QCD

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4^{th} Lecture

Latin American School of High Energy Physics — March 2023

Parton shower & Monte Carlo methods

- today one can compute some IR-safe quantities at NNLO and very few ones at N^{3}LO. Difficult to expect much more in the coming years.
- we have also seen that sometimes large logarithms spoil the convergence of PT, fixed-order becomes unreliable (divergent)
- now we adopt a different approach: we seek for an approximate result such that enhanced terms are taken into account to all orders
- this will lead to a 'parton shower' picture, which can be implemented in computer simulations, usually called Monte Carlo programs or event generators

Monte Carlos enter any experimental study at current colliders

Perturbative evolution

In exact analogy with what done for parton densities inside hadrons we want to write an evolution equation for the probability to have partons at the momentum scale Q^2 with momentum fraction z during PT branching

Start from DGLAP equation

$$Q^2 \frac{\partial f(x,Q^2)}{\partial Q^2} = \int_0^1 dz \frac{\alpha_s}{2\pi} \hat{P}(z) \left(\frac{1}{z} f\left(\frac{x}{z}, Q^2\right) - f(x,Q^2) \right)$$

Introduce a cut-off to regulate divergences

$$Q^2 \frac{\partial f(x,Q^2)}{\partial Q^2} = \int_0^{1-\epsilon} \frac{dz}{z} \frac{\alpha_s}{2\pi} \hat{P}(z) f\left(\frac{x}{z}, Q^2\right) - f(x,Q^2) \int_0^{1-\epsilon} dz \frac{\alpha_s}{2\pi} \hat{P}(z)$$

Introduce a Sudakov form factor

$$\Delta(Q^2) = exp\left\{ - \int_{Q_0}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^{1-\epsilon} dz \frac{\alpha_s}{2\pi} \hat{P}(z) \right\}$$

Perturbative evolution

The DGLAP equation becomes

$$Q^2 \frac{\partial}{\partial Q^2}\left(\frac{f(x,Q^2)}{\Delta(Q^2)}\right) = \frac{1}{\Delta(Q^2)}\int_0^{1-\epsilon}\frac{dz}{z}\frac{\alpha_s}{2\pi}\hat{P}(z) f\left(\frac{x}{z},Q^2\right)$$

Integrating the above equation one gets

$$f(x,Q^2) = f(x,Q_0^2)\frac{\Delta(Q^2)}{\Delta(Q_0^2)} + \int_{Q^0}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\frac{\Delta(Q^2)}{\Delta(k_\perp^2)}\int_0^{1-\epsilon}\frac{dz}{z}\frac{\alpha_s}{2\pi}\hat{P}(z) f\left(\frac{x}{z},k_\perp^2\right)$$

This equation has a *probabilistic interpretation*

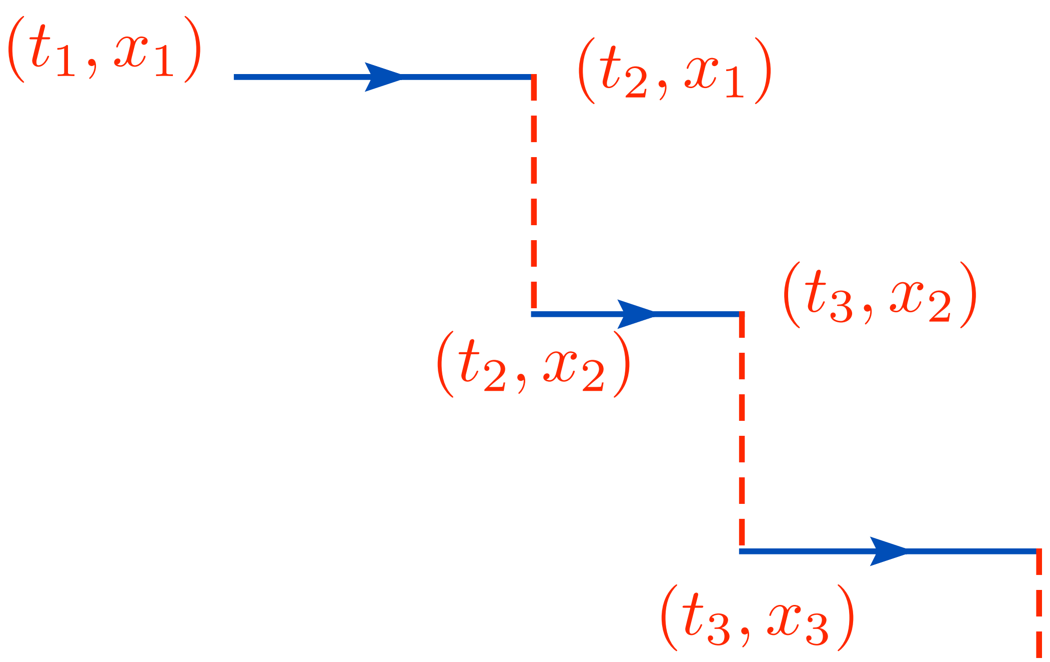
- First term: probability of evolving from Q_0^2 to Q^2 without emissions (ratio of Sudakovs $\Delta(Q^2)/\Delta(Q_0^2)$)
- Second term: emission at scale $k_\perp^2$ and evolution from $k_\perp^2$ to Q^2 without further emissions

Multiple branchings

Multiple branching can now be described using the above probabilistic equation

Denote by t the evolution variable (e.g $t = Q^2$)
Start from one parton at scale t_1 and momentum fraction x_1

One needs to generate the values of $\{t_2, x_2, \varphi_2\}$ with the correct probability



Multiple branchings

1. t_2 generated with the correct probability by solving the equation (r = random number in [0,1])

$$\Delta(t_1)/\Delta(t_2) = r$$

   If t_2 smaller than cut-off evolution stops (no further branching)

2. Else, generate momentum fraction z = x_2/x_1 with $\mathrm{Prob.} \sim \frac{\alpha_s}{2\pi} P(z)$

$$\int_\epsilon^{x_2/x_1} dz \frac{\alpha_s}{2\pi} P(z) = r' \int_\epsilon^{1-\epsilon} dz \frac{\alpha_s}{2\pi} P(z)$$

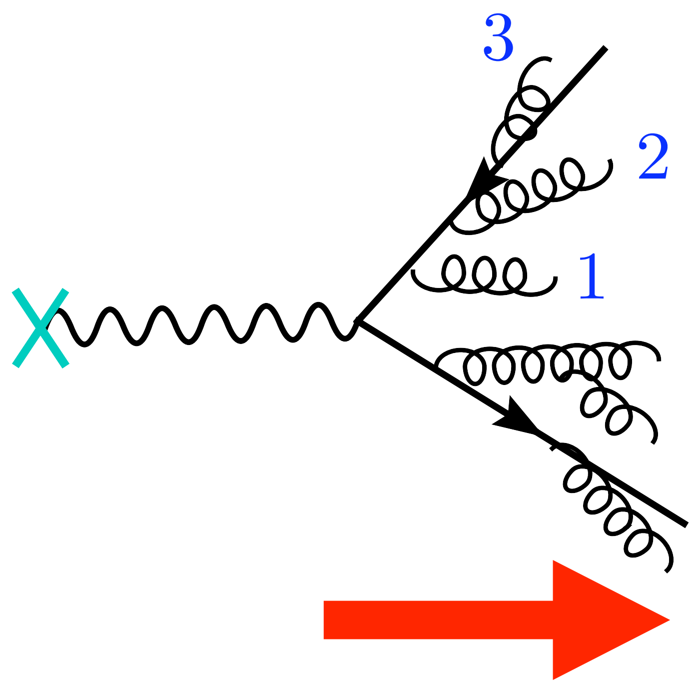
   ε: IR cut-off for resolvable branching

3. Azimuthal angles: generated uniformly in $(0,2\pi)$ (or taking into account polarization correlations)

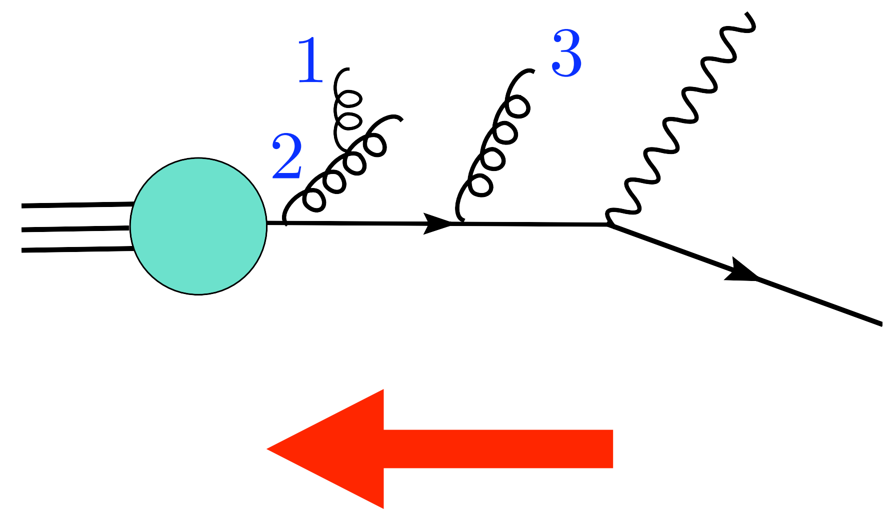
Space-like vs time-like evolution

Time-like: t evolves from a hard-scale downwards to an IR cut-off

Space-like: t increases in the evolution up to the hard scale Q^2



$$Q > t_1 > t_2 > \cdots > Q_0$$



$$Q_0 < t_1 < t_2 < \ldots, Q$$

Each outgoing parton becomes a source of the new branching until the "no-branching" step is met (cut-off essential in parton shower)

⇒ a parton cascade develops, when all branchings are done partons are converted into hadrons via a hadronization model

Accuracy of Monte Carlos

Formally, Monte Carlos are Leading Logarithmic (LL) showers

- because they don't include any higher order corrections to the $1\to 2$ splitting
- because they don't have any $1\to 3$ splittings
- ....

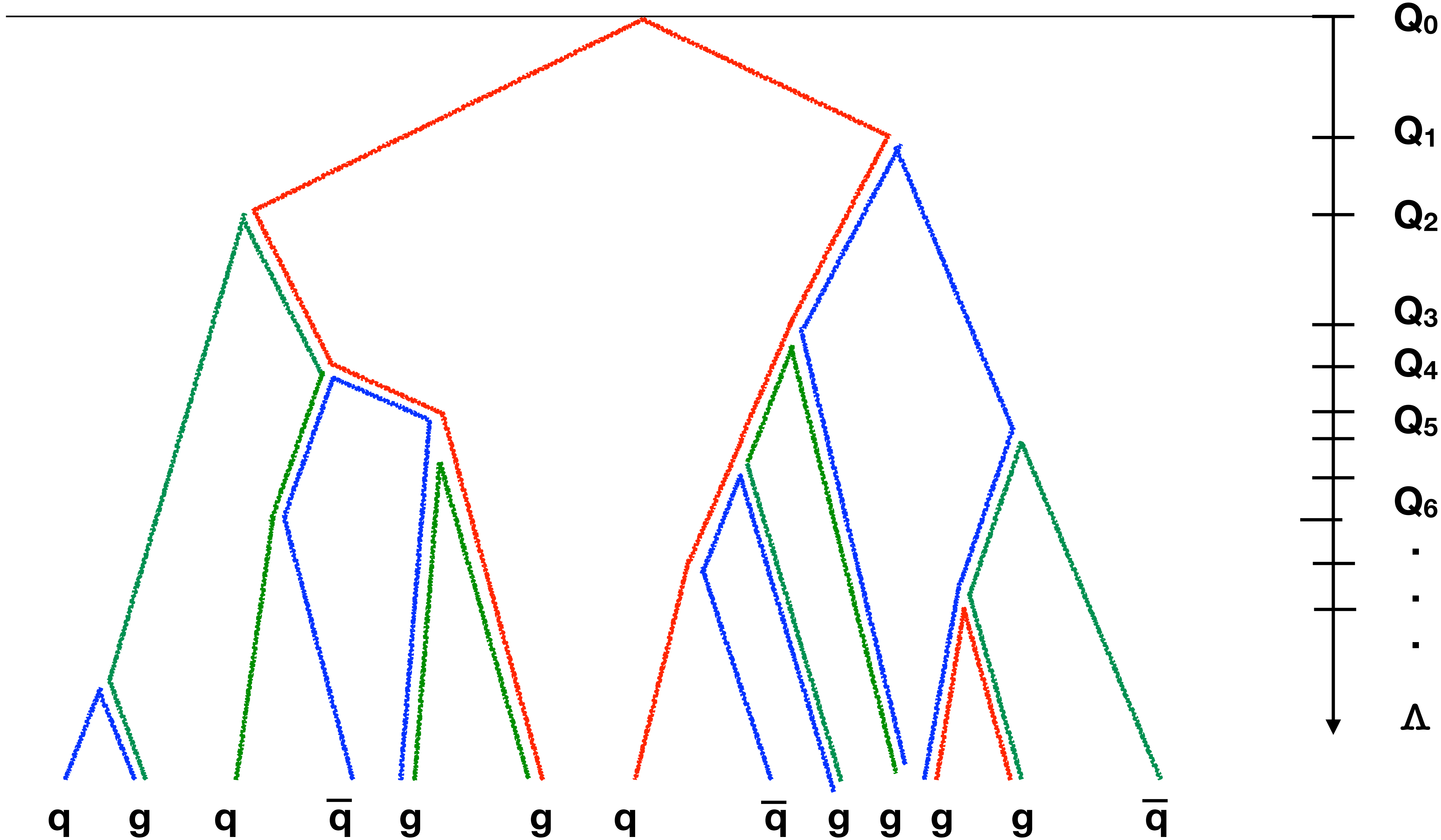
However, they fare better than analytic Leading Log calculations, because

- they have energy conservation (NLO effect) implemented
- they have coherence
- they have optimized choices for the coupling
- they provide an exclusive description of the final state

So, despite not guaranteeing NLL accuracy, traditional parton showers fare better than LL analytic calculations

The real issue is hard to estimate the uncertainty

Illustration of shower evolution



Choices in parton showers

Most relevant choices in the definition of a parton shower:

- Evolution type: DGLAP evolution (e.g. Herwig Pythia) vs dipole/antenna type evolution (Vincia, Dire, Sherpa, Herwig-dipole…)
- Evolution variable: virtuality ordered, angular ordered, k_t ordered…
- Kinematic mapping: how to go from n to (n+1) particles? (local or global schemes)
- Treatment of recoil: how to select the emitter, how to absorb the recoil (e.g. in the dipole frame or in the centre of mass frame)

Choices in parton showers

Considerable progress in recent years in understanding

- What are the best choices (there are wrong/bad choices)
- Assessing the logarithmic accuracy and uncertainty of parton shower predictions, at least for some observables
- Matching parton shower and fixed-order NNLO QCD predictions
- Matching to NLO EW predictions
- Inclusion of colour- and spin-correlation effects