Nature is so extra!

An Introduction to the Standard Model of Particle Physics
Lecture II: A closer look

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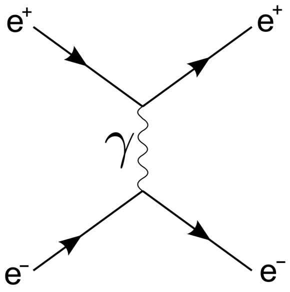
many diagrams from *Modern Particle Physics* by M. Thomson

What we know: forces through boson *exchange*

- fundamental particles *interact*: scatter, decay, annihilate...



Feynman diagram for Bhabha scattering ($e^+e^- \to e^+e^-$)

- basic interactions (EM, weak, strong) understood as due to boson exchange (γ, $W^\pm$ or Z, g)

Interaction by particle exchange

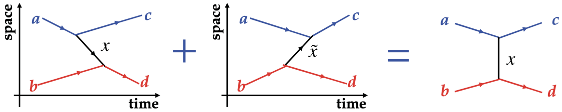
- QM: transition probability (rate) from one state to another given by Fermi's Golden Rule

$$\Gamma_{fi} = 2\pi |T_{fi}|^2 \rho(E_f)$$

- T_{fi} encodes the fundamental physics in an amplitude, i.e., couplings/charges...
- $\rho(E_f)$ density of available states ($\frac{dN}{dE}$) for f at E_f

Interaction by particle exchange: toy example

- Toy example: *charged* particles exchange *spinless* boson x



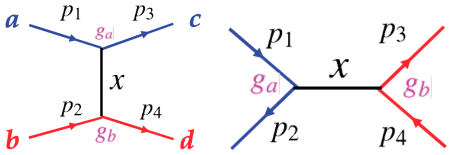
- transitions from $i \rightarrow f$ can proceed via two *time orderings*
- summing the from each leads to manifestly Lorentz-invariant matrix-element

$$\mathcal{M}_{fi} = \frac{g_A g_B}{q^2 - m^2}$$

- factor of $\frac{1}{q^2 - m^2}$ is called the *propagtor* arises naturally from picture of interaction by particle exchange
- $\approx$ amplitude for x to be found at the second spacetime point

Interaction by particle exchange: toy example

- $a + b \to c + d$ can also proceed via *annihilation*

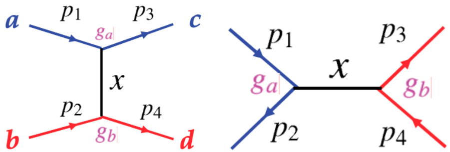


- expression for $\mathcal{M}_{fi}$ unchanged

$$\mathcal{M}_{fi} = \frac{g_A g_B}{q^2 - m^2}$$

- However, let's take a closer look at q^2

Interaction by particle exchange: toy example



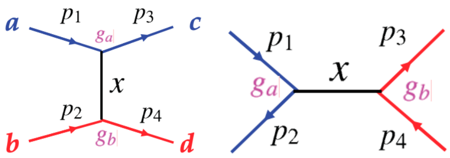
- left: $q^2 = (p_1 - p_3)^2 = (p_2 - p_4)^2 = t$
- right: $q^2 = (p_1 + p_2)^2 = (p_3 + p_4)^2 = s$

$$\mathcal{M}_{fi} = \frac{g_A g_B}{q^2 - m^2}$$

- referred to as $t-$ and $s-$channel diagrams respectively
- t and s are two of the Mandelstam* variables

*Bio of Stanley Mandelstam, South African

Interaction by particle exchange: toy example

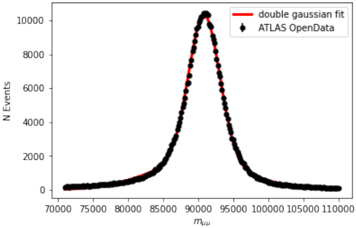


$$\mathcal{M}_{fi} = \frac{g_A g_B}{q^2 - m^2}$$

- Same form for $\mathcal{M}_{fi}$ in $t-$ and $s-$channel diagrams but significant kinematic differences:
  - **s-channel**: observed final state system has: $M_X^2 = (p_1 + p_2)^2$
  - If X is massive, e.g $M_X \approx y$, peak appears in $|\mathcal{M}_{fi}|^2$ at $q \approx y$
    - probability for $q\bar{q} \to Z \to \mu^+\mu^-$ peaks when $q \approx M_Z$

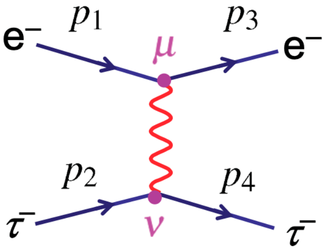
Shameless advertising!

- Why not study $q\bar{q} \to Z \to \mu^+\mu^-$ yourself?
- Fully documented Python Jupyter Notebook to study $q\bar{q} \to Z \to \mu^+\mu^-$ publicly available at this link
- Study of Z and Higgs bosons with ATLAS OpenData forms a third-year lab at UCT
- Try it out!



The basic QED interaction

- We have surprisingly little to do in order to make the toy example into a real expression for the basic QED process
- Consider the interaction between e and τ leptons by the exchange of a photon.
- Same ideas apply, but now we must account for the spin of the e,τ and also the spin (polarization) of the virtual photon.

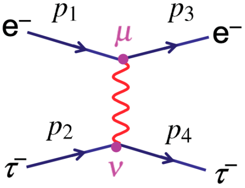


The basic QED interaction

$$\mathcal{M}_{fi} = \left[Q_e e \bar{u}_e(p_3)\gamma^\mu u_e(p_1)\right]\left[\sum_\lambda \frac{\epsilon_\mu^\lambda(\epsilon_\nu^\lambda)^*}{q^2}\right]\left[Q_\tau e \bar{u}_\tau(p_4)\gamma^\nu u_\tau(p_2)\right]$$

$$\mathcal{M}_{fi} = \left[Q_e e \bar{u}_e(p_3)\gamma^\mu u_e(p_1)\right]\left[\frac{-g_{\mu\nu}}{q^2}\right]\left[Q_\tau e \bar{u}_\tau(p_4)\gamma^\nu u_\tau(p_2)\right]$$

- interaction of $e^\pm$ with photon
- massless photon propagator summing over polarisations[†]
- interaction of $\tau^\pm$ with photon

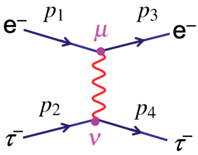


[†] not trivial, Thomson Appendix D.4

The basic QED interaction

$$\mathcal{M}_{fi} = \left[Q_e e \bar{u}_e(p_3)\gamma^\mu u_e(p_1)\right] \left[\frac{-g_{\mu\nu}}{q^2}\right] \left[Q_\tau e \bar{u}_\tau(p_4)\gamma^\nu u_\tau(p_2)\right]$$

- $\left[\bar{u}_e(p_3)\gamma^\mu u_e(p_1)\right]$ four-vector "current" j_e^μ
- $\left[\bar{u}_\tau(p_4)\gamma^\nu u_\tau(p_2)\right]$ four-vector "current" j_τ^ν



- Identification of four-vector currents allows manifestly LI

$$\mathcal{M} = -\frac{e^2}{q^2} j_e^\mu \cdot j_\tau^\nu$$

Recap: Parity

- The parity *operation* is equivalent to spatial inversion through the origin: $x \to -x$
- So the QM parity operator $\hat{P}$ behaves as: $\psi(x,t) \to \psi'(x,t) = \hat{P}\psi(x,t) = \psi(-x,t)$
- $\hat{P}$ is a Hermitian operator - corresponds to an observable property (real eigenvalues), clearly $\hat{P}\hat{P} = I$

Scalars, pseudoscalars, vectors and axial vectors

(High-school physics+)

- Physical quantities classified by rank and $\hat{P}$ inversion properties
  - **Scalar:** invariant under $\hat{P}$, e.g. mass, temperature
    - Can also be formed from, scalar product of two vectors, e.g $P^{\mu}P_{\mu} = m^2c^2$
  - **Vector:** sign change under $\hat{P}$, e.g. position, momentum
  - **Axial vector:** vectors, but invariant under $\hat{P}$, e.g. $\vec{L} = \vec{x} \times \vec{p}$
  - **Pseudoscalar:** single-valued, but sign change under $\hat{P}$, e.g. $h = \vec{S} \cdot \vec{p}$

Table 11.1 The parity properties of scalars, pseudoscalars, vectors and axial vectors.

	Rank	Parity	Example
Scalar	0	+	Temperature, T
Pseudoscalar	0	−	Helicity, h
Vector	1	−	Momentum, $\mathbf{p}$
Axial vector	1	+	Angular momentum, $\mathbf{L}$

Parity operator as a chirality test of a theory

- Chirality - the inherent handedness of a fundamental particle.
- Do our theories care about Chirality?
- Applying the parity operator to the theory gives us the answer...
  - How? $\rightarrow$ Parity operation changes handedness
- Don't confuse Chirality with Helicity
  - **chirality** - determined by transformation properties of ψ
  - **helicity** - (frame dependent) projection of spin vector on momentum vector

Parity conservation in QED

$$\mathcal{M} = -\frac{e^2}{q^2} j_e^{\mu} \cdot j_\tau^{\nu}$$

Let's apply $\hat{P}$ to $\mathcal{M}$ see what happens to $\mathcal{M}$

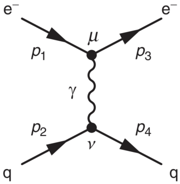
Only consider the product of the currents $j_e \cdot j_q$

As $j_e \cdot j_q = j_e^0 \cdot j_q^0 - j_e^k \cdot j_q^k$

It's clear that $j_e \cdot j_q$ transforms as

$$\hat{P}(j_e \cdot j_q) = j_e^0 \cdot j_q^0 - (-j_e^k \cdot -j_q^k)$$
$$= j_e \cdot j_q$$

parity is conserved in QED interactions

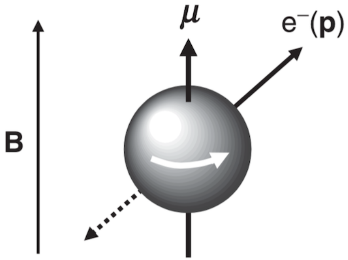


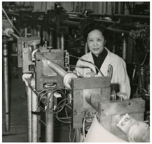
As QCD interactions have the same form:

parity is conserved in QCD interactions

Parity in nuclear β-decay

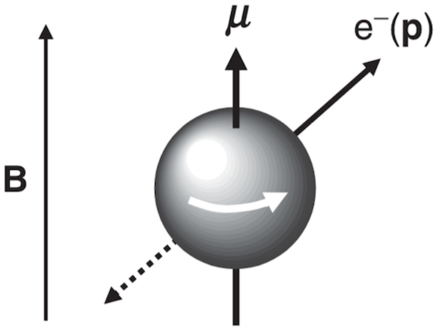
- Recall β decay involves the emission of a $W^{\pm}$ boson by a quark - clearly a weak interaction
- 1957 - C.S. Wu wiki et al. studied the parity structure of a particular β-decay
  - $^{60}Co \rightarrow ^{60}Ni^{*} + e^{-} + \bar{\nu}_e$
- Co nuclei possess permanent mag. moment $\vec{\mu}$ aligned in a strong magnetic field





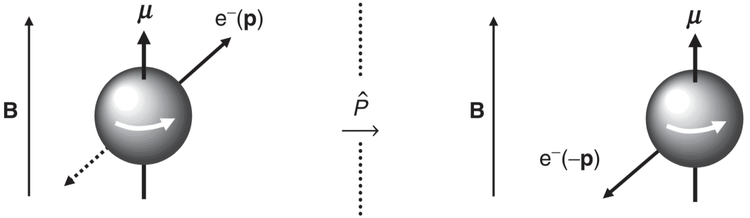
Parity in nuclear β-decay

- β electrons detected at various polar angles
- typical decay shown
- $\vec{B}$ and μ are axial vectors
- $\rightarrow$ only $\vec{p}_e$ changes sign under $\hat{P}$
- dashed line shows $\vec{p}_e$ after $\hat{P}$



Parity in nuclear β-decay

- if parity is conserved, the rates of the transformed and untransformed should be identical
- but they are not - e^- emitted in the opposite direction of $\vec{B}$ field much more often
- Thus parity is not conserved in the weak interaction
- clearly the weak interaction **cannot** have currents of the form $j^\mu = \bar{u}\gamma^k\gamma^0\gamma^0 u$



Parity in weak interactions

- From the Wu experiment we weak interactions can't be described with the parity conserving currents of QED/QCD
- What other form might the current take?
- Only 5 ways of combining the spinors to form currents that transform as to allow Lorenz Invariant amplitudes.

Type	Form	Components	Boson spin
Scalar	$\overline{\psi}\phi$	1	0
Pseudoscalar	$\overline{\psi}\gamma^5\phi$	1	0
Vector	$\overline{\psi}\gamma^\mu\phi$	4	1
Axial vector	$\overline{\psi}\gamma^\mu\gamma^5\phi$	4	1
Tensor	$\overline{\psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\phi$	6	2

Parity in weak interactions

- As the $W^{\pm}$ is spin-1, the answer must involved vector and axial vector currents.
- Most generic solution is just a linear combination of the two

$$j^{\mu} \propto \bar{u}(p')(g_V\gamma^{\mu} + g_A\gamma^{\mu}\gamma^{5})u(p) = g_V j_V^{\mu} + g_A j_A^{\mu}$$

- g_V and g_A are vector and axial-vector coupling constants. ‡
- Can a combination of vector and axial vector currents give the parity-violating amplitude we need?

‡ Full description given in Thomson pg. 290-292

Parity and $j.j$

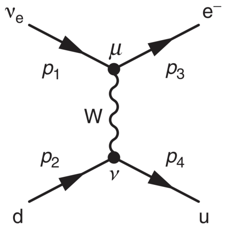
- Both $(j_V.j_V)$ and $(j_A.j_A)$ are invariant under parity transformations.
- What if $j = g_V j_V^\mu + g_A j_A^\mu$?
- Exercise:
  Show that $(j_V.j_A)$ transforms to $(-j_V.j_A)$ (**not parity invariant**)
- linear combination of vector and axial-vector current provides a mechanism to explain the observed parity violation in weak interactions.
- Let's try this out for a basic weak interaction!

The basic weak interaction

- Consider a basic weak interaction (inverse β decay, $\nu_e d \to e^- u$)
- We assume the currents have currents of the form

$$j^\mu_{\nu e} = \bar{u}(p_3)(g_V\gamma^\mu + g_A\gamma^\mu\gamma^5)u(p_1) = g_V j^V_{\nu e} + g_A j^A_{\nu e}$$

$$j^\mu_{du} = \bar{u}(p_4)(g_V\gamma^\mu + g_A\gamma^\mu\gamma^5)u(p_2) = g_V j^V_{du} + g_A j^A_{du}$$



- The amplitude will be proportional to the product of the two currents

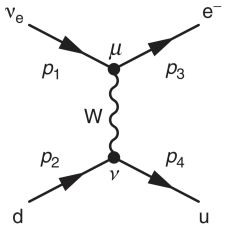
$$\mathcal{M} \propto j_{\nu e}.j_{du} = g_V^2(j^V_{\nu e}.j^V_{du}) + g_A^2(j^A_{\nu e}.j^A_{du}) + g_V g_A(j^V_{\nu e}.j^A_{du} + j^A_{\nu e}.j^V_{du})$$

The basic weak interaction under a parity transformation

- The VV and AA terms do not change sign under the parity transformation, but the AV term does:

$$j_{\nu e} . j_{du} \xrightarrow{\hat{P}}$$

$$g_V^2 j_{\nu e}^V . j_{du}^V + g_A^2 j_{\nu e}^A . j_{du}^A - g_V g_A (j_{\nu e}^V . j_{du}^A + j_{\nu e}^A . j_{du}^V)$$



- Ratio of parity-violating to non parity-conserving parts of the amplitude given by:

$$\frac{g_V g_A}{g_V^2 + g_A^2}$$

- Maximal parity violation when $g_V = g_A$

Parity in weak interactions

- From experiment, we know [§]
  - weak current due to $W^{\pm}$ bosons is a vector minus axial vector (V - A) interaction of the form $(\gamma^\mu - \gamma^\mu \gamma^5)$
  - $g_V = g_A$ Maximal parity violation!
- The corresponding vertex factor is:

$$\frac{-g_W}{\sqrt{2}} \bar{u}(p') \frac{1}{2} \gamma^\mu (1 - \gamma^5) u(p)$$

[§] Interesting history on the deduction of the V-A form link

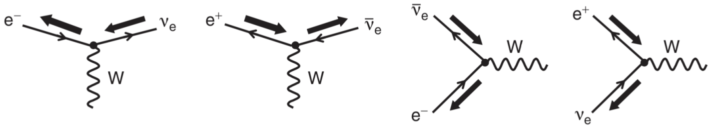
A closer look at the weak vertex factor

$$\frac{-g_W}{\sqrt{2}}\bar{u}(p')\frac{1}{2}\gamma^\mu(1-\gamma^5)u(p)$$

- g_W - the weak coupling strength
- $\frac{1}{2}\gamma^\mu(1-\gamma^5)$: the *chiral projection* operator
- Any Dirac spinor can be decomposed in to left and right-handed components with these operators
- $\frac{1}{2}\gamma^\mu(1-\gamma^5)$: the *left-handed* projection operator
- $\frac{1}{2}\gamma^\mu(1+\gamma^5)$: the *right-handed* projection operator
- This makes the **chiral** nature of the weak interaction explicit:
    - Only the left-handed part of the currents participate in the weak interaction!

Chirality in the weak vertex factor

- The presence of $\hat{P}_L$ makes the **chiral** nature of the weak interaction explicit:
  - Only the left-handed part of the fermion currents participate in the weak interaction!
- For anti-particle spinors, $\hat{P}_L$ projects out the right-handed states
  - Only the right-handed part of the anti-fermion currents participate in the weak interaction!



allowed helicity states for basic weak interactions

The W boson propagator

- The propagator term for the virtual photon exchanged in QED interactions is

$$\frac{-ig_{\mu\nu}}{q^2}$$

- Charged weak interactions are mediated by *massive* W bosons hence we must again sum over polarisation states of the $W^{\pm}$[¶], yielding a vertex factor:

$$\frac{-i}{q^2 - m_W^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}\right)$$

- For $q^2 << m_W$[‖], so the propagator is approximately

$$\frac{ig_{\mu\nu}}{m_W^2}$$

[¶] Thomson Appendix D

[‖] Thomson pg. 295

Fermi theory of β decay (1934)

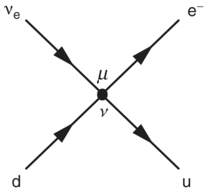
- Long before the discovery of parity violation or even the $W^{\pm}$ boson...
- low energy β-decay described as a *contact interaction* governed by the *Fermi coupling* G_F

$$\mathcal{M} = G_F g_{\mu\nu}[\bar{u}_3\gamma^{\mu}u_1][\bar{u}_4\gamma^{\nu}u_2]$$

- G_F is really small $\approx 1.16 \times 10^{-5} GeV^{-2}$
- After the discovery of parity violation, the amplitude is rewritten as:

$$\mathcal{M} = \frac{1}{\sqrt{2}}G_F g_{\mu\nu}[\bar{u}_3\gamma^{\mu}(1-\gamma^5)u_1][\bar{u}_4\gamma^{\nu}(1-\gamma^5)u_2]$$

- $\frac{1}{\sqrt{2}}$ appears to from the additional current and to keep G_F at the same value



Is the weak interaction weak?

- precise measurements of G_F and m_W allow us to extract the fundamental weak coupling constant g_W and dimensionless version α_W**

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$$

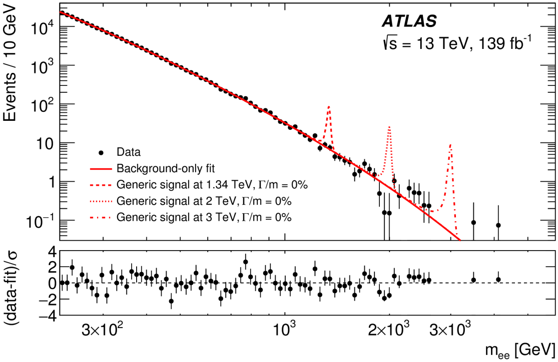
$$\alpha_W = \frac{g_W^2}{4\pi} \approx \frac{1}{30}$$

- The *weak* interaction is intrinsically stronger than the QED interaction $(\alpha_W > \alpha)$
- Before the W was discovered, the *weak* interaction seemed weak because some unknown physics at an experimentally-inaccessible energy was suppressing its effect...

**Thomson pg 297, 298

History often repeats itself...

- Could new physics be just around the corner?
- **Are you the physicist who will discover it?**



- Don't hesitate to ask/email if you are interested in particle physics and/or ATLAS!

Backup slides

Parity conservation in QED

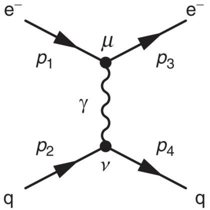
- Let's recall a basic QED interaction: electron-quark scattering.

We can write down matrix element as follows:

$$\mathcal{M} = \frac{Q_q e^2}{q^2} j_e \cdot j_q$$

where j_e^μ and j_q^ν are the electron and quark 4-vector currents.

$$j_e^\mu = \bar{u}(p_3)\gamma^\mu u(p_1), \; j_q^\nu = \bar{u}(p_4)\gamma^\mu u(p_2)$$



- Let's apply the parity operation to $\mathcal{M}$ and see what happens!

Parity conservation in QED

We apply the γ^0 operation to the spinors of the currents

Spinors transform as $\hat{P}u = \gamma^0 u$

Adjoint spinors transform as $\hat{P}\bar{u} = \bar{u}\gamma^0$

So the currents transform as

$$\hat{P}j_e^\mu = \bar{u}\gamma^0\gamma^\mu\gamma^0 u$$

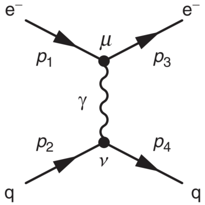
As $\gamma^0\gamma^0 = I$, the timelike (0) component of $\hat{P}j_e^\mu$ is

$$\hat{P}j_e^0 = \bar{u}\gamma^0\gamma^0\gamma^0 u = \bar{u}\gamma^0 u$$

the spacelike components (1,2,3) of $\hat{P}j_e^\mu$ are

$$\begin{aligned}\hat{P}j_e^k &= \bar{u}\gamma^0\gamma^k\gamma^0 u \\ &= -\bar{u}\gamma^k\gamma^0\gamma^0 u \\ &= -\bar{u}\gamma^k u\end{aligned}$$

$$(\textbf{Recall}\ \ \gamma^0\gamma^k = -\gamma^k\gamma^0)$$



Fermi theory of β decay (1934)

- Fermi theory says: $\mathcal{M} = \frac{1}{\sqrt{2}} G_F g_{\mu\nu} [\bar{u_3} \gamma^\mu (1-\gamma^5) u_1][\bar{u_4} \gamma^\nu (1-\gamma^5) u_2]$
- Weak theory says:

$$\mathcal{M} = \left[\frac{-g_W}{\sqrt{2}} \bar{u_3} \frac{1}{2} \gamma^\mu (1-\gamma^5) u_1 \right] \left[\frac{g_{\mu\nu} - q_\mu q_\nu / m_W^2}{q^2 - m_W^2} \right] \left[\frac{-g_W}{\sqrt{2}} \bar{u_4} \frac{1}{2} \gamma^\nu (1-\gamma^5) u_2 \right]$$

- if $q^2 << m_W^2$, Weak theory says:

$$\mathcal{M} = \frac{g_W^2}{8 m_W^2} g_{\mu\nu} \left[\bar{u_3} \gamma^\mu (1-\gamma^5) u_1 \right] \left[\bar{u_4} \gamma^\nu (1-\gamma^5) u_2 \right]$$

- so if $q^2 << m_W^2$ both theories give similar forms for $\mathcal{M}$

Fermi theory of β decay (1934)

- As Fermi's and the weak give almost identical results that agree with data when $q^2 << m_W^2$
- we can relate g_W to G_F. [††]

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$$

- The apparent *weakness* of the weak interaction at low energies is nothing more than an artefact of the large W boson mass!
- Fermi's theory had *absorbed* effect of the large m_W in the effective coupling G_F
- Fermi's theory ignores the $W^\pm$ boson propagtor...
  - When and how will Fermi's theory breakdown?

[††] Thomson pg 296, 297

Parity and axial vector currents

- We already saw how **vector** currents like those in QED and QCD transform under parity.
- What about **axial-vector** currents?

$$j_A^\mu = \bar{u}\gamma^\mu\gamma^5 u$$

$$j_A^\mu \xrightarrow{\hat{P}} \bar{u}\gamma^0\gamma^\mu\gamma^5\gamma^0 u = -\bar{u}\gamma^0\gamma^\mu\gamma^0\gamma^5 u$$

- **recall** $\gamma^5\gamma^0 = -\gamma^0\gamma^5$

Parity and axial vector currents

- So the time-like component of the axial-vector current transforms as

$$j_A^0 \xrightarrow{\hat{P}} -\bar{u}\gamma^0\gamma^0\gamma^0\gamma^5 u = -\bar{u}\gamma^0\gamma^5 u = -j_A^0$$

- and the space-like components of the axial-vector current transforms as

$$j_A^k \xrightarrow{\hat{P}} -\bar{u}\gamma^0\gamma^k\gamma^0\gamma^5 u = \bar{u}\gamma^k\gamma^5 u = j_A^k$$

- Recall $\gamma^0\gamma^k = -\gamma^k\gamma^0$
- summarising:

$$j_V^0 \xrightarrow{\hat{P}} j_V^k, j_V^k \xrightarrow{\hat{P}} -j_V^k \text{ and } j_A^0 \xrightarrow{\hat{P}} -j_A^0, j_A^k \xrightarrow{\hat{P}} j_A^k$$

Parity and $j.j$

- Our $t-$ and $s-$ channel amplitudes depend on scalar products of currents, i.e. $j.j$
- So the scalar product of two axial vectors is invariant under $\hat{P}$

$$j_{A,1} \cdot j_{A,2} = j^0_{A,1} j^0_{A,2} - j^k_{A,1} j^k_{A,2} \xrightarrow{\hat{P}} (-j^0_{A,1})(-j^0_{A,2}) - j^k_{A,1} j^k_{A,2} = j_{A,1} \cdot j_{A,2}$$