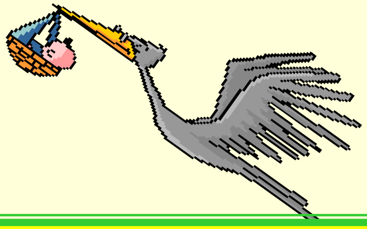
An Introduction to Theoretical Fluid and Plasma Physics



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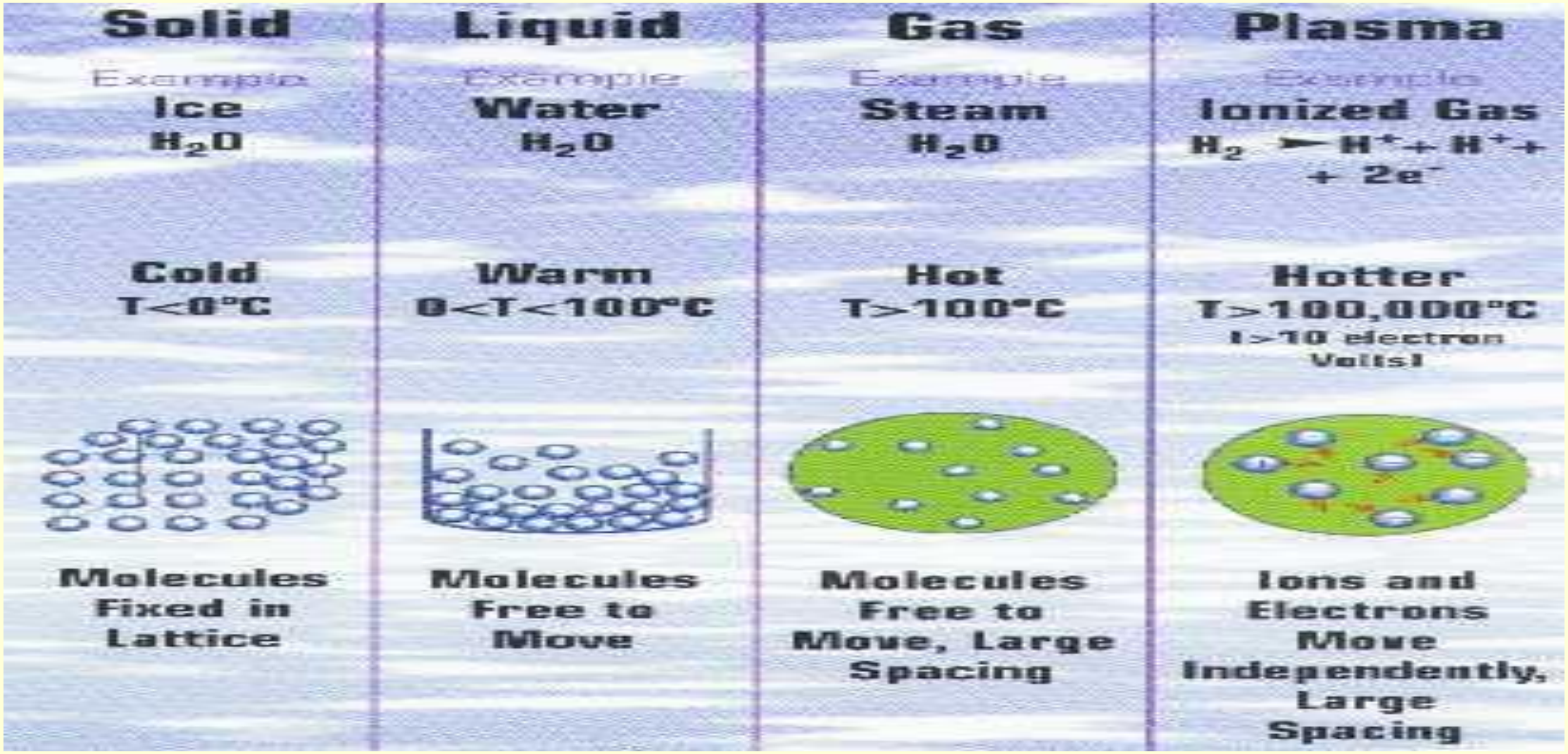
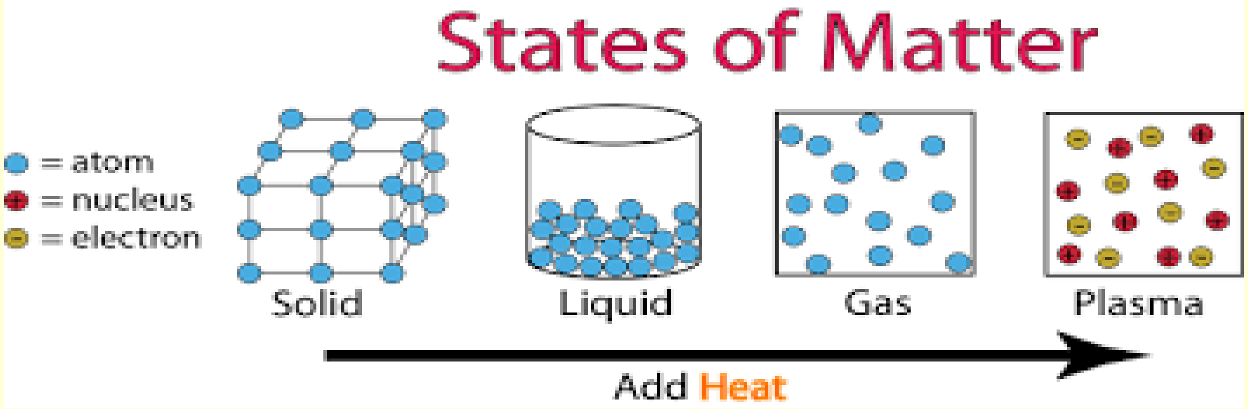
The "Fourth State" of the Matter

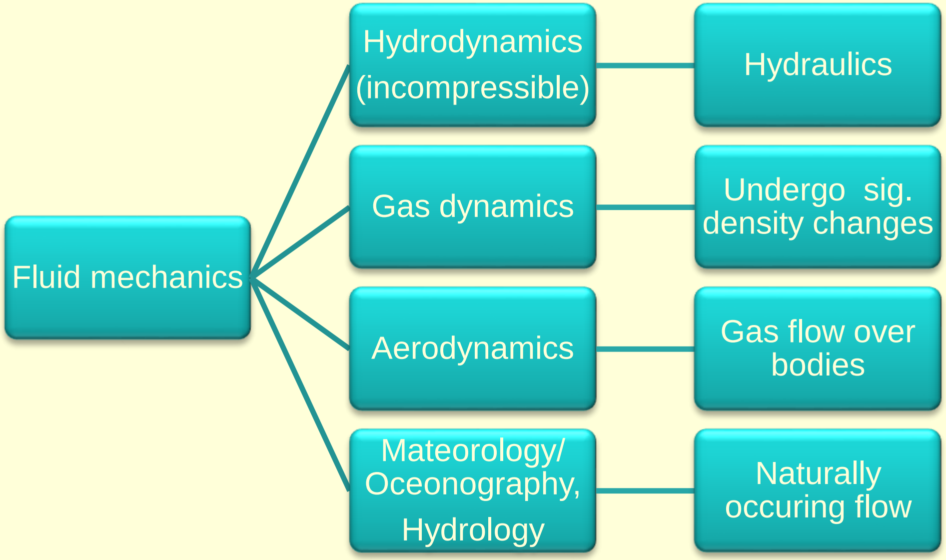




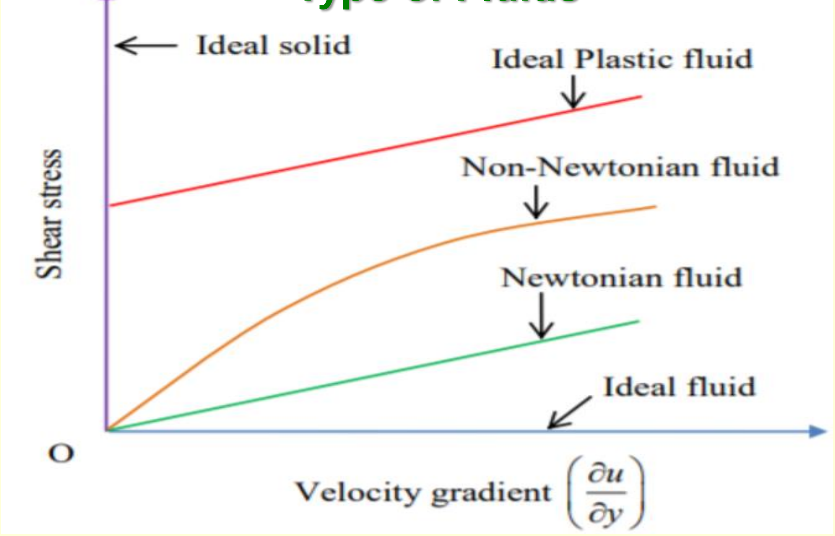
Fluid: Materials that can flow:

Liquid, Gas and Plasma.

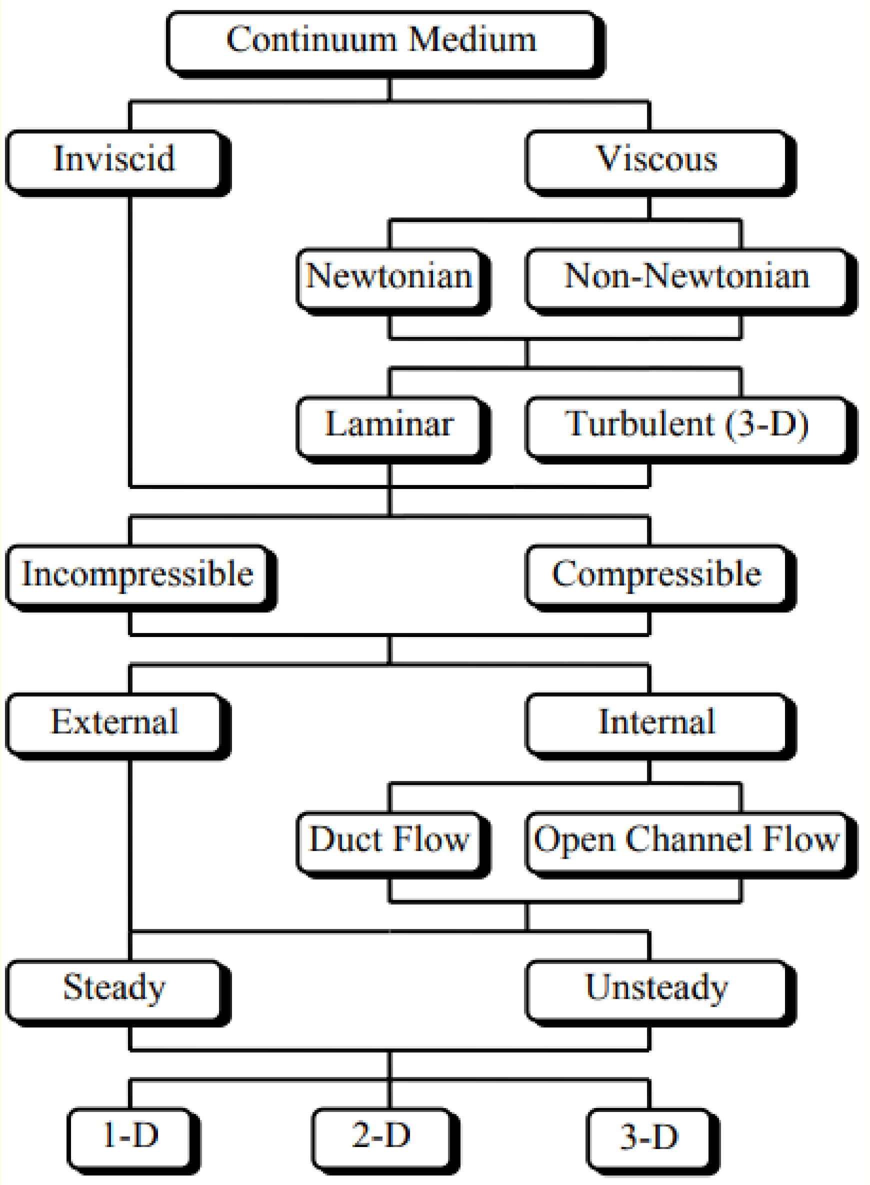
Fluid Mechanics



Type of Fluids



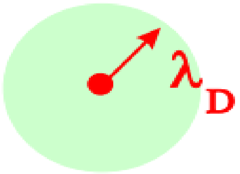
Classification of Fluid Motions



Standard Definition of Plasma

- "Plasma" named by Irving Langmuir in 1920's
- The standard definition of a plasma is as the 4th state of matter (solid, liquid, gas, plasma), where the material has become so hot that (at least some) electrons are no longer bound to individual nuclei. Thus a plasma is electrically conducting, and can exhibit collective dynamics.
- I.e., a plasma is an ionized gas, or a partially-ionized gas.
- Implies that the potential energy of a particle with its nearest neighboring particles is weak compared to their kinetic energy (otherwise electrons would be bound to ions). → Ideal "weakly-coupled plasma" limit. (There are also more-exotic strongly-coupled plasmas, but we won't discuss those.)
- Even though the interaction between any pair of particles is typically weak, the collective interactions between many particles is strong.
  2 examples: Debye Shielding & Plasma Oscillations.

DEBYE NUMBER



- Consider a sphere of radius the Debye length λ_{D}.
  It contains $N_{\mathrm{D}} \equiv \frac{4}{3}\pi\lambda_{\mathrm{D}}^{3} n_{\mathrm{e}}$ electrons: the Debye number.

- The Debye number is the number of electrons in the "coat" shielding any ion in the plasma.

- The Debye number is a measure of the importance of collective effects in the plasma.

- If $N_{\mathrm{D}} < 1$ there are no collective effects. The "plasma" is merely a collection of individual particles.

- If $N_{\mathrm{D}} > 1$ it is a true plasma and cooperative effects are important.

- Usually $N_{\mathrm{D}} \gg 1$, with N_{D} ranging from 10^{4} (laboratory) to 10^{32} (cluster of galaxies).

Plasma Applications

Applications of Plasma range from **energy production by thermonuclear fusion to laboratory astrophysics, creation of intense sources of high-energy particle and radiation beams, and fundamental studies involving high-field quantum electrodynamics**.

Plasma is being used in many high tech industries.

- It is used in **making many microelectronic or electronic devices such as semiconductors**.
- It can help make features on chips for computers.
- Plasma is also used in making transmitters for microwaves or high temperature films.

Plasma research is leading to profound new insights on the inner workings of the Sun and other stars, and fascinating astrophysical objects such as black holes and neutron stars. The study of plasma is **enabling prediction of space weather, medical treatments, and even water purification**.

Examples of naturally occurring plasmas:

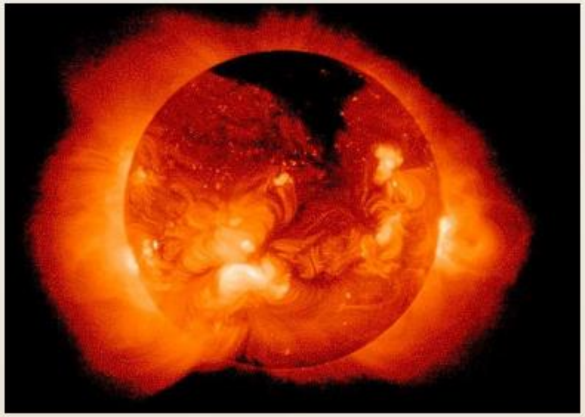
- (99% of the visible universe is a plasma)

Gas Nebula

Lightning

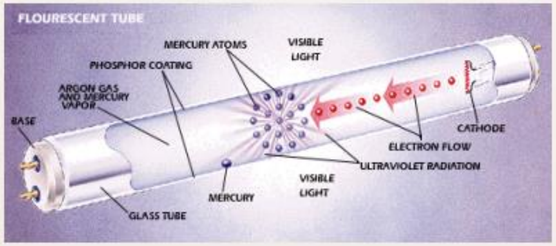
Flames

Aurora Borealis

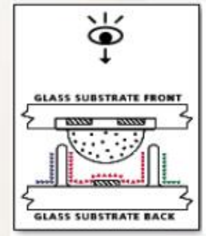


Solar Corona

Examples of man-made plasmas:

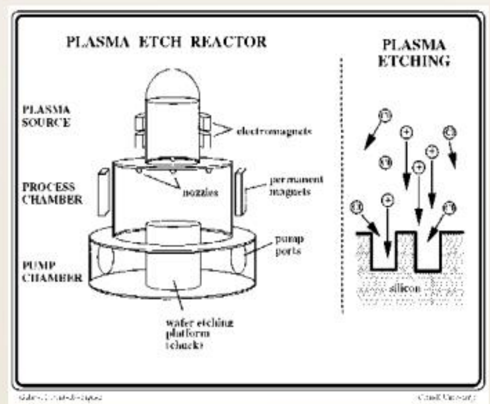


Fluorescent lamps
(glow discharge)

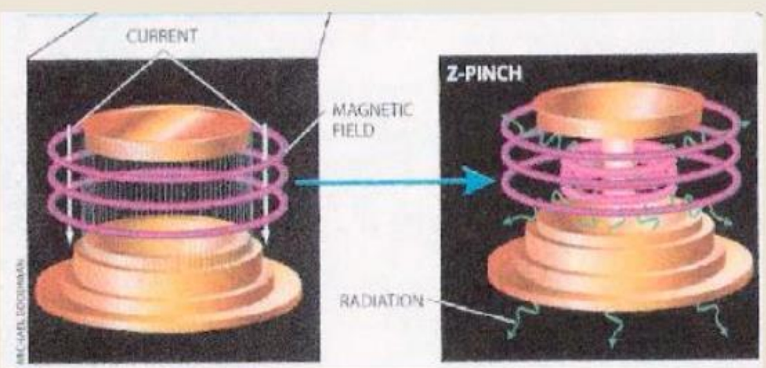


Flat panel plasma display

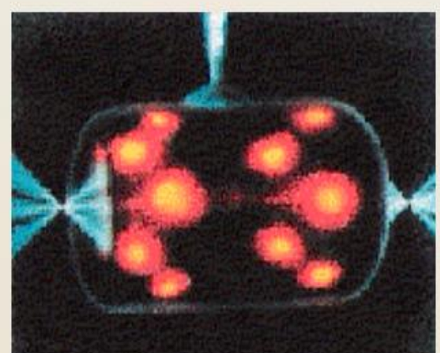
Plasma torch



Plasma etching reactor
(plasmas play important role in the manufacturing of integrated circuits)



Z-pinch



Laser-created plasmas

Fluid Description of Plasma

Plasma phenomena can be explained by a fluid model, in which the identity of the individual particle is neglected, and only the motion of fluid elements is taken into account

The theoretical study of plasma as a fluid is governed by the concept of magnetohydrodynamics (MHD) which involved a combination of conservation equations of conducting fluid mass, charges and momentum coupled with state equation and Maxwell equations of electromagnetism

Plasma may involve the dynamics **positively charged ion fluid** and **negatively charged electron fluid**. In a partially ionized gas, the dynamics of **fluid of neutral atoms** may also be involved. The neutral fluid will interact with the ions and electrons only through collisions. The ion and electron fluids will interact with each other even in the absence of collisions due to the generation of the electric and magnetic fields

Magneto-Fluid Dynamics

- Magneto-fluid dynamics (MFD) is the study of the flow of electrically conducting fluids in a magnetic field.
- MFD is derived from three words; magneto – magnetic field, fluids, and dynamics – movement.
- It covers phenomena where electrically conducting ionized fluids, with velocity field **V,** and the magnetic field **B** are coupled.
- Any movement of a conducting material in a magnetic field generates electric currents **j**, which in turn induce
  - their own magnetic fields, and
  - **j** x **B** forces on the medium known as *Lorentz* force.

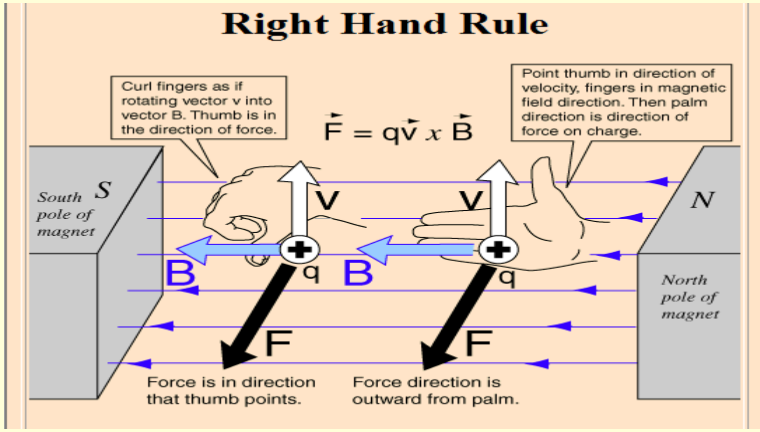
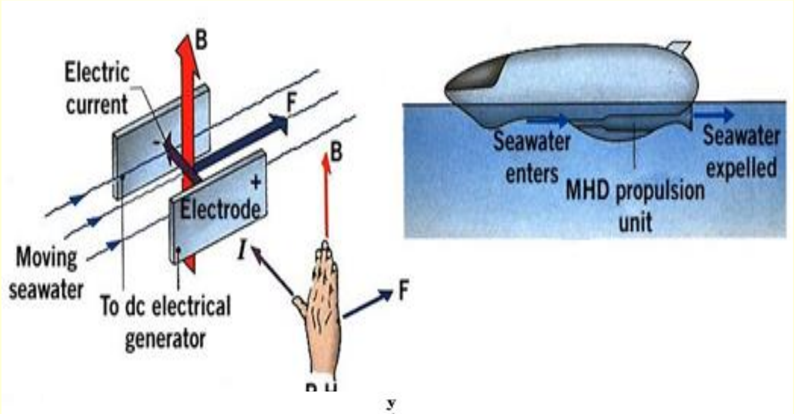
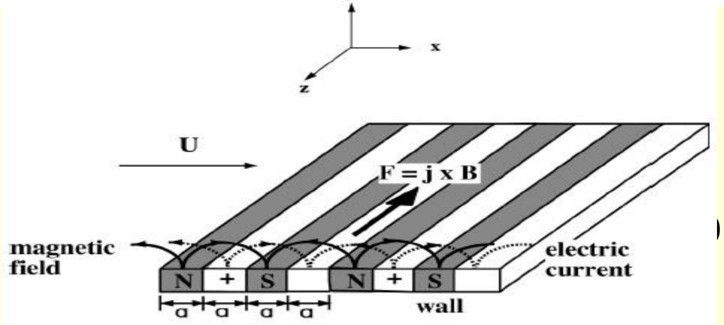


Fig.1: The Right Hand Rule





Hannes Alfvén (1908-1995), winning the Nobel Prize for his work on Magnetohydrodynamics.

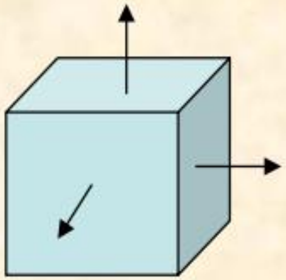
MHD equations: conservation laws

…for mass, momentum, energy, and magnetic flux.

Mass conservation:

Rate of change of mass in a volume is divergence of fluxes through surface

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0}$$



ρ = mass density

$\mathbf{v}$ = velocity

$\frac{\partial}{\partial t}$ = Eulerian derivative (at a fixed point in space)

$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$ = Lagrangian derivative (moving with flow)

MagnetoHydrodynamics Equations (Liquid and Gas)

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho U)=0$$ **Continuty Equation**

$$\frac{\partial U}{\partial t}+(U\cdot\nabla)U=-\frac{1}{\rho}\nabla p+\frac{\mu}{\rho}\nabla^2 U+\frac{1}{\rho}\mathbf{j}\times\mathbf{B},$$ **Momentum Equation**

$$(\rho C_p)\left(\frac{\partial T}{\partial t}+(U\cdot\nabla)T\right)=k\nabla^2 T+\mu\Phi+\frac{j^2}{\sigma}\pm q'''$$ **Energy Equation**

$$\frac{\partial \mathbf{B}}{\partial t}+(U\cdot\nabla)\mathbf{B}=(\mathbf{B}\cdot\nabla)U+\upsilon_e\nabla^2\mathbf{B},\ \nabla\cdot\mathbf{B}=0,$$ (Maxwell Equation)

$\mathbf{j}=\mu_e^{-1}\nabla\times\mathbf{B}$ (Ampere's law), $\dfrac{\partial \mathbf{B}}{\partial t}=-\nabla\times\mathbf{E}$ (Faraday's law), $\mathbf{j}=\sigma(\mathbf{E}+\mathbf{V}\times\mathbf{B})$ (Ohm's law)

Some Plasma Properties

Mass density $\rho_m = n_e m_e + n_i m_i$

Charge density $\sigma = q_e n_e + q_i n_i$

Mass velocity $V = (n_e m_e v_e + n_i m_i v_e) / \rho_m$

Current density $\mathbf{j} = q_e n_e v_e + q_i n_i v_i = q_e n_e (v_e - v_i)$

Total pressure $p = p_e + p_i$

where the subscripts *i* and *e* represent the ions and electrons, respectively.

Magnetohydrodynamics (MHD) Equations for Plasma

(1) $$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (nV) = 0,$$ (Mass Conservation Equation)

(2) $$\frac{\partial \sigma}{\partial t} + \nabla \cdot (n\mathbf{j}) = 0,$$ (Charge Conservation Equation)

(3) $$\underbrace{\rho_m\left(\frac{\partial V}{\partial t} + \mathrm{V} \cdot \nabla V\right)}_{\substack{\textit{rate}\text{ of change of} \\ \text{total momentum density}}} = \underbrace{\sigma E}_{\substack{\textit{Electric} \\ \text{body force}}} + \underbrace{\mathbf{j} \times B}_{\substack{\textit{Magnetic}\text{ force} \\ \text{on current}}} - \underbrace{\nabla P}_{\text{Pr}\textit{essure}},$$ (Momentum Equation)

or

$$\rho_m\left(\frac{\partial V}{\partial t} + \mathrm{V} \cdot \nabla V\right) = \sigma(E + V \times B) - \nabla P + \rho_m F, \quad F = -\upsilon_{jk}(v_j - v_k),$$

Maxwell Equations

(4) $$\nabla \times B = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial E}{\partial t}, \quad \nabla \times E = -\frac{\partial B}{\partial t},$$

$$\nabla \cdot B = 0, \ \nabla \cdot (\epsilon_0 \ E) = \sigma, \quad E + V \times B = \eta \mathbf{j} + \frac{\mathbf{j} \times B - \nabla p_e}{ne},$$

where B is the magnetic field strength, E is the electric field, n is the particle density and η is the resistivity.

Flux conservation:

Given by Maxwell's equations:

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}$$

$$\nabla \cdot \mathbf{B} = 0$$ (constraint rather than evolutionary equation)

From Ohm's Law, the current and electric field are related by

$$\mathbf{J} = \sigma \left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right)$$

For a fully conducting plasma, $\sigma \rightarrow \infty$

So c**E** = -(**v** x **B**).

$$\boxed{\frac{\partial B}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0}$$

Equation of state (EOS)

- An equation of state is a thermodynamic equation describing the state of matter under a given set of physical conditions.

$$p = p(n, T), \qquad \varepsilon = \varepsilon(n, T)$$

- Isothermal EOS for slow time variations, where temperatures are allowed to equilibrate. In this case, the fluid can exchange energy with its surroundings.

$$\boxed{p = nkT, \qquad \nabla p = kT\nabla n} \qquad n_{\mathrm{g}}\,(\mathrm{cm}^{-3}) \approx 3.250 \times 10^{16}\, p\,(\mathrm{Torr})$$

→ The energy conservation equation needs to be solved to determine *p* and *T*.

- Adiabatic EOS for fast time variations, such as in waves, when the fluid does not exchange energy with its surroundings

$$\boxed{p = Cn^{\gamma}, \qquad \frac{\nabla p}{p} = \gamma \frac{\nabla n}{n}} \qquad \gamma = \frac{C_p}{C_v} \quad \text{(specific heat ratio)}$$

→ The energy conservation equation is not required.

- Specific heat ratio vs degree of freedom (f) $\qquad \gamma = 1 + \dfrac{2}{f}$

Equation of state

Usually adopt the ideal gas law $\boxed{P=nkT}$

In thermal equilibrium, each internal degree of freedom has energy (kT/2). Thus, internal energy density for an ideal gas with *m* internal degrees of freedom

$$e = nm(kT/2).$$

Combining, $\boxed{P = (\gamma\text{-}1)e}$ where $\gamma = (m+2)/m$

For monoatomic gas (H), $\gamma=5/3$ ($m=3$)
diatomic gas (H_2), $\gamma=7/5$ ($m=5$)

Also common to use isothermal EOS $\boxed{P = C^2\rho}$ where C=isothermal sound speed when (radiative cooling time) << (dynamical time)

In some circumstances, an ideal gas law is not appropriate, and must use more complex (or tabular) EOS (e.g. for degenerate matter)

MODELLING IN FLUIDS AND PLASMA PHYSICS

Experimental vs. Theoretical Analysis

Fluids and plasma systems can be studied either *experimentally* (testing and taking measurements) or theoretically (by analysis or calculations).

- The **experimental approach** has the advantage that we deal with the actual physical system, and the desired quantity is determined by measurement, within the limits of experimental error. However, this approach is expensive, time-consuming, and often impractical.

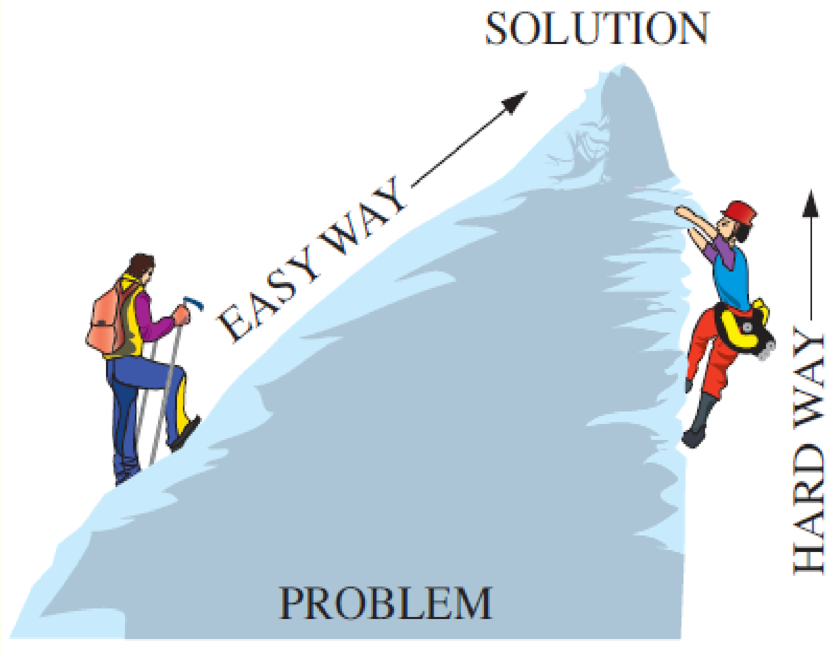
- The **theoretical approach** (including the numerical approach) has the advantage that it is fast and inexpensive, but the results obtained are subject to the accuracy of the assumptions, approximations, and idealizations made in the analysis.

PROBLEM-SOLVING TECHNIQUE

- Step 1: Problem Statement
- Step 2: Schematic
- Step 3: Assumptions and Approximations
- Step 4: Physical Laws
- Step 5: Properties
- Step 6: Calculations
- Step 7: Reasoning, Verification, and Discussion

The assumptions made while solving fluids and plasma physics problem must be reasonable and justifiable.

A step-by-step approach can greatly simplify problem solving.

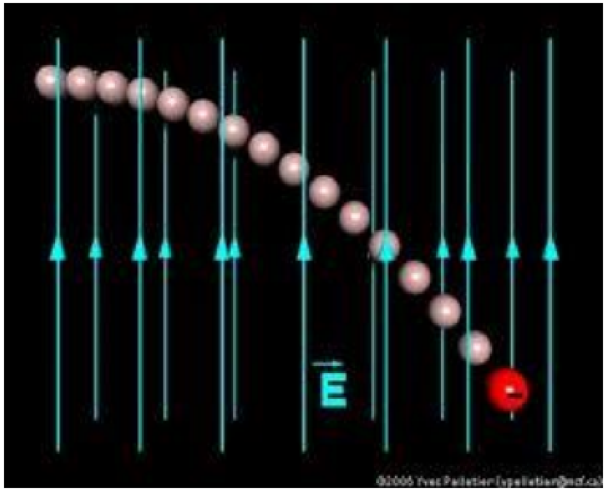


Motions of a charged particle in uniform electric field

- Equation of motion of a charged particle in fields

$$m\frac{d\boldsymbol{v}}{dt} = q[\boldsymbol{E}(\boldsymbol{r},t) + \boldsymbol{v} \times \boldsymbol{B}(\boldsymbol{r},t)], \qquad \frac{d\boldsymbol{r}}{dt} = \boldsymbol{v}(t)$$

- Motion in constant electric field



  ✓ For a constant electric field $\boldsymbol{E} = \boldsymbol{E}_0$ with $\boldsymbol{B} = 0$,

$$\boldsymbol{r}(t) = \boldsymbol{r}_0 + \boldsymbol{v}_0 t + \frac{q\boldsymbol{E}_0}{2m}t^2$$

  ✓ Electrons are easily accelerated by electric field due to their smaller mass than ions.

  ✓ Electrons (Ions) move against (along) the electric field direction.

  ✓ The charged particles get kinetic energies.

Motions of a charged particle in uniform magnetic field

- Motion in constant magnetic field

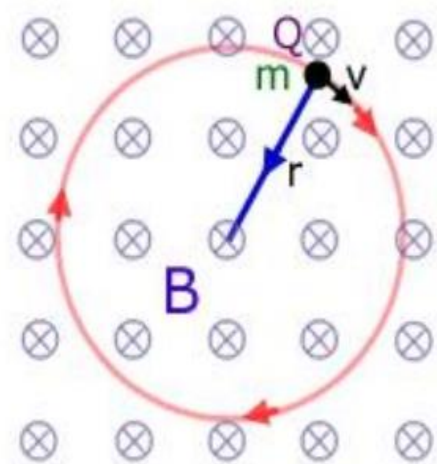
$$m\frac{d\boldsymbol{v}}{dt} = q\boldsymbol{v} \times \boldsymbol{B}$$

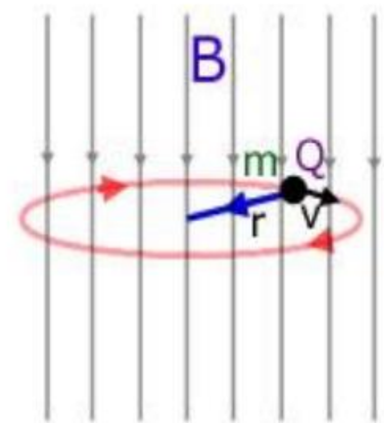
- For a constant magnetic field $\boldsymbol{B} = B_0\boldsymbol{z}$ with $\boldsymbol{E} = 0$,

$$m\frac{dv_x}{dt} = qB_0v_y$$

$$m\frac{dv_y}{dt} = -qB_0v_x$$

$$m\frac{dv_z}{dt} = 0$$





- Cyclotron (gyration) frequency

$$\frac{d^2v_x}{dt^2} = -\omega_c^2v_x$$

$$\boxed{\omega_c = \frac{|q|B_0}{m}}$$

Motions of a charged particle in uniform magnetic field

- Particle velocity

$$v_x = v_\perp \cos(\omega_c t + \phi_0)$$
$$v_y = -v_\perp \sin(\omega_c t + \phi_0)$$
$$v_z = v_{z0}$$

- Particle position

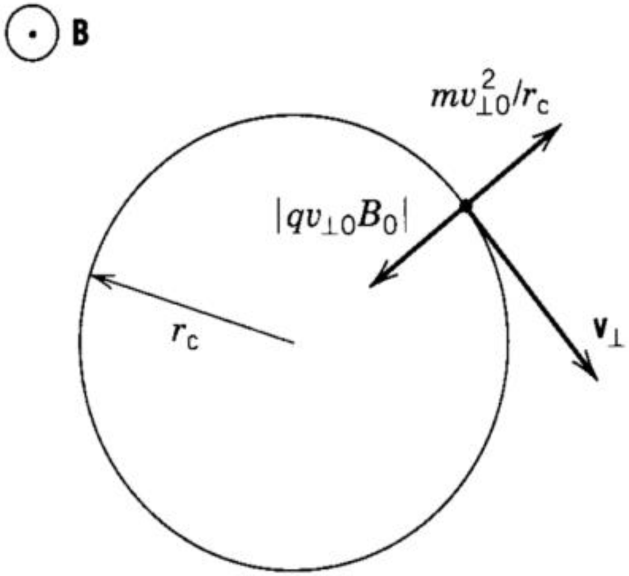
$$x = x_0 + r_c \sin(\omega_c t + \phi_0)$$
$$y = y_0 + r_c \cos(\omega_c t + \phi_0)$$
$$z = z_0 + v_{z0} t$$

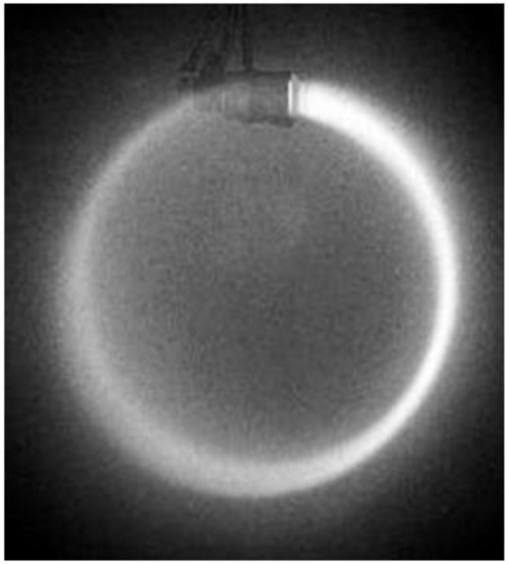
- Guiding center

$$(x_0, y_0, z_0 + v_{z0} t\ )$$

- Larmor (gyration) radius

$$\boxed{r_c = r_\mathrm{L} = \frac{v_\perp}{\omega_c} = \frac{m v_\perp}{|q| B_0}}$$

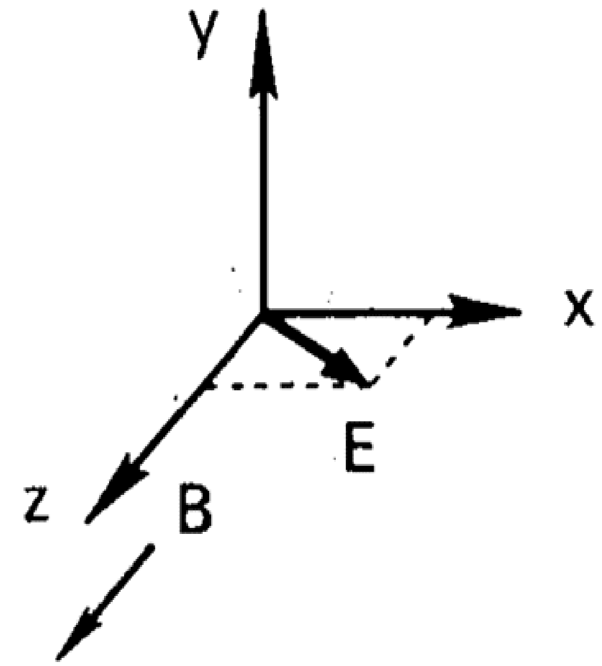




Motions of a charged particle in uniform E and B fields

- Equation of motion

$$m\frac{d\boldsymbol{v}}{dt} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$



- Parallel motion: $\boldsymbol{B} = B_0\boldsymbol{z}$ and $\boldsymbol{E} = E_0\boldsymbol{z}$,

$$m\frac{dv_z}{dt} = qE_z$$

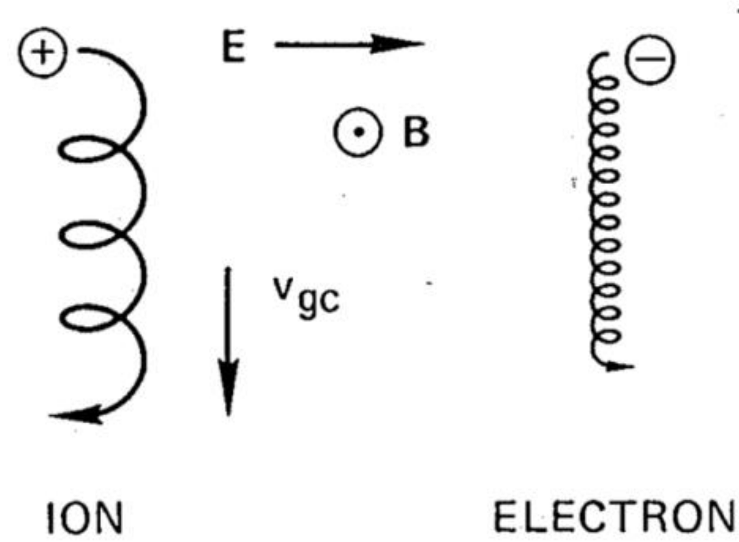
$$v_z = \frac{qE_z}{m}t + v_{z0}$$

→ Straightforward acceleration along B

$\boldsymbol{E}\times\boldsymbol{B}$ drift

- Transverse motion: $\boldsymbol{B} = B_0\boldsymbol{z}$ and $\boldsymbol{E} = E_0\boldsymbol{x}$,

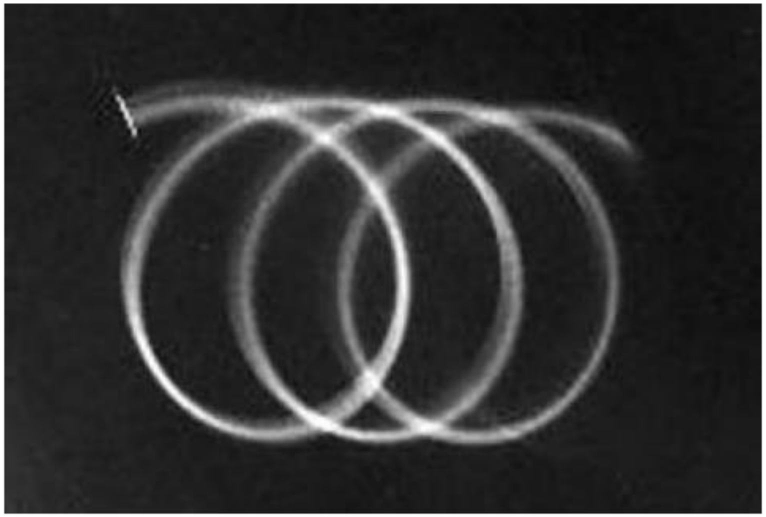
$$m\frac{dv_x}{dt} = qE_0 + qB_0v_y$$
$$m\frac{dv_y}{dt} = -qB_0v_x$$



- Differentiating,

$$\frac{d^2v_x}{dt^2} = -\omega_c^2 v_x$$
$$\frac{d^2v_y}{dt^2} = -\omega_c^2\left(\frac{E_0}{B_0} + v_y\right)$$

$$\boxed{\boldsymbol{v}_E = \frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2}}$$



- Particle velocity

$$v_x = v_\perp\cos(\omega_c t + \phi_0)$$
$$v_y = -v_\perp\sin(\omega_c t + \phi_0) - \frac{E_0}{B_0}$$

v_{gc}

Diffusion and mobility

- The fluid equation of motion including collisions

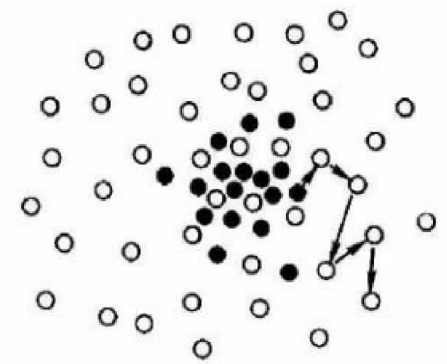
$$mn\frac{d\boldsymbol{u}}{dt} = mn\left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}\right] = qn\boldsymbol{E} - \boldsymbol{\nabla}p - mn\nu_m\boldsymbol{u}$$

- In steady-state, for isothermal plasmas

$$\boldsymbol{u} = \frac{1}{mn\nu_m}(qn\boldsymbol{E} - \boldsymbol{\nabla}p) = \frac{1}{mn\nu_m}(qn\boldsymbol{E} - kT\boldsymbol{\nabla}n)$$

$$= \frac{q}{m\nu_m}\boldsymbol{E} - \frac{kT}{m\nu_m}\frac{\boldsymbol{\nabla}n}{n} = \pm\mu\boldsymbol{E} - D\frac{\boldsymbol{\nabla}n}{n}$$

Drift Diffusion



Diffusion is a random walk process.

- In terms of particle flux

$$\boldsymbol{\Gamma} = n\boldsymbol{u} = \pm n\mu\boldsymbol{E} - D\boldsymbol{\nabla}n$$

$\mu = \frac{|q|D}{kT}$: Einstein relation

$\mu = \frac{|q|}{m\nu_m}$: Mobility

$D = \frac{kT}{m\nu_m}$: Diffusion coefficient

Ambipolar diffusion

- The flux of electrons and ions out of any region must be equal such that charge does not build up. Since the electrons are lighter, and would tend to flow out faster in an unmagnetized plasma, an electric field must spring up to maintain the local flux balance.

$$\Gamma_i = +n\mu_i E - D_i \nabla n \qquad\qquad \Gamma_e = -n\mu_e E - D_e \nabla n$$

- Ambipolar electric field for $\Gamma_i = \Gamma_e$

$$E = \frac{D_i - D_e}{\mu_i + \mu_e} \frac{\nabla n}{n}$$

- The common particle flux

$$\Gamma = -\frac{\mu_e D_i + \mu_i D_e}{\mu_i + \mu_e} \nabla n = -D_a \nabla n$$

- The ambipolar diffusion coefficient for weakly ionized plasmas

$$D_a = \frac{\mu_e D_i + \mu_i D_e}{\mu_i + \mu_e} \approx D_i + \frac{\mu_i}{\mu_e} D_e \approx D_i \left(1 + \frac{\mathrm{T_e}}{\mathrm{T_i}}\right)$$

Decay of a plasma by diffusion in a slab

- Diffusion equation

Derived from the continuity equation

$$\frac{\partial n}{\partial t} - D_a \nabla^2 n = 0 \quad \Leftarrow \quad n_i \approx n_e = n, \quad D = D_a$$

- In Cartesian coordinates,

$$\boxed{\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial x^2}}$$

- Find $n(x,t)$ under the boundary conditions [H/W]

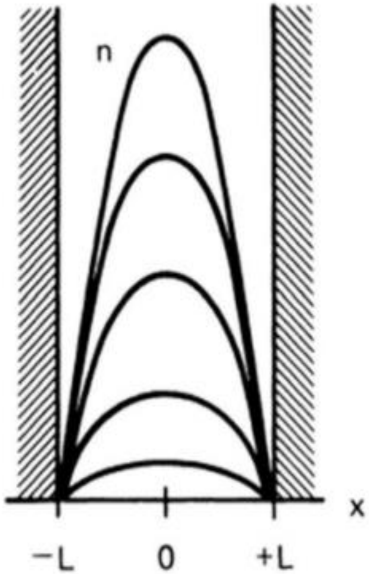
$$n(x = \pm L, t) = 0$$

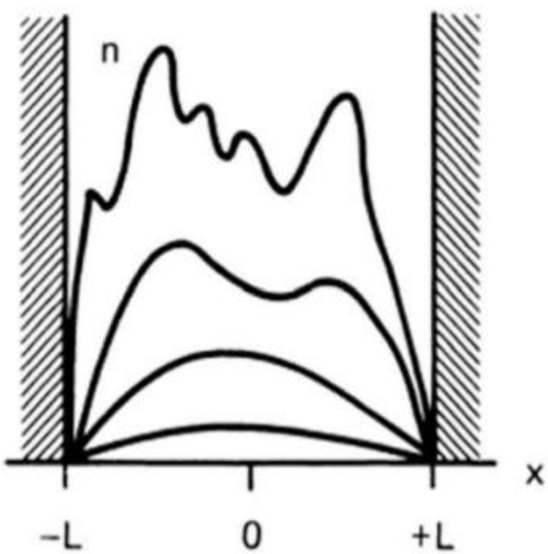
$$n(x, t = 0) = n_0(1 - (x/L)^2)$$

- In general

$$n = n_0 \left(\sum_l a_l e^{-t/\tau_l} \cos \frac{(l + \frac{1}{2})\pi x}{L} + \sum_m b_m e^{-t/\tau_m} \sin \frac{m\pi x}{L} \right)$$

$$\tau_l = \left[\frac{L}{(l + \frac{1}{2})\pi} \right]^2 \frac{1}{D}$$





Steady-state solution in cylindrical geometry

- Diffusion equation

volume source and sink

$$\frac{\partial n}{\partial t} - D_a \nabla^2 n = G - L = \nu_{iz} n - \alpha n^2 \quad \Leftarrow \quad n_i \approx n_e = n$$

- In steady-state, ignoring volume recombination

$$\boxed{\nabla^2 n + \frac{\nu_{iz}}{D} n = 0}$$

where, $D = D_a$ and ν_{iz} is the ionization frequency

- In cylindrical coordinates,

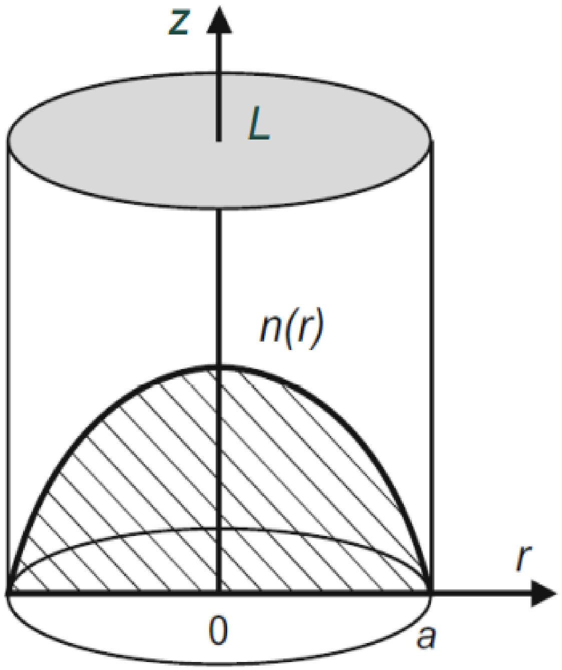
$$\frac{\partial^2 n}{\partial r^2} + \frac{1}{r}\frac{\partial n}{\partial r} + \frac{\partial^2 n}{\partial z^2} + \frac{\nu_{iz}}{D} n = 0$$

- Find $n(r, z)$ under the boundary conditions [H/W]

$$n(r = R, z) = 0$$

$$n(r, z = 0) = 0$$

$$n(r, z = L) = 0$$



Computational Demonstration-1

Decay of a Plasma in a Slab

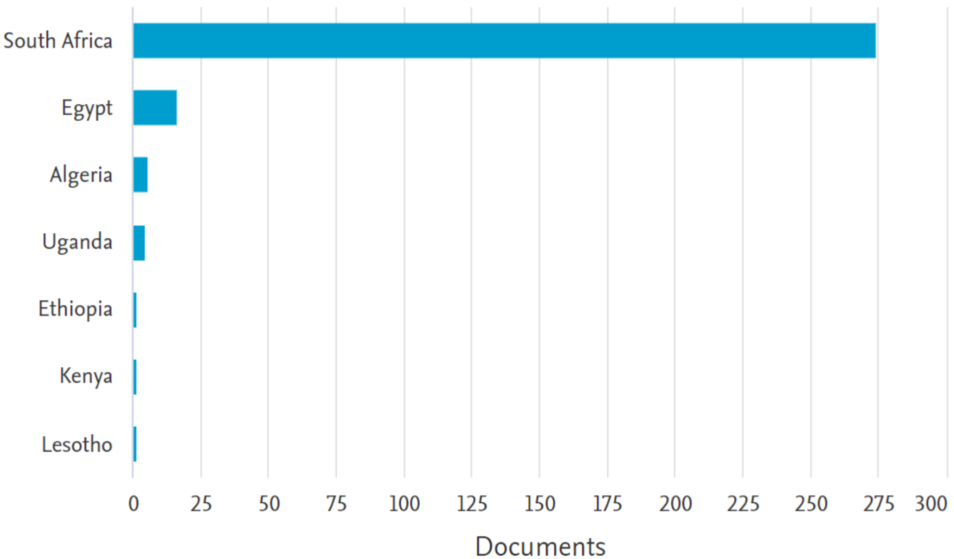
$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2},\ -1 \le x \le 1,\ t \ge 0,$$

$$n(-1,t) = n(1,t) = 0,\ n(x,0) = 0.5(1-x^2)$$

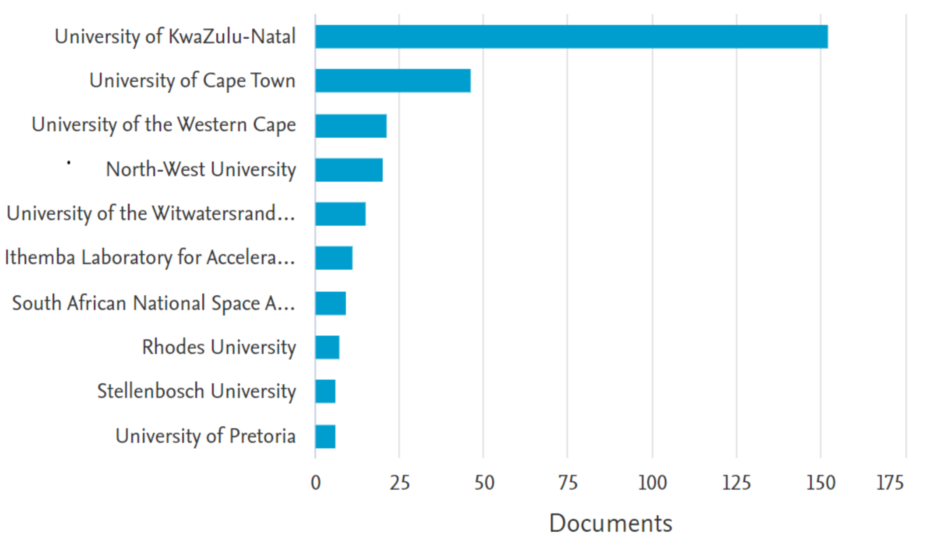
(Using MAPLE Software)

Africa Research Output in Fluid & Plasma Physics

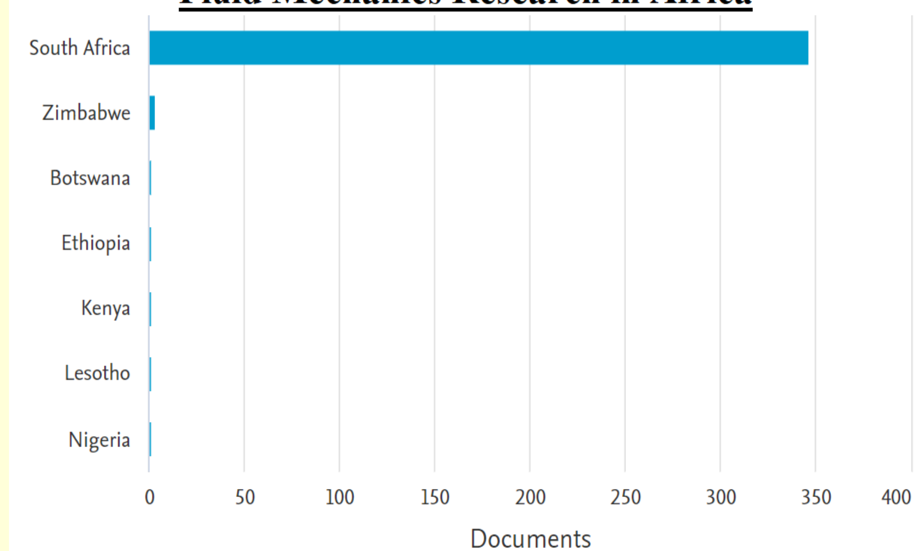
Fluid & Plasma Physics Research in Africa



Plasma Research in South Africa Institutions



Fluid Mechanics Research in Africa



Useful Constants and Formulae

Table 1: Commonly used physical constants

Name	Symbol	Value (SI)	Value (cgs)
Boltzmann constant	k_{B}	1.38×10^{-23} J K^{-1}	1.38×10^{-16} erg K^{-1}
Electron charge	e	1.6×10^{-19} C	4.8×10^{-10} statcoul
Electron mass	m_{e}	9.1×10^{-31} kg	9.1×10^{-28} g
Proton mass	m_{p}	1.67×10^{-27} kg	1.67×10^{-24} g
Planck constant	h	6.63×10^{-34} J s	6.63×10^{-27} erg-s
Speed of light	c	3×10^{8} m s^{-1}	3×10^{10} cm s^{-1}
Dielectric constant	ε_0	8.85×10^{-12} F m^{-1}	—
Permeability constant	μ_0	$4\pi \times 10^{-7}$	—
Proton/electron mass ratio	$m_{\mathrm{p}}/m_{\mathrm{e}}$	1836	1836
Temperature = 1eV	e/k_{B}	11 604 K	11 604 K
Avogadro number	N_{A}	6.02×10^{23} mol^{-1}	6.02×10^{23} mol^{-1}
Atmospheric pressure	1 atm	1.013×10^{5} Pa	1.013×10^{6} dyne cm^{-2}

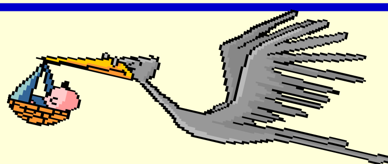
Table 2: Formulae in SI and cgs units

Name	Symbol	Formula (SI)	Formula (cgs)
Debye length	λ_{D}	$\left(\frac{\varepsilon_0 k_{\mathrm{B}} T_{\mathrm{e}}}{e^2 n_{\mathrm{e}}}\right)^{1/2}$ m	$\left(\frac{k_{\mathrm{B}} T_{\mathrm{e}}}{4\pi e^2 n_{\mathrm{e}}}\right)^{1/2}$ cm
Particles in Debye sphere	N_{D}	$\frac{4\pi}{3}\lambda_{\mathrm{D}}^3$	$\frac{4\pi}{3}\lambda_{\mathrm{D}}^3$
Plasma frequency (electrons)	ω_{pe}	$\left(\frac{e^2 n_{\mathrm{e}}}{\varepsilon_0 m_{\mathrm{e}}}\right)^{1/2}$ s^{-1}	$\left(\frac{4\pi e^2 n_{\mathrm{e}}}{m_{\mathrm{e}}}\right)^{1/2}$ s^{-1}
Plasma frequency (ions)	ω_{pi}	$\left(\frac{Z^2 e^2 n_{\mathrm{i}}}{\varepsilon_0 m_{\mathrm{i}}}\right)^{1/2}$ s^{-1}	$\left(\frac{4\pi Z^2 e^2 n_{\mathrm{i}}}{m_{\mathrm{i}}}\right)^{1/2}$ s^{-1}
Thermal velocity	$v_{\mathrm{te}} = \omega_{\mathrm{pe}}\lambda_{\mathrm{D}}$	$\left(\frac{k_{\mathrm{B}} T_{\mathrm{e}}}{m_{\mathrm{e}}}\right)^{1/2}$ m s^{-1}	$\left(\frac{k_{\mathrm{B}} T_{\mathrm{e}}}{m_{\mathrm{e}}}\right)^{1/2}$ cm s^{-1}
Electron gyrofrequency	ω_{c}	eB/m_{e} s^{-1}	eB/m_{e} s^{-1}
Electron–ion collision frequency	ν_{ei}	$\frac{\pi^{3/2} n_{\mathrm{e}} Z e^4 \ln\Lambda}{2^{1/2}(4\pi\varepsilon_0)^2 m_{\mathrm{e}}^2 v_{\mathrm{te}}^3}$ s^{-1}	$\frac{4(2\pi)^{1/2} n_{\mathrm{e}} Z e^4 \ln\Lambda}{3 m_{\mathrm{e}}^2 v_{\mathrm{te}}^3}$ s^{-1}
Coulomb logarithm	$\ln\Lambda$	$\ln\frac{9N_{\mathrm{D}}}{Z}$	$\ln\frac{9N_{\mathrm{D}}}{Z}$

CONCLUSION

Both theoretical and experimental research are very essential in fluid and plasma physics.

- **Complex fluid and plasma physics problems can be easily investigated theoretically using modelling and computational approach.**
- **Modelling helps to make things better, faster, safer and cheaper through simulation of complex phenomena and the reduction of the flood of data with visualisation**
- An important feature of the application of modelling and computations to fluid and plasma physics problem is that, it enables us to make scientific predictions that are to draw on the basis of logic and with the aid of mathematical methods, correct conclusions whose agreement with reality is then confirmed by experience, experiment and practice leading to innovation and national development



THE END

THANK YOU VERY MUCH

ALL THE BEST AND GOD BLESS