Accelerator Design

JUAS'22 - Lecture and Mini-Workshop - Bastian Haerer (KIT)

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Joint Universities Accelerator School

The logical path to Accelerator Design

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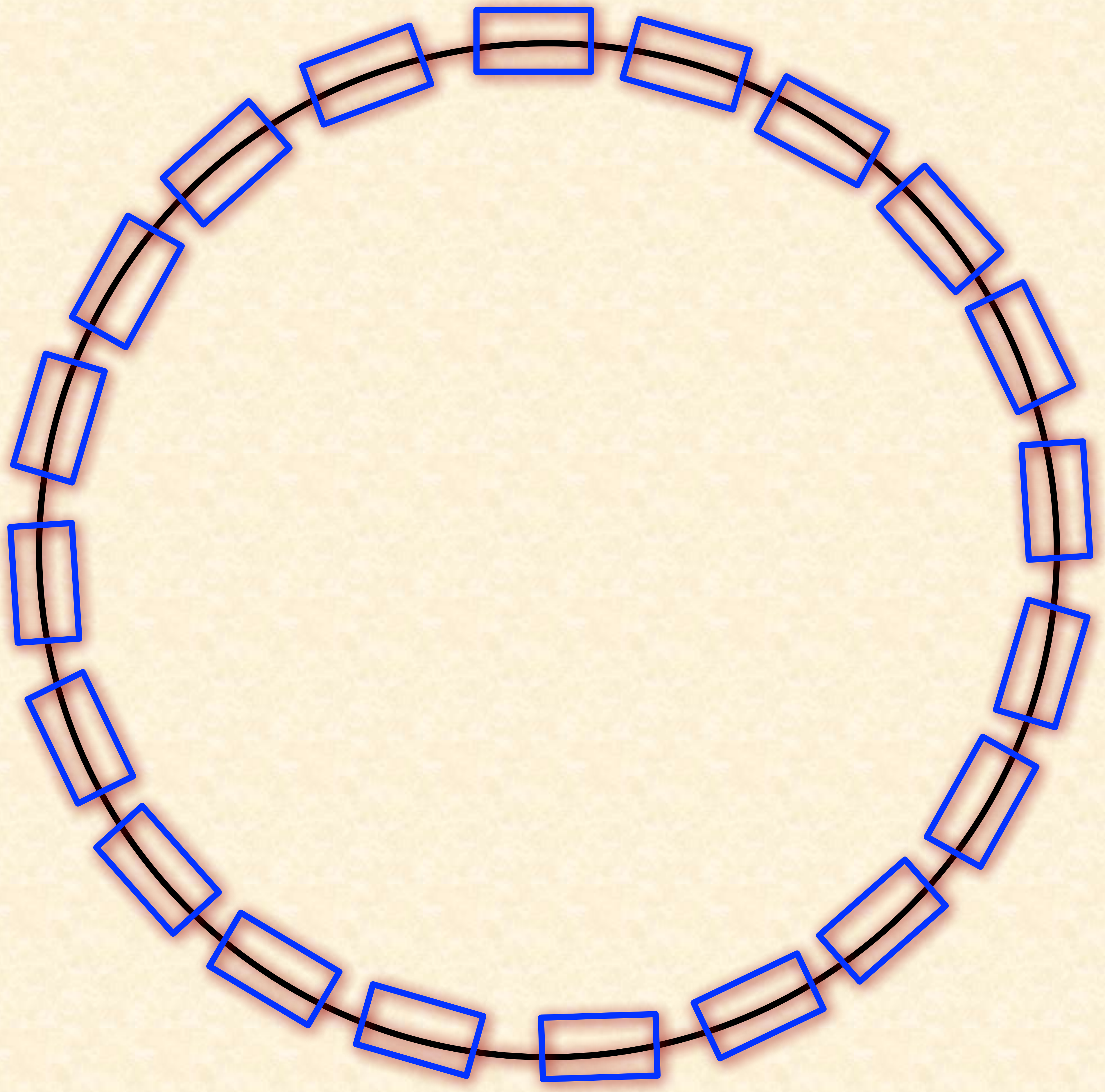
1.) determine particle type & energy ✓

2.) beam rigidity —> calculate integrated dipole field

magnet technology
dipole length & number ✓
size of the ring
arrangement of the dipoles in the ring



The logical path to Accelerator Design

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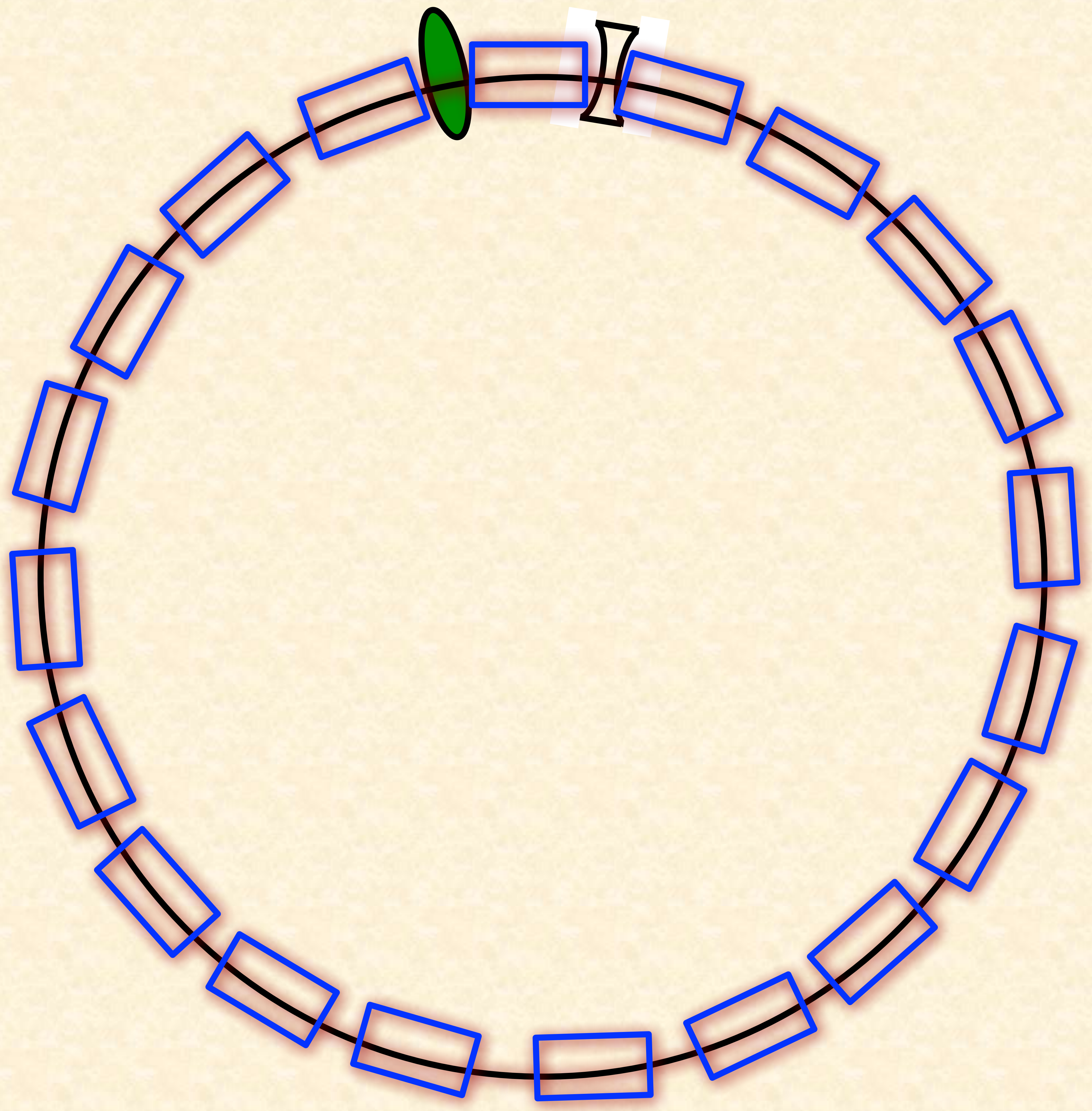
3.) determine the focusing structure of the basic cell
— FODO, DBA — etc. etc. ✓

calculate the optics parameters of the basic cell
beam dimension
vacuum chamber
magnet aperture & design ✓
tune



Recap: Hadron and electron storage rings

Hadron storage rings

- Heavy particles require strong B fields
- Push for highest B fields up to technical limit
- Energy limit given by maximum acceptable circumference

$$2\pi\frac{p_0}{e} = \int B\,\mathrm{d}l$$

$$\epsilon_x \propto \frac{1}{\beta\gamma}$$

$$\epsilon_x \approx \epsilon_y$$

Electron storage rings

- Synchrotron light dominated
- Push for small B fields thus large bending radius
- Energy limit given by synchrotron radiation power

$$P_\gamma = \propto \frac{\gamma^4}{\rho^2}$$

$$\epsilon_x = \frac{C_q}{J_x}\gamma^2\frac{\mathcal{I}_5}{\mathcal{I}_2} \propto \gamma^2$$

$$\epsilon_y \approx 0.1 - 1\,\%\,\epsilon_x$$

Recap: e+e- colliders vs. synchrotron light sources

Collider

- High dipole filling factor → FODO structure
- High energy → large circumference

  → Naturally small emittance

$$\mathscr{L} = \frac{N_1 N_2 n_b f}{4\pi\sigma_x^* \sigma_y^*}$$

N particles per bunch
n_b number of bunches
f revolution frequency

Jean-Luc Revol: ESRF - The European Synchrotron, JUAS 2020

Synchrotron light source

- Small footprint desired
- Low emittance beams for high brilliance

$$B(\lambda) = \frac{F(\lambda)}{(2\pi)^2 \sigma_x \sigma_{x'} \sigma_y \sigma_{y'}} \propto \frac{1}{\epsilon_x \epsilon_y}$$ with photon flux $F(\lambda)$ [1]

$$[F] = \frac{\text{photons}}{s\ 0.1\,\%\,\text{BW A}}$$

- Achromat structures

[1] K. Wille: The Physics of Particles accelerators - an introduction

Today:

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- RF sections
- Dispersion suppressor
- Matching sections
- Interaction regions and mini-beta insertions

- Adrian: Details to groups, exercises and examination
- Start of the workshop! :-)

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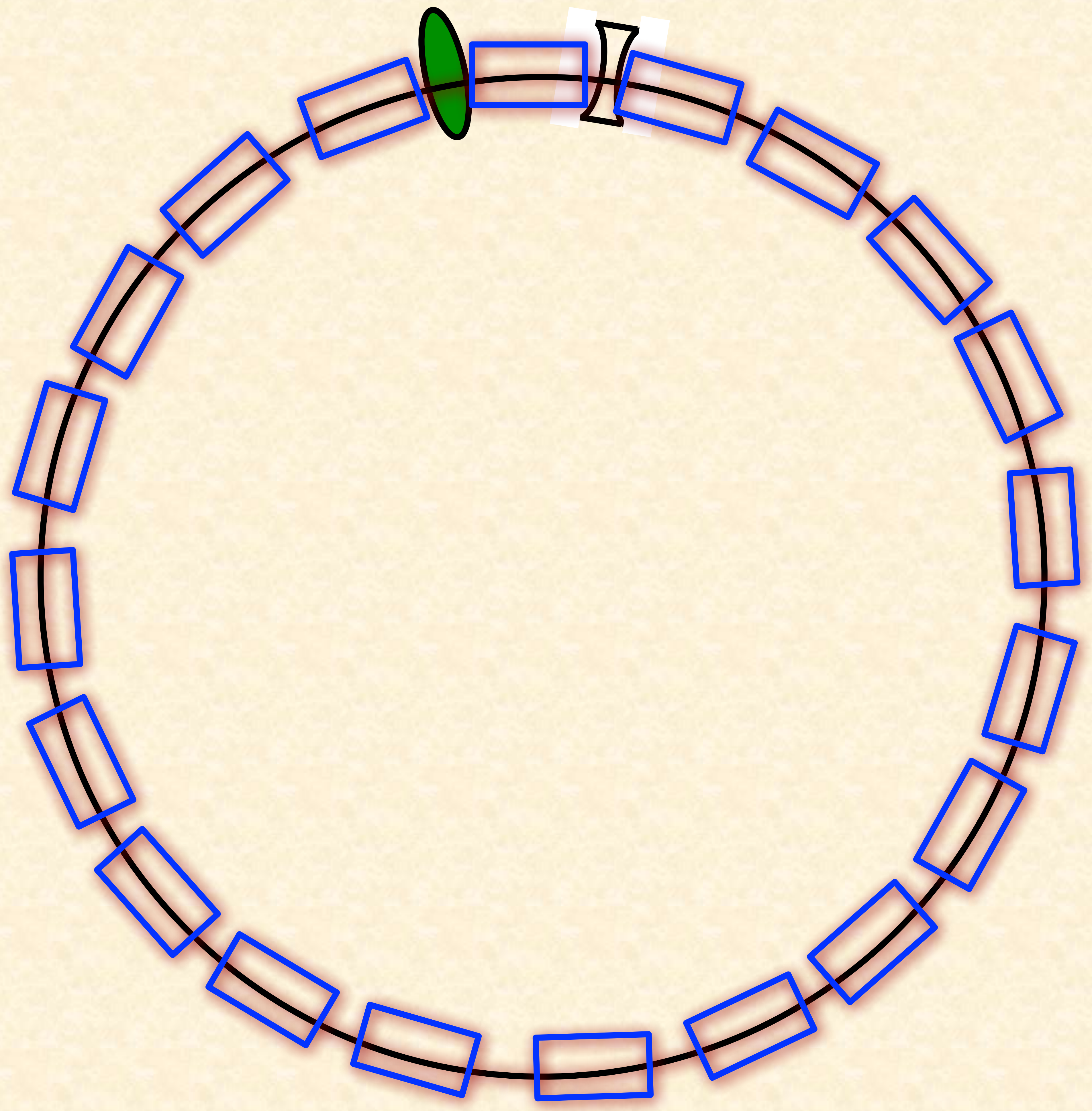
4.) Determine the radiation losses
- **Energy loss per turn**
- **Power loss frequency**
- **—> electrons radiate !!**
- **—> protons do not !!**

5.) Determine the parameters for the RF system
- **Frequency, overall voltage,**
- **space needed in the lattice**
- **for the cavities**



Energy loss and orbit offset

Energy loss per turn: $$U_0 = \frac{C_\gamma}{2\pi} E^4 \mathcal{I}_2$$

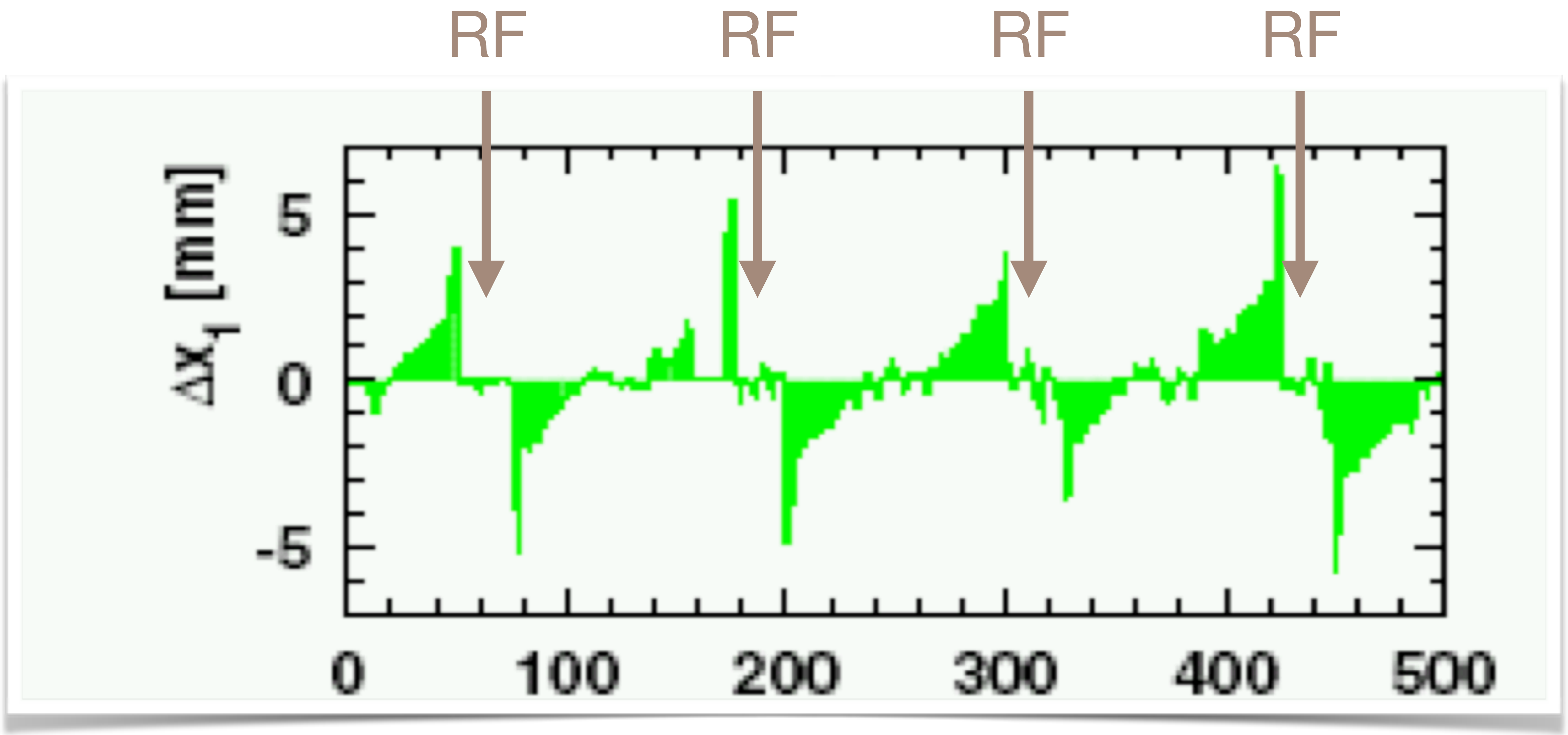
$$\mathcal{I}_2 = \oint \frac{1}{\rho^2}\,\mathrm{d}s \qquad C_\gamma = \frac{e^2}{3\epsilon_0}\frac{1}{(m_e c^2)^4} = 8.8460 \times 10^{-5} \frac{\mathrm{m}}{\mathrm{GeV}^3}$$

	particle	beam energy	U_0
LHC	p	7 TeV	~10 keV
FCC-hh	p	100 TeV	~ 5 MeV
KARA	e	2.5 GeV	0.63 MeV
LEP	e	100 GeV	2.85 GeV
FCC-ee	e	182.5 GeV	9 GeV

Example hadron electron LS, HE electron

Sawtooth orbit:

$$x(s) = x_\beta + D_x \delta$$



LEP sawtooth orbit

Acceleration

deflection/focussing

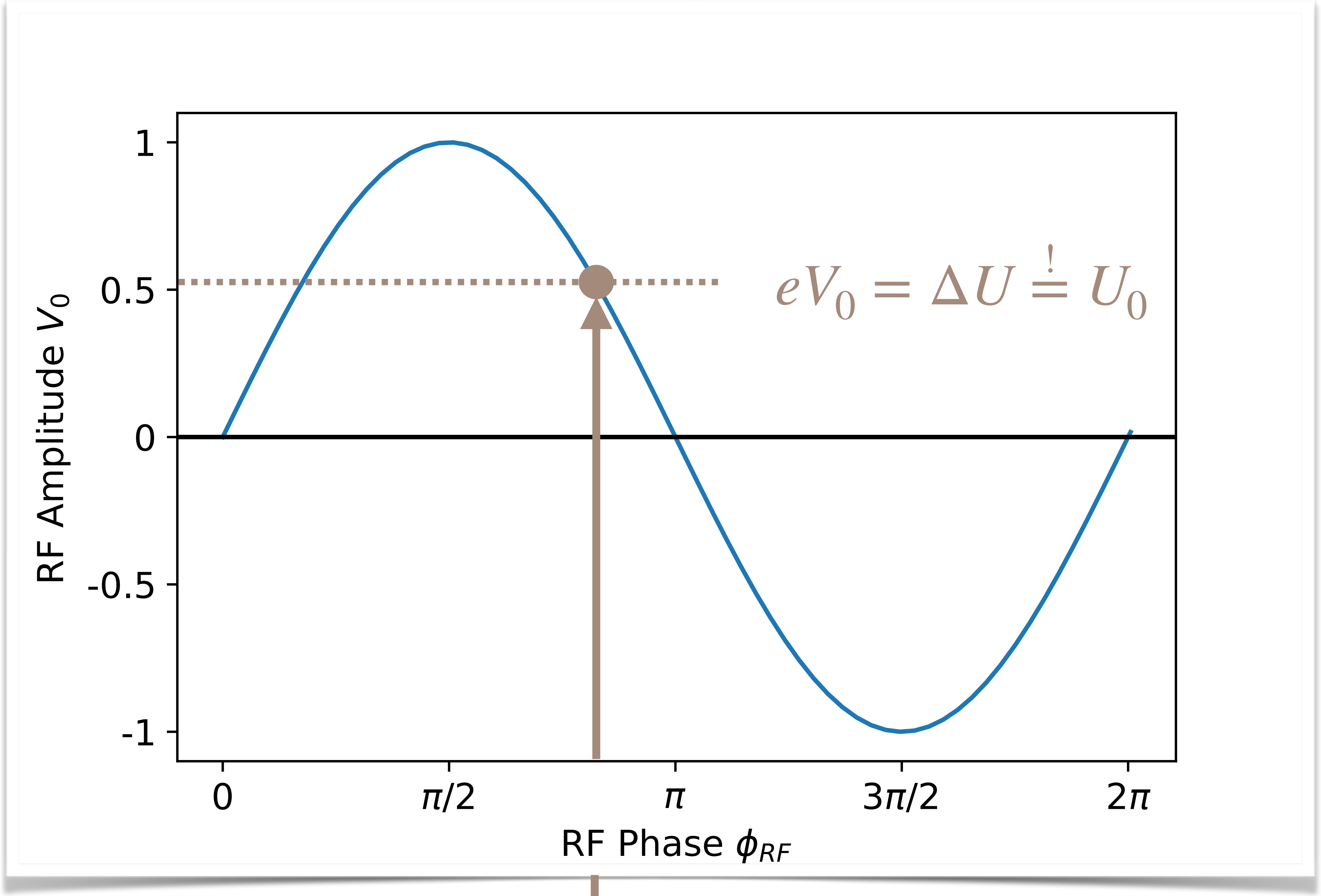
- Lorentz force: $\vec{F}_L = e(\vec{E} + \vec{v} \times \vec{B})$

energy gain

- Oscillating *E* fields in RF cavities
- Energy gain in cavity is given by

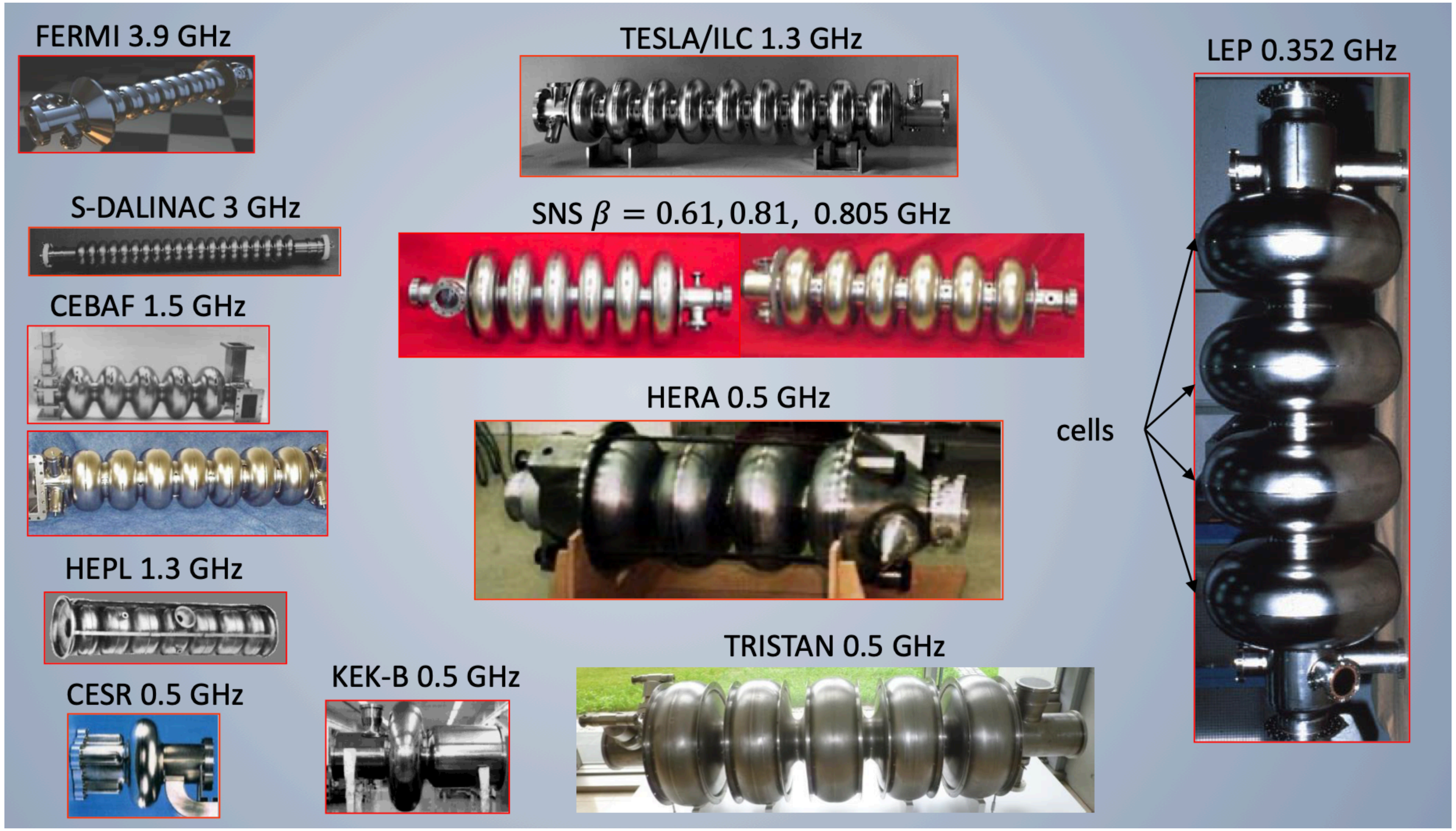
$$\Delta U = eV_{RF} \sin(\phi_{RF} - hf_0 t)$$

h harmonic number
f_0 revolution frequency



synchronous/RF phase ϕ_{RF}

RF cavities

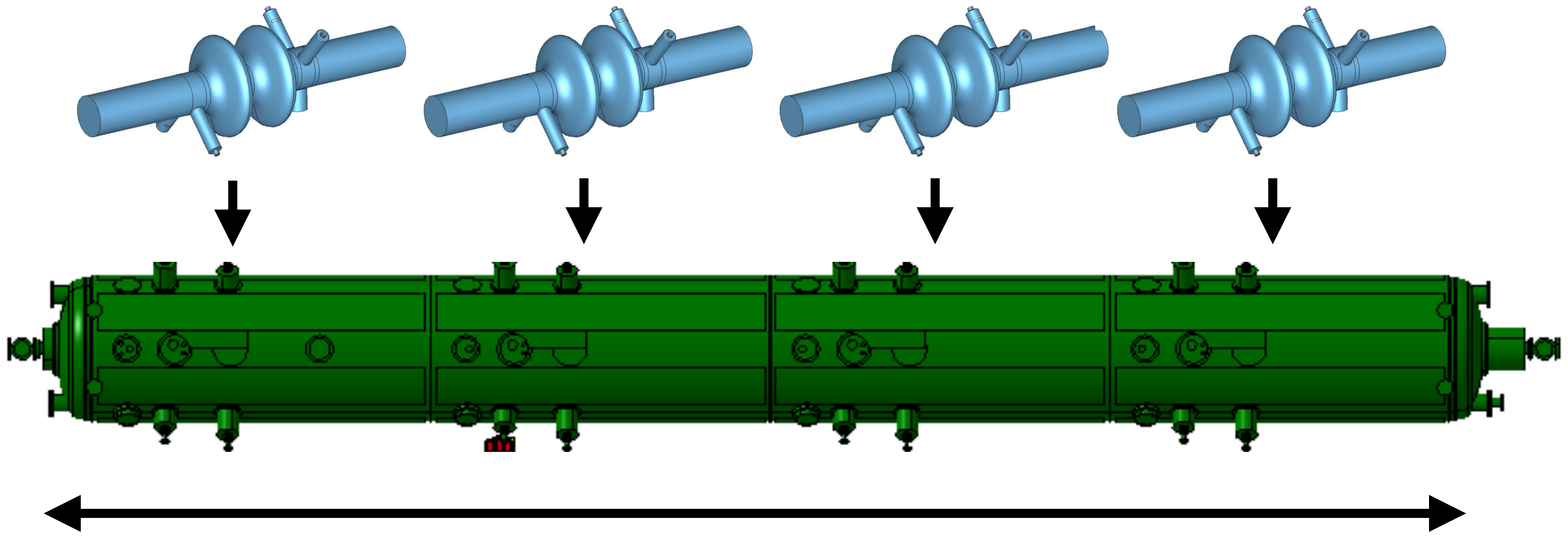


high power dc type operation:

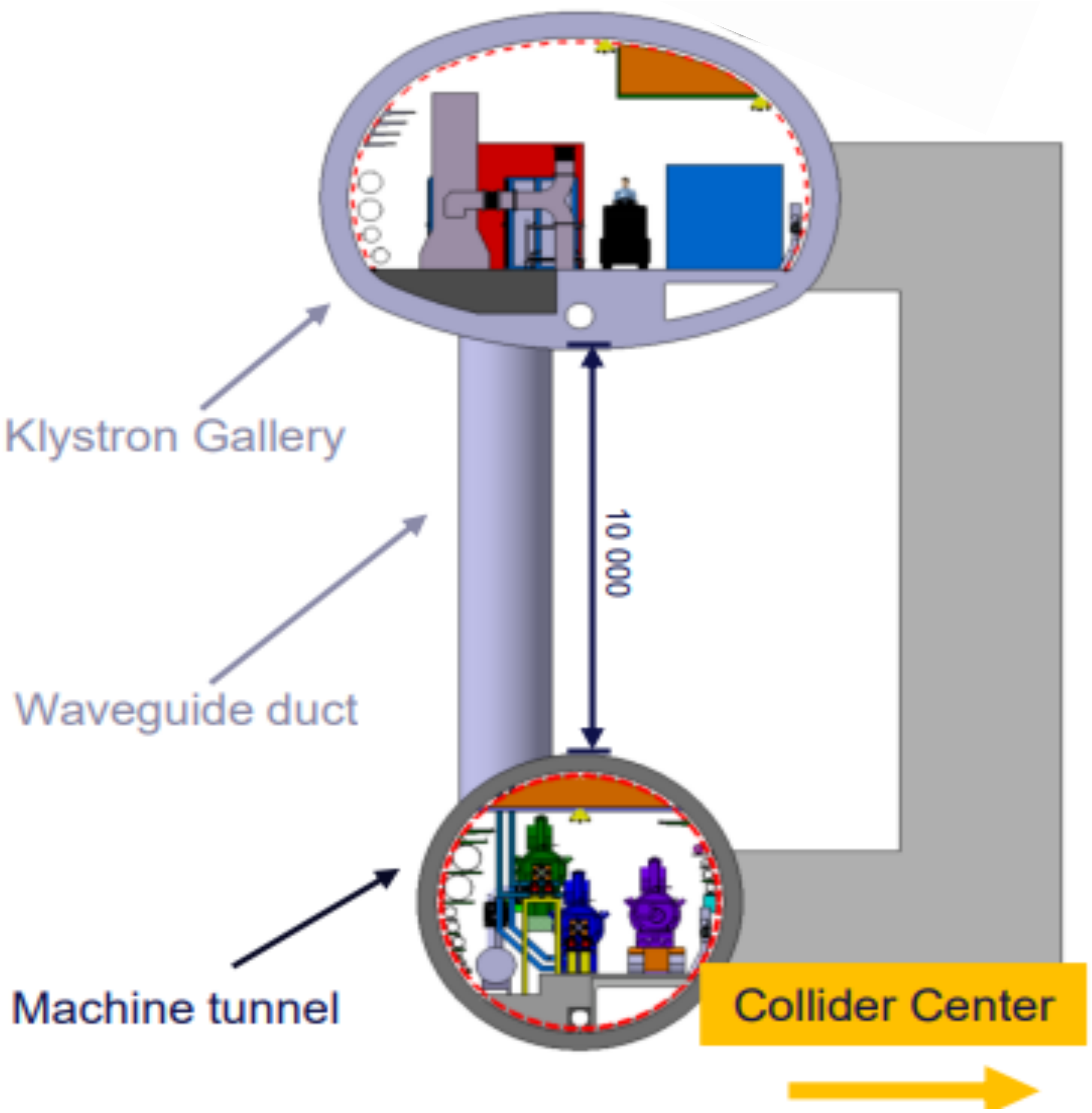
sc. cavities preferred, operational frequency range: ≈ 1 GHz

RF sections

- Reserve space for RF in straight sections
- Acceleration gradient ~20 MV/m
- Cavity consists of cells of length $L_{\mathrm{cav}} = \lambda_{\mathrm{RF}}/2$
- Superconducting cavities have to be installed in a cryomodule



FCC-ee 400 MHz Cavity Cryomodule: L = 12 m



Images from F. Valchkova, O. Brunner

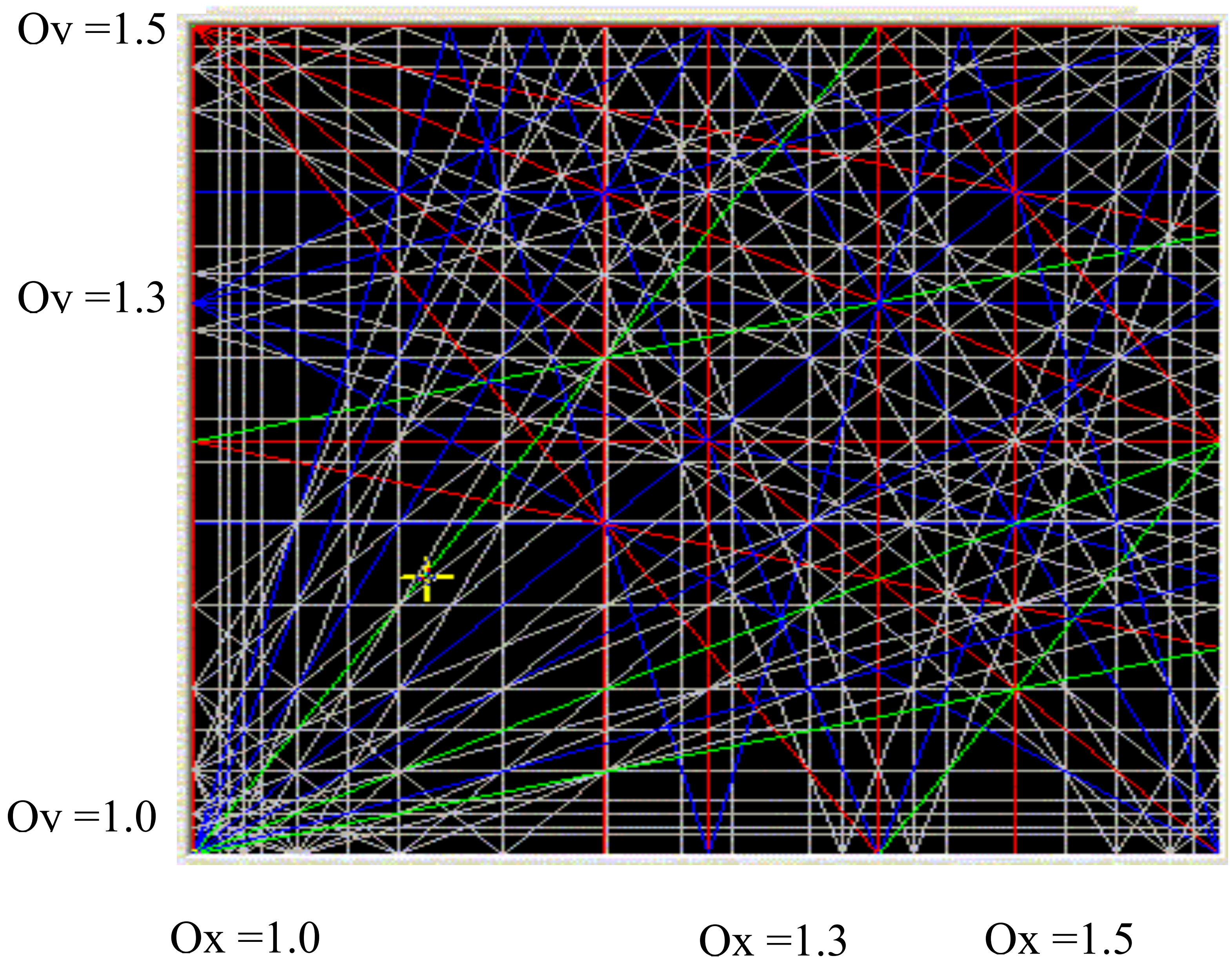
One word about RF Cavities

RF Cavities must be installed in …

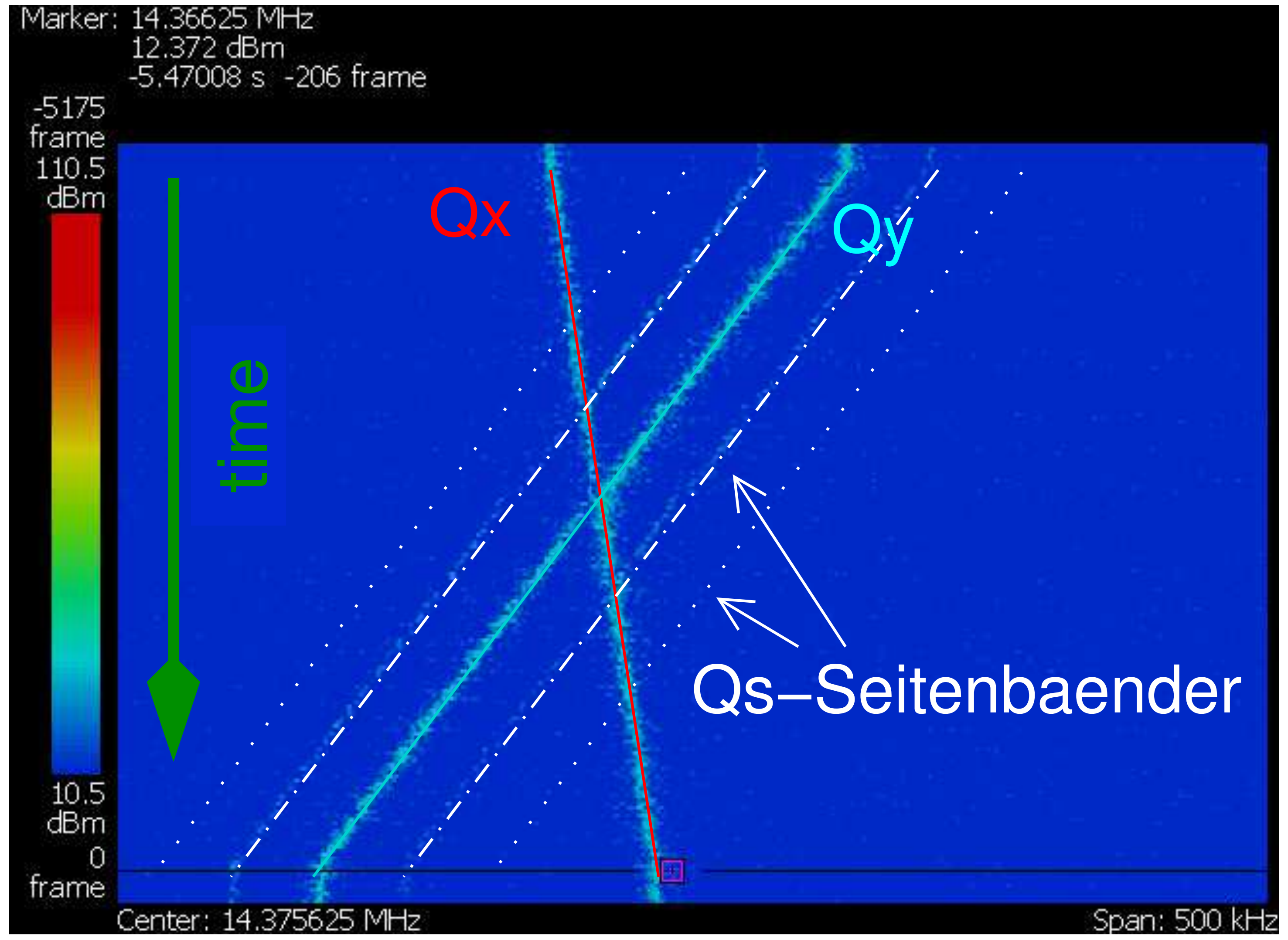
- - - - > Dispersion free sections < - - - -

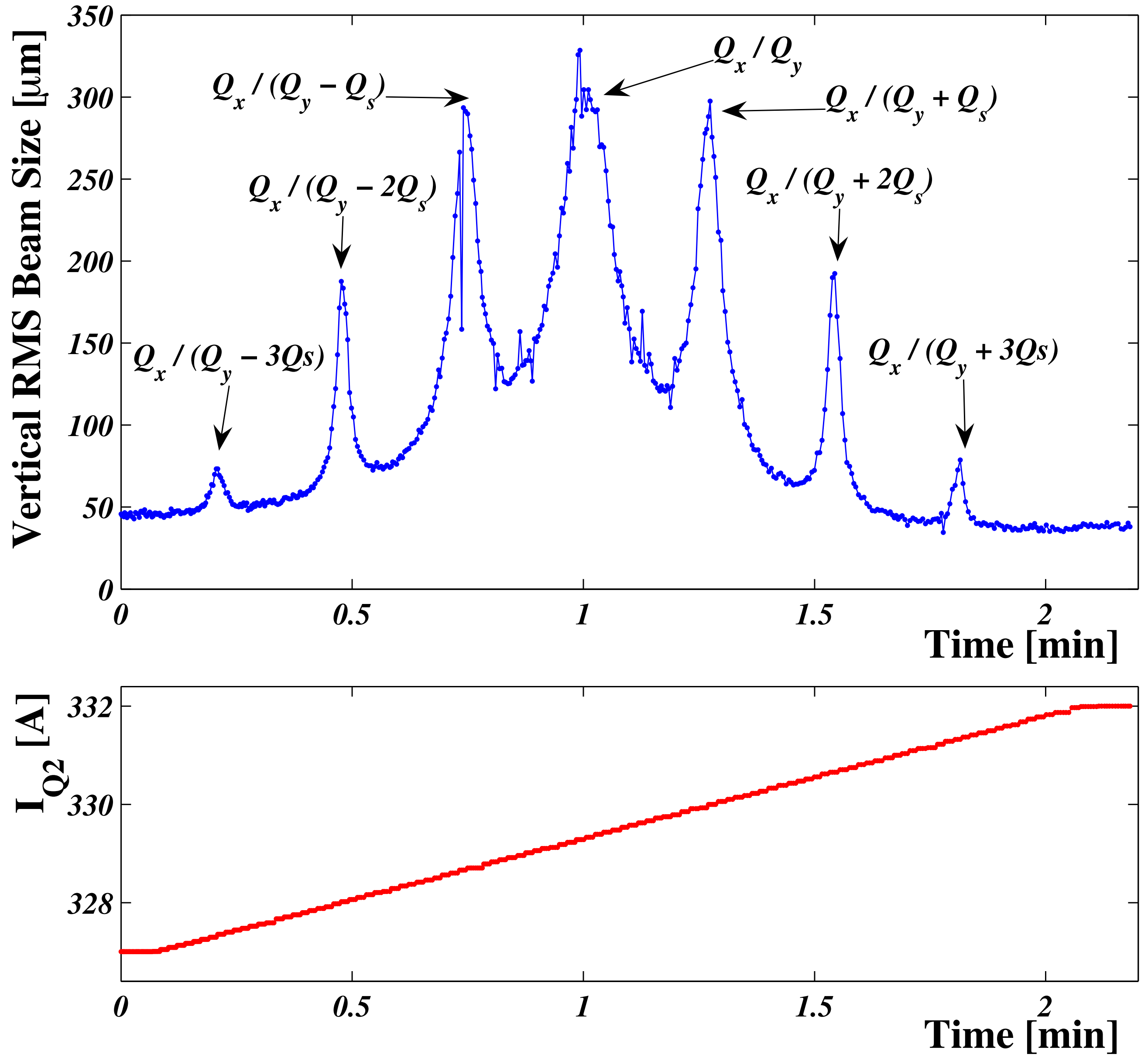
of the storage ring.

… in order to suppress coupling between the longitudinal motion (energy gain) and the transverse motion via dispersion.



$$m\,Q_x + n\,Q_y + l\;Q_s = \text{integer}$$





The logical path to Accelerator Design

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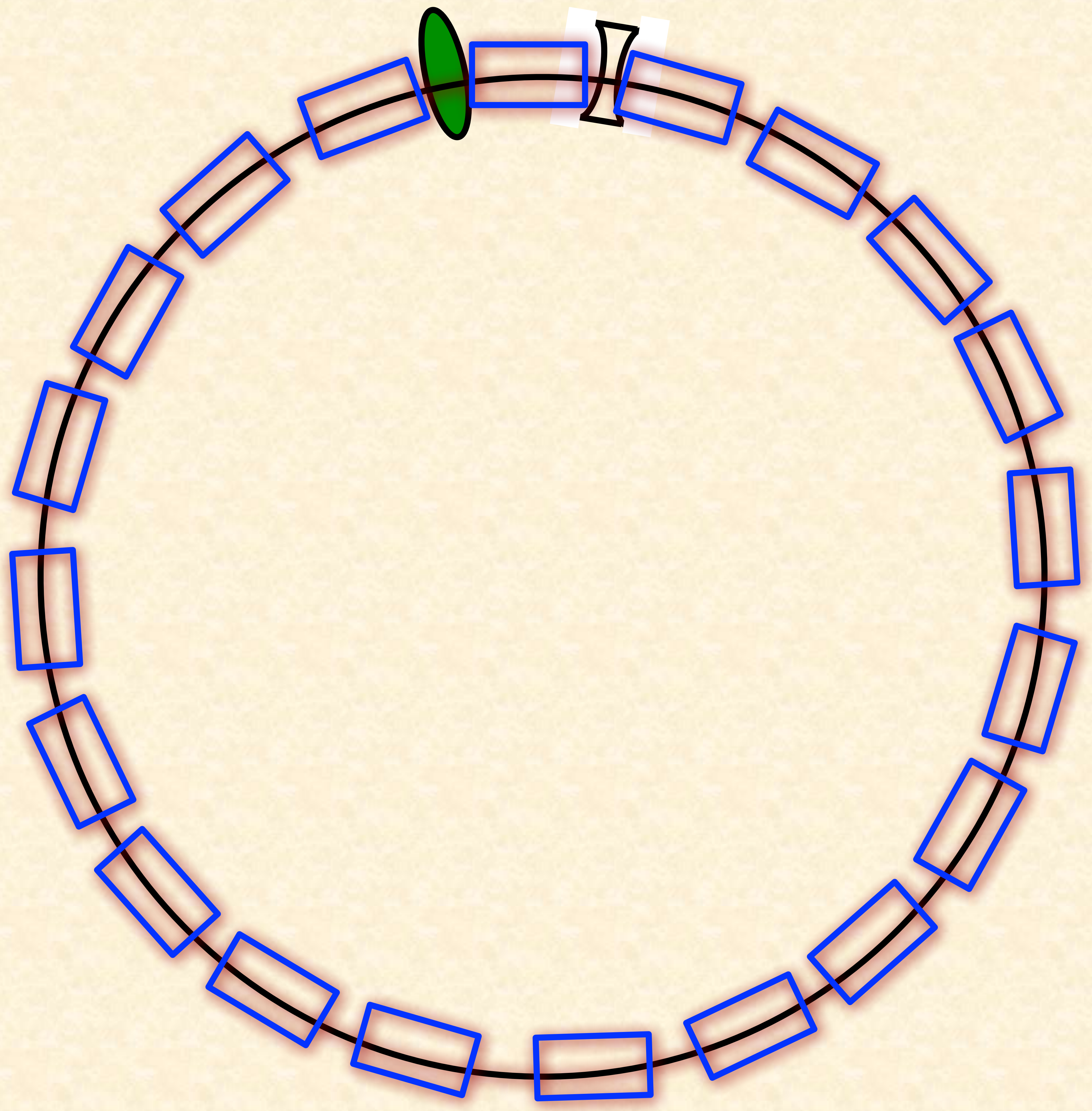
4.) Determine the radiation losses
Energy loss per turn
Power loss frequency
—> electrons radiate !!
—> protons do not !!
✓

5.) Determine the parameters for the RF system
Frequency, overall voltage,
space needed in the lattice
for the cavities
✓

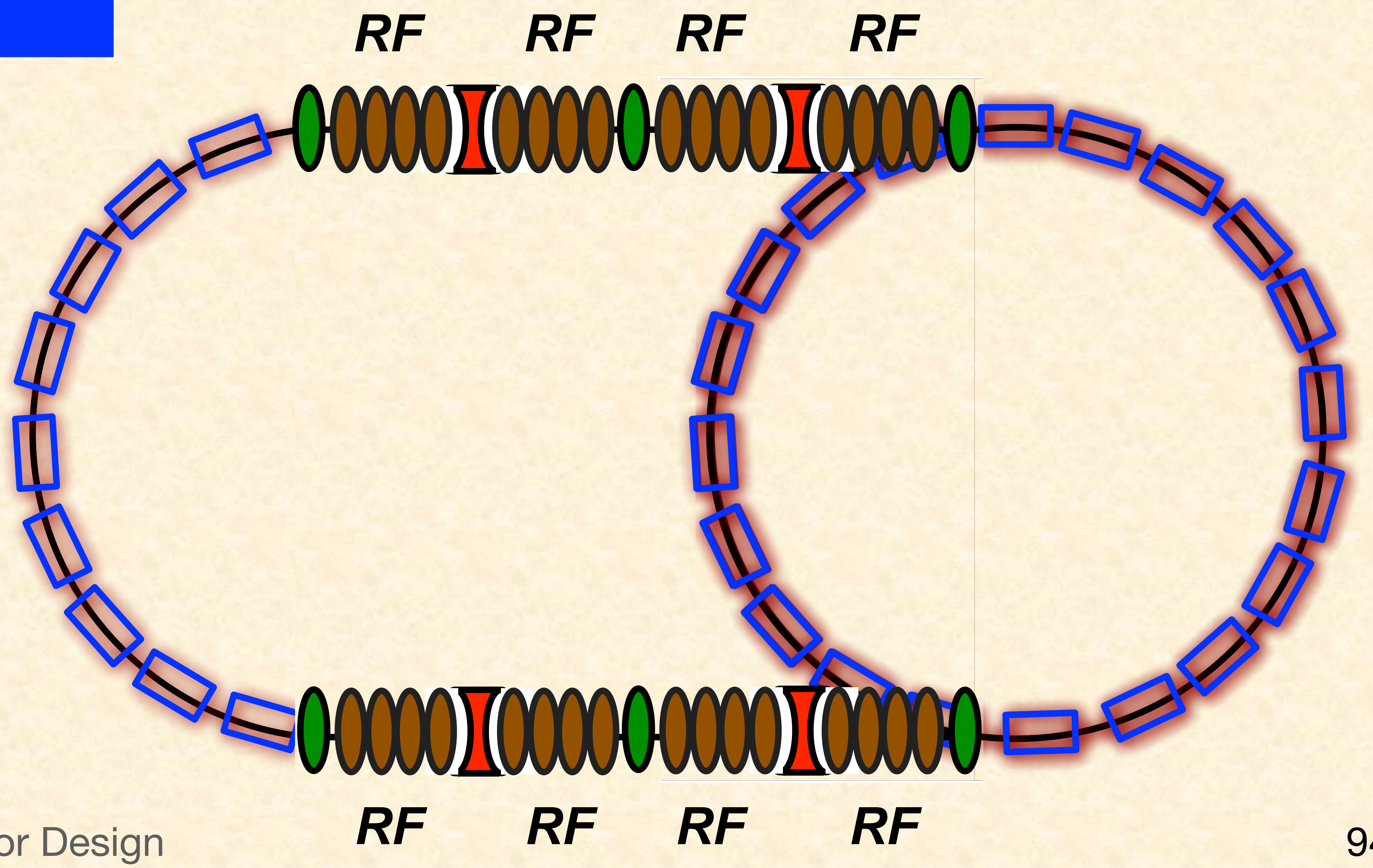


The logical path to Accelerator Design

6.) Open the lattice structure to install
- straight sections for the RF system
- optimise the phase advance per cell
- connect the straight sections to the arc lattice with dispersion suppressors
- choose which type fits best
- add eventually a matching section

Guideline:

Reserve ~20 % of circumference for RF, sections, injection&extraction, experiments, etc.



Dispersion function

Momentum spread of the beam adds a term on the r.h.s. of the equation of motion.
—> *inhomogeneous differential equation.*

$$x'' + x\left(\frac{1}{\rho^2} - k\right) = \frac{\Delta p}{p_0}\frac{1}{\rho}$$

—> *additional term in the solution for the particle's amplitude*

$$x(s) = x_\beta(s) + x_D(s)$$

$$x_D(s) = D(s)\frac{\Delta p}{p}$$

Normalise with respect to Δp/p:

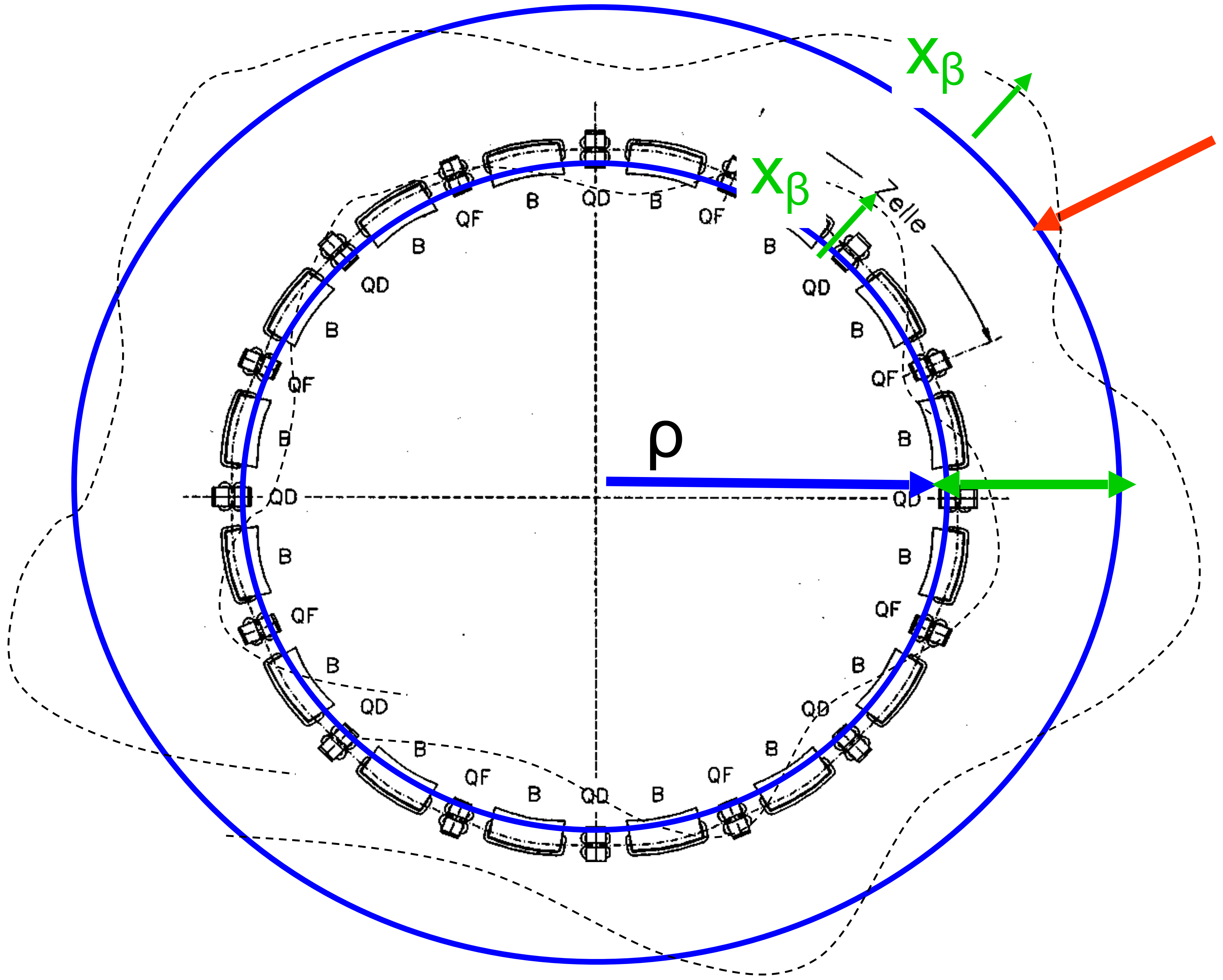
$$D(s) = \frac{x_i(s)}{\Delta p / p}$$

Dispersion function D(s) …

… is that special orbit, an ideal particle would have for Δp/p = 1

The orbit of any particle is the sum of the well known x_β and the dispersion

Dispersion orbit with homogeneous dipole field



Closed orbit for Δp/p > 0

$$x_i(s) = D(s) \cdot \frac{\Delta p}{p}$$

Example HERA

$$x_\beta = 1 \ldots 2\, mm$$

$$D(s) \approx 1 \ldots 2\, m$$

$$\Delta p / p \approx 1 \cdot 10^{-3}$$

contribution due to Dispersion ≈ beam size

—> Dispersion must vanish at the collision point

$$\sigma = \sqrt{\epsilon\beta + D^2\delta^2}$$

Dispersion suppressor - idea

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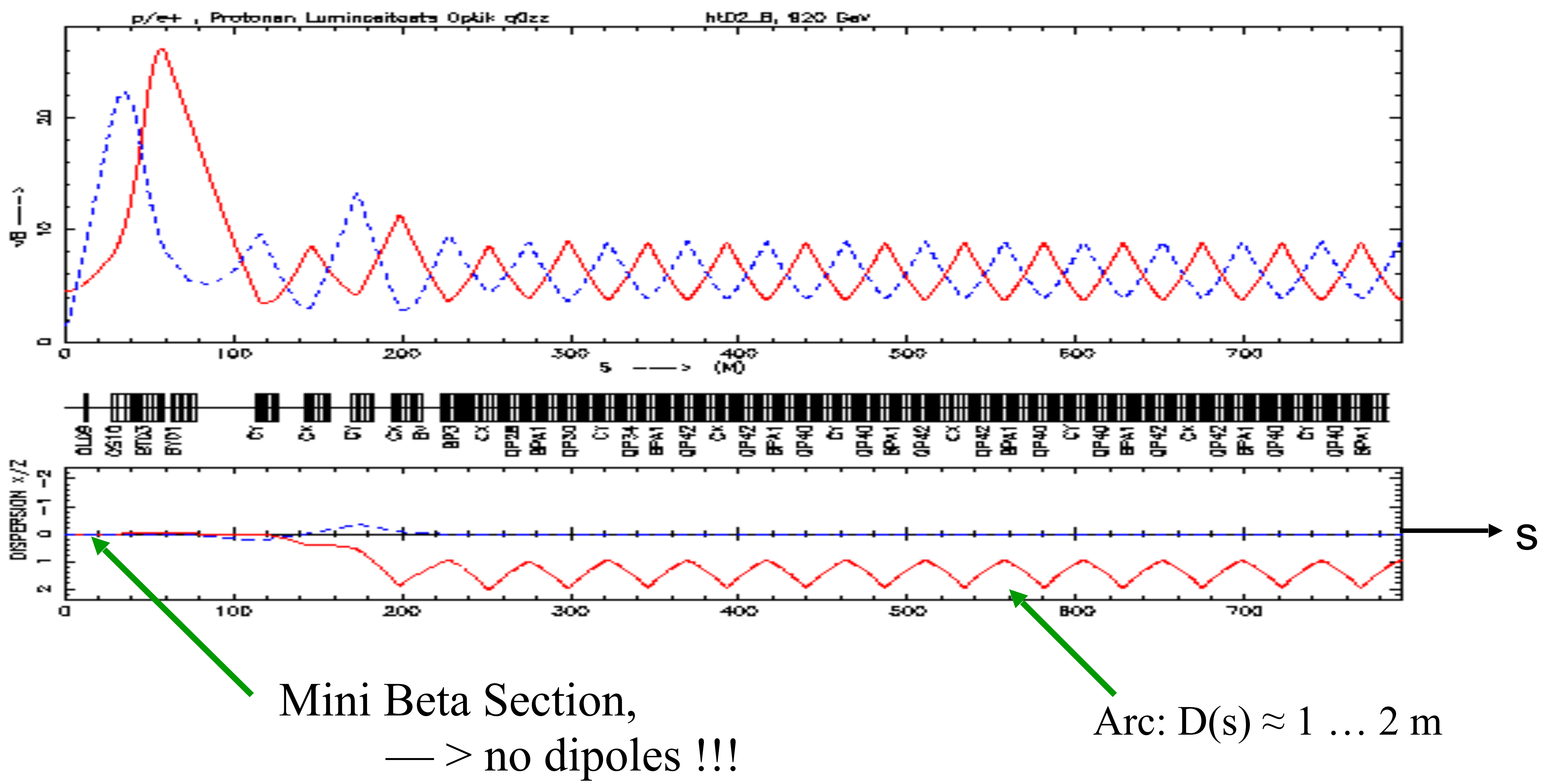
D(s) is created by the dipole magnets

… and afterwards focused by the quadrupole fields



Think right —> left :

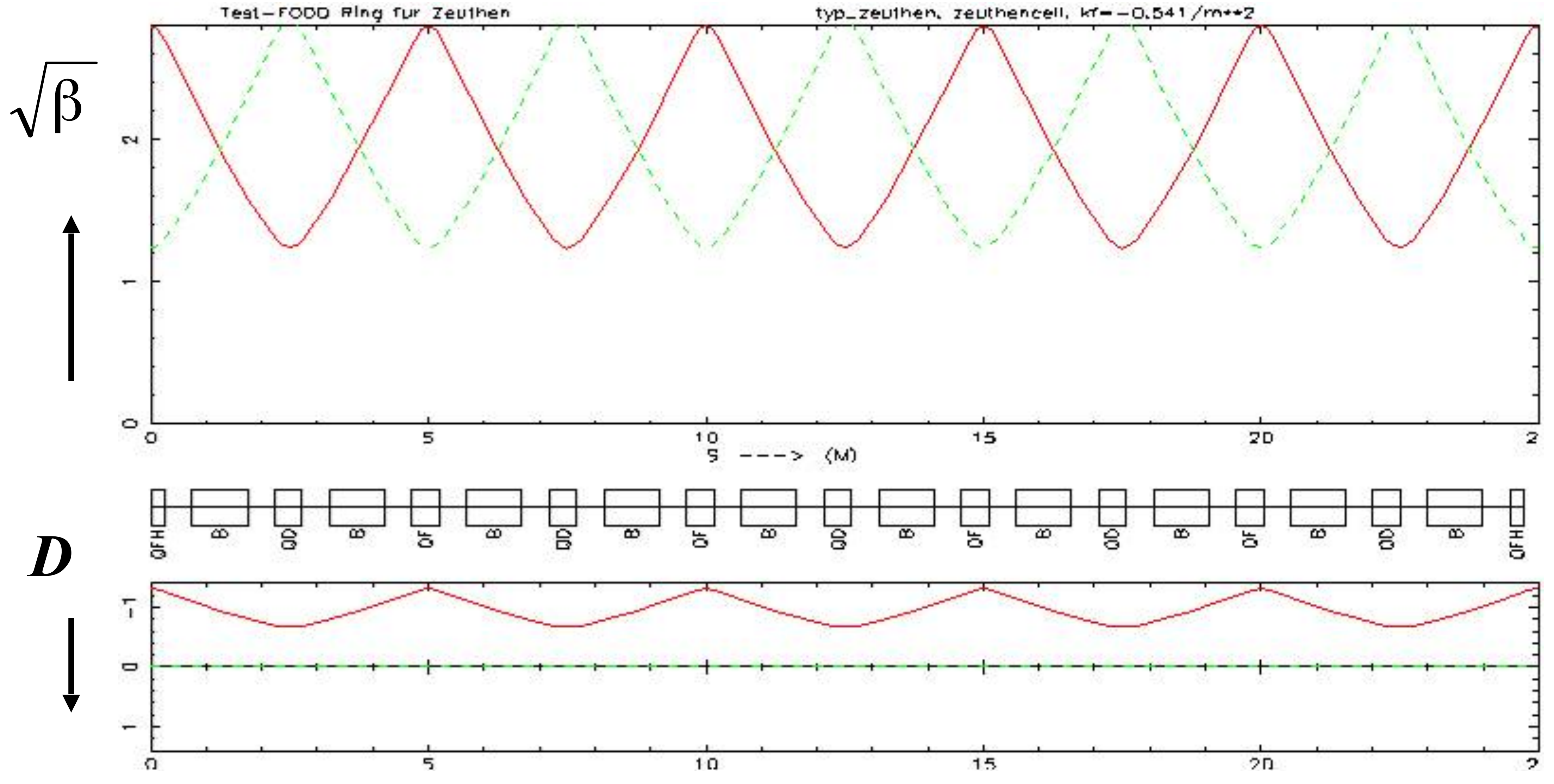
by clever arrangement of dipole fields & quadrupole strengths we can make D(s) vanish.

Quadrupole-based dispersion suppressor

The straight forward one: use *additional quadrupole lenses* to match the optical parameters ... including the D(s), D´(s) terms

Quadrupoles have an influence on
the optics (β function)
the phase advance
but also … the orbit.

And dispersion is "just another orbit".



FoDo cell: regular structure in the arc

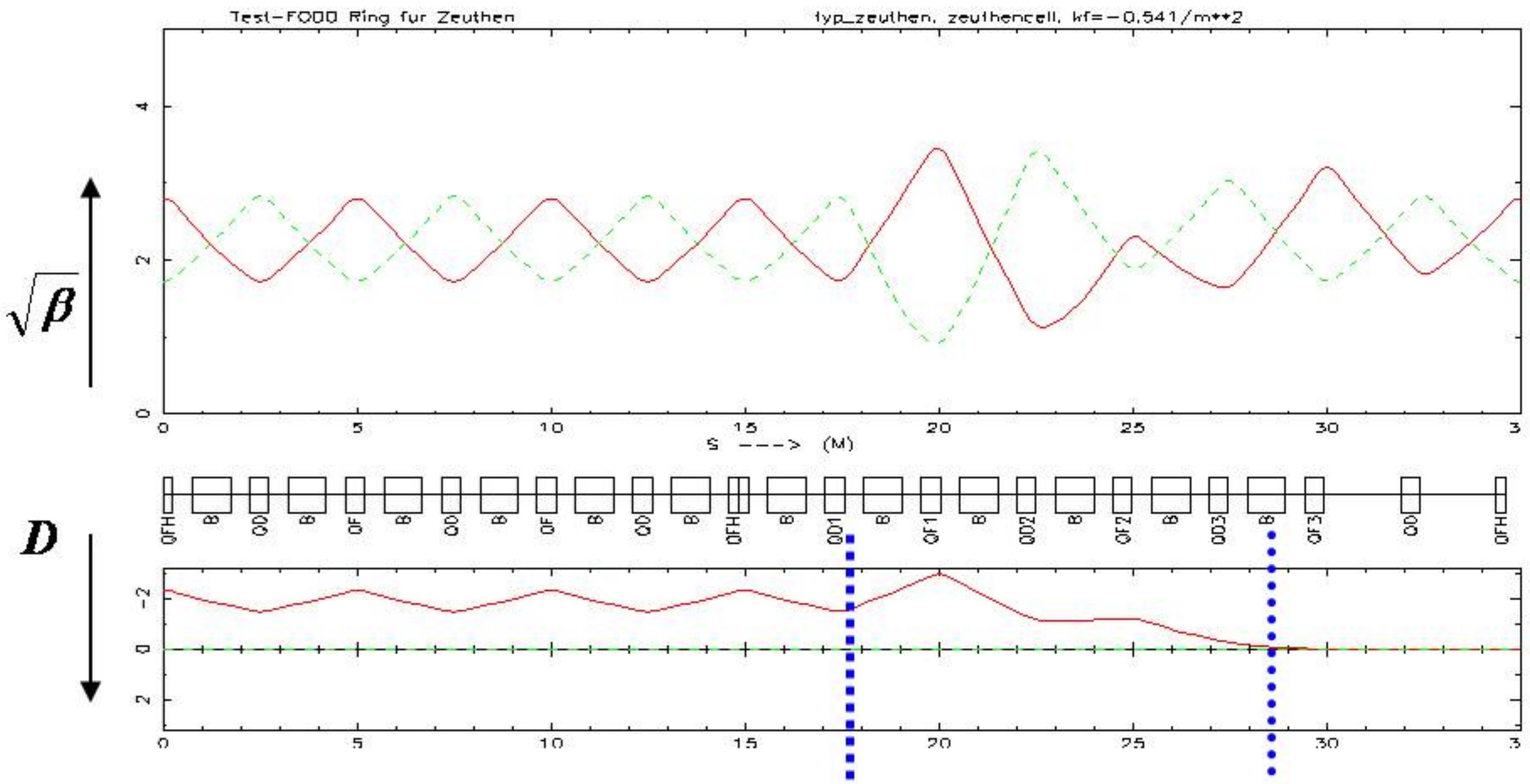
Correct the dispersion D and D' to zero by 2 quadrupole lenses,

Restore (match back) β and α to the values of the periodic solution by 4 additional quadrupoles

$$\left.\begin{array}{l} D(s), \quad D'(s) \\ \beta_x(s), \alpha_x(s) \\ \beta_y(s), \alpha_y(s) \end{array}\right\} \rightarrow$$

6 additional independent quadrupole lenses required

Quadrupole-based dispersion suppressor



periodic FoDo structure

matching section including 6 additional quadrupoles

dispersion free section, regular FoDo without dipoles

Advantage:

! easy

! flexible: it works for any phase advance per cell

! does not change the geometry of the storage ring

! can be used to match between different lattice structures (i.e. phase advances)

Disadvantage:

! additional power supplies needed (→ expensive)

! requires stronger quadrupoles

! due to higher β values: more aperture required

Dipole-based schemes

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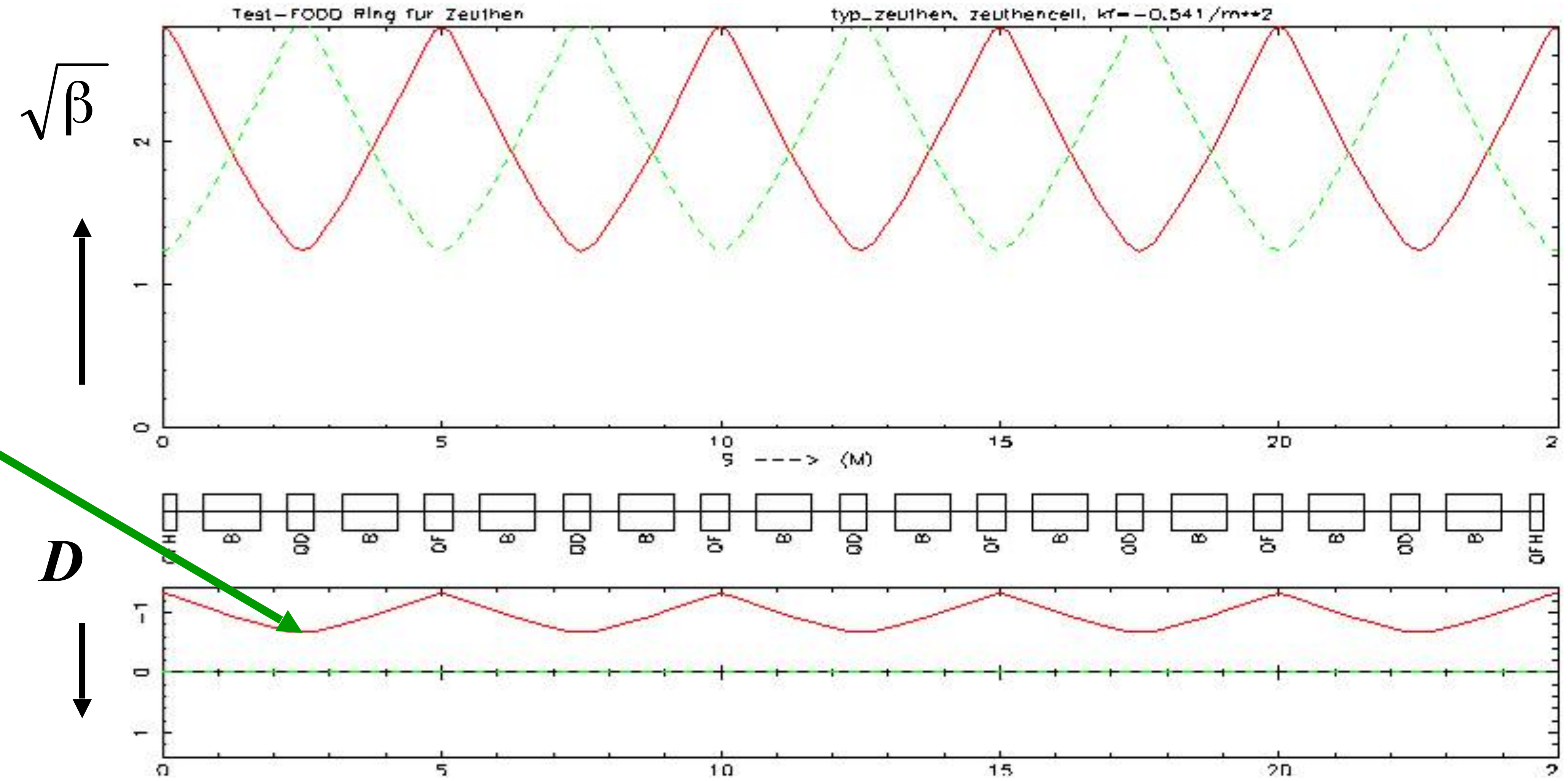
Dipole based schemes: the clever way

periodic dispersion in the arc

(FoDo in thin lens approx)

$$\hat{D} = \frac{L^2}{\rho} \frac{\left(1 + \frac{1}{2}\sin(\mu/2)\right)}{4\sin^2(\mu/2)}, \quad D' = 0$$



Think right —> left :

arrange a number of dipoles to build up — from zero — dispersion that fits to the periodic solution

$$D(s) = S(s) * \int \frac{1}{\rho(\tilde{s})} C(\tilde{s})\, d\tilde{s} - C(s) * \int \frac{1}{\rho(\tilde{s})} S(\tilde{s})\, d\tilde{s}$$

Arrange the dipole fields (1/ρ) in a way to create at the beginning of the regular arc cells $\hat{D}$ and D’=0.

… how this is done in detail —> see appendix

Half-bend dispersion suppressor

condition for vanishing dispersion:

so if we require $D = 0$,

$$2 * \delta_{supr} * \sin^2\left(\frac{n\Phi_c}{2}\right) = \delta_{arc}$$

with δ_{supp} = dipole strength in the suppressor region

with δ_{arc} = dipole strength in the arc structure

with Φ_c = phase advance per cell

... proof ... is easy but lengthy
—> appendix

and we can set $\delta_{supr} = \frac{1}{2} * \delta_{arc}$ ***—> we get*** $\sin^2\left(\frac{n\Phi_c}{2}\right) = 1$

and equivalent for D'=0 ***—> we get*** $\sin(n\Phi_c) = 0$

For a given phase advance per cell we just have to add up the number of cells to get theses conditions fulfilled.

Which means ... $n\Phi_c = k * \pi, \quad k = 1,3,\ldots$

In the n suppressor cells the phase advance has to accumulate to a odd multiple of π

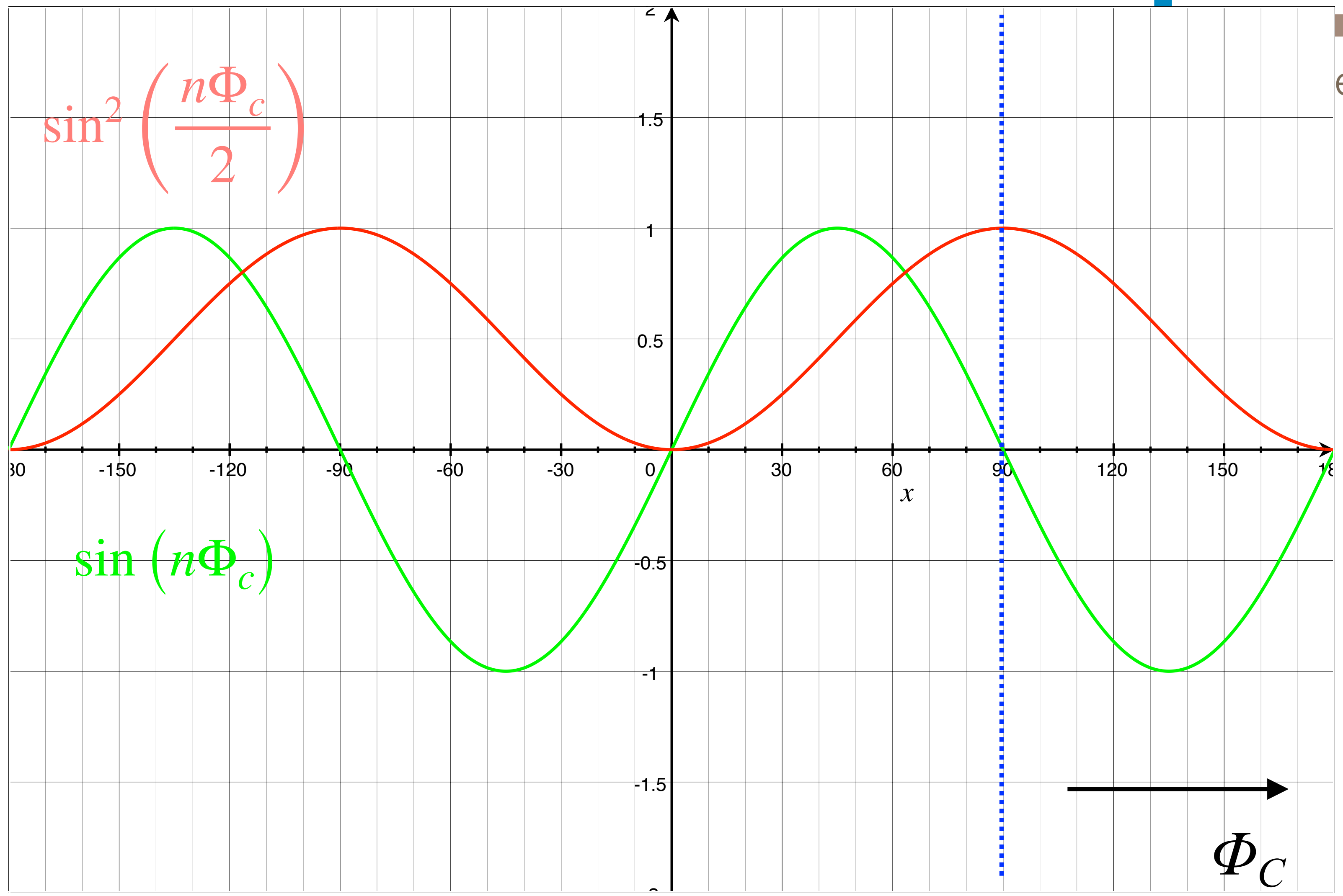
Half-bend dispersion suppressor - II

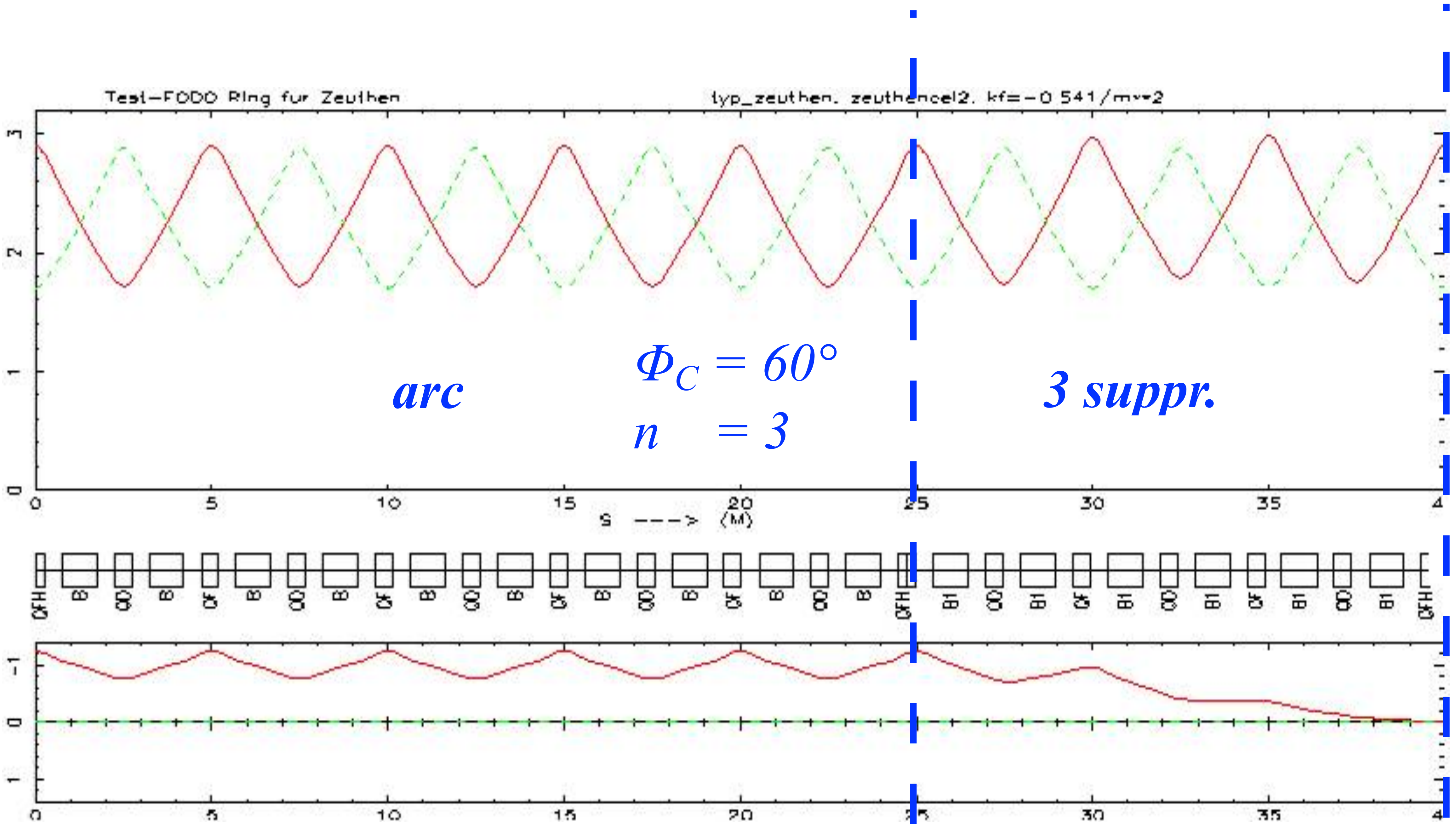
$$\sin^2\left(\frac{n\Phi_c}{2}\right) = 1$$

$$\sin(n\Phi_c) = 0$$

$\Phi_C = 90°$
$n = 2$

—> $n\Phi_c = k * \pi\,, \qquad k = 1,3,\ldots$





strength of suppressor dipoles half as strong as that of arc dipoles

Example:

phase advance in the arc
$\Phi_C = 90°$
number of suppressor cells $n = 2$

phase advance in the arc
$\Phi_C = 60°$
number of suppressor cells $n = 3$

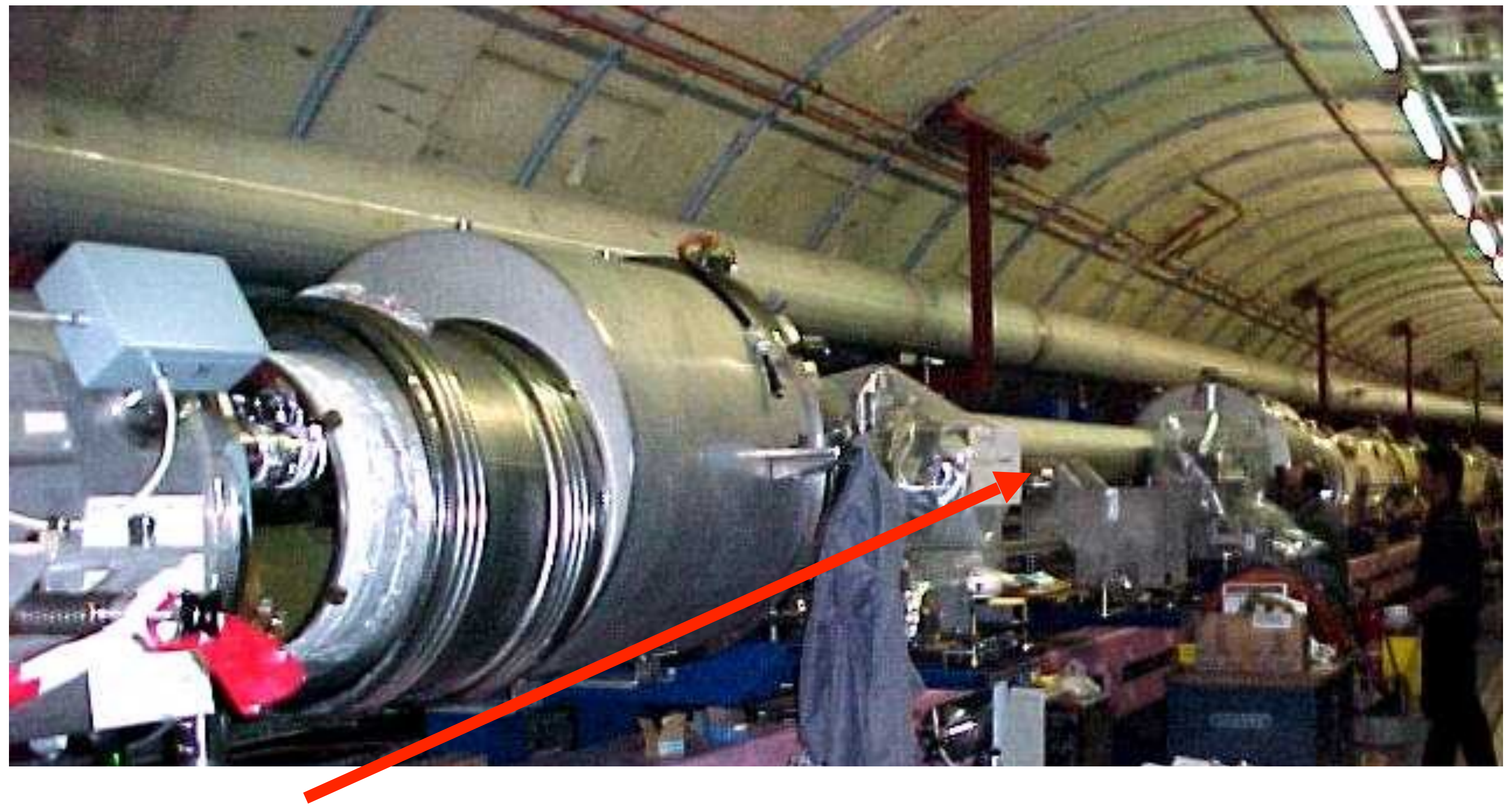
Missing-bend dispersion suppressor

conditions for the (missing) dipole fields:

$$\frac{2m+n}{2}\Phi_C = (2k+1)\frac{\pi}{2}$$

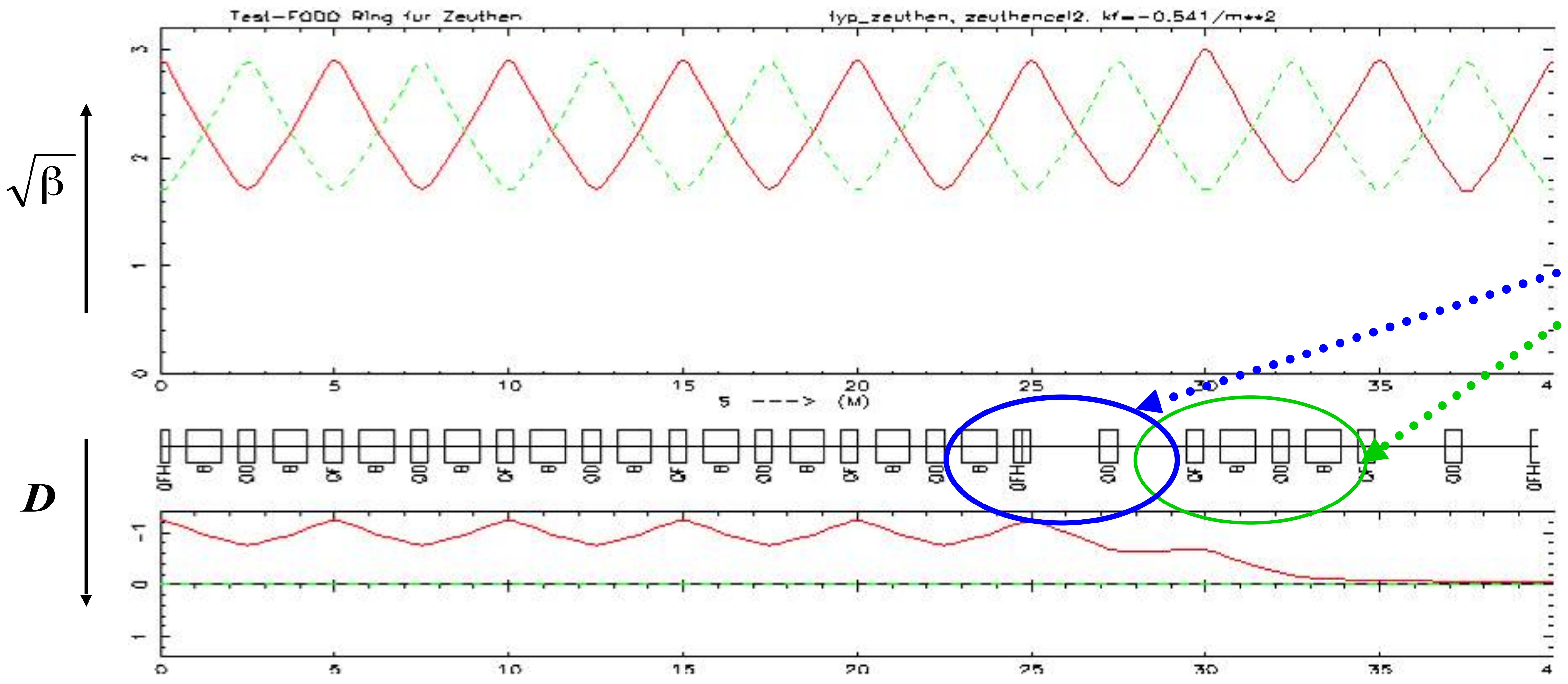
$$\sin\frac{n\Phi_C}{2} = \frac{1}{2}\ ,\ k = 0,2\ \ldots\ \textit{or}$$

$$\sin\frac{n\Phi_C}{2} = \frac{-1}{2}\ ,\ k = 1,3\ \ldots$$



Empty cell suppressor in HERA

m = number of cells without dipoles
followed by
n regular arc cells.

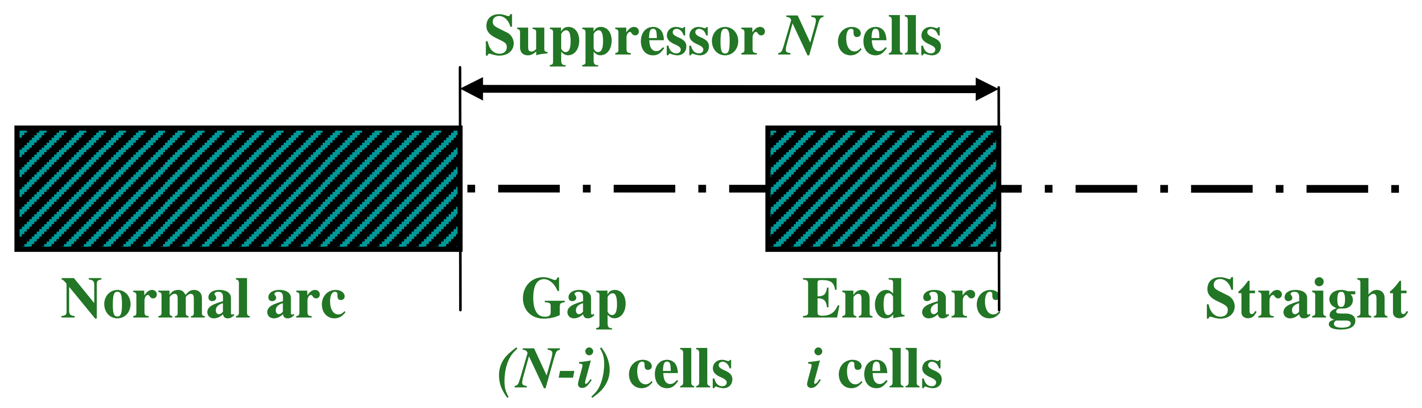


Example:

phase advance in the arc $\Phi_C = 60°$

number of suppr. cells $m = 1$
number of regular cells $n = 1$

Dispersion suppressors



- Missing-magnet suppressors for FODO arcs (Fquad. + Dipole + Dquad. + Dipole):

N	*Gap*	*i*	$\Delta\mu$	End arc dipole θ
2	1	1	60°	L/ρ
3	1	2	45°	$(L/\rho)/\sqrt{2}$
4	2	2	30°	$(L/\rho)/2$

- Half-field suppressors for FODO arcs ($N = i$, <u>no gap</u>)

N=i	*Gap*	$\Delta\mu$	End arc dipole θ
2	0	90°	$(L/\rho)/2$
3	0	60°	$(L/\rho)/2$
4	0	45°	$(L/\rho)/2$

Half-field is useful in electron machines as it reduces the synchrotron radiation into the experimental region.

JUAS17_02- P.J. Bryant - Lecture 2
Lattice Design I - Slide18

(Phil Bryant)

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Comments:

- Dipole-based dispersion suppressors affect the geometry of the ring
- … but not the optics!
- If the footprint of a new accelerator is pre-defined (e. g. existing tunnel,…), this concept cannot be fully exploited.

→ Dispersion suppressor has to be supported by quadrupoles.

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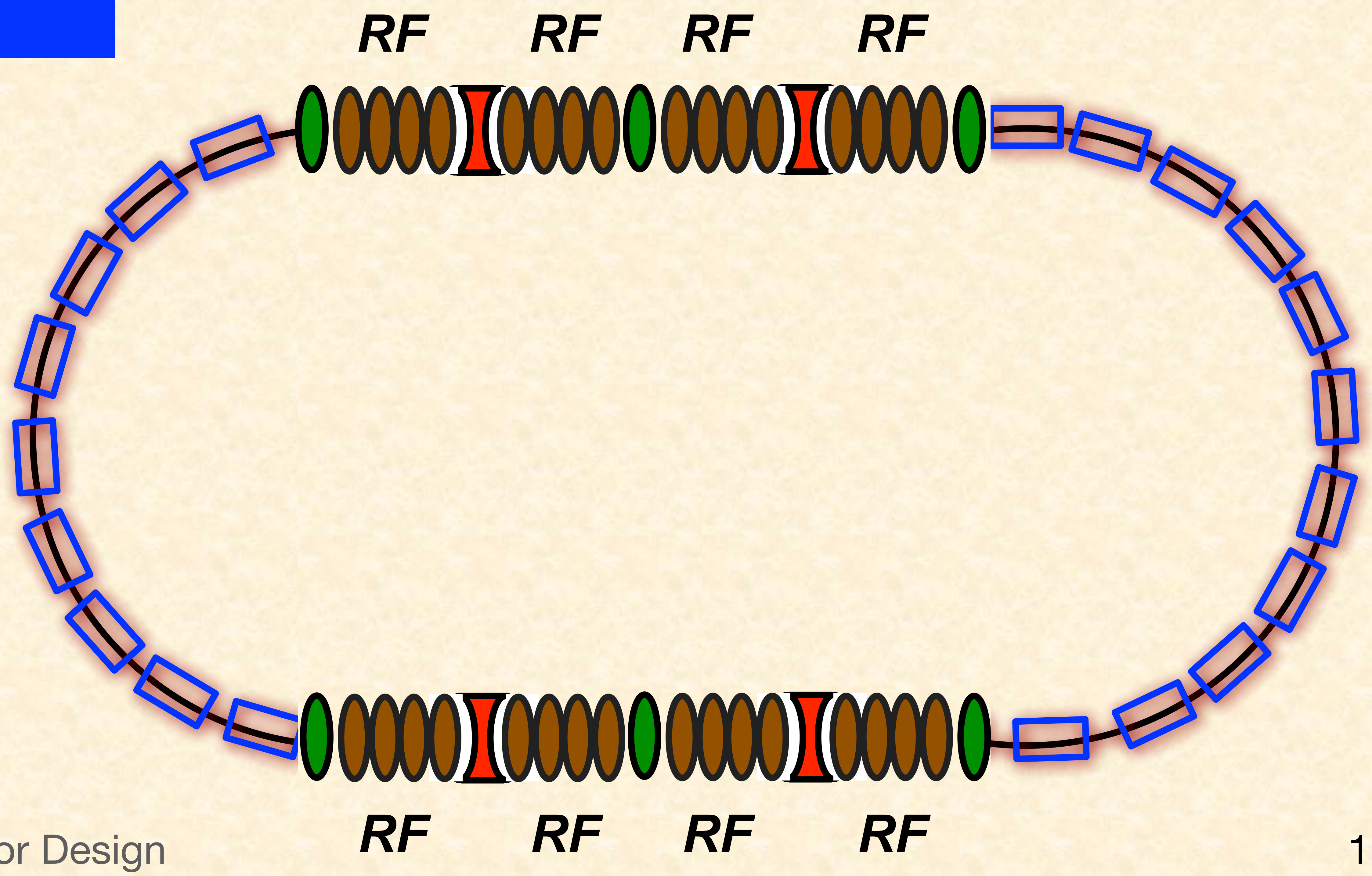
6.) Open the lattice structure to install
straight sections for the RF system
optimise the phase advance per cell
connect the straight sections to the arc lattice with
dispersion suppressors
choose which type fits best ✓
add eventually a matching section



General Remark:

Whenever we combine two different lattice structures we need a

"matching section"

in between to adapt the optics functions between the two lattices.

Example: Change of phase advance per cell

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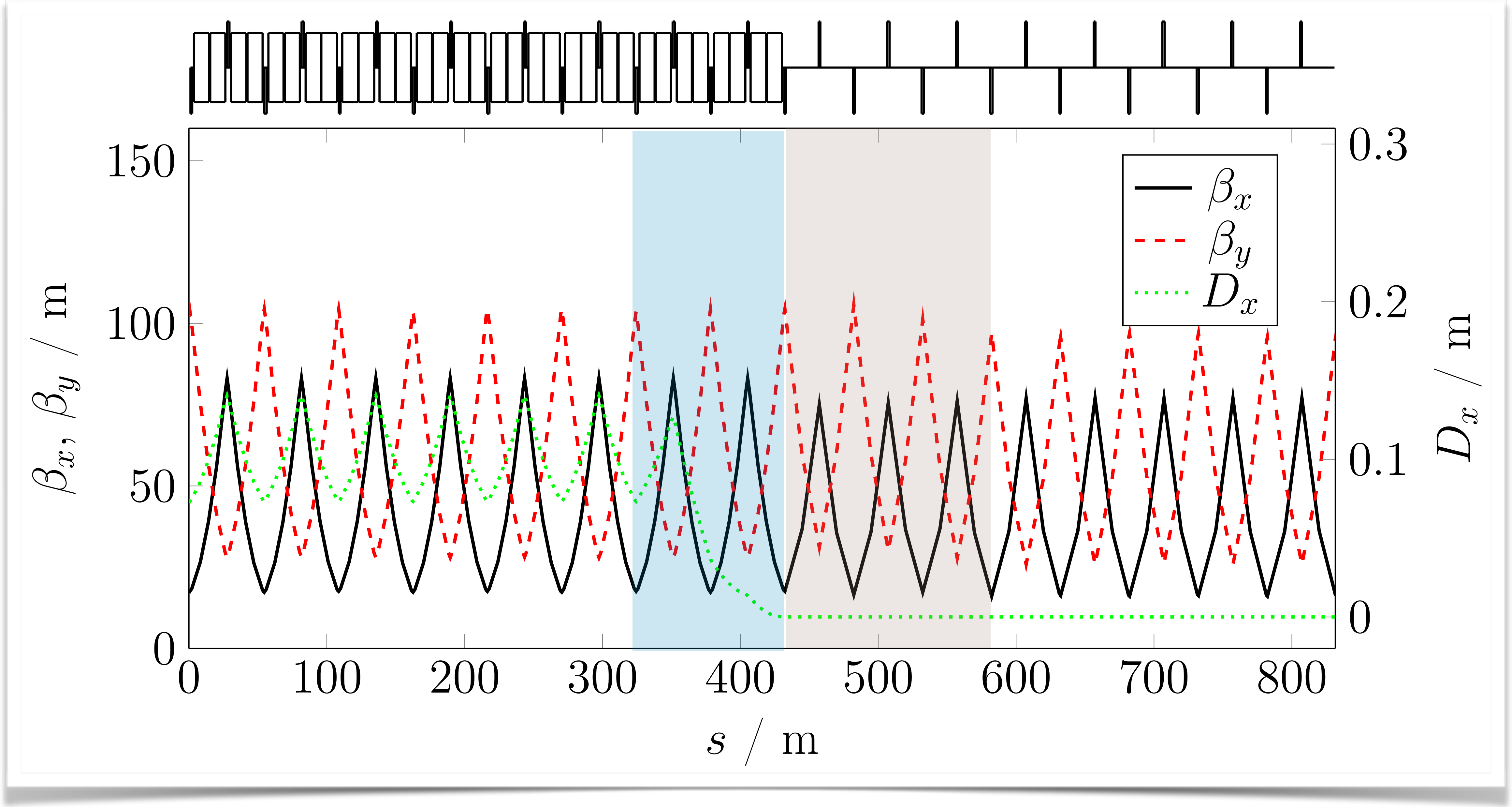
54 m arc cell ⇒ 50 m straight cell

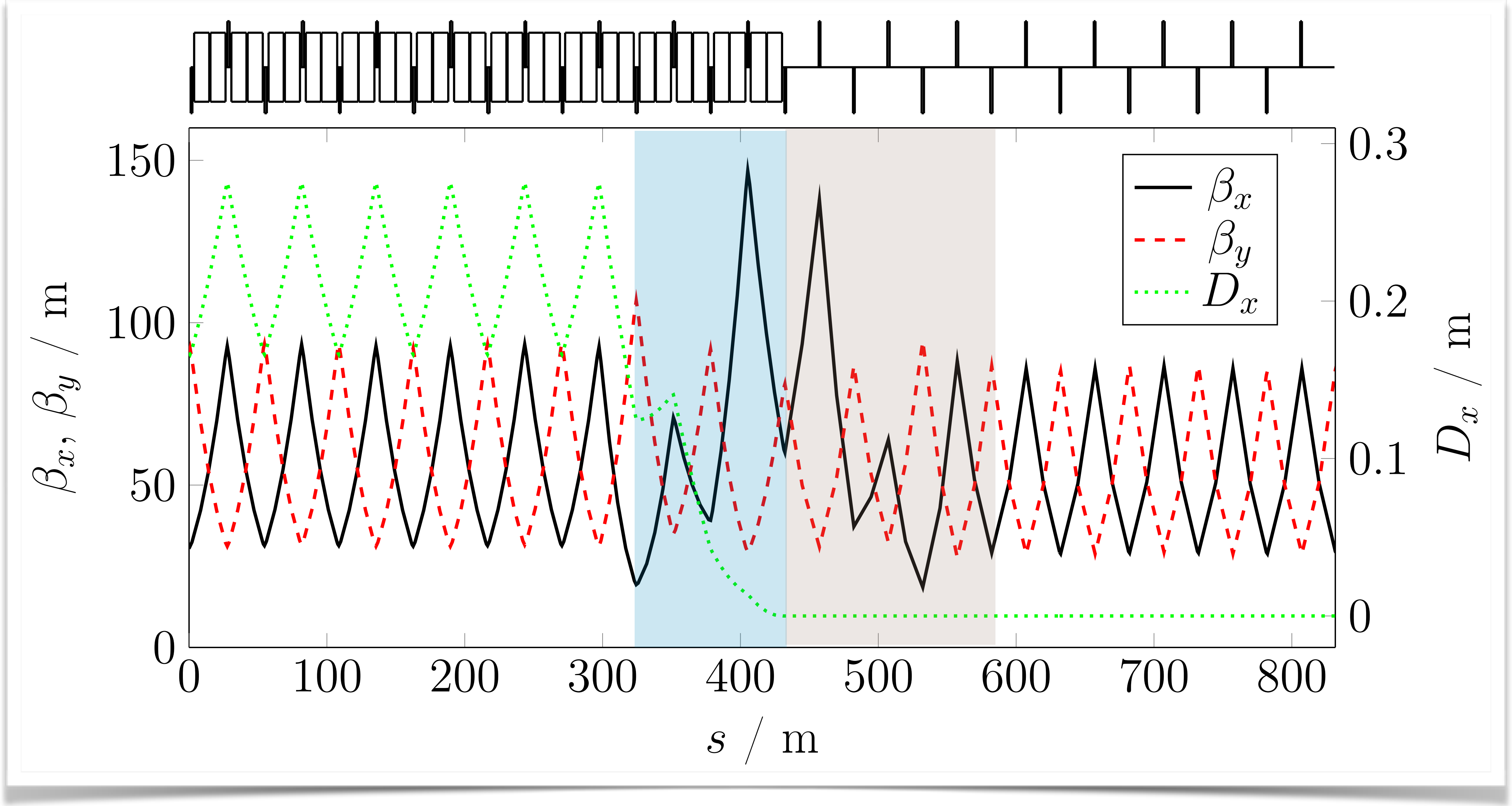
- $\mu = 90°$



half-bend DS
(4 quadrupoles)

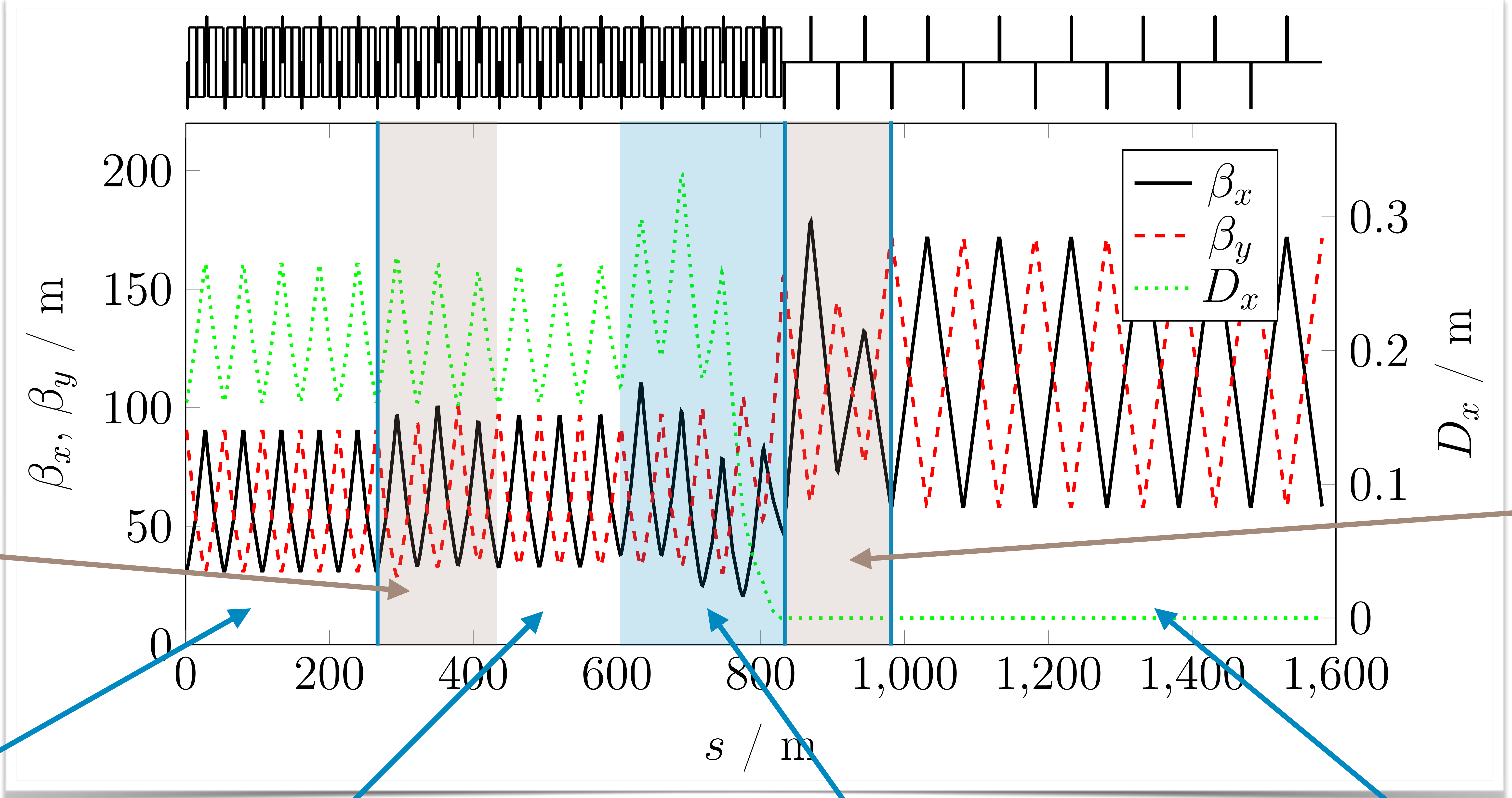
matching section
(5 quadrupoles)

- switch to $\mu = 60°$



half-bend DS needs to be supported by quadrupoles

Example: FCC-ee top-up booster synchrotron



Matching section
6 quadrupoles

Matching section
4 quadrupoles
L_{cell} = 75 m

16 km arc
L_{cell} = 54 m
rho = 13.15 km

FCC-hh dispersion suppressor section
L_{cell} = 56.6 m
rho = 15.06 km

Dispersion suppressor
8 quadrupoles
L_{cell} = 56.6 m
rho = 15.06 km

Straight section for RF installation
L_{cell} = 100 m
(no contribution to ϵ_x)

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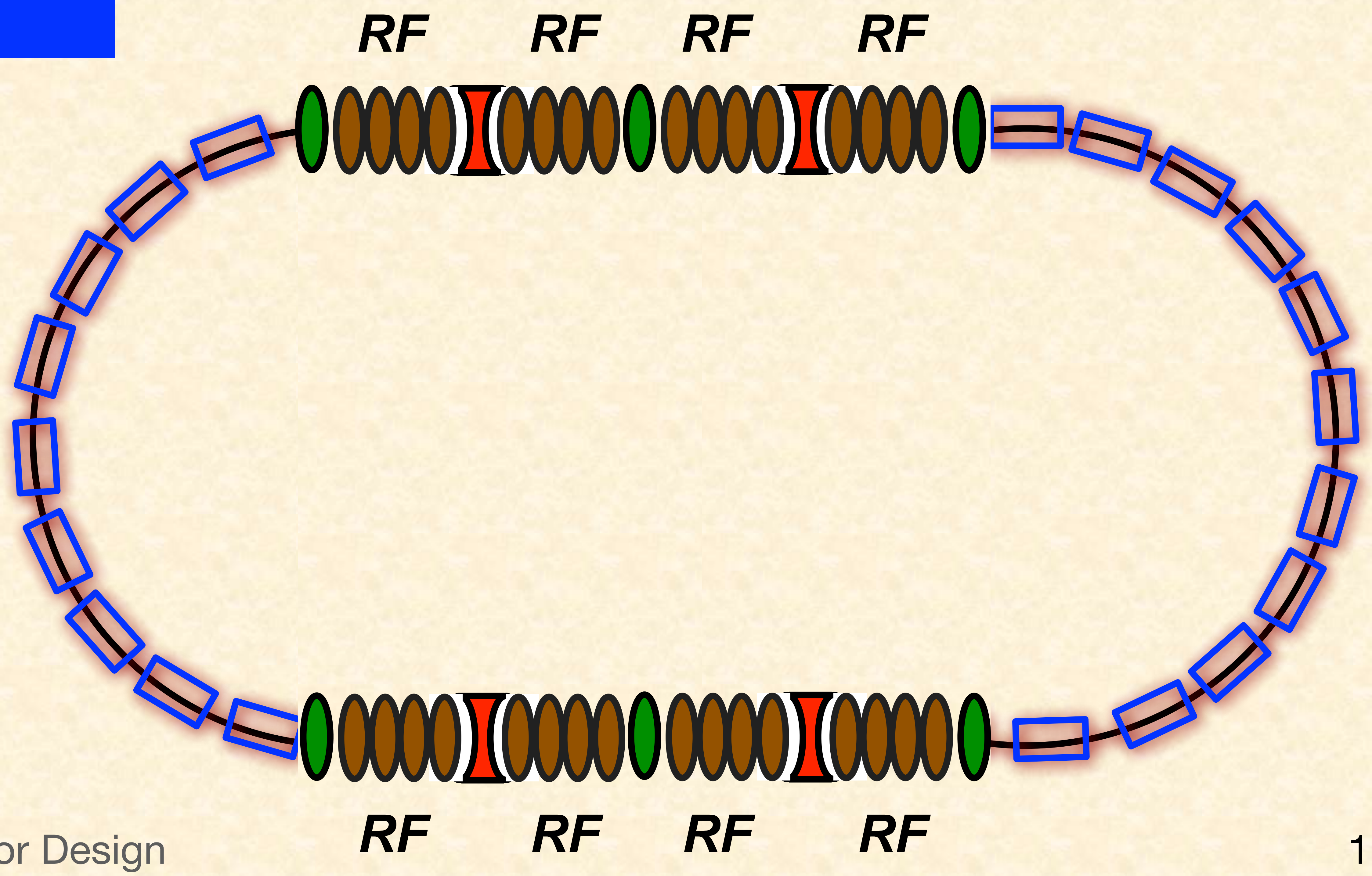
dispersion suppressors
choose which type fits best ✓
add eventually a matching section ✓



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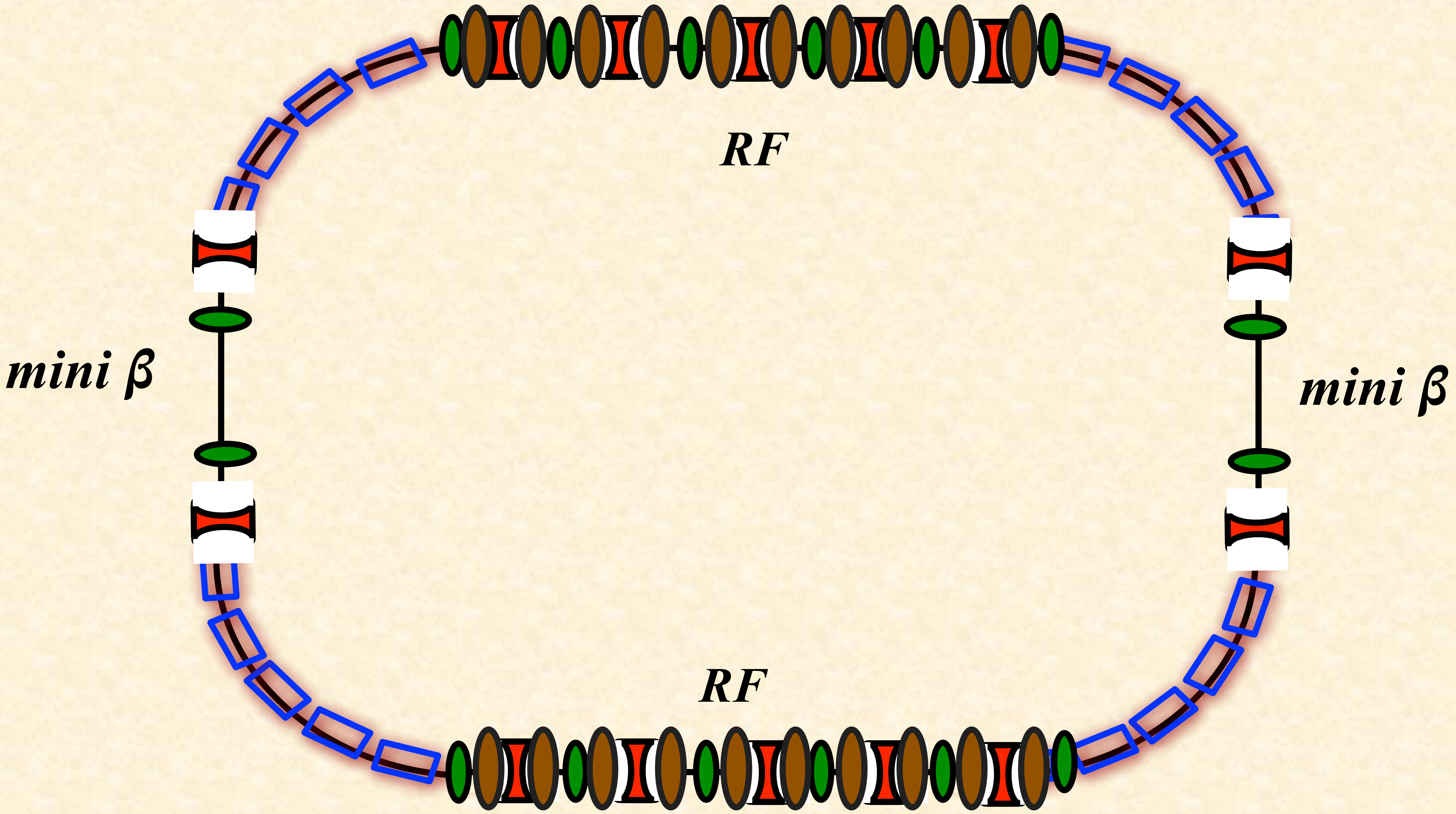
7.) Open the lattice structure to install
- a dispersion free straight section for the mini beta insertion
- define independent quadrupoles (four if D_x=0)
- connect the straight sections to the arc lattice with mini-beta quadrupoles and matching quadrupoles
- match to the desired β^*



Prepare for beam collisions

… there is just a little problem

Problem: Our particles are VERY small!

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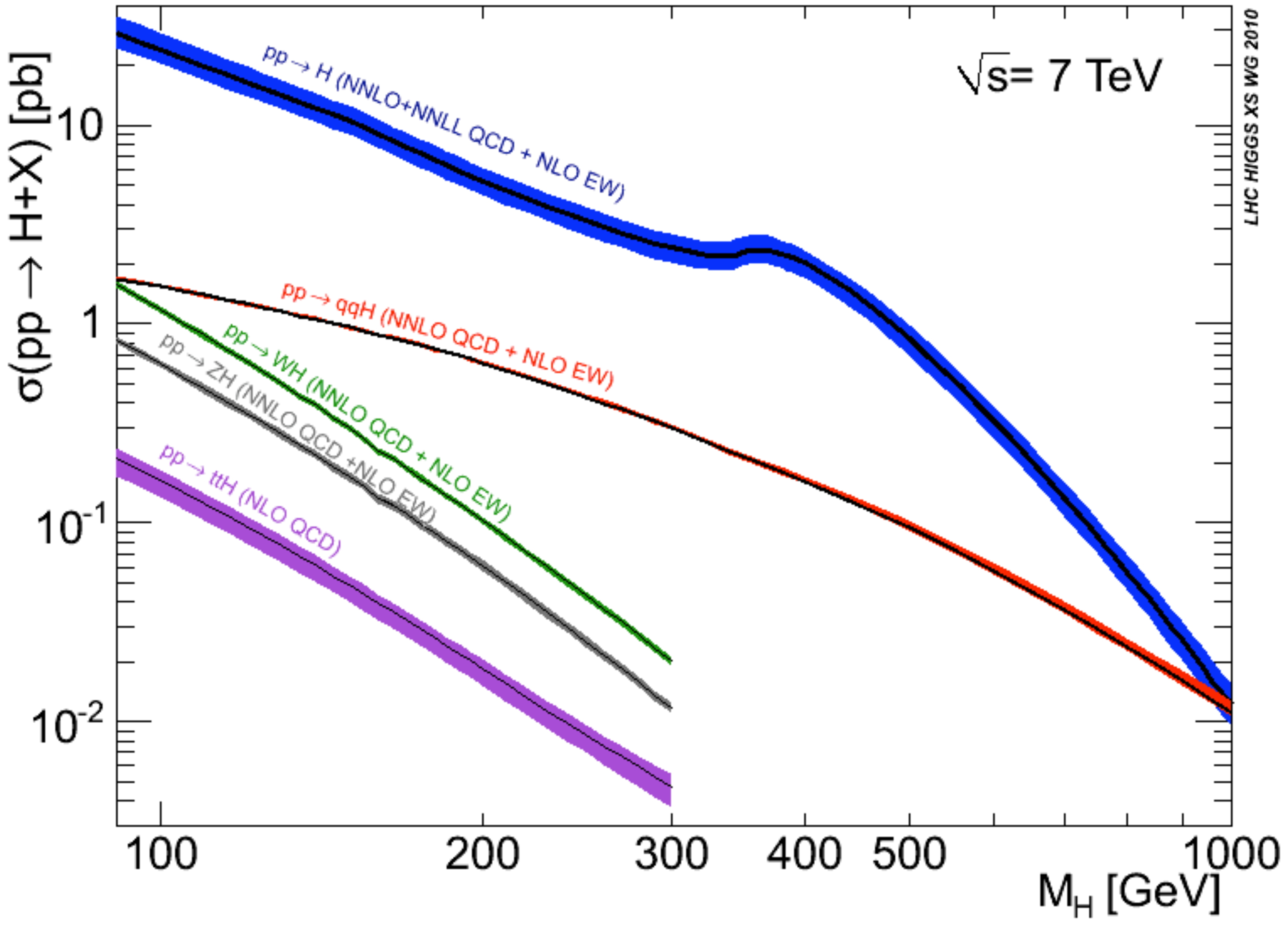
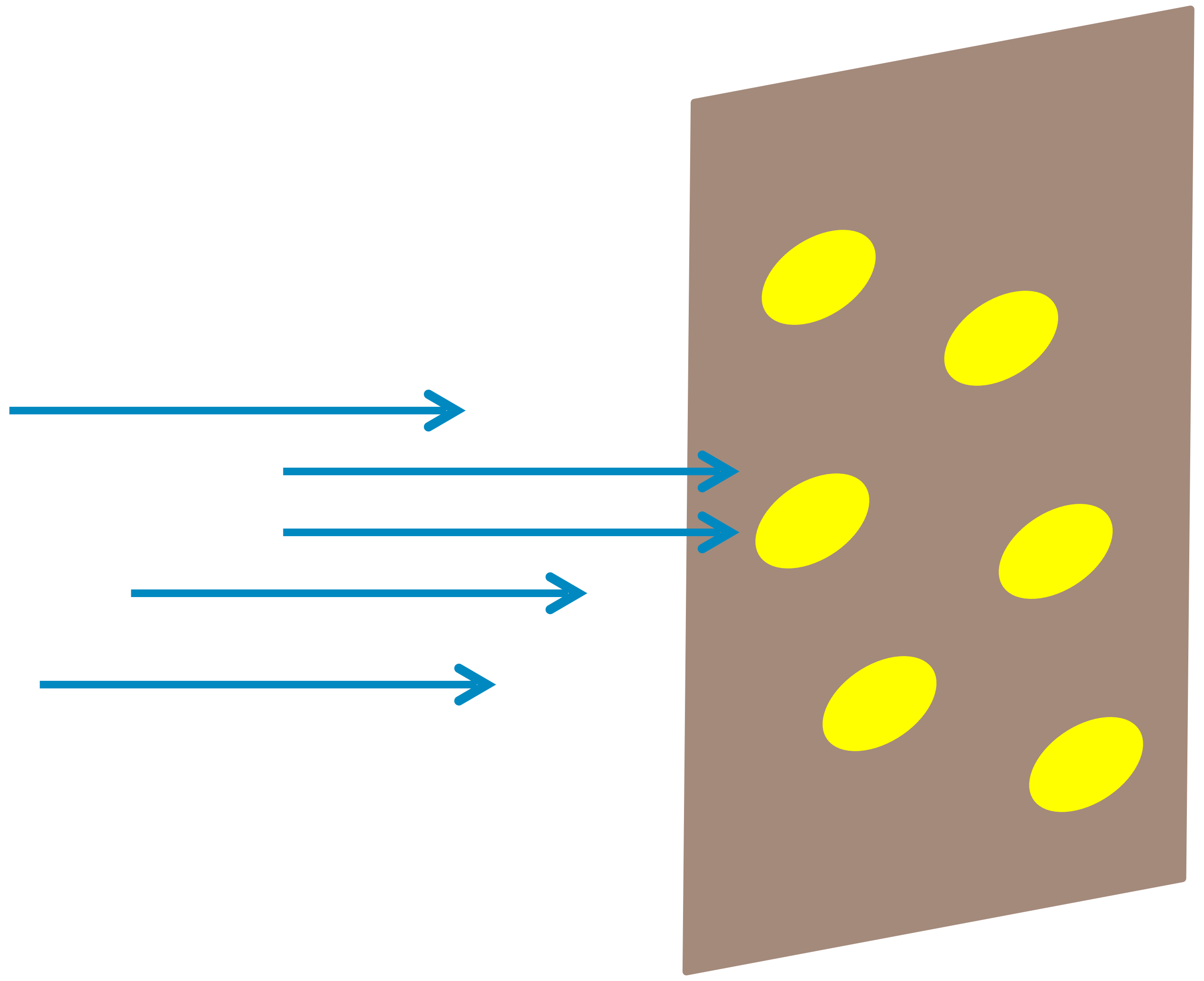
Overall cross section of the Higgs:



$$\Sigma_{react} \approx 1\ \text{pb}$$



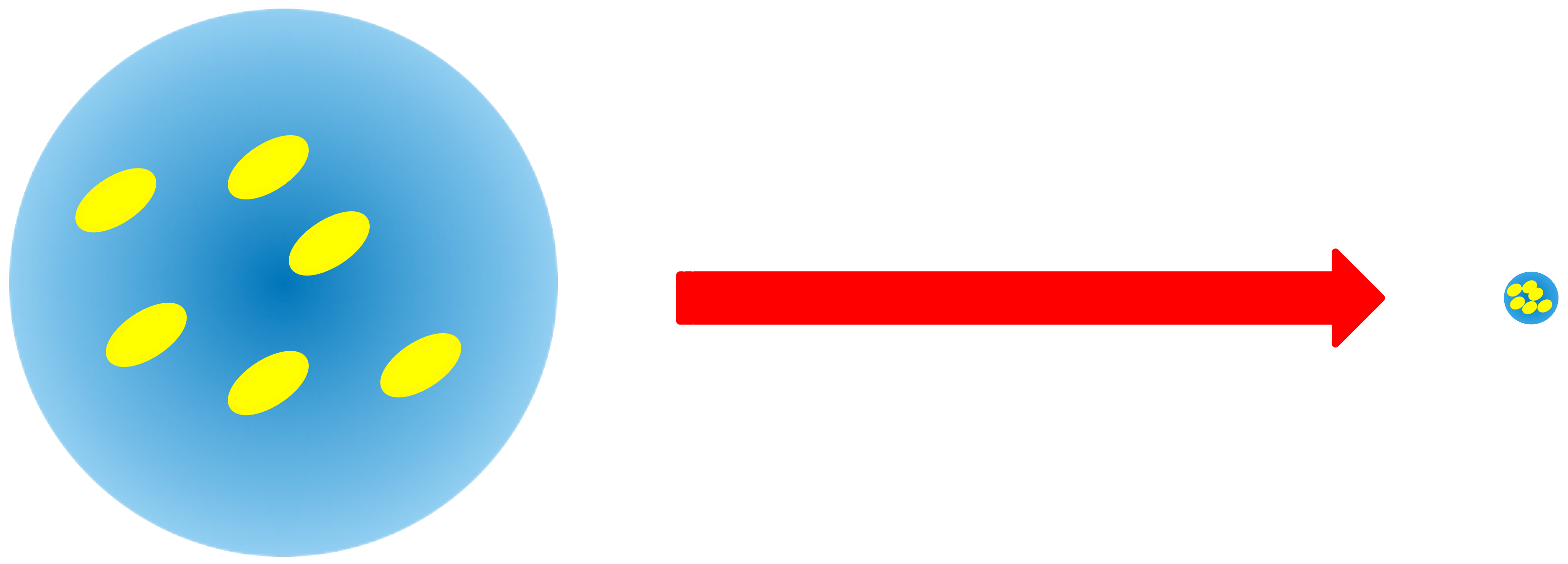
$$1\ \text{b} = 10^{-24}\ \text{cm}^2$$

$$1\ \text{pb} = 10^{-12} \cdot 10^{-24}\ \text{cm}^2$$

$$1\ \text{pb} = \frac{1}{\text{mio}} \cdot \frac{1}{\text{mio}} \cdot \frac{1}{\text{mio}} \cdot \frac{1}{\text{mio}} \cdot \frac{1}{\text{mio}} \cdot \frac{1}{10000}\ \text{mm}^2$$

The only chance we have:

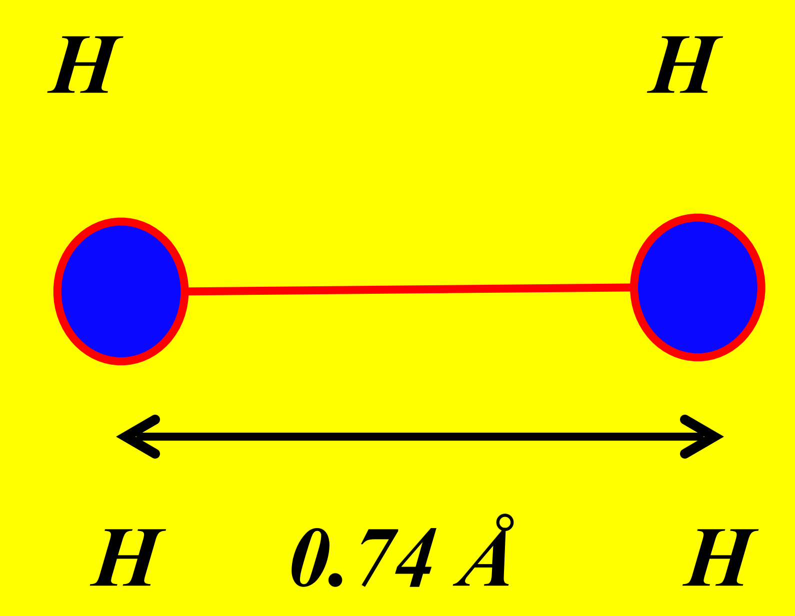
compress the transverse beam size … at the IP



LHC typical:

***σ* = 0.1 mm → 16 µm**

Particle density in matter

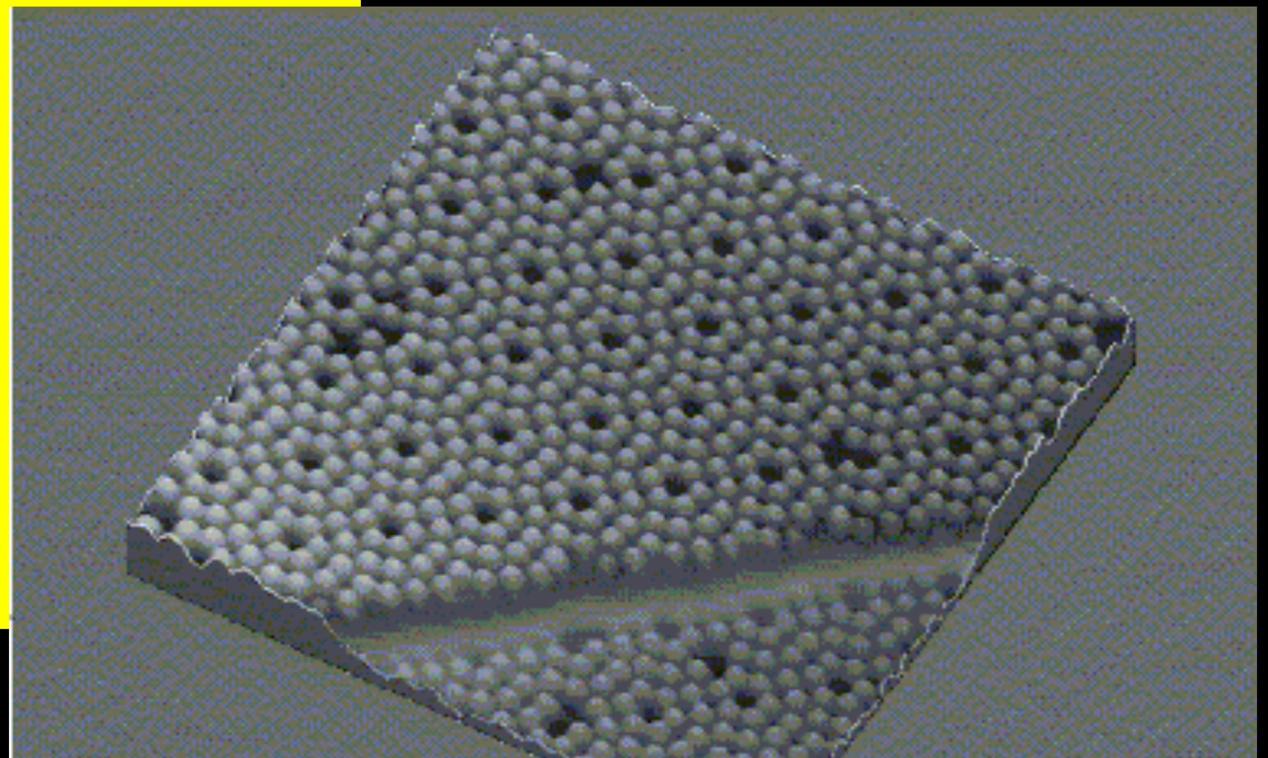


Atomic Distance in Hydrogen Molecule

$R_B \approx 0.5$ Å

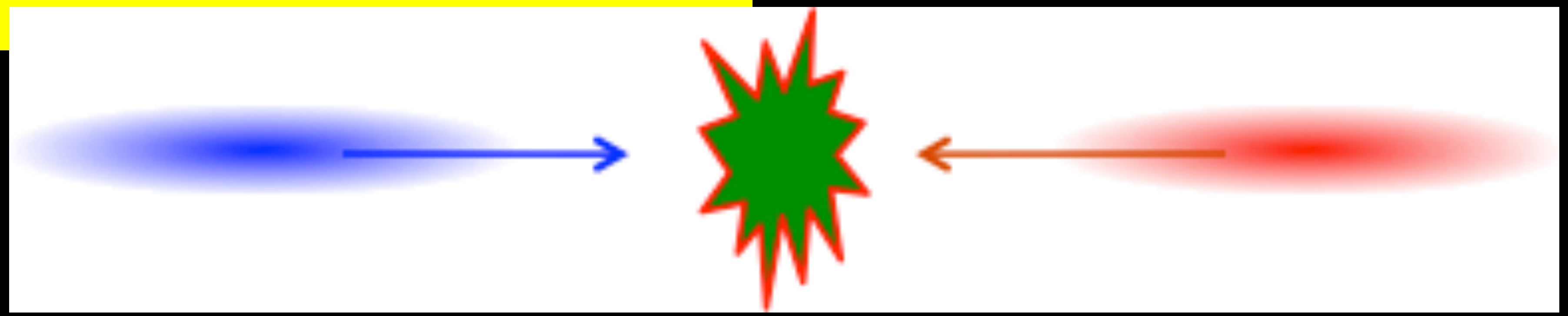
in solids / fluids $\lambda \approx 1 \ldots 3$ Å

in gases $\lambda \approx 35$ Å $= 3.5$ nm



Raster Tunnel e-Microscope

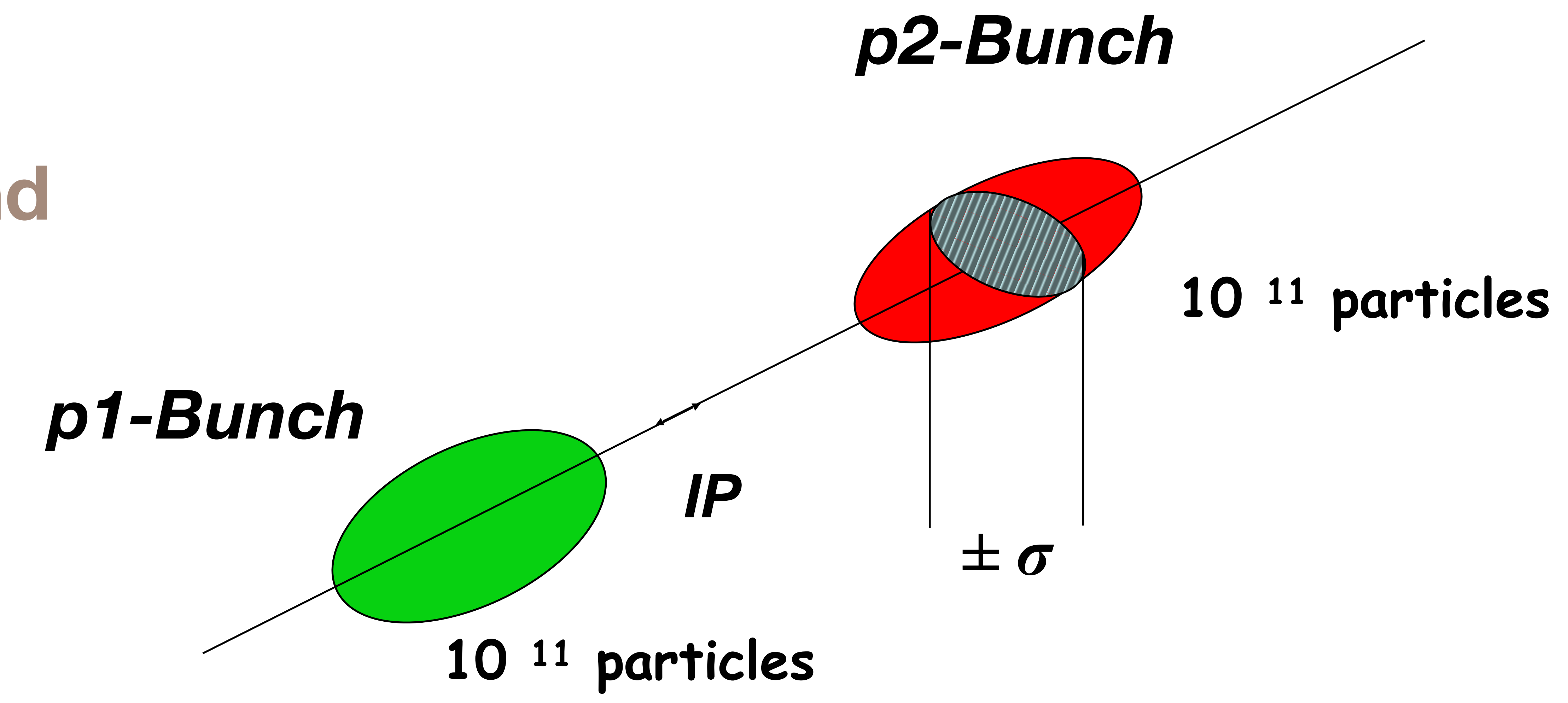
Particle Distance in Accelerators: $\lambda \approx 600$ nm (Arc) … 300nm (IP LEP)
$= 3000$ Å



Luminosity

Event Rate: "Physics" per Second

$$R = \Sigma_{react} \cdot L$$



Example: Luminosity run at LHC

$\beta_{x,y} = 0.55\ m$

$f_0 = 11.245\ kHz$

$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\ m$

$n_b = 2808$

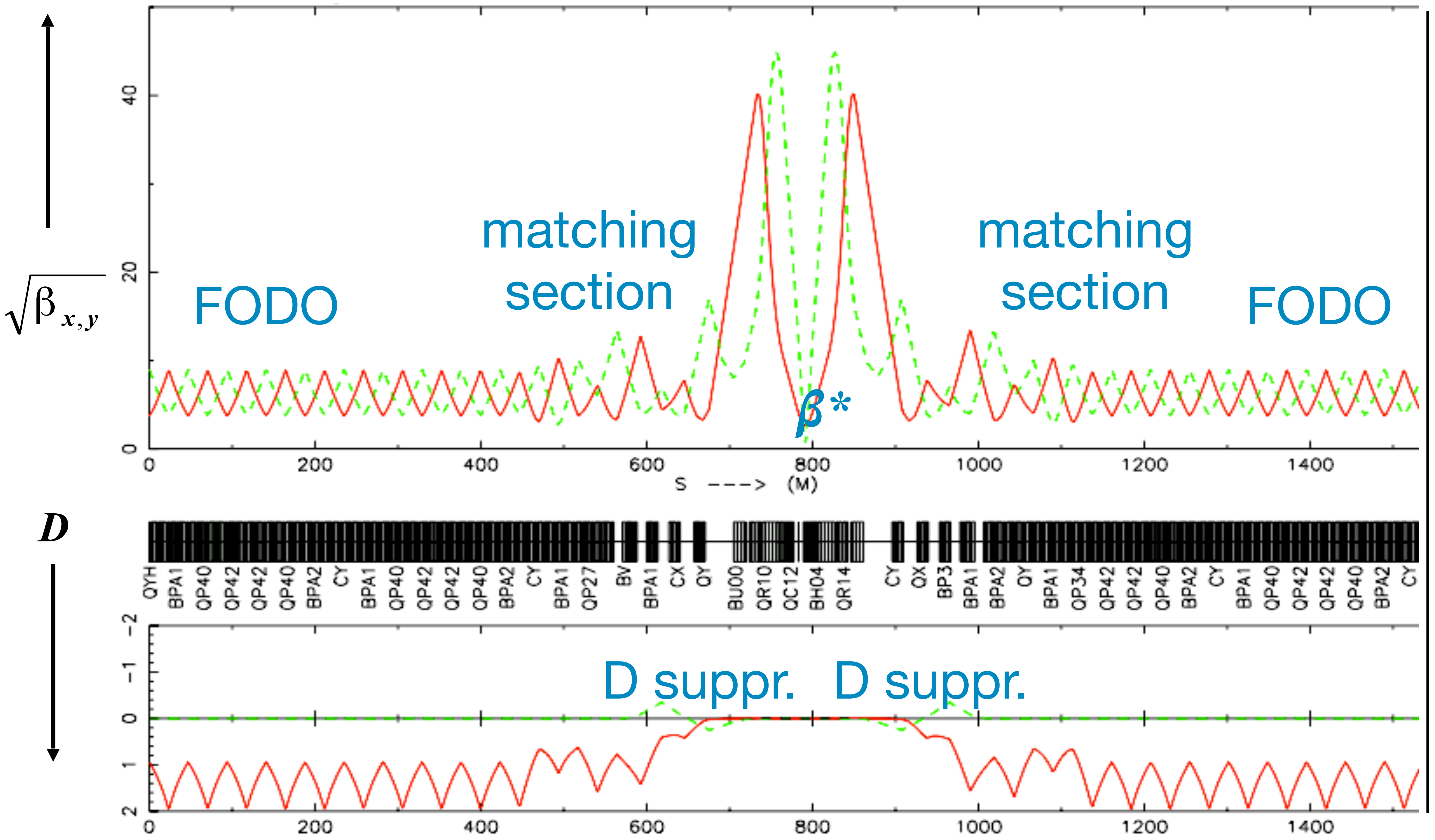
$\sigma_{x,y} = 17\ \mu m$

$I_p = 584\ mA$

$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

$$L = 1.0 * 10^{34}\ \frac{1}{cm^2 s}$$

Layout of the HERA mini-beta insertion



Mini-beta insertion: phase space

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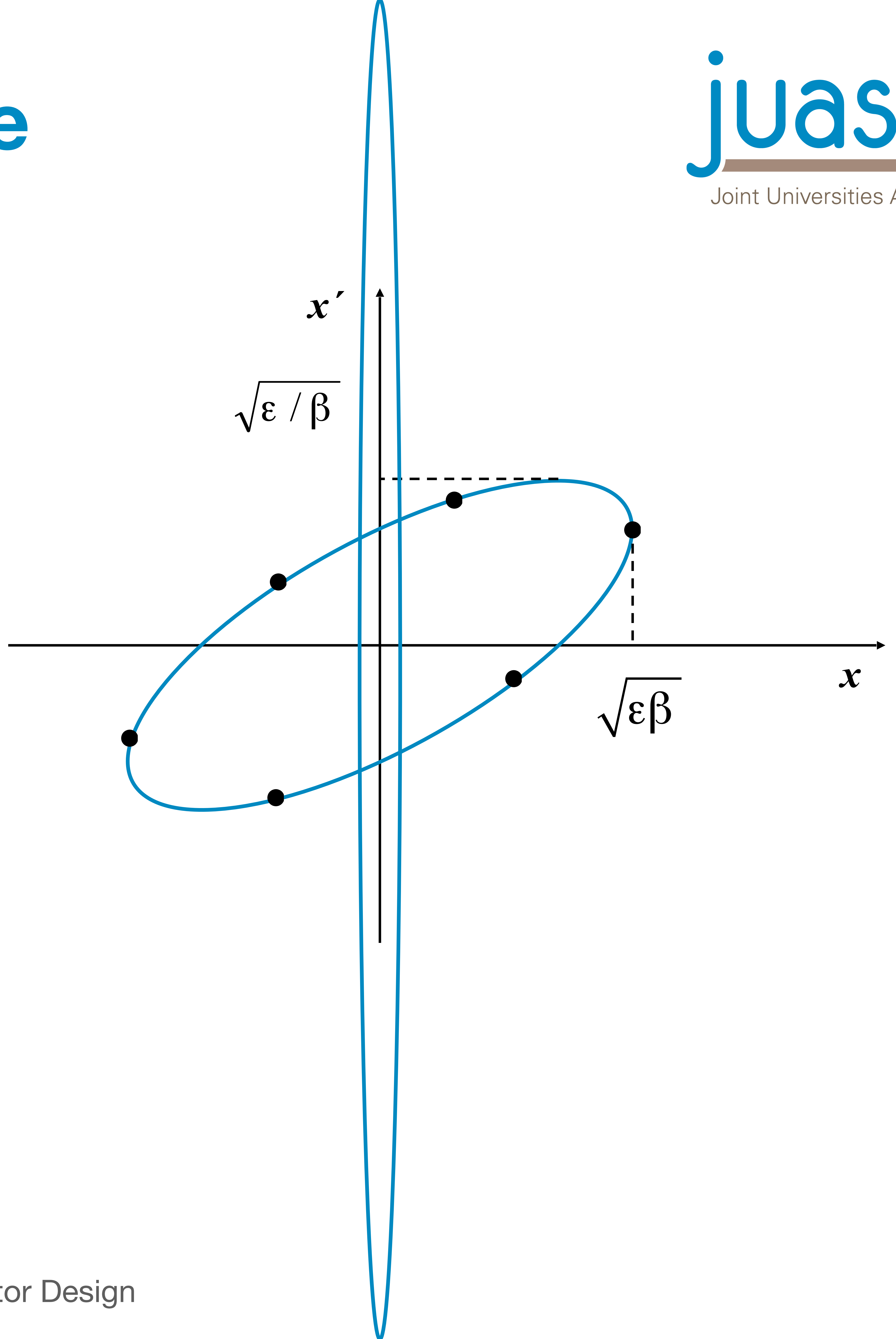
Symmetry point of a drift space: $\alpha^* = 0$

$$\epsilon = \gamma(s)x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^2(s)$$

Greetings from Liouville:

the smaller the beam size

the larger the beam divergence



Beam collisions: mini-beta insertion

A mini-beta-insertion is basically just a long
drift space, embedded in the storage ring lattice.

transformation rule for the optics parameters:

$$\begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}_{s2} = \begin{pmatrix} m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\ -m_{11}m_{21} & m_{12}m_{21}+m_{11}m_{22} & -m_{12}m_{22} \\ m_{12}^2 & -2m_{22}m_{21} & m_{22}^2 \end{pmatrix}_{s1} = \begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}_{s1}$$

with the matrix elements given by the product-matrix of the lattice elements

$$M_{total} = \dots M_{QD}\cdot M_{Drift}\cdot M_B\cdot M_{Drift}\cdot M_{QF} = \begin{pmatrix} m_{11} & m_{12}\\ m_{21} & m_{22}\end{pmatrix}$$

transfer matrix for a drift:

$$M_{drift} = \begin{pmatrix} m_{11} & m_{12}\\ m_{21} & m_{22}\end{pmatrix} = \begin{pmatrix} 1 & s\\ 0 & 1\end{pmatrix}$$

transferring from $0 \rightarrow s$

$$\beta(s) = \beta_0 - 2a_0\cdot s + \gamma_0\cdot s^2$$

$$\alpha(s) = \alpha_0 - \gamma_0\cdot s$$

$$\gamma(s) = \gamma_0$$

Betafunction in mini-beta insertion

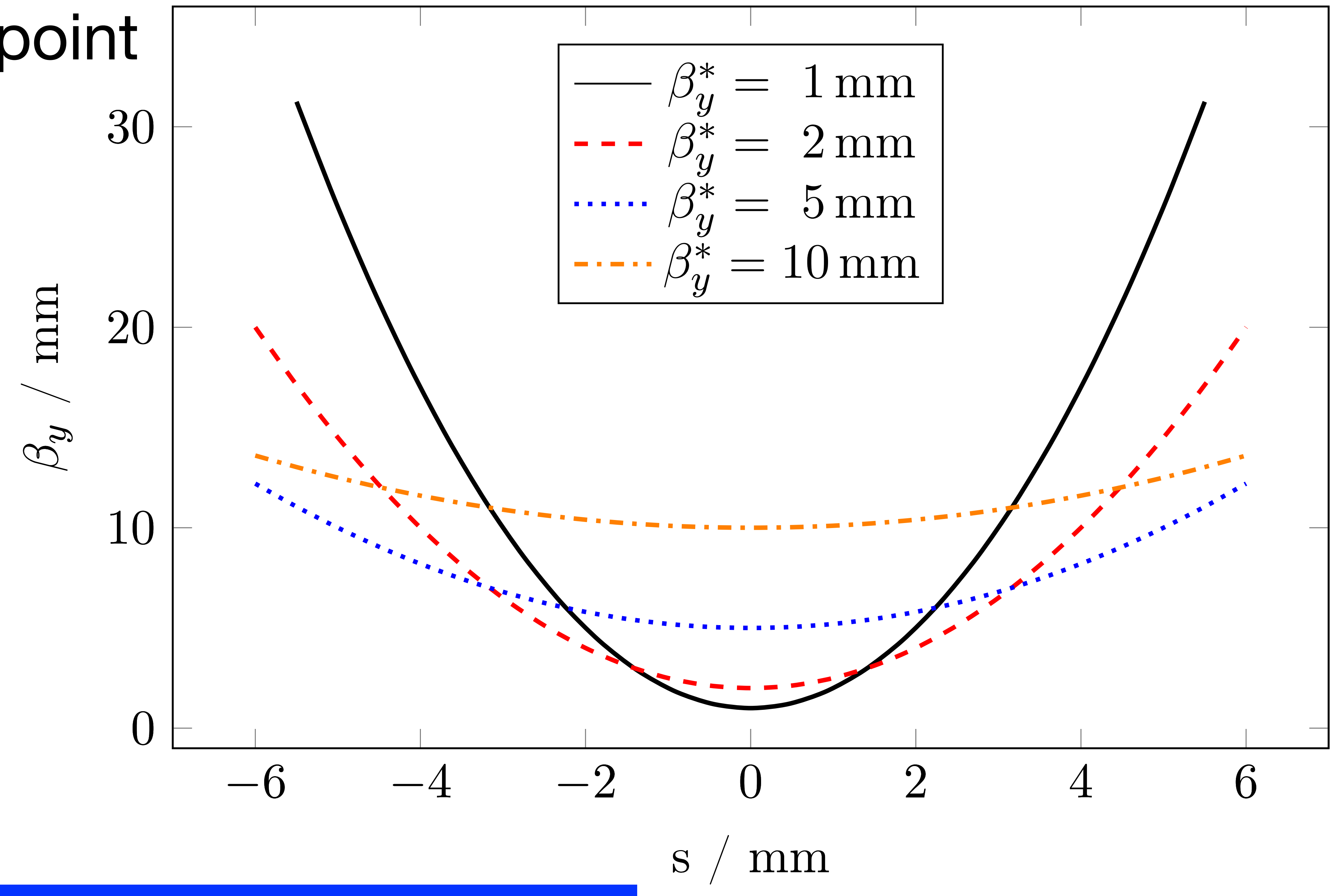
Let's assume we are at a symmetry point in the center of a drift.

$$\beta(s) = \beta_0 - 2\alpha_0 s + \gamma_0 s^2 \quad \text{… as} \quad \alpha_0 = 0, \quad \rightarrow \quad \gamma_0 = \frac{1+\alpha_0^2}{\beta_0} = \frac{1}{\beta_0}$$

we get for the β function in the neighbourhood of the symmetry point

$$\beta(s) = \beta_0 + \frac{s^2}{\beta_0}$$

! ! !



Nota bene:

1.) this is very bad !!!

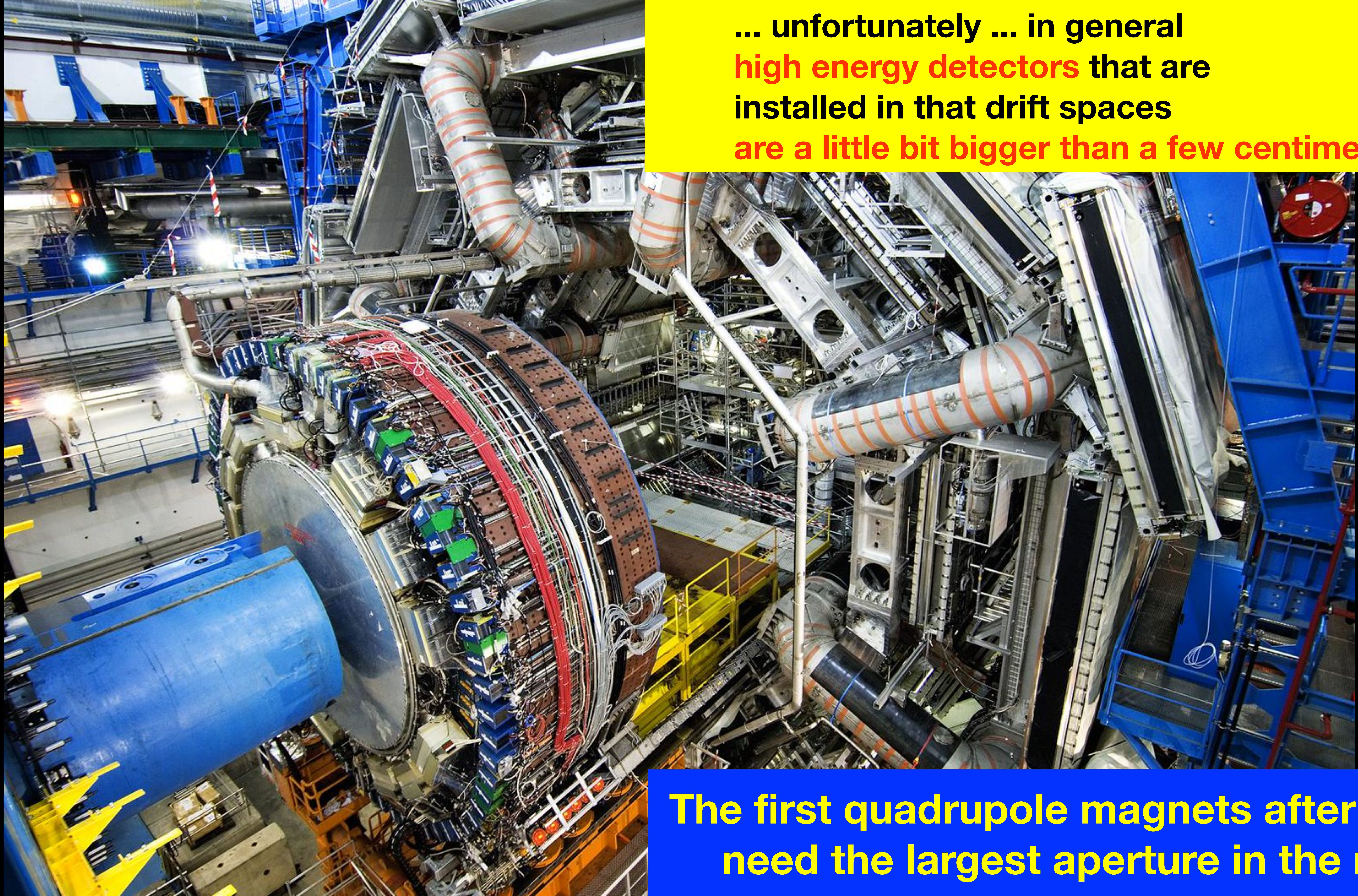
2.) this is a direct consequence of the conservation of phase space density (… in our words: ε = const) … and there is no way out.

3.) Thank you, Mr. Liouville !!!

In any case: keep s as SMALL as possible !!!

Example: Luminosity optics at LHC: $\beta^* = 55$ cm
for smallest β_{max} we have to limit the overall length
and keep the distance "s" as small as possible.

... clearly there is a problem !!!



... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...

The first quadrupole magnets after the IP
need the largest aperture in the ring.

Mini-beta insertions: phase advance

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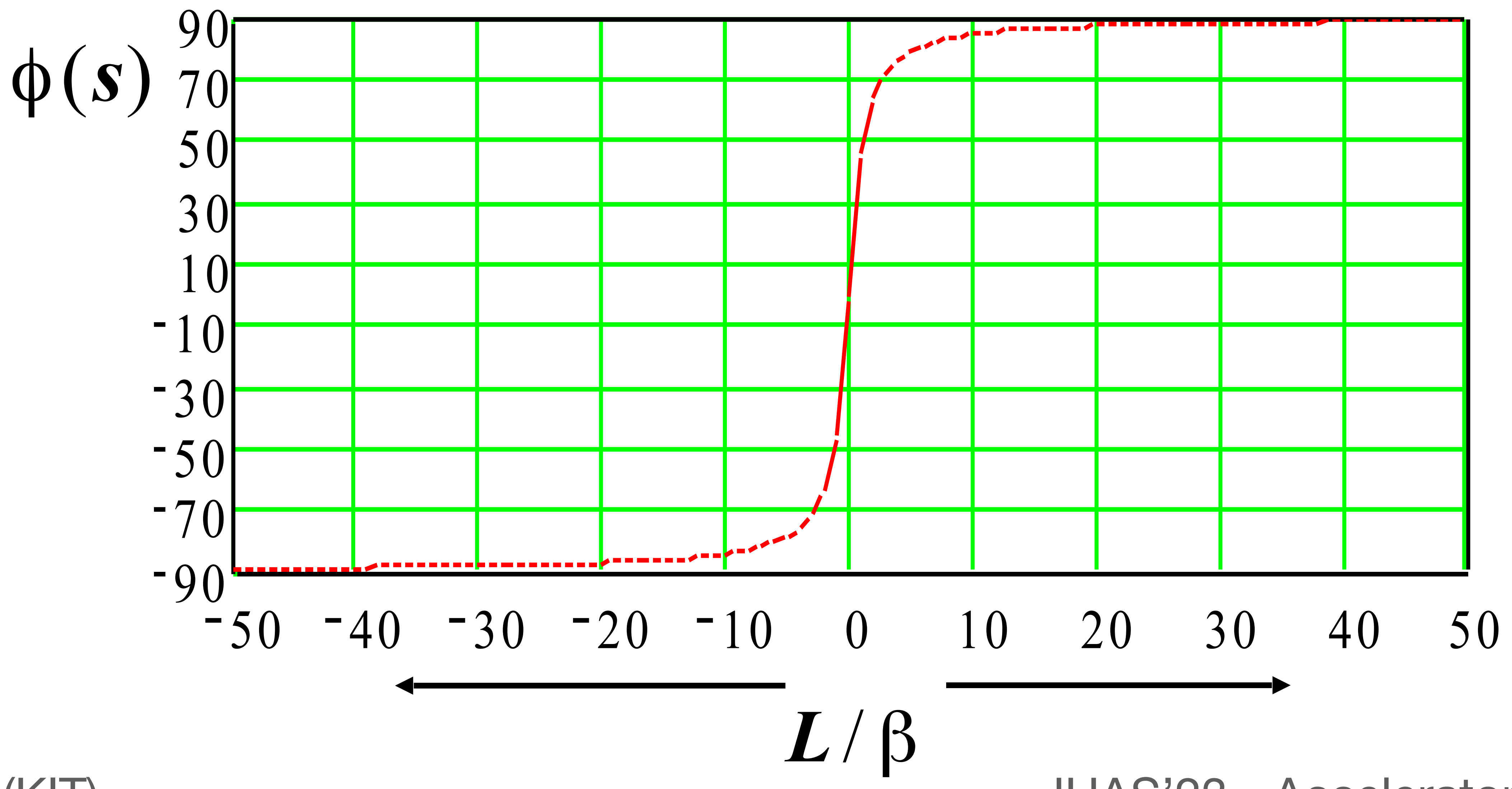
By definition the phase advance is given by:

$$\mu(s) = \int \frac{1}{\beta(s)} \mathrm{ds}$$

Now in a mini-beta insertion:

$$\beta(s) = \beta_0 \left(1 + \frac{s^2}{\beta_0^2}\right)$$

$$\rightarrow \mu(s) = \frac{1}{\beta_0} \int_0^L \frac{1}{1 + s^2/\beta_0^2} \mathrm{ds} = \arctan \frac{L}{\beta_0}$$



Consider the drift spaces on both sides of the IP:

the phase advance of a mini β insertion is approximately π,

in other words: the tune will increase by half an integer.

Mini-beta insertion: Guidelines

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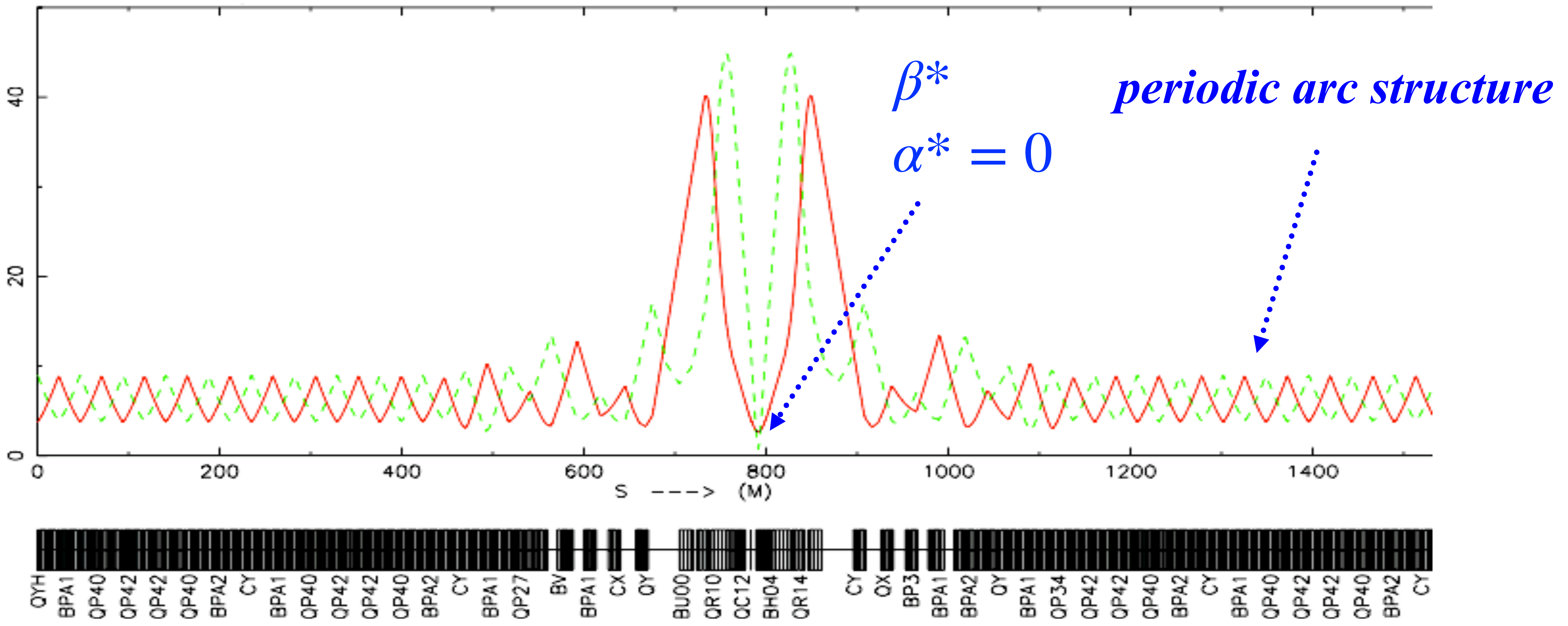
* calculate the periodic solution in the arc

* introduce the drift space needed for the insertion device (detector ...)

* put a quadrupole doublet (triplet ?) as close as possible

* introduce additional quadrupole lenses to match the beam parameters to the values at the beginning of the arc structure



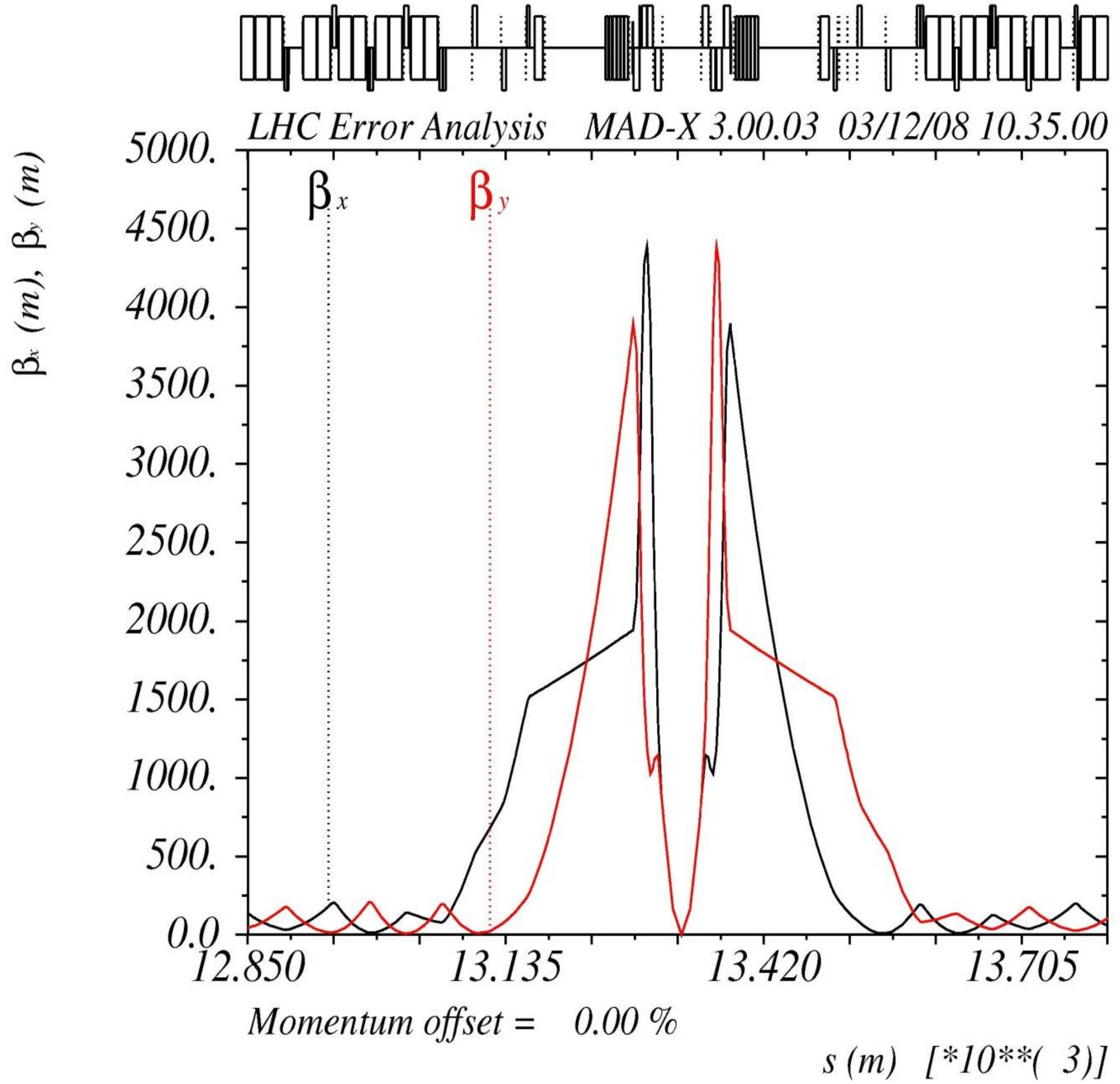
Parameters to be optimised & matched to the periodic solution:

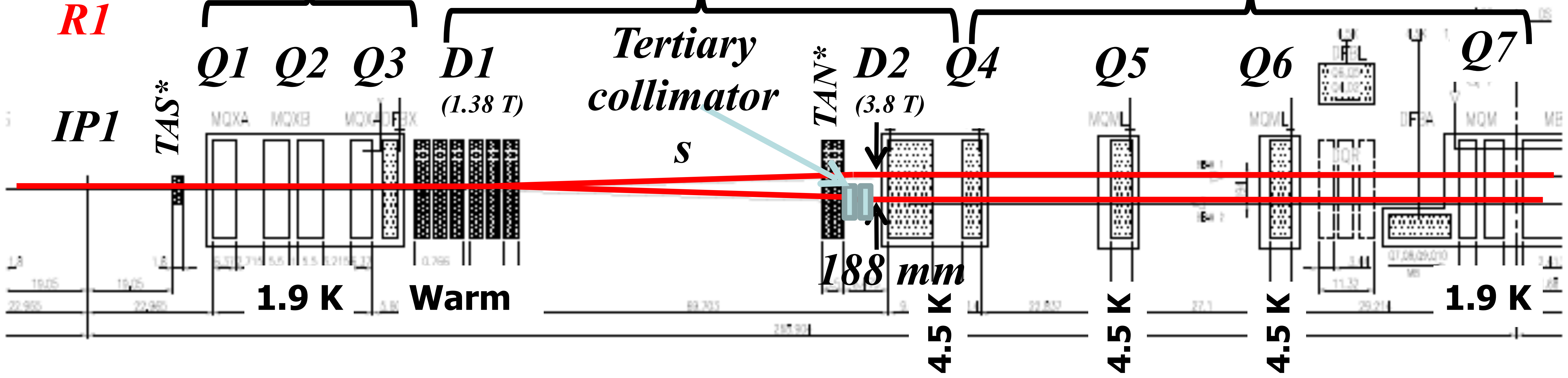
$$\alpha_x,\ \beta_x \qquad D_x,\ D_x'$$
$$\alpha_y,\ \beta_y \qquad Q_x,\ Q_y$$

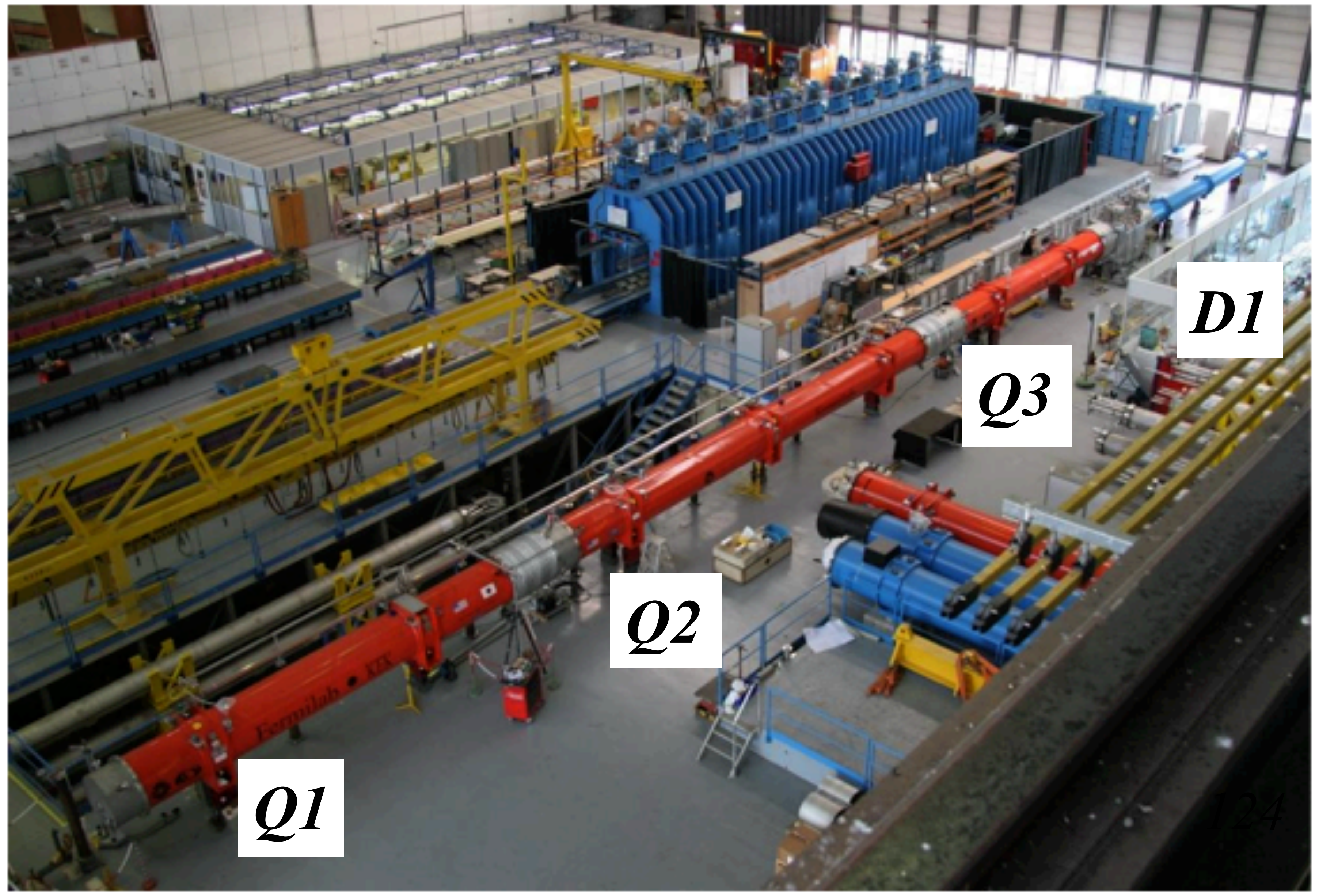
8 individually powered quadrupole magnets are needed to match the insertion (... at least)

The LHC mini-beta insertions

mini-beta optics







One word about mini-beta insertions:

Mini-beta insertions must be installed in

… **straight sections** (no dipoles that drive dispersion)

… that are **dispersion free**

… that are connected to the arc lattice by **dispersion suppressors**

if not, the dispersion dilutes the particle density and increases the effective transverse beam size.

One word about limitations:

It looks like we can get infinite luminosities by

… creating smallest β* at the IP

… and accumulating infinite bunch intensities.

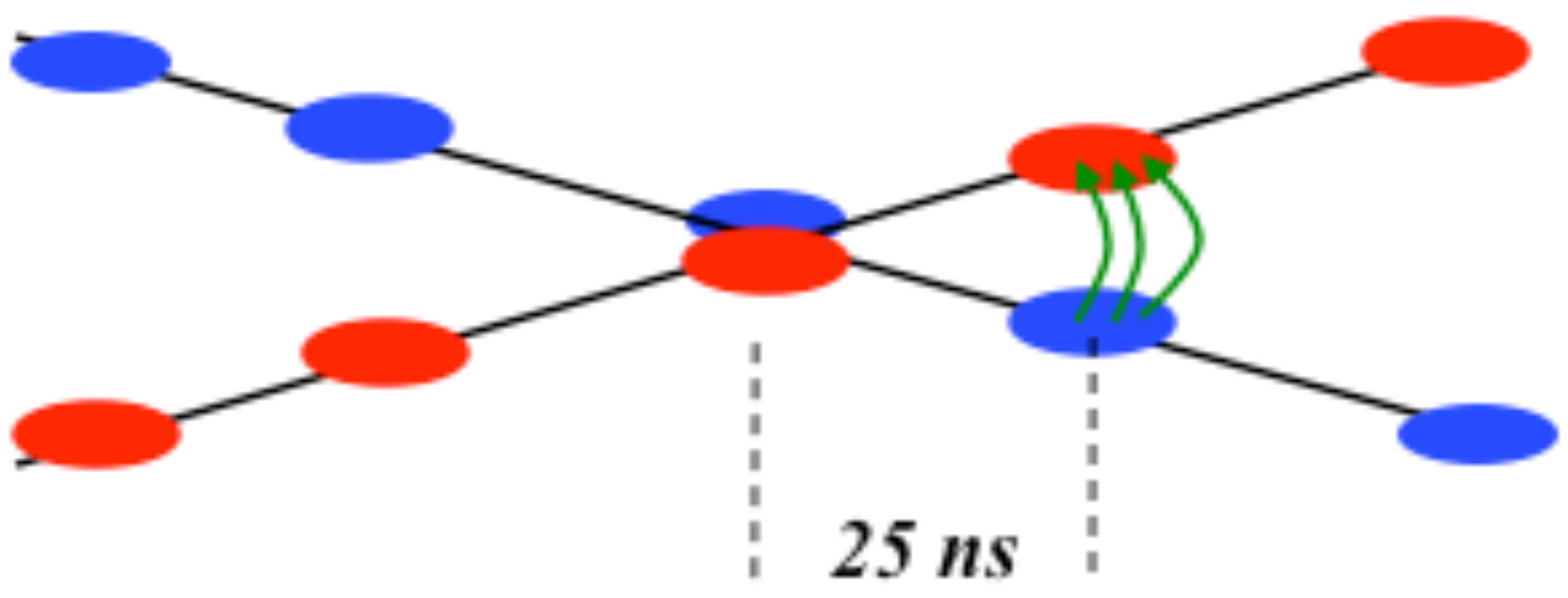
However, that is not how life is.

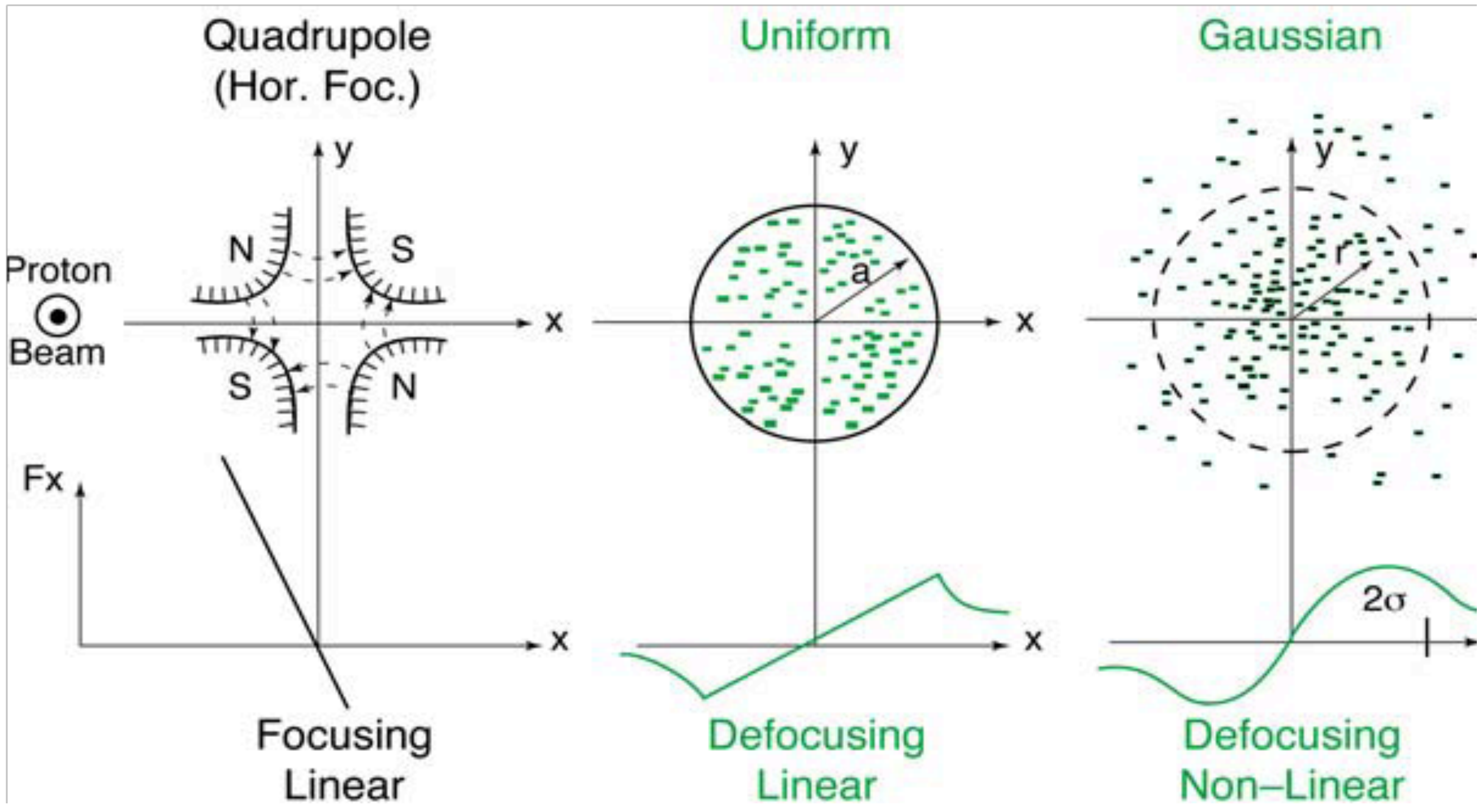
Luminosity limit due to Beam-Beam Effect

the colliding bunches influence each other (space charge)

⇒ change the focusing properties of the ring !!

for LHC a strong non-linear defocussing effect





court. K. Schindl

most simple case:

linear beam-beam tune-shift

→ puts a limit to N_p

$$\Delta Q_y = \frac{r_e}{2\pi\gamma_e} \cdot \frac{\beta_y^*}{\sigma_y} \cdot \frac{N_e}{(\sigma_x + \sigma_y)}$$

Particles are pushed onto resonances and are lost.

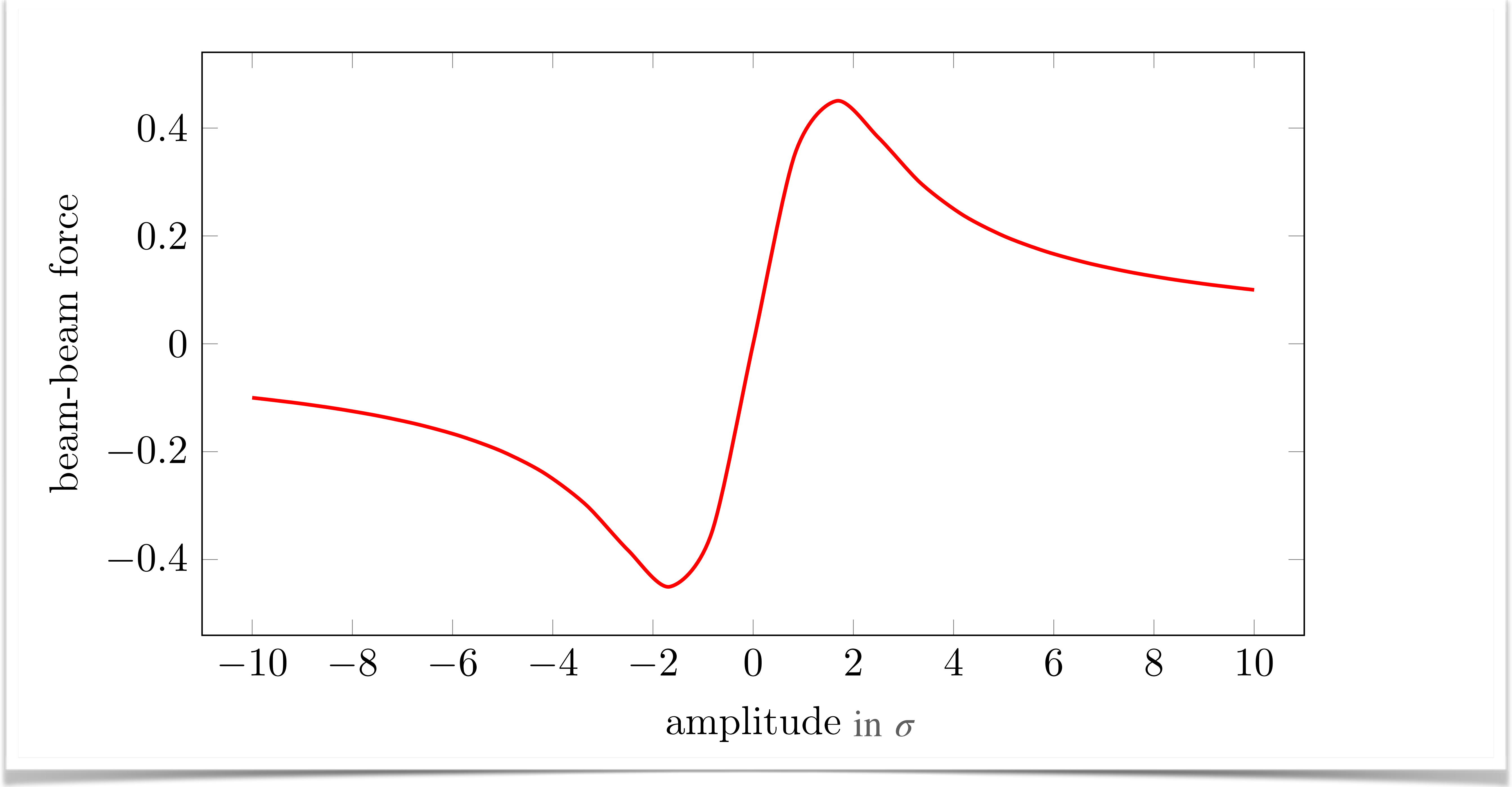
Beam-beam parameter

- For small amplitudes the tune shift is equal to the linear beam-beam parameter:

$$\xi_u = \frac{N r_e \beta_u^*}{2\pi\gamma\sigma_u^*(\sigma_x + \sigma_u)}$$

- It is often used to quantify the strength of the beam-beam interaction.
- However, it does not reflect its non-linear nature.

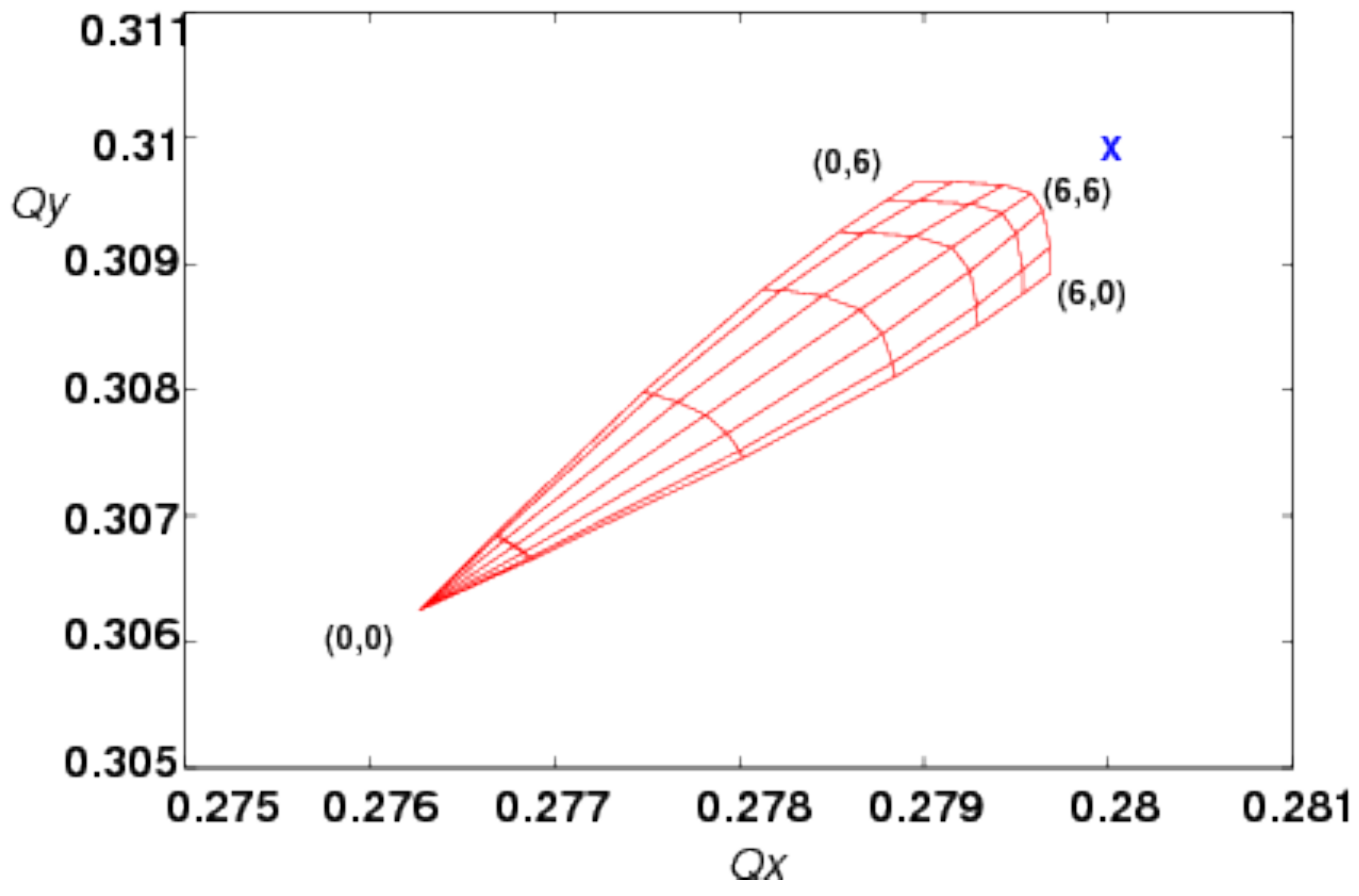
Important: $$\xi_u \propto \frac{N}{\epsilon_u}$$



Beam-beam force for round beams in arbitrary units

Beam-beam: Tune footprint and Luminosity

- Beam-beam parameter: $\xi_u \propto \frac{N}{\epsilon_u}$



- What are the implications for the luminosity?

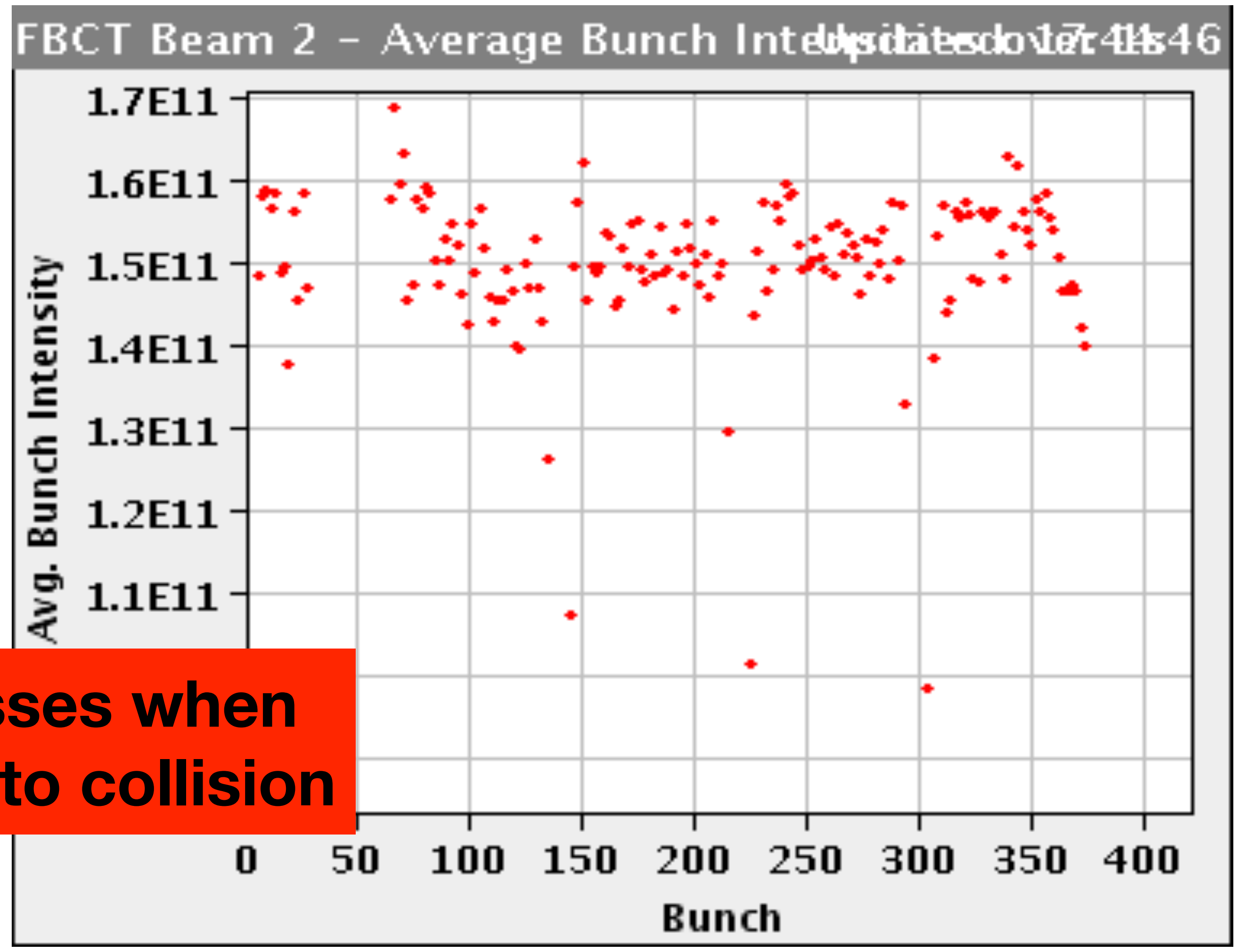
$$\mathscr{L} = \frac{1}{4\pi e^2 f_0 n_b} \frac{I_1 \cdot I_2}{\sigma_x^* \cdot \sigma_y^*}$$

with $I = n_b\ N_p\ e\ f_0$ and assuming equal beam sizes

$$\Rightarrow \mathscr{L} = \frac{N_1 N_2 n_b f}{4\pi \sigma_x^* \sigma_y^*}$$

$\Rightarrow$ Number of particles per bunch is limited!!!

$$\mathcal{O}(N) = 10^{11}$$



observed particle losses when beams are brought into collision

The logical path to Accelerator Design

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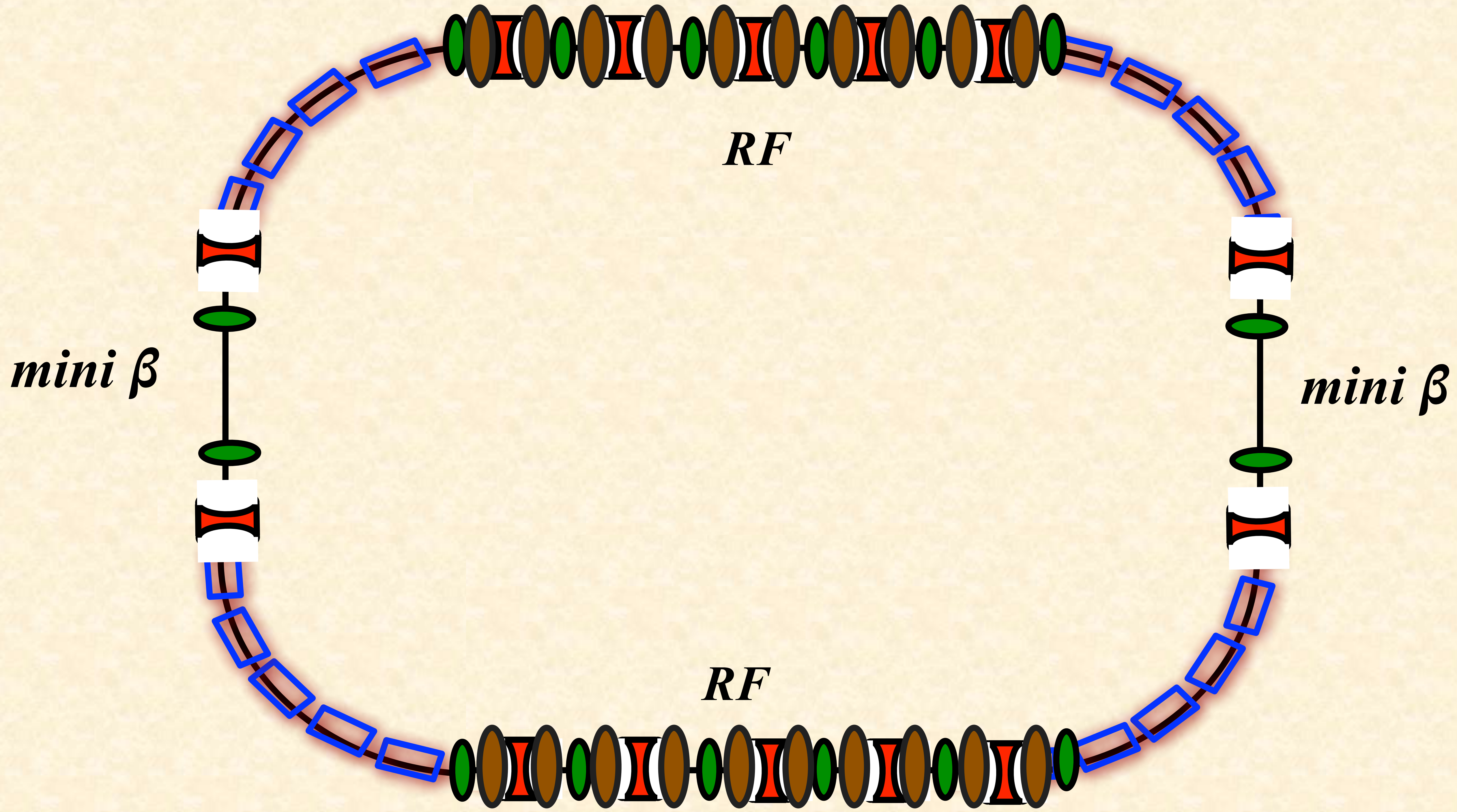
7.) Open the lattice structure to install
a dispersion free straight section for the mini beta insertion
define independent quadrupoles (four if D_x=0)
connect the straight sections to the arc lattice with mini-beta quadrupoles and matching quadrupoles
match to the desired β*

✓



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… and then you just turn the key and run the machine.

Now you are experts. ⇒ Let’s start the workshop!

Accelerator Design

JUAS'22 - Lecture and Mini-Workshop - Bastian Haerer (KIT)

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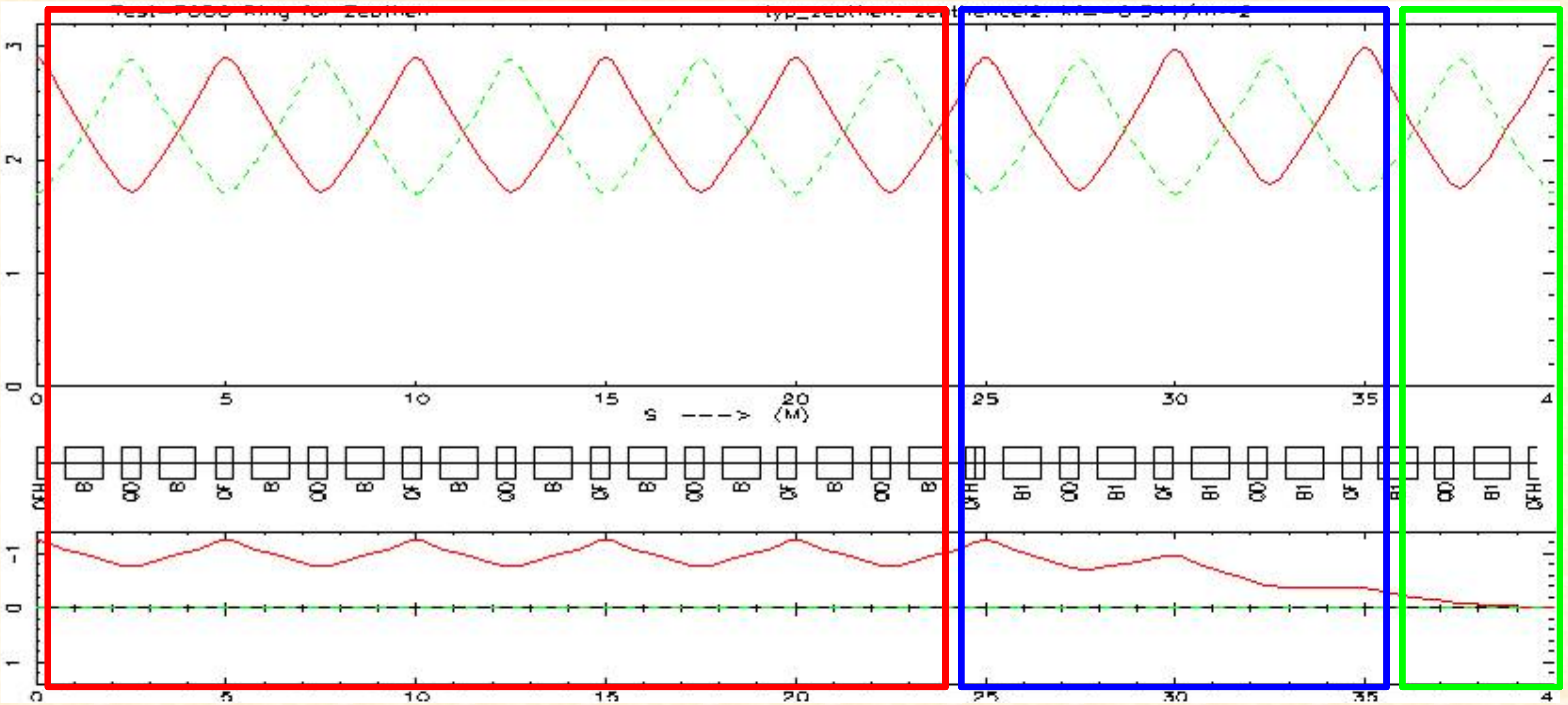
Appendix: Dispersion Suppressors

... the calculation of the half bend scheme in full detail (for purists only)

1.) the lattice is split into 3 parts: *(Gallia divisa est in partes tres)*

* periodic solution of the arc	periodic β, periodic dispersion D
* section of the dispersion suppressor	periodic β, dispersion vanishes
* FoDo cells without dispersion	periodic β, D = D´ = 0



2.) calculate the dispersion D in the periodic part of the lattice

transfer matrix of a periodic cell:

$$M_{0\to S} = \begin{pmatrix} \sqrt{\frac{\beta_S}{\beta_0}}(\cos\phi + \alpha_0 \sin\phi) & \sqrt{\beta_S\beta_0}\sin\phi \\ \frac{(\alpha_0 - \alpha_S)\cos\phi - (1+\alpha_0\alpha_S)\sin\phi}{\sqrt{\beta_S\beta_0}} & \sqrt{\frac{\beta_S}{\beta_0}}(\cos\phi - \alpha_S \sin\phi) \end{pmatrix}$$

for the transformation from one symmetriy point to the next (i.e. one cell) we have:
Φ_C = phase advance of the cell, α = 0 at a symmetry point. The index "*c*" refers to the periodic solution of one cell.

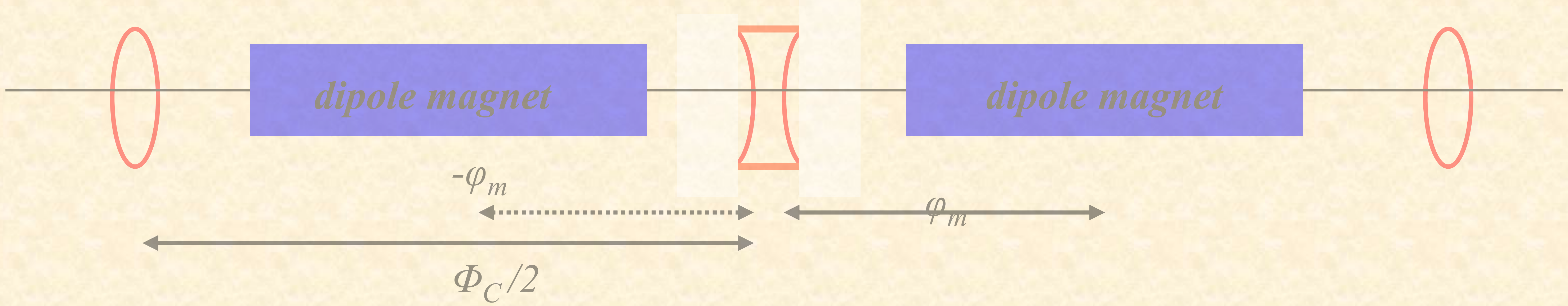
$$M_{Cell} = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos\Phi_C & \beta_C \sin\Phi_C & D(l) \\ \frac{-1}{\beta_C}\sin\Phi_C & \cos\Phi_C & D'(l) \\ 0 & 0 & 1 \end{pmatrix}$$

The matrix elements D and D' are given by the C and S elements in the usual way:

$$D(l) = S(l) * \int_0^l \frac{1}{\rho(\tilde{s})} C(\tilde{s})\, d\tilde{s} - C(l) * \int_0^l \frac{1}{\rho(\tilde{s})} S(\tilde{s})\, d\tilde{s}$$

$$D'(l) = S'(l) * \int_0^l \frac{1}{\rho(\tilde{s})} C(\tilde{s})\, d\tilde{s} - C'(l) * \int_0^l \frac{1}{\rho(\tilde{s})} S(\tilde{s})\, d\tilde{s}$$

here the values C(l) and S(l) refer to the symmetry point of the cell (middle of the quadrupole) and the integral is to be taken over the dipole magnet where $\rho \neq 0$. For ρ = const the integral over C(s) and S(s) is approximated by the values in the middle of the dipole magnet.



Transformation of C(s) from the symmetry point to the center of the dipole:

$$C_m = \sqrt{\frac{\beta_m}{\beta_C}} \cos \Delta\Phi = \sqrt{\frac{\beta_m}{\beta_C}} \cos(\frac{\Phi_C}{2} \pm \varphi_m) \qquad S_m = \beta_m \beta_C \sin(\frac{\Phi_C}{2} \pm \varphi_m)$$

where β_C is the periodic β function at the beginning and end of the cell, β_m its value at the middle of the dipole and φ_m the phase advance from the quadrupole lens to the dipole center.

Now we can solve the intergal for D and D':

$$D(l) = S(l) * \int_0^l \frac{1}{\rho(\tilde{s})} C(\tilde{s}) d\tilde{s} - C(l) * \int_0^l \frac{1}{\rho(\tilde{s})} S(\tilde{s}) d\tilde{s}$$

$$D(l) = \beta_C \sin \Phi_C * \frac{L}{\rho} * \sqrt{\frac{\beta_m}{\beta_C}} * \cos(\frac{\Phi_C}{2} \pm \varphi_m) - \cos \Phi_C * \frac{L}{\rho} \sqrt{\beta_m \beta_C} * \sin(\frac{\Phi_C}{2} \pm \varphi_m)$$

$$D(l) = \delta\sqrt{\beta_m\beta_C}\left\{\sin\Phi_C\left[\cos(\frac{\Phi_C}{2}+\varphi_m)+\cos(\frac{\Phi_C}{2}-\varphi_m)\right]-\right.$$

$$\left.-\cos\Phi_C\left[\sin(\frac{\Phi_C}{2}+\varphi_m)+\sin(\frac{\Phi_C}{2}-\varphi_m)\right]\right\}$$

I have put δ = L/ρ for the strength of the dipole

remember the relations $\cos x + \cos y = 2\cos\frac{x+y}{2} * \cos\frac{x-y}{2}$

$$\sin x + \sin y = 2\sin\frac{x+y}{2} * \cos\frac{x-y}{2}$$

$$D(l) = \delta\sqrt{\beta_m\beta_C}\left\{\sin\Phi_C * 2\cos\frac{\Phi_C}{2} * \cos\varphi_m - \cos\Phi_C * 2\sin\frac{\Phi_C}{2} * \cos\varphi_m\right\}$$

$$D(l) = 2\delta\sqrt{\beta_m\beta_C} * \cos\varphi_m\left\{\sin\Phi_C * \cos\frac{\Phi_C}{2} * -\cos\Phi_C * \sin\frac{\Phi_C}{2}\right\}$$

remember: $\sin 2x = 2\sin x * \cos x$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$D(l) = 2\delta\sqrt{\beta_m\beta_C} * \cos\varphi_m\left\{2\sin\frac{\Phi_C}{2} * \cos^2\frac{\Phi_C}{2} - (\cos^2\frac{\Phi_C}{2} - \sin^2\frac{\Phi_C}{2}) * \sin\frac{\Phi_C}{2}\right\}$$

$$D(l) = 2\delta\sqrt{\beta_m\beta_C} * \cos\varphi_m * \sin\frac{\Phi_C}{2}\left\{2\cos^2\frac{\Phi_C}{2} - \cos^2\frac{\Phi_C}{2} + \sin^2\frac{\Phi_C}{2}\right\}$$

$$D(l) = 2\delta\sqrt{\beta_m\beta_C} * \cos\varphi_m * \sin\frac{\Phi_C}{2}$$

in full analogy one derives the expression for D‘:

$$\boldsymbol{D}'(\boldsymbol{l}) = 2\delta\sqrt{\beta_{\boldsymbol{m}} / \beta_{\boldsymbol{c}}} * \cos\varphi_{\boldsymbol{m}} * \cos\frac{\Phi_{\boldsymbol{c}}}{2}$$

As we refer the expression for D and D‘ to a periodic struture, namly a FoDo cell we require periodicity conditons:

$$\begin{pmatrix} D_C \\ D'_C \\ 1 \end{pmatrix} = M_C * \begin{pmatrix} D_C \\ D'_C \\ 1 \end{pmatrix}$$

and by symmetry: $D'_C = 0$

With these boundary conditions the Dispersion in the FoDo is determined:

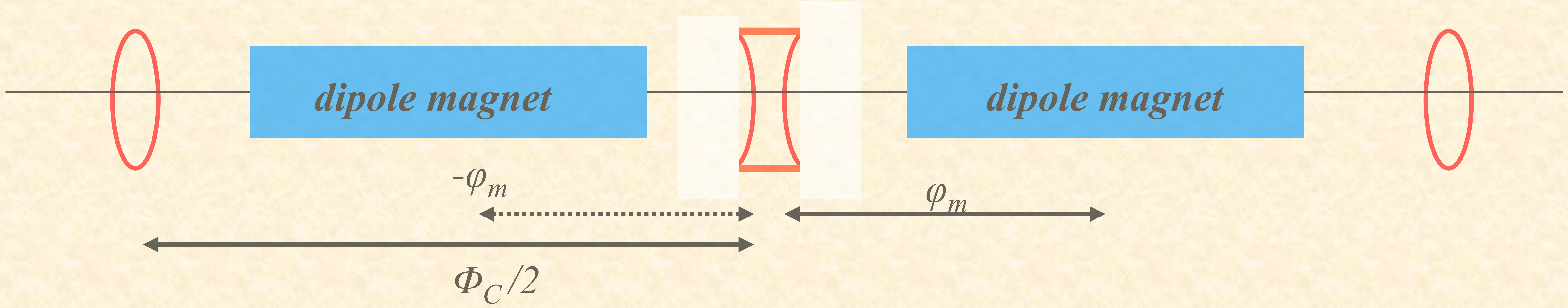
$$D_C * \cos\Phi_C + \delta\sqrt{\beta_m\beta_C} * \cos\varphi_m * 2\sin\frac{\Phi_C}{2} = D_C$$

(A1)
$$D_C = \delta\sqrt{\beta_m \beta_C} * \cos\varphi_m / \sin\frac{\Phi_C}{2}$$

This is the value of the periodic dispersion in the cell evaluated at the position of the dipole magnets.

3.) Calculate the dispersion in the suppressor part:

We will now move to the second part of the dispersion suppressor: The section where ... starting from D=D'=0 the dispesion is generated ... or turning it around where the Dispersion of the arc is reduced to zero.
The goal will be to generate the dispersion in this section in a way that the values of the periodic cell that have been calculated above are obtained.



The relation for D, generated in a cell still holds in the same way:

$$D(l) = S(l) * \int_0^l \frac{1}{\rho(\tilde{s})} C(\tilde{s}) d\tilde{s} - C(l) * \int_0^l \frac{1}{\rho(\tilde{s})} S(\tilde{s}) d\tilde{s}$$

as the dispersion is generated in a number of *n* cells the matrix for these *n* cells is

$$M_n = M_C^n = \begin{pmatrix} \cos n\Phi_C & \beta_C \sin n\Phi_C & D_n \\ \frac{-1}{\beta_C}\sin n\Phi_C & \cos n\Phi_C & D'_n \\ 0 & 0 & 1 \end{pmatrix}$$

$$D_n = \beta_C \sin n\Phi_C * \delta_{\sup r} * \sum_{i=1}^{n} \cos(i\Phi_C - \frac{1}{2}\Phi_C \pm \varphi_m) * \sqrt{\frac{\beta_m}{\beta_C}} -$$

$$- \cos n\Phi_C * \delta_{\sup r} * \sum_{i=1}^{n} \sqrt{\beta_m \beta_C} * \sin(i\Phi_C - \frac{1}{2}\Phi_C \pm \varphi_m)$$

$$D_n = \sqrt{\beta_m \beta_C} * \sin n\Phi_C * \delta_{\sup r} * \sum_{i=1}^{n} \cos((2i-1)\frac{\Phi_C}{2} \pm \varphi_m) - \sqrt{\beta_m \beta_C} * \delta_{\sup r} * \cos n\Phi_C \sum_{i=1}^{n} \sin((2i-1)\frac{\Phi_C}{2} \pm \varphi_m)$$

remember: $\sin x + \sin y = 2\sin\frac{x+y}{2} * \cos\frac{x-y}{2}$ $\quad$ $\cos x + \cos y = 2\cos\frac{x+y}{2} * \cos\frac{x-y}{2}$

$$D_n = \delta_{\sup r} * \sqrt{\beta_m \beta_C} * \sin n\Phi_C * \sum_{i=1}^{n} \cos((2i-1)\frac{\Phi_C}{2}) * 2\cos\varphi_m -$$

$$-\delta_{\sup r} * \sqrt{\beta_m \beta_C} * \cos n\Phi_C \sum_{i=1}^{n} \sin((2i-1)\frac{\Phi_C}{2}) * 2\cos\varphi_m$$

$$D_n = 2\delta_{\sup r} * \sqrt{\beta_m \beta_C} * \cos\varphi_m \left\{ \sum_{i=1}^{n} \cos((2i-1)\frac{\Phi_C}{2}) * \sin n\Phi_C - \sum_{i=1}^{n} \sin((2i-1)\frac{\Phi_C}{2}) * \cos n\Phi_C \right\}$$

$$D_n = 2\delta_{\sup r} * \sqrt{\beta_m \beta_C} * \cos\varphi_m \left\{ \sin n\Phi_C \left\{ \frac{\sin\frac{n\Phi_C}{2} * \cos\frac{n\Phi_C}{2}}{\sin\frac{\Phi_C}{2}} \right\} - \cos n\Phi_C * \left\{ \frac{\sin\frac{n\Phi_C}{2} * \sin\frac{n\Phi_C}{2}}{\sin\frac{\Phi_C}{2}} \right\} \right\}$$

$$D_n = \frac{2\delta_{\sup r} * \sqrt{\beta_m \beta_C} * \cos\varphi_m}{\sin\frac{\Phi_C}{2}} \left\{ \sin n\Phi_C * \sin\frac{n\Phi_C}{2} * \cos\frac{n\Phi_C}{2} - \cos n\Phi_C * \sin^2\frac{n\Phi_C}{2} \right\}$$

set for more convenience $x = n\Phi_C/2$

$$D_n = \frac{2\delta_{\sup r} * \sqrt{\beta_m \beta_C} * \cos\varphi_m}{\sin\frac{\Phi_C}{2}} \left\{ \sin 2x * \sin x * \cos x - \cos 2x * \sin^2 x \right\}$$

$$D_n = \frac{2\delta_{\sup r} * \sqrt{\beta_m \beta_C} * \cos\varphi_m}{\sin\frac{\Phi_C}{2}} \left\{ 2\sin x \cos x * \cos x \sin x - (\cos^2 x - \sin^2 x)\sin^2 x \right\}$$

(A2)

$$D_n = \frac{2\delta_{supr} * \sqrt{\beta_m \beta_C} * \cos\varphi_m}{\sin\frac{\Phi_C}{2}} * \sin^2\frac{n\Phi_C}{2}$$

and in similar calculations:

$$D'_n = \frac{2\delta_{supr} * \sqrt{\beta_m \beta_C} * \cos\varphi_m}{\sin\frac{\Phi_C}{2}} * \sin n\Phi_C$$

This expression gives the dispersion generated in a certain number of *n* cells as a function of the dipole kick δ in these cells.
At the end of the dispersion generating section the value obtained for D(s) and D'(s) has to be equal to the value of the periodic solution:

→equating *(A1)* and *(A2)* gives the conditions for the matching of the periodic dispersion in the arc to the values D = D'= 0 afte the suppressor.

$$D_n = \frac{2\delta_{supr} * \sqrt{\beta_m \beta_C} * \cos\varphi_m}{\sin\frac{\Phi_C}{2}} * \sin^2\frac{n\Phi_C}{2} = \delta_{arc}\sqrt{\beta_m \beta_C} * \frac{\cos\varphi_m}{\sin\frac{\Phi_C}{2}}$$

$$\left.\begin{array}{l} \rightarrow 2\delta_{\sup r}\sin^2(\frac{n\Phi_C}{2}) = \delta_{arc} \\ \rightarrow \sin(n\Phi_C) = 0 \end{array}\right\} \delta_{\sup r} = \frac{1}{2}\delta_{arc}$$

and at the same time the phase advance in the arc cell has to obey the relation:

$$n\Phi_C = k*\pi,\ k = 1, 3,\ \ldots$$