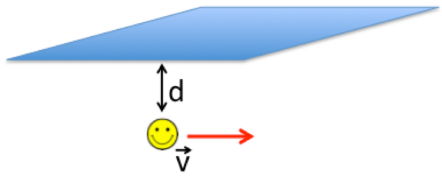
Some old JUAS excercises
(M. Migliorati)

- Obtain the coherent and incoherent betatron tune shift of a uniform proton beam of radius $a = 5$ mm, length $l_0 = 1$ m, inside a circular perfectly conducting pipe of radius $b = 35$ mm, with constant kinetic energy $E_0 = 2$ GeV (other parameters: total number of protons $N = 1 \times 10^{11}$, machine bending radius $\rho_x = 100$ m, betatron tune $Q_0 = 4.15$, classical radius of proton $r_p = 1.53 \times 10^{-18}$ m, proton rest mass = 0.938 GeV).

$$\Delta Q_c = -\frac{\rho_x^2 N r_p}{b^2 \beta^2 \gamma Q_0 l_0} = -0.107, \qquad \Delta Q_{inc} = -\frac{\rho_x^2 N r_p}{a^2 \beta^2 \gamma^3 Q_0 l_0} = -0.536$$

- Evaluate the electromagnetic force acting on a charge q at a distance d from an infinite, perfectly conducting plane, moving with a relativistic velocity v parallel to the plane (see figure).



It is the same as that of two charges at distance $2d$ moving with the same velocity on parallel trajectories:

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{\gamma 4 d^2}$$

- Obtain the vertical indirect space charge force of a uniform d.c. current (uniform beam distribution in longitudinal and transverse plane) inside a dipole magnet, which can be considered as two parallel plates of ferromagnetic material knowing that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ under the approximation that $g \gg a$ with $2g$ the dipole gap and a the beam radius.

The magnetic field can be obtained by removing the screens and considering image currents flowing in the same direction. If the current is along ‘z’, then the horizontal magnetic field in a position ‘y’ inside the beam is

$$B_x^{im}(z,y) = \frac{\mu_0 \beta c \bar{\lambda}(z)}{2\pi} \sum_{n=1}^{\infty} \left[\frac{1}{2ng - y} - \frac{1}{2ng + y} \right]$$

By using the approximation *h>>a>y*, the magnetic field due to the image currents is

$$B_x^{im}(z,y) \cong \frac{\mu_0 \beta c \bar{\lambda}(z) y}{4\pi g^2} \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\mu_0 \beta c \bar{\lambda}(z) \pi^2 y}{24\pi g^2}$$

and the corresponding force is $F_y^{im}(z,y) = \frac{e\beta^2 \bar{\lambda}(z) \pi^2}{24\pi\varepsilon_0 g^2} y$

- The linac of the Next Linear Collider has a length of 10 km and accelerates Gaussian bunches with $\sigma_z = 150\ \mu$m containing 1.1e10 electrons from 10 to 500 GeV. Assume a uniform betatron focusing with 100 betatron oscillations in the linac. The accelerating structure has a transverse wake that, inside the bunch, can be linearly approximated. The slope of the transverse wake per unit length of the linac is: $W_\perp(z)/(zL_w)$ = -5.2e6 V/pC/m^3, where z is the negative distance from the head of the bunch, which, for the Gaussian shape, is supposed to be $3\sigma_z$ ahead of the center. Compute the growth of the oscillation amplitude of the tail with respect to the head at the end of the linac with the two-particle model, evaluating the transverse wake at $z = -5\sigma_z$.

$$\left(\frac{\hat{y}_2 - \hat{y}_1}{\hat{y}_1}\right) = \frac{cNeW_\perp(z)L_L}{4\omega_y(E_0/e)L_w}\frac{E_0}{E_f}\ln\frac{E_f}{E_0}$$

$$\omega_y = c\,k_y = c\,2\pi\frac{100}{L_L}$$

$$\frac{W_\perp}{L_w} = -5.2\times10^6\times(-5\sigma_z)\times10^{12}\frac{V}{Cm^2}$$

$$\frac{E_0}{E_f}\ln\frac{E_f}{E_0} = 0.0782$$

$$\rightarrow\left(\frac{\hat{y}_2 - \hat{y}_1}{\hat{y}_1}\right) = \frac{1.1\mathrm{e}10\times1.6\mathrm{e}(-19)\times5.2\mathrm{e}6\times5\times150(\mathrm{e}-6)\times10^{12}L_L^2}{8\pi\ 100(E_0/e)}0.0782 = 2.1$$

- The longitudinal wake field of a pill box cavity of length g at high frequency ($\omega \gg c/b$, diffraction model), with b the pipe radius, can be written as

$$w_{//} = \frac{Z_0c\sqrt{2g}}{2\pi^2b}\frac{1}{\sqrt{z}}$$

Evaluate the total energy lost by a uniform bunch distribution of length l_0 with total charge q due to this longitudinal wake field.

From the tutorial, the energy lost by a point charge inside the distribution is

$$U(z) = -\frac{2eqZ_0c\sqrt{g(l_0-2z)}}{l_02\pi^2b}$$

and the total energy lost by the bunch is

$$U_{bunch} = \frac{1}{e}\int_{-\infty}^{\infty}U(z)\lambda(z)dz = -\frac{q^2Z_0c\sqrt{g}}{l_0^2\pi^2b}\int_{-\frac{l_0}{2}}^{\frac{l_0}{2}}\sqrt{l_0-2z}\,dz$$

$$= \frac{q^2Z_0c\sqrt{g}}{l_0^2\pi^2b}\frac{1}{3}(l_0-2z)^{\frac{3}{2}}\Big|_{-\frac{l}{2}}^{\frac{l}{2}} = -\frac{2}{3}\frac{q^2Z_0c}{\pi^2b}\sqrt{\frac{2g}{l_0}}$$

- In the past at CERN the beam was directly injected from the LINAC2 into the PS machine. However, the space charge tune spread was too big, and it was decided to build the PSB machine (with a radius 4 times smaller than the PS) to reduce it. Can you explain why that could help?

- For both longitudinal and transverse planes, discuss whether the space charge forces can have only defocusing effect, a focusing effect, or both. Motivate the answer.