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Introduction to Fluid Dynamics and Computational Fluid Dynamics (CFD)

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Course Outline I

- Introduction to CFD
- Mathematical Conventions
- Basic Fluid Flow (Single Phase)
- Governing Equations: Continuity, Momentum and Energy Equations
- Inviscid Flow, Newtonian and Non-Newtonian Fluids
- Generalised Scalar Transport Equation
- Compressible and Incompressible Flows
- Steady and Unsteady Flows
- Introduction to Turbulence
- Navier-Stokes Equations and Laminar Flow
- RANS Equations
- The Finite Volume Method (in brief): Generalised Scalar Transport Equation $\rightarrow$ Discretised Domain $\rightarrow$ Discretised Form of the Generalised Scalar Transport Equation $\rightarrow$ Numerical Solution

Introduction to CFD I

- The governing equations for fluid flow (single and multiphase), heat and mass transfer have no analytic solutions for the general case [1, 2, 3, 4]
- Analytic/exact solutions exist for simple cases [1, 2, 3, 4]
- Simplifying assumptions are made in these cases such as 2D, symmetric, constant density etc. [1, 2, 3, 4]
- Most related engineering problems are not as simple [1, 2, 3, 4]
- Numerical analysis provides a plausible option for solving complex engineering problems related to fluid flow, heat and mass transfer [1, 2, 3, 4]
- CFD refers to numerical solution of the governing equations for fluid flow, heat and mass transfer [1, 2, 3, 4]
- Many commercial CFD codes available: Star-CD, Star-CCM+, PHOENICS, Flow EFD, Flow 3D as well as ANSYS Fluent and CFX

Introduction to CFD II

- Relatively complete and widely used open source code - OPENFOAM
- Most codes are based on the Finite Volume Method (FVM) [1, 2, 3, 4]
- Codes based on the Finite Element Method (FEM) are available [1, 2, 3, 4]
- Lattice Boltzmann Method (LBM) is becoming popular - permeating into industrial application
- LBM - simplicity in algorithm implementation and high level of parallelism, however, multi-physics and boundary conditions can be 'tricky'
- Smooth Particle Hydrodynamics (SPH) becoming increasingly popular due to increase in computing power and with the advent of large scale GPU computing
- Can write your own custom code

Introduction to CFD III

- CFD has been around for decades [1, 2, 3, 4]
- However, due to computational restrictions was limited in usage [1, 2, 3, 4]
- With advent of cheap and easily available large scale computing power use of CFD widespread [1, 2, 3, 4]
- The widespread use and industrial demand for short turn around times has led to some problems for the CFD analyst [1, 2, 3, 4]
- Many CFD analyst have only received an undergraduate qualification [1, 2, 3, 4]
- Thus, many analysts do not have a sufficient background in advanced fluid dynamics, thermodynamics, heat and mass transfer to produce high quality CFD solutions [1, 2, 3, 4]
- Furthermore, financial and time constraints prevent further training for the analyst

Introduction to CFD IV

- In addition, these constraints prevent mesh and turbulence sensitivity studies as well as the use of "full" physics in CFD models
- Purpose of this course is to address first of the two issues
- Many companies prefer to hire CFD analysts with at least a Masters background in CFD
- The ideal skill set for a CFD analyst [5]:
  - Fluid Dynamics, Heat and Mass Transfer, Thermodynamics, Thermofluids - Transport Phenomena
  - Mathematics and Applied Mathematics
  - Computer Science - Programming
  - Computer Hardware and Architecture
  - Parallel computing

Mathematical Conventions I

You should know this by now but here is a reminder.

For a given scalar [6, 7]:

$$\phi = \phi(x, y, z)$$

The gradient is defined as [6, 7]:

$$\nabla \phi \equiv grad(\phi) = \frac{\partial \phi}{\partial x}\overrightarrow{i} + \frac{\partial \phi}{\partial y}\overrightarrow{j} + \frac{\partial \phi}{\partial z}\overrightarrow{k}$$

For a vector [6, 7]:

$$\overrightarrow{\phi} = \phi_x \overrightarrow{i} + \phi_y \overrightarrow{j} + \phi_z \overrightarrow{k}$$

The gradient (which is a second order tensor) is defined as [6, 7]:

$$\nabla \overrightarrow{\phi} \equiv grad\left(\overrightarrow{\phi}\right) = \left[\frac{\partial}{\partial x}\overrightarrow{i} + \frac{\partial}{\partial y}\overrightarrow{j} + \frac{\partial}{\partial z}\overrightarrow{k}\right]\left[\phi_x \overrightarrow{i} + \phi_y \overrightarrow{j} + \phi_z \overrightarrow{k}\right]$$

Mathematical Conventions II

The above tensor can be written as [6, 7]:

$$\begin{pmatrix} \frac{\partial \phi_x}{\partial x} & \frac{\partial \phi_x}{\partial y} & \frac{\partial \phi_x}{\partial z} \\ \frac{\partial \phi_y}{\partial x} & \frac{\partial \phi_y}{\partial y} & \frac{\partial \phi_y}{\partial z} \\ \frac{\partial \phi_z}{\partial x} & \frac{\partial \phi_z}{\partial y} & \frac{\partial \phi_z}{\partial z} \end{pmatrix}$$

The divergence of a vector is given as [6, 7]:

$$\nabla \cdot \overrightarrow{\phi} \equiv div\left(\overrightarrow{\phi}\right) = \frac{\partial \phi_x}{\partial x} + \frac{\partial \phi_y}{\partial y} + \frac{\partial \phi_z}{\partial z}$$

The Laplacian is defined as [6, 7]:

$$\nabla \cdot \nabla \phi \equiv \nabla^2 \phi = \frac{\partial^2 \phi}{\partial x} + \frac{\partial^2 \phi}{\partial y} + \frac{\partial^2 \phi}{\partial z}$$

NOT THE SAME AS $(\nabla \phi)^2$ [6, 7]:

Mathematical Conventions III

$$(\nabla\phi)^2 = \left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2$$

Dot product between two vectors $\overrightarrow{a} = \{a_1, a_2, ..., a_n\}^T$ and $\overrightarrow{b} = \{b_1, b_2, ..., b_n\}^T$ is defined as [6, 7]:

$$\overrightarrow{a} \cdot \overrightarrow{b} = a_i b_i, \qquad \forall i = 1, 2, ..., n$$

Cross product between two vectors $\overrightarrow{a} = \{a_1, a_2, a_3\}^T$ and $\overrightarrow{b} = \{b_1, b_2, b_3\}^T$ is defined as [6, 7]:

$$\overrightarrow{a} \otimes \overrightarrow{b} = \begin{pmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{pmatrix}$$

Governing Equations I

- The governing equations are derived based on a continuum approach as opposed to a molecular dynamics approach [1, 2, 3, 4]
- The governing equations for fluid flow are conservation laws [1, 2, 3, 4]
- The first governing equation, in vector form, is the mass conservation (continuity) equation [1, 2, 3, 4]:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

$$\text{where } \vec{u} = \{u, v, w\}^T$$

- In Cartesian Tensor Notation [1, 2, 3, 4]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0$$

$\forall i = 1, 2, 3$. Where $u_1 = u, u_2 = v, u_3 = w$ and $x_1 = x, x_2 = y, x_3 =$

Governing Equations II

- The momentum equations, in Cartesian reference frame, are as follows (in vector form) [1, 2, 3, 4]:

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u \vec{u}) = \frac{\partial(-p + \tau_{xx})}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + B_x + S_{M_x}$$

$$\frac{\partial \rho v}{\partial t} + \nabla \cdot (\rho v \vec{u}) = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial(-p + \tau_{yy})}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + B_y + S_{M_y}$$

$$\frac{\partial \rho w}{\partial t} + \nabla \cdot (\rho w \vec{u}) = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial(-p + \tau_{zz})}{\partial z} + B_z + S_{M_z}$$

- The term on the RHS is the rate of change of x, y and z -momentum of the fluid particle [1, 2, 3, 4]:
- The terms B_x, B_y, B_z are the body force terms and the $S_{M_x}, S_{M_y}, S_{M_z}$ are the source terms [1, 2, 3, 4]:

Governing Equations III

- The other terms represent the surface forces - combination of pressure and viscous stresses [1, 2, 3, 4]:
- Note nine viscous stress components
- Can combine the source terms and body force terms into one source term [1, 2, 3, 4]
- Body forces include - gravity force, centrifugal force, Coriolis force, electromagnetic force [1, 2, 3, 4]
- The final governing equation is the (internal) energy equation and is given by [1, 2, 3, 4]:

$$\frac{\partial \rho i}{\partial t} + \nabla \cdot (\rho i \overrightarrow{u}) = -p \nabla \cdot (\overrightarrow{u}) + \nabla \cdot (k \nabla T) + \tau_{xx} \frac{\partial u}{\partial x} + \tau_{yx} \frac{\partial u}{\partial y} + \tau_{zx} \frac{\partial u}{\partial z} +$$

$$\tau_{xy} \frac{\partial v}{\partial x} + \tau_{yy} \frac{\partial v}{\partial y} + \tau_{zy} \frac{\partial v}{\partial z} + \tau_{xz} \frac{\partial w}{\partial x} + \tau_{yz} \frac{\partial w}{\partial y} + \tau_{zz} \frac{\partial w}{\partial z} + S_i$$

Governing Equations IV

- Can convert the above to temperature (T), total energy (E) or total enthalpy (h_0) equation using, respectively [1, 2, 3, 4]:

$$i = C_V T,$$

$$i = E - \frac{1}{2}(u^2 + v^2 + w^2) \text{ or}$$

$$i = h - \frac{p}{\rho} \text{ where } h = h_0 - \frac{1}{2}(u^2 + v^2 + w^2)$$

- The Equations of state are also needed [1, 2, 3, 4]:

$$p = p(\rho, T) \text{ and } i = i(\rho, T)$$

Inviscid Flow I

- For flows where the viscous forces are negligible - can assume shearing stresses are negligible [1, 2, 3, 4]:
- Thus, $\tau_{ij} = 0, \forall i, j = x, y, z$ [1, 2, 3, 4]
- Thus, momentum equations reduce to [1, 2, 3, 4]:

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u \overrightarrow{u}) = -\frac{\partial p}{\partial x} + B_x + S_{M_x}$$

$$\frac{\partial \rho v}{\partial t} + \nabla \cdot (\rho v \overrightarrow{u}) = -\frac{\partial p}{\partial y} + B_y + S_{M_y}$$

$$\frac{\partial \rho w}{\partial t} + \nabla \cdot (\rho w \overrightarrow{u}) = -\frac{\partial p}{\partial z} + B_z + S_{M_z}$$

- known as Euler's equations [1, 2, 3, 4]
- The energy equation reduces to [1, 2, 3, 4]:

$$\frac{\partial \rho i}{\partial t} + \nabla \cdot (\rho i \overrightarrow{u}) = -p \nabla \cdot (\overrightarrow{u}) + \nabla \cdot (k \nabla T) + S_i$$

Inviscid Flow II

- Equations are much simpler for inviscid flow [1, 2, 3, 4]
- However, still are a set of non-linear partial differential field equations [1, 2, 3, 4]
- No general solution [1, 2, 3, 4]

Newtonian Fluid I

- For most (isotropic) fluids, the shear stress is proportional to the viscosity and shear rate [1, 2, 3, 4]
- The above is Newton's law of viscosity [1, 2, 3, 4]:
- To define the shear stresses - need to define linear elongating deformations, linear shearing deformations and volumetric deformation
- Linear elongating deformations [1, 2, 3, 4]:

$$e_{xx} = \frac{\partial u}{\partial x}$$

$$e_{yy} = \frac{\partial v}{\partial y}$$

$$e_{zz} = \frac{\partial w}{\partial z}$$

Newtonian Fluid II

- Linear shearing deformations [1, 2, 3, 4]:

$$e_{xy} = e_{yx} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$e_{xz} = e_{zx} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$e_{yz} = e_{zy} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

- Volumetric deformation [1, 2, 3, 4]:

$$e_{xx} + e_{yy} + e_{xx} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla \cdot (\overrightarrow{u})$$

- Shear stresses [1, 2, 3, 4]:

$$\tau_{ii} = 2\mu e_{ii} + \lambda \nabla \cdot (\overrightarrow{u})$$

$$\tau_{ij} = \tau_{ji} = 2\mu e_{ij}$$

Newtonian Fluid III

- λ is the second viscosity [1, 2, 3, 4]:
- Usually effect of λ is negligible [1, 2, 3, 4]:
- Not necessary for incompressible flows ($\nabla \cdot (\overrightarrow{u}) = 0$) [1, 2, 3, 4]:
- For gases can use $\lambda = -2/3\mu$ [1, 2, 3, 4]:
- After substitution and lots of algebra momentum equations reduce to [1, 2, 3, 4]:

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u \overrightarrow{u}) = -\frac{\partial p}{\partial x} + \nabla \cdot (\mu \nabla u) + B_x + S_{M_x}$$

$$\frac{\partial \rho v}{\partial t} + \nabla \cdot (\rho v \overrightarrow{u}) = -\frac{\partial p}{\partial y} + \nabla \cdot (\mu \nabla v) + B_y + S_{M_y}$$

$$\frac{\partial \rho w}{\partial t} + \nabla \cdot (\rho w \overrightarrow{u}) = -\frac{\partial p}{\partial z} + \nabla \cdot (\mu \nabla w) + B_z + S_{M_z}$$

Newtonian Fluid IV

- Known as Navier-Stokes Equations [1, 2, 3, 4]
- The energy equation reduces to [1, 2, 3, 4]:

$$\frac{\partial \rho i}{\partial t} + \nabla \cdot (\rho i \overrightarrow{u}) = -p\nabla \cdot (\overrightarrow{u}) + \nabla \cdot (k\nabla T) + \Phi + S_i$$

- Φ is the viscous dissipation (viscous heating term) [1, 2, 3, 4]
- Φ - can be ignored for flows where $Br = (\mu U_e)/(k\Delta T) << 1$ [6]
- Also a set of non-linear partial differential field equations [1, 2, 3, 4]
- No general solution [1, 2, 3, 4]
- Thus, above equations with the continuity equation form the governing equations for (Newtonian) fluid flow

Non-Newtonian Fluid

- Newtonian fluids - the shear stress is proportional to the viscosity and shear rate [1, 2, 3, 4]
- Certain fluids have significantly different behaviour [8]
- These are known as Non-Newtonian fluids [8]
- Three main categories for Non-Newtonian behaviour [8]:
  - time independent behaviour - viscosity (apparent) that varies as the shear rate varies
  - time dependent behaviour - viscosity (apparent) changes with time at constant shear rate
  - visco-elastic behaviour - fluid bahaviour between pure liquid and pure solid
- Treatment beyond scope of this course - can consult literature such as [8]

Generalised Scalar Transport Equation I

- The governing equations for fluid dynamics and heat transfer, in conservative form, are [1, 2, 3, 4]

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u \vec{u}) = -\frac{\partial p}{\partial x} + \nabla \cdot (\mu \nabla u) + B_x + S_{M_x}$$

$$\frac{\partial \rho v}{\partial t} + \nabla \cdot (\rho v \vec{u}) = -\frac{\partial p}{\partial y} + \nabla \cdot (\mu \nabla v) + B_y + S_{M_y}$$

$$\frac{\partial \rho w}{\partial t} + \nabla \cdot (\rho w \vec{u}) = -\frac{\partial p}{\partial z} + \nabla \cdot (\mu \nabla w) + B_z + S_{M_z}$$

$$\frac{\partial \rho i}{\partial t} + \nabla \cdot (\rho i \vec{u}) = -p \nabla \cdot (\vec{u}) + \nabla \cdot (k \nabla T) + \Phi + S_i$$

Generalised Scalar Transport Equation II

- Equations can be re-written in the form of a generalised scalar transport equation for the general transport variable named ϕ [1, 2, 3, 4]

$$\frac{\partial \rho \phi}{\partial t} + \nabla \cdot (\rho \phi \vec{u}) = \nabla \cdot (\Gamma \nabla \phi) + S_\phi$$

- Meaning [1, 2, 3, 4]: Rate of increase of ϕ of fluid element + Net rate of flow of ϕ out of fluid element = Rate of increase of ϕ due to diffusion + Rate of increase of ϕ due to sources
- Important Concept in CFD
- Keep in mind - will need it later

Compressible and Incompressible Flow I

- Till now dealt with general compressible flow
- For incompressible flow can simplify equations using:

$$\frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial \rho \phi}{\partial t} = \rho \frac{\partial \phi}{\partial t} + \phi \frac{\partial \rho}{\partial t} = \rho \frac{\partial \phi}{\partial t}$$

$$\nabla \cdot (\rho \phi \overrightarrow{u}) = \rho \nabla \cdot (\phi \overrightarrow{u})$$

$$\nabla \cdot (\rho \overrightarrow{u}) = \rho \nabla \cdot (\overrightarrow{u})$$

Compressible and Incompressible Flow II

- Thus the governing equations become:

$$\rho\nabla\cdot(\overrightarrow{u}) = 0,\ \text{Thus,}\ \nabla\cdot(\overrightarrow{u}) = 0$$

$$\rho\frac{\partial u}{\partial t} + \rho\nabla\cdot(u\overrightarrow{u}) = -\frac{\partial p}{\partial x} + \nabla\cdot(\mu\nabla u) + B_x + S_{M_x}$$

$$\rho\frac{\partial v}{\partial t} + \rho\nabla\cdot(v\overrightarrow{u}) = -\frac{\partial p}{\partial y} + \nabla\cdot(\mu\nabla v) + B_y + S_{M_y}$$

$$\rho\frac{\partial w}{\partial t} + \rho\nabla\cdot(w\overrightarrow{u}) = -\frac{\partial p}{\partial z} + \nabla\cdot(\mu\nabla w) + B_z + S_{M_z}$$

$$\rho\frac{\partial i}{\partial t} + \rho\nabla\cdot(i\overrightarrow{u}) = -p\nabla\cdot(\overrightarrow{u}) + \nabla\cdot(k\nabla T) + \Phi + S_i$$

Steady and Unsteady Flow I

- Till now dealt with general unsteady flow
- For steady flow can simplify equations using:

$$\frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial \rho \phi}{\partial t} = 0$$

- Thus the governing equations become:

$$\nabla \cdot (\rho \overrightarrow{u}) = 0$$

$$\nabla \cdot (\rho u \overrightarrow{u}) = -\frac{\partial p}{\partial x} + \nabla \cdot (\mu \nabla u) + B_x + S_{M_x}$$

$$\nabla \cdot (\rho v \overrightarrow{u}) = -\frac{\partial p}{\partial y} + \nabla \cdot (\mu \nabla v) + B_y + S_{M_y}$$

$$\nabla \cdot (\rho w \overrightarrow{u}) = -\frac{\partial p}{\partial z} + \nabla \cdot (\mu \nabla w) + B_z + S_{M_z}$$

$$\nabla \cdot (\rho i \overrightarrow{u}) = -p \nabla \cdot (\overrightarrow{u}) + \nabla \cdot (k \nabla T) + \Phi + S_i$$

Introduction to Turbulence I

- Most flows are turbulent [1, 9]
- Thus, need to quantify the effects of turbulence in CFD models [1, 9]
- For this need sound understanding of the physics of turbulence [1, 9]
- What is turbulence?
- Simple definition [1]:
  "A chaotic and random state of motion develops in which the velocity and pressure change continuously with time within substantial regions of flow"
- To characterise the onset of turbulence - use Reynolds number
- Is turbulence good or a bad?
- In other words to we want turbulence in a system or not?
- The answer depends on the purpose of the system

Introduction to Turbulence II

- Turbulence by nature has a "three dimensional spatial character" [1]

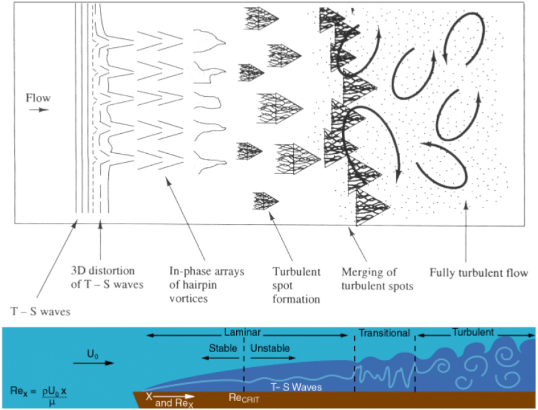

Figure: Transition in a Boundary Layer over a Flat Plate [10]

Introduction to Turbulence III

- This is the case even when the mean velocities and pressure vary in one or two dimensions
- An important aspect of turbulence is the formation of rotational flow structures [1, 9]
- These are known as eddies
- The turbulent eddies vary in length scale
- Large scale eddies are anisotropic and are highly flow dependent - inertia effects are dominant
- Small scale eddies - viscous effects are dominant
- Small scale eddies - isotropic for high Reynolds number flows
- Transition - Don't confuse with transient
- Transition - intermediate region between stable laminar flow and unstable turbulent flow
- Characterised by Turbulent spot formation and merging of turbulent spots
- Transition - still very difficult physical phenomenon to model in CFD

Navier-Stokes Equations and Laminar Flow I

- Navier-Stokes Equations for Newtonian Fluid [1, 2, 3, 4]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u \vec{u}) = -\frac{\partial p}{\partial x} + \nabla \cdot (\mu \nabla u) + B_x + S_{M_x}$$

$$\frac{\partial \rho v}{\partial t} + \nabla \cdot (\rho v \vec{u}) = -\frac{\partial p}{\partial y} + \nabla \cdot (\mu \nabla v) + B_y + S_{M_y}$$

$$\frac{\partial \rho w}{\partial t} + \nabla \cdot (\rho w \vec{u}) = -\frac{\partial p}{\partial z} + \nabla \cdot (\mu \nabla w) + B_z + S_{M_z}$$

- Above applicable for laminar and turbulent flow.
- However as it stands the above equations, without expanding are equations for laminar flow

Reynolds Averaged Navier-Stokes Equations I

- To include effects of turbulence express instantaneous velocities (u, v, w) and pressure p as superposition of two components [1, 9]:

$$\phi(t) = \bar{\phi} + \phi'(t)$$

- The first component is the mean flow component and the second term is the random fluctuating component (due to turbulence) [1, 9]
- The mean flow component is given by [1, 9]:

$$\bar{\phi} = \frac{1}{\Delta t} \int_0^{\Delta t} \phi(t) dt$$

- The time average of the fluctuating component is given by [1, 9]:

$$\bar{\phi}' = \frac{1}{\Delta t} \int_0^{\Delta t} \phi'(t) dt \equiv 0$$

Reynolds Averaged Navier-Stokes Equations II

- Thus, use the RMS of the fluctuating components [1]:

$$\sqrt{\overline{(\phi')^2}} = \frac{1}{\Delta t}\int_0^{\Delta t} (\phi'(t))^2 dt$$

- Use the above to derive the time-averaged or Reynolds averaged Navier-Stokes equations [1, 9]
- Navier-Stokes Equations for Newtonian Fluid [1, 2, 3, 4]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{u}) = 0$$

$$\frac{\partial \rho \bar{u}}{\partial t} + \nabla \cdot (\rho \bar{u} \bar{\mathbf{u}}) = -\frac{\partial \bar{p}}{\partial x} + \nabla \cdot (\mu \nabla \bar{u}) + B_x + S_{M_x}$$

$$- \left[\frac{\partial \rho \overline{u'^2}}{\partial x} + \frac{\partial \rho \overline{u' v'}}{\partial y} + \frac{\partial \rho \overline{u' w'}}{\partial z} \right]$$

$$\frac{\partial \rho \bar{v}}{\partial t} + \nabla \cdot (\rho \bar{v} \bar{\mathbf{u}}) = -\frac{\partial \bar{p}}{\partial y} + \nabla \cdot (\mu \nabla \bar{v}) + B_y + S_{M_y}$$

Reynolds Averaged Navier-Stokes Equations III

$$- \left[\frac{\partial \rho \overline{u'v'}}{\partial x} + \frac{\partial \rho \overline{v'^2}}{\partial y} + \frac{\partial \rho \overline{v'w'}}{\partial z} \right]$$

$$\frac{\partial \rho \bar{w}}{\partial t} + \nabla \cdot (\rho \bar{w} \bar{\mathbf{u}}) = -\frac{\partial \bar{p}}{\partial z} + \nabla \cdot (\mu \nabla \bar{w}) + B_z + S_{M_z}$$

$$- \left[\frac{\partial \rho \overline{u'w'}}{\partial x} + \frac{\partial \rho \overline{v'w'}}{\partial y} + \frac{\partial \rho \overline{w'^2}}{\partial z} \right]$$

- By averaging - we gain nine extra terms $-pu_i u_j$
- Known as Reynolds stresses
- Under-determined system of equations
- Thus, need closure model
- Thus, the need for turbulence models
- Turbulence modelling is central to CFD
- Models built to provide closure by introducing additional equations These models are developed either empirically from experiment or rigorous mathematical methods

Reynolds Averaged Navier-Stokes Equations IV

- Thus, they vary in applicability based on the assumptions about the fluid physics
- Thus, must choose appropriate model for specific flow problem
- Direct Numerical Simulation (DNS) resolves all scales of turbulence - costly
- Other models resolve some scales and model other scales

Reynolds Averaged Navier-Stokes Equations V

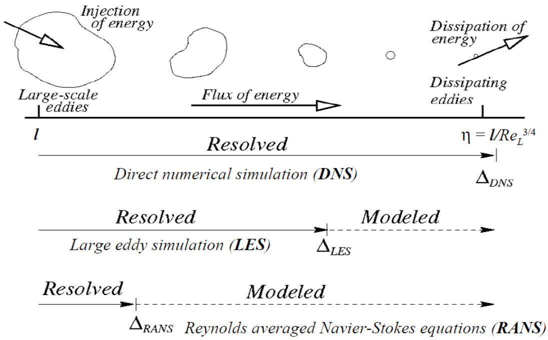

Figure: Turbulence Models - Resolved Scales vs. Modelled Scales [10]

The Finite Volume Method (in brief) I

- Remember the Generalised Scalar transport Equation:
- Equations can be re-written in the form of a generalised scalar transport equation for the general transport variable named ϕ [1, 2, 3, 4]

$$\frac{\partial \rho\phi}{\partial t} + \nabla \cdot (\rho\phi\overrightarrow{u}) = \nabla \cdot (\Gamma\nabla\phi) + S_\phi$$

- Discretise Domain into discrete (control) volumes (cells/voxels):
- Discretise the generalised scalar transport equation:
- $$\frac{\partial(\rho\phi)}{\partial t}V + \sum_{f}^{N_{faces}} \rho_f\phi_f\mathbf{u}_f.\mathbf{A}_f = \sum_{f}^{N_{faces}} \Gamma_f\nabla\phi_f.\mathbf{A}_f + S_\phi V$$
- Write above for each cell in the computational domain, for each transport variable ϕ [2, 1, 11, 6, 12].

The Finite Volume Method (in brief) II

- Apply boundary and/or initial conditions.
- The resulting equations are solved for all the transport variables ϕ at the cell center for all cells [2, 1, 11, 6, 12].

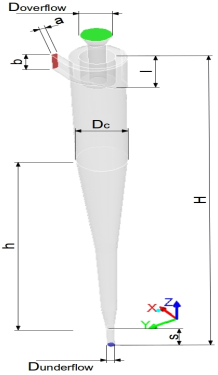

Figure: Hydrocyclone geometry [13]

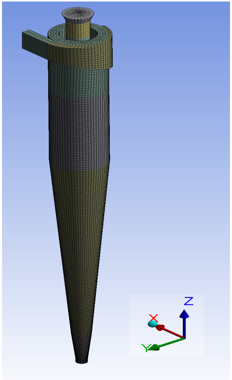

Figure: Computational grid (mesh) [13]

The Finite Volume Method (in brief) III

- Thus, providing an approximate solution for the given problem [2, 1, 11, 6, 12].
- The FVM has more complexity, however, above is a basic overview.
- Important to keep in mind - CFD is an approximation.
- CFD solutions have sources of error:
  - Modelling error: as a result of model assumptions/simplifications
  - Computing error: as a result of numerical methods and computer round-off (finite precision machine)

Other Numerical Methods or Approached I

- Can use the Finite Difference Method (FDM) or the Finite Element Method (FEM)
- other approaches:
  - the Lattice Boltzmann Method (LBM)
  - Smooth Particle Hydrodynamics (SPH)

Bibliography and Recommended Reading I

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