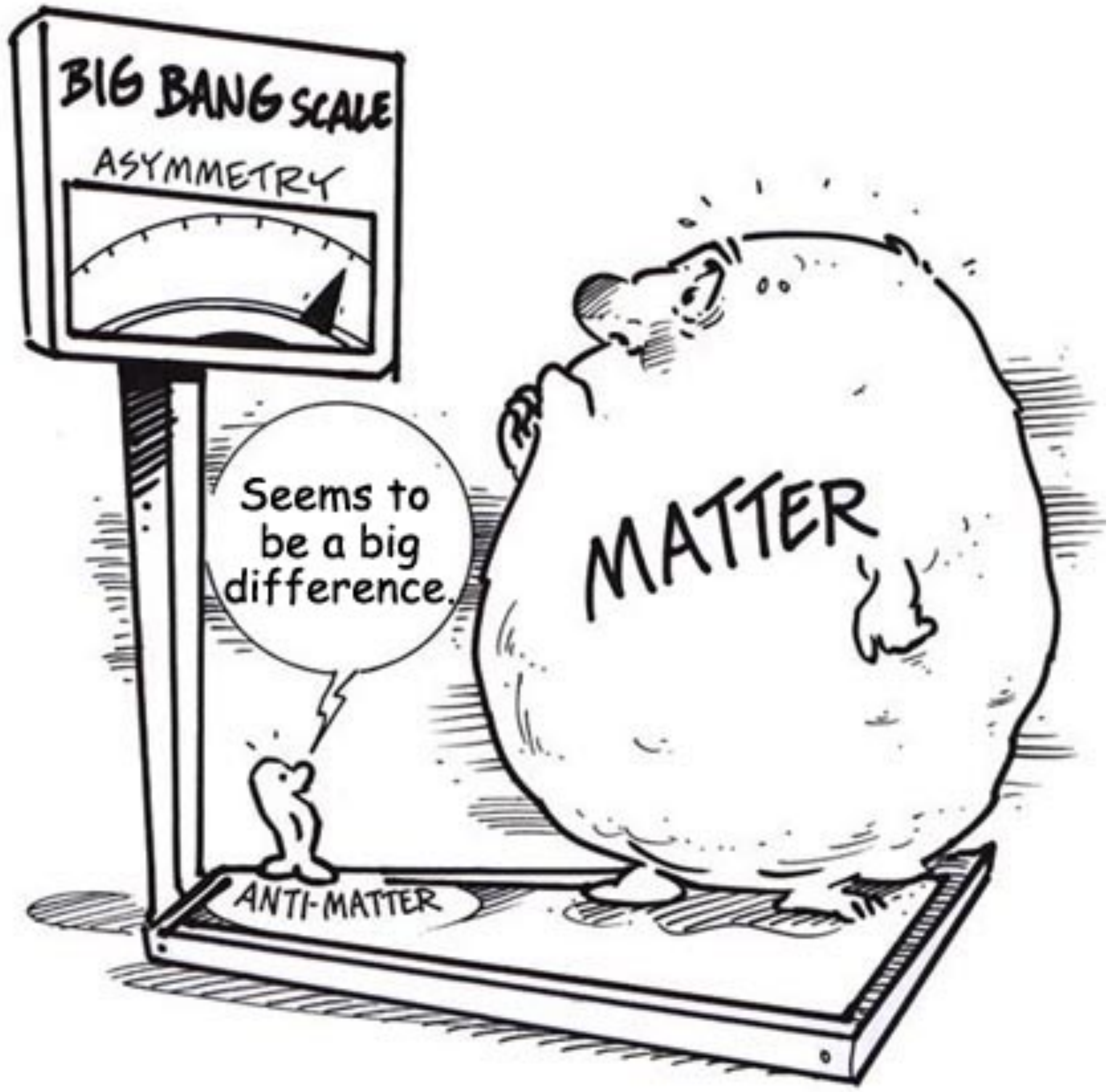
PROPERTIES OF THE CKM MATRIX

This freedom can be used to eliminate $(2n_g - 1)$ phases, leaving:

$$\frac{n_g(n_g-1)}{2}$$ Euler angles

$$\frac{(n_g-1)(n_g-2)}{2}$$ complex phases



- minimal model containing a complex phase has $n_g = 3$ generations!
- allows for an absolute distinction between matter and antimatter!

PROPERTIES OF THE CKM MATRIX

This freedom can be used to eliminate $(2n_g - 1)$ phases, leaving:

$$\frac{n_g(n_g-1)}{2} \quad \text{Euler angles}$$

$$\frac{(n_g-1)(n_g-2)}{2} \quad \text{complex phases}$$

- minimal model containing a complex phase has $n_g = 3$ generations!

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of *CP*-violation are studied. It is concluded that no realistic models of *CP*-violation exist in the quartet scheme without introducing any other new fields. Some possible models of *CP*-violation are also discussed.

[...]

As was pointed out, in this case we cannot absorb all phases of matrix elements into the phase convention and can take, for example, the following expression:

$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3+\sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3+\cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\sin\theta_3 e^{i\delta} \end{pmatrix}. \tag{13}$$

Then, we have *CP*-violating effects through the interference among these different current components.

PROPERTIES OF THE CKM MATRIX

Many equivalent parameterizations of CKM exist; **standard parameterization** is (with $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$):

$$V_{\text{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- observed hierarchy:

$$s_{13} \ll s_{23} \ll s_{12} \ll 1$$

Wolfenstein parameterization:

$$V_{\text{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- where:

$$s_{12} = \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}} \qquad \lambda = 0.22500(67)$$

$$s_{23} = A\lambda^2 = \lambda \left| \frac{V_{cb}}{V_{us}} \right| \qquad A = 0.826(17)$$

$$s_{13}e^{i\delta} = V_{ub}^* = A\lambda^3(\rho + i\eta) \qquad \rho = 0.163(10), \quad \eta = 0.357(10)$$

UNITARITY RELATIONS

Unitarity of the CKM matrix implies:

$$V^{-1} = V^\dagger \quad \Rightarrow \quad VV^\dagger = V^\dagger V = 1$$

- in components:

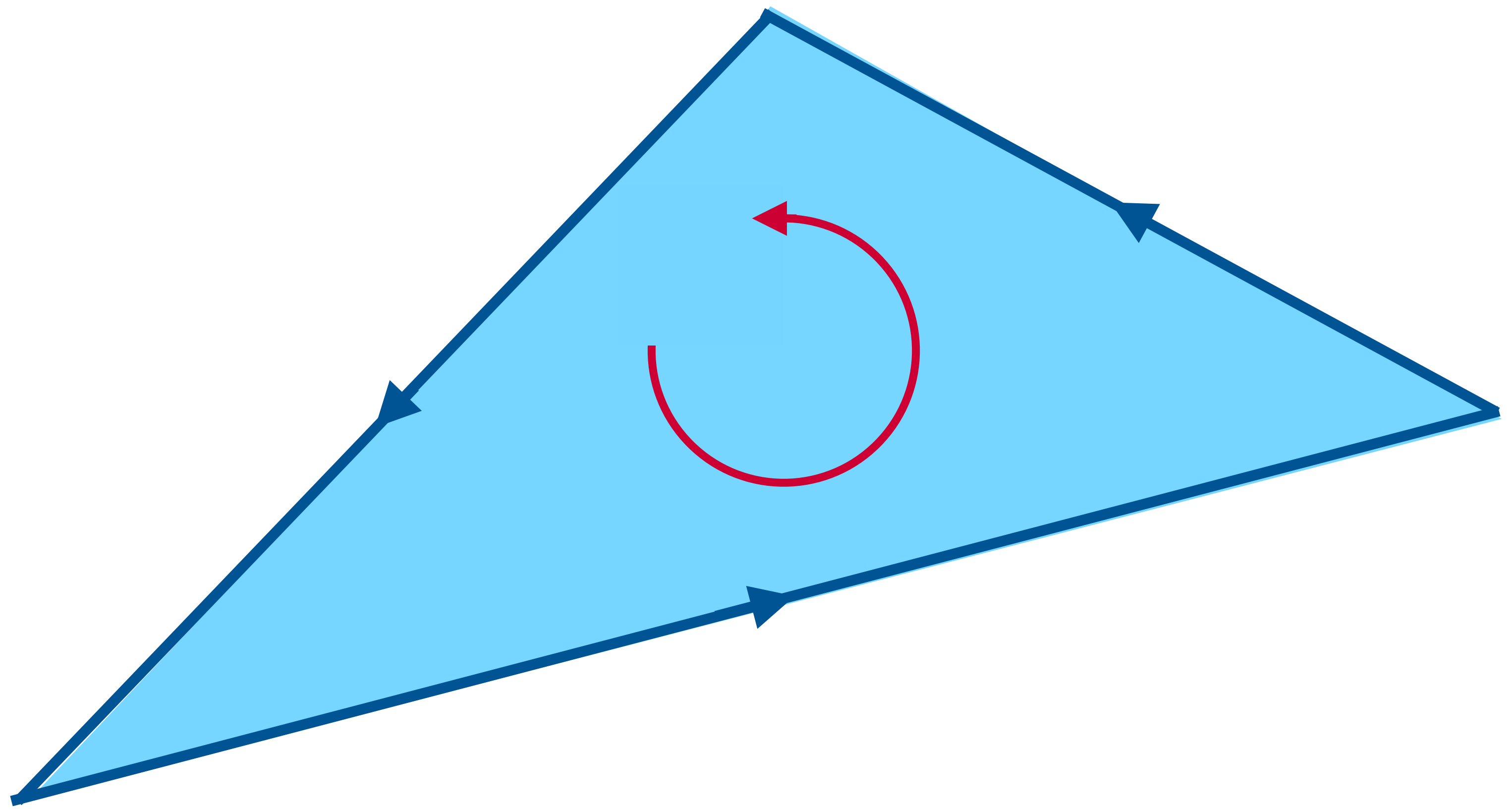
$$\sum_k V_{ik} V_{jk}^* = \delta_{ij}\,, \qquad \sum_k V_{ki} V_{kj}^* = \delta_{ij}$$

- row/column relations ($i = j$):

$$\sum_k |V_{ik}|^2 = \sum_k |V_{ki}|^2 = 1$$

- triangle relations ($i \neq j$):

$$\sum_k V_{ik} V_{jk}^* = 0\,, \qquad \sum_k V_{ki} V_{kj}^* = 0$$



- 6 triangles, all with the same area $\frac{J}{2}$
- under phase redefinitions, the triangles turn in the complex plane

UNITARITY RELATIONS

Jarlskog invariant:

$$\mathrm{Im}(V_{ij}V_{kl}V_{il}^*V_{kj}^*) = J\sum_{m,n}\epsilon_{ikm}\epsilon_{jln}$$

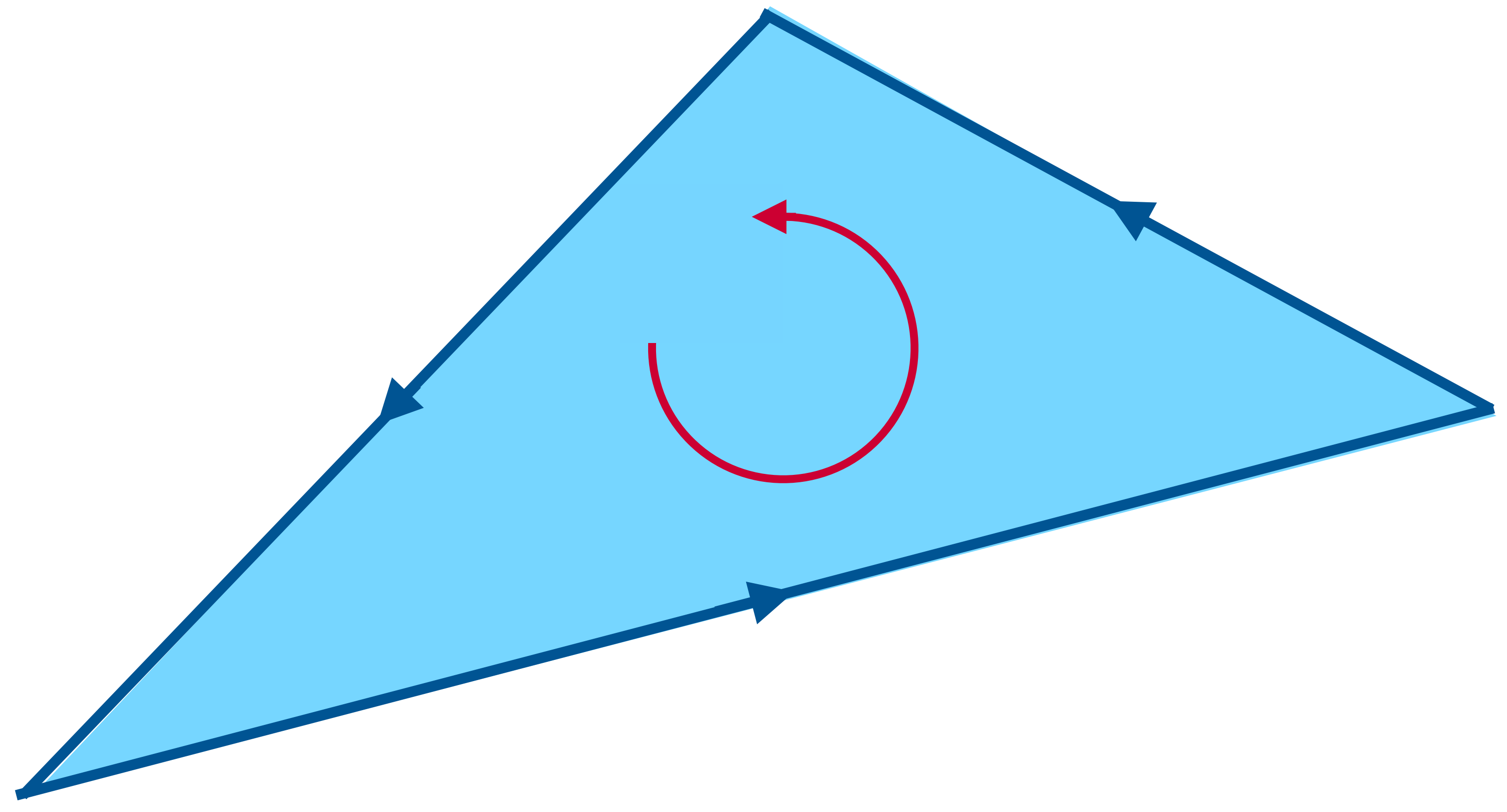
- invariant measure of CP violation
- in Wolfenstein parameterization:

$$J = A^2\lambda^6\eta + \mathcal{O}(\lambda^8)$$

- CP in the SM is suppressed!

- triangle relations ($i \neq j$):

$$\sum_k V_{ik}V_{jk}^* = 0\,, \qquad \sum_k V_{ki}V_{kj}^* = 0$$



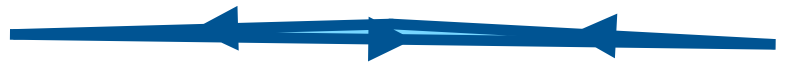
- 6 triangles, all with the same area $\dfrac{J}{2}$
- under phase redefinitions, the triangles turn in the complex plane

UNITARITY TRIANGLES & 1ST-ROW UNITARITY RELATION

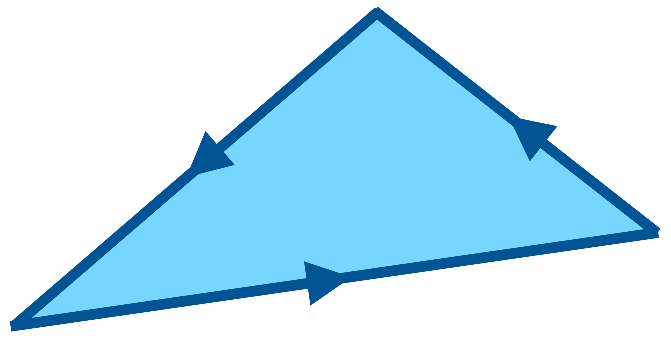
Up-sector triangle relations:

Down-sector triangle relations:

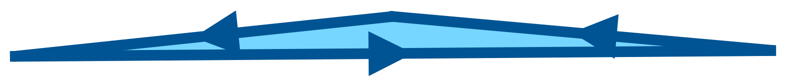
$$\underset{\sim\lambda}{V_{ud}V_{cd}^*} + \underset{\sim\lambda}{V_{us}V_{cs}^*} + \underset{\sim\lambda^5}{V_{ub}V_{cb}^*} = 0 \qquad\qquad \underset{\sim\lambda}{V_{ud}V_{us}^*} + \underset{\sim\lambda}{V_{cd}V_{cs}^*} + \underset{\sim\lambda^5}{V_{td}V_{ts}^*} = 0$$



$$\boxed{\underset{\sim\lambda^3}{V_{ud}V_{td}^*} + \underset{\sim\lambda^3}{V_{us}V_{ts}^*} + \underset{\sim\lambda^3}{V_{ub}V_{tb}^*} = 0} \qquad\qquad \boxed{\underset{\sim\lambda^3}{V_{ud}V_{ub}^*} + \underset{\sim\lambda^3}{V_{cd}V_{cb}^*} + \underset{\sim\lambda^3}{V_{td}V_{tb}^*} = 0}$$



$$\underset{\sim\lambda^4}{V_{cd}V_{td}^*} + \underset{\sim\lambda^2}{V_{cs}V_{ts}^*} + \underset{\sim\lambda^2}{V_{cb}V_{tb}^*} = 0 \qquad\qquad \underset{\sim\lambda^4}{V_{us}V_{ub}^*} + \underset{\sim\lambda^2}{V_{cs}V_{cb}^*} + \underset{\sim\lambda^2}{V_{ts}V_{tb}^*} = 0$$



The Unitarity Triangle (UT)

First-row unitarity relation:

$$|V_{ud}|^2 + |V_{us}|^2 + |\cancelto{0}{V_{ub}}|^2 = 1$$

⇒ currently a 3σ deficit is observed!

LOOP-INDUCED FCNC AND THE GIM MECHANISM

Flavor-changing neutral currents beyond tree level:

- no longer forbidden, since two charged-current interactions can add up to give a neutral-current interaction
- many important applications to rare decays of K and B mesons, as well as to $K-\bar{K}$ and $B-\bar{B}$ mixing

PHYSICAL REVIEW D VOLUME 2, NUMBER 7 1 OCTOBER 1970

Weak Interactions with Lepton-Hadron Symmetry*

S. L. Glashow, J. Iliopoulos, and L. Maiani†

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139

(Received 5 March 1970)

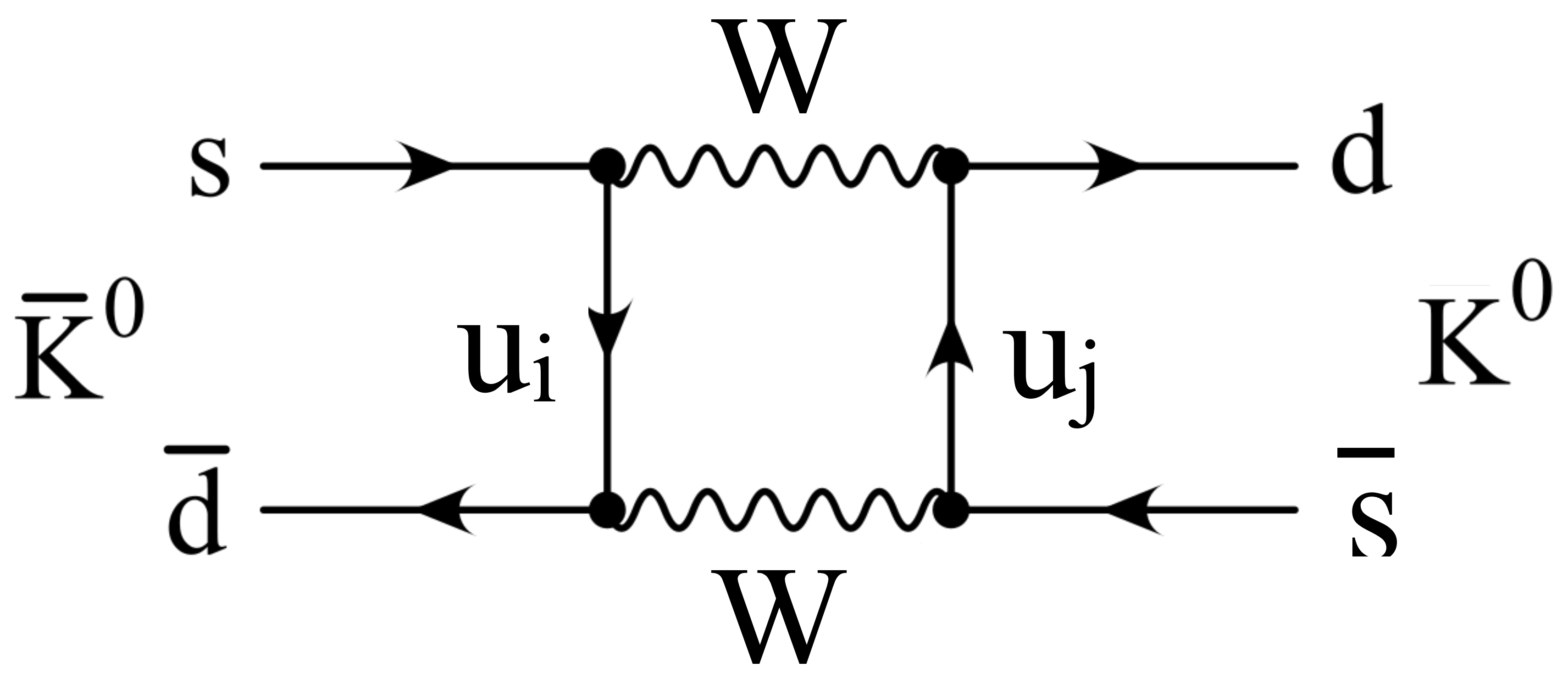
We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Milis theory is discussed.

- prior to discovery of the 4$^{\text{th}}$ quark and the W and Z bosons, this paper predicted the charm quark to explain the smallness of FCNC processes

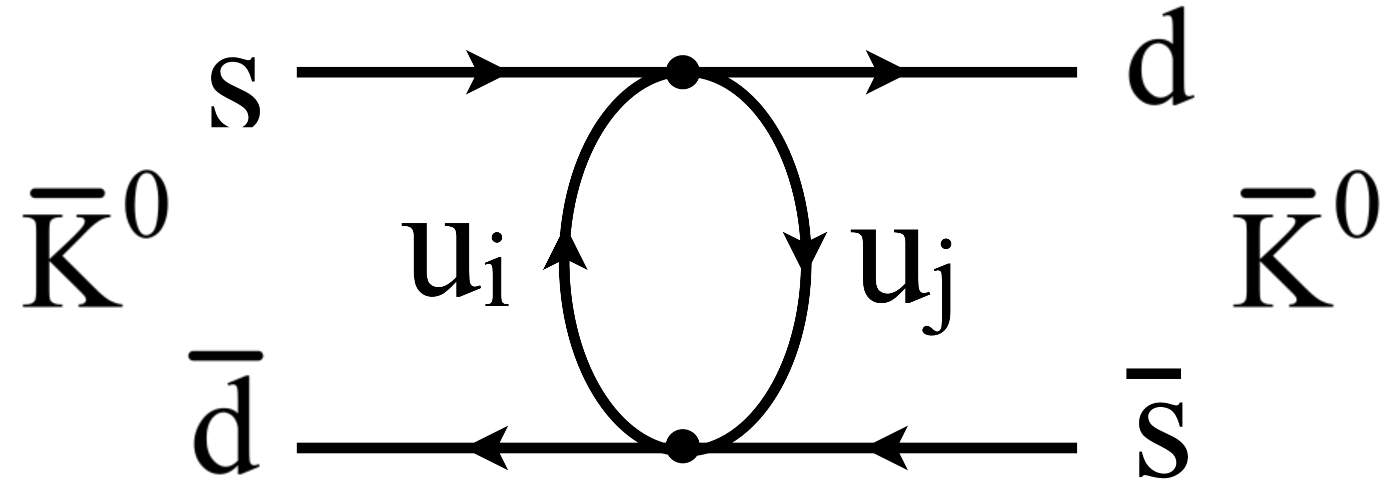
LOOP-INDUCED FCNC AND THE GIM MECHANISM

Example: $K-\bar{K}$ mixing ($\Delta F=2$)

- for $m_W \to \infty$ (Fermi theory), the mixing amplitudes in the 3-quark theory (1970) are quadratically divergent
- postulating a 4th quark, GIM found that the unitarity of the 2x2 Cabibbo matrix renders the amplitude finite and strongly suppressed: loop x CKM x (m_c^2/m_W^2)



$\Downarrow$ $m_W \to \infty$



$$\sim \frac{G_F^2}{16\pi^2}\,\sin^2\theta_C\cos^2\theta_C\,m_c^2$$

LOOP-INDUCED FCNC AND THE GIM MECHANISM

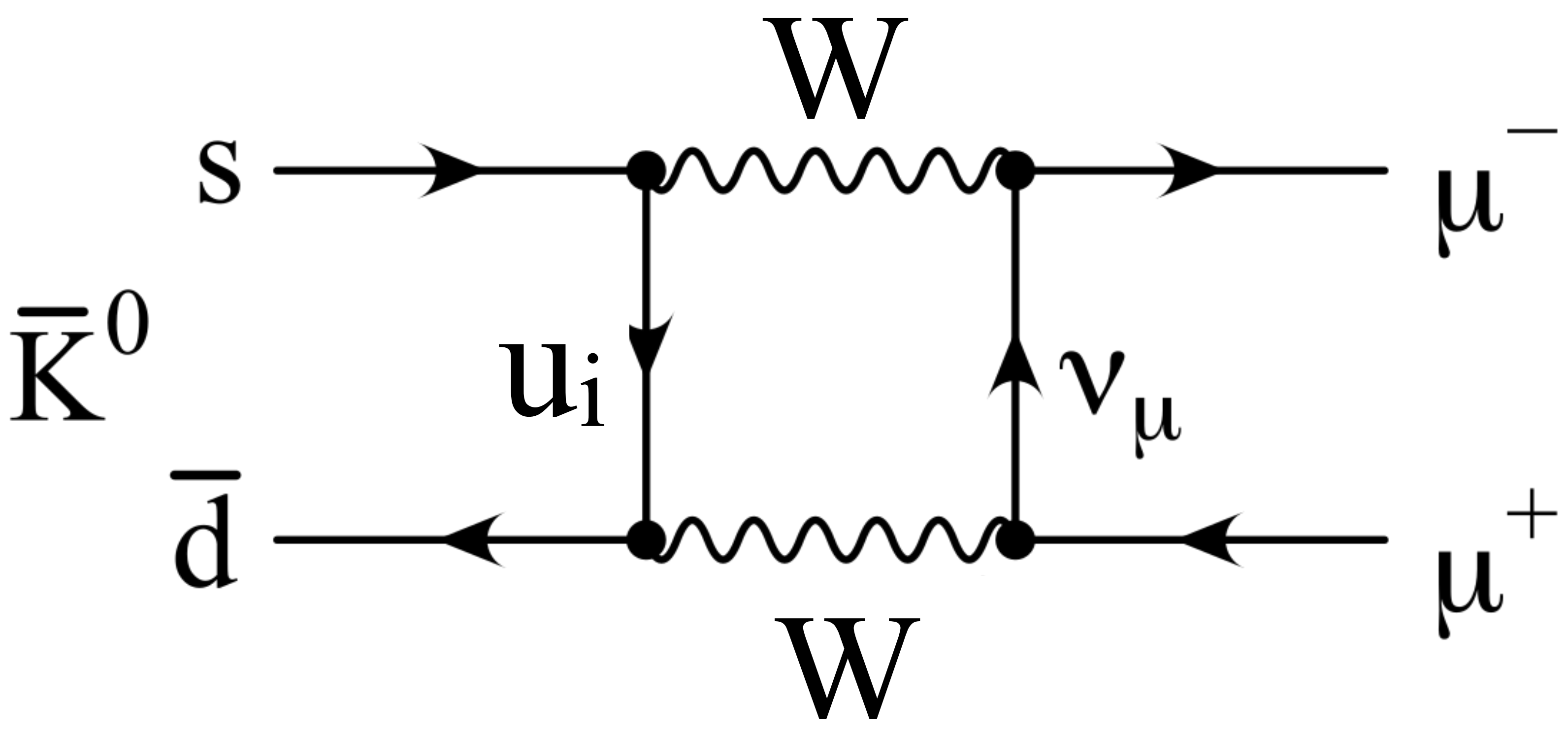
Example: structure of the $\bar{K}^0 \to \mu^-\mu^+$ decay amplitude in the SM (with $x_{u_i} = m_{u_i}^2/m_W^2$):

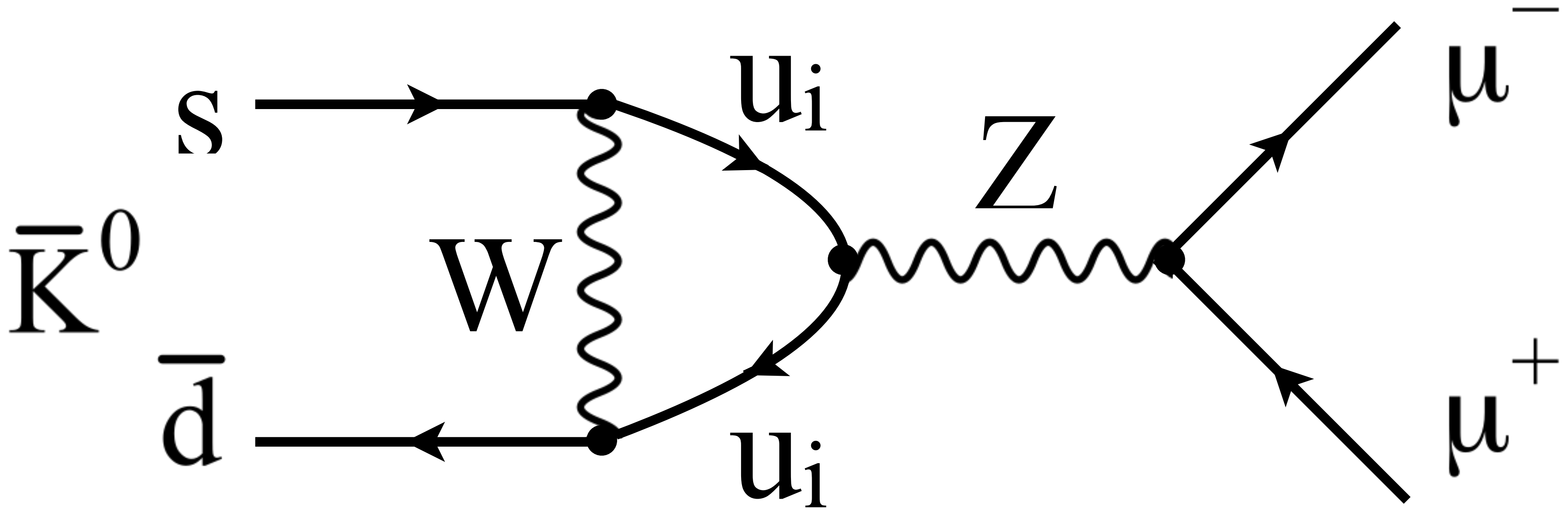
$$\mathcal{A} \sim \frac{g_L^4}{16\pi^2} \sum_i V_{u_i s} V_{u_i d}^* \, f(x_{u_i})$$

$$V_{us}V_{ud}^* + V_{cs}V_{cd}^* + V_{ts}V_{td}^* = 0$$

$$= \frac{g_L^4}{16\pi^2} \Big\{ V_{cs}V_{cd}^* \left[f(x_c) - \cancelto{0}{f(x_u)} \right] + V_{ts}V_{td}^* \left[f(x_t) - \cancelto{0}{f(x_u)} \right] \Big\}$$

$$\simeq \frac{g_L^4}{16\pi^2} \Big\{ \underbrace{\underbrace{V_{cs}V_{cd}^*}_{\approx 0.22} f(x_c)}_{\approx 2 \cdot 10^{-5}} + \underbrace{\underbrace{V_{ts}V_{td}^*}_{\approx 3.5 \cdot 10^{-4}} f(x_t)}_{\approx 3.3 \cdot 10^{-4}} \Big\}$$

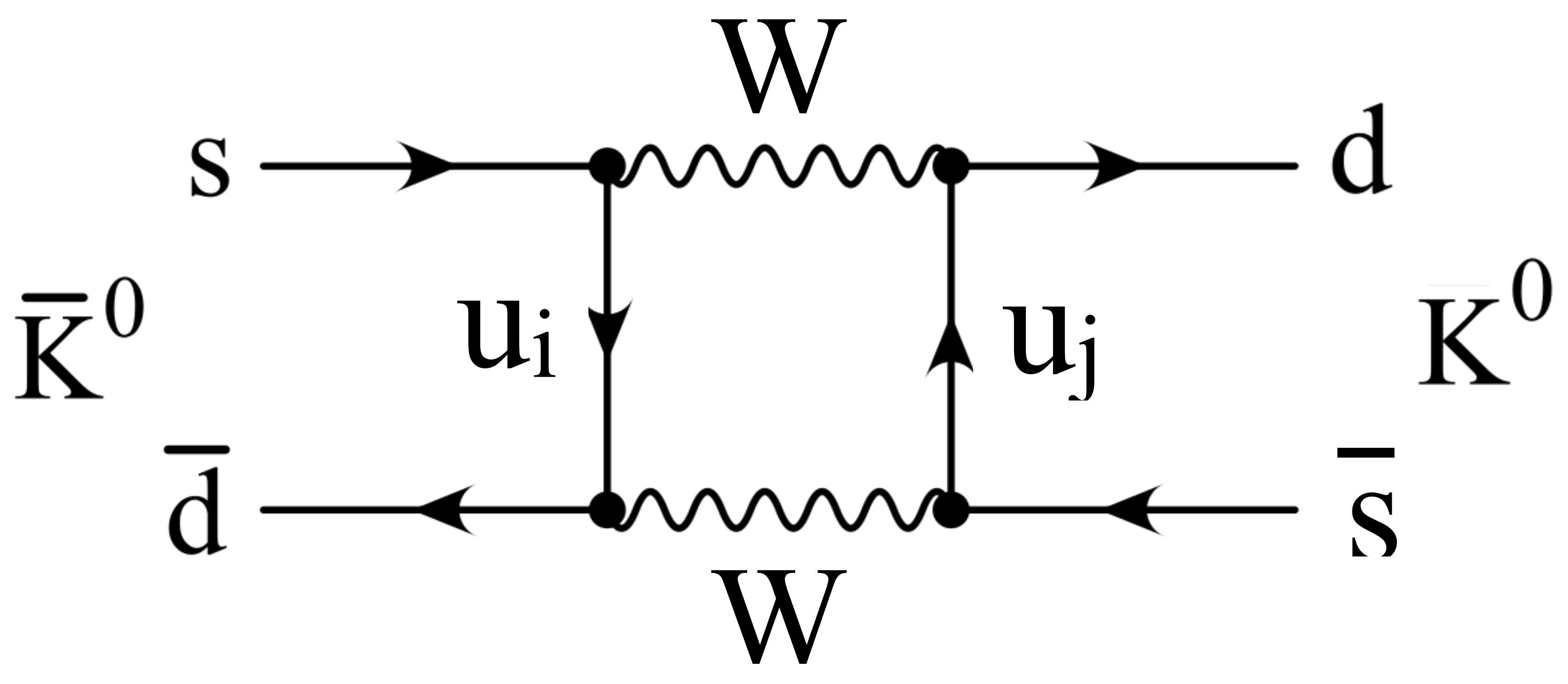




$$f(x) = \frac{x}{8} \left[\frac{4-x}{1-x} + \frac{3x}{(1-x)^2} \ln x \right]$$

LOOP-INDUCED FCNC AND THE GIM MECHANISM

Example: $K - \bar{K}$ mixing in the SM



$$\mathcal{A} \sim \frac{g_L^4}{16\pi^2} \sum_{i,j} V_{u_i s} V^*_{u_i d} V_{u_j s} V^*_{u_j d}\, f(x_{u_i}, x_{u_j})$$

$V_{us}V^*_{ud} + V_{cs}V^*_{cd} + V_{ts}V^*_{td} = 0$

$$= \frac{g_L^4}{16\pi^2} \Big[\underbrace{(V_{cs}V^*_{cd})^2}_{\approx 9.0 \cdot 10^{-6}} S_0(x_c) + \underbrace{(V_{ts}V^*_{td})^2}_{\approx 2.8 \cdot 10^{-7}} S_0(x_t)$$

$$+2 \underbrace{(V_{cs}V^*_{cd})(V_{ts}V^*_{td})}_{\approx 2.6 \cdot 10^{-7}} S_0(x_c, x_t) \Big]$$

$$S_0(x_c) \doteq x_c$$

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \ln x_t}{2(1-x_t)^3}$$

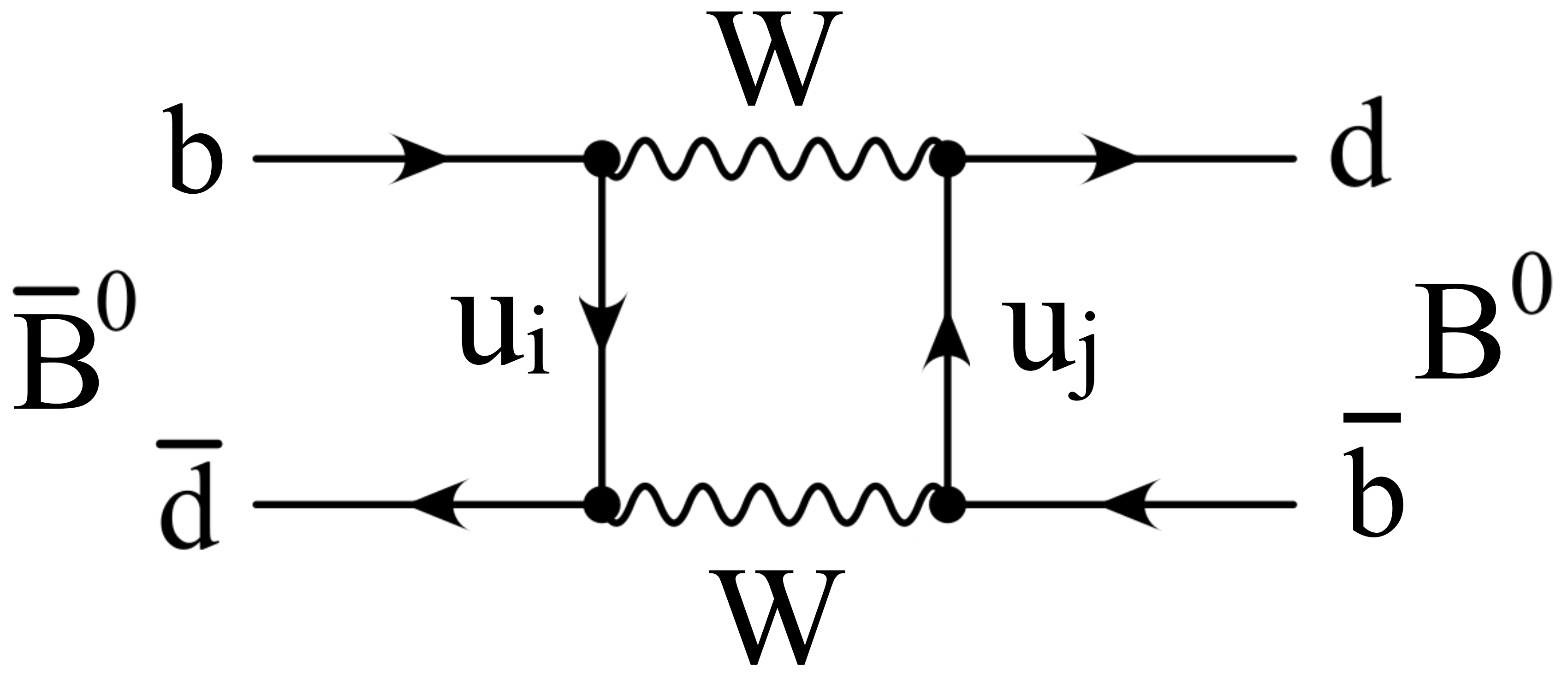
$$S_0(x_c, x_t) = x_c \left[\ln \frac{x_t}{x_c} - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2 \ln x_t}{4(1-x_t)^2} \right]$$

$$\approx \frac{g_L^4}{16\pi^2} (V_{cs}V^*_{cd})^2 \frac{m_c^2}{m_W^2}$$

→ third generation can be neglected to good approximation

LOOP-INDUCED FCNC AND THE GIM MECHANISM

Example: $B-\bar{B}$ mixing in the SM



$$\mathcal{A} \sim \frac{g_L^4}{16\pi^2} \sum_{i,j} V_{u_i b} V_{u_i d}^* V_{u_j b} V_{u_j d}^* \, f(x_{u_i}, x_{u_j})$$

$V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^* = 0$

$$= \frac{g_L^4}{16\pi^2} \Big[\underbrace{(V_{cb}V_{cd}^*)^2}_{\approx 1.7\cdot 10^{-8}} S_0(x_c) + \underbrace{(V_{tb}V_{td}^*)^2}_{\approx 1.7\cdot 10^{-4}} S_0(x_t)$$

$$+2 \underbrace{(V_{cb}V_{cd}^*)(V_{tb}V_{td}^*)}_{\approx 2.8\cdot 10^{-7}} S_0(x_c, x_t) \Big]$$

$$\approx \frac{g_L^4}{16\pi^2} (V_{tb}V_{td}^*)^2 S_0(x_t)$$ → top-quark contributions dominant!

$$S_0(x_c) \doteq x_c$$

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \ln x_t}{2(1-x_t)^3}$$

$$S_0(x_c, x_t) = x_c \left[\ln\frac{x_t}{x_c} - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2 \ln x_t}{4(1-x_t)^2} \right]$$

HINT FOR A HEAVY TOP QUARK

Volume 192, number 1,2 PHYSICS LETTERS B 25 June 1987

OBSERVATION OF B^0–$\bar{B}^0$ MIXING

ARGUS Collaboration

H. ALBRECHT, A.A. ANDAM [1], U. BINDER, P. BÖCKMANN, R. GLÄSER, G. HARDER, A. NIPPE, M. SCHÄFER, W. SCHMIDT-PARZEFALL, H. SCHRÖDER, H.D. SCHULZ, R. WURTH, A. YAGIL [2,3]
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[…]

Received 9 April 1987

Using the ARGUS detector at the DORIS II storage ring we have searched in three different ways for B^0–$\bar{B}^0$ mixing in Υ (4S) decays. One explicitly mixed event, a decay $\Upsilon\,(4S) \to B^0 B^0$, has been completely reconstructed. Furthermore, we observe a 4.0 standard deviation signal of 24.8 events with like-sign lepton pairs and a 3.0 standard deviation signal of 4.1 events containing one reconstructed $B^0(\bar{B}^0)$ and an additional fast $\ell^+(\ell^-)$. This leads to the conclusion that B^0–$\bar{B}^0$ mixing is substantial. For the mixing parameter we obtain $r = 0.21 \pm 0.08$.

$$B_1^0 \to D_1^{*-} \mu_1^+ \nu_1 \qquad B_2^0 \to D_2^{*-} \mu_2^+ \nu_2$$

$$D_1^{*-} \to \pi_1^- \bar{D}^0 \qquad D_2^{*-} \to \pi^0 D^-$$

$$\bar{D}^0 \to K_1^+ \pi_1^- \qquad D^- \to K_2^+ \pi_2^- \pi_2^-$$

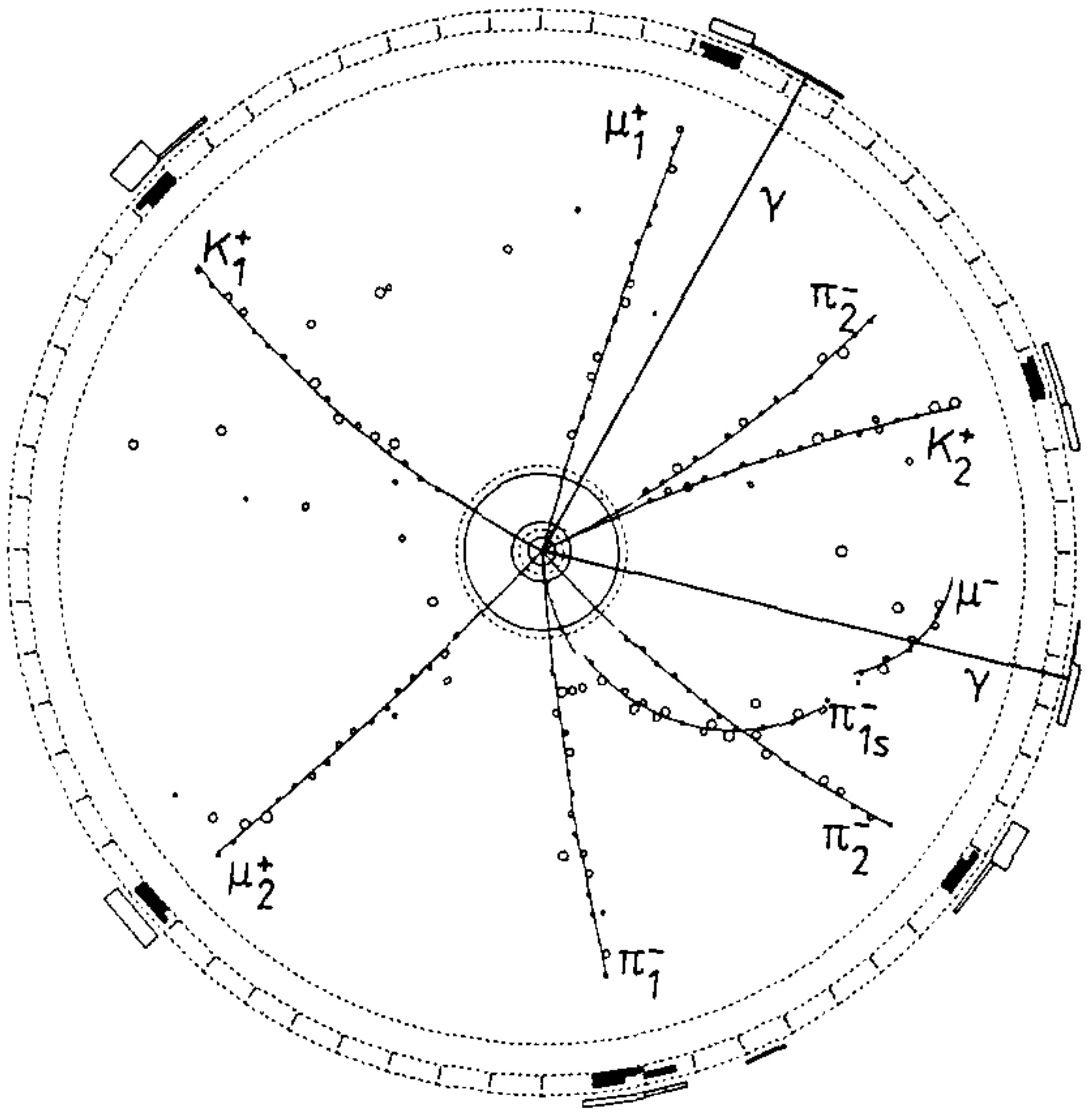

Fig. 2. Completely reconstructed event consisting of the decay $\Upsilon\,(4S) \to B^0 B^0$.

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- quote from the preprint:

In the framework of the Standard Model, with three families of quarks [15], the mixing is expected to be dominated by the contribution of the t quark to the second order weak box diagram [16]. The $B^0 - \overline{B^0}$ is probably governed by

$$x = \frac{\Delta M}{\Gamma} = 32\pi \frac{B f_B^2 m_t^2 m_b}{m_\mu^5} \frac{\tau_b}{\tau_\mu} \mid V_{td} \mid^2 \eta_{QCD}$$

related to experiment by

$$r = \frac{x^2}{x^2+2}$$

and for which we obtain the value $x = 0.73 \pm 0.28$.

Prior to this measurement, predicted values of x for this process were very much smaller -- for example $x = 0.12$ for $m_t = 60 GeV/c^2$ [17]. Already, however, a steady flow of preprints is appearing which show that our new experimental result can readily be incorporated into the Standard Model without pushing the other parameters into regions of any great controversy (for example [18]). The abundant flexibility of the Standard Model is thus demonstrated once again.

$$\Rightarrow m_t \approx 150\,\text{GeV} \quad (!)$$

(much larger than assumed at the time)

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- quote from the publication:

We discuss our result in the framework of the standard model with three generations. Assuming dominance of the box diagram, mixing is described by the parameter x [1]:

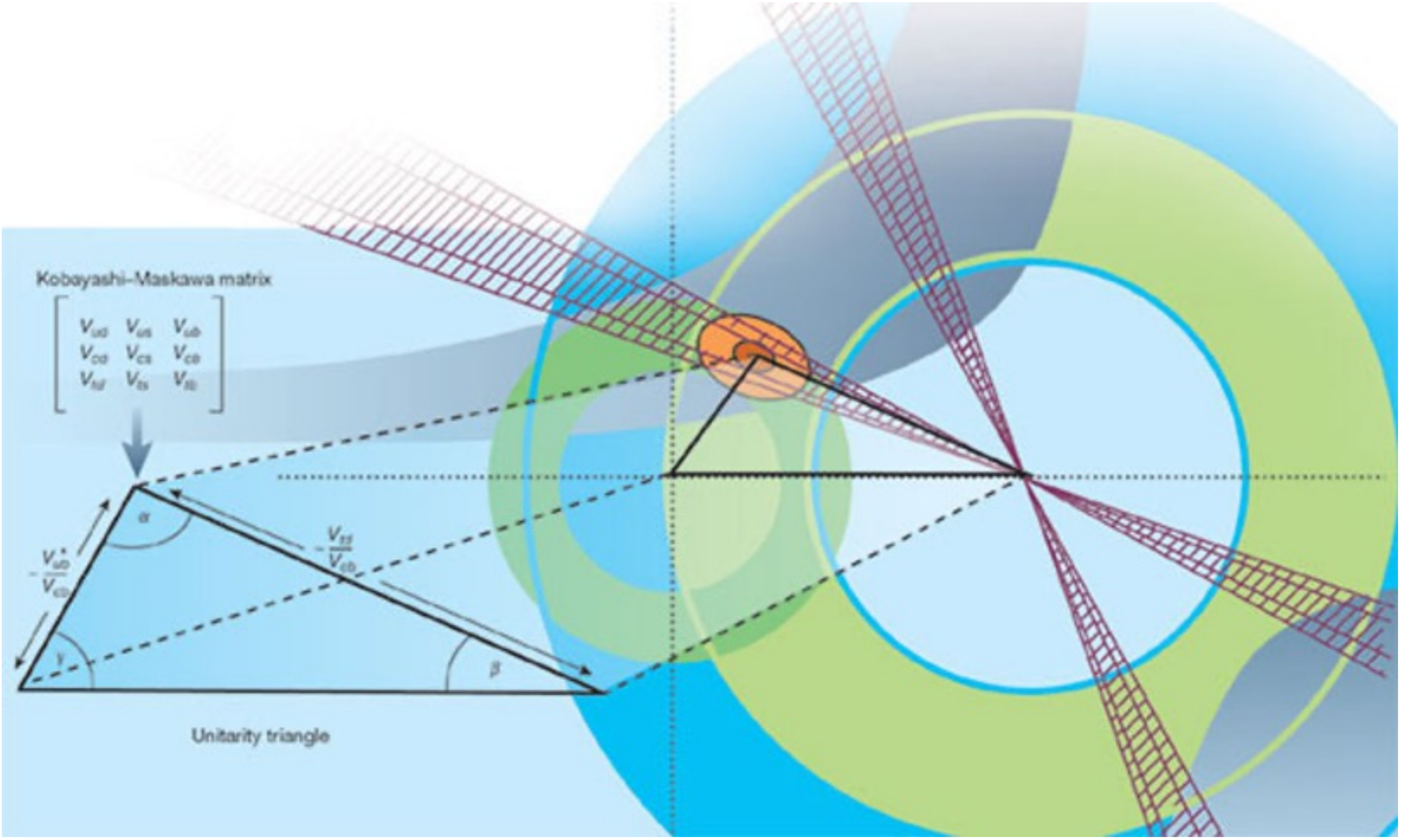
$$x = \frac{\Delta M}{\Gamma} = 32\pi \, \frac{B f_B^2 m_t^2 m_b}{m_\mu^5} \, \frac{\tau_b}{\tau_\mu} \, |V_{td}|^2 \eta_{QCD} ,$$

and related to experiment by

$$r = \frac{x^2}{x^2 + 2} .$$

The rate of B^0–$\bar{B}^0$ mixing provides a strong constraint on parameters of the standard model. Specifically, our result shows that the Kobayashi–Maskawa element V_{td} is non-zero. The observed value of r can still be accommodated by the standard model within the present knowledge of its parameters. As an illustration, one example of a set of limits is given in table 3.

(no mentioning of the top-quark mass ...)



II. PHENOMENOLOGY OF WEAK DECAYS AND MESON OSCILLATIONS

"THE" UNITARITY TRIANGLE

Unitarity relation:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

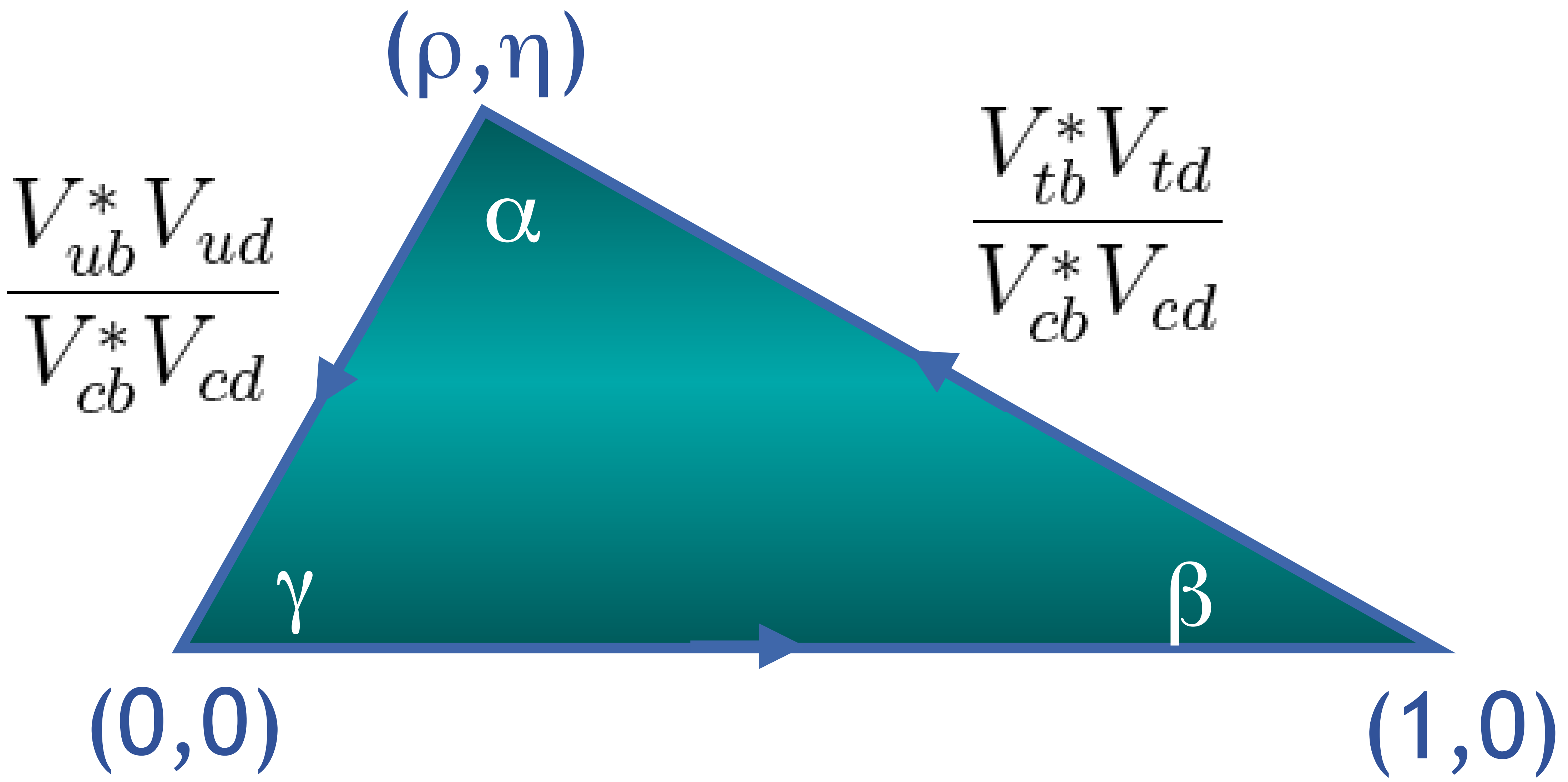
- rewrite this as:

$$1 + \frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} + \frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} = 0$$

invariant under phase redefinitions

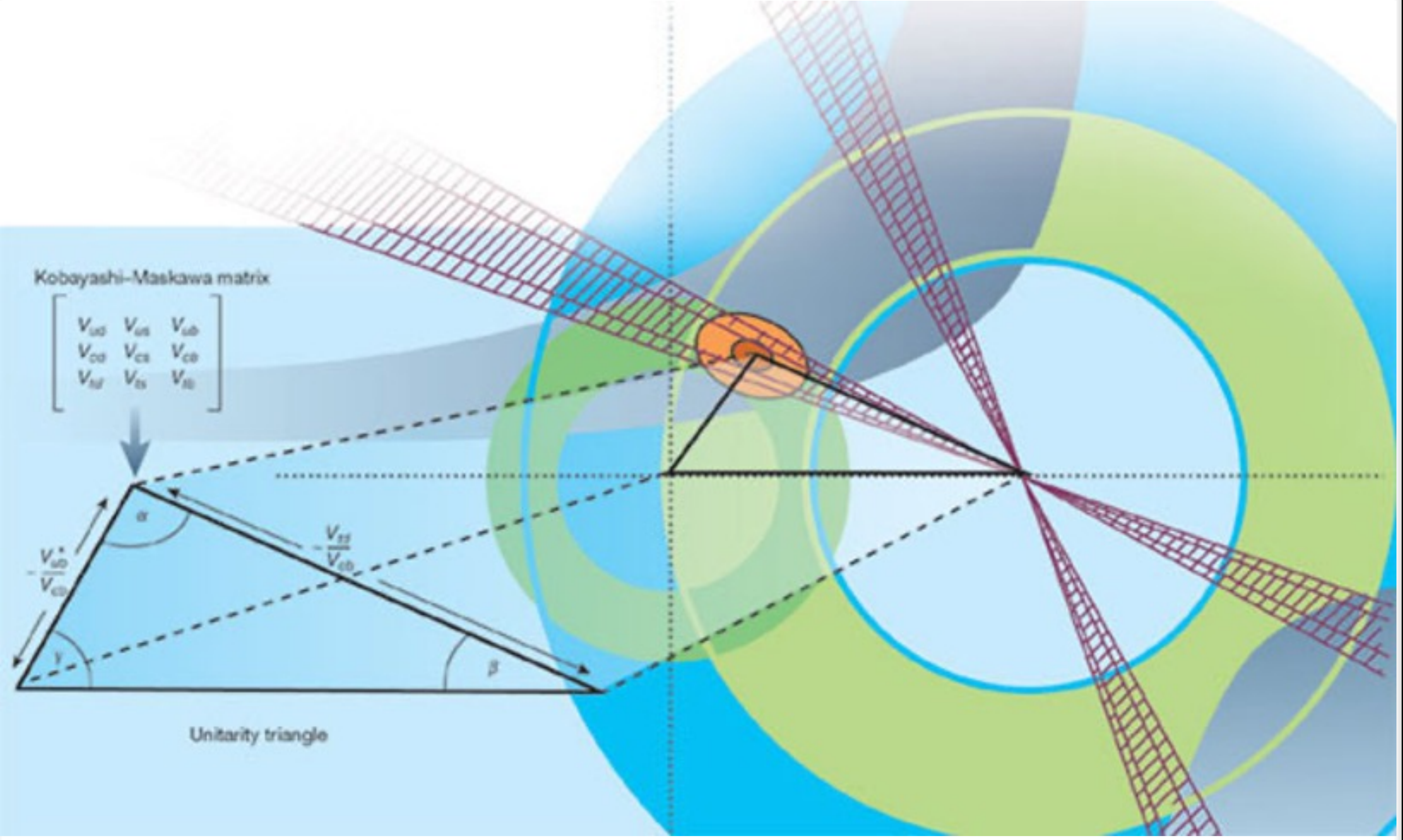
- Wolfenstein parameterization:

$$1 - (1 - \rho - i\eta) - (\rho + i\eta) = 0$$



"THE" UNITARITY TRIANGLE

- unitarity triangle as a target for precision studies in quark flavor sector
- determinations of sides & angles
- plethora of possibilities for powerful new-physics searches
- pivotal for establishing the notion of an "intensity frontier" in high-energy physics, complementing – and often surpassing – searches at the energy frontier

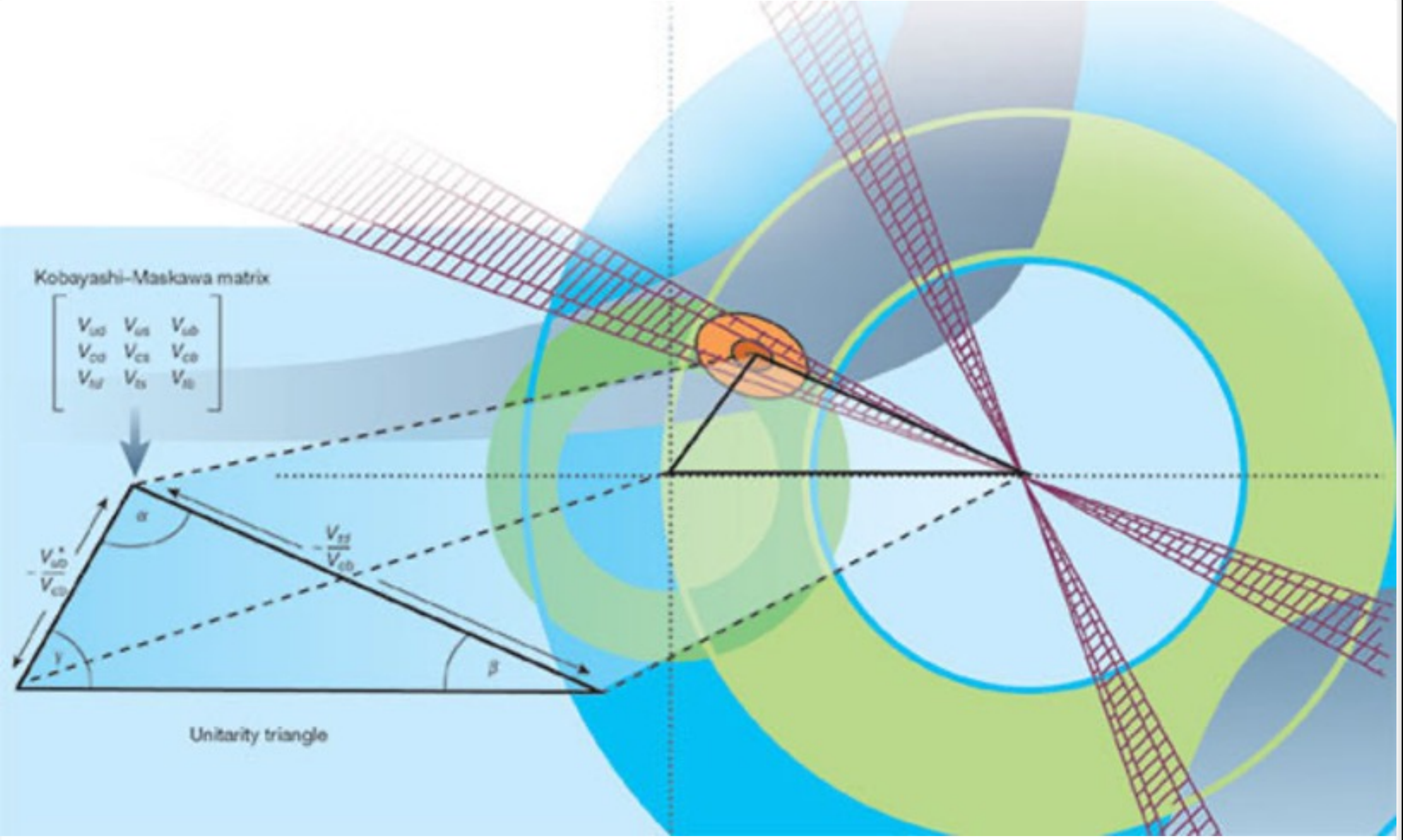


$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3+\sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3+\cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\cos\theta_3 e^{i\delta} \end{pmatrix}. \tag{13}$$

Then, we have CP-violating effects through the interference among these different current components.

PUSHING THE LUMINOSITY FRONTIER

- tremendous experimental advances:
  - 1. gen.: ARGUS & CLEO, LEP expts.
  - 2. gen.: BaBar & Belle, LHCb, CMS, …
  - 3. gen.: Belle II, LHCb upgrade, …
- precise measurement of CKM elements $|V_{cb}|, |V_{ub}|, |V_{td}|, |V_{ts}|$ involving third-generation quarks
- precise determinations of angles (CPV)
- New Physics searches using FCNC processes



$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3 + \sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3 + \cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3 - \cos\theta_2\cos\theta_3 e^{i\delta} \end{pmatrix}. \tag{13}$$

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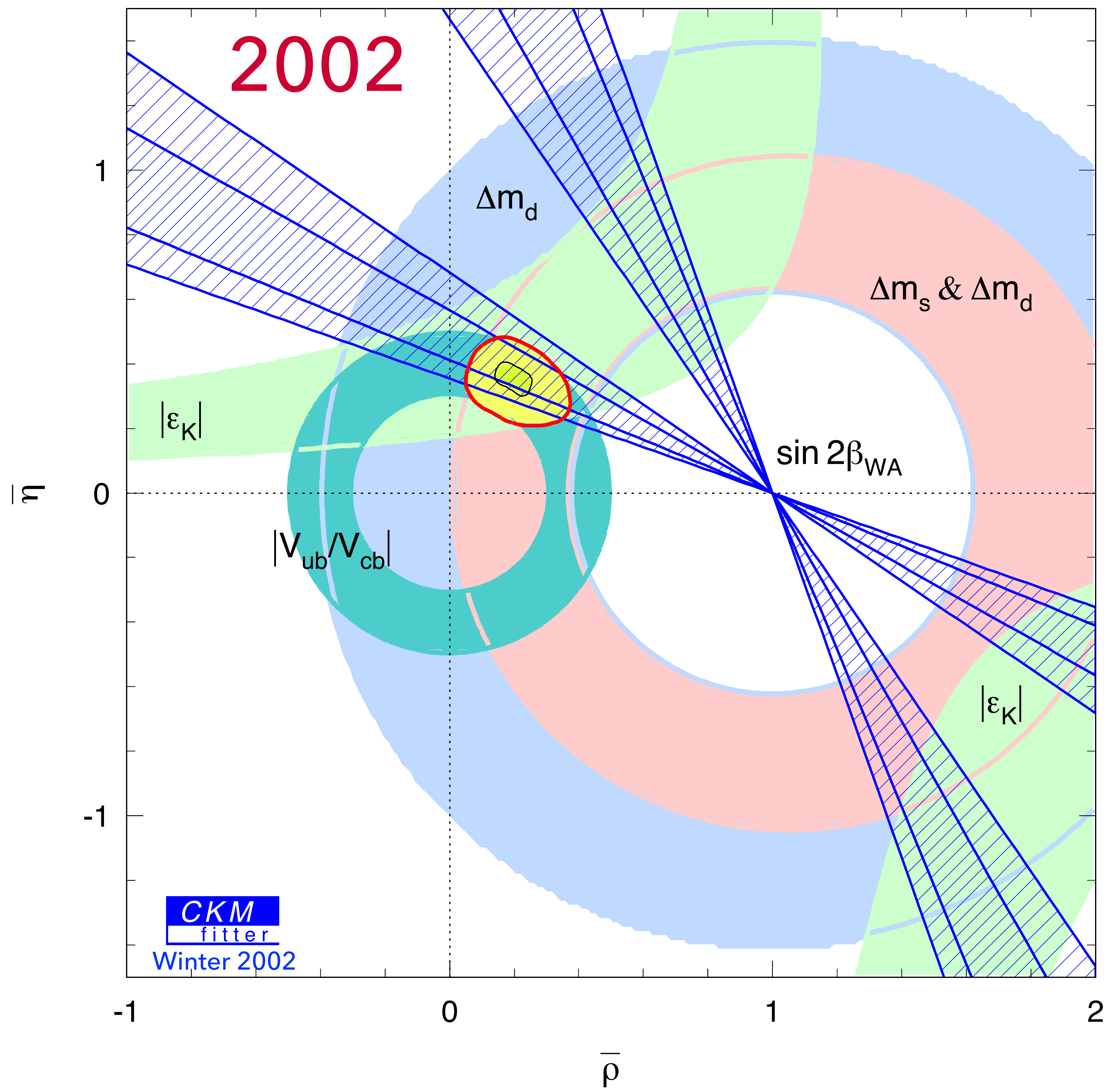
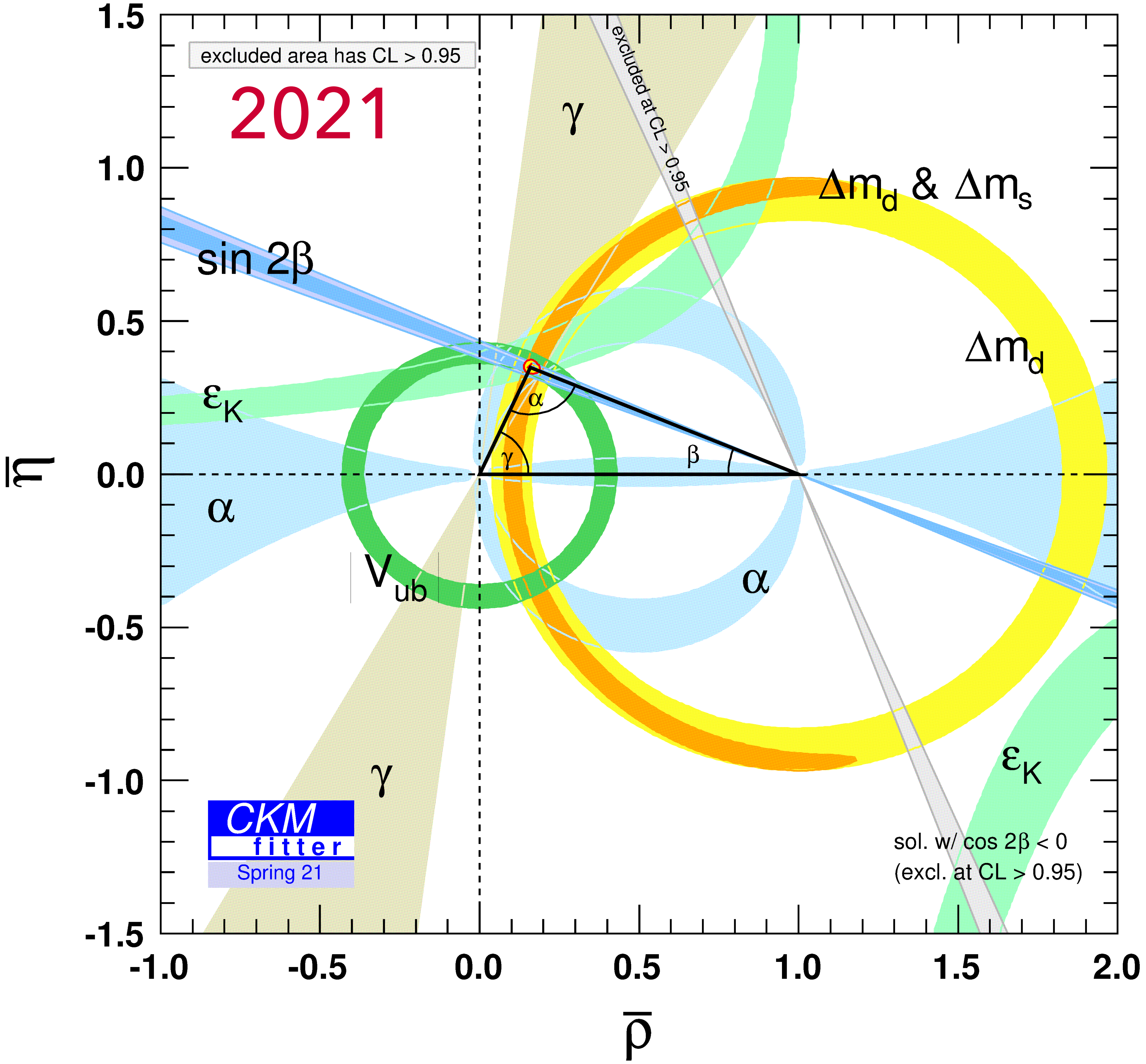
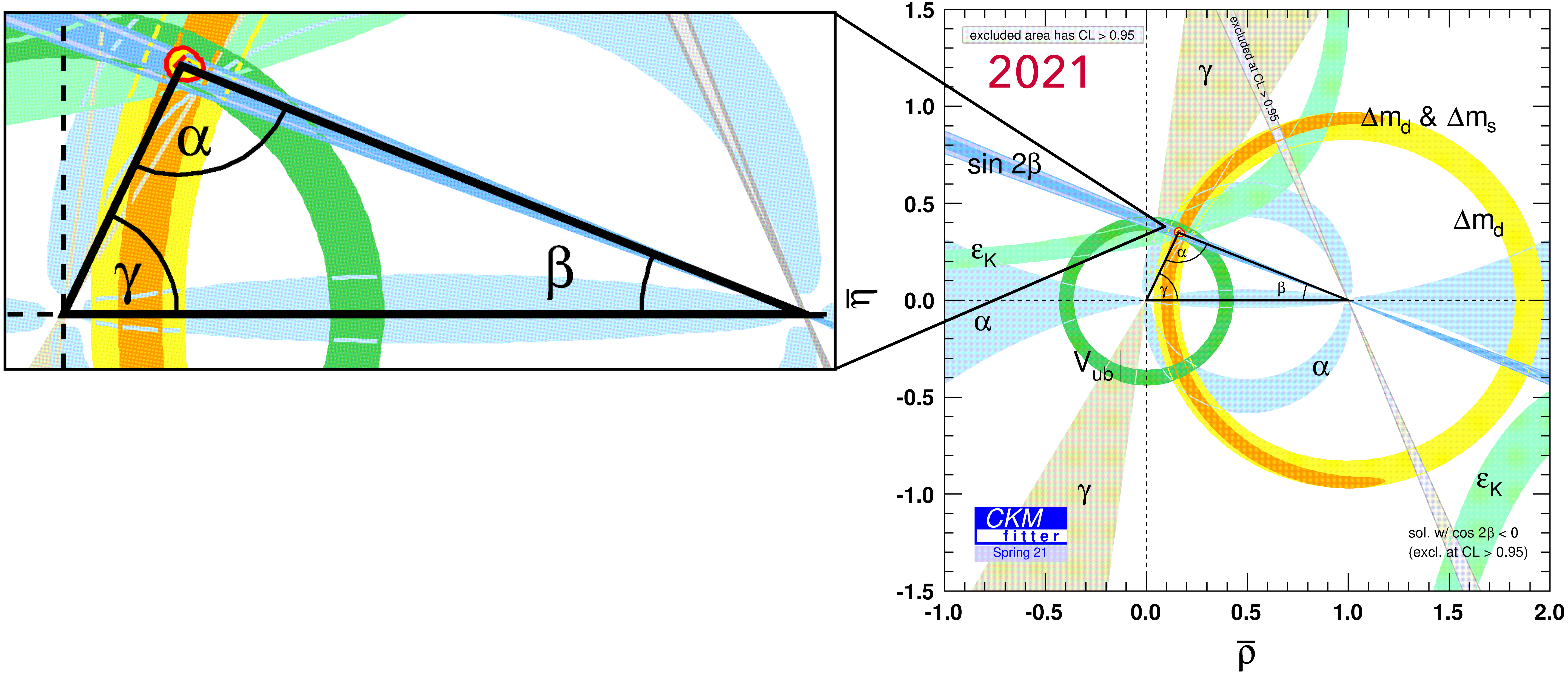


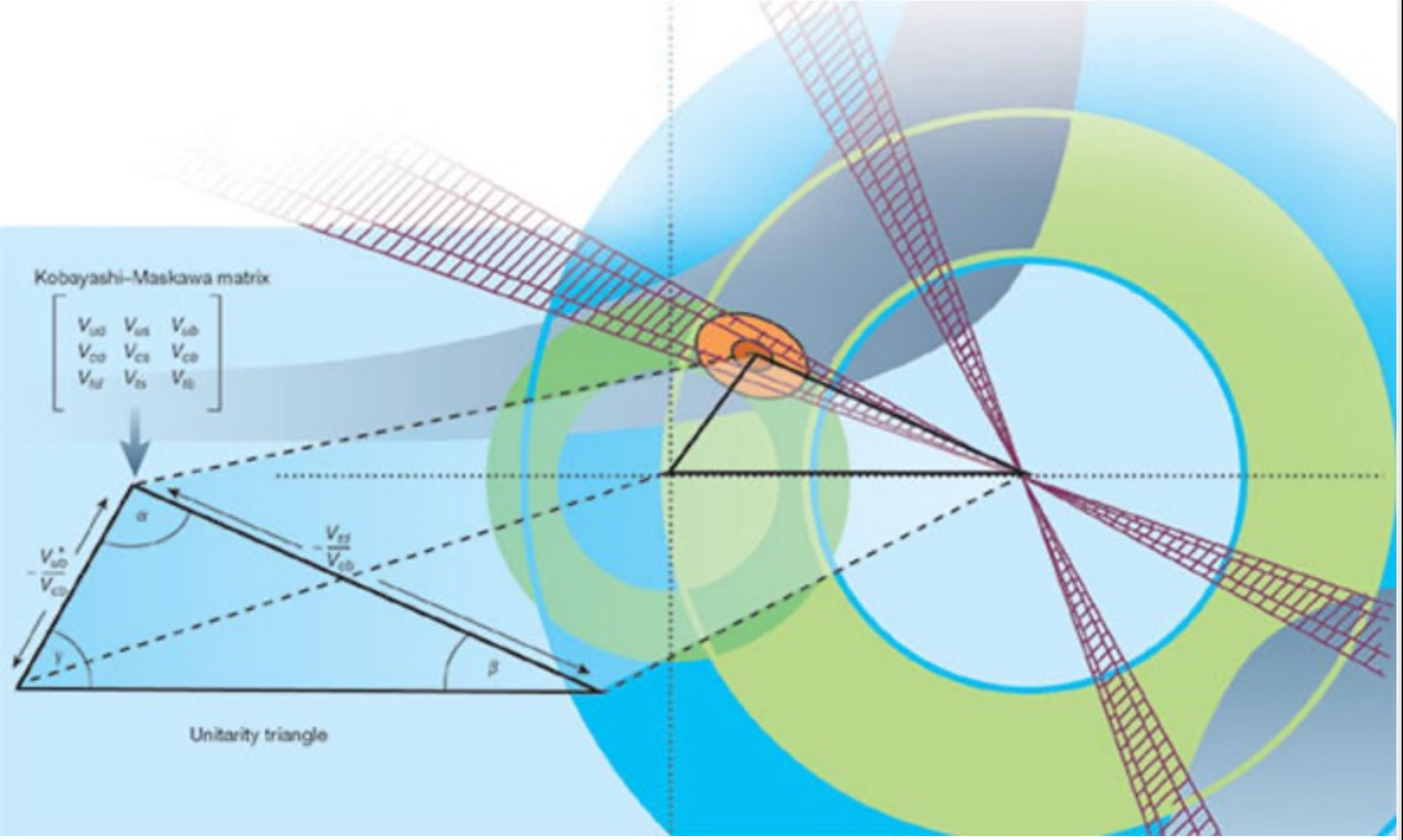


PUSHING THE LUMINOSITY FRONTIER



PUSHING THE LUMINOSITY FRONTIER

- matching the incredible precision of the *B*-factories required a revolution in theory
- concerted effort of theory community was an important indirect consequence of the *B*-factory program
- breakthrough came from using effective field theories (EFTs):
  - $\mathscr{H}_{\rm eff}^{\rm weak}$, HQET, QCDF, SCET



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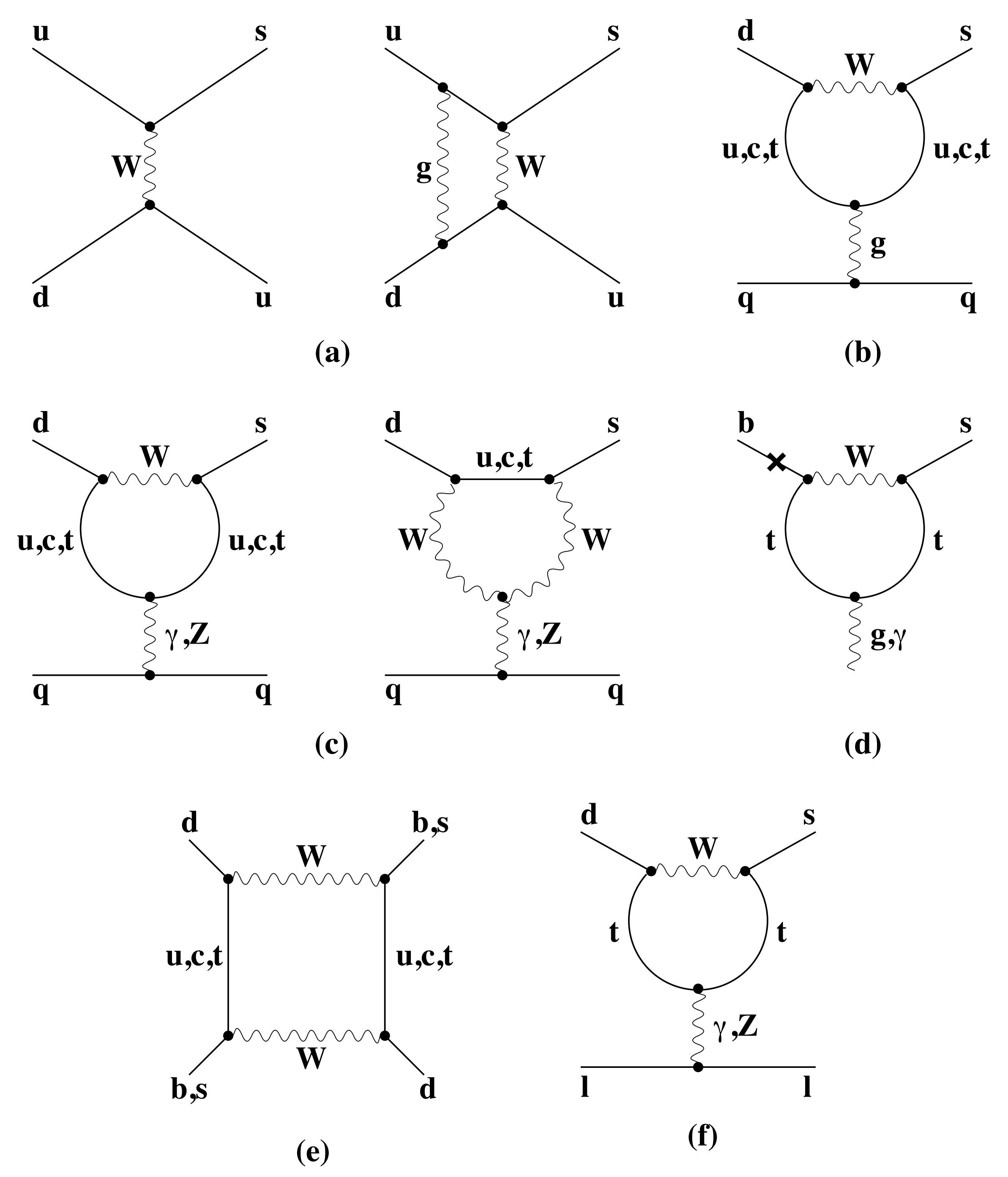
Then, we have CP-violating effects through the interference among these different current components.

EFFECTIVE WEAK HAMILTONIAN

- systematic method to separate short-distance effects (weak scale and beyond) from long-distance hadronic dynamics
- **BUT**: the challenge is to evaluate the hadronic matrix elements of the quark-gluon operators $Q_i(\mu)$ in all but the simplest cases
- powerful theoretical tools exist (lattice QCD, EFTs, dispersive methods …)



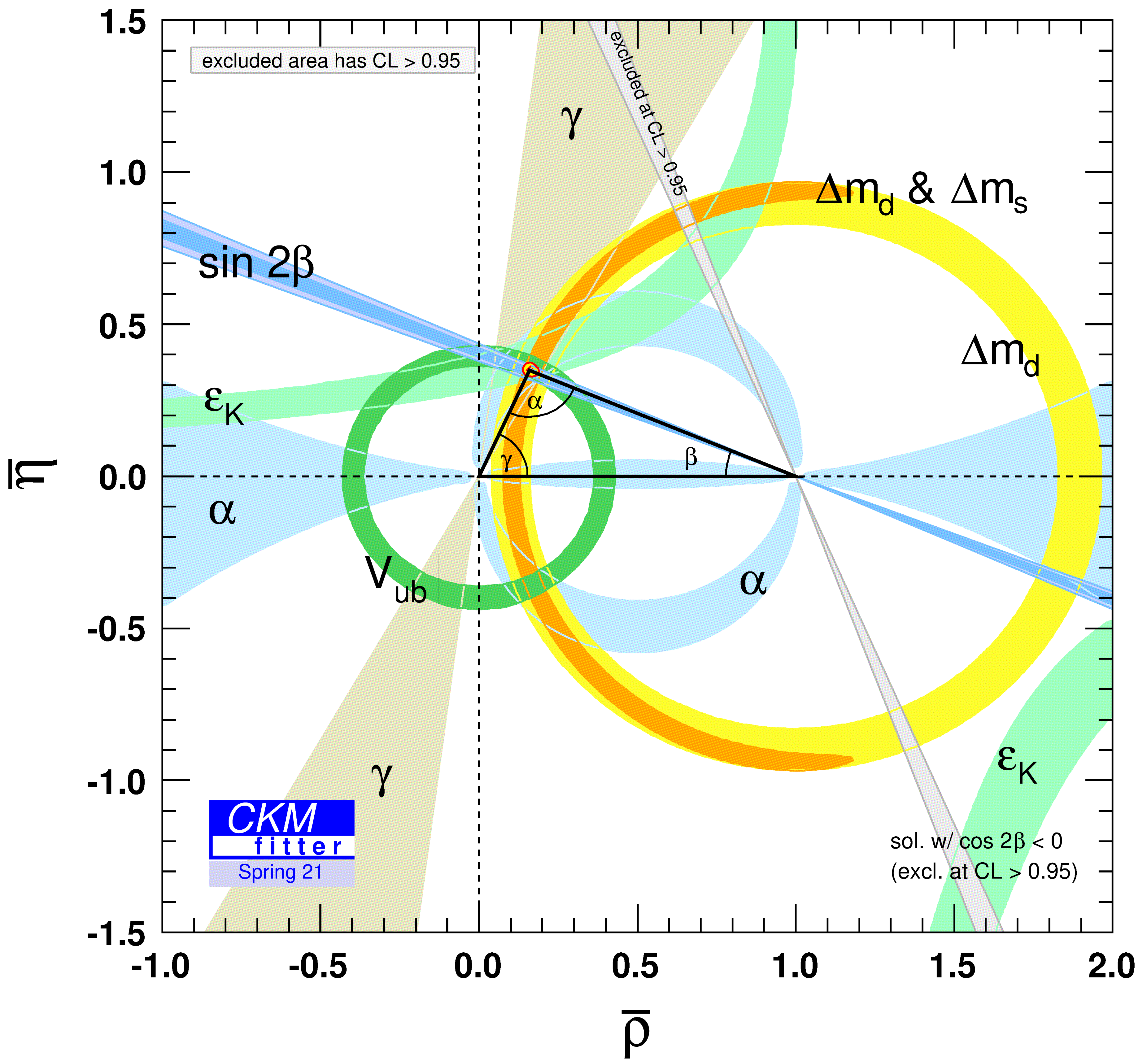
$$
\begin{aligned}
Q_1^q &= (\bar{b}_i q_j)_{\mathrm{V-A}}\,(\bar{q}_j d_i)_{\mathrm{V-A}} \\
Q_2^q &= (\bar{b} q)_{\mathrm{V-A}}\,(\bar{q} d)_{\mathrm{V-A}} \\
Q_3 &= (\bar{b} d)_{\mathrm{V-A}} \sum_q (\bar{q} q)_{\mathrm{V-A}} \\
Q_4 &= (\bar{b}_i d_j)_{\mathrm{V-A}} \sum_q (\bar{q}_j q_i)_{\mathrm{V-A}} \\
Q_5 &= (\bar{b} d)_{\mathrm{V-A}} \sum_q (\bar{q} q)_{\mathrm{V+A}} \\
Q_6 &= (\bar{b}_i d_j)_{\mathrm{V-A}} \sum_q (\bar{q}_j q_i)_{\mathrm{V+A}} \\
Q_7 &= \frac{3}{2}\,(\bar{s} d)_{\mathrm{V-A}} \sum_q e_q\,(\bar{q} q)_{\mathrm{V+A}} \\
Q_8 &= \frac{3}{2}\,(\bar{s}_i d_j)_{\mathrm{V-A}} \sum_q e_q\,(\bar{q}_j q_i)_{\mathrm{V+A}} \\
Q_9 &= \frac{3}{2}\,(\bar{s} d)_{\mathrm{V-A}} \sum_q e_q\,(\bar{q} q)_{\mathrm{V-A}} \\
Q_{10} &= \frac{3}{2}\,(\bar{s}_i d_j)_{\mathrm{V-A}} \sum_q e_q\,(\bar{q}_j q_i)_{\mathrm{V-A}}
\end{aligned}
$$

$$
\mathcal{H}_{\mathrm{eff}}(\Delta S = 1) = \frac{G_F}{\sqrt{2}} V_{\mathrm{us}}^* V_{\mathrm{ud}} \sum_{i=1}^{10} \left(z_i(\mu) + \tau\, y_i(\mu) \right) Q_i(\mu)
$$

$$
\tau = \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}}
$$

[Gilman, Wise (1979); Buras et al. (1990s)]

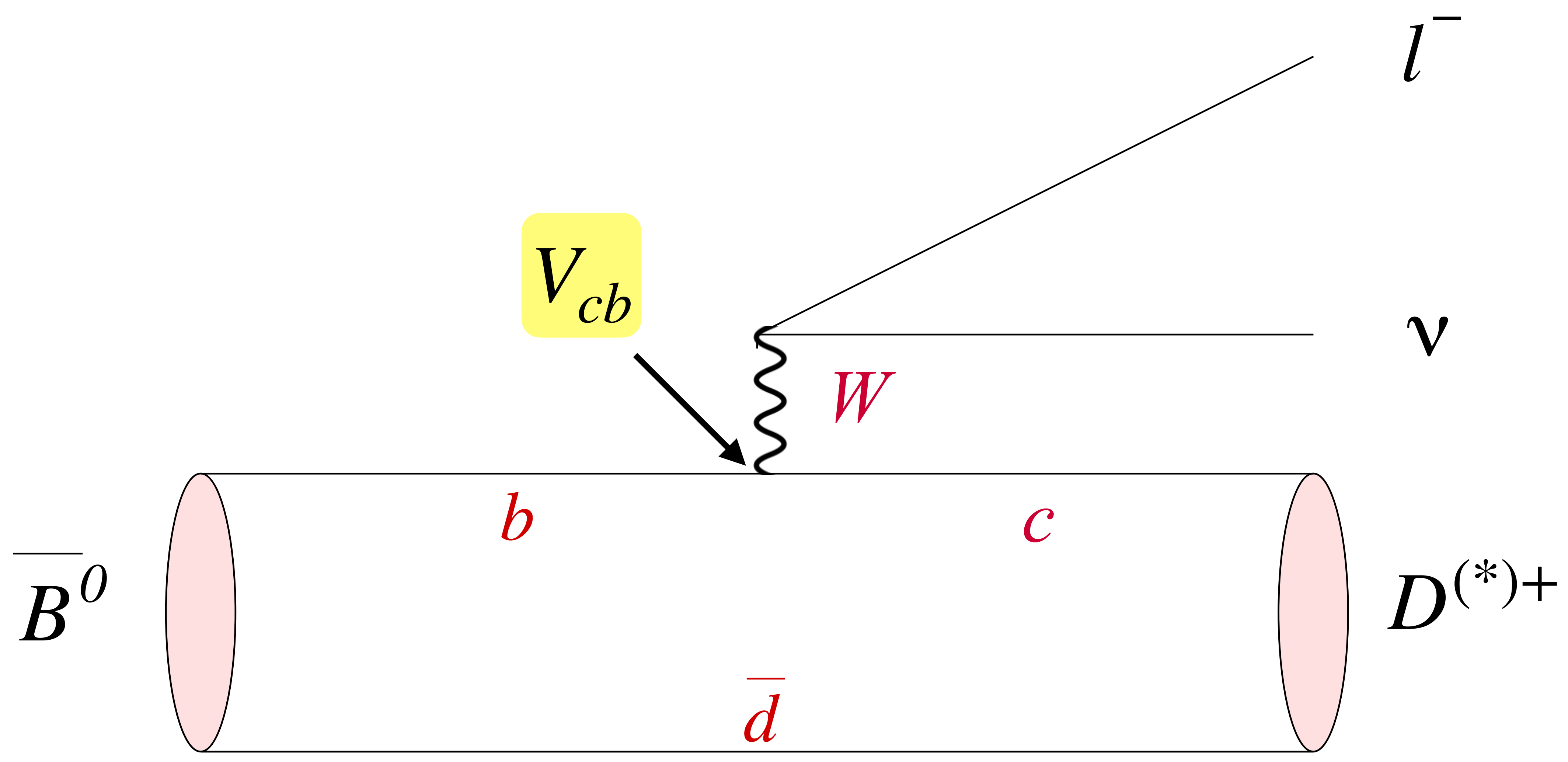
CONSTRAINTS ON THE UNITARITY TRIANGLE



CONSTRAINTS ON THE UNITARITY TRIANGLE

Determination of $|V_{cb}|$:

- needed to normalize the base of the unitarity triangle to 1
- beautiful application of heavy-quark symmetry and the heavy-quark expansion
- extraction of $|V_{cb}|$ from both exclusive decays $B \to D^{(*)}\ell\bar{\nu}_\ell$ and inclusive decay $B \to X_c\ell\bar{\nu}_\ell$
- relevant "flavor-flow" diagram:



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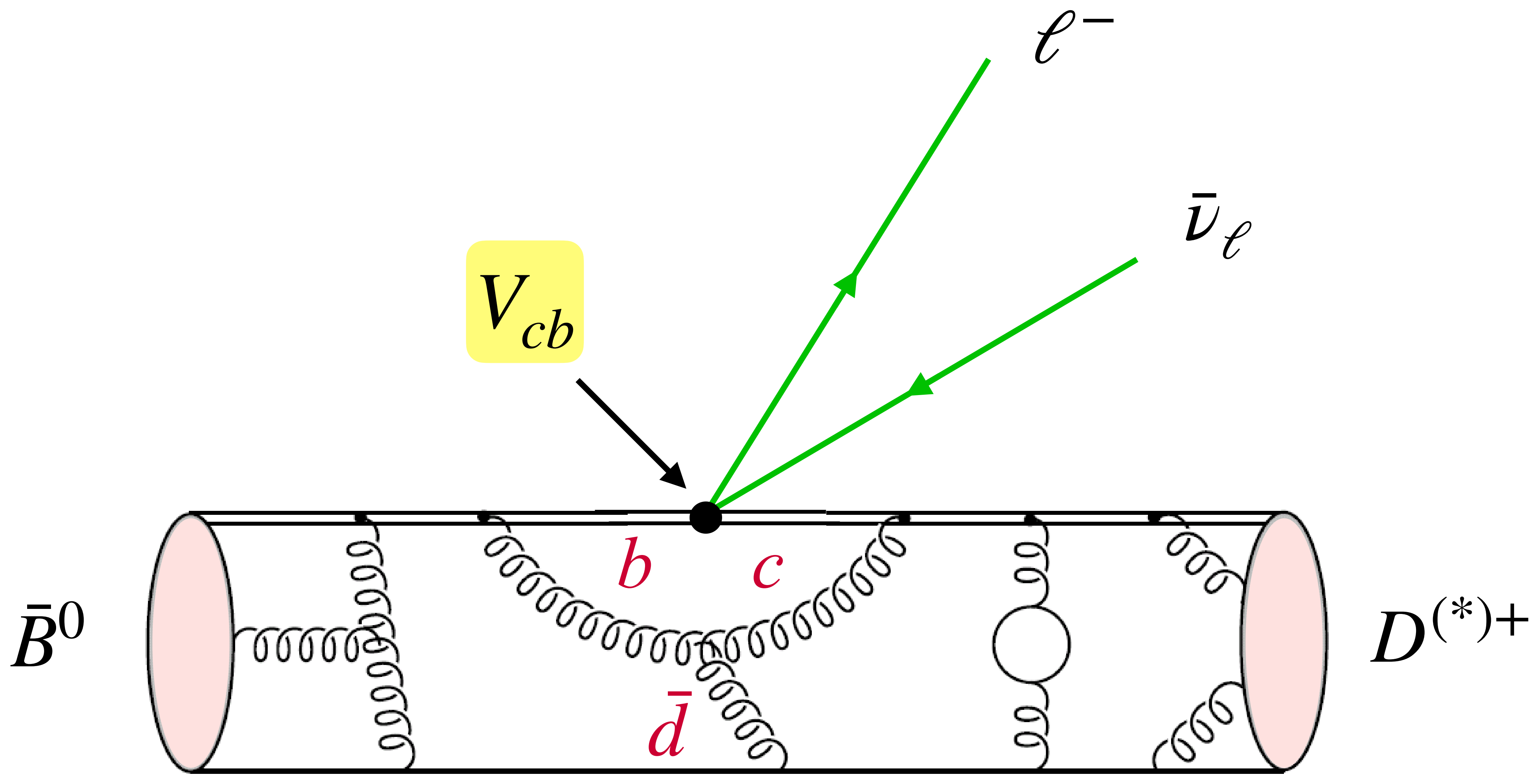
- more appropriate representation:



- nonperturbative hadronic form factor

HEAVY QUARK SYMMETRY

- hadronic bound states containing a heavy quark obey an approximate spin-flavor symmetry
- many predictions for spectroscopy of heavy hadrons [Shuryak (1980)]
- symmetry relations among $B \to D^{(*)}$ form factors, including symmetry-breaking corrections $\sim \alpha_s(m_Q)$ or $\Lambda_{\rm QCD}/m_Q$

[Isgur, Wise (1990)]

Relations between level spacings in bottom and charm systems, e.g.:

- $m_{B^*}^2 - m_B^2 \approx 0.49\,\text{GeV}^2$ vs. $m_{D^*}^2 - m_D^2 \approx 0.55\,\text{GeV}^2$
- $m_{B_s} - m_B \approx m_{D_s} - m_d \approx 0.10\,\text{GeV}$
- $m_{B_2^*}^2 - m_{B_1}^2 \approx m_{D_2^*}^2 - m_{D_1}^2 \approx 0.17\,\text{GeV}^2$

Form-factor relations:

$$\langle D(v')|V^\mu|B(v)\rangle = h_+(w)\,(v+v')^\mu + h_-(w)\,(v-v')^\mu$$

$$\langle D^*(v',\epsilon)|V^\mu|B(v)\rangle = i h_V(w)\,\epsilon^{\mu\nu\alpha\beta}\epsilon_\nu^* v'_\alpha v_\beta$$

$$\langle D^*(v',\epsilon)|A^\mu|B(v)\rangle = h_{A_1}(w)(w+1)\epsilon^{*\mu} - [h_{A_2}(w)v^\mu + h_{A_3}(w)v'^\mu]\,\epsilon^*\cdot v$$

with ($w = v\cdot v'$):

$$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w) \quad \text{and} \quad \xi(1) = 1$$

$$h_-(w) = h_{A_2}(w) = 0$$

MODEL-INDEPENDENT DETERMINATION OF $|V_{CB}|$

- extrapolate observed $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$ spectrum in $w = v \cdot v'$ to zero recoil ($w = 1$)
- account for α_s corrections using QCD perturbation theory
- leading $\Lambda_{\rm QCD}/m_Q$ corrections vanish at this point (Luke's theorem) [Luke (1990)]
- alternative methods: lattice QCD, dispersive constraints on form factors

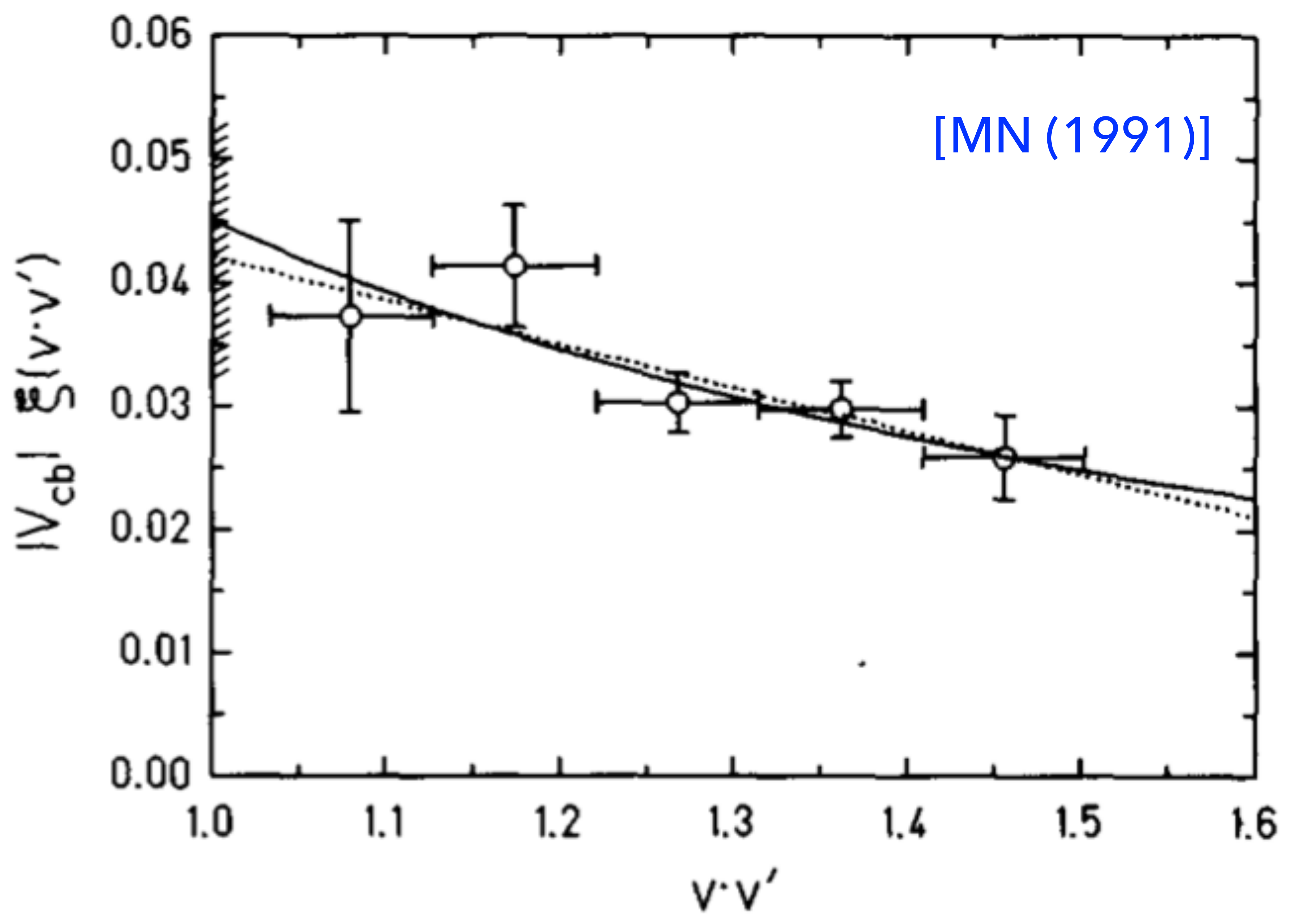
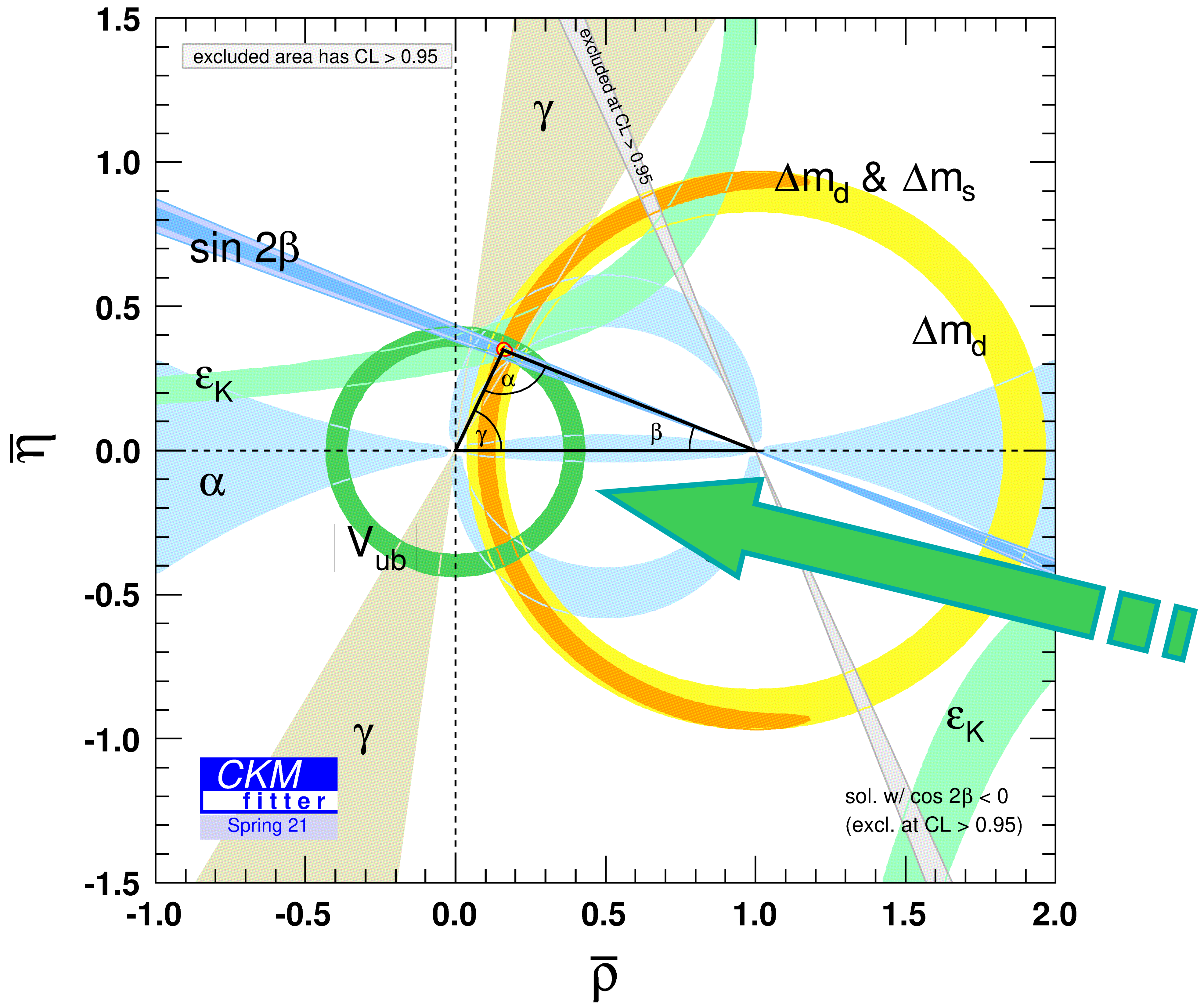


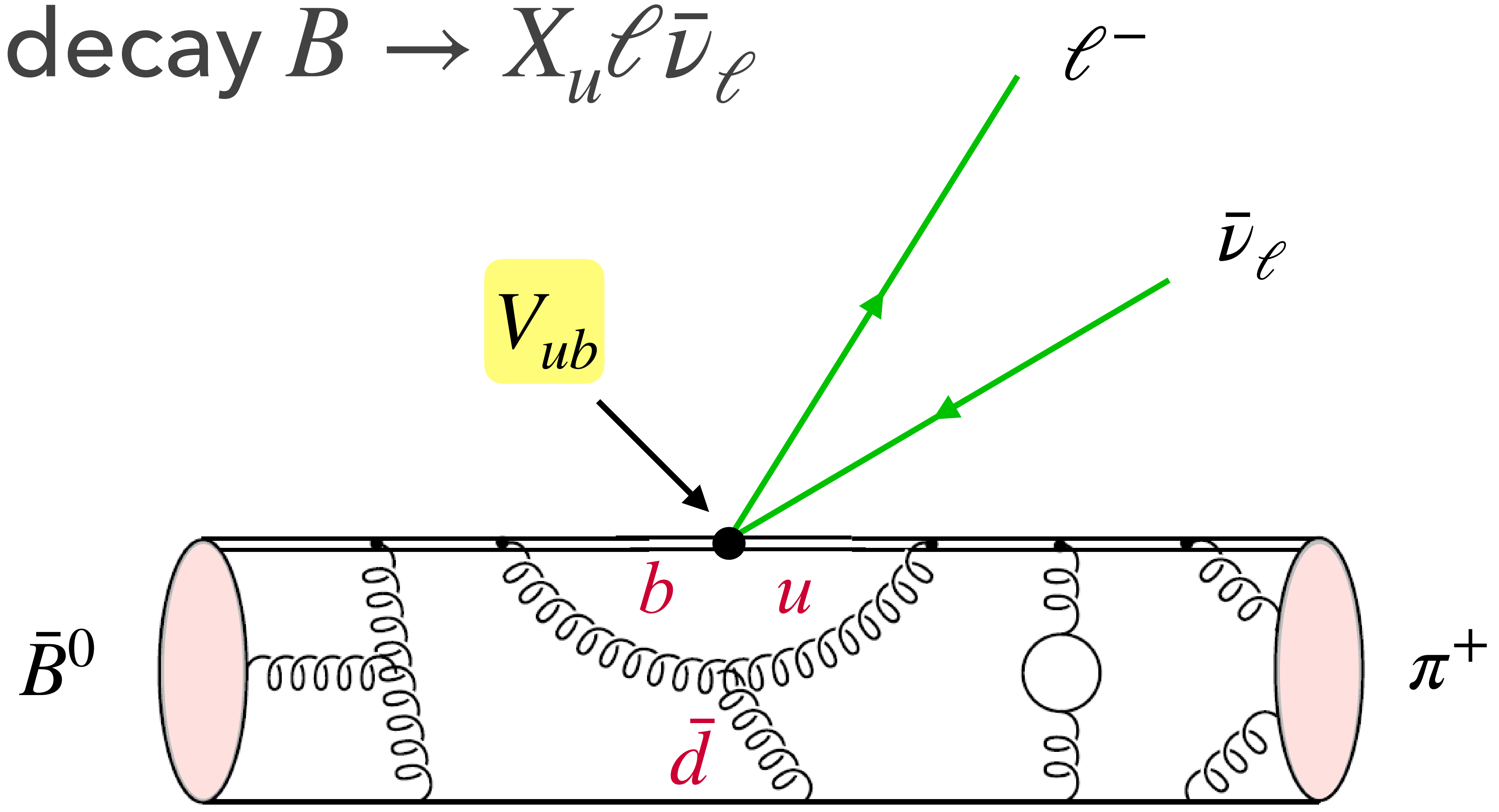
Fig. 1. Extraction of $|V_{cb}|$ and the Isgur–Wise function from $\bar{B}^0 \to D^{*+}\ell\bar{\nu}_\ell$ decays. The data are taken from ref. [16]. $\tau_{B^0} = 1.18$ ps is assumed. $|V_{cb}|$ follows from an extrapolation of the data to $v\cdot v' = 1$. Its currently best value is indicated as a shaded area on the vertical axis.

CONSTRAINTS ON THE UNITARITY TRIANGLE

Determination of $|V_{ub}|$ from semileptonic decays

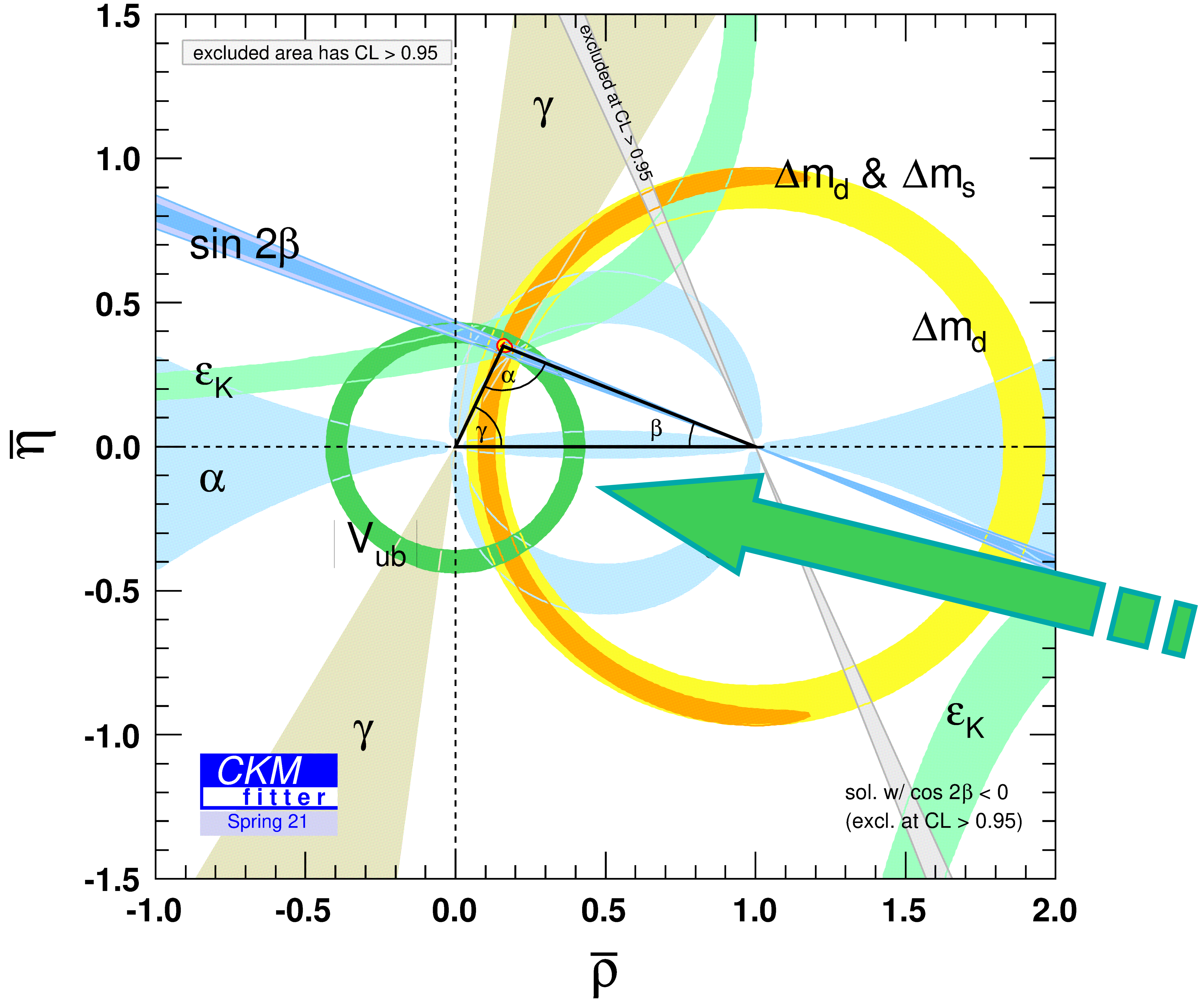


- extraction of $|V_{ub}|$ from exclusive decay $B \to \pi \ell \bar{\nu}_\ell$ and inclusive decay $B \to X_u \ell \bar{\nu}_\ell$

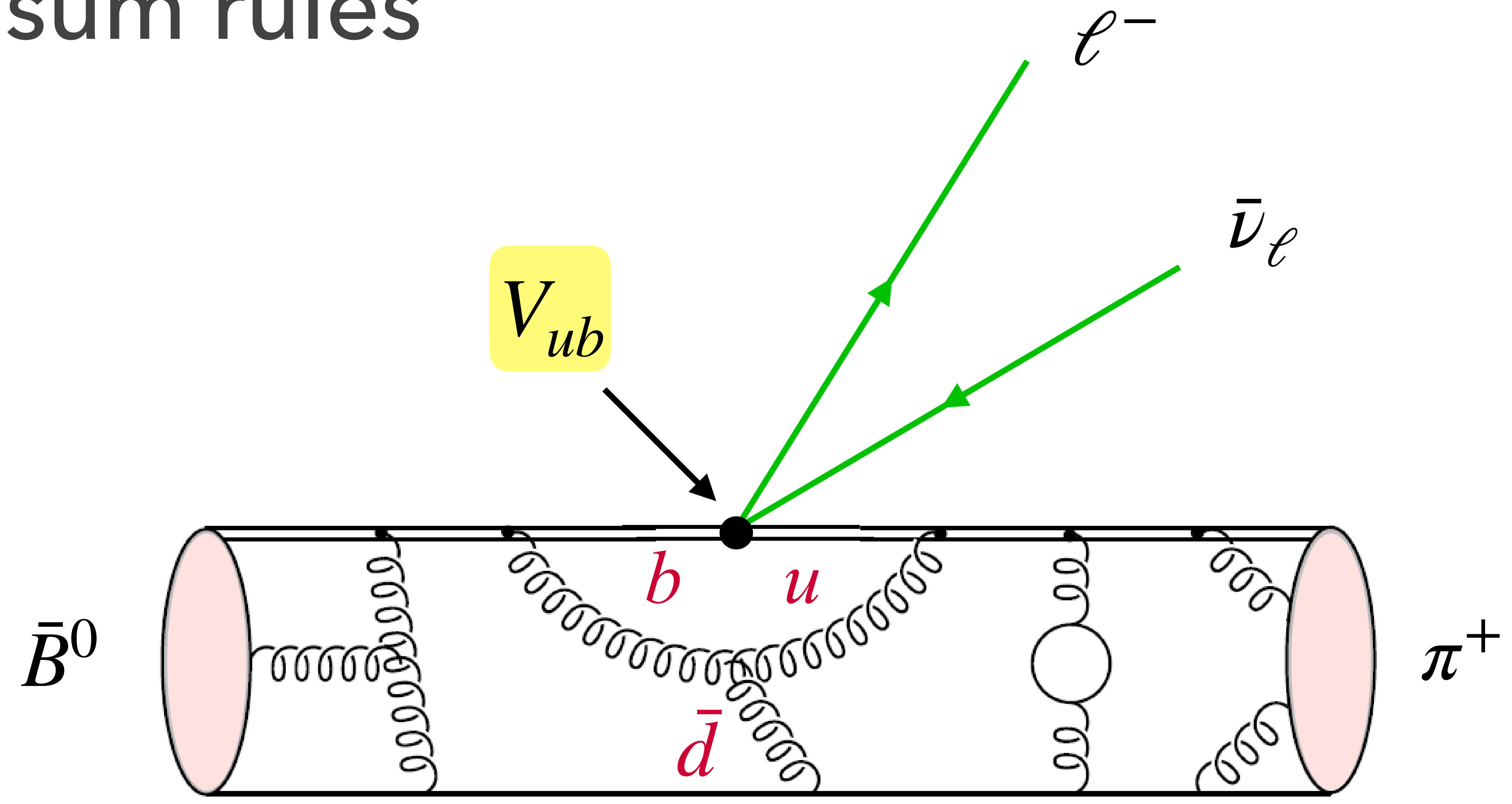


CONSTRAINTS ON THE UNITARITY TRIANGLE

Determination of $|V_{ub}|$ from semileptonic decays



- hadronic form factor from lattice QCD, SCET and light-cone QCD sum rules



HEAVY-QUARK EXPANSION FOR INCLUSIVE DECAYS

OPE based on optical theorem

$$\Gamma(H_b) = \frac{1}{M_{H_b}} \mathrm{Im}\, \langle H_b | \, i \int \mathrm{d}^4x \, T\left\{ \mathcal{H}_{\mathrm{eff}}(x), \mathcal{H}_{\mathrm{eff}}(0) \right\} | H_b \rangle$$

$$= \frac{G_F^2 m_b^5 |V_{cb}|^2}{192\pi^3} \left[c_1 + c_2 \frac{\mu_\pi^2(H_b)}{2m_b^2} + c_3 \frac{\mu_G^2(H_b)}{2m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right) \right]$$

[Bigi, Shifman, Uraltsev, Vainshtein (1993); Manohar, Wise (1993)]

- non-local matrix elements appear when one calculates decay spectra, e.g.:

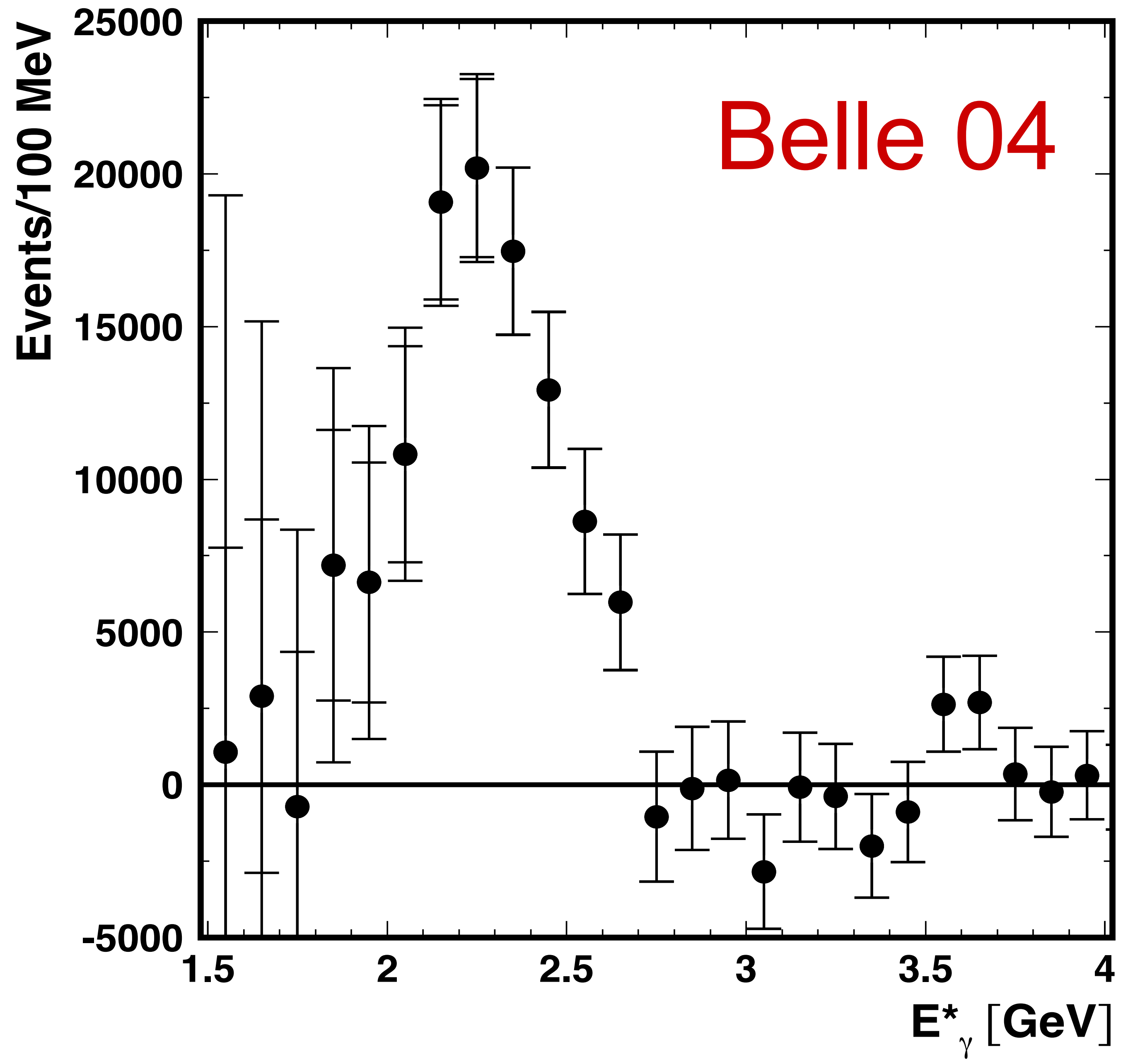
[MN (1993); Bigi, Shifman, Uraltsev, Vainshtein (1993)]

$$\frac{d\Gamma}{dE_\gamma} = \frac{G_F^2 \alpha}{2\pi^4} |V_{tb} V_{ts}^*|^2 \, m_b^2 \, E_\gamma^3$$

$$\times \, |H_\gamma(\mu)|^2 \int_0^{M_B - 2E_\gamma} d\hat{\omega} \, m_b \, J(m_b(M_B - 2E_\gamma - \hat{\omega}), \mu) \, \boxed{S(\hat{\omega}, \mu)} + \dots$$

- shape function (PDF):

$$\boxed{S(\omega)} = \int \frac{dt}{4\pi} \, e^{-i\omega t} \, \langle \bar{B}(v) | \, \bar{h}_v(tn) Y_n(tn) Y_n^\dagger(0) h_v(0) | \bar{B}(v) \rangle$$



FIRST OBSERVATION OF A NON-ZERO $|V_{ub}|$

CLEO, PRL **64** (1990) 16, Received 8 Nov 1989 $(212+101\,\mathrm{pb}^{-1})$

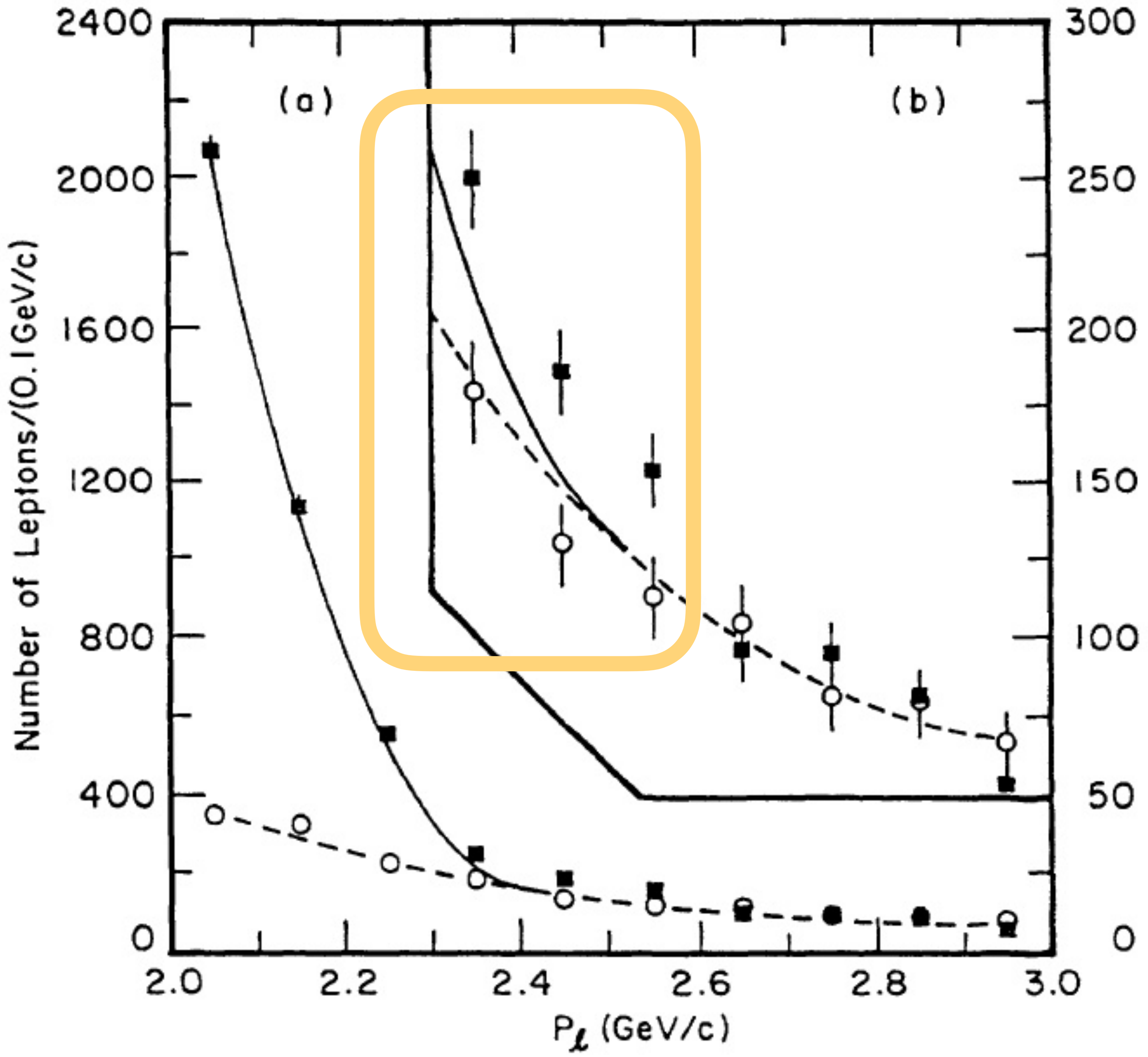


FIG. 1. Sum of the e and μ momentum spectra for ON data (filled squares), scaled OFF data (open circles), the fit to the OFF data (dashed line), and the fit to the OFF data plus the $b\to c l\nu$ yield (solid line). Note the different vertical scales in (a) and (b).

ARGUS, PLB **234** (1990) 409, Received 28 Nov 1989 $(201+69\,\mathrm{pb}^{-1})$

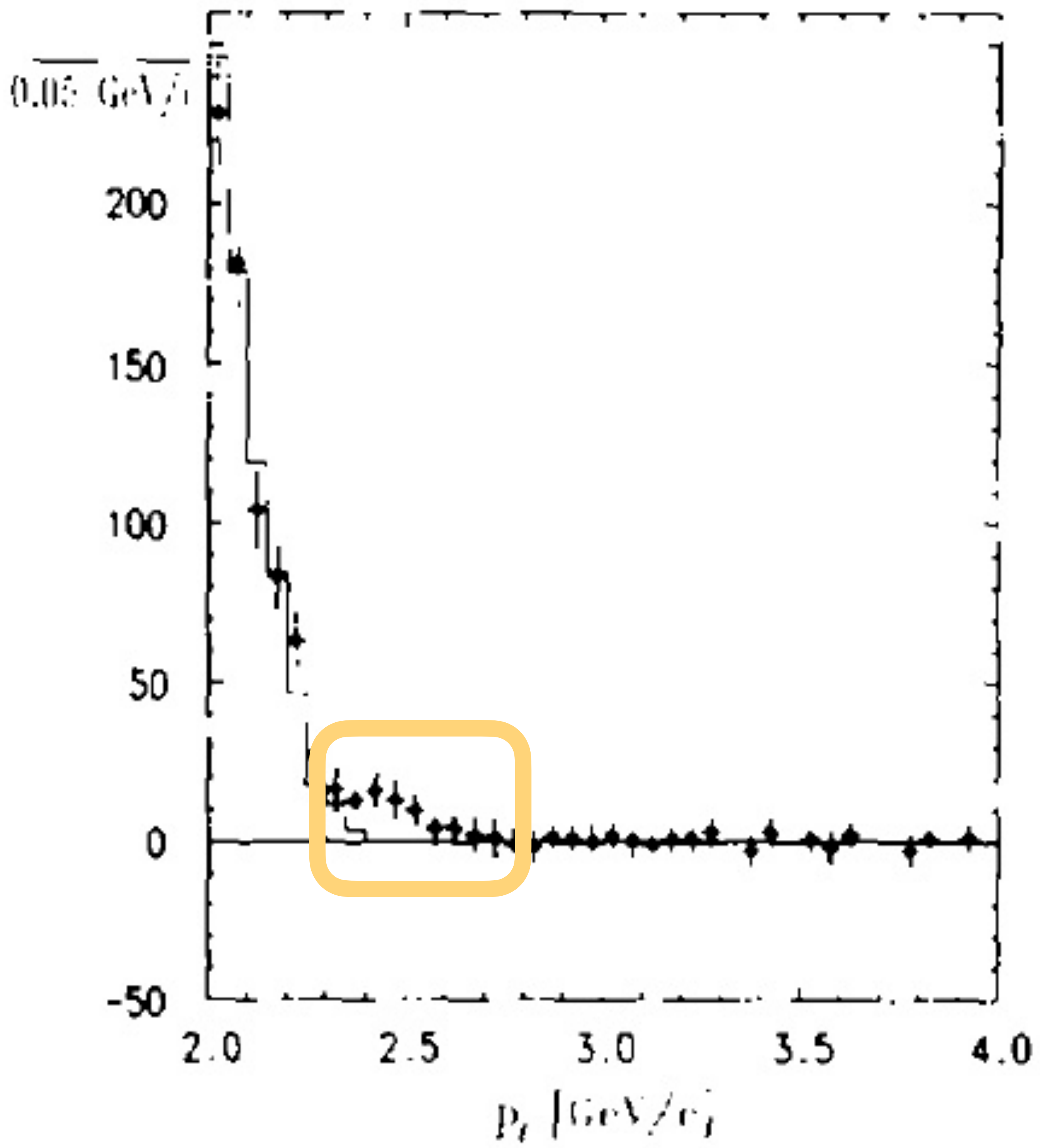


Fig. 5. Combined lepton momentum spectrum for direct $\Upsilon(4S)$ decays: the histogram is a $b\to c$ contribution normalized in the region 2.0–2.3 GeV/c.