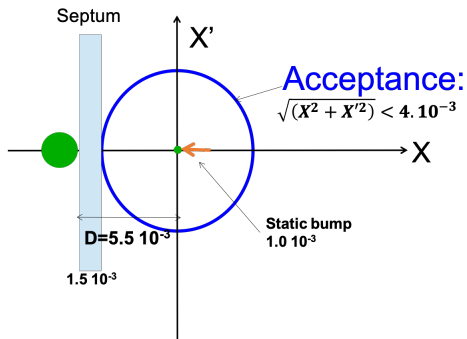
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Exercise: single kicker injection



Can we improve a bit this result?

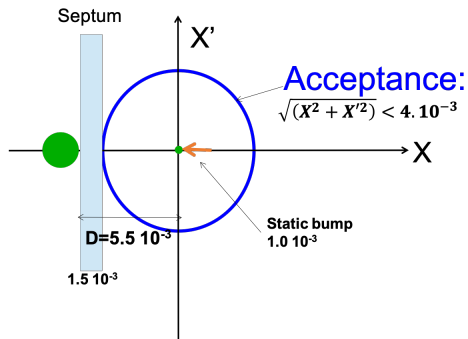
Exercise: single kicker injection



Can we improve a bit this result?

- Yes! We can do a static bump!

Exercise: single kicker injection

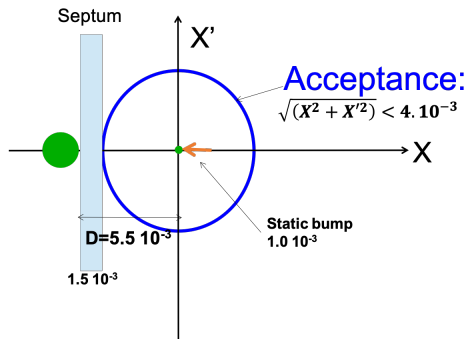


Can we improve a bit this result?

- Yes! We can do a static bump!

So we can now inject with 100% injection efficiency!

Exercise: single kicker injection



Can we improve a bit this result?

- Yes! We can do a static bump!

So we can now inject with 100% injection efficiency!

Of course all this solutions have the draw back that the stored beam is perturbed for a few ms at each injection.

Thanks for your attention!



Many thanks to T. Perron and L. Farvacque.