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Nuclear Reaction, Structure and Astrophysics

African School of Physics,
NMU 2022

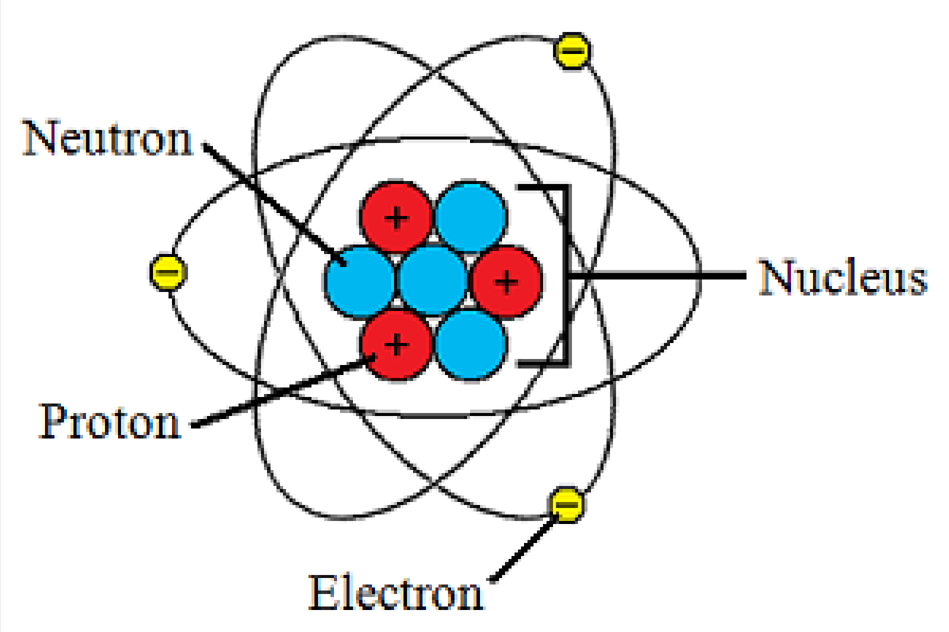
ASP

NRF
National Research Foundation
iThemba LABS
Laboratory for Accelerator Based Sciences

UNIVERSITY OF THE WITWATERSRAND
JOHANNESBURG

science & innovation
Department:
Science and Innovation
REPUBLIC OF SOUTH AFRICA

Outline



Part I

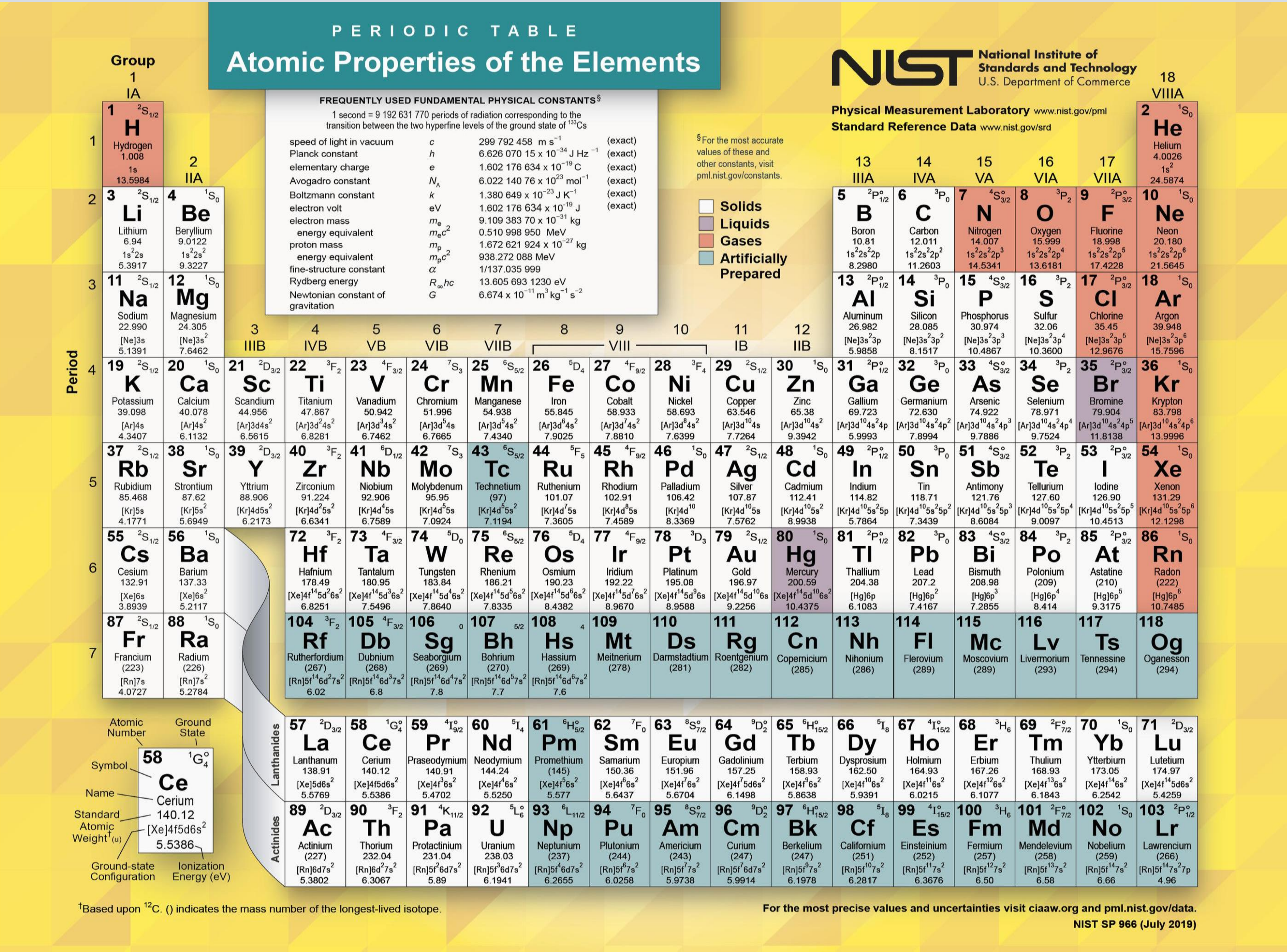
- Nuclear Properties
- Nuclear Reactions
- Nuclear Structure and Astrophysics

Experimental facilities at iThemba LABS

Part II

- Introduction to Nuclear Reaction Codes

Are we on the verge of a whole new beginning?
Newton's physics got us to the Moon and Mars;
Einstein's physics points us to the stars. We live in an exciting time!



PERIODIC TABLE
Atomic Properties of the Elements

National Institute of Standards and Technology
U.S. Department of Commerce

Physical Measurement Laboratory www.nist.gov/pml
Standard Reference Data www.nist.gov/srd

FREQUENTLY USED FUNDAMENTAL PHYSICAL CONSTANTS§

1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs

speed of light in vacuum	c	299 792 458 m s^{-1}	(exact)
Planck constant	h	6.626 070 15 x 10^{-34} J Hz^{-1}	(exact)
elementary charge	e	1.602 176 634 x 10^{-19} C	(exact)
Avogadro constant	N_A	6.022 140 76 x 10^{23} mol^{-1}	(exact)
Boltzmann constant	k	1.380 649 x 10^{-23} J K^{-1}	(exact)
electron volt	eV	1.602 176 634 x 10^{-19} J	(exact)
electron mass	m_e	9.109 383 70 x 10^{-31} kg	
energy equivalent	m_ec^2	0.510 998 950 MeV	
proton mass	m_p	1.672 621 924 x 10^{-27} kg	
energy equivalent	m_pc^2	938.272 088 MeV	
fine-structure constant	α	1/137.035 999	
Rydberg energy	$R_\infty hc$	13.605 693 1230 eV	
Newtonian constant of gravitation	G	6.674 x 10^{-11} m^3 kg^{-1} s^{-2}	

§For the most accurate values of these and other constants, visit pml.nist.gov/constants.

- ☐ **Solids**
- ▨ **Liquids**
- 🟥 **Gases**
- 🟦 **Artificially Prepared**

Period	Group 1 IA	2 IIA	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB	8	9 VIII	10	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
1	1 $^2S_{1/2}$ **H** Hydrogen 1.008 $1s$ 13.5984																	2 $^1S_0$ **He** Helium 4.0026 $1s^2$ 24.5874
2	3 $^2S_{1/2}$ **Li** Lithium 6.94 $1s^22s$ 5.3917	4 $^1S_0$ **Be** Beryllium 9.0122 $1s^22s^2$ 9.3227											5 $^2P^\circ_{1/2}$ **B** Boron 10.81 $1s^22s^22p$ 8.2980	6 $^3P_0$ **C** Carbon 12.011 $1s^22s^22p^2$ 11.2603	7 $^4S^\circ_{3/2}$ **N** Nitrogen 14.007 $1s^22s^22p^3$ 14.5341	8 $^3P_2$ **O** Oxygen 15.999 $1s^22s^22p^4$ 13.6181	9 $^2P^\circ_{3/2}$ **F** Fluorine 18.998 $1s^22s^22p^5$ 17.4228	10 $^1S_0$ **Ne** Neon 20.180 $1s^22s^22p^6$ 21.5645
3	11 $^2S_{1/2}$ **Na** Sodium 22.990 $[Ne]3s$ 5.1391	12 $^1S_0$ **Mg** Magnesium 24.305 $[Ne]3s^2$ 7.6462											13 $^2P^\circ_{1/2}$ **Al** Aluminum 26.982 $[Ne]3s^23p$ 5.9858	14 $^3P_0$ **Si** Silicon 28.085 $[Ne]3s^23p^2$ 8.1517	15 $^4S^\circ_{3/2}$ **P** Phosphorus 30.974 $[Ne]3s^23p^3$ 10.4867	16 $^3P_2$ **S** Sulfur 32.06 $[Ne]3s^23p^4$ 10.3600	17 $^2P^\circ_{3/2}$ **Cl** Chlorine 35.45 $[Ne]3s^23p^5$ 12.9676	18 $^1S_0$ **Ar** Argon 39.948 $[Ne]3s^23p^6$ 15.7596
4	19 $^2S_{1/2}$ **K** Potassium 39.098 $[Ar]4s$ 4.3407	20 $^1S_0$ **Ca** Calcium 40.078 $[Ar]4s^2$ 6.1132	21 $^2D_{3/2}$ **Sc** Scandium 44.956 $[Ar]3d4s^2$ 6.5615	22 $^3F_2$ **Ti** Titanium 47.867 $[Ar]3d^24s^2$ 6.8281	23 $^4F_{3/2}$ **V** Vanadium 50.942 $[Ar]3d^34s^2$ 6.7462	24 $^7S_3$ **Cr** Chromium 51.996 $[Ar]3d^54s$ 6.7665	25 $^6S_{5/2}$ **Mn** Manganese 54.938 $[Ar]3d^54s^2$ 7.4340	26 $^5D_4$ **Fe** Iron 55.845 $[Ar]3d^64s^2$ 7.9025	27 $^4F_{9/2}$ **Co** Cobalt 58.933 $[Ar]3d^74s^2$ 7.8810	28 $^3F_4$ **Ni** Nickel 58.693 $[Ar]3d^84s^2$ 7.6399	29 $^2S_{1/2}$ **Cu** Copper 63.546 $[Ar]3d^{10}4s$ 7.7264	30 $^1S_0$ **Zn** Zinc 65.38 $[Ar]3d^{10}4s^2$ 9.3942	31 $^2P^\circ_{1/2}$ **Ga** Gallium 69.723 $[Ar]3d^{10}4s^24p$ 5.9993	32 $^3P_0$ **Ge** Germanium 72.630 $[Ar]3d^{10}4s^24p^2$ 7.8994	33 $^4S^\circ_{3/2}$ **As** Arsenic 74.922 $[Ar]3d^{10}4s^24p^3$ 9.7886	34 $^3P_2$ **Se** Selenium 78.971 $[Ar]3d^{10}4s^24p^4$ 9.7524	35 $^2P^\circ_{3/2}$ **Br** Bromine 79.904 $[Ar]3d^{10}4s^24p^5$ 11.8138	36 $^1S_0$ **Kr** Krypton 83.798 $[Ar]3d^{10}4s^24p^6$ 13.9996
5	37 $^2S_{1/2}$ **Rb** Rubidium 85.468 $[Kr]5s$ 4.1771	38 $^1S_0$ **Sr** Strontium 87.62 $[Kr]5s^2$ 5.6949	39 $^2D_{3/2}$ **Y** Yttrium 88.906 $[Kr]4d5s^2$ 6.2173	40 $^3F_2$ **Zr** Zirconium 91.224 $[Kr]4d^25s^2$ 6.6341	41 $^6D_{1/2}$ **Nb** Niobium 92.906 $[Kr]4d^45s$ 6.7589	42 $^7S_3$ **Mo** Molybdenum 95.95 $[Kr]4d^55s$ 7.0924	43 $^6S_{5/2}$ **Tc** Technetium (97) $[Kr]4d^55s^2$ 7.1194	44 $^5F_5$ **Ru** Ruthenium 101.07 $[Kr]4d^75s$ 7.3605	45 $^4F_{9/2}$ **Rh** Rhodium 102.91 $[Kr]4d^85s$ 7.4589	46 $^1S_0$ **Pd** Palladium 106.42 $[Kr]4d^{10}$ 8.3369	47 $^2S_{1/2}$ **Ag** Silver 107.87 $[Kr]4d^{10}5s$ 7.5762	48 $^1S_0$ **Cd** Cadmium 112.41 $[Kr]4d^{10}5s^2$ 8.9938	49 $^2P^\circ_{1/2}$ **In** Indium 114.82 $[Kr]4d^{10}5s^25p$ 5.7864	50 $^3P_0$ **Sn** Tin 118.71 $[Kr]4d^{10}5s^25p^2$ 7.3439	51 $^4S^\circ_{3/2}$ **Sb** Antimony 121.76 $[Kr]4d^{10}5s^25p^3$ 8.6084	52 $^3P_2$ **Te** Tellurium 127.60 $[Kr]4d^{10}5s^25p^4$ 9.0097	53 $^2P^\circ_{3/2}$ **I** Iodine 126.90 $[Kr]4d^{10}5s^25p^5$ 10.4513	54 $^1S_0$ **Xe** Xenon 131.29 $[Kr]4d^{10}5s^25p^6$ 12.1298
6	55 $^2S_{1/2}$ **Cs** Cesium 132.91 $[Xe]6s$ 3.8939	56 $^1S_0$ **Ba** Barium 137.33 $[Xe]6s^2$ 5.2117		72 $^3F_2$ **Hf** Hafnium 178.49 $[Xe]4f^{14}5d^26s^2$ 6.8251	73 $^4F_{3/2}$ **Ta** Tantalum 180.95 $[Xe]4f^{14}5d^36s^2$ 7.5496	74 $^5D_0$ **W** Tungsten 183.84 $[Xe]4f^{14}5d^46s^2$ 7.8640	75 $^6S_{5/2}$ **Re** Rhenium 186.21 $[Xe]4f^{14}5d^56s^2$ 7.8335	76 $^5D_4$ **Os** Osmium 190.23 $[Xe]4f^{14}5d^66s^2$ 8.4382	77 $^4F_{9/2}$ **Ir** Iridium 192.22 $[Xe]4f^{14}5d^76s^2$ 8.9670	78 $^3D_3$ **Pt** Platinum 195.08 $[Xe]4f^{14}5d^96s$ 8.9588	79 $^2S_{1/2}$ **Au** Gold 196.97 $[Xe]4f^{14}5d^{10}6s$ 9.2256	80 $^1S_0$ **Hg** Mercury 200.59 $[Xe]4f^{14}5d^{10}6s^2$ 10.4375	81 $^2P^\circ_{1/2}$ **Tl** Thallium 204.38 $[Hg]6p$ 6.1083	82 $^3P_0$ **Pb** Lead 207.2 $[Hg]6p^2$ 7.4167	83 $^4S^\circ_{3/2}$ **Bi** Bismuth 208.98 $[Hg]6p^3$ 7.2855	84 $^3P_2$ **Po** Polonium (209) $[Hg]6p^4$ 8.414	85 $^2P^\circ_{3/2}$ **At** Astatine (210) $[Hg]6p^5$ 9.3175	86 $^1S_0$ **Rn** Radon (222) $[Hg]6p^6$ 10.7485
7	87 $^2S_{1/2}$ **Fr** Francium (223) $[Rn]7s$ 4.0727	88 $^1S_0$ **Ra** Radium (226) $[Rn]7s^2$ 5.2784		104 $^3F_2$ **Rf** Rutherfordium (267) $[Rn]5f^{14}6d^27s^2$ 6.02	105 $^4F_{3/2}$ **Db** Dubnium (268) $[Rn]5f^{14}6d^37s^2$ 6.8	106 $_0$ **Sg** Seaborgium (269) $[Rn]5f^{14}6d^47s^2$ 7.8	107 $_{5/2}$ **Bh** Bohrium (270) $[Rn]5f^{14}6d^57s^2$ 7.7	108 $_4$ **Hs** Hassium (269) $[Rn]5f^{14}6d^67s^2$ 7.6	109 **Mt** Meitnerium (278)	110 **Ds** Darmstadtium (281)	111 **Rg** Roentgenium (282)	112 **Cn** Copernicium (285)	113 **Nh** Nihonium (286)	114 **Fl** Flerovium (289)	115 **Mc** Moscovium (289)	116 **Lv** Livermorium (293)	117 **Ts** Tennessine (294)	118 **Og** Oganesson (294)

Lanthanides	57 $^2D_{3/2}$ **La** Lanthanum 138.91 $[Xe]5d6s^2$ 5.5769	58 $^1G^\circ_4$ **Ce** Cerium 140.12 $[Xe]4f5d6s^2$ 5.5386	59 $^4I^\circ_{9/2}$ **Pr** Praseodymium 140.91 $[Xe]4f^36s^2$ 5.4702	60 $^5I_4$ **Nd** Neodymium 144.24 $[Xe]4f^46s^2$ 5.5250	61 $^6H^\circ_{5/2}$ **Pm** Promethium (145) $[Xe]4f^56s^2$ 5.577	62 $^7F_0$ **Sm** Samarium 150.36 $[Xe]4f^66s^2$ 5.6437	63 $^8S^\circ_{7/2}$ **Eu** Europium 151.96 $[Xe]4f^76s^2$ 5.6704	64 $^9D^\circ_2$ **Gd** Gadolinium 157.25 $[Xe]4f^75d6s^2$ 6.1498	65 $^6H^\circ_{15/2}$ **Tb** Terbium 158.93 $[Xe]4f^96s^2$ 5.8638	66 $^5I_8$ **Dy** Dysprosium 162.50 $[Xe]4f^{10}6s^2$ 5.9391	67 $^4I^\circ_{15/2}$ **Ho** Holmium 164.93 $[Xe]4f^{11}6s^2$ 6.0215	68 $^3H_6$ **Er** Erbium 167.26 $[Xe]4f^{12}6s^2$ 6.1077	69 $^2F^\circ_{7/2}$ **Tm** Thulium 168.93 $[Xe]4f^{13}6s^2$ 6.1843	70 $^1S_0$ **Yb** Ytterbium 173.05 $[Xe]4f^{14}6s^2$ 6.2542	71 $^2D_{3/2}$ **Lu** Lutetium 174.97 $[Xe]4f^{14}5d6s^2$ 5.4259
Actinides	89 $^2D_{3/2}$ **Ac** Actinium (227) $[Rn]6d7s^2$ 5.3802	90 $^3F_2$ **Th** Thorium 232.04 $[Rn]6d^27s^2$ 6.3067	91 $^4K_{11/2}$ **Pa** Protactinium 231.04 $[Rn]5f^26d7s^2$ 5.89	92 $^5L^\circ_6$ **U** Uranium 238.03 $[Rn]5f^36d7s^2$ 6.1941	93 $^6L_{11/2}$ **Np** Neptunium (237) $[Rn]5f^46d7s^2$ 6.2655	94 $^7F_0$ **Pu** Plutonium (244) $[Rn]5f^67s^2$ 6.0258	95 $^8S^\circ_{7/2}$ **Am** Americium (243) $[Rn]5f^77s^2$ 5.9738	96 $^9D^\circ_2$ **Cm** Curium (247) $[Rn]5f^76d7s^2$ 5.9914	97 $^6H^\circ_{15/2}$ **Bk** Berkelium (247) $[Rn]5f^97s^2$ 6.1978	98 $^5I_8$ **Cf** Californium (251) $[Rn]5f^{10}7s^2$ 6.2817	99 $^4I^\circ_{15/2}$ **Es** Einsteinium (252) $[Rn]5f^{11}7s^2$ 6.3676	100 $^3H_6$ **Fm** Fermium (257) $[Rn]5f^{12}7s^2$ 6.50	101 $^2F^\circ_{7/2}$ **Md** Mendelevium (258) $[Rn]5f^{13}7s^2$ 6.58	102 $^1S_0$ **No** Nobelium (259) $[Rn]5f^{14}7s^2$ 6.66	103 $^2P^\circ_{1/2}$ **Lr** Lawrencium (266) $[Rn]5f^{14}7s^27p$ 4.96

Atomic Number	58
Ground State	$^1G^\circ_4$
Symbol	**Ce**
Name	Cerium
Standard Atomic Weight†(u)	140.12
Ground-state Configuration	$[Xe]4f5d6s^2$
Ionization Energy (eV)	5.5386

†Based upon ^{12}C. () indicates the mass number of the longest-lived isotope.

For the most precise values and uncertainties visit ciaaw.org and pml.nist.gov/data.

NIST SP 966 (July 2019)

Nuclear Properties

- The Nucleus and its properties
- Nuclear Binding energy and Mass defects
- Separation energies
- Nuclear Angular Momentum, Spin and Parity
- Nuclear Excited States

The Nucleus and its properties

The Nucleons

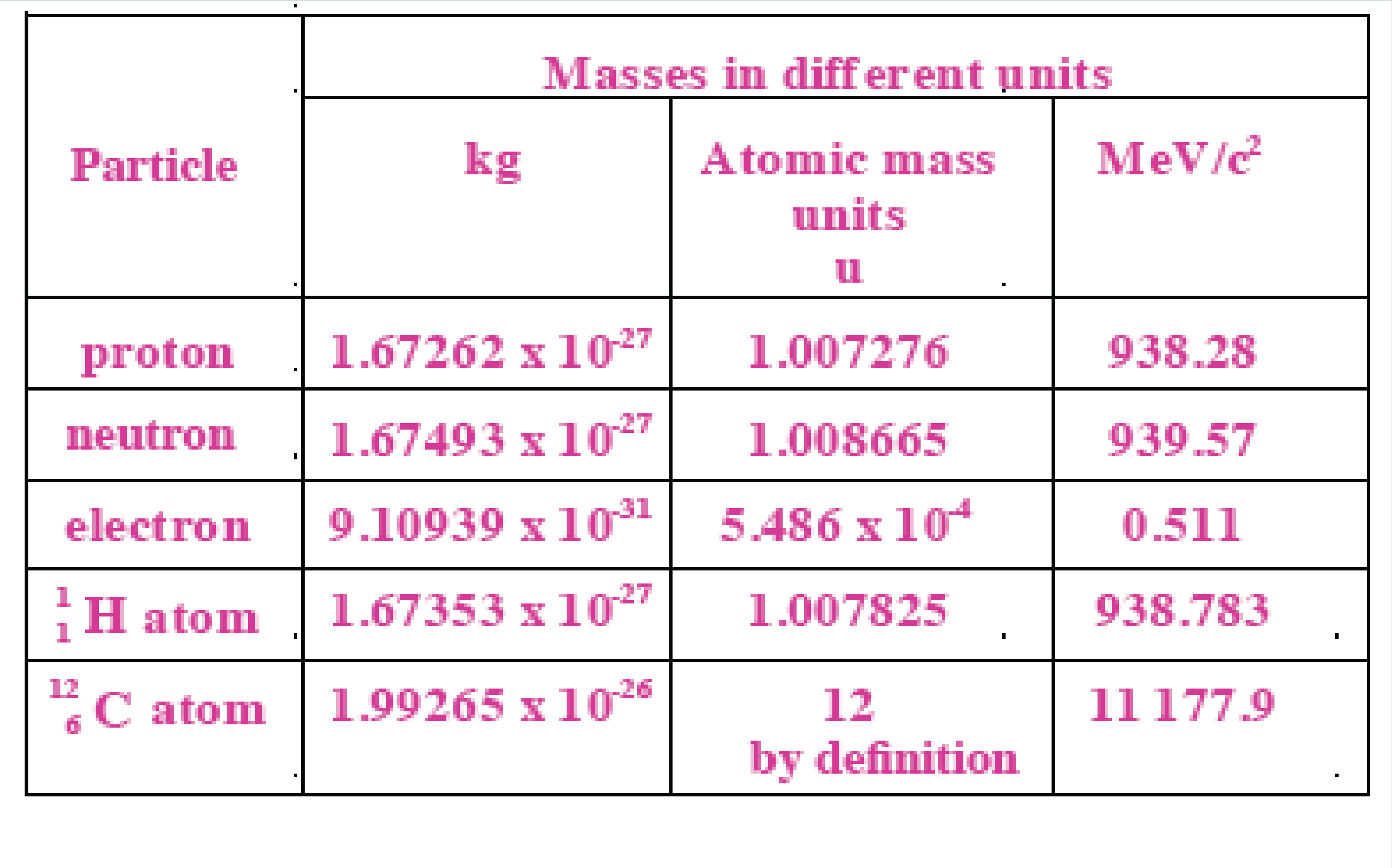
At Subatomic level, All nuclei are composed of:

- Positively charged Protons
- Neutral Neutrons
- Protons and Neutrons are collectively referred to as Nucleons.

	Proton	Neutron
Charge	$+1.6 \times 10^{-19}\ C$	0
Rest Mass (amu)	1.007277	1.008665
Spin	$1/2$	$1/2$
Magnetic Moment	$+2.7928\ \mu_n$	$-1.9128\ \mu_n$

- Where amu is the atomic mass unit
- μ_n is the Nuclear magneton

The Nucleus and its properties



Particle	Masses in different units		
	kg	Atomic mass units u	MeV/c^2
proton	1.67262×10^{-27}	1.007276	938.28
neutron	1.67493×10^{-27}	1.008665	939.57
electron	9.10939×10^{-31}	5.486×10^{-4}	0.511
$^{1}_{1}$H atom	1.67353×10^{-27}	1.007825	938.783
$^{12}_{6}$C atom	1.99265×10^{-26}	12 by definition	11 177.9

$$1 \text{ amu} = 931.502 \text{ MeV}/c^2 = 1.66054 \times 10^{-27} \text{ kg}$$

- A nucleus is identified by its atomic number Z (i.e., the number of protons), the neutron number, N, and the mass number A, where A = Z + N.

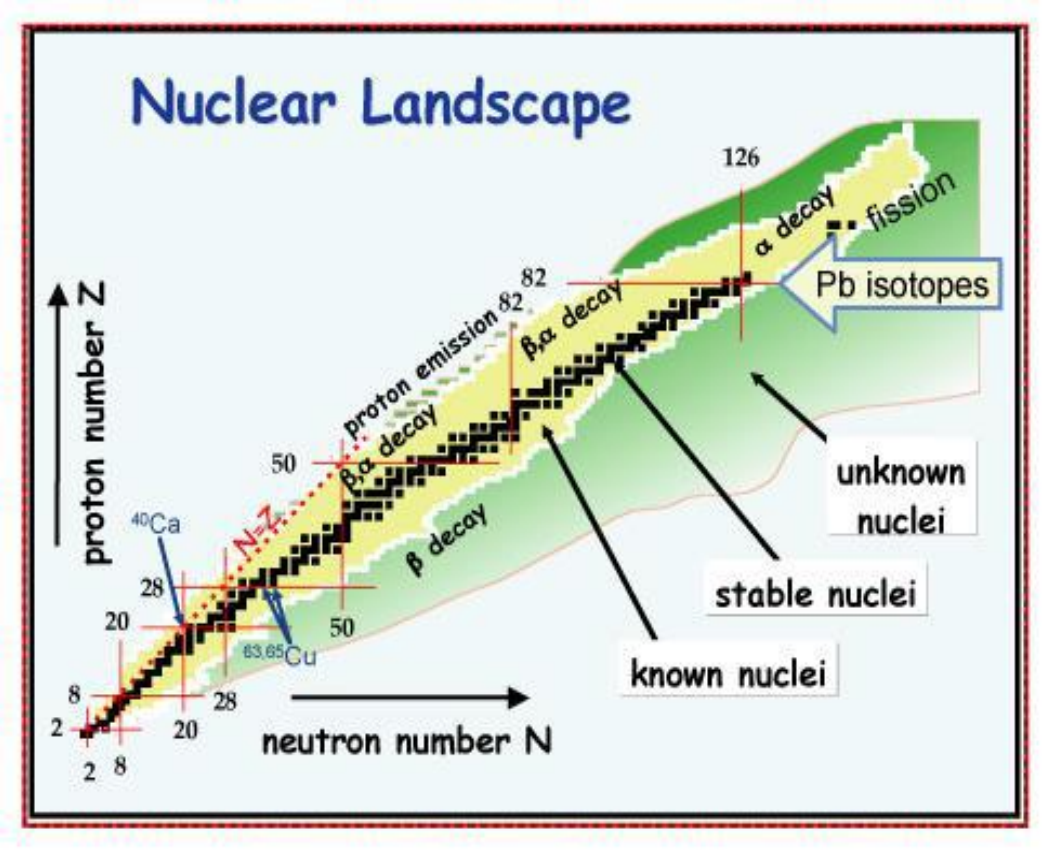
Isotopes

- **Isotopes:** Same element with the same number of protons (Z) but different mass (A).
- There are three isotopes of the element hydrogen:
  Hydrogen, Deuterium, and Tritium.
- Their nuclear symbols are therefore ^{1}H, ^{2}H, and ^{3}H or ^{1}H, D, and T.

- **Isotones:** nuclides with the same neutron N
  e.g. ^{2}H (d) and ^{3}He

- **Isobars:** nuclides with the same mass A
  e.g. ^{10}Be and ^{10}B, ^{14}C and ^{14}N

Nuclear Stability, Chart of Nuclides



- As Z increases, the long range Coulomb repulsion between protons is balanced by the presence of additional neutrons to provide additional short-range attractive forces.

- The particles are bounded together by the so called Strong Nuclear Force.

Mass and Abundance of Nuclides

- Refer to the Table of Isotopes for nuclear masses and abundances of various stable and radioactive nuclei
- To determine the nuclear masses to a precision of order 10^{-6}, a sophisticated instrument known as Accelerator Mass Spectrometer (AMS) is required (available at iThemba LABS Gauteng)
- The nuclear reaction using Radioactive Ion Beam (RIB) allows the measurement of masses of unstable nuclides which can not be directly measured using AMS
- Atomic Mass Evaluation https://www-nds.iaea.org/amdc/

The Nucleus as a Sphere

Size How big is a nucleus? $R = r_o A^{1/3}$,
where R = Radius (fm), A = mass number, $r_o = 1.2 \times 10^{-15}\ m$,
1 fm = 10^{-15} m

Shape
If the density of nuclear matter is assumed to be constant, the Volume of a nucleus will be directly proportional to the number of nucleons

For Spherical symmetry, $V = \left(\frac{4}{3}\pi {r_o}^3\right) A$

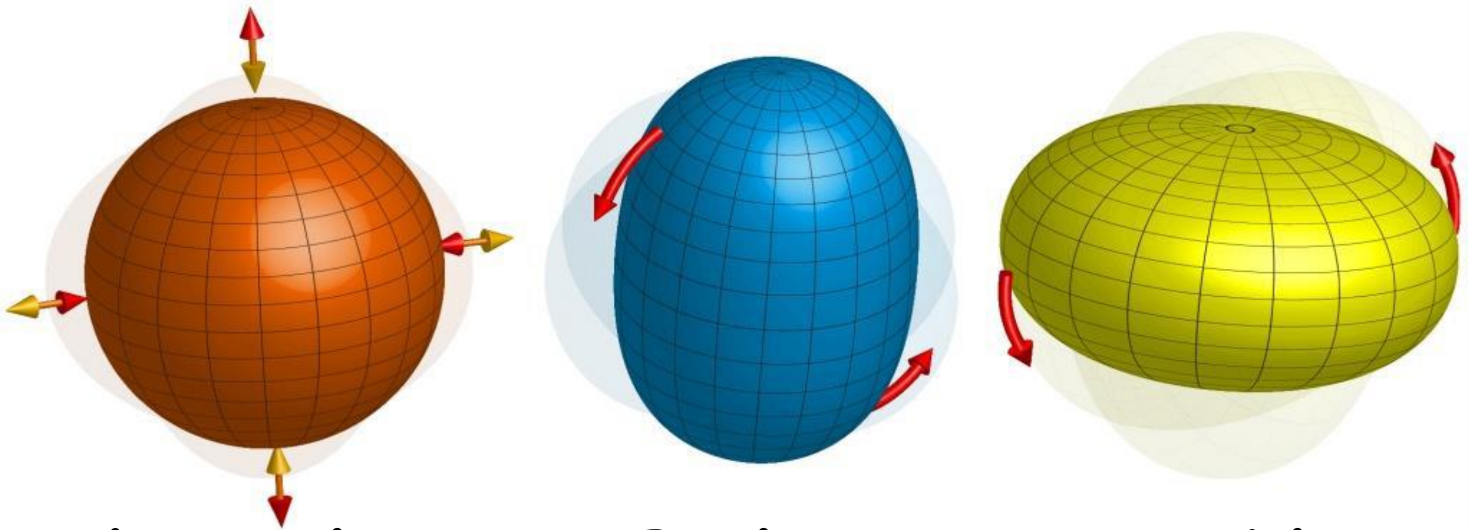
The Nucleus as a Sphere

For a nucleus as a sphere with uniformly distributed charge Ze;

The electrostatic energy $E_C = \frac{3}{5}\frac{kZ(Z-1)e^2}{R}$

This relationship provides a method for determining the size of nuclear charge distribution

Shape



Spherical Prolate Oblate

Binding Energy

- **Binding Energy** is defined as the amount of energy that must be supplied to a nucleus to completely separate its nucleons.

- The total nuclear binding energy is given by the difference between the rest energies of the constituent nucleons and the rest energy of the final nucleus

- The Binding Energy of a nucleus ${}^{A}_{Z}\mathrm{X}_{N}$ is given by:

$$B = \left\{ Zm({}^{1}\mathrm{H}) + Nm_{\mathrm{n}} - m({}^{A}\mathrm{X}) \right\} c^2$$

Note use neutral atomic masses, electron masses cancel

Binding Energy

Example: Calculate the binding energy of deuterium (isotope of hydrogen).

$$B = \left\{ Zm\left({}^{1}\mathrm{H} \right) + Nm_{\mathrm{n}} - m\left({}^{A}X \right) \right\} c^2$$

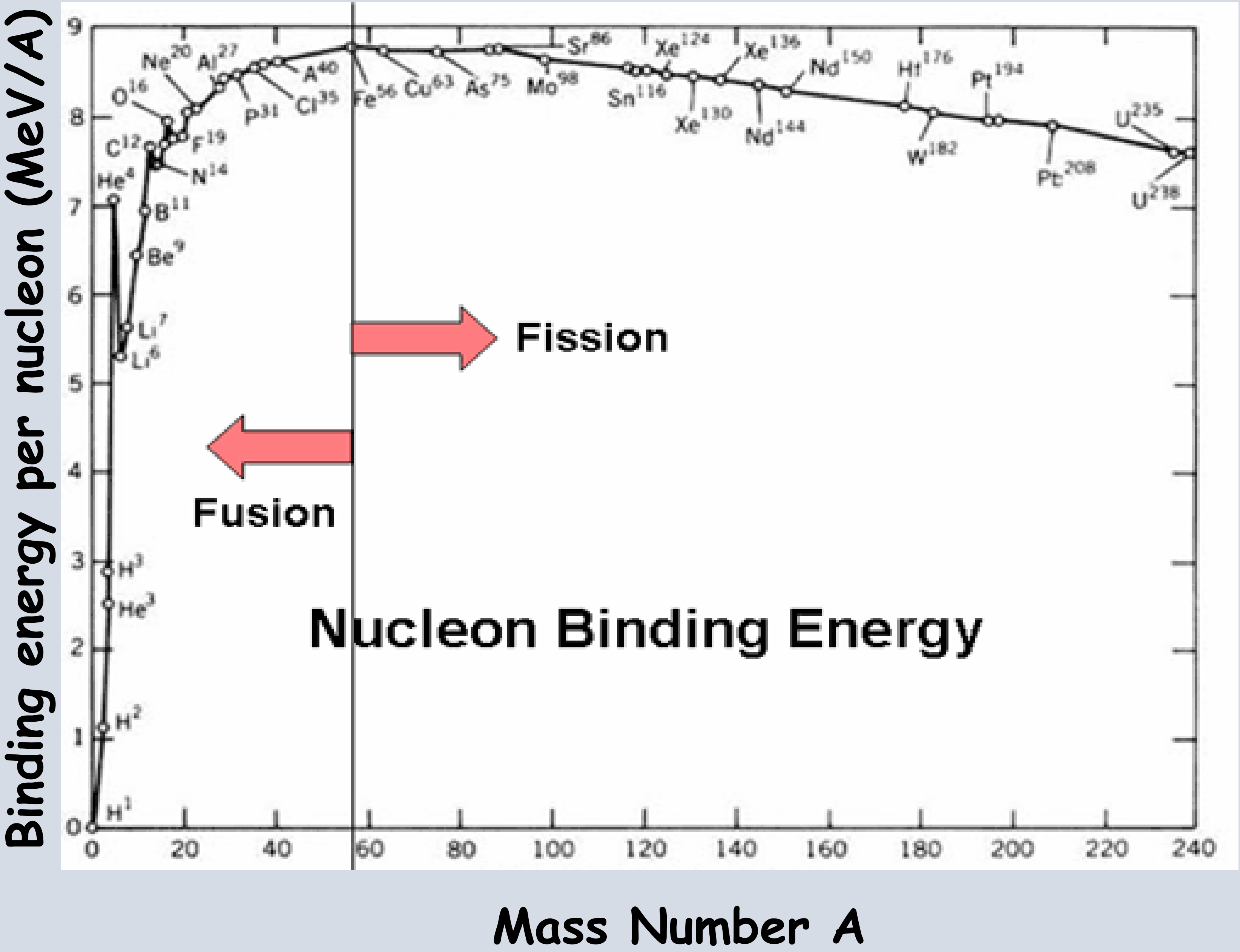
$$B = \{1(1.007825) + 1(1.008665) - 2.014102\}\ 931.502$$

Binding Energy of deuterium = 0.002388 x 931.502 = 2.22 MeV

- Note that the Binding Energy is the total energy required to pull apart the nucleus into its constituent nucleons.

- Conversely, it is the energy which binds the nucleons, and is therefore a measure of nuclear stability.

Nuclear Binding Energy Curve



Mass of the Nucleus

From the curve of the average binding energy per nucleon (B/A) of nuclei for different mass A, the following conclusions can be drawn.

- The curve is relatively constant except for the light nuclei
- The average binding energy of most nuclei is, to within 10%, about 8 MeV per nucleon
- There is a broad maximum of B/A for nuclei around A ~ 60 where the nuclei are most tightly bound and this roughly divides nuclear species into those which will release energy when undergoing fusion, and those which release energy on fission into smaller nuclei.

Proton and Neutron Separation Energies

- The neutron separation energy S_n is the amount of energy that is needed to remove a neutron from a nucleus, equal to the difference in binding energies between ${}^{A}_{Z}X_{n}$ and ${}^{A-1}_{Z}X_{n-1}$

- The proton separation energy S_p is the amount of energy that is needed to remove a proton from a nucleus, equal to the difference in binding energies between ${}^{A}_{Z}X_{n}$ and ${}^{A-1}_{Z-1}X_{n}$

- Likewise, one can calculate S_{2p} and S_{2n}

- The separation energies show evidence for nuclear shell structure that is similar to atomic shell structure

- Valueable information can be gained in nuclear structure from a systematic study of nuclear binding energy through nuclear models.

Proton and Neutron Separation Energies

Example: Calculate the neutron and proton Separation energy of ${}^{40}_{20}Ca$

$$S_n\left({}^{A}_{Z}X_n\right) = \left[m\left({}^{A-1}_{Z}X_{n-1}\right) + m_n - m\left({}^{A}_{Z}X_n\right)\right]c^2$$

$$S_n({}^{40}_{20}Ca) = \mathbf{15.43\ MeV}$$

$$S_p\left({}^{A}_{Z}X_n\right) = \left[m\left({}^{A-1}_{Z-1}X_n\right) + m_H - m\left({}^{A}_{Z}X_n\right)\right]c^2$$

$$S_p({}^{40}_{20}Ca) = \mathbf{8.435\ MeV}$$

Nuclear Angular Momentum, Spin and Parity

- The coupling of Orbital Angular Momentum l with spin s gives Total Angular Momentum of a single nucleon j
- In the quantum mechanical sense, every nucleon can be labelled with the corresponding quantum numbers l, s, and j
- The Total Angular Momentum of a nucleus containing A nucleons can be written as the vector sum of angular momenta of all the nucleons. This is called the **Nuclear Spin** I

$$I^2 = \hbar^2 I(I+1), \qquad \text{And } I_z = m\hbar \quad (m = -I \;\ldots..+I)$$

- If a single valence particle determines all of the nuclear properties, then, $I = j$
- For two valence particles, $I = j_1 + j_2$

Nuclear Spin

- The measured values for Nuclear Spin I can tell us a great deal about the nuclear structure

- The values of I are Integer if A is even, and Half-integer if A is odd

- For example, even Z-even N nuclei have spin-0 ground state (which indicates that identical nucleons tend to pair their angular momenta in the opposite directions – pairing effect)

- The ground state spin of odd-A nuclei must be equal to the j of the odd proton or neutron which is 1/2. Nuclear spin coincides with the last unpaired nucleon

- For Deuteron, $\boldsymbol{I = s_p + s_n + l}$

Nuclear Parity

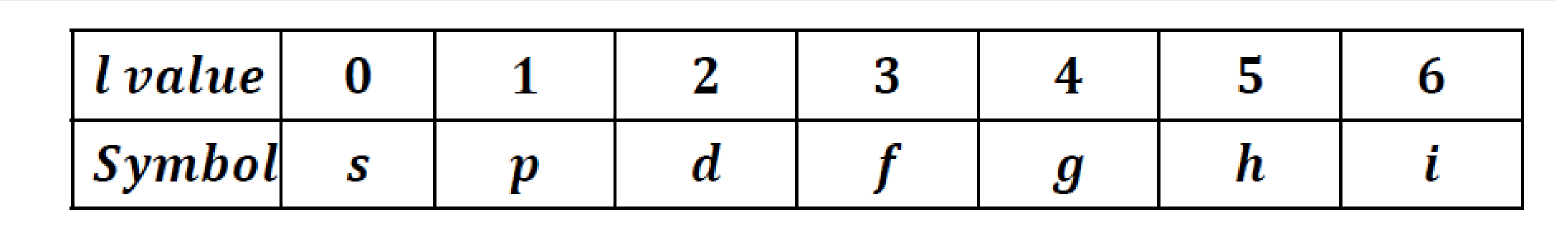
- Parity π can be used to label nuclear states

- The parity associated with orbital motion is $(-1)^l$ either $+$ (even) or $-$ (odd) values

- The parity π is regarded as an overall property of the whole nucleus

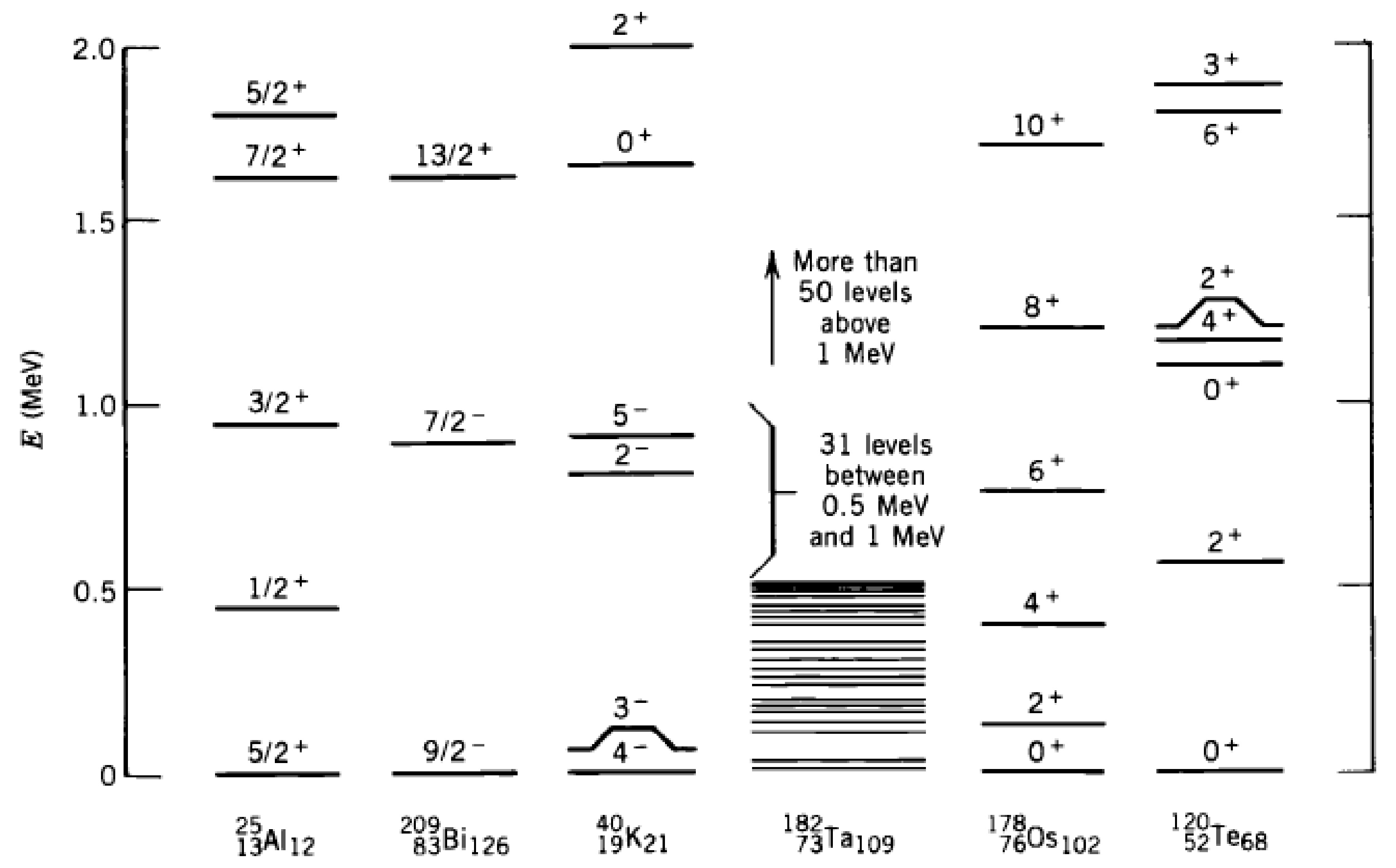
- The parity is denoted by a $+$ or $-$ superscript to the nuclear spin j^π. Examples are $0^+, 2^-, 1/2^-, 3/2^-$

Spectroscopic Notation



l value	0	1	2	3	4	5	6
Symbol	s	p	d	f	g	h	i

Nuclear Excited States



Nuclear Reactions

- Types of Reactions and Conservation Laws
- Energetics of Nuclear Reactions
- Reaction Cross Sections

Nuclear Reactions

- A nuclear reaction is described by identifying the incident particle, target nucleus, and reaction products.
- A typical nuclear reaction is written as:

$$a + X \rightarrow b + Y$$

where $a =$ the accelerated projectile

$X =$ the target (stationary in the Laboratory frame)

Y and b are the reaction products

- Usually, Y will be a heavy product that stops in the target and is not directly observed, while b is the light particle that can be detected and measured
- Alternatively, $X(a,b)Y$ gives a natural way to refer to a general class of reaction

Types of Nuclear Reaction

If the incident and outgoing particles are the same

- **Elastic scattering:** Projectile and Target (scattering partners) remain the same as reaction products and in their ground state. If Y and b are in their ground states:

$$p + {}^{40}Ca \rightarrow {}^{40}Ca + p$$

- **Inelastic scattering:** Projectile and Target (scattering partners) remain the same as reaction products but in an excited state. If Y or b is in an excited state:

$$p + {}^{40}Ca \rightarrow {}^{40}Ca + p'$$

Types of Nuclear Reaction

- **Knockout Reaction:** When a and b are the same particle, but the reaction causes yet another nucleon to be ejected separately, so that there are three particles in the final state

$$p + {}^{40}Ca \rightarrow {}^{39}Ca + p' + n$$

- **Transfer Reaction:** When one or two nucleons are transferred between the projectile and the target
- **Pickup Reaction:** Projectile picks up nucleons leaving target with less nucleons.

$$^{16}O(d,t)^{15}O$$

- **Stripping Reaction:** Projectile is stripped of nucleons leaving target with more nucleons.

$$^{90}Zr(d,p)^{91}Zr$$

Types of Nuclear Reaction

In terms of the mechanism that governs the process, we have;

- **Direct Reactions:** Only very few nucleons take part in the reaction, with the remaining target nucleons serving as passive spectators. Many excited states of Y can be reached in these reactions.

- **Compound nucleus Reactions:** The incoming projectile and target merge briefly for a complete sharing of energy before the outgoing nucleon is ejected.

- **Resonance Reactions:** The incoming particle forms a “quasibond” state before the outgoing particle is ejected.

- **Compound nucleus Reaction** is a reaction in which two nuclei combine into a single excited nucleus; the excited nucleus lives for a relatively long time and "forgets" how it was formed. The decay from this state of excitation is by "evaporation" of nucleons from the heated liquid drop of the compound nucleus, by gamma decay, or by fission of the compound nucleus.

$$
\begin{aligned}
{}^{19}F + p \;\rightarrow\; {}^{20}Ne^{*} \;&\rightarrow {}^{19}F + p \\
&\rightarrow {}^{19}Ne + n \\
&\rightarrow {}^{18}F + d \\
&\rightarrow {}^{17}F + t \\
&\rightarrow {}^{17}O + {}^{3}He \\
&\rightarrow {}^{16}O + \alpha \\
&\rightarrow {}^{20}Ne + \gamma
\end{aligned}
$$

- **Fusion reactions** are the combination of two light nuclei to form a more massive nucleus. Many fusion reactions release large amounts of energy.

$$^{3}H + \, ^{2}H \rightarrow \, ^{4}He + n + 17.6 \, MeV$$

Another example of fusion is the reaction set that powers the Sun and other low-mass stars:

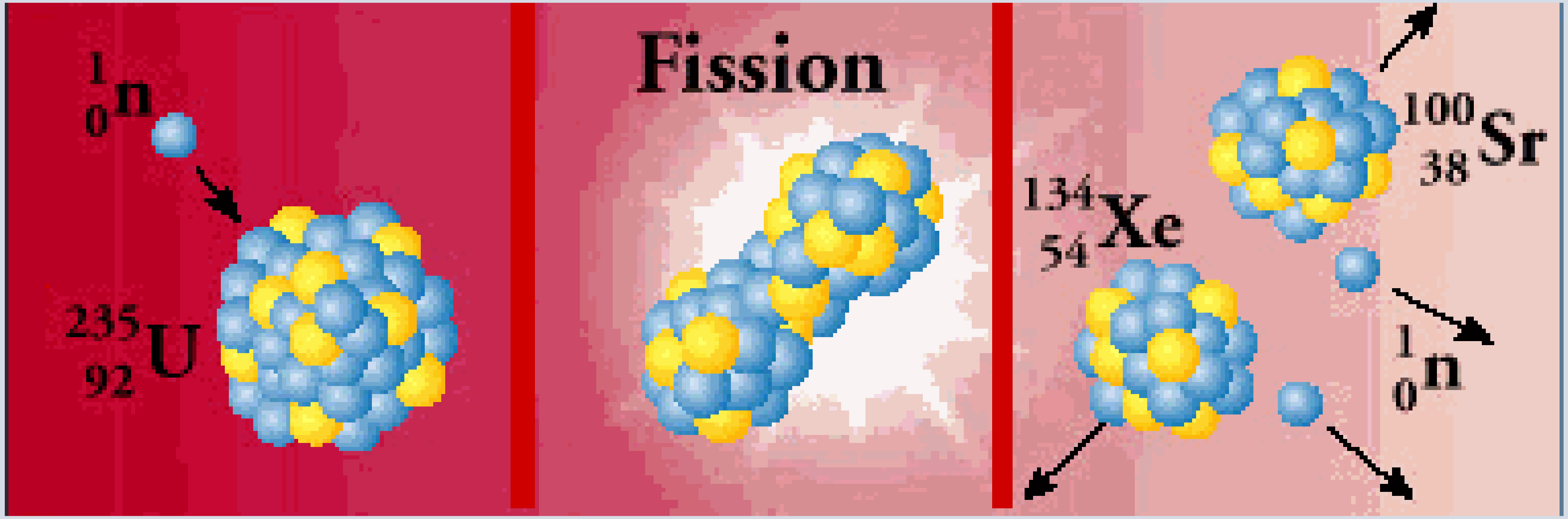
$$^{1}H + \, ^{1}H \rightarrow \, ^{2}H + \, e^{+} + \nu_{e}$$

$$^{2}H + \, ^{1}H \rightarrow \, ^{3}H + \, \gamma$$

$$^{3}He + \, ^{3}He \rightarrow \, ^{4}He + \, ^{1}H + \, ^{1}H$$

The net energy output from this chain is $26.7 \, MeV$ for each Helium-4 nucleus formed.

- **Neutron-induced fission** of massive nuclei into two lower-mass nuclei plus neutrons is also an energy source for power generation.



$$n + {}^{235}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2n + 200\ MeV$$

Fundamental Laws

- Conservation of **Nucleons (protons and neutrons):**
  Total "*A*" remains the same. The mass number *A* and the charge must balance on each side of the reaction arrow.

- Conservation of **Total Energy and Linear momentum:** This can be used to relate the unknown energies of the products to the known and controllable energy of the projectile. The total energy includes the particle kinetic energies plus the energy equivalent of the particle rest masses $E = mc^2$

Fundamental Laws

- Conservation of Angular momentum: This enables us to relate the spin assignments of the reacting particles and the orbital angular momentum carried by the outgoing particles, which can be deduced by measuring its angular distribution.

- Conservation of Parity: The net parity before the reaction must be equal the net parity after the reaction.

Total "I^{π}" remains the same. Quantum rules govern the balancing of the angular momentum, parity, and spin of the nuclear levels.

Q-value

The reaction Q-value is defined as the initial mass energy minus final mass energy

$$\begin{aligned} Q &= (m_{initial} - m_{final})c^2 \\ &= (m_a + m_X - m_Y - m_b)c^2 \end{aligned}$$

which is the same as the excess kinetic energy of the final product

$$\begin{aligned} Q &= (T_{final} - T_{initial}) \\ &= (T_Y + T_b - T_X - T_a) \end{aligned}$$

- **Exothermic** reaction produces energy

If Q-value > 0 (Exothermic), this means that the nuclear mass or binding energy is released as the kinetic energy of final products

- **Endothermic** reaction requires energy

If Q-value < 0 (Endothermic); this means that the initial kinetic energy is converted in to nuclear mass or binding energy

Examples of Q-value calculation

Exothermic $${}^{9}_{4}Be + {}^{4}_{2}He \rightarrow {}^{12}_{6}C + n$$

$$Q = [\,M({}^{9}_{4}Be) + M({}^{4}_{2}He) - M({}^{12}_{6}C) - m_n]c^2$$

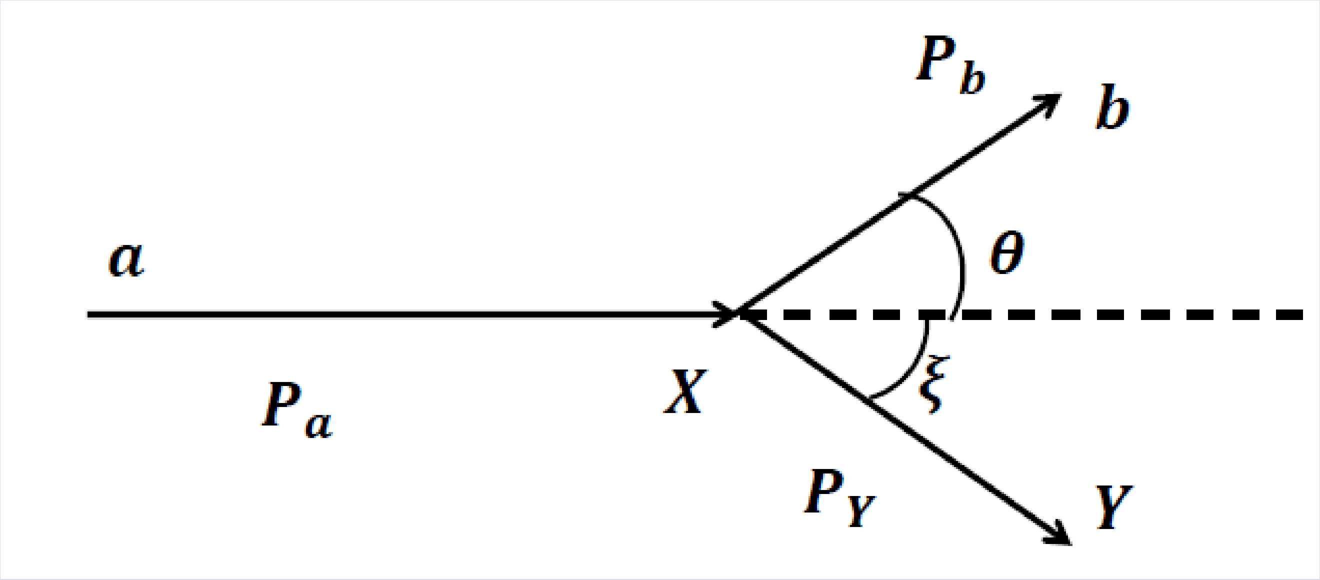
$$Q = 5.702\ MeV$$

Endothermic $${}^{16}_{8}O + n \rightarrow {}^{13}_{6}C + {}^{4}_{2}He$$

$$Q = [\,M({}^{16}_{8}O) + m_n - M({}^{13}_{6}C) - M({}^{4}_{2}He)]c^2$$

$$Q = -2.215\ MeV$$

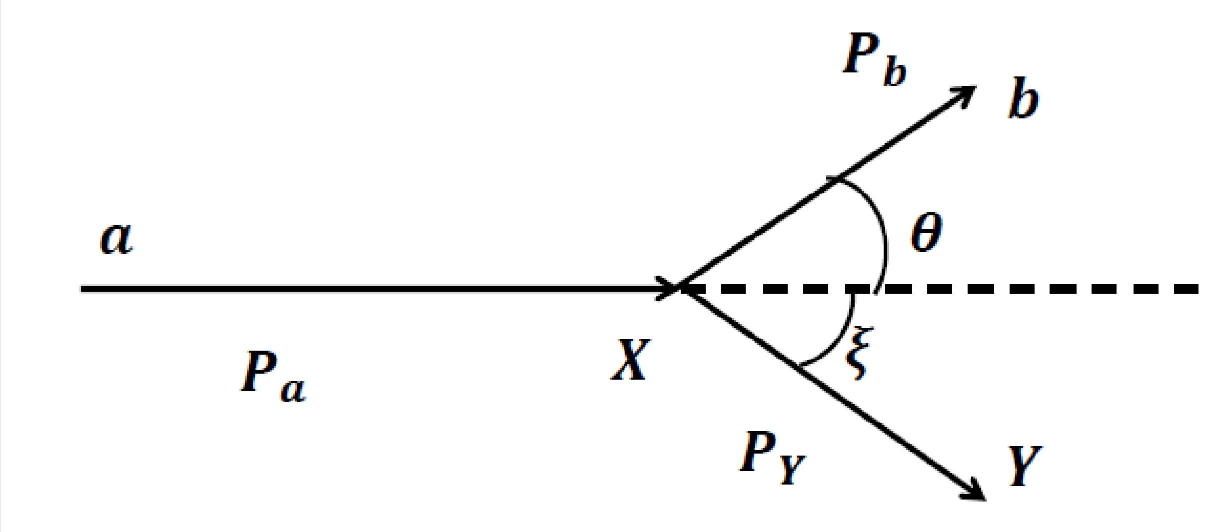
Energetics of Nuclear Reactions



Basic reaction geometry for $a + X \rightarrow b + Y$

- Conservation of total relativistic energy gives $m_X c^2 + T_X + m_a c^2 + T_a = m_Y c^2 + T_Y + m_b c^2 + T_b$
- where T's are kinetic energies (for which we can use the nonrelativistic approximation $\frac{1}{2}mv^2$ at low energy
- m's are the rest masses

Energetics of Nuclear Reactions



Basic reaction geometry for $a + X \rightarrow b + Y$

Conservation of Linear momentum in both horizontal and vertical direction to the projectile gives

$$p_a = p_b cos\theta + p_Y cos\xi$$

$$0 = p_b sin\theta - p_Y sin\xi$$

And $$Q = (T_Y + T_b - T_X - T_a)$$

So we have three equations with four unknowns (θ, ξ, T_b, T_Y)

Energetics of Nuclear Reactions

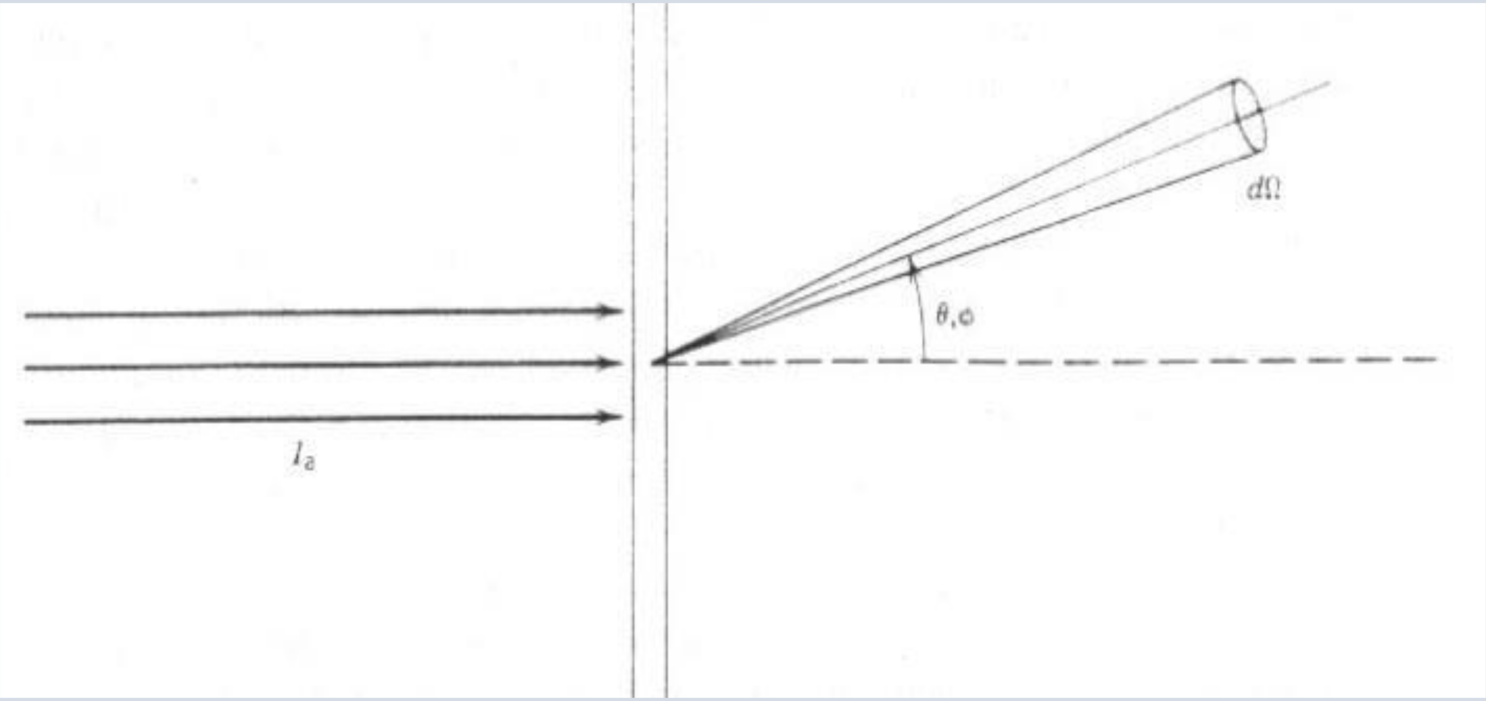
- If the reaction reaches excited states of Y, the Q-value equation should include the mass energy of the excited state

$$\begin{aligned} Q_{ex} &= (m_X + m_a - m_{Y*} - m_b)c^2 \\ &= Q_o - E_{ex} \end{aligned}$$

Where Q_o is the Q-value corresponding to the ground state of Y, and $m_{Y*}c^2 = m_Y c^2 + E_{ex}$ is the mass energy of the excited state;
E_{ex} is the excitation energy above the ground state

Reaction Cross-Sections

- Cross section is a measure of the relative probability of the reaction to occur.



Reaction geometry showing incident beam Ia, target and outgoing beam into solid angle $d\Omega$ at (θ, ϕ)

- A detector is placed to record particle b emitted in the direction (θ, ϕ) with respect to the beam direction
- The detector defines a solid angle $d\Omega$ at the target nucleus

Reaction Cross-Sections

- Let the current of incident particles is Ia particles per unit time
- And the target shows to the beam N target nuclei per unit area
- If the outgoing particles appear at a rate R_b
- Therefore, Cross section $\sigma = \frac{R_b}{I_a N}$
- Cross section σ has a dimension of area in $barn\,(b)$

$$100\ fm^2 = 1\ b$$

- The detector occupies only a small solid angle $d\Omega$
- Therefore, only a small fraction dR_b of the outgoing particles can be observed
- Hence, only a fraction of the cross section $\mathrm{d}\sigma$ can be deduced

Reaction Cross-Sections

- The outgoing particles will not be emitted uniformly in all directions, but will have an angular distribution $r(\theta,\phi)$

- Then, $dR_b = \frac{r(\theta,\phi)d\phi}{4\pi}$
- The quantity Differential cross section $\frac{d\sigma}{d\Omega} = \frac{r(\theta,\phi)}{4\pi I_a N}$
- $\frac{d\sigma}{d\Omega}$ has a unit of $barn/steradian$
- The reaction cross section σ can be determined by integrating $\frac{d\sigma}{d\Omega}$ over all angles, with $d\Omega = sin\theta d\theta d\phi$

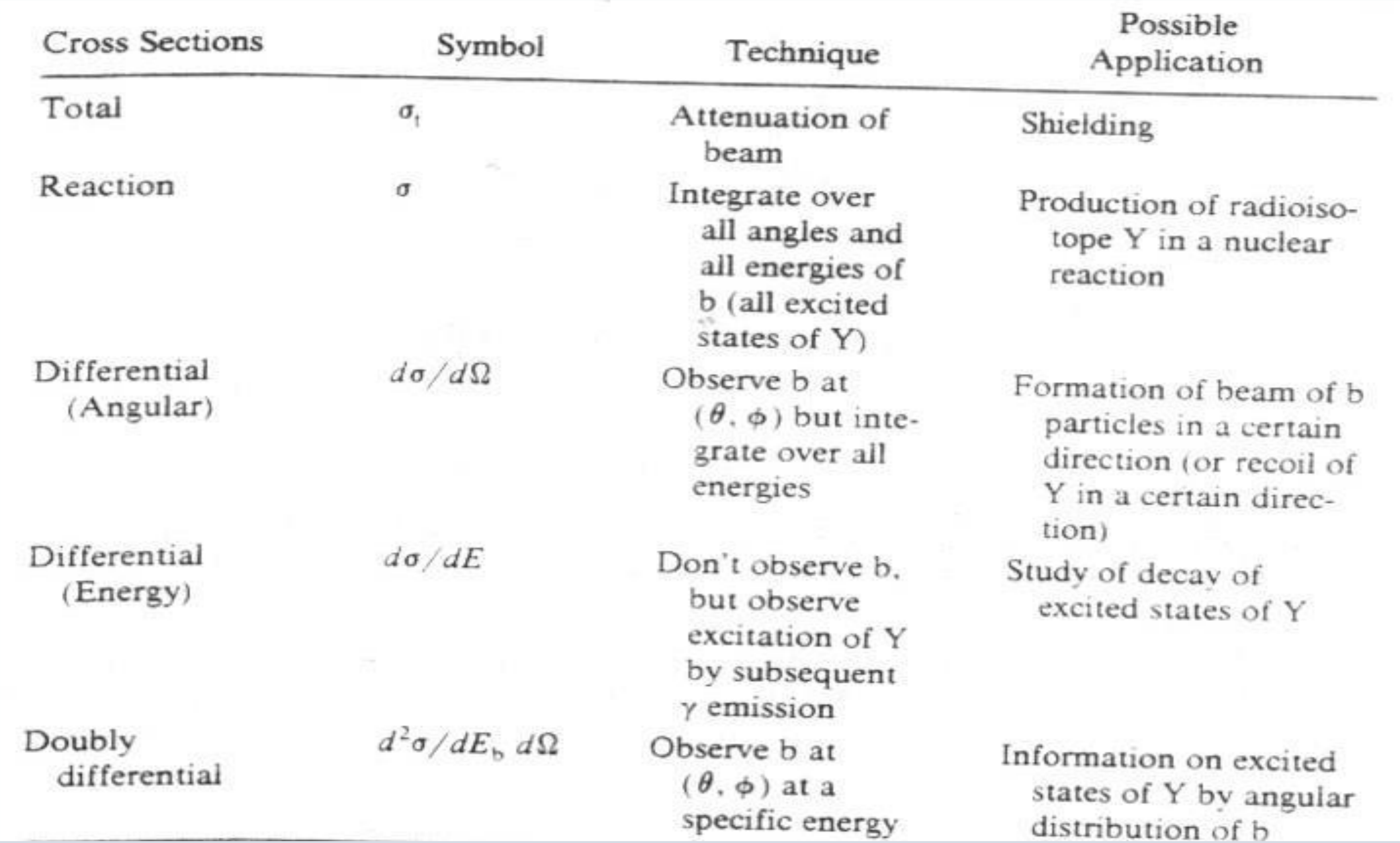
$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \int_0^{\pi} sin\theta d\theta \int_0^{2\pi} d\phi \frac{d\sigma}{d\Omega}$$

Reaction Cross-Sections

- If the probability to find particle b at a certain angle and at certain energy that corresponds to a particular energy of the residual nucleus Y,
- Then we have Double Differential cross
- Double Differential cross section $\dfrac{d^2\sigma}{d\Omega dE_b}$
- This is the probability to observe b in the angular range $d\Omega$ and in the energy range dE_b

- If we add all the reaction cross sections σ for all possible different outgoing particles b, no matter what their direction or energy are for a specific incident particle a
- Then, we have Total reaction cross section σ_t

Reaction Cross-Sections



Cross Sections	Symbol	Technique	Possible Application
Total	σ_t	Attenuation of beam	Shielding
Reaction	σ	Integrate over all angles and all energies of b (all excited states of Y)	Production of radioisotope Y in a nuclear reaction
Differential (Angular)	$d\sigma/d\Omega$	Observe b at (θ, ϕ) but integrate over all energies	Formation of beam of b particles in a certain direction (or recoil of Y in a certain direction)
Differential (Energy)	$d\sigma/dE$	Don't observe b, but observe excitation of Y by subsequent γ emission	Study of decay of excited states of Y
Doubly differential	$d^2\sigma/dE_b\,d\Omega$	Observe b at (θ, ϕ) at a specific energy	Information on excited states of Y by angular distribution of b

Nuclear Models

- Liquid Drop Models
- Fermi-Gas Model
- Shell Model
  - Infinite Square Well
  - Harmonic Oscillator
  - Spin-Orbit Potential
  - Predictions of the Shell Model
- Collective Model
- Superdeformed Nuclei

Nuclear Shell Model

- The semiempirical liquid drop model or binding-energy formula gives a good overall picture of the trends of nuclear binding energies. But does not account for finer details.

- Nuclear Model is a simplified view of nuclear structure that contains the essentials of nuclear physics

- Ultimately, we want to try to model the behaviour of the nucleus:

- What kind of potential do the nucleons 'feel' ?

- Can we reproduce/predict the important nuclear parameters such as:
    - Nuclear spin
    - Nuclear magnetic moments
    - Nuclear quadrupole moments
    - Magic numbers

Nuclear Shell Model

- Atomic shell structure ↔ Nuclear levels also exhibit shell-like structure

- Double shell arrangement for two classes of nucleons (protons and neutrons)

- Nuclei with Z or N values that correspond to complete shells (completely filled) are highly stable. This is in analogy with atoms with complete electronic shells (inert gases).

- These Z and N values are called " Magic Numbers" 2, 8, 20, 28, 50, 82 and 126

- Spectroscopic Notation:

l value	0	1	2	3	4	5	6
Symbol	s	p	d	f	g	h	i

Nuclear Shell Model

- Solve Schrodinger equation using potentials:
    - Infinite well
    - Harmonic oscillator

Level	l	*Parity*	$2(2l + 1)$
g	4	+	18
f	3	–	14
d	2	+	10
p	1	–	6
s	0	+	2

- *Parity* $\pi = (-1)^l$
- Maximum number of particles in each level $2(2l + 1)$

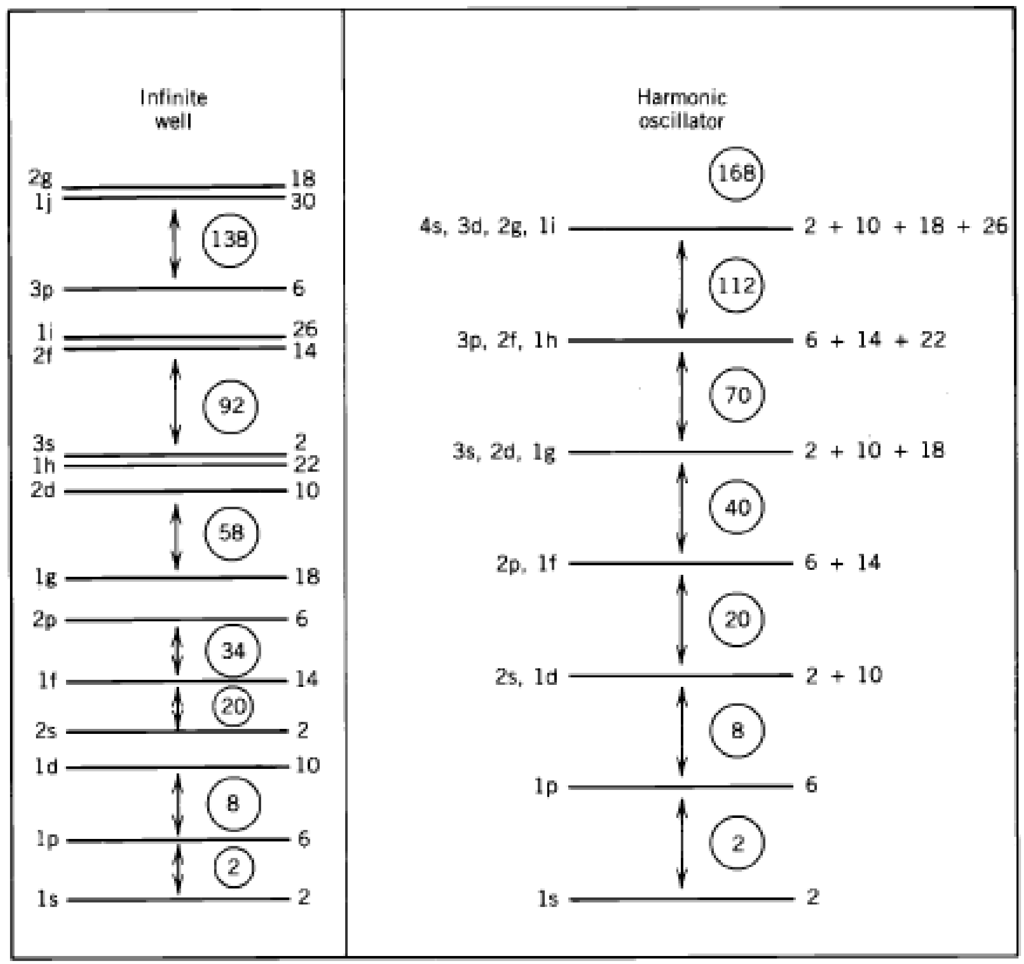


Figure 5.4 Shell structure obtained with infinite well and harmonic oscillator potentials. The capacity of each level is indicated to its right. Large gaps occur between the levels, which we associate with closed shells. The circled numbers indicate the total number of nucleons at each shell closure.

Nuclear Shell Model

Spin orbit potential

- Spin orbit interaction: $V_{so} = \vec{l}.\vec{s}$; $\vec{l}.\vec{s}$ causes reordering of levels
- Total angular momentum: $\vec{j} = \vec{l} + \vec{s}$
- Nucleons have $s = \frac{1}{2}$; $j = l + \frac{1}{2}$ OR $j = l - \frac{1}{2}$
- Degeneracy $= 2j + 1$

Nuclear Shell Model

Spin orbit potential

- A completely filled shell for all even-N even-Z nucleus have no unpaired nucleons, so $j^{\pi} = 0^{+}$ in its ground state

- If either proton or neutron is unpaired for odd-A nucleus, the spin of either proton or neutron determines the j^{π} value of the nucleus; $J = j;\ \pi = (-1)^{l}$

- If both proton and neutron are unpaired especially for odd-N and odd-Z nucleus, then the spin is the vector sum of j for last proton and neutron

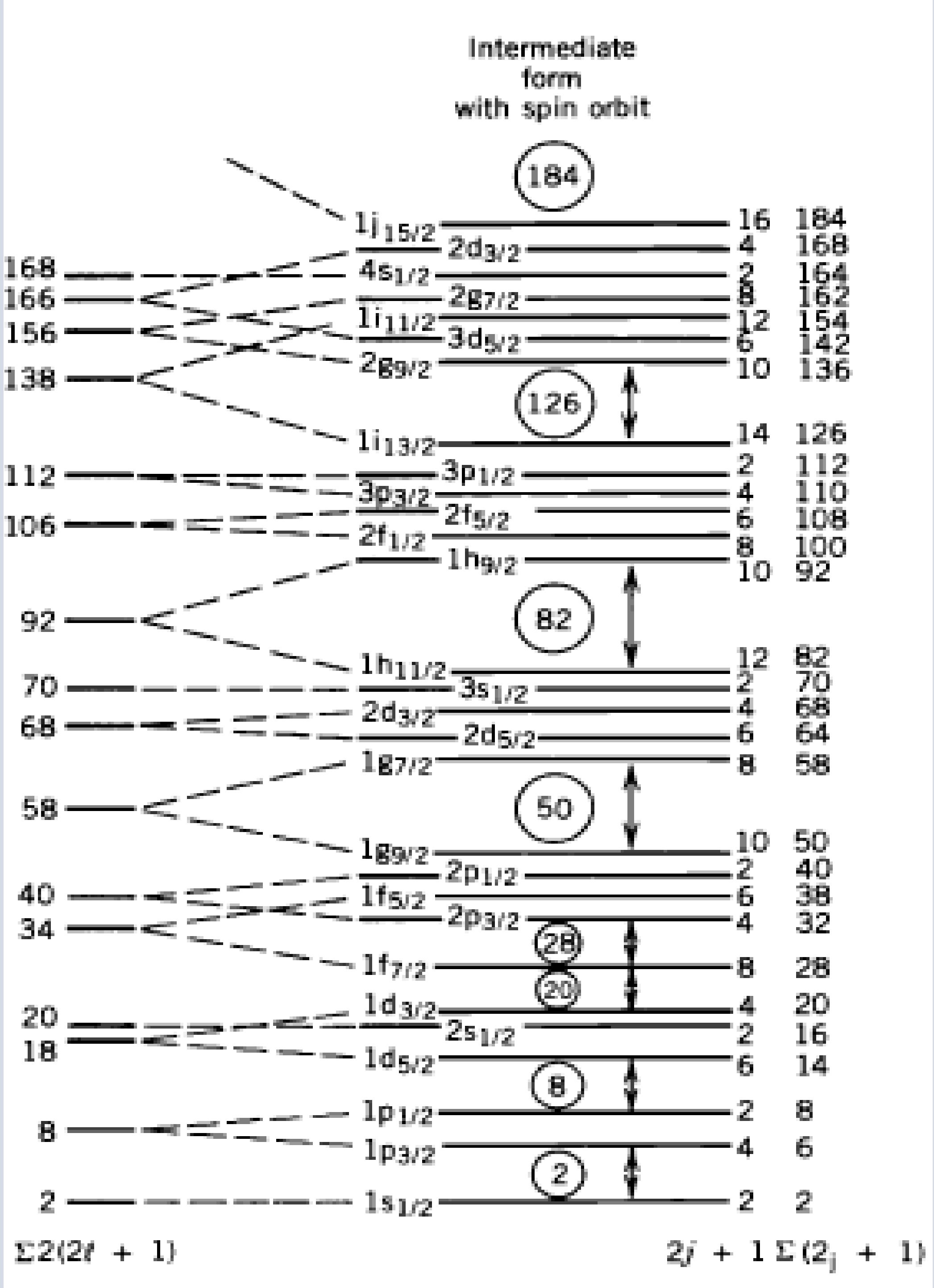
$$J = j_p + j_n$$

$$|j_p - j_n| \le J \le j_p + j_n$$

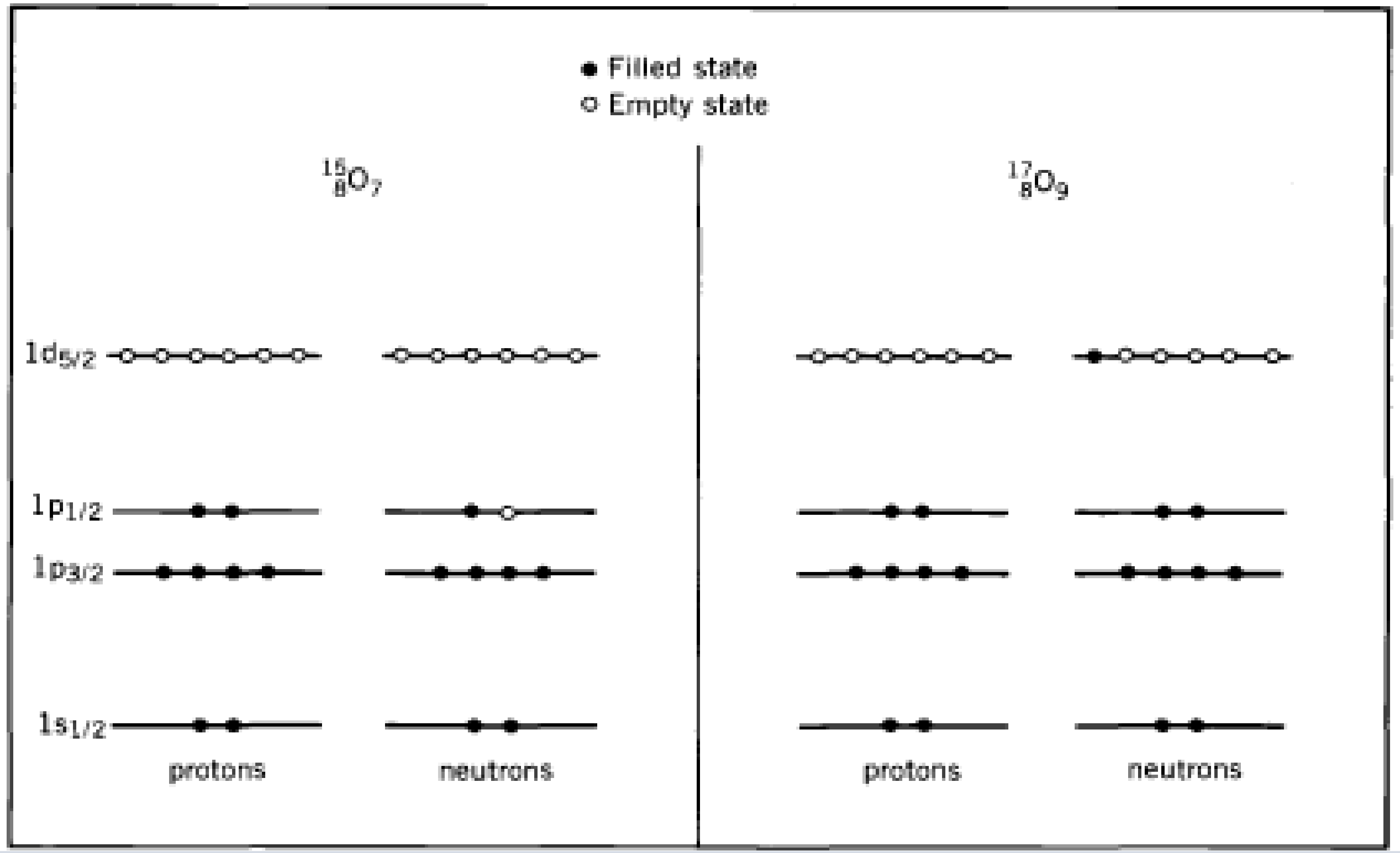
$$\pi_{nucleus} = \pi_p \times \pi_n$$

The Shell Model

The effect of the spin-orbit interaction, which splits the levels with $l > 0$ into two new levels. The magic numbers are exactly reproduced.



Filling of shells in ^{15}O and ^{17}O



- The filled proton shell do not contribute to the structure.
- The properties of the ground state are determined primarily by the neutrons

Experimental Techniques

To do precision spectroscopy of the outgoing particle b and the residual nucleus Y, the beam must satisfy the following criteria;

- The beam must be highly collimated and focussed
- It must have a sharply defined energy
- It must be of high intensity
- In order to measure the lifetime of excited states of Y, the beam must be sharply pulsed to provide a reference signal for the formation of the states

Experimental Techniques

- Under normal operation mode, the accelerator beam should be easily selectable;
- The intensity of the incident beam should be nearly constant and easily measurable
- The beam may be polarised or unpolarised according to the desire of the experimentalists
- The beam must be transported to the target through high vacuum system so as to prevent beam degradation and production of unwanted products by collision with air molecules.

Charged Particles and Photons Detectors in Nuclear Physics Experiments

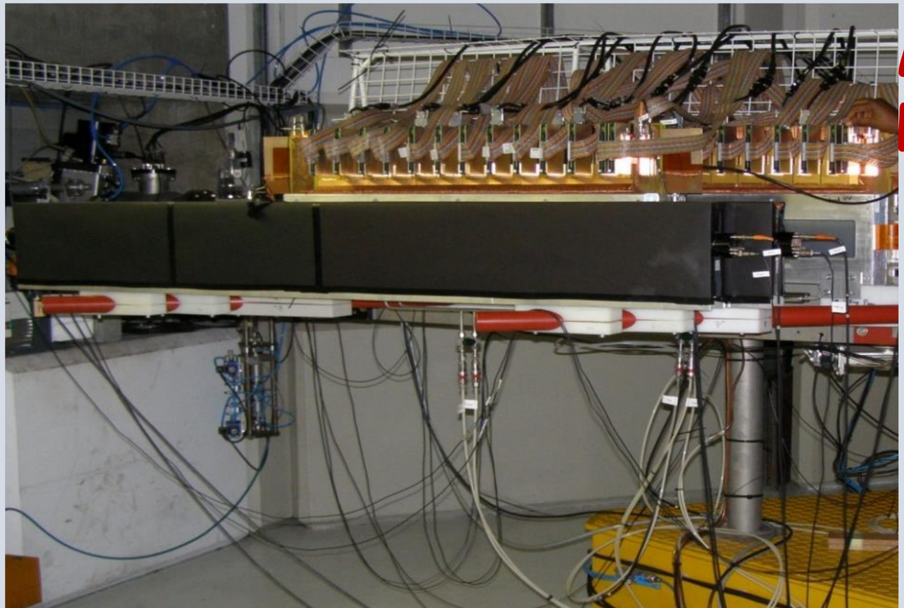
- **Particle Detector or Detector Telescopes**: This is used to determine the energy and type of the outgoing particles.
  Example: Magnetic Spectrometers for good energy resolution necessary to identify close-lying excited states of Y.

- **Position Sensitive Detectors**: This is used to do accurate angular distribution as well as transfer reaction experiments.
  Example: Multi-Wire Drift Chambers (MWDC),
  Double-Sided Surface Silicon Detectors (DSSSD)

- **Gamma-ray Detectors**: This is used to observe the de-excitation of the excited states of Y possibly in coincidence with particle b.
- Example: Hyper-Pure Germanium (HPGe), NaI(Tl) and $LaBr_3$(Ce)

K600 Magnetic Spectrometer and Associated Detectors

GAMKA



MWDC and Paddles

CAKE

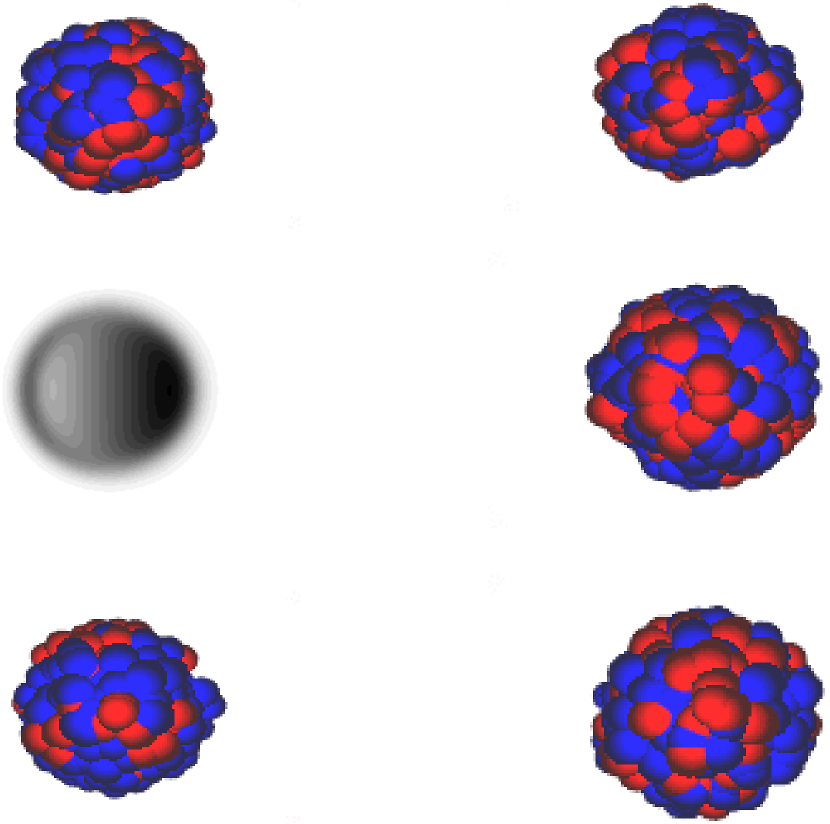
Nuclear and Radiation Physics @ WITS

https://www.wits.ac.za/physics/research-areas/nuclear-and-radiation-physics/

Research areas include:

- **Nuclear Structure**: Light-ion induced medium-energy reactions with investigations ranging from nuclear structure, nuclear reactions, nuclear cluster and nuclear astrophysics using K600 Magnetic Spectrometer at iThemba LABS SSC, Cape Town and TANDEM Accelerator iThemba LABS Gauteng.
- **Applied Nuclear Physics**: Environmental Radiation in collaboration with Centre for Nuclear Safety and Security (CNSS) at NNR, Nuclear Forensics, Medical Physics and Nuclear Medicine.
- **Reactor Physics**: Reactor Safety Analysis using MCNP and SCALE-VI in collaboration with NECSA RRT department, Nuclear Material Research using SAFARI-1 Reactor Neutron Activation Analysis (NAA) at Necsa, and Power Reactor Accidental Consequence Analysis with CNSS at NNR.

Giant Resonances



Isoscalar

Isovector

Monopole
$\Delta L = 0$

Dipole
$\Delta L = 1$

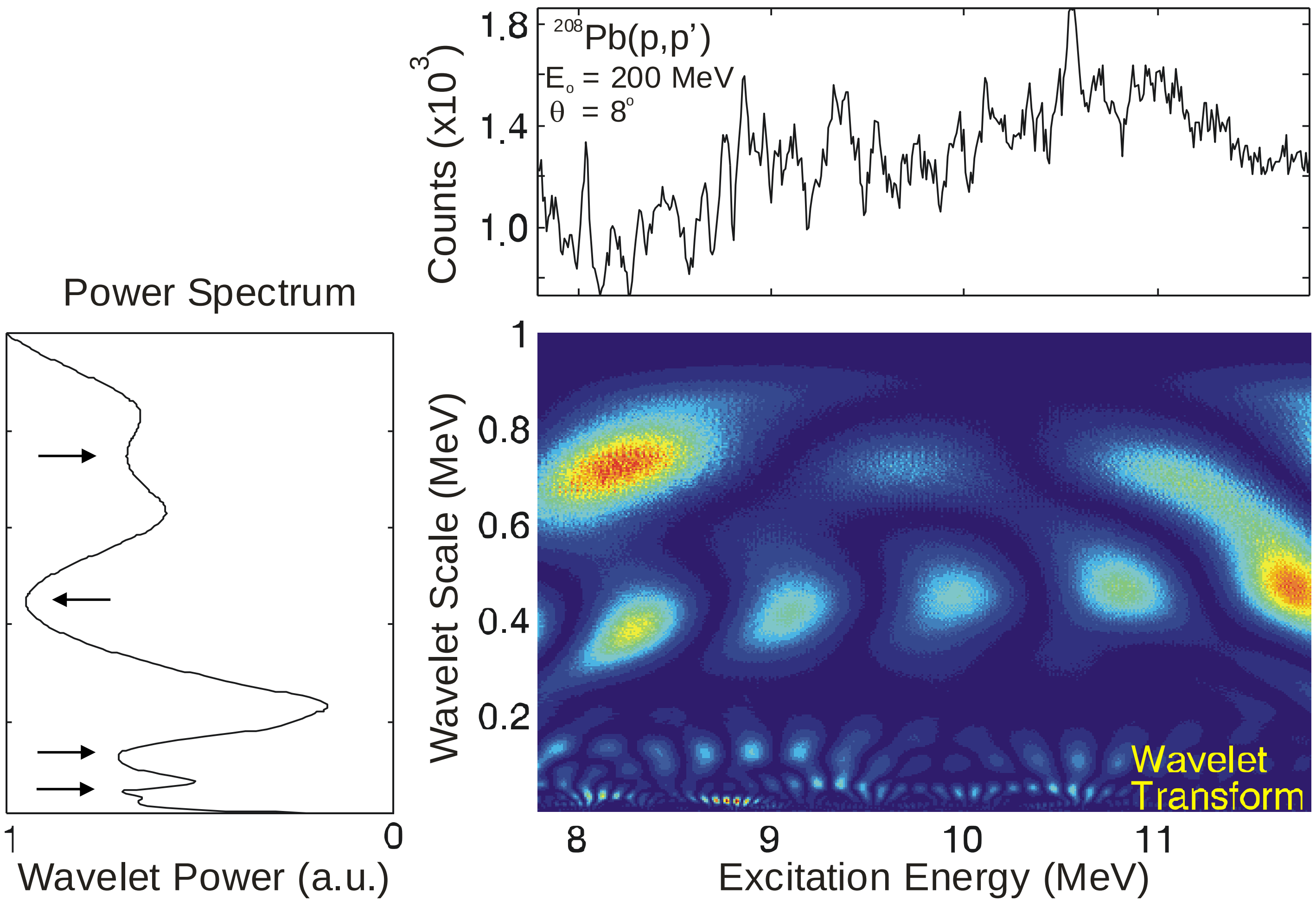
Quadrupole
$\Delta L = 2$

$\Delta T = 0$
$\Delta S = 0$

$\Delta T = 1$
$\Delta S = 0$

Courtesy of P. Adrich

^{208}Pb(p,p´) at iThemba LABS



Introduction to Nuclear Reaction Codes

- **CatKin**: An Excel-based kinematics spreadsheet for the calculation of simple two-body kinematics, fully relativistic and using accurate atomic masses http://personal.ph.surrey.ac.uk/~phs1wc/kinematics/

- **VIKAR**: is designed primarily to simulate two-body nuclear reactions, in normal or inverse kinematics, from which charged particles are detected. https://sites.google.com/a/nuclearemail.org/vikar/home

- **DWUCK4/5**: A code for zero-range or finite-range Distorted Wave Born Approximation (DWBA) calculations. The simplest useful reaction model, which assumes a direct, one-step transfer process where the transfers to specific states are individually weak and may be treated using perturbation theory. This requires optical model potentials that describe the appropriate elastic and inelastic entrance and exit channels.

Nuclear Kinematics software

CatKin

https://witscloud-my.sharepoint.com/personal/a0000891_wits_ac_za/Documents/Documents/My%20Files/My_Documents/ASP_2022/catkin2.03.xlsx

Nuclear Astrophysics Codes

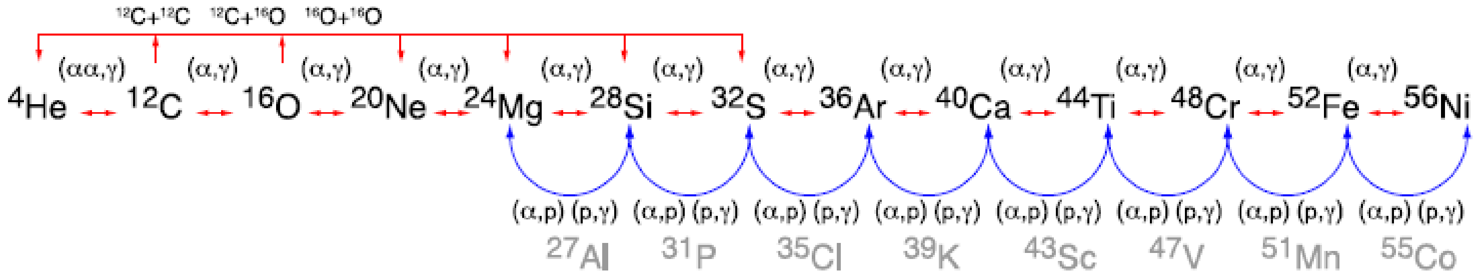
https://orruba.org/software/

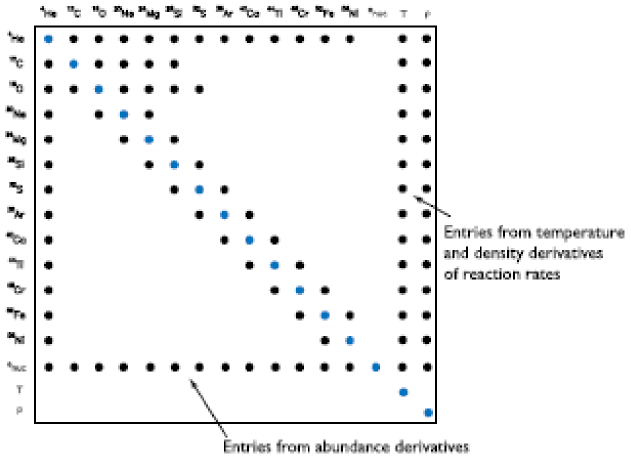
- **TALYS**: An opensource software package for the simulation of nuclear reactions for astrophysical applications.
  https://tendl.web.psi.ch/tendl_2019/talys.html
- **FRESCO**: A flexible universal nuclear reaction code used for transfer processes. http://www.fresco.org.uk/

Nuclear Astrophysics: Alpha-chain reaction network

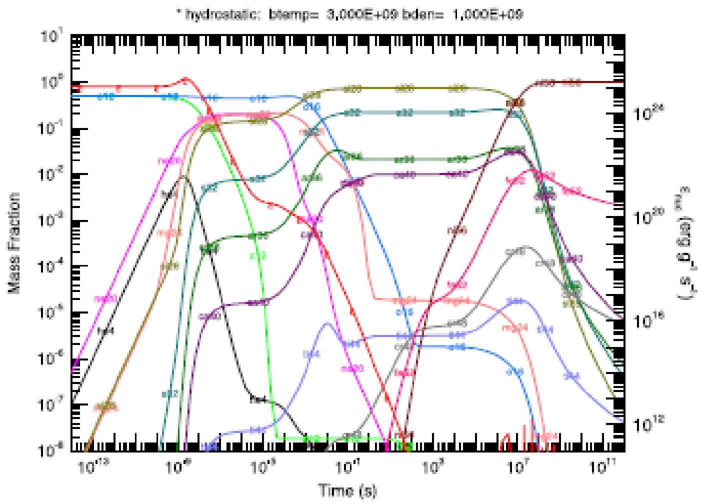
https://www.jinaweb.org/science-research/scientific-resources

13 isotope approximation network





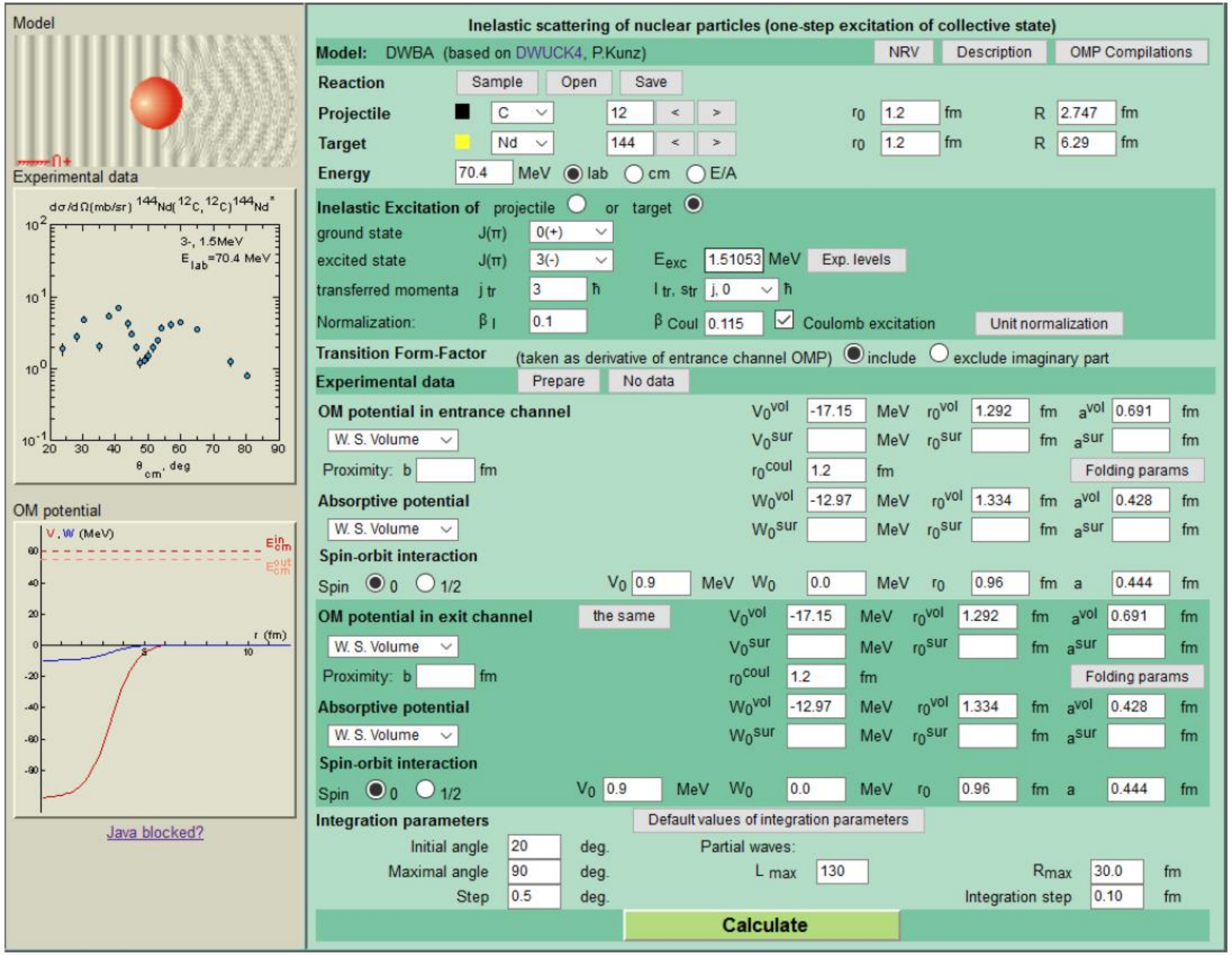
Jacobian



A mild c+o burn

Nuclear Reaction Codes

http://nrv.jinr.ru/



Inelastic scattering of nuclear particles (one-step excitation of collective state)

Model: DWBA (based on DWUCK4, P.Kunz) [NRV] [Description] [OMP Compilations]

Reaction [Sample] [Open] [Save]

Projectile C 12 < > r_0 1.2 fm R 2.747 fm

Target Nd 144 < > r_0 1.2 fm R 6.29 fm

Energy 70.4 MeV ● lab ○ cm ○ E/A

Inelastic Excitation of projectile ○ or target ●

ground state J(π) 0(+)

excited state J(π) 3(-) E_{exc} 1.51053 MeV [Exp. levels]

transferred momenta j tr 3 ħ l tr, str j, 0 ħ

Normalization: β_l 0.1 β Coul 0.115 ☑ Coulomb excitation [Unit normalization]

Transition Form-Factor (taken as derivative of entrance channel OMP) ● include ○ exclude imaginary part

Experimental data [Prepare] [No data]

OM potential in entrance channel V_0^{vol} -17.15 MeV r_0^{vol} 1.292 fm a^{vol} 0.691 fm

W. S. Volume V_0^{sur} MeV r_0^{sur} fm a^{sur} fm

Proximity: b fm r_0^{coul} 1.2 fm [Folding params]

Absorptive potential W_0^{vol} -12.97 MeV r_0^{vol} 1.334 fm a^{vol} 0.428 fm

W. S. Volume W_0^{sur} MeV r_0^{sur} fm a^{sur} fm

Spin-orbit interaction

Spin ● 0 ○ 1/2 V_0 0.9 MeV W_0 0.0 MeV r_0 0.96 fm a 0.444 fm

OM potential in exit channel [the same] V_0^{vol} -17.15 MeV r_0^{vol} 1.292 fm a^{vol} 0.691 fm

W. S. Volume V_0^{sur} MeV r_0^{sur} fm a^{sur} fm

Proximity: b fm r_0^{coul} 1.2 fm [Folding params]

Absorptive potential W_0^{vol} -12.97 MeV r_0^{vol} 1.334 fm a^{vol} 0.428 fm

W. S. Volume W_0^{sur} MeV r_0^{sur} fm a^{sur} fm

Spin-orbit interaction

Spin ● 0 ○ 1/2 V_0 0.9 MeV W_0 0.0 MeV r_0 0.96 fm a 0.444 fm

Integration parameters [Default values of integration parameters]

Initial angle 20 deg.
Maximal angle 90 deg.
Step 0.5 deg.
Partial waves: L max 130
R_{max} 30.0 fm
Integration step 0.10 fm

Calculate

Set your reaction

Set properties of excited state

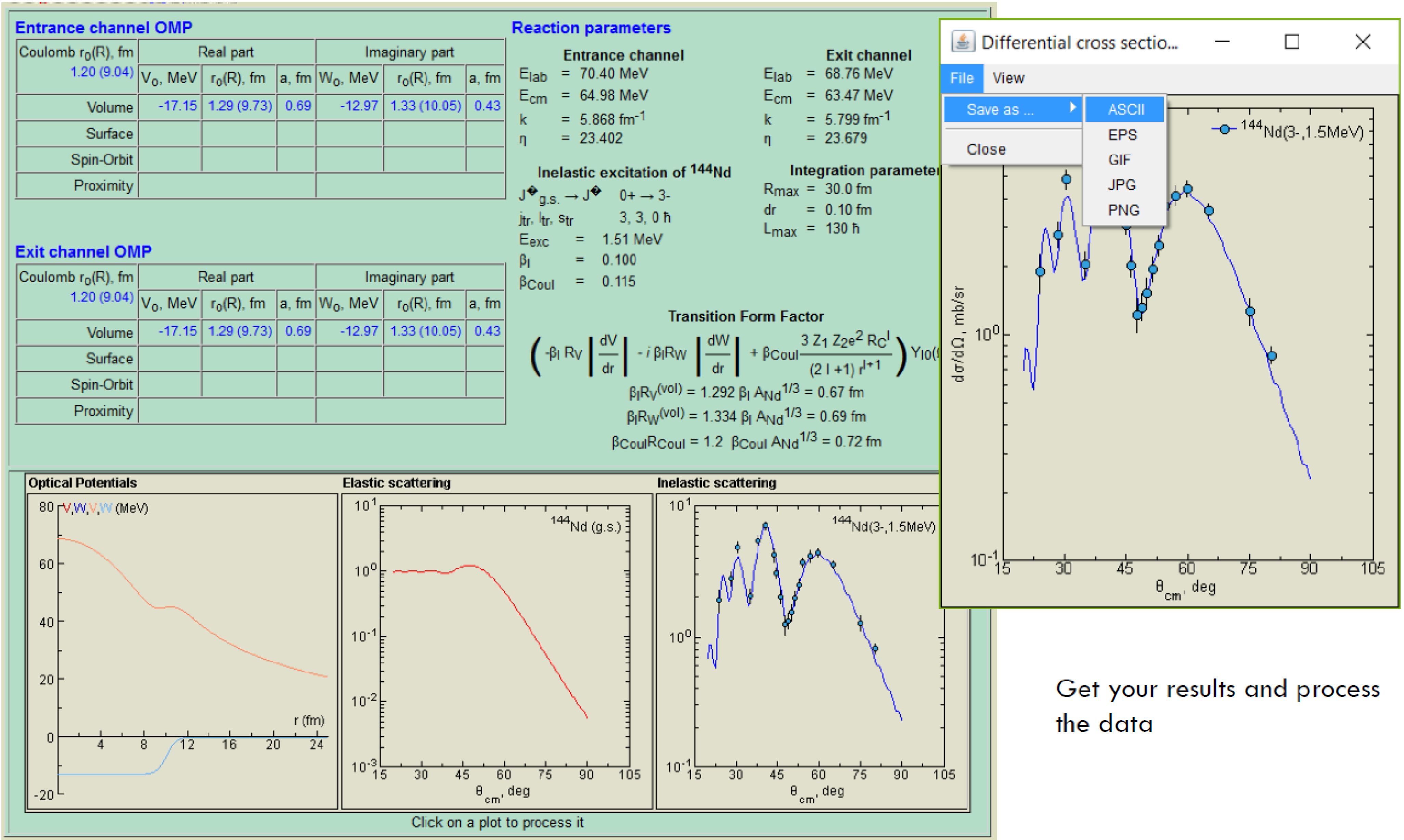
Set exp. data

Set OMP for in-channel

Set OMP for out-channel

Set integration params

Nuclear Reaction Codes



Entrance channel OMP

Coulomb $r_0(R)$, fm 1.20 (9.04)	Real part			Imaginary part		
	V_0, MeV	$r_0(R)$, fm	a, fm	W_0, MeV	$r_0(R)$, fm	a, fm
Volume	-17.15	1.29 (9.73)	0.69	-12.97	1.33 (10.05)	0.43
Surface						
Spin-Orbit						
Proximity						

Exit channel OMP

Coulomb $r_0(R)$, fm 1.20 (9.04)	Real part			Imaginary part		
	V_0, MeV	$r_0(R)$, fm	a, fm	W_0, MeV	$r_0(R)$, fm	a, fm
Volume	-17.15	1.29 (9.73)	0.69	-12.97	1.33 (10.05)	0.43
Surface						
Spin-Orbit						
Proximity						

Reaction parameters

Entrance channel

E_{lab} = 70.40 MeV
E_{cm} = 64.98 MeV
k = 5.868 fm^{-1}
η = 23.402

Exit channel

E_{lab} = 68.76 MeV
E_{cm} = 63.47 MeV
k = 5.799 fm^{-1}
η = 23.679

Inelastic excitation of ^{144}Nd

$J^{\text{�}}_{g.s.} \rightarrow J^{\text{�}}$ 0+ → 3-
j_{tr}, l_{tr}, s_{tr} 3, 3, 0 ħ
E_{exc} = 1.51 MeV
β_l = 0.100
β_{Coul} = 0.115

Integration paramete

R_{max} = 30.0 fm
dr = 0.10 fm
L_{max} = 130 ħ

Transition Form Factor

$$\left(-\beta_l R_V \left| \frac{dV}{dr} \right| - i\, \beta_l R_W \left| \frac{dW}{dr} \right| + \beta_{Coul} \frac{3 Z_1 Z_2 e^2 R_C{}^l}{(2l+1)\, r^{l+1}} \right) Y_{l0}($$

$$\beta_l R_V{}^{(vol)} = 1.292\ \beta_l\ A_{Nd}{}^{1/3} = 0.67 \text{ fm}$$

$$\beta_l R_W{}^{(vol)} = 1.334\ \beta_l\ A_{Nd}{}^{1/3} = 0.69 \text{ fm}$$

$$\beta_{Coul} R_{Coul} = 1.2\ \ \beta_{Coul}\ A_{Nd}{}^{1/3} = 0.72 \text{ fm}$$

Get your results and process the data

Thank you for listening!!!

iyabo.usman@wits.ac.za