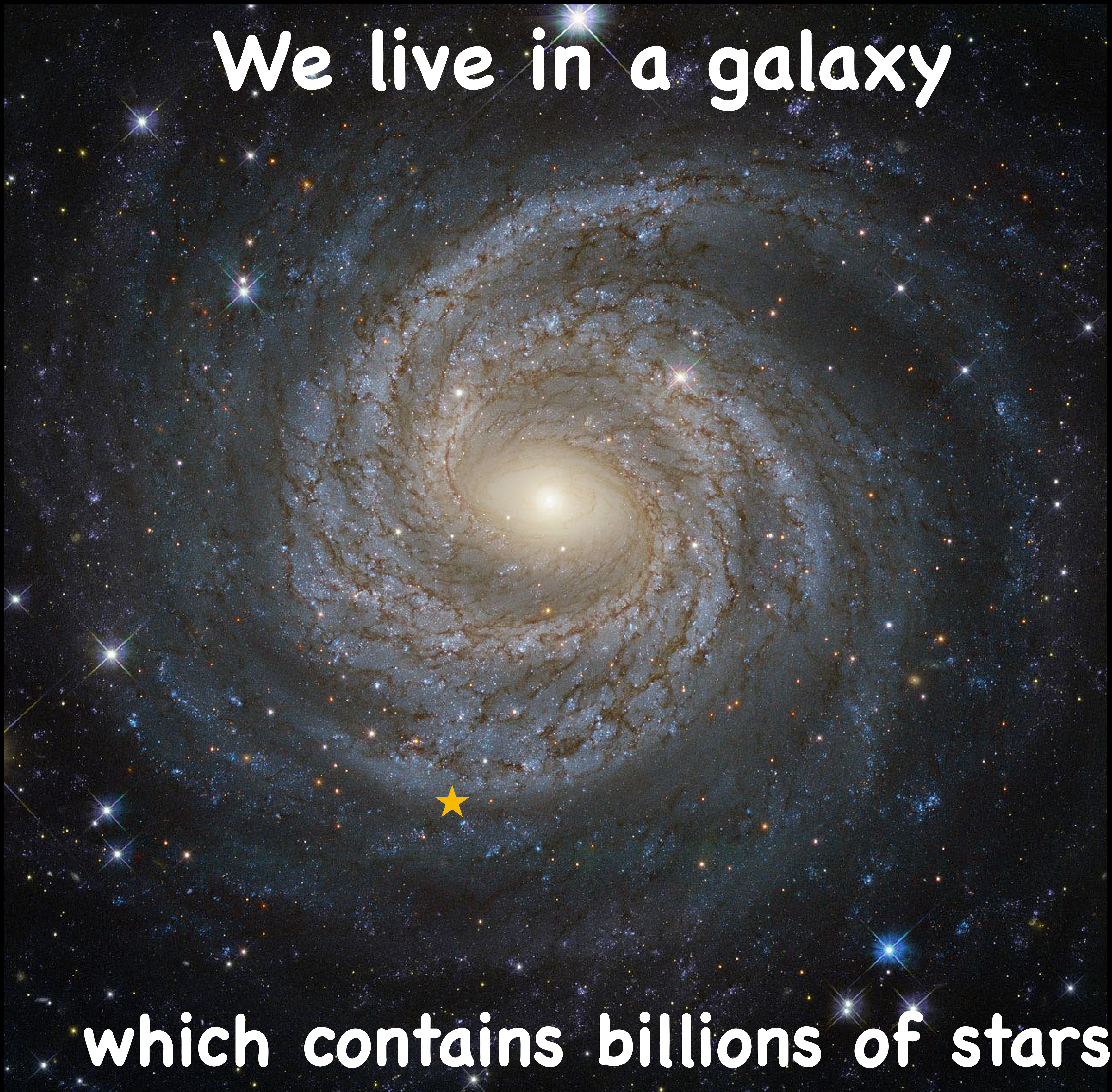
Cosmology

Prof Celine Boehm

THE UNIVERSITY OF SYDNEY

CLASHEP2023


Norris-2016-Dawes-Review-Aboriginal-Astronomy.pdf


We live in a galaxy
which contains billions of stars

Our place in the Universe

Galaxies within galaxies

LMC and SMC are galaxies within the Milky Way and many more

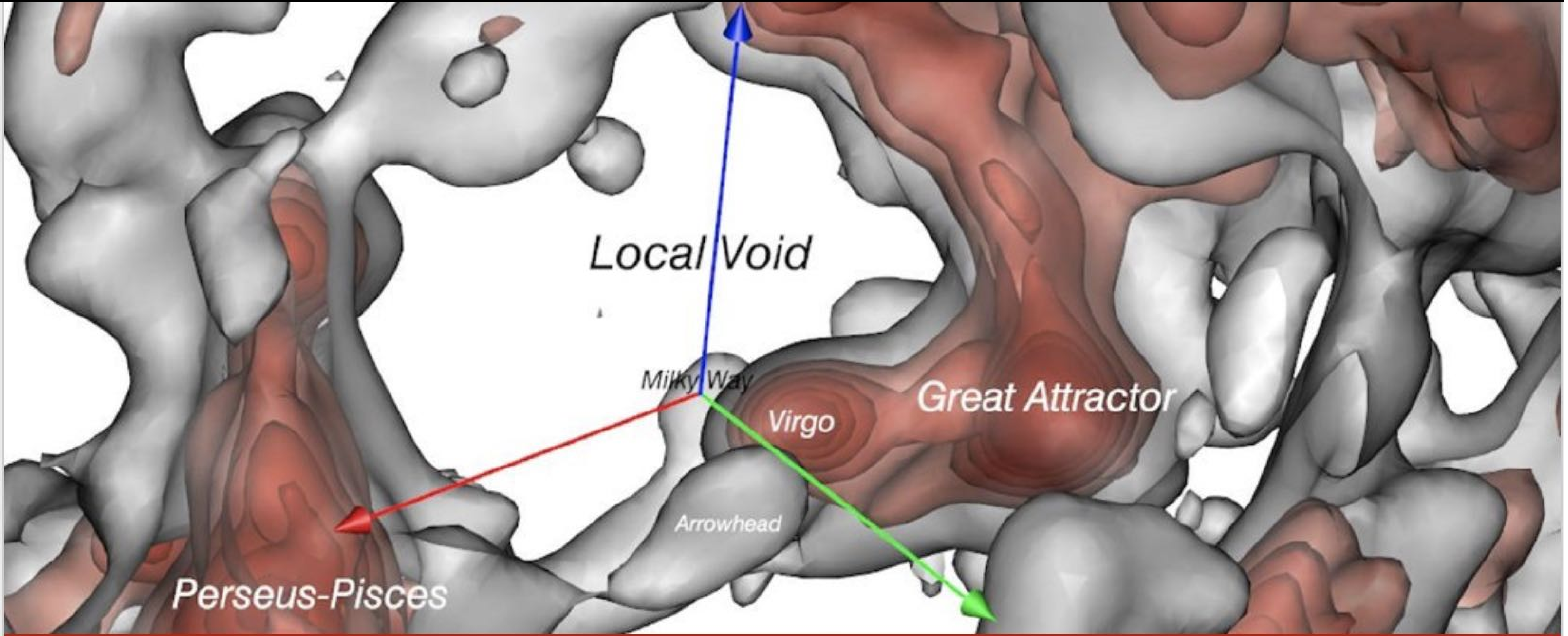
Clusters of galaxies

Abell 370, credit Nasa

Our Local Group belongs to an even larger structure, a supercluster with the rich **Virgo cluster** of galaxies at its centre, about 70 million light years (about 21 **Mpc**) away. (Courtesy Astronomy @ Swinburn uni)

Laniakea, our super cluster

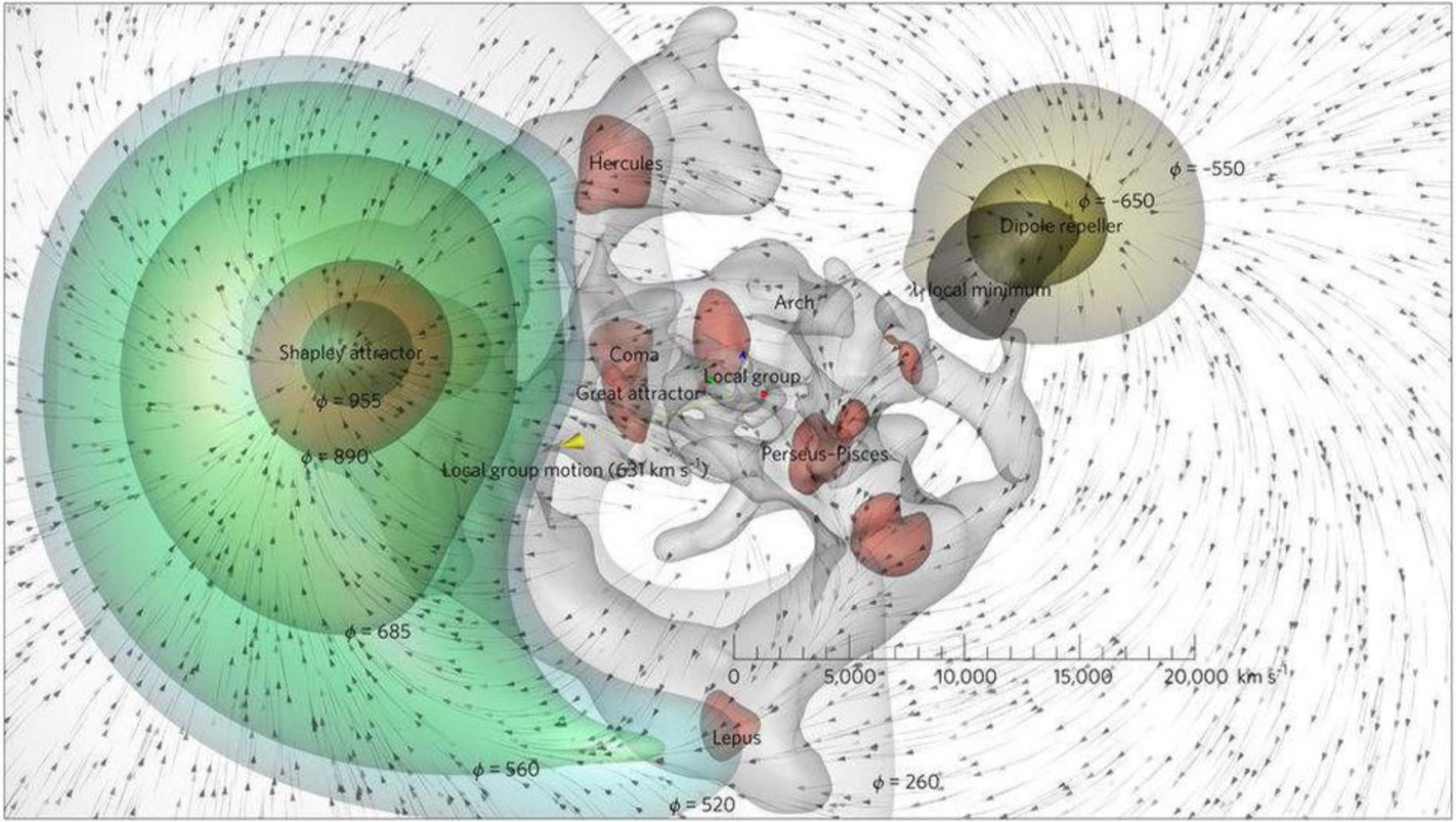
sciencealert Trending



SPACE

There's a Huge Void Near Our Galaxy. Its Mysterious Depths Have Just Been Measured

EVAN GOUGH, UNIVERSE TODAY 23 JUL 2019



The relative attractive and repulsive effects of overdense and underdense regions on the Milky Way...

[+] YEHUDA HOFFMAN, DANIEL POMARÈDE, R. BRENT TULLY, AND HÉLÈNE COURTOIS, NATURE ASTRONOMY 1, 0036 (2017)

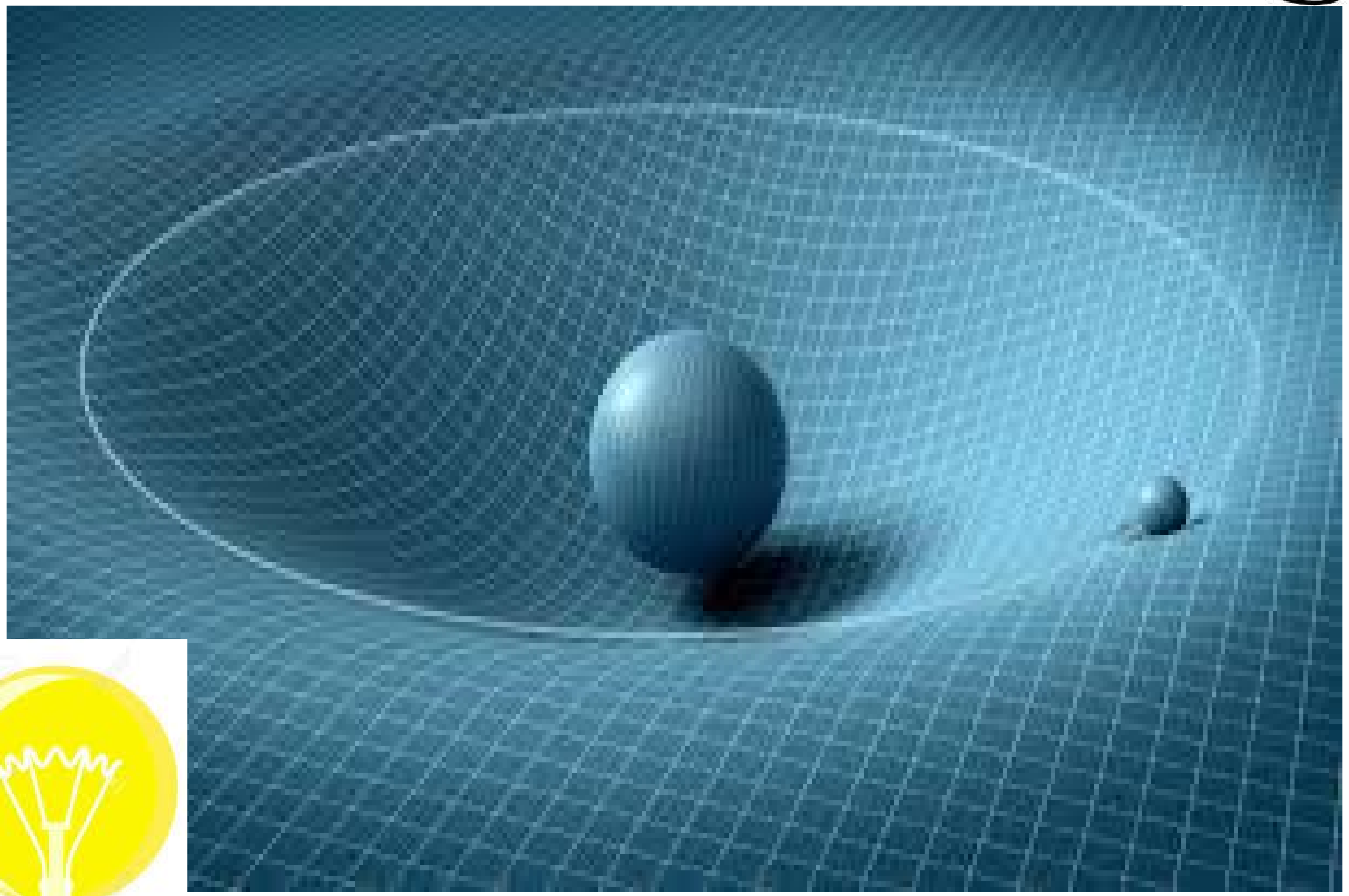
The cosmic web

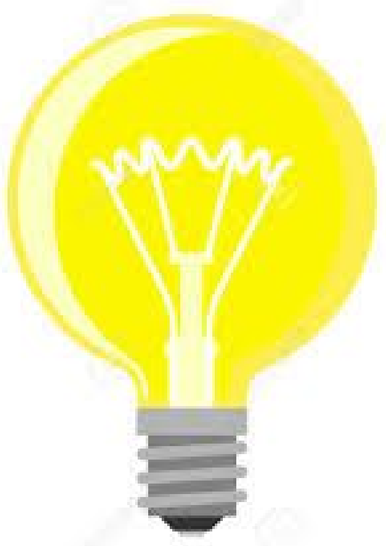
How did we get this particular Universe?

This is first and foremost

A story about photons making their way to us

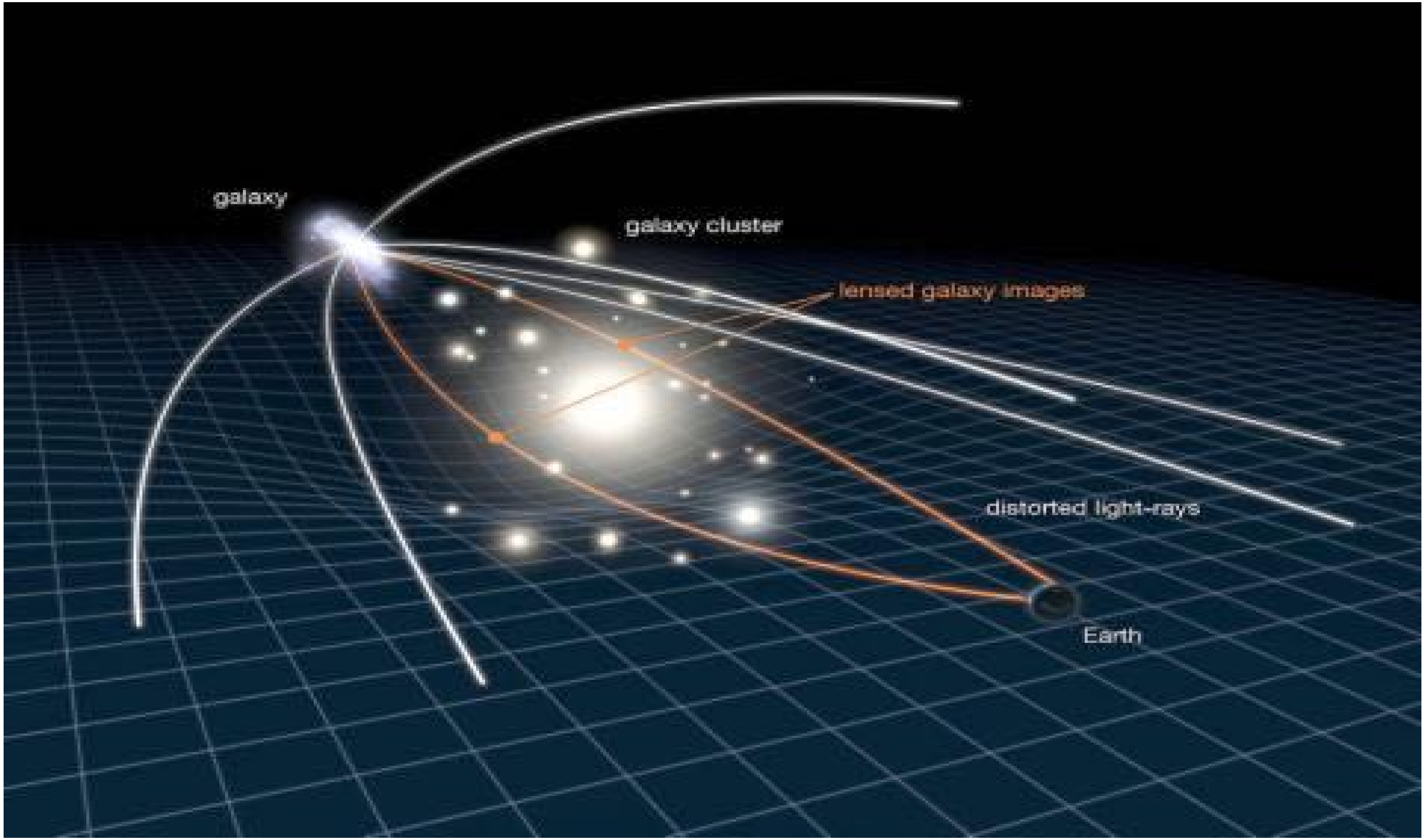
Does primordial light travel straight?



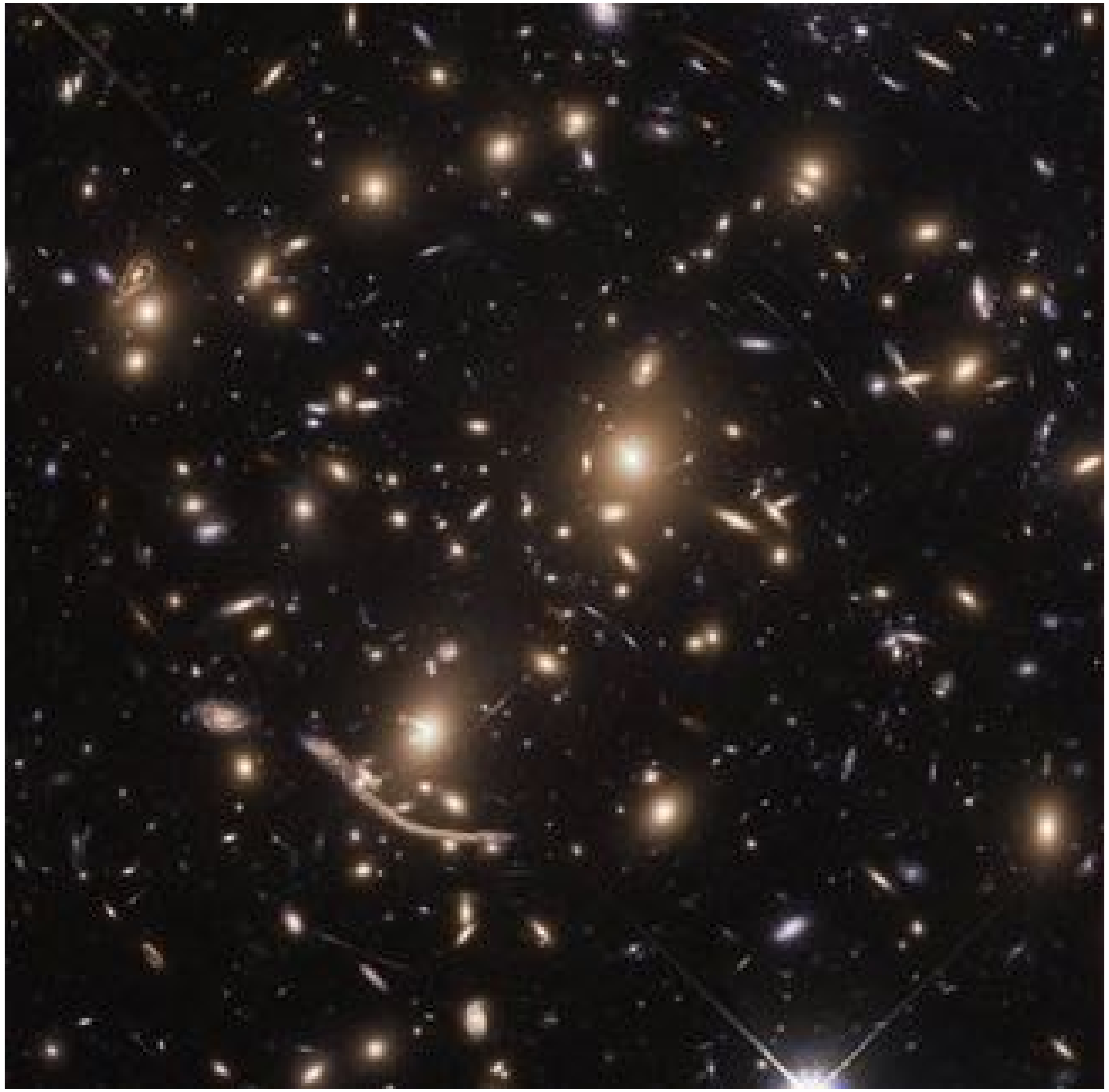


Light follows space-time but its path will be distorted and therefore its path is curved.

We know that matter curves space-time

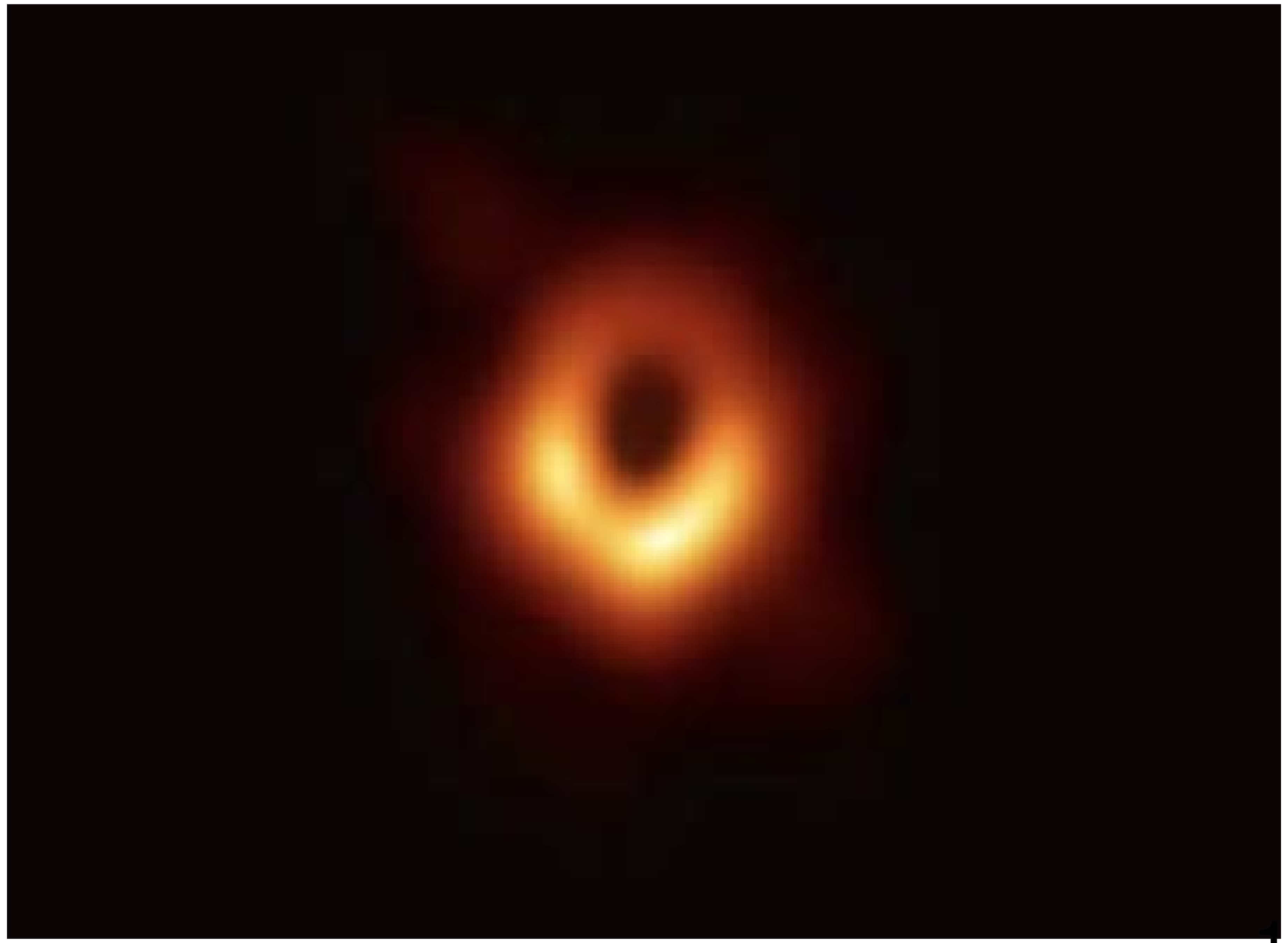


We know that matter curves space-time



The matter inside is curving space
So all objects appear distorted
One can use these distortions to reconstruct the invisible mass

Event horizon is the same phenomena
EHT collaboration 2019



Can we use primordial light to measure the content of the Universe?

Geometry of the Universe

Content of the Universe

CMB & Formation of structures

The invisible (challenges)

Part I. Geometry

Light travels means we need to define a metric

Minkowski metric $$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Metric associated with a flat space-time

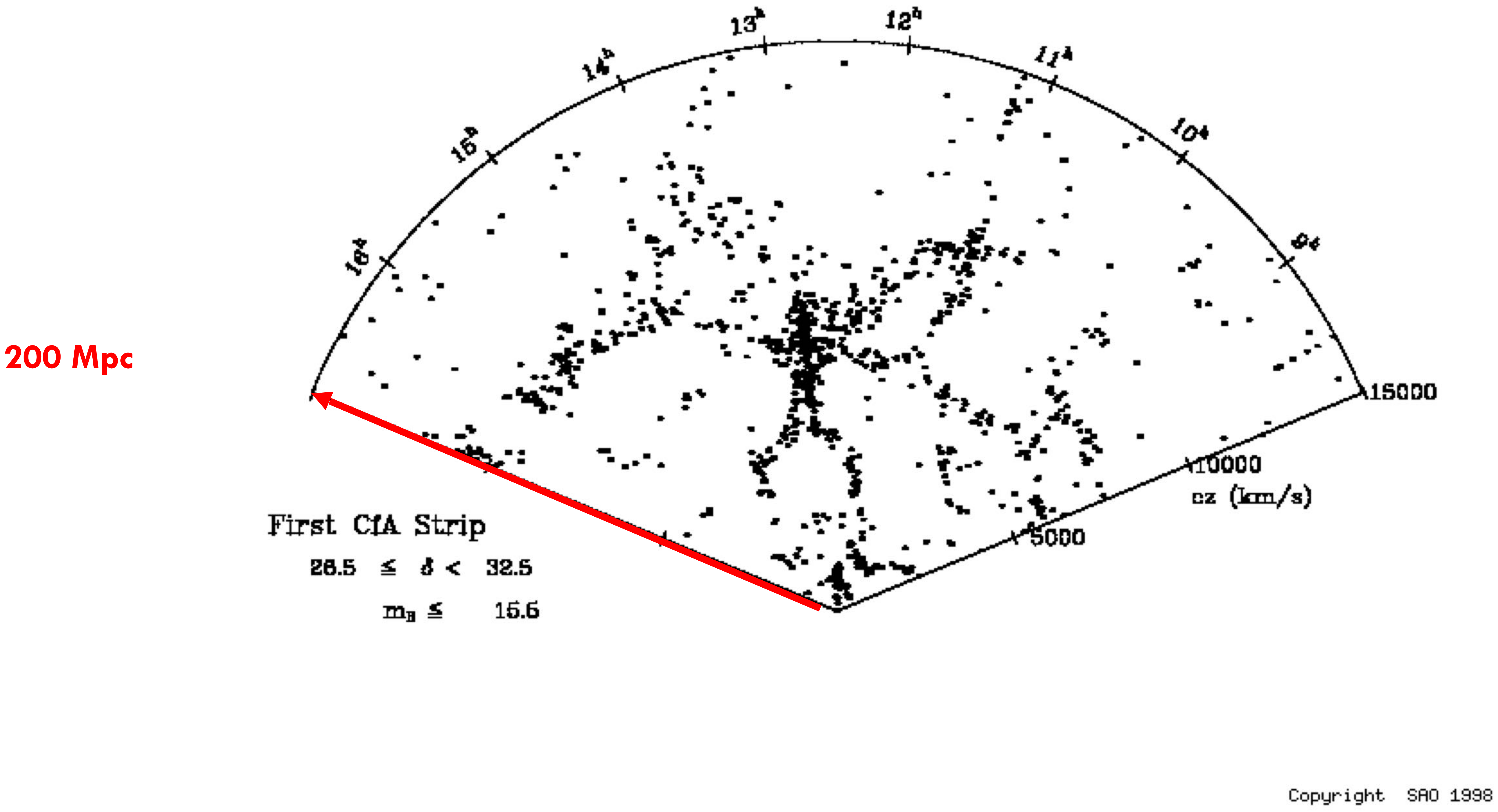
But this is not taking into account the key principles from GR and the fact that matter can curve space.

How do we generalise? $$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

with $g_{\mu\nu} = \ldots$

Part I. Geometry

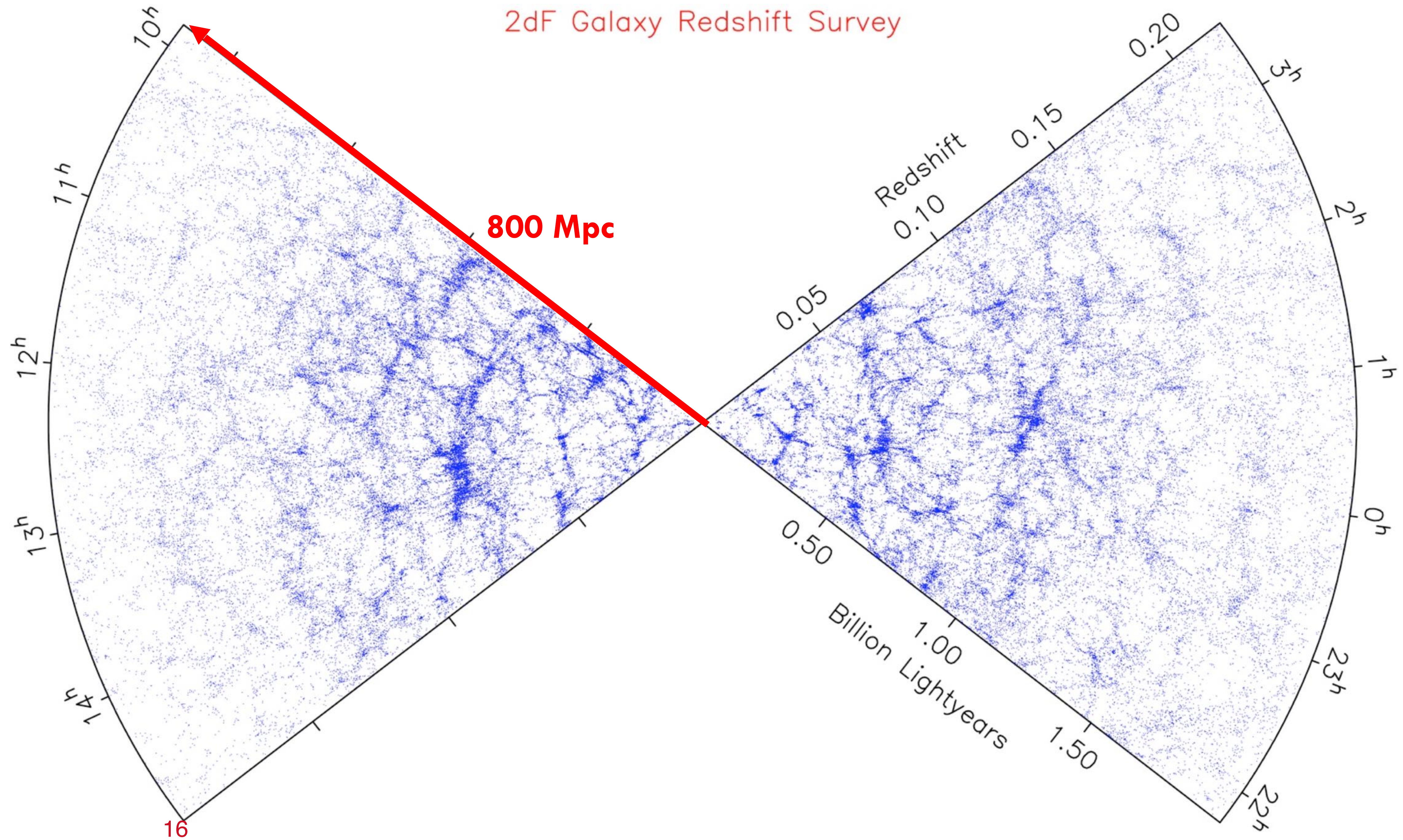
The Universe is mostly homogeneous and isotropic



Center for Astrophysics (CfA) Survey: Geller & Huchra 1989

Part I. Geometry

The Universe is mostly homogeneous and isotropic

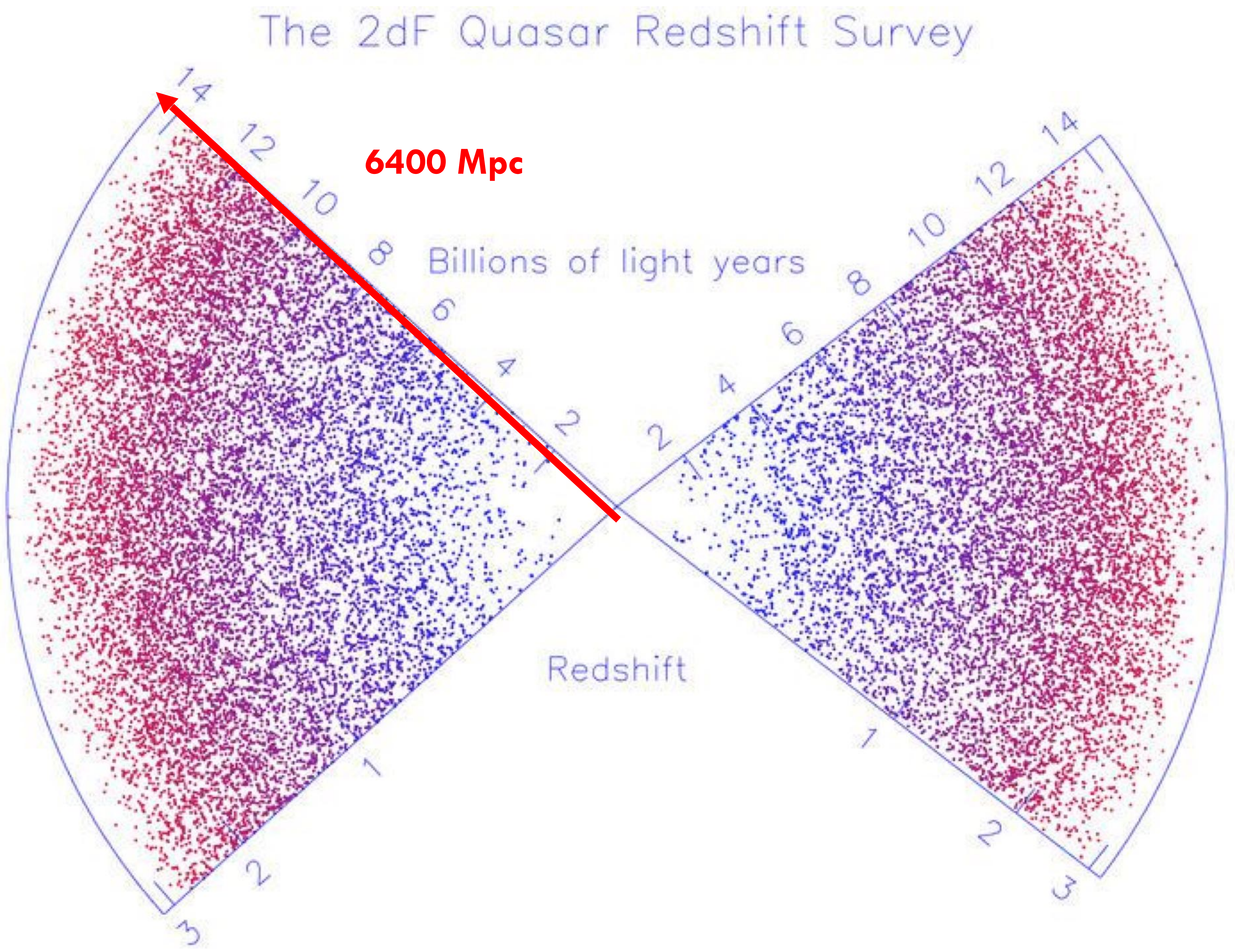


16

2dF Galaxy&Redshift Survey (2dFGRS): Colless et al 2001

Part I. Geometry

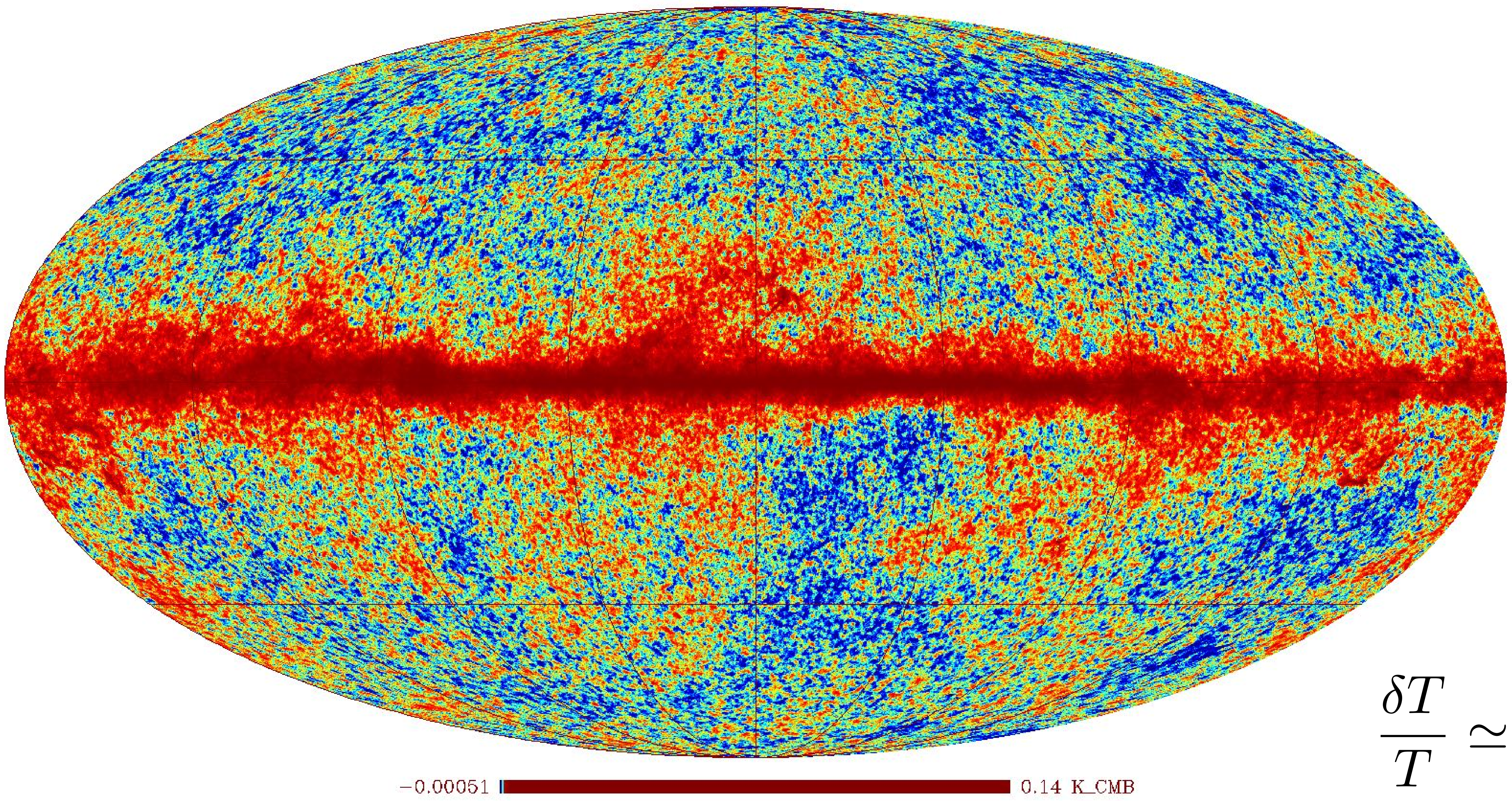
The Universe is mostly homogeneous and isotropic



Part I. Geometry

The Universe is mostly homogeneous and isotropic even at very large redshift

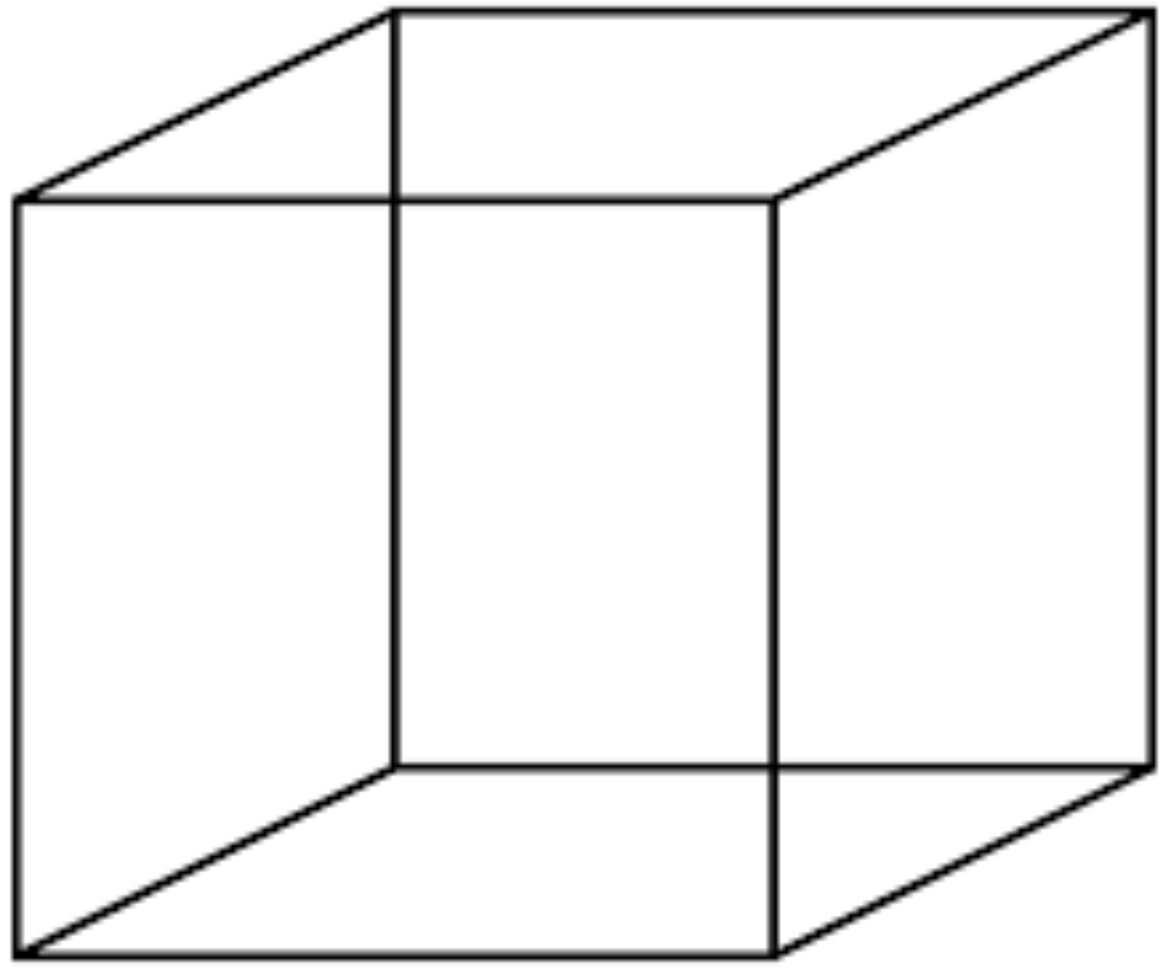
Planck 2018

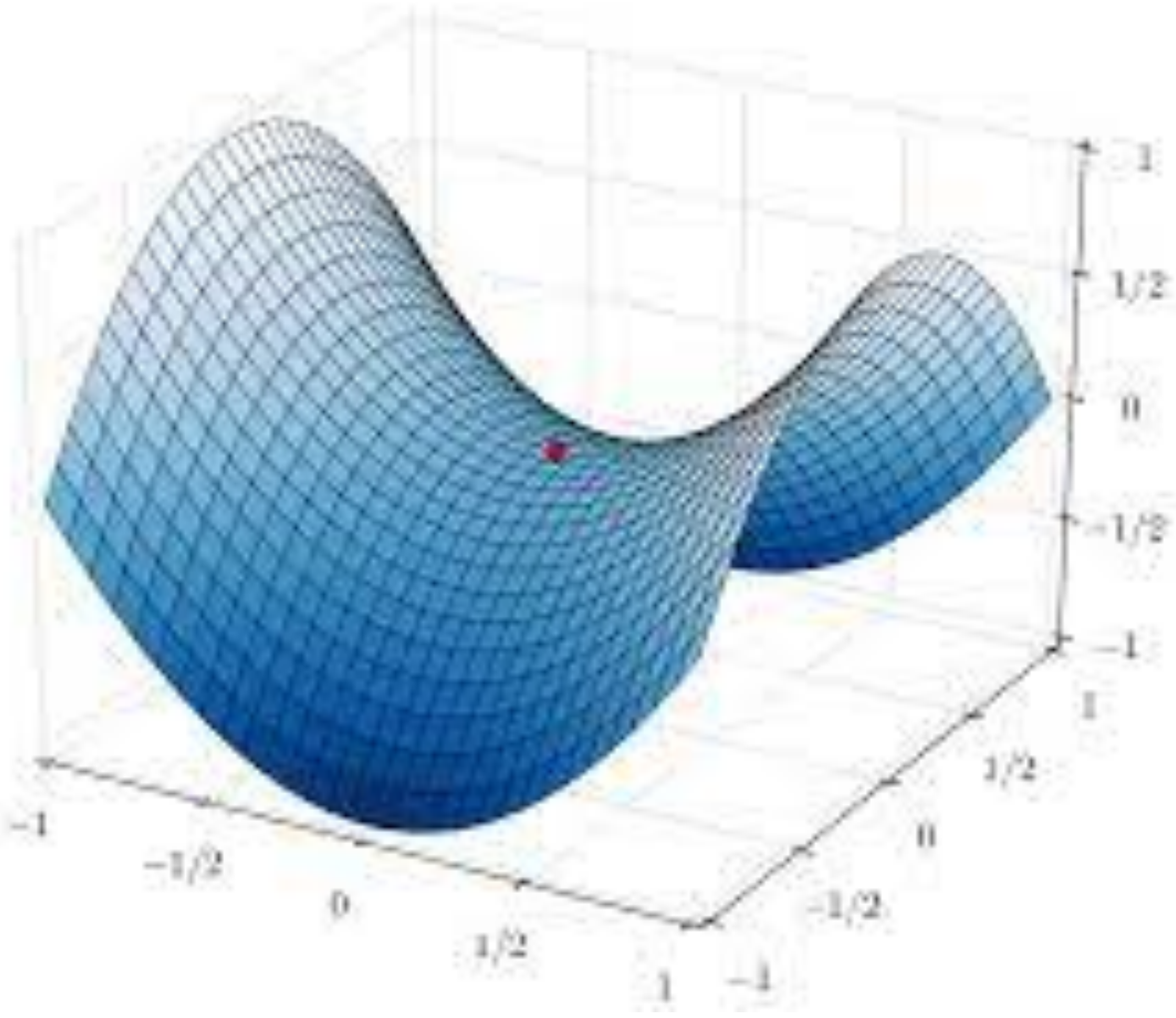


$$\frac{\delta T}{T} \simeq 10^{-5}$$

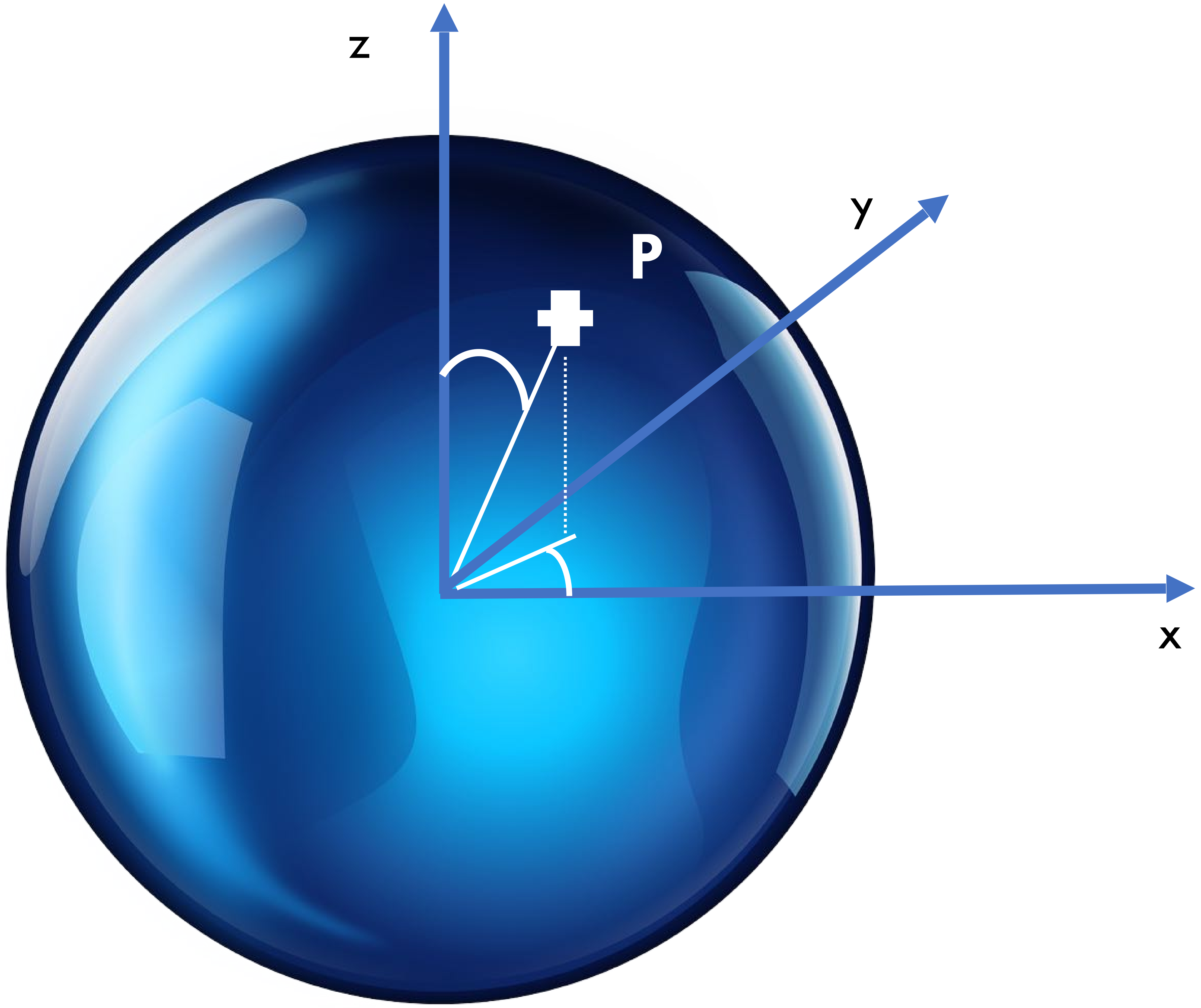
Part I. Geometry

The Universe is mostly homogeneous and isotropic means





Part I. Geometry



P is located on a sphere of radius R

$$ds^2 = x_p^2 + y_p^2 + z_p^2$$

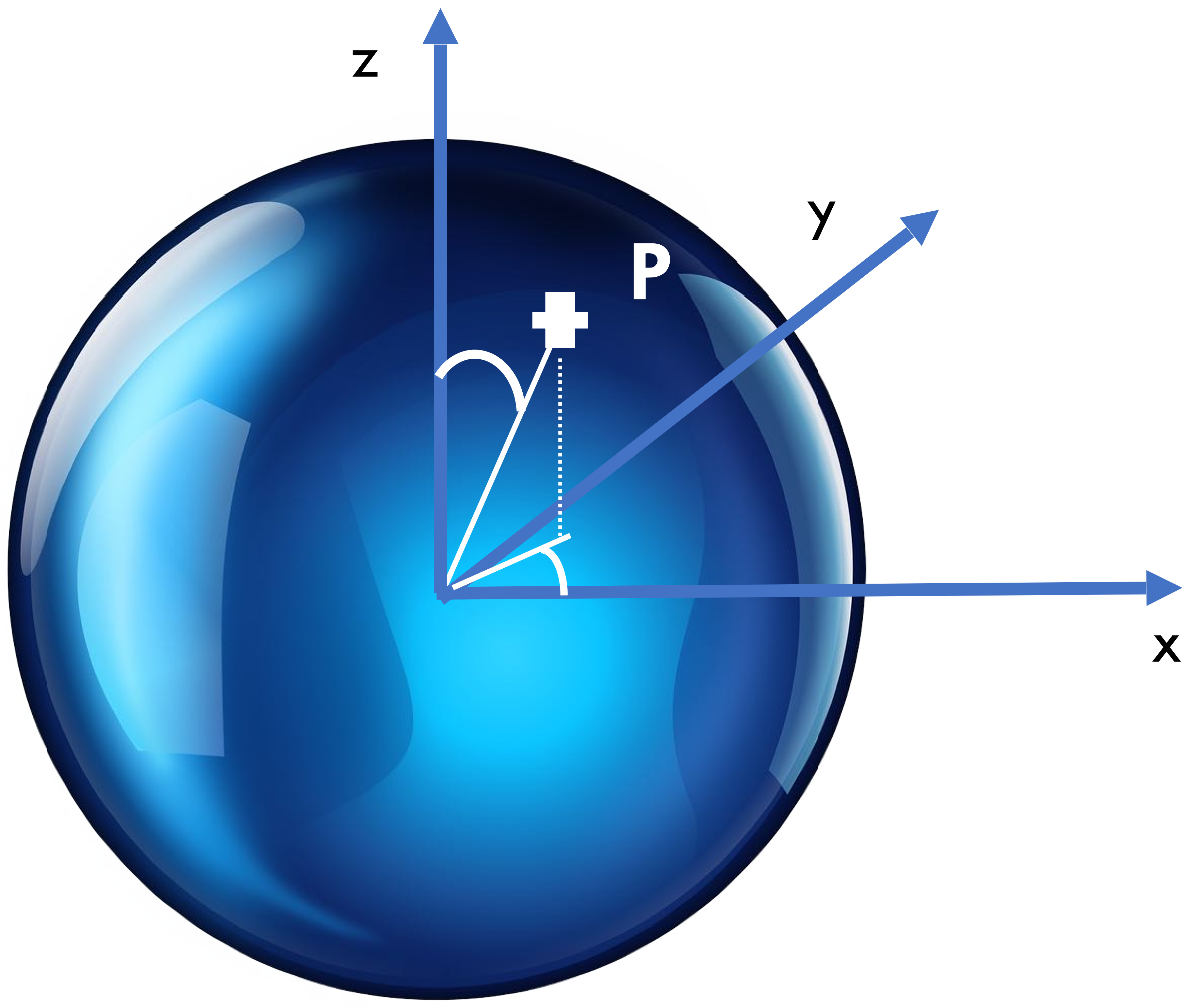
Can be reinterpreted by defining the coordinated on the sphere

$$x_p = R \ \sin\theta \cos\phi$$
$$y_p = R \ \sin\theta \sin\phi$$
$$z_p = R \ \cos\theta$$

$$ds^2 = \underbrace{(x_p - x_0)^2}_{(dx)^2} + \underbrace{(y_p - y_0)^2}_{(dy)^2} + \underbrace{(z_p - z_0)^2}_{(dz)^2} \Rightarrow ds^2 = dx^2 + dy^2 + dz^2$$

Part I. Geometry

$$ds^2 = dx^2 + dy^2 + dz^2$$



$$x_p = R\ \sin\theta\cos\phi$$
$$y_p = R\ \sin\theta\sin\phi$$
$$z_p = R\ \cos\theta$$

derivative

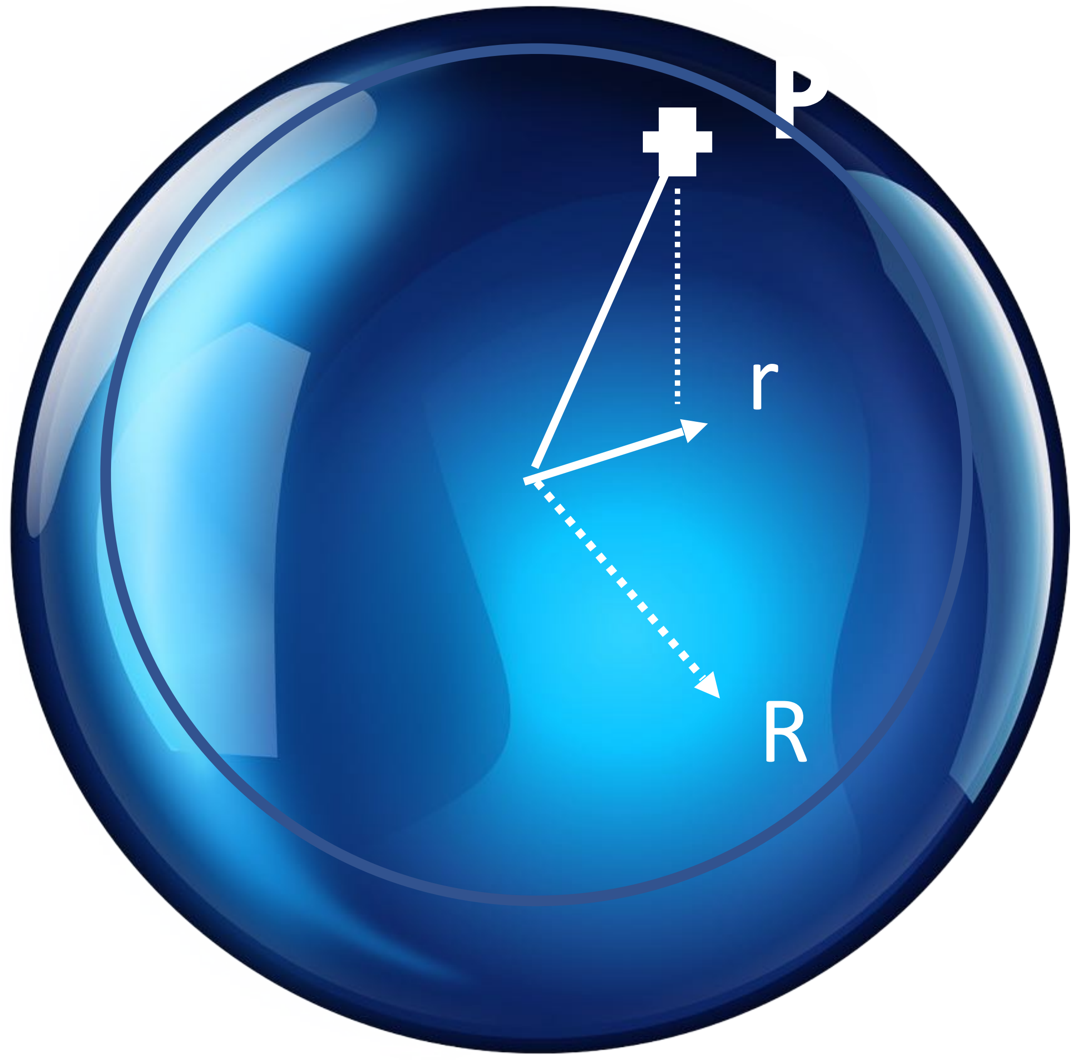
$$dx = \underbrace{dR\sin\theta\cos\phi}_{=0} + R\ \cos\theta d\theta\cos\phi - R\sin\theta\sin\phi d\phi$$
$$dy = \underbrace{dR\sin\theta\cos\phi}_{=0} + R\ \cos\theta d\theta\sin\phi + R\sin\theta\cos\phi d\phi$$
$$dz = -R\sin\theta\, d\theta$$

$$ds^2 = R^2\left(d\theta^2 + \sin^2\theta\ d\phi^2\right)$$

Only geometrical information

Size of the sphere

Part I. Geometry   Projection from 3d to 2d



$$z = \sqrt{R^2 - x^2 - y^2}$$

$$dz = -\frac{(xdx + ydy)}{\sqrt{R^2 - x^2 - y^2}}$$

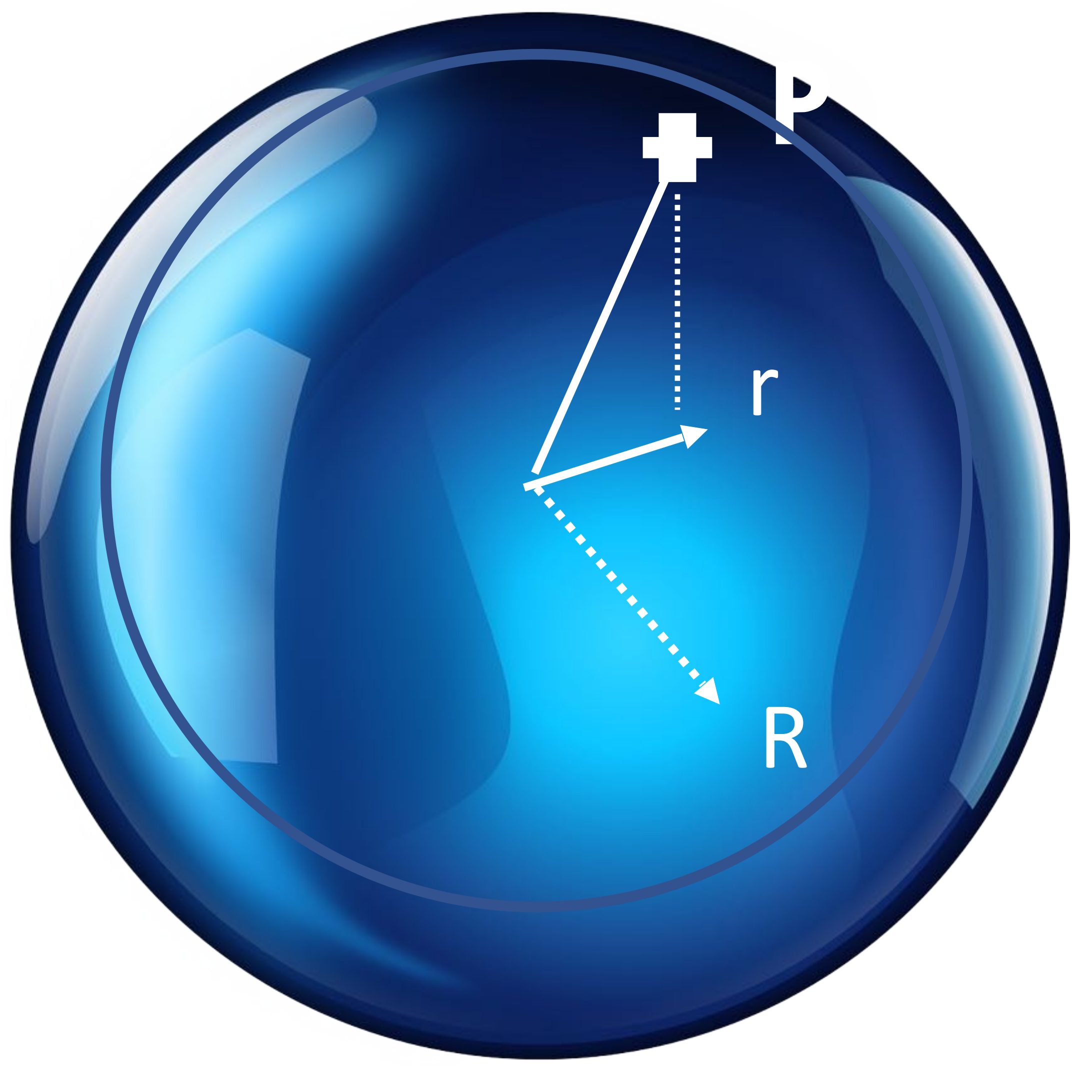
$$ds^2 = dx^2 + dy^2 + \frac{(xdx + ydy)^2}{(R^2 - x^2 - y^2)}$$

$$x = r\ \cos\theta$$
$$y = r\ \sin\theta$$
$$dx^2 + dy^2 = dr^2 + r^2 d\theta^2$$

$$dz^2 = \frac{r^2 dr^2}{(R^2 - r^2)} \Rightarrow ds^2 = dr^2 + r^2 d\theta^2 + \frac{r^2\ dr^2}{R^2 - r^2}$$

Part I. Geometry

Projection from 3d to 2d



$$ds^2 = dr^2 + r^2 d\theta^2 + \frac{r^2 \; dr^2}{R^2 - r^2}$$

Let us define r'= r/R

$$ds^2 = R^2 \left(\frac{dr'^2}{1 - r'^2} + r'^2 d\theta^2 \right)$$

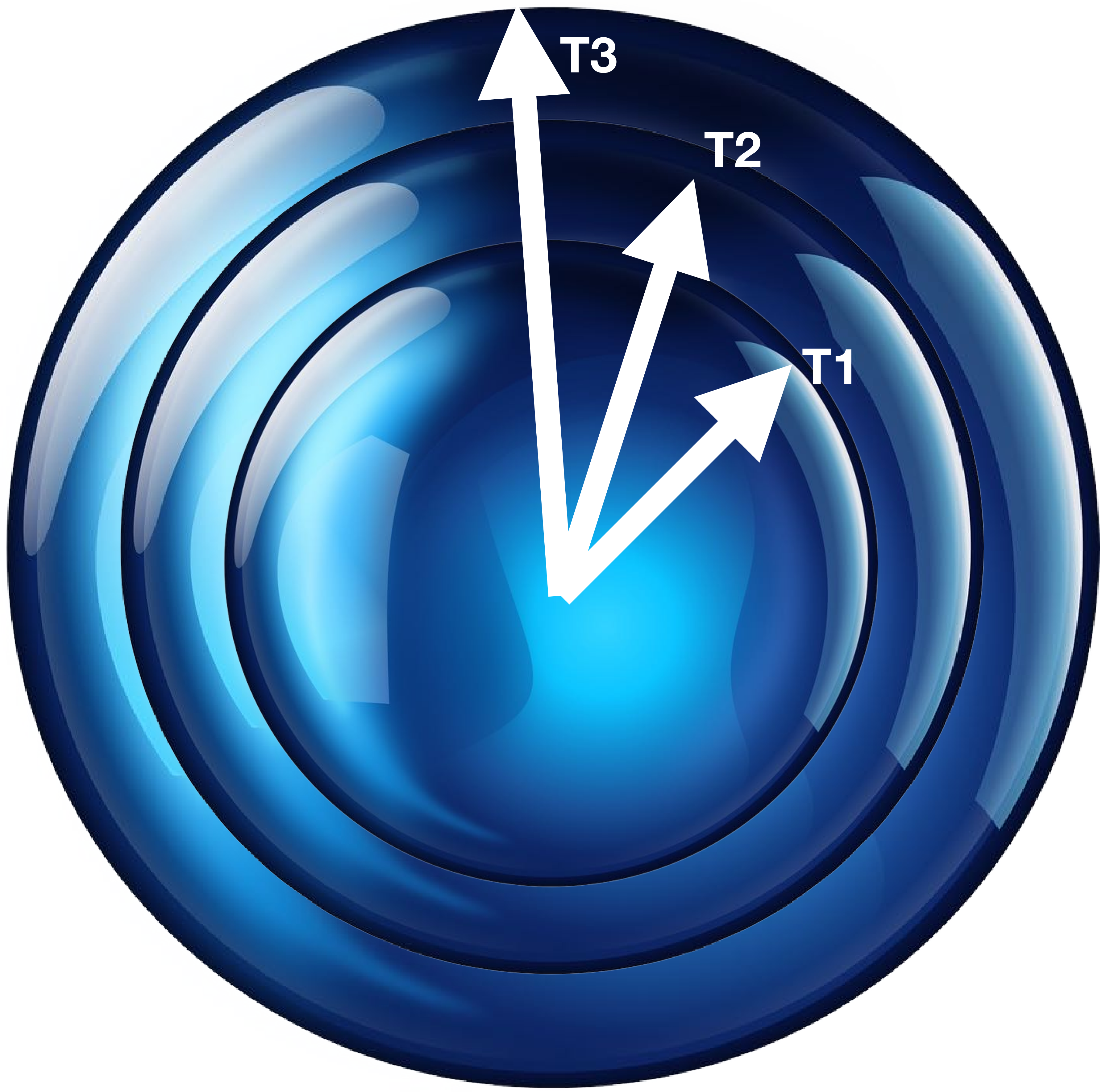
with $r' = \frac{r}{R}$

Singularity

What if we live in 4d but do not see the 4th dimension?

Part I. Geometry

Projection from 4d to 3d



4th dimension of time added

Spheres can grow

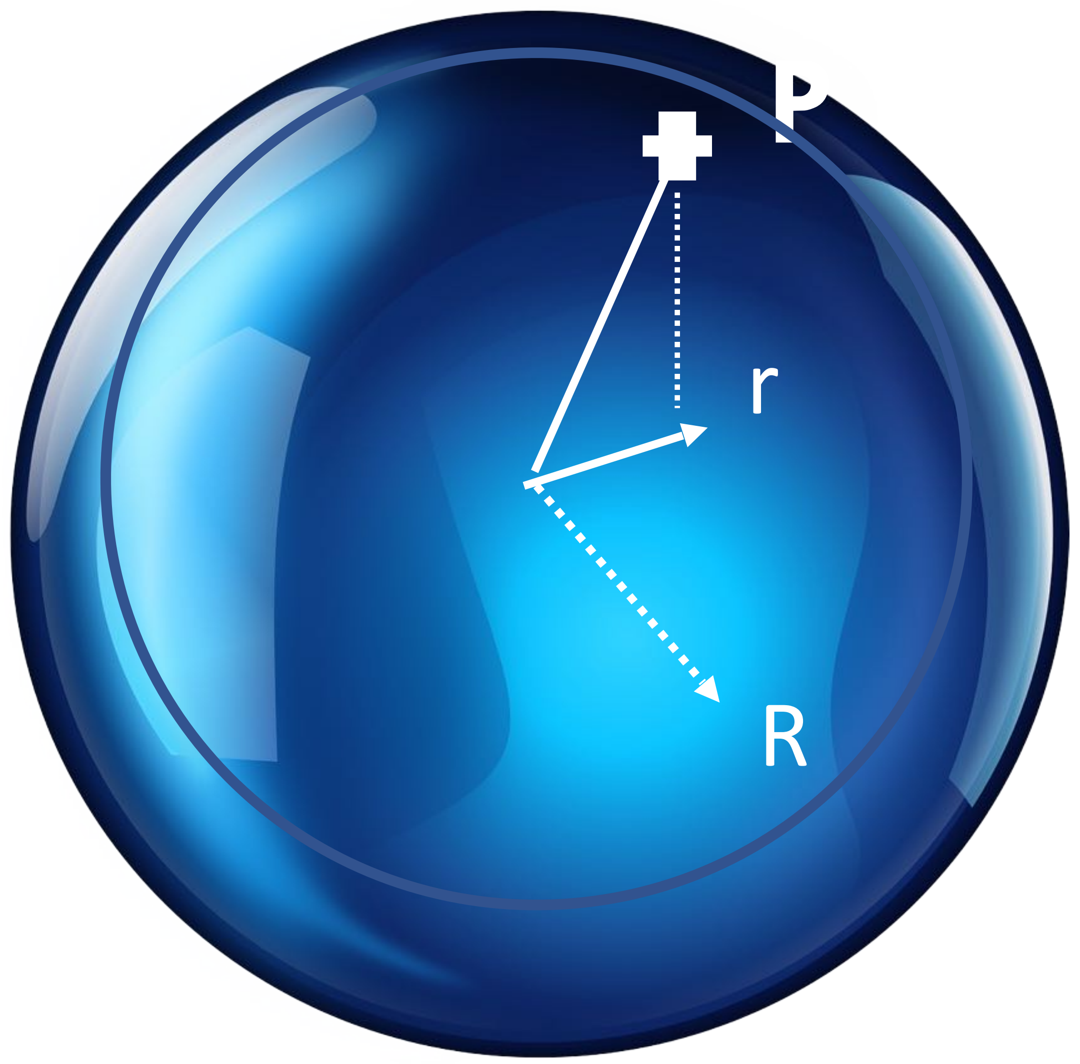
$$x^2 + y^2 + z^2 + w^2 = R^2$$

$$w = \sqrt{R^2 - (x^2 + y^2 + z^2)}$$

$$\begin{aligned} x &= R \sin\theta \sin\phi \sin\chi \\ y &= R \sin\theta \cos\phi \sin\chi \\ z &= R \cos\theta \sin\chi \\ w &= R \cos\chi \end{aligned}$$

Part I. Geometry

Projection from 4d to 3d



Step 1: Getting rid off the 4th dimensions

$$x^2 + y^2 + z^2 + w^2 = R^2$$

$$w = \sqrt{R^2 - (x^2 + y^2 + z^2)}$$

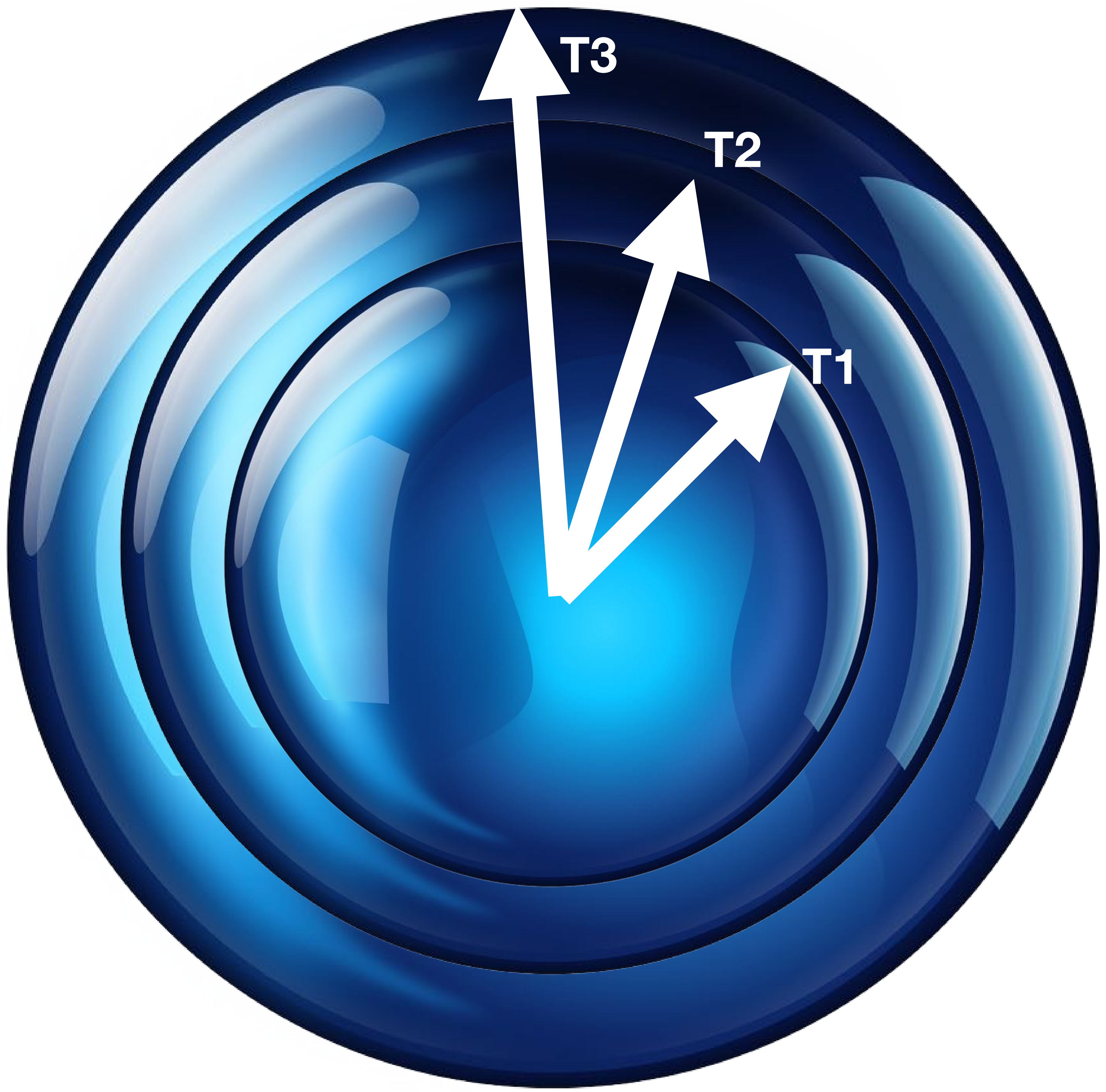
$$r^2 = (x^2 + y^2 + z^2)$$

$$w = \sqrt{R^2 - r^2}$$

$$r^2 = (x^2 + y^2 + z^2) \Rightarrow r^2 = \sum_i x_i^2 \Rightarrow r^2 = r^2 \sum_i {x'}_i^2$$

Part I. Geometry

Projection from 4d to 3d



Step 2: taking derivatives

$$ds^2 = dx^2 + dy^2 + dz^2 + dw^2$$
$$w = \sqrt{R^2 - (x^2 + y^2 + z^2)}$$

R fixed

$$ds^2 = \frac{r^2 dr^2}{R^2 - r^2} + dr^2 \sum_i x'^2_i + r^2 \underbrace{\sum_i dx'^2_i}_{\text{3d-sphere}}$$

Part I. Geometry

Projection from 4d to 3d

Step 3: use spherical coordinates

$$ds^2 = \frac{r^2 dr^2}{R^2 - r^2} + dr^2 \sum_i {x'}_i^2 + r^2 \underbrace{\sum_i {dx'}_i^2}_{\text{3d-sphere}}$$

with

$$\sum_i {dx'}_i^2 = (d\theta^2 + \sin^2\theta \, d\phi^2)$$

$$\sum_i {x'}_i^2 = 1$$

$$dr^2 \sum_i {x'}_i^2 = dr^2$$

$$r^2 \sum_i {dx'}_i^2 = r^2 \, (d\theta^2 + sin^2\theta d\phi^2)$$

Part I. Geometry

Projection from 4d to 3d

$$ds^2 = \frac{r^2 dr^2}{R^2 - r^2} + dr^2 \sum_i x'^2_i + r^2 \underbrace{\sum_i dx'^2_i}_{\text{3d-sphere}}$$

with

$$\sum_i dx'^2_i = (d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$\sum_i x'^2_i = 1$$

$$ds^2 = \frac{R^2 dr^2}{R^2 - r^2} + r^2 \underbrace{\sum_i dx'^2_i}_{\text{3d-sphere}} \quad \Rightarrow \quad ds^2 = \frac{R^2 dr^2}{R^2 - r^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

$$ds^2 = R^2 \left[\frac{dr'^2}{1 - r'^2} + r'^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

Interpretation of the metric (from 4d to 3d)

Part I. Geometry

Projection from 4d to 3d

$$ds^2 = R^2 \left[\frac{dr'^2}{1 - r'^2} + r'^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

$$w = \sqrt{R^2 - (x^2 + y^2 + z^2)}$$ and $$w = R\cos\chi$$

$$w = R\cos\chi = \sqrt{R^2 - r^2} \Rightarrow r = R\sin\chi \Rightarrow r' = \frac{r}{R} = \sin\chi$$

$$\frac{dr'^2}{1 - r'^2} = \frac{(\cos\chi\, d\chi)^2}{\cos^2\chi}$$

Part I. Geometry

Projection from 4d to 3d

$$ds^2 = R^2 \left[\frac{dr'^2}{1 - r'^2} + r'^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

with $\dfrac{dr'^2}{1 - r'^2} = \dfrac{(\cos\chi \; d\chi)^2}{\cos^2\chi} = d\chi^2$ and $r' = \dfrac{r}{R} = \sin\chi$

$$ds^2 = R^2 \left[d\chi^2 + r'^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

$$ds^2 = R^2 \left[d\chi^2 + sin^2\chi \; (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

Projection from 4d to 3d

Projection from 3d to 2d

$$ds^2 = R^2 \left(\underbrace{\frac{dr^2}{1 - r'^2}}_{\text{Singularity}} + r'^2 d\theta^2 \right) \quad \text{with} \quad r' = \frac{r}{R}$$

Projection from 4d to 3d

$$ds^2 = R^2 \left[\underbrace{d\chi^2}_{\text{Singularity}} + r'^2 (d\theta^2 + \sin^2\theta d\phi^2) \right] \quad \text{with} \quad d\chi^2 = \frac{dr'^2}{1 - r'^2}$$

Part I. Geometry

Projection from 4d to 3d

$$ds^2 = R^2\left[\frac{dr'^2}{1-r'^2} + r'^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

$$ds^2 = R^2\left[d\chi^2 + sin^2\chi\ d\Omega^2\right]$$

(“chi”) is the angle associated
With 4th dimension

Adding time

$$ds^2 = -c^2dt^2 + R^2\left[d\chi^2 + \sin^2\chi\ d\Omega^2\right]$$

$c = 1$

$$ds^2 = -dt^2 + R^2\left[d\chi^2 + sin^2\chi\ d\Omega^2\right]$$

Part I. Geometry

Friedman-Robertson-LeMaitre-Walker

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + dw^2$$

$$ds^2 = -dt^2 + R^2 \left[d\chi^2 + \boxed{\sin^2\chi} \, d\Omega^2 \right]$$

$$ds^2 = -c^2 dt^2 + R^2 \left[d\chi^2 + f_k(\chi) \, d\Omega^2 \right]$$

Part I. Geometry

Projection from 4d to 3d

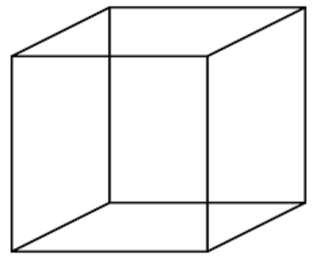
$$ds^2 = -dt^2 + R^2\left[d\chi^2 + sin^2\chi\ d\Omega^2\right]$$

Spherical metric in 4d

$$d\chi^2 = \frac{dr'^2}{1 - r'^2}$$

$$ds^2 = -dt^2 + R^2\left[d\chi^2 + \ d\Omega^2\right]$$

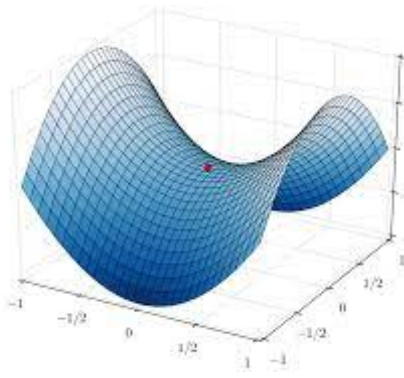
Flat metric in 4d



$$d\chi^2 = dr'^2$$

$$ds^2 = -dt^2 + R^2\left[d\chi^2 + \sinh^2\chi\ d\Omega^2\right]$$

Open metric in 4d



$$d\chi^2 = \frac{dr'^2}{1 + r'^2}$$

Generalisation to FRLW metric

$$ds^2 = -dt^2 + R^2 \left[d\chi^2 + f_k(\chi) d\Omega^2\right]$$

$$d\chi^2 = \frac{dr'^2}{1 - k\ r'^2}$$

$k = 1 \Rightarrow r' = sin\chi$ Spherical

$k = 0 \Rightarrow r' = \chi$ Flat

$k = -1 \Rightarrow r' = sinh\chi$ Hyperbolic

$f_k(\chi) = r'^2$

$$ds^2 = R^2 \left[-d\eta^2 + \delta_{ij} dx^i dx^j\right]$$

assuming homogeneous and isotropic

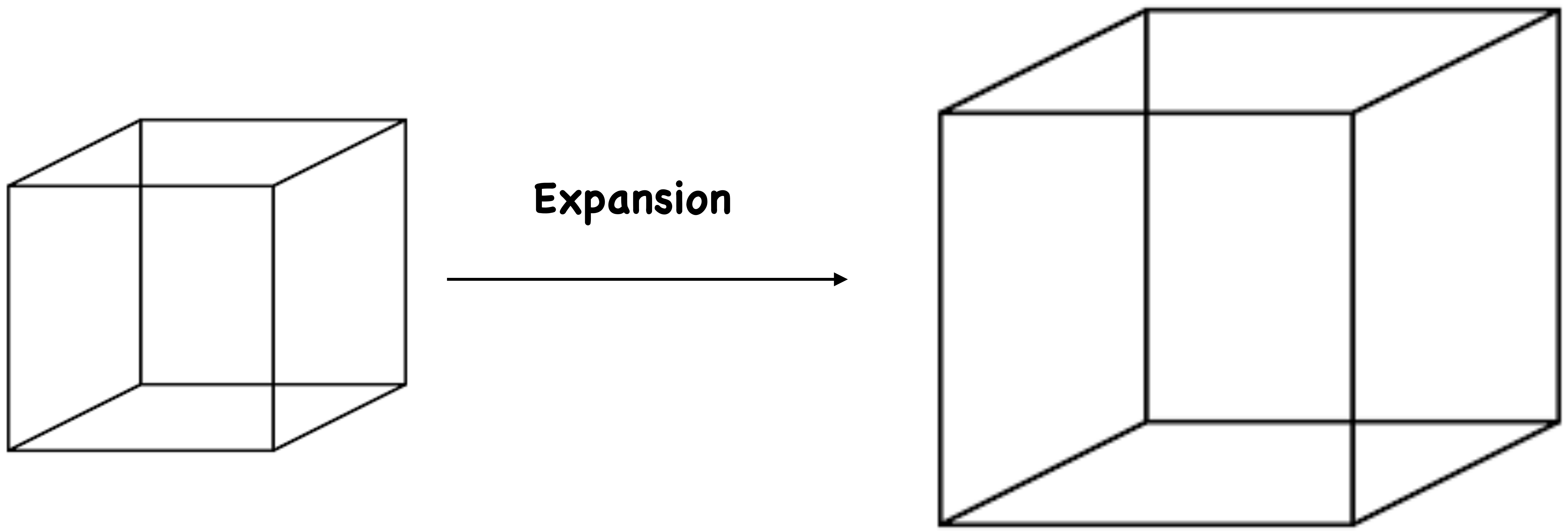
Part I. Geometry — Generalisation to FRLW metric

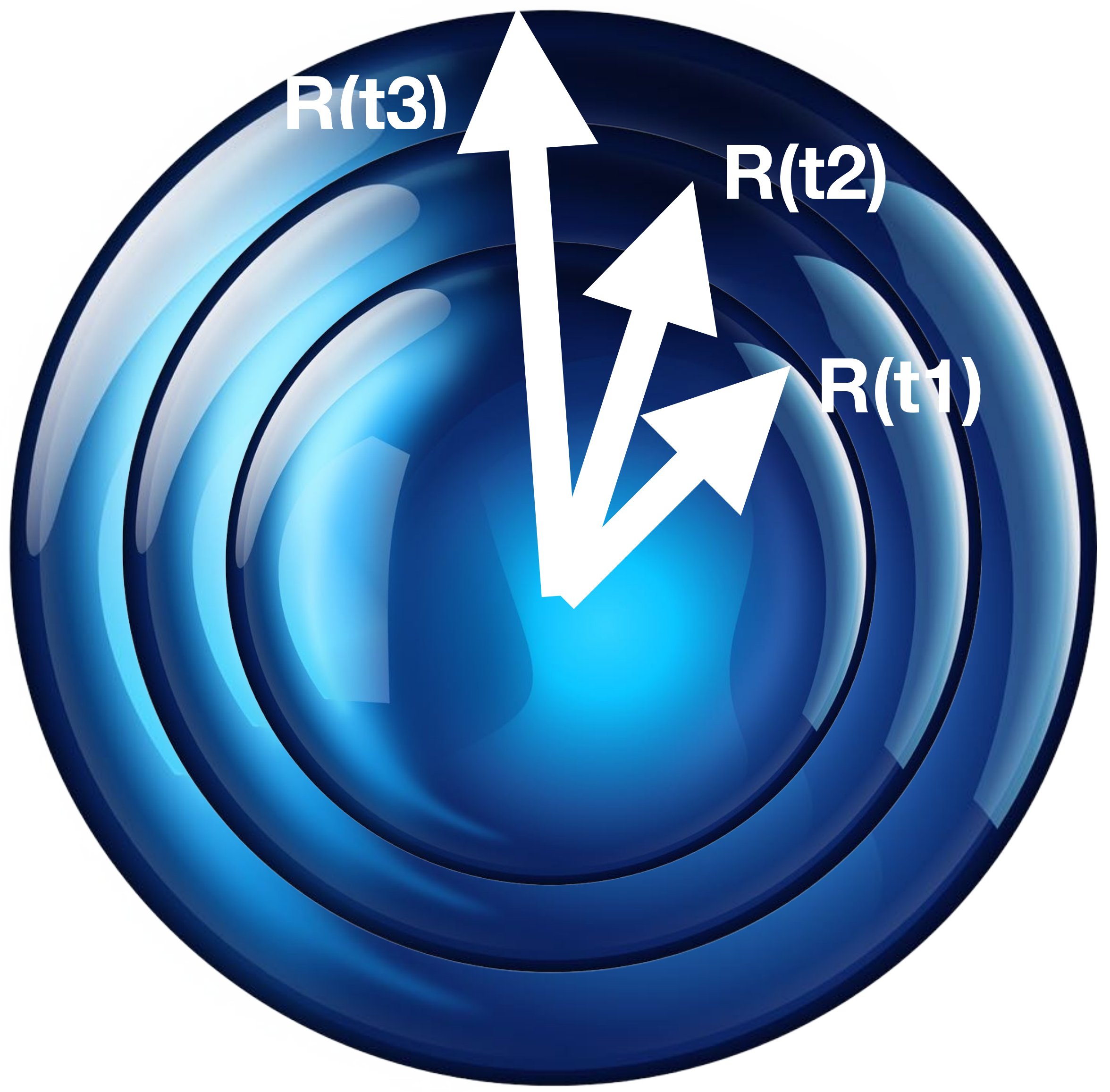
$$d\chi^2 = \frac{dr'^2}{1 - k\ r'^2}$$

Singularity in 4d but now the 4th dimension represents the time evolution projected onto the Universe at a given time.

$$ds^2 = -dt^2 + R^2 \left[d\chi^2 + f_k(\chi) d\Omega^2\right]$$

with R(t) the scale factor





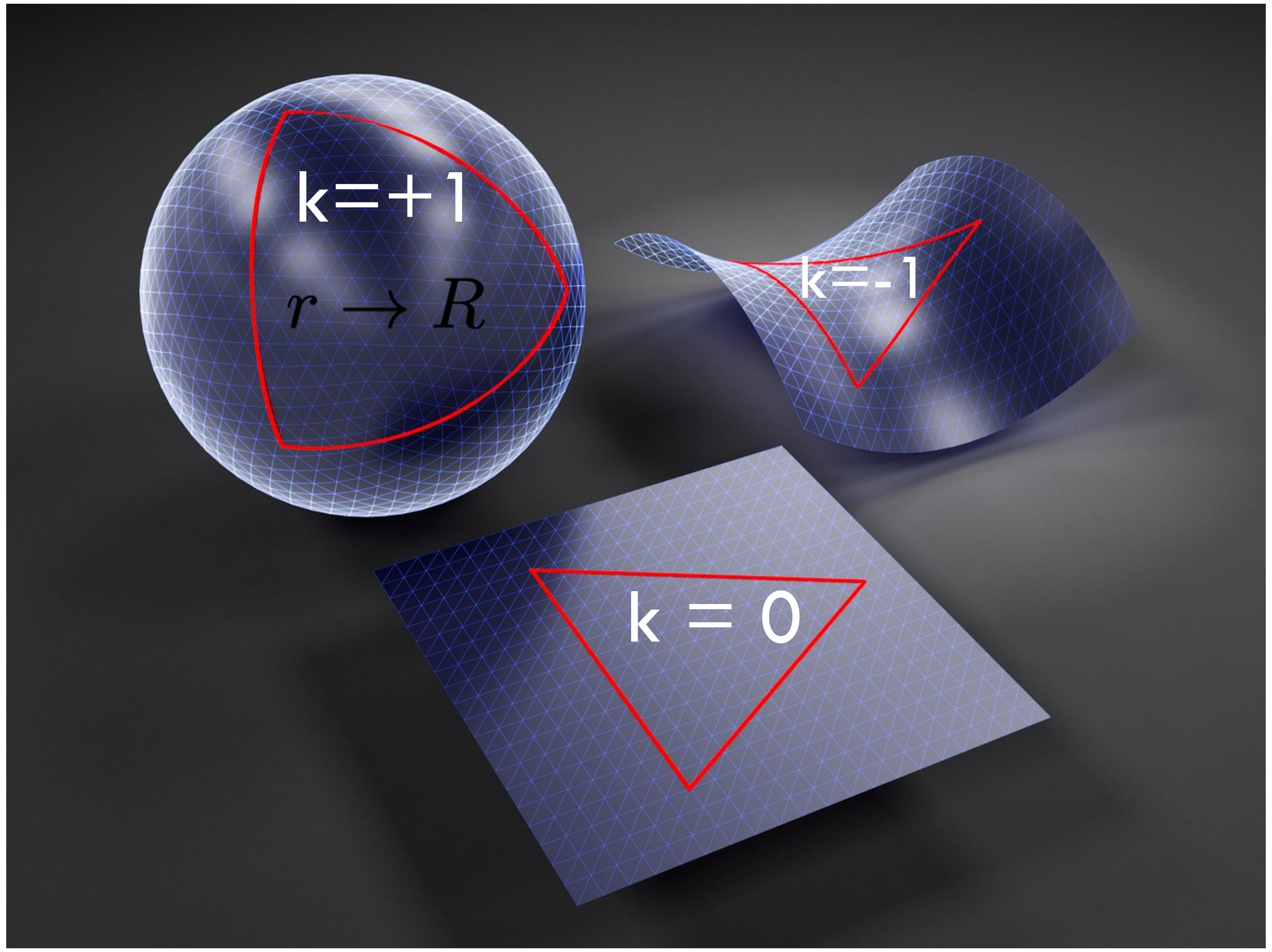
Part I. Geometry — Generalisation to FRLW metric

$$ds^2 = -dt^2 + R^2\left[d\chi^2 + f_k(\chi)d\Omega^2\right]$$

Grows with time

Independent of time evolution “comoving”

$$d\chi^2 = \frac{dr'^2}{1 - k\ r'^2}$$



Part I. Geometry

Meaning of FRLW Metric

$$ds^2 = -c^2 dt^2 + R^2 \left[d\chi^2 + f_k(\chi)\ d\Omega^2 \right]$$

Scale factor; expansion

Independent of time evolution
"comoving"

$$ds = 0$$

$$c^2 dt^2 = R^2 \left[d\chi^2 + f_k(\chi)\ d\Omega^2 \right]$$

Geometry but isotropic so not important
to define distance and time

Part I. Geometry

Consequences of FRLW Metric

$$c^2dt^2 = R^2\left[d\chi^2 + f_k(\chi)\ d\Omega^2\right]$$

$$d\chi^2 = \frac{dr'^2}{1-k\ r'^2} \quad \Rightarrow \quad c^2dt^2 = R^2 d\chi^2$$

$$|\chi| = c\int\frac{dt}{R} + cst$$

Measure the expansion

and

$$|\chi| = \int\frac{dr'}{1-kr'} + cst$$

Measure the curvature …

Part I. Geometry — Consequences of FRLW Metric

$$ds^2 = c^2 dt^2 - R^2 \left[d\chi^2 + f_k(\chi)\ d\Omega^2 \right] \qquad R \rightarrow \frac{\dot{R}}{R} \equiv \left(\frac{\dot{a}}{a} \right)$$

- The photon (i.e. light) defines **OUR** space-time!
- All your coordinates are defined with respect to the light in the Universe!

Metric = contains an information about the size of the Universe today

Part I. Geometry

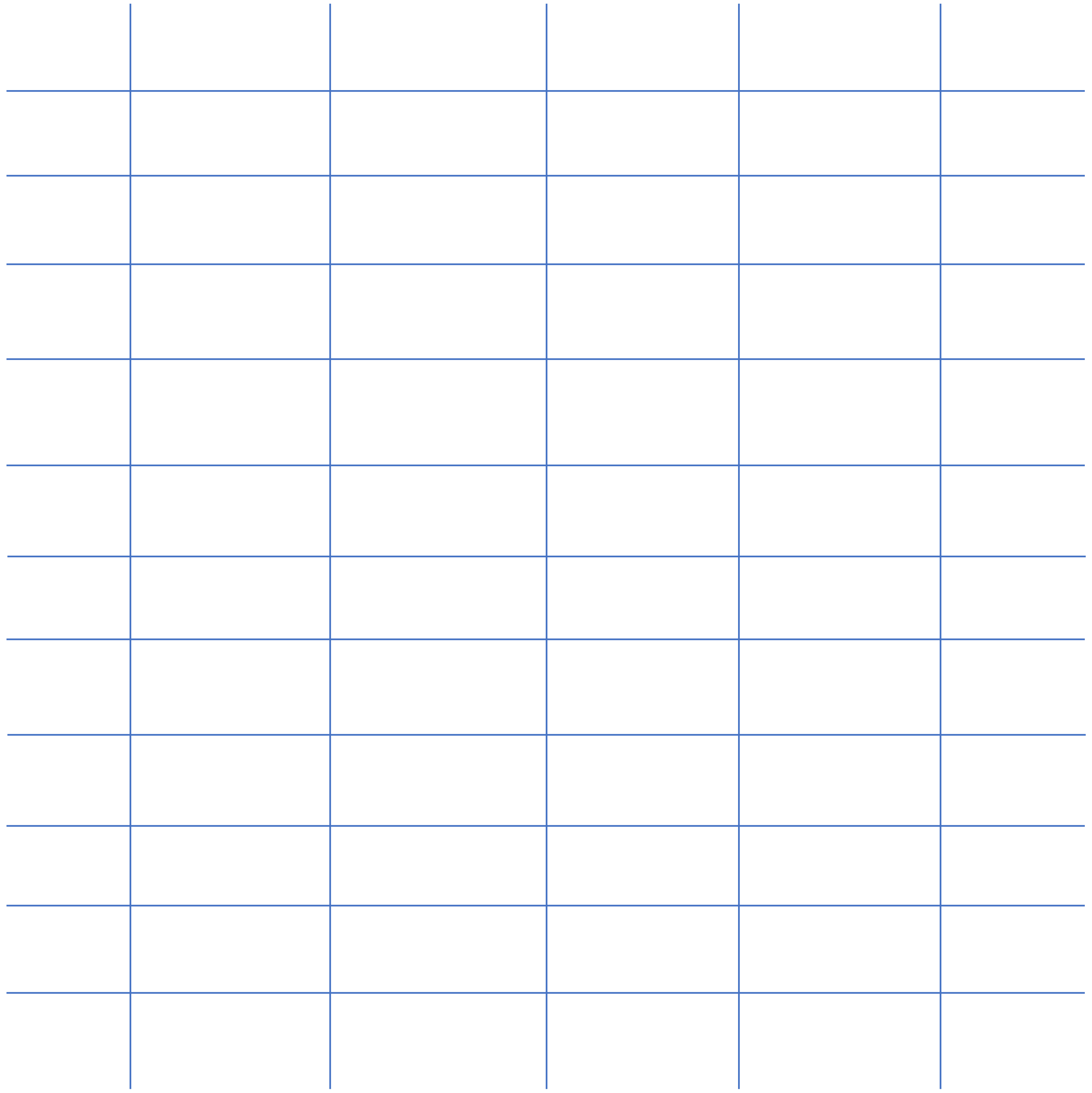
Consequences of FRLW Metric

$$ds^2 = -c^2 dt^2 + R^2 \left[d\chi^2 + f_k(\chi)\ d\Omega^2 \right]$$

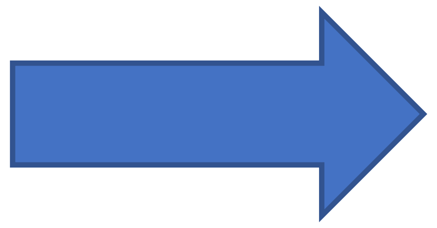
- Possible singularity which occur when r tends to R (closed Universe)
- This singularity does not exist in a flat (Euclidian/Minkowski) or hyperbolic metric
- Current paradigm: current curvature = 0 but R = R(t) and R(t=0) ~ 0.
- Analogy of a balloon that keeps growing.

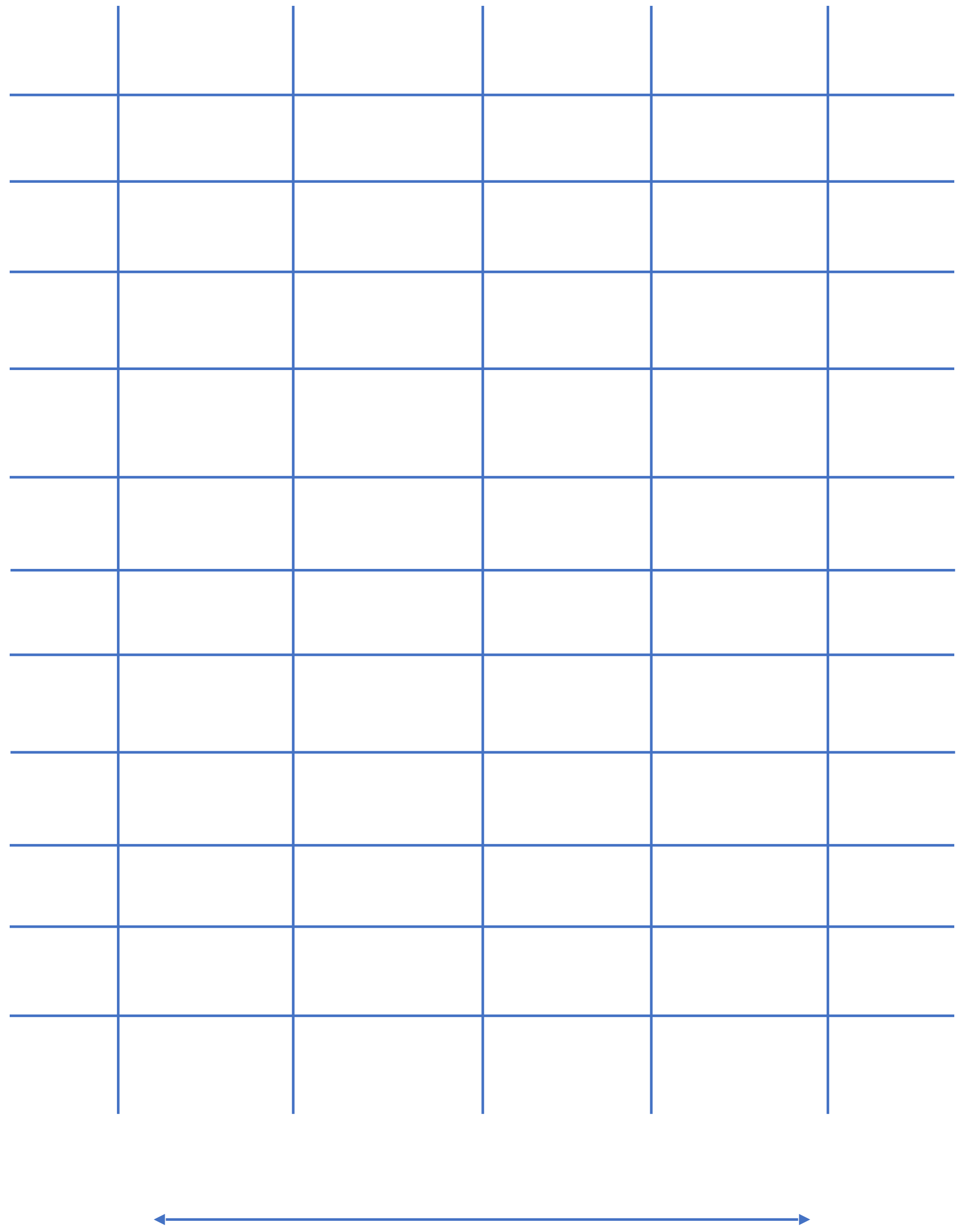
Part I. Geometry

R times original distances



expansion





Original distance that get stretched

$$l = l_{\text{comoving}}$$

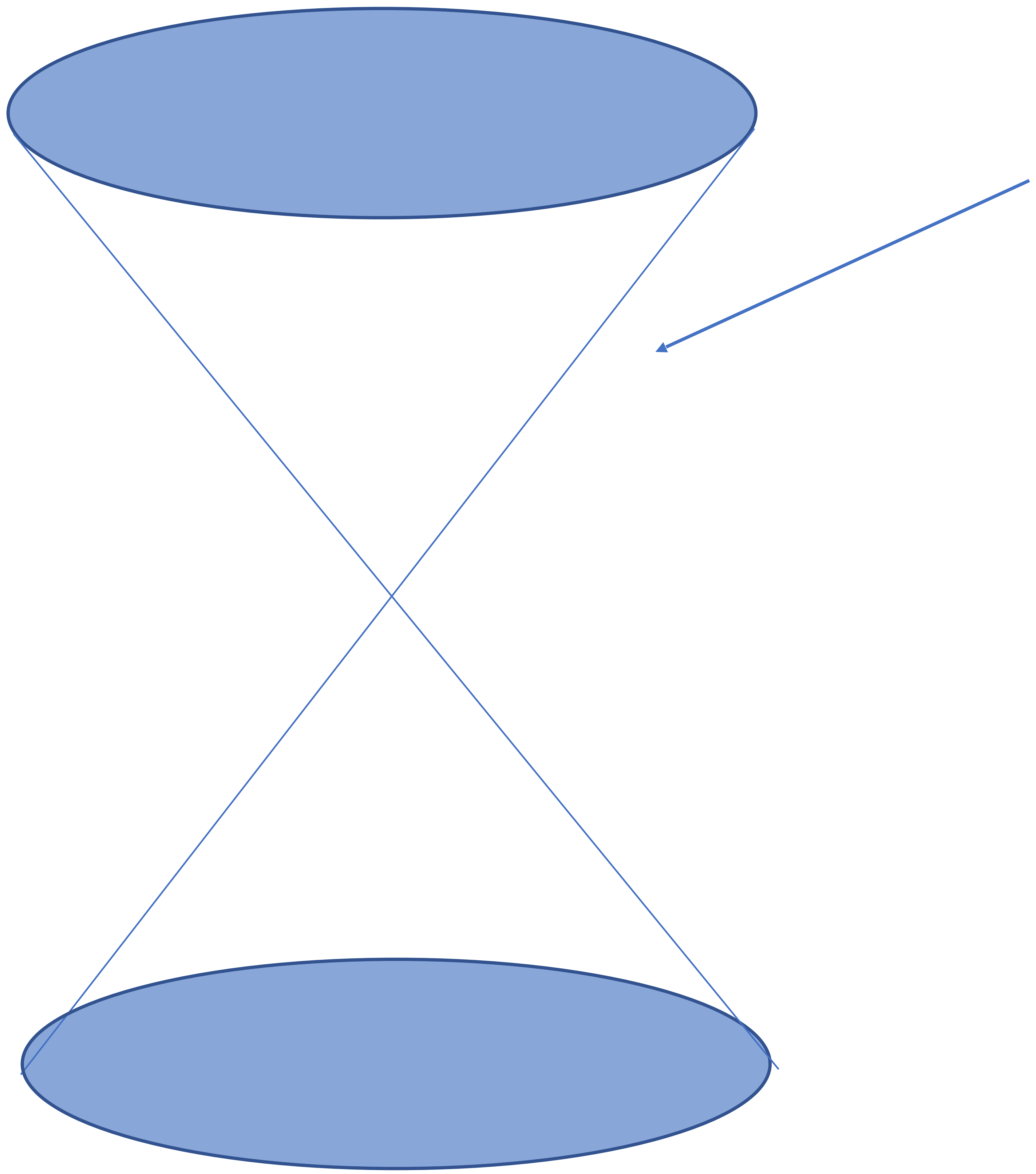
$$r = R\, l_{comoving}$$

$$ds^2 = -dt^2 + R^2 \underbrace{\left[d\chi^2 + f_k(\chi) d\Omega^2\right]}_{l_{\text{comoving}}}$$

Part I. Geometry

Horizon

$$ds^2 = -c^2dt^2 + R^2\left[d\chi^2 + f_k(\chi)\ d\Omega^2\right]$$



Light cone, ds= 0 $\quad d = c\,t$

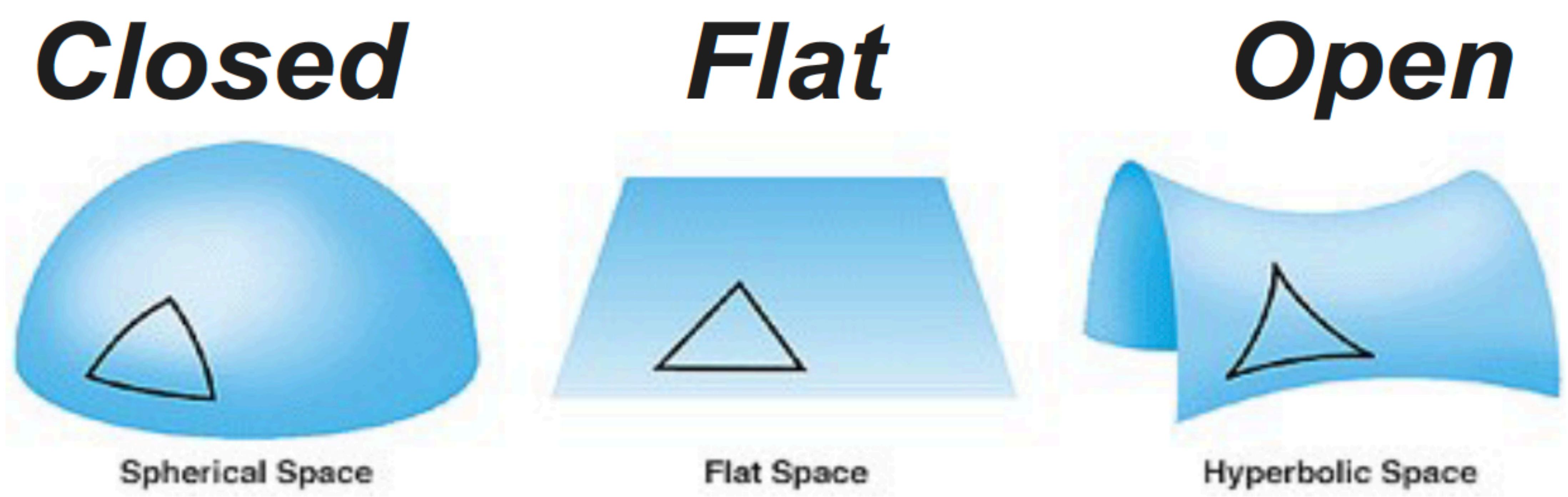
Inside cone, particles (v<c) $\quad ds^2 > 0$

Outside cone, tachyons (v>c) $\quad ds^2 < 0$

Tachyons are not physical
(we have never seen v>c)

Part I. Geometry

Summary



Curvature:	+	0	--
Sum of angles of triangle:	> 180º	= 180º	< 180º
Circumference of circle:	< 2 π *r*	= 2 π *r*	> 2 π *r*
Parallel lines:	converge	remain parallel	diverge
Size:	finite	infinite	infinite
Edge:	no	no	no

Singularity + 3 possible geometries… how do we find out which one is correct?