

Lecture II: QCD scattering theory

- ❖ Physical states of QCD, Feynman rules and color algebra
- ❖ Kinematics, colour structure and amplitudes of typical processes
- ❖ Lorentz-invariant phase space and the parton-level cross section
- ❖ Soft gluon radiation and resummation
- ❖ Hadronisation and confinement

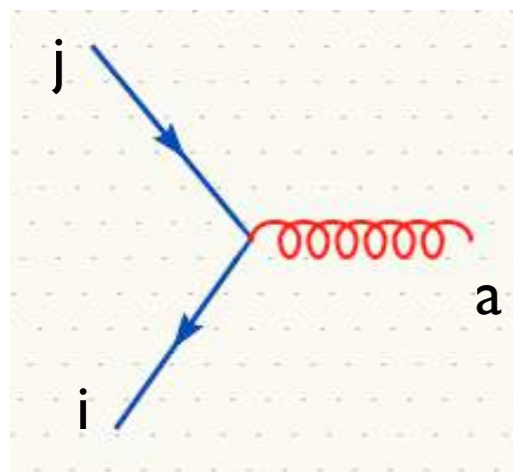
From QED to QCD interactions: 3-gluon vertex

Particles carry non-Abelian charge “color”: they transform under a nontrivial representation R of a non-Abelian group G ($=SU(3)$ for quarks!)

The standard currents operator transforms as $R \otimes \bar{R}$: the only representation that belongs to this product for any R is the adjoint representation of group G

$$3 \otimes 3^* = 1 \oplus 8$$

The field that couples to the current must transform under adjoint rep of G !



$$q\bar{q} \rightarrow gg$$

$$-ig_s t_{ij}^a \gamma^\mu$$

$$[t^a, t^b] = if^{abc} t^c$$

$$\frac{i}{g_s^2} M_g \equiv (t^b t^a)_{ij} D_1 + (t^a t^b)_{ij} D_2$$

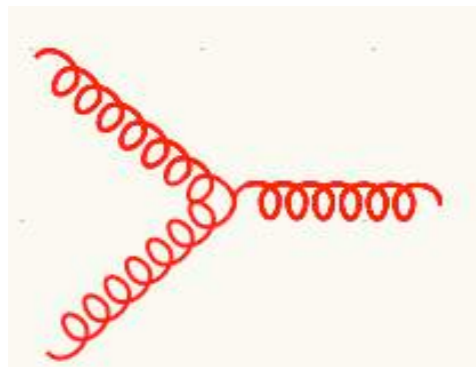
$$M_g = (t^a t^b)_{ij} M_\gamma - g^2 f^{abc} t_{ij}^c D_1$$

The gauge invariance must still work:

$$k_1^\mu \epsilon_2^\nu M_g^{\mu,\nu} = k_2^\nu \epsilon_1^\mu M_g^{\mu,\nu} = 0$$

BUT! One piece has to be compensated

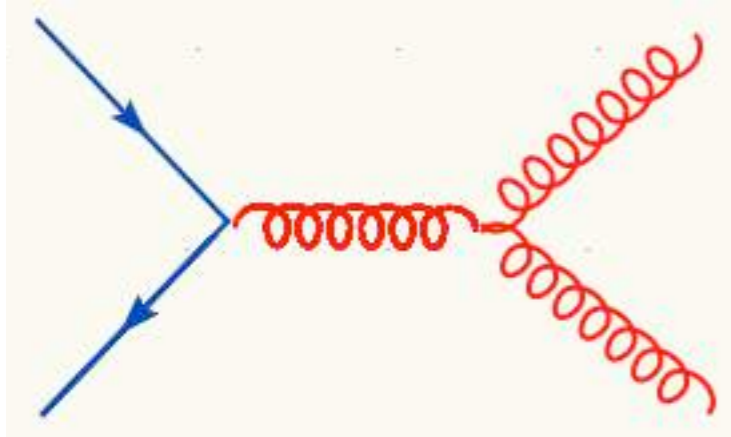
$$k_{1\mu} M_g^\mu = i(-g_s f^{abc} \epsilon_2^\mu) (-ig_s t_{ij}^c \bar{v}_i(\bar{q}) \gamma_\mu u_i(q))$$



$$-g_s f^{abc} V_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)$$

new 3-gluon vertex!

From QED to QCD interactions: 3-gluon vertex



$$-ig_s^2 D_3 = (-ig_s t_{ij}^a \bar{v}_i(\bar{q}) \gamma^\mu u_j(q)) \times \left(\frac{-i}{p^2} \right) \times (-gf^{abc} V_{\mu\nu\rho}(-p, k_1, k_2) \epsilon_1^\nu(k_1) \epsilon_2^\rho(k_2))$$

How do we write the Lorentz structure of such a new interaction?

- ❖ Lorentz invariance: built out of the terms of the form $g_{\mu_1\mu_2} p_{\mu_3}$
- ❖ Bose symmetry: V is fully anti-symmetric w.r.t. permutations $(\mu_i, p_i) \leftrightarrow (\mu_j, p_j)$
- ❖ Dimensional analysis: only one power of momentum!
- ❖ Up to an overall factor:

$$V_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) = V_0 [(p_1 - p_2)_{\mu_3} g_{\mu_1\mu_2} + (p_2 - p_3)_{\mu_1} g_{\mu_2\mu_3} + (p_3 - p_1)_{\mu_2} g_{\mu_3\mu_1}]$$

$$k_1 \cdot D_3 = g^2 f^{abc} t^c V_0 \left[\bar{v}(\bar{q}) \not{\epsilon}_2 u(q) - \frac{k_2 \cdot \epsilon_2}{2k_1 \cdot k_2} \bar{v}(\bar{q}) \not{k}_1 u(q) \right]$$

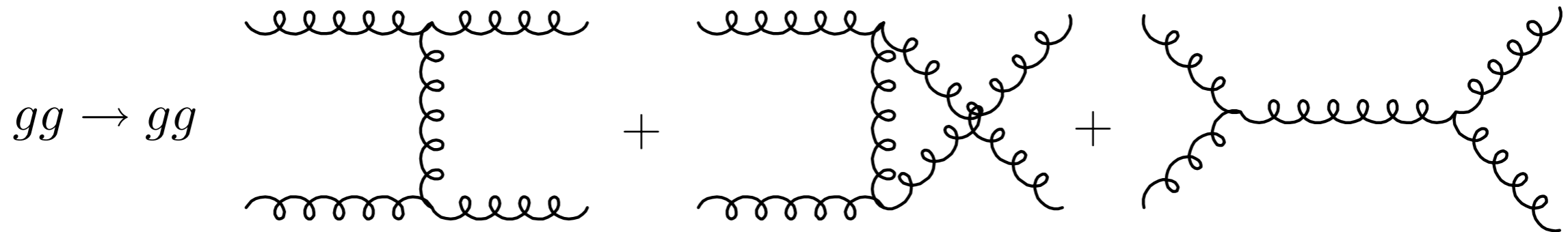
Gauge invariance is satisfied for physical external gluons:

$$k_2 \cdot \epsilon_2 = 0$$

with $V_0 = 1$

4-gluon vertex

- ❖ Similarly to the 3-gluon vertex, the gauge invariance of the gluon scattering



requires the presence of an additional contribution given by the 4-gluon vertex:

$$\begin{aligned}
 &= -ig^2 f^{xac} f^{xbd} \left(g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\delta} g^{\beta\gamma} \right) \\
 &\quad -ig^2 f^{xad} f^{xbc} \left(g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\gamma} g^{\beta\delta} \right) \\
 &\quad -ig^2 f^{xab} f^{xcd} \left(g^{\alpha\gamma} g^{\beta\delta} - g^{\alpha\delta} g^{\beta\gamma} \right)
 \end{aligned}$$

The diagram shows a four-gluon vertex with four external wavy lines. The top-left line is labeled a, α , the top-right line is labeled b, β , the bottom-left line is labeled c, γ , and the bottom-right line is labeled d, δ .

No extra vertices have to be introduced – all QCD amplitudes are automatically gauge-invariant with this set of Feynman rules!

Physical states of QCD

❖ For Abelian symmetries (such as in QED)

$$\sum_{\epsilon_1} |M|^2 = \left(\sum_{\epsilon_1} \epsilon_1^\mu \epsilon_1^{\nu*} \right) M_\mu M_\nu^*$$

ALL polarisations!

Two independent physical polarisations:

$$\epsilon_{L,R}^\mu = (0; 1, \pm i, 0) / \sqrt{2}$$

$$\epsilon_L \cdot \epsilon_L^* = -1 = \epsilon_R \cdot \epsilon_R^* \quad \epsilon_L \cdot \epsilon_R^* = 0$$

❖ Sum over physical polarisations:

$$k_\mu M^\mu = 0 \quad M_0 = M_3$$

$$k = (k_0; 0, 0, k_0) \quad \bar{k} = (k_0; 0, 0, -k_0)$$

$$\sum_{i=L,R} \epsilon_i^\mu \epsilon_i^{\nu*} \equiv \begin{pmatrix} 0 & \vec{0} \\ \vec{0} & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{pmatrix} = -g_{\mu\nu} + \frac{k_\mu \bar{k}_\nu + k_\nu \bar{k}_\mu}{k \cdot \bar{k}}$$

$$\sum_{i=L,R} |\epsilon_i \cdot M|^2 = |M_1|^2 + |M_2|^2 = |M_1|^2 + |M_2|^2 + |M_3|^2 - |M_0|^2 \equiv -g^{\mu\nu} M_\mu M_\nu^*$$

In QED: can be safely dropped!

Production of space-like and longitudinal photons cancel out: only physical states are produced!

❖ NOT true in non-Abelian theories such as QCD!!!

$$k_1 \cdot M \propto \epsilon_2 \cdot k_2$$

Vanishes ONLY for physical ϵ_2 !!!

Production of a pair of unphysical gluons is allowed in QCD!

$$\sum_{\text{non-physical}} |\epsilon_1^\mu \epsilon_2^\nu M_{\mu\nu}|^2 = \left| i g^2 f^{abc} \lambda^c \frac{1}{2k_1 k_2} \bar{v}(\bar{q}) \not{k}_1 u(q) \right|^2$$

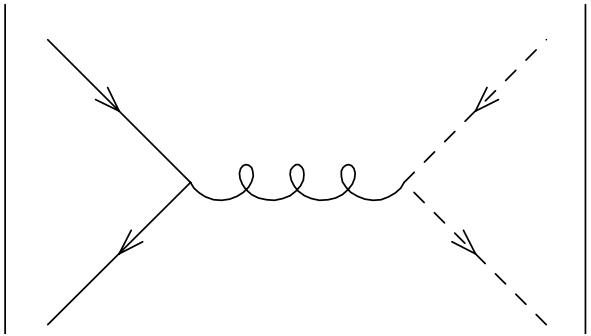
The role of ghosts

- ❖ The choice of physical gauges: remove ALL unphysical d.o.f. from the final states (and off-shell propagators they are coupled to) explicitly imposing

$$\sum \epsilon_\mu \epsilon_\nu^* = -g_{\mu\nu} \quad \underline{\text{Unitarity is imposed!}}$$

- ❖ Alternatively, gauge fixing in non-physical gauges (e.g. covariant gauge): appearance of two colour octet scalar d.o.f. only coupling to gluons (their loop comes with -1 as if they obey Fermi statistics!) which are pair-produced and exist in loops

- ❖ Due to violation of the spin-statistics theorem, the ghosts lead to negative probability contributions necessary to cancel unphysical gluons' effect!



$$\left| \text{Diagram} \right|^2 = - \left| i g^2 f^{abc} \lambda^c \frac{1}{2k_1 k_2} \bar{v}(\bar{q}) \not{k}_1 u(q) \right|^2 \quad \underline{\text{Unitarity is restored!}}$$

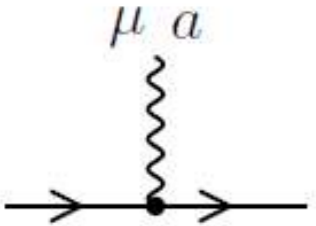
exactly cancels the unphysical gluon term!

QCD Feynman rules

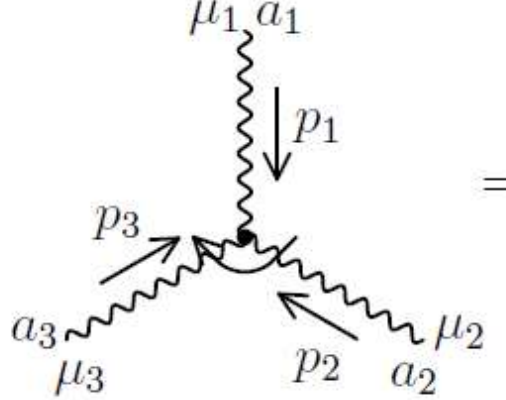
Free fields:

- $u(p, s)$ incoming fermion
- $\bar{u}(p, s)$ outgoing fermion
- $\bar{v}(p, s)$ incoming antifermion
- $v(p, s)$ outgoing antifermion
- $\epsilon(p, s)$ incoming vector boson
- $\epsilon^*(p, s)$ outgoing vector boson

Bare vertices:

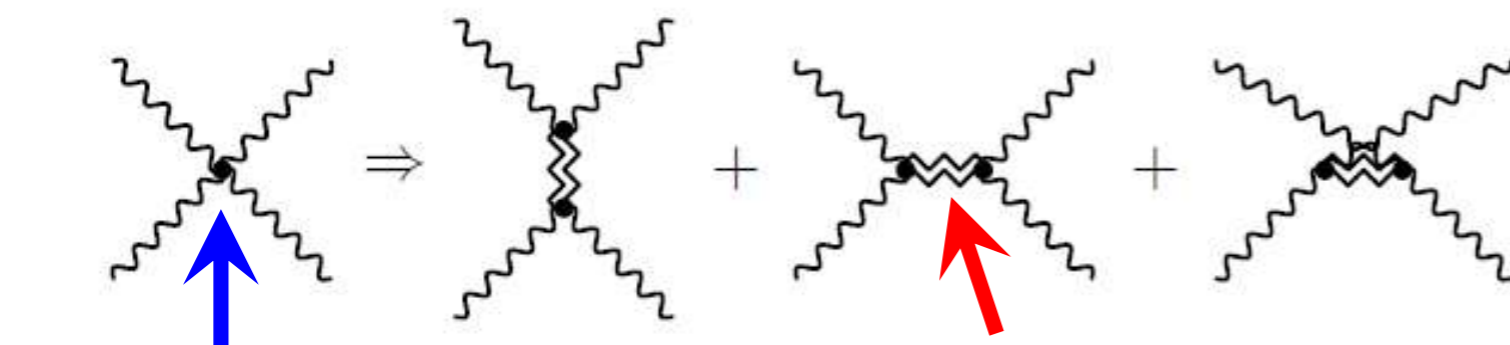


$$= t^a \times ig_0 \gamma^\mu$$



$$= if^{a_1 a_2 a_3} \times ig_0 V^{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)$$

$$V^{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3) = (p_3 - p_2)^{\mu_1} g^{\mu_2 \mu_3} + (p_1 - p_3)^{\mu_2} g^{\mu_3 \mu_1} + (p_2 - p_1)^{\mu_3} g^{\mu_1 \mu_2}$$



second order in coupling

auxiliary field

in order to define the gluon propagator \rightarrow fix the gauge!

popular Feynman gauge: $a_0 = 1$

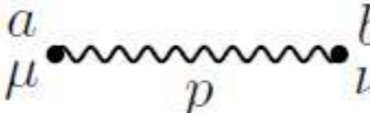
Free propagators in covariant gauges:



$$= iS_0(p)$$

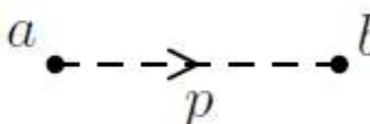
$$S_0(p) = \frac{1}{\not{p}} = \frac{\not{p}}{p^2}$$

massless quark!



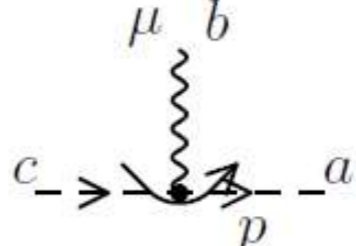
$$= -i\delta^{ab} D_{\mu\nu}^0(p)$$

$$D_{\mu\nu}^0(p) = \frac{1}{p^2} \left[g_{\mu\nu} - (1 - a_0) \frac{p_\mu p_\nu}{p^2} \right]$$




$$= i\delta^{ab} G_0(p)$$

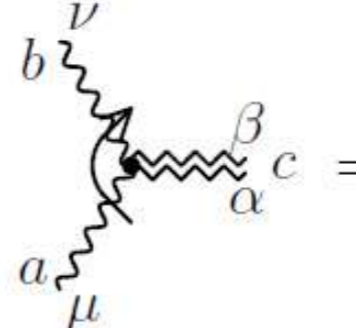
$$G_0(p) = \frac{1}{p^2}$$



$$= if^{abc} \times ig_0 p^\mu$$



$$= i\delta^{ab} (g^{\mu\alpha} g^{\nu\beta} - g^{\mu\beta} g^{\nu\alpha})$$



$$= if^{abc} \times g_0 g^{\mu\alpha} g^{\nu\beta}$$

Covariant gauges:

ghost: scalar field obeying Fermi statistics

Color factors calculus: Cvitanović algorithm

Graphical representation of the color factors:

...eliminating gluons from color diagrams!

$$\begin{aligned} \text{Tr } 1 &= N_c & \text{or } \text{circle} &= N_c, \\ \text{Tr } t^a &= 0 & \text{or } \text{circle with wavy line} &= 0, \\ \text{Tr } t^a t^b &= T_F \delta^{ab} & \text{or } \text{circle with two wavy lines} &= T_F \text{ wavy line} \end{aligned}$$

$$(t^a)^i_j (t^a)^k_l = T_F \left[\delta_l^i \delta_j^k - \frac{1}{N_c} \delta_j^i \delta_l^k \right]$$

$$\text{quark-antiquark loop} = T_F \left[\text{quark-antiquark} - \frac{1}{N_c} \text{color singlet} \right]$$

Quark-antiquark Color singlet

Example 1: number of gluons

$$\begin{aligned} N_g &= \text{gluon loop} = \frac{1}{T_F} \text{quark-antiquark loop} = \frac{1}{T_F} \text{color singlet loop} \\ &= \text{quark-antiquark loop} - \frac{1}{N_c} \text{circle} = N_c^2 - 1, \end{aligned}$$

Example 2: Casimir operator in the fundamental rep.

$$\begin{aligned} \text{quark self-energy} &= T_F \left[\text{quark-antiquark loop} - \frac{1}{N_c} \text{circle} \right] \\ &= C_F \text{ quark line} \quad \text{or } t^a t^a = C_F \quad C_F = T_F \left(N_c - \frac{1}{N_c} \right) \\ &\quad (t^a t^a)_{ij} = C_F \delta_{ij} \end{aligned}$$

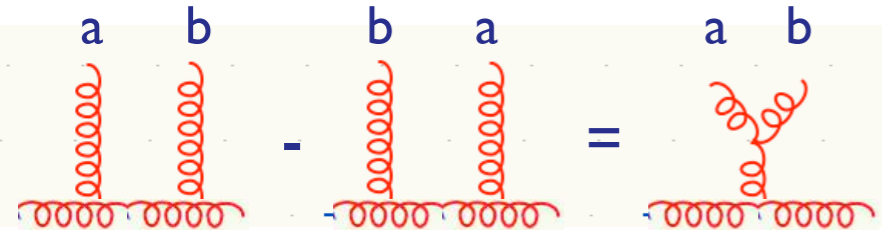
...eliminating 3-gluon vertices!

Example 3: Casimir operator in the adjoint rep.

$$\text{3-gluon vertex} = \frac{1}{T_F} \left[\text{quark-antiquark loop with 3 wavy lines} - \text{quark-antiquark loop with 3 wavy lines} \right]$$

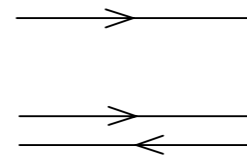
$$\begin{aligned} \text{gluon loop} &= C_A \text{ wavy line} \quad \text{or } i f^{acd} i f^{bdc} = C_A \delta^{ab} \\ i f^{abc} (t^b t^c)_{ij} &= \frac{C_A}{2} t^a_{ij} \quad C_A = 2 T_F N_c \end{aligned}$$

More color algebra



Generators in the adjoint representation satisfy:

$$[F^a, F^b] = i f^{abc} F^c$$



fermion
gluon

In the large N-limit
gluons behaves
as a quark-antiquark
pair!

$$\frac{1}{\sqrt{2}} \left(\begin{array}{c} \text{Feynman diagram 1} \\ - \frac{1}{N} \text{Feynman diagram 2} \end{array} \right)$$

Fermion-Gluon Vertex (t^a)

$$\frac{1}{\sqrt{2}} \left(\begin{array}{c} \text{Feynman diagram 3} \\ - \text{Feynman diagram 4} \end{array} \right)$$

3-Gluon Vertex (f^{abc})

$$i \xrightarrow{\lambda^a} \text{gluon loop} \xrightarrow{\lambda^a} j = \frac{1}{\sqrt{2}} \left(\begin{array}{c} \text{Feynman diagram 1} \\ - \frac{1}{N} \text{Feynman diagram 2} \end{array} \right) \times \frac{1}{\sqrt{2}} \left(\begin{array}{c} \text{Feynman diagram 1} \\ - \frac{1}{N} \text{Feynman diagram 2} \end{array} \right)$$

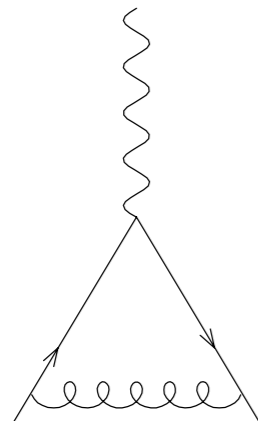
$$= \frac{1}{2} \left(\begin{array}{c} \text{Feynman diagram 5} \\ - \frac{1}{N} \text{Feynman diagram 6} \\ - \frac{1}{N} \text{Feynman diagram 7} \end{array} \right) +$$

$$\left. \frac{1}{N^2} \text{Feynman diagram 8} \right) = \delta^{ij} \frac{N^2 - 1}{2N}$$

Many algorithms and codes
work in the large-N limit
(e.g. parton showers)

Even more color algebra

- ❖ The gluon exchange between the quark and anti-quark in the color-singlet state:



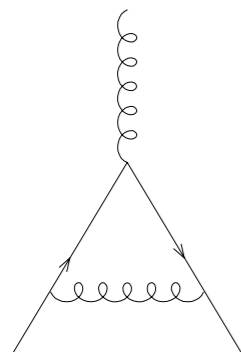
$$= \frac{1}{2} \left(\text{triangle diagram} - \frac{1}{N} \text{triangle diagram} \right) = \frac{1}{2} \frac{N^2 - 1}{N} \delta_{ij} = C_F \delta_{ij}$$

Bound states are formed,
with attractive potential

has positive sign, so the interaction is attractive!

$$V(r) \simeq -C_F \frac{\alpha_S(1/r)}{r}$$

- ❖ The gluon exchange between the quark and anti-quark in the color-octet state:



$$= \frac{1}{\sqrt{2}} \left(\text{triangle diagram} - \frac{1}{N} \text{triangle diagram} \right) \times \frac{1}{2} \left(\text{triangle diagram} - \frac{1}{N} \text{triangle diagram} \right)$$

$$= \frac{1}{2\sqrt{2}} \left(\text{triangle diagram} - \frac{1}{N} \text{circle diagram} - \frac{1}{N} \text{triangle diagram} + \frac{1}{N^2} \text{triangle diagram} \right)$$

$$= -\frac{1}{2N} \frac{1}{\sqrt{2}} \left(\text{triangle diagram} - \frac{1}{N} \text{triangle diagram} \right) = -\frac{1}{2N} \text{triangle diagram}$$

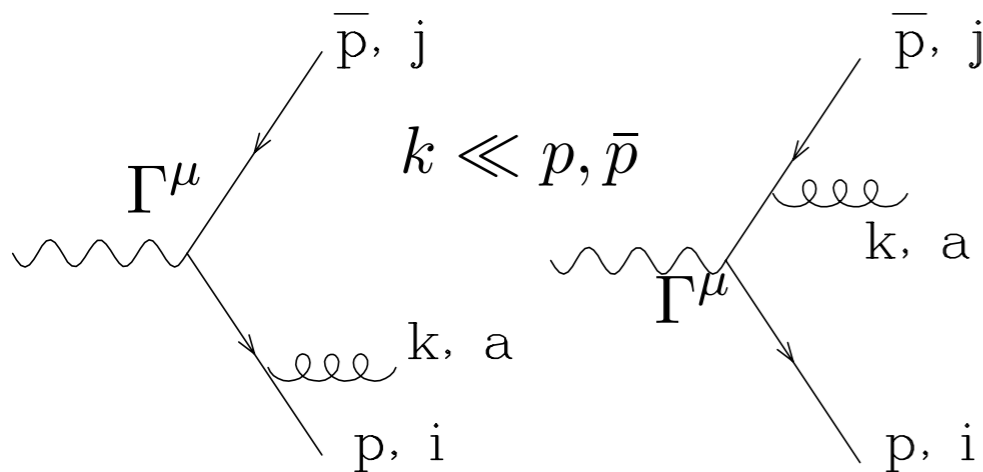
has negative sign,
so the interaction
is repulsive!

$$(t^b t^a t^b)_{ij} = (C_F - \frac{C_A}{2}) t^a_{ij}$$

No colour-octet
bound states
exist!

Soft gluon emission off a singlet quark-antiquark pair

- ❖ Emission of soft (=long wavelength) gluons plays a fundamental role in evolution of the final state (e.g. it defines final-state multiplicity of hadrons)
- ❖ Soft gluons are insensitive to the spin of the partons and cannot distinguish features of the interactions taking place at time scales shorter than their wavelengths (phenomenon of QCD factorisation)



Amplitude:

$$A_{soft} = \bar{u}(p)\epsilon(k)(ig)\frac{-i}{\not{p} + \not{k}}\Gamma^\mu v(\bar{p})\lambda_{ij}^a + \bar{u}(p)\Gamma^\mu\frac{i}{\not{p} + \not{k}}(ig)\epsilon(k)v(\bar{p})\lambda_{ij}^a$$

$$= \left[\frac{g}{2p \cdot k} \bar{u}(p)\epsilon(k)(\not{p} + \not{k})\Gamma^\mu v(\bar{p}) - \frac{g}{2\bar{p} \cdot k} \bar{u}(p)\Gamma^\mu(\not{p} + \not{k})\epsilon(k)v(\bar{p}) \right] \lambda_{ij}^a$$

- ❖ Using the Dirac equation

$$A_{soft} = g\lambda_{ij}^a \left(\frac{p \cdot \epsilon}{p \cdot k} - \frac{\bar{p} \cdot \epsilon}{\bar{p} \cdot k} \right) A_{Born} \quad \text{Factorises!}$$

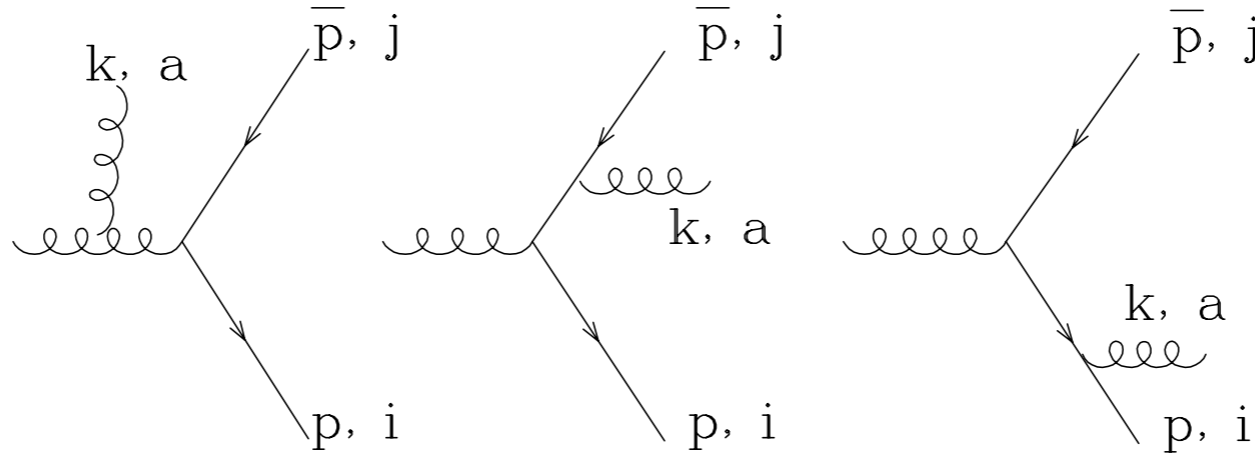
- ❖ The Feynman rule for the soft photon radiation

$$p, j \xrightarrow{\text{gluon}} p, i = g \lambda_{ij}^a 2p^\mu$$

- ❖ Analogically

$$c, \nu \xrightarrow{\text{gluon}} b, \rho = igf^{abc} 2p^\mu g^{\nu\rho}$$

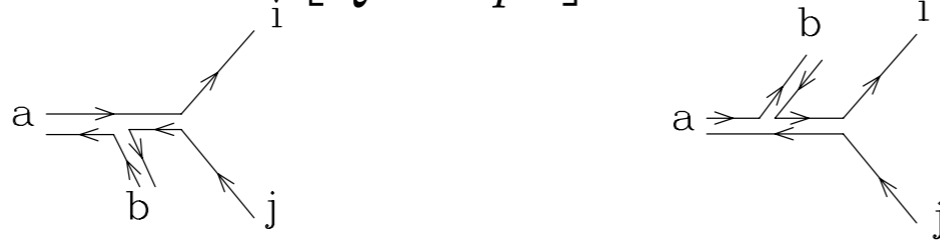
Soft gluon emission off an octet quark-antiquark pair



$$\stackrel{k \rightarrow 0}{=} \left[i g f^{abc} \lambda_{ij}^c \left(\frac{Q\epsilon}{Qk} \right) + g (\lambda^b \lambda^a)_{ij} \left(\frac{p\epsilon}{pk} \right) - g (\lambda^a \lambda^b)_{ij} \left(\frac{\bar{p}\epsilon}{pk} \right) \right] A_{Born}$$

$$= g (\lambda^a \lambda^b)_{ij} \left[\frac{Q\epsilon}{Qk} - \frac{\bar{p}\epsilon}{pk} \right] + g (\lambda^b \lambda^a)_{ij} \left[\frac{p\epsilon}{pk} - \frac{Q\epsilon}{Qk} \right]$$

Color flow:

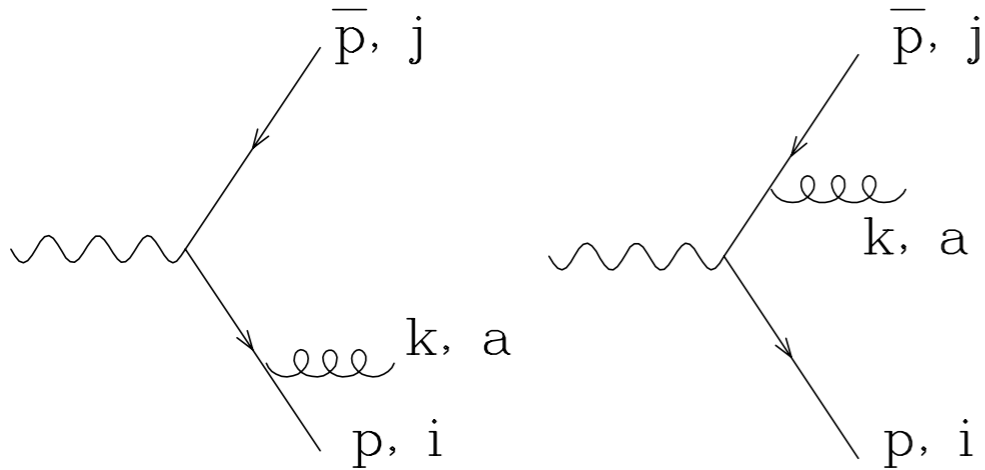


In the emission probability, the interference terms are color-suppressed:

$$\sum_{a,b,i,j} |(\lambda^a \lambda^b)_{ij}|^2 = \sum_{a,b} \text{tr} (\lambda^a \lambda^b \lambda^b \lambda^a) = \frac{N^2 - 1}{2} C_F = \mathcal{O}(N^3)$$

$$\sum_{a,b,i,j} (\lambda^a \lambda^b)_{ij} [(\lambda^b \lambda^a)_{ij}]^* = \sum_{a,b} \text{tr}(\lambda^a \lambda^b \lambda^a \lambda^b) = \frac{N^2 - 1}{2} \underbrace{\left(C_F - \frac{C_A}{2} \right)}_{-\frac{1}{2N}} = \mathcal{O}(N)$$

Angular ordering



$$\theta_{\alpha\beta} = \theta_\alpha - \theta_\beta \quad \begin{array}{l} i, j, k \\ q, \bar{q}, g \end{array}$$

Including the gluon phase space:

$$\begin{aligned} d\sigma_g &= \sum |A_{soft}|^2 \frac{d^3k}{(2\pi)^3 2k^0} \\ &= \sum |A_0|^2 \frac{-2p^\mu \bar{p}^\nu}{(pk)(\bar{p}k)} g^2 \sum \epsilon_\mu \epsilon_\nu^* \frac{d^3k}{(2\pi)^3 2k^0} \\ &= d\sigma_0 \frac{2(p\bar{p})}{(pk)(\bar{p}k)} g^2 C_f \left(\frac{d\phi}{2\pi} \right) \frac{k^0 dk^0}{8\pi^2} d \cos \theta \\ &= d\sigma_0 \frac{\alpha_s C_F}{\pi} \frac{dk^0}{k^0} \frac{d\phi}{2\pi} \frac{1 - \cos \theta_{ij}}{(1 - \cos \theta_{ik})(1 - \cos \theta_{jk})} d \cos \theta \end{aligned}$$

The radiation probabilities from the quark and antiquark lines

$$\frac{1 - \cos \theta_{ij}}{(1 - \cos \theta_{ik})(1 - \cos \theta_{jk})} = \frac{1}{2} \left[\frac{\cos \theta_{jk} - \cos \theta_{ij}}{(1 - \cos \theta_{ik})(1 - \cos \theta_{jk})} + \frac{1}{1 - \cos \theta_{ik}} \right] + \frac{1}{2} [i \leftrightarrow j] \equiv W_{(i)} + W_{(j)}$$

Azimuthal average around q-direction

...are singular in the limit of parallel emission off the respective (anti)quark!

$$\int \frac{d\phi}{2\pi} W_{(i)} = \begin{cases} \frac{1}{1 - \cos \theta_{ik}} & \text{if } \theta_{ik} < \theta_{ij} \\ 0 & \text{otherwise} \end{cases}$$

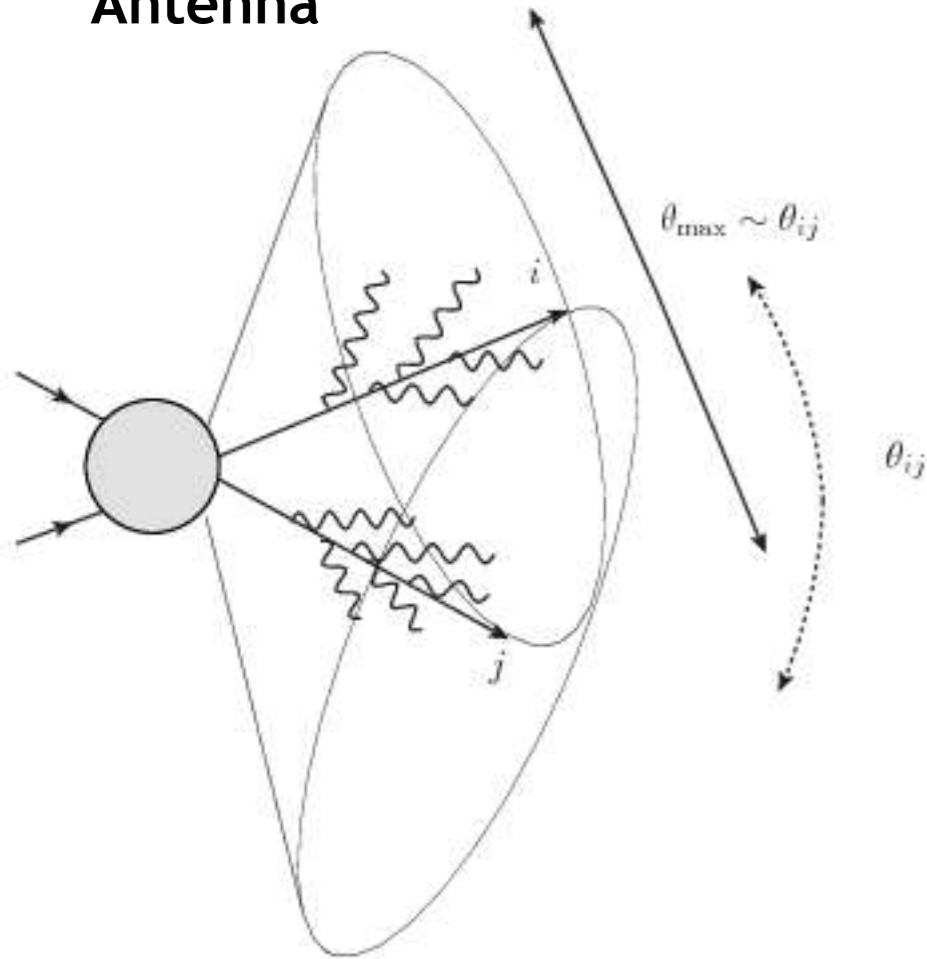
(i) No emission outside the two cones obtained by rotating the antiquark direction around the quark one, and vice versa (quantum coherence)

$$\int \frac{d\phi}{2\pi} W_{(j)} = \begin{cases} \frac{1}{1 - \cos \theta_{jk}} & \text{if } \theta_{jk} < \theta_{ij} \\ 0 & \text{otherwise} \end{cases}$$

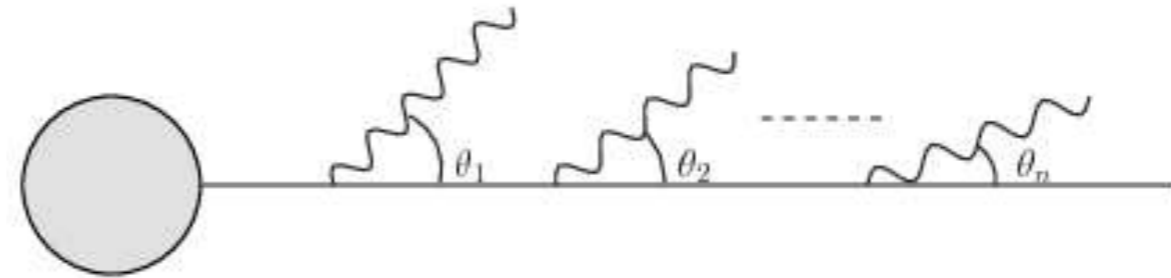
(ii) Uncorrelated emission inside the cones!

Angular ordering

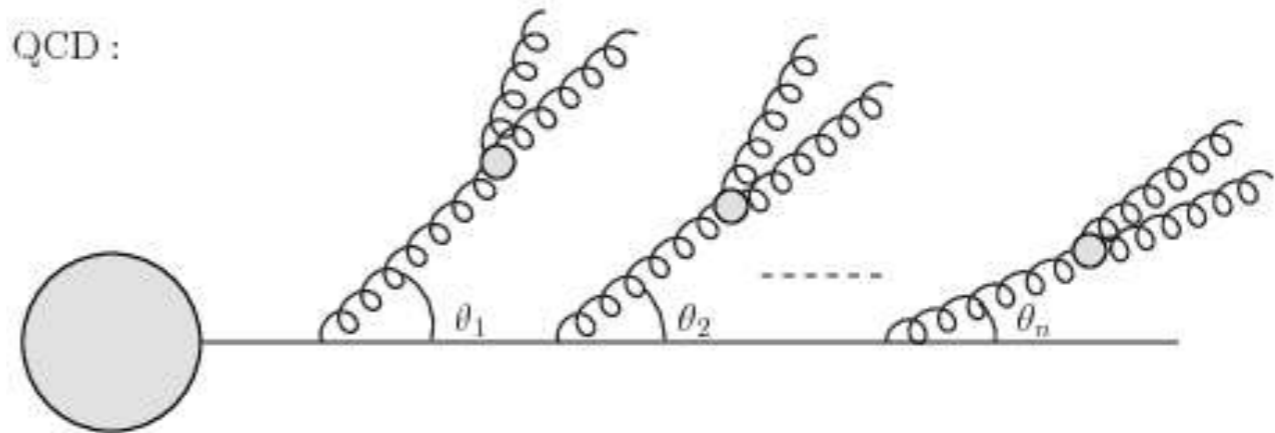
“Antenna”



QED :

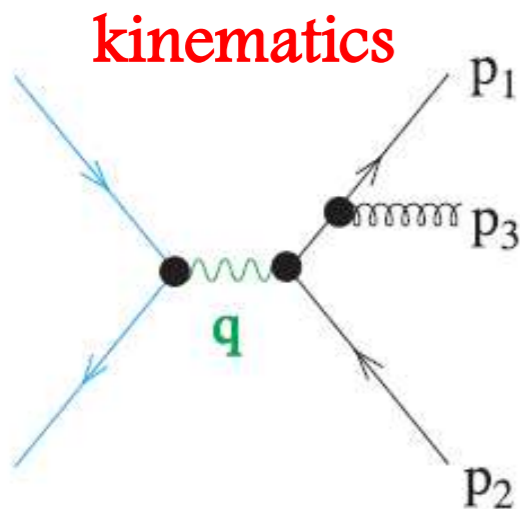


QCD :



- ❖ Color always flows directly from the emitting parton to the emitted one
- ❖ The radiation gets softer at later stages of the evolution
- ❖ Partons forming the color-singlet clusters appear to be close in phase space
- ❖ Hadronisation happens locally inside the jet: only pairs of nearby partons are involved!

Soft and collinear divergences



$q^\mu q_\mu = s$ *c.m.s energy squared*
 $E_i = p_i^0$ *final states energies*

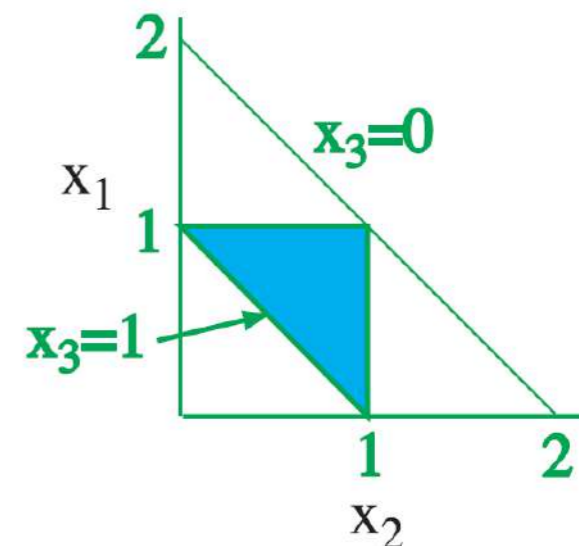
energy fractions
 $x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2p_i \cdot q}{s} \quad 0 \leq x_i \leq 1$
 $\sum_i x_i = \frac{2(\sum p_i) \cdot q}{s} = 2 \quad x_3 = 2 - x_1 - x_2$

angles between final momenta

$x_1 x_2 (1 - \cos \theta_{12}) = 2(1 - x_3)$
 $x_2 x_3 (1 - \cos \theta_{23}) = 2(1 - x_1)$
 $x_3 x_1 (1 - \cos \theta_{31}) = 2(1 - x_2)$

collinear partons

$\theta_{12} \rightarrow 0 \Leftrightarrow x_3 \rightarrow 1$
 $\theta_{23} \rightarrow 0 \Leftrightarrow x_1 \rightarrow 1$
 $\theta_{31} \rightarrow 0 \Leftrightarrow x_2 \rightarrow 1$



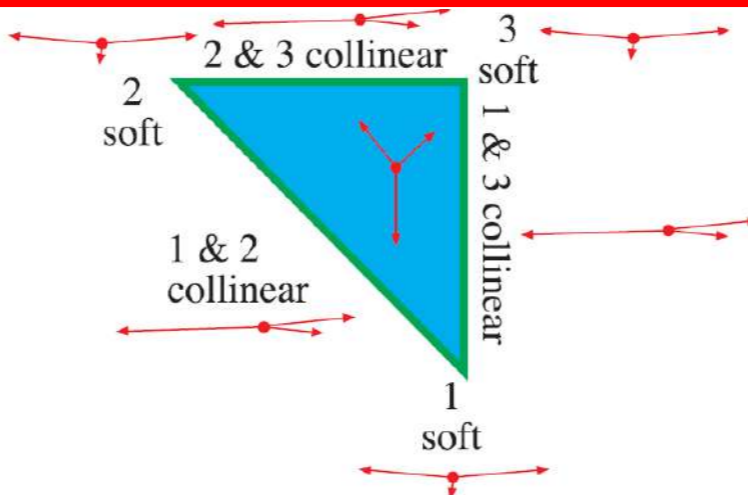
Differential 2→3 cross section

$$\frac{1}{\sigma_0} \frac{d\sigma}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$\sigma_0 = (4\pi\alpha^2/s) \sum Q_f^2 \quad C_F = 4/3$

Collinear divergences

$(1 - x_1) \rightarrow 0$
 $(1 - x_2) \rightarrow 0$



Soft divergence

$x_3 \rightarrow 0$

$\frac{1}{\sigma_0} \frac{d\sigma}{dE_3 d \cos \theta_{31}} = \frac{\alpha_s}{2\pi} C_F \frac{f(E_3, \theta_{31})}{E_3 (1 - \cos \theta_{31})}$

$d\sigma \sim \int E_3 dE_3 d\theta_{31}^2 d\phi \left[\frac{\theta_{31}}{E_3 \theta_{31}^2} \right]^2 \sim \int \frac{dE_3}{E_3} \frac{d\theta_{31}^2}{\theta_{31}^2} d\phi$

double log behavior!

Cancelation of soft/collinear divergences

- ❖ Correction to the $e^+e^- \rightarrow q\bar{q}$ cross section due to soft one-gluon emission

$$d\sigma_g = \sigma_0 \frac{2\alpha_s}{\pi} C_F \frac{dk_0}{k_0} \frac{d\cos\theta}{1 - \cos^2\theta} \quad q \cdot \bar{q} = 2q_0\bar{q}_0, \quad q \cdot k = q_0k_0(1 - \cos\theta),$$

soft div.
collinear div.

$$\bar{q}k = \bar{q}_0k_0(1 + \cos\theta)$$

- ❖ Since the total cross section is finite, the soft and collinear divergences must cancel!
- ❖ Such cancellation indeed happens when summed together with the virtual contribution which is the soft gluon limit has the form

$$\frac{d^2\sigma_v}{dk_0 d\cos\theta} = -\sigma_0 \frac{2\alpha_s}{\pi} C_F \int_0^{\sqrt{s/2}} \frac{dk'_0}{k'_0} \int_{-1}^1 \frac{d\cos\theta'}{(1 - \cos^2\theta')} \times \frac{1}{2} \delta(k_0) [\delta(1 - \cos\theta) + \delta(1 + \cos\theta)]$$

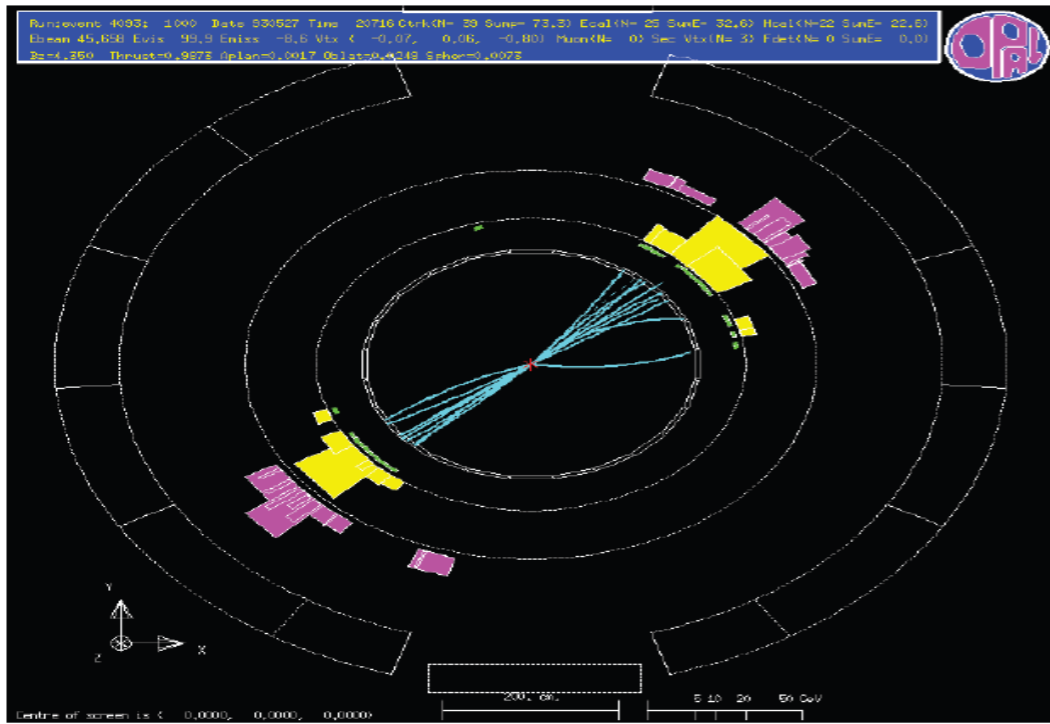
...plus finite terms

such that

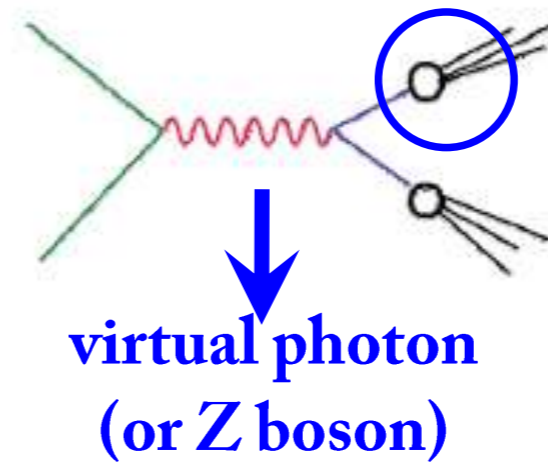
$$\int_0^{\sqrt{s/2}} dk_0 \int_{-1}^1 d\cos\theta \left[\frac{d^2\sigma_g}{dk_0 d\cos\theta} + \frac{d^2\sigma_v}{dk_0 d\cos\theta} \right] = \text{finite}$$

Jets in electron-positron annihilation

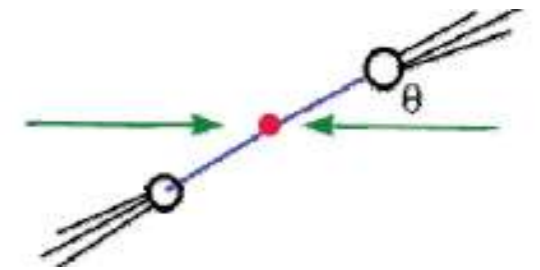
Di-jet LEP event



Jet: collinear spray of hadrons!

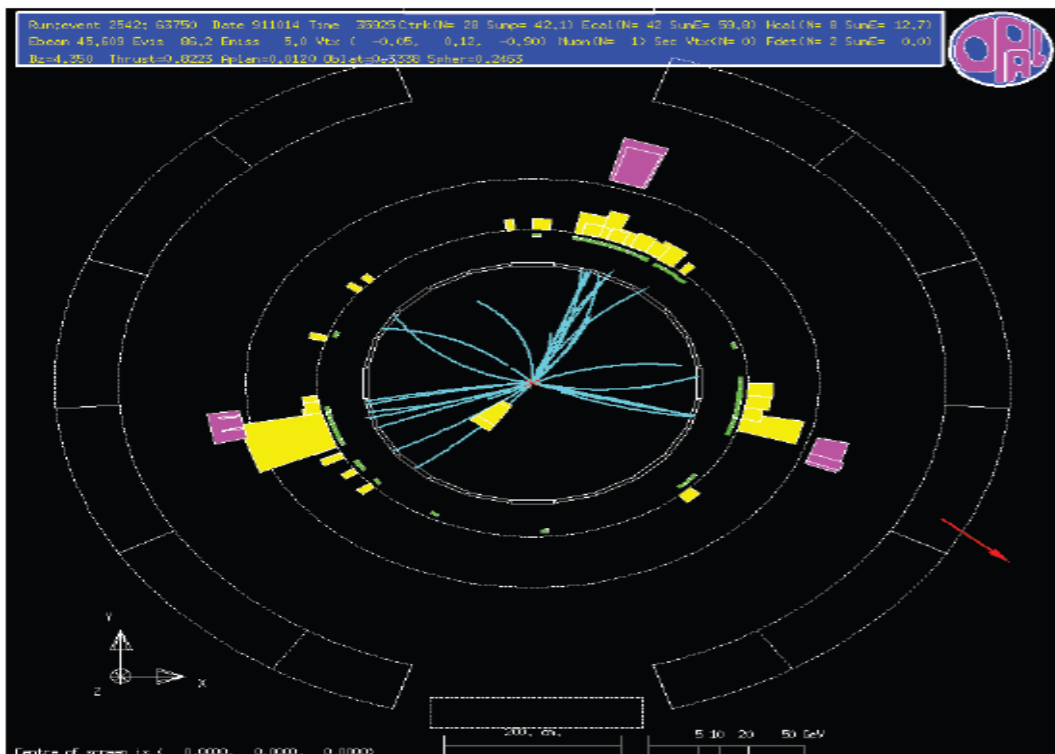


hadronisation of quarks



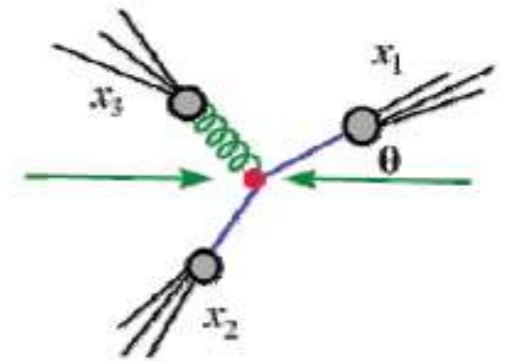
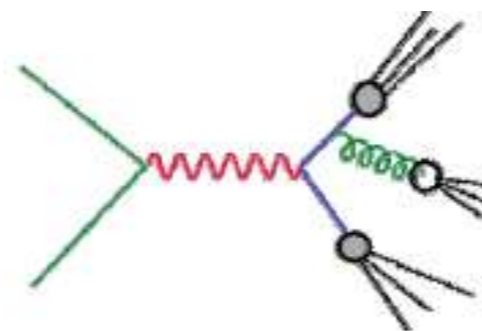
resembles all the features of $2 \rightarrow 2$ hard process (e.g. angular distribution inherent to spin-1/2 particles)

Three-jet LEP event



first observation of gluons!

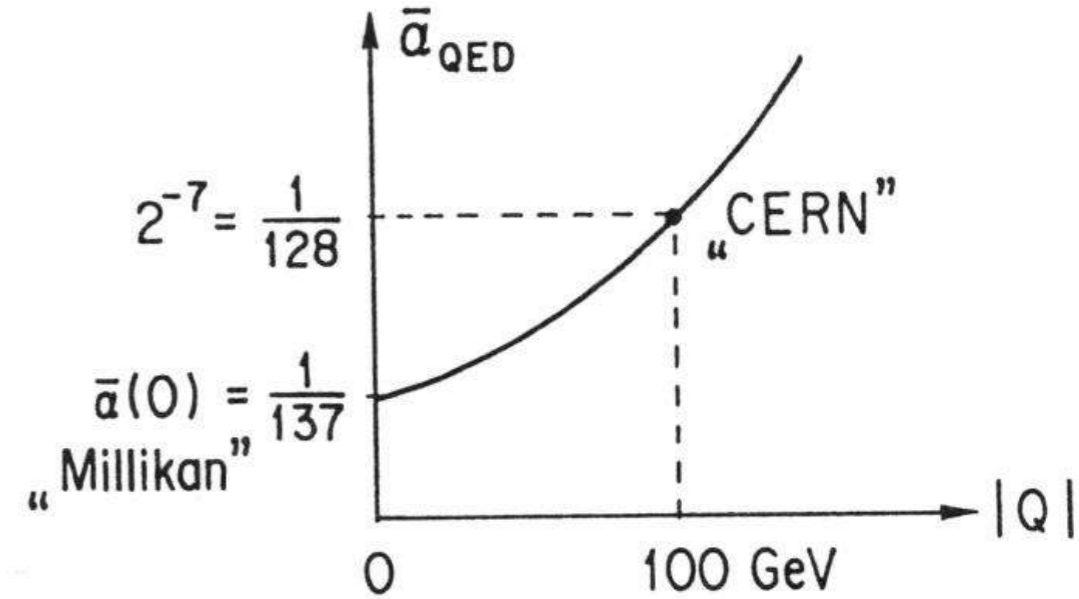
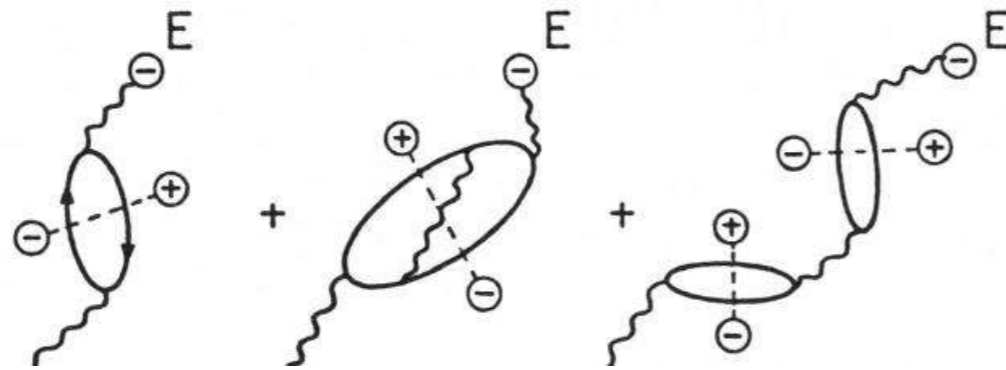
$$e^+e^- \rightarrow q\bar{q}g$$



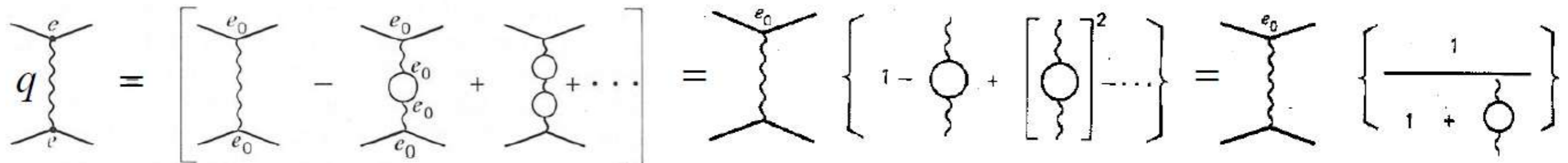
- *of higher order (extra coupling factor) as compared to the previous case (10 % of events) $\alpha_s \approx 0.1$*
- *angular dependence as expected from spin-1 gluon*

What is the fundamental role of the vacuum polarisation?

Screening of electric charge in QED



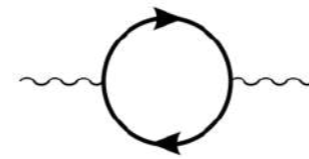
Higher-order corrections to the e+e- annihilation



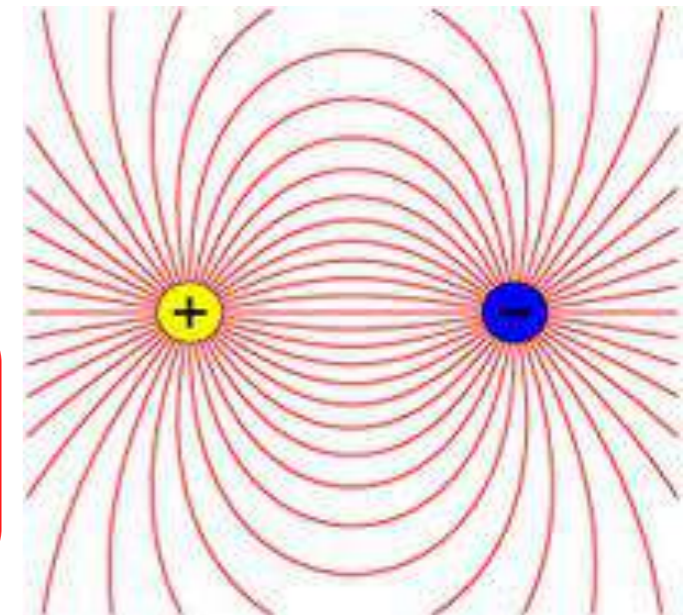
“Running” electric charge $e^2(Q^2 = -q^2) = e_0^2 \left(\frac{1}{1 + I(Q^2)} \right)$

$$I(Q^2) = \int_{m_e}^M \frac{dp^2}{p^2} \dots = -\frac{\alpha_0}{3\pi} \log \left(\frac{Q^2}{M^2} \right)$$

UV cut-off



Long range EM interactions



At larger distances EM interaction becomes weaker!

$$e \rightarrow e(r) = e \left\{ 1 - \frac{\alpha}{3\pi} \ln \frac{r}{r_e} + O(\alpha^2) \right\}$$

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log \left(\frac{Q^2}{\mu^2} \right)}$$

increases with scale!

Why we do not see quarks?

Running QCD coupling

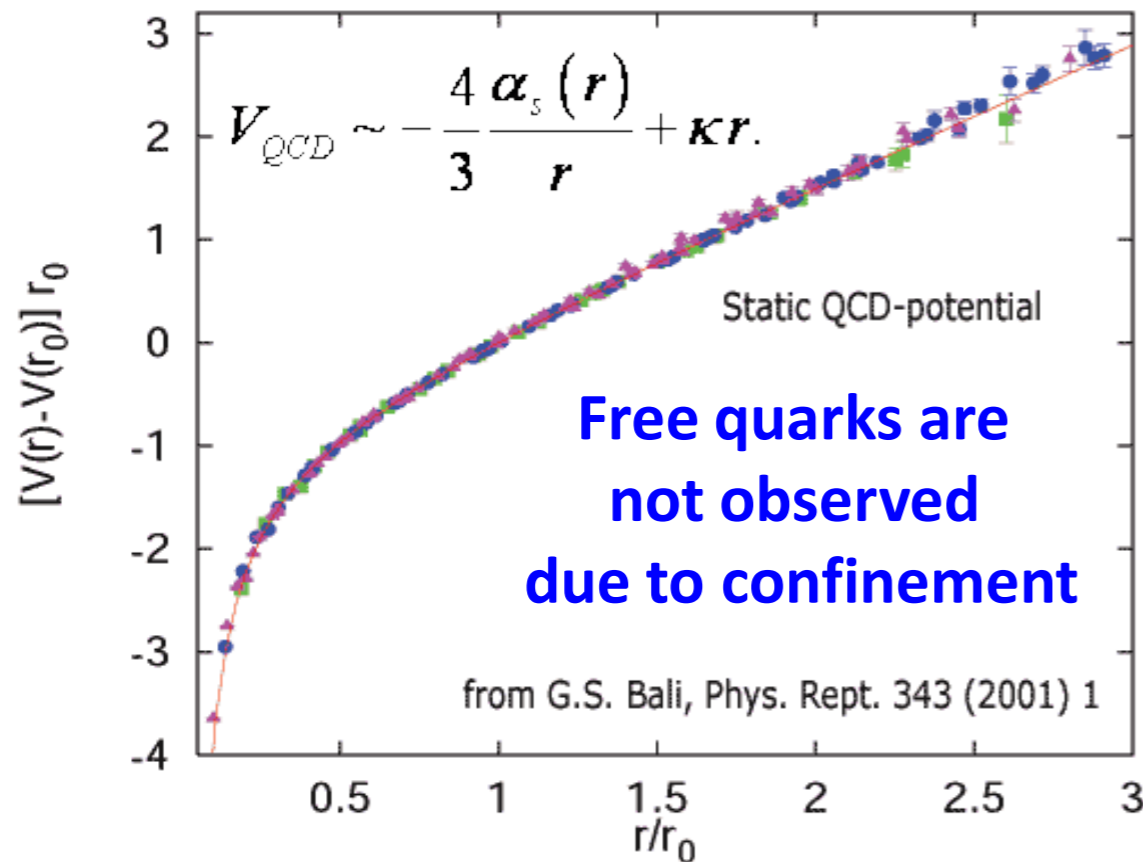
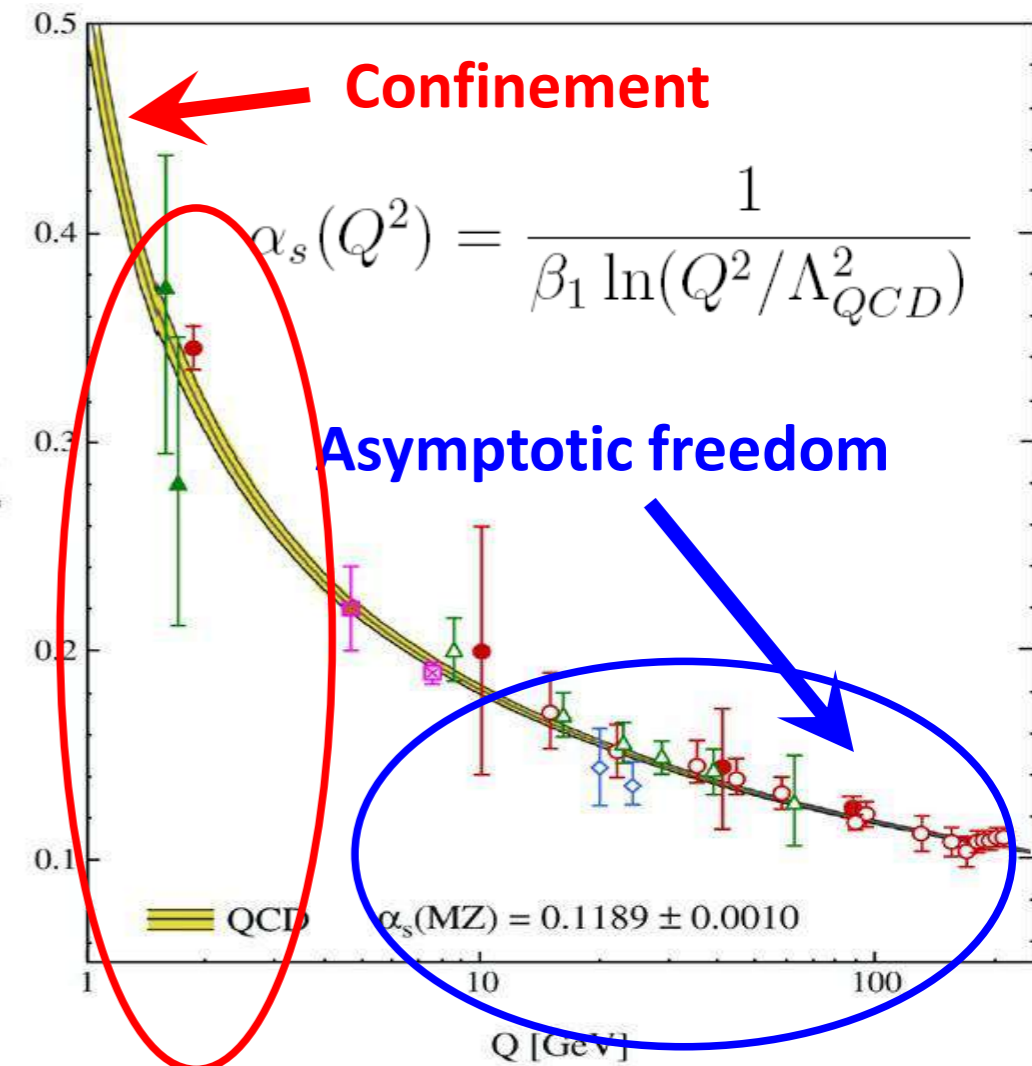
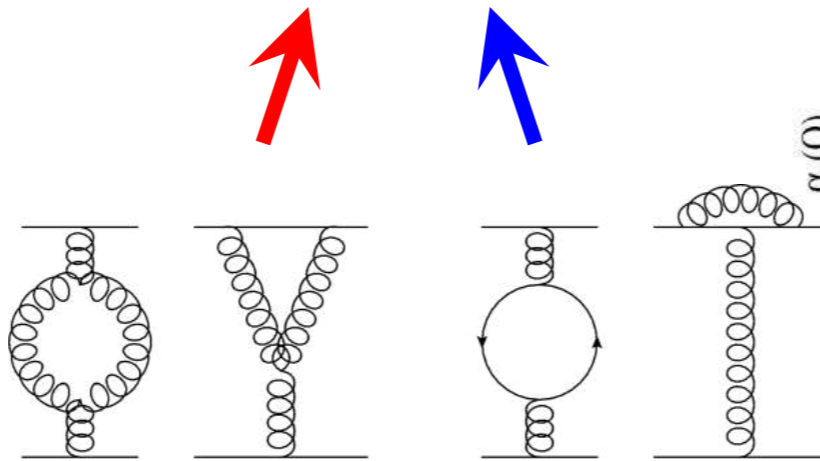
$$\alpha_s = \frac{g^2}{4\pi}$$

Color charge anti-screening

$$\mu^2 \frac{d\alpha_s}{d\mu^2} = \beta(\alpha_s). \quad \beta(\alpha_s) = - \left(11 - \frac{2n_f}{3} \right) \frac{\alpha_s^2}{2\pi}$$

Nobel Prize 2004:

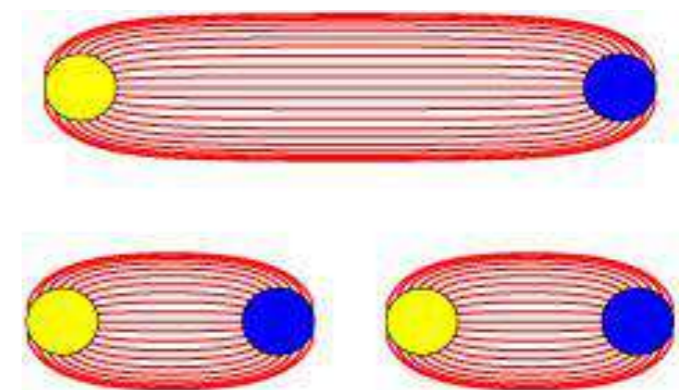
Gross, Wilczek; Politzer



Soft QCD (models)

Hard QCD (PT)

Short range strong interactions



basis for hadronisation models.....

Thank you

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