The Skyrmion in Little Higgs Models: a Dark Matter Candidate?

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Introduction

In many BSM models, the Higgs is a pseudo-Goldstone boson of a new symmetry: technicolor, composite Higgs, little Higgs models, ...

The Higgs sector of these models is described at energies below the cutoff $\Lambda \sim 4\pi f$ by a σ-model,

$$\mathscr{L} = \frac{f^2}{4} \operatorname{Tr}\left(\partial_\mu \Phi \, \partial^\mu \Phi^\dagger\right),$$

with two consequences:

1. The scalar field $\Phi(x)$ is much lighter than the scale f (it describes the "pions", in analogy with QCD)
2. The model may also contain topological solitons called skyrmions (the "baryons" of the theory)

Outline

The chiral effective theory

In the limit of massless quarks, QCD has a global $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry, broken down to $SU(N_f)_V$ by a non-zero quark condensate $\langle \overline{q} q \rangle \neq 0$.

The low-energy degrees of freedom of QCD are the pions, described by a σ-model

$$\mathscr{L}_{\chi PT} = \frac{f_\pi^2}{4} \operatorname{Tr}\left(\partial_\mu \Phi \, \partial^\mu \Phi^\dagger\right), \qquad \Phi(x) \in SU(N_f).$$

In the two-flavour case $N_f = 2$, $\Phi = \exp\left(i \, \boldsymbol{\pi} \cdot \boldsymbol{\sigma} / f_\pi\right)$ and

$$\mathscr{L}_{\chi PT} = \frac{1}{2} \left|\partial_\mu \boldsymbol{\pi}\right|^2 + \frac{1}{6 f_\pi^2} \left[\left|\boldsymbol{\pi} \cdot \partial_\mu \boldsymbol{\pi}\right|^2 - \boldsymbol{\pi}^2 \left|\partial_\mu \boldsymbol{\pi}\right|^2\right] + \ldots$$

Topological solitons

The σ-model also contains topological solitons, the skyrmions:

- classical static solutions of the field equations, with finite size and finite energy,
- topologically stable, since they cannot be deformed into the true vacuum by infinitesimal transformation.

The presence of skyrmions in a theory depends on the topology of the vacuum manifold, described by the third homotopy group π_3 of the target space. For QCD,

$$\pi_3(SU(N)) = \mathbb{Z}\,.$$

Winding number integral:

$$B(\Phi) = \frac{1}{24\pi^2}\epsilon_{ijk}\int d^3x\,\mathrm{Tr}\left(\Phi^\dagger\partial_i\Phi\right)\left(\Phi^\dagger\partial_j\Phi\right)\left(\Phi^\dagger\partial_k\Phi\right) \in \mathbb{Z}\,.$$

The Skyrme model of baryons

B is actually the baryon number of the theory, and the skyrmions represent baryon states. Witten (1983) Adkins, Nappi, Witten (1983)

Higher-derivative terms are needed to stabilize the skyrmion.

G.H. Derrick (1964)

The Skyrme Lagrangian:

$$\mathscr{L}_{\text{Skyrme}} = \frac{f_\pi^2}{4}\,\text{Tr}\left(\partial_\mu \Phi\, \partial^\mu \Phi^\dagger\right) + \frac{1}{32e^2}\,\text{Tr}\left|\left[\Phi^\dagger \partial_\mu \Phi, \Phi^\dagger \partial_\nu \Phi\right]\right|^2$$

T.H.R. Skyrme (1961)

With $e \sim 5$, the baryons properties are reproduced with about 20% precision: mass, radius, magnetic moment, ...

Topology of little Higgs models

The skyrmions in little Higgs model can be fundamentaly different from the QCD ones, because of a different topology.

Symmetry	Models	π_3
$\frac{SU(N)\times SU(N)}{SU(N)}$	"Minimal moose" models Arkani-Hamed *et al.* (2002) – with exact DM parity Freitas, Schwaller, Wyler (2009)	$\mathbb{Z}$
$\frac{SO(N)\times SO(N)}{SO(N)}$	– with custodial symmetry Chang, Wacker (2004) "Bestest LH" Schmaltz, Stolarski, Thaler (2010)	$\mathbb{Z}$
$SU(N)/SO(N)$	Littlest Higgs Arkani-Hamed *et al.* (2002) – with T-parity Cheng, Low (2003)	$\mathbb{Z}_2$
$SU(2N)/Sp(2N)$	"Antisymmetric cond." Low, Skiba, Tucker-Smith (2002)	0
$SU(N)/SU(N-1)$	LH from a simple group Kaplan, Schmaltz (2003)	0

Bryan, Carroll, Pyne (1993)

The littlest Higgs model

The littlest Higgs is described by a field $\Sigma(x)$ in the two-index symmetric representation of $SU(5)$, with a vev $\langle\Sigma\rangle = \mathbb{1}$ breaking the symmetry down to $SO(5)$:

$$\mathscr{L}_{LH} = \frac{f^2}{4}\,\mathrm{Tr}\left(\partial_\mu\Sigma\,\partial^\mu\Sigma^\dagger\right) + \frac{1}{32e^2}\,\mathrm{Tr}\left|\left[\Sigma^\dagger\partial_\mu\Sigma, \Sigma^\dagger\partial_\nu\Sigma\right]\right|^2$$

$\rightarrow$ we add the Skyrme term as in QCD.

We use the Cartan embedding $\Sigma(x) = \Phi(x)\Phi(x)^T$ and a "hedgehog ansatz" $\Phi(x) = \exp\left[i\,F(r)\,\hat{x}_i\,T_i\right]$ with

$$T_i = \frac{1}{4}\begin{pmatrix} \sigma_i & 0 & i\,\sigma_i \\ 0 & 0 & 0 \\ -i\sigma_i & 0 & \sigma_i \end{pmatrix}$$

so that the field has $B = 1$ and spherical symmetry.

The "littlest skyrmion"

The energy density is a functional of F:

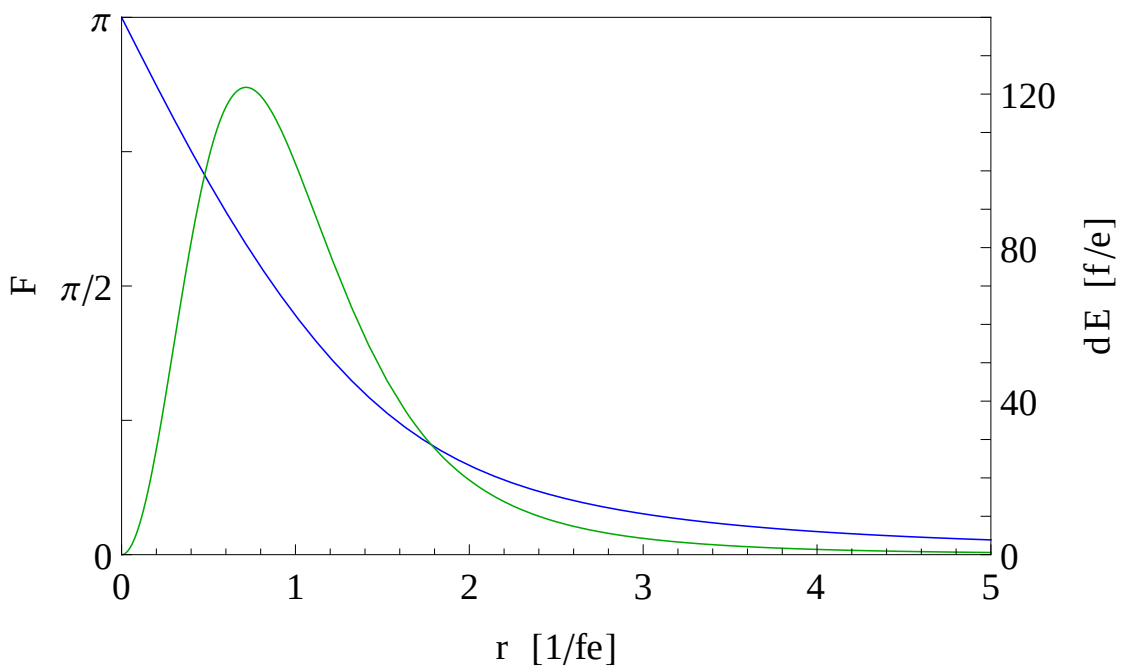
$$E[F] = 4\pi \frac{f}{e} \int_0^\infty d\tilde{r} \left[\left(\tilde{r}^2 + 2\sin^2 F \right) F'^2 + \left(2\tilde{r}^2 + \sin^2 F \right) \frac{\sin^2 F}{\tilde{r}^2} \right]$$

minimized by solving the Euler-Lagrange equation for F

$\Downarrow$

$$M = 145.8 \, \frac{f}{e}$$

$$\langle r^2 \rangle = \left(\frac{1.058}{f\, e} \right)^2$$



The effects of gauge fields

The global $SU(5)$ symmetry is explicitly broken by gauging a $[SU(2) \times U(1)]^2$ subgroup.

$\Rightarrow$ The skyrmion can decay with the help of an instanton.

D'Hoker, Farhi (1984)

Still, the extremely small tunneling probability makes the skyrmion quasi-stable, with lifetime

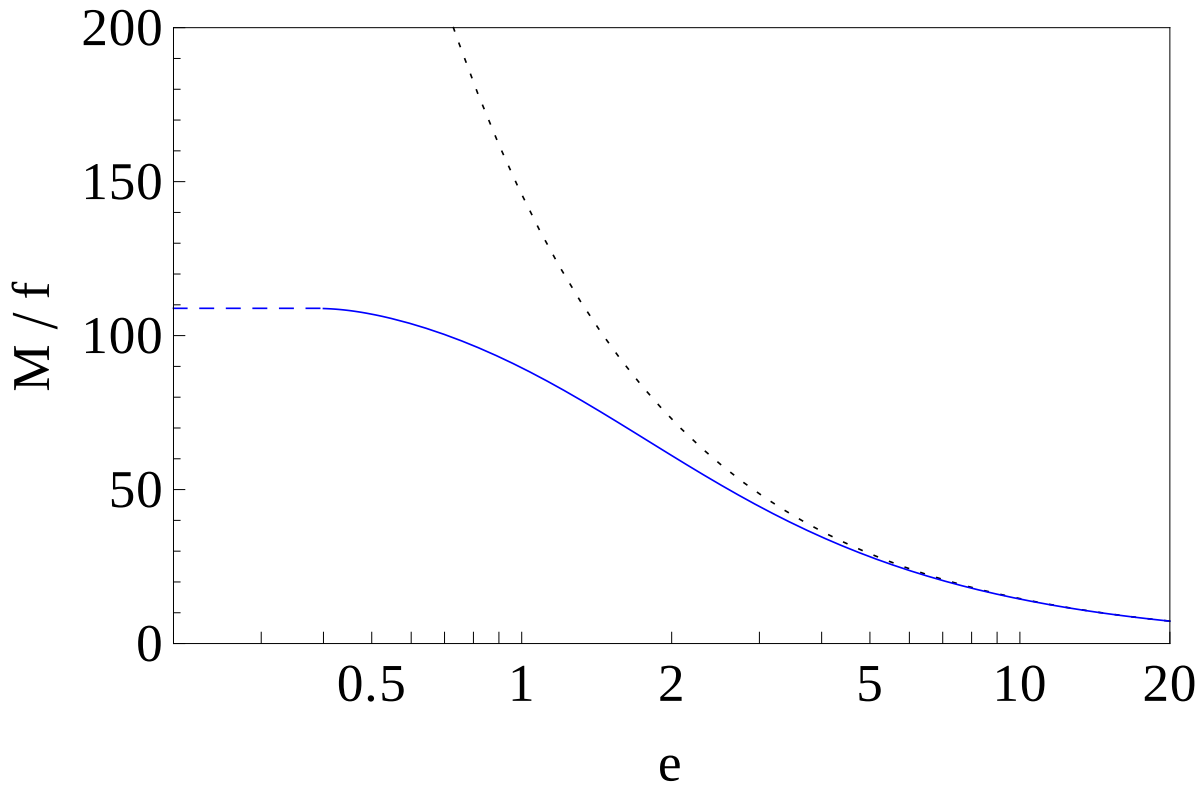
$$\tau \sim \frac{e^{16\pi^2/g^2}}{M} \gg \tau_{\text{universe}}$$

Also, the gauge fields lower the mass of the skyrmion, without spoiling its spherical symmetry.

Mass of the gauged skyrmion

- Tends to $M_{\text{ungauged}} = 145.8\, f/e$ at large e
- Upper bound in the limit of small e:

$$M_{e\to 0} = 16\sqrt{2}\,\pi\, f/g \cong 108.9\, f$$



Relic density of skyrmions

The peculiar $\mathbb{Z}_2$ topology of the littlest Higgs implies that:

- skyrmions are produced and annihilated in pairs of identical particles,
- no need to generate an asymmetry to explain the skyrmion relic density in the universe.

Skyrmion pair-production and annihilation cannot be computed perturbatively. A naive estimate:

$$\sigma_A = \pi \langle r^2 \rangle \cong \frac{\pi}{(f\, e)^2}$$

The WIMP relic density $\Omega h^2 \cong \frac{3{\cdot}10^{-27}\mathrm{cm}^3/\mathrm{s}}{\langle \sigma v \rangle} \cong 0.1$ is satisfied provided $f\, e \sim 35$ TeV

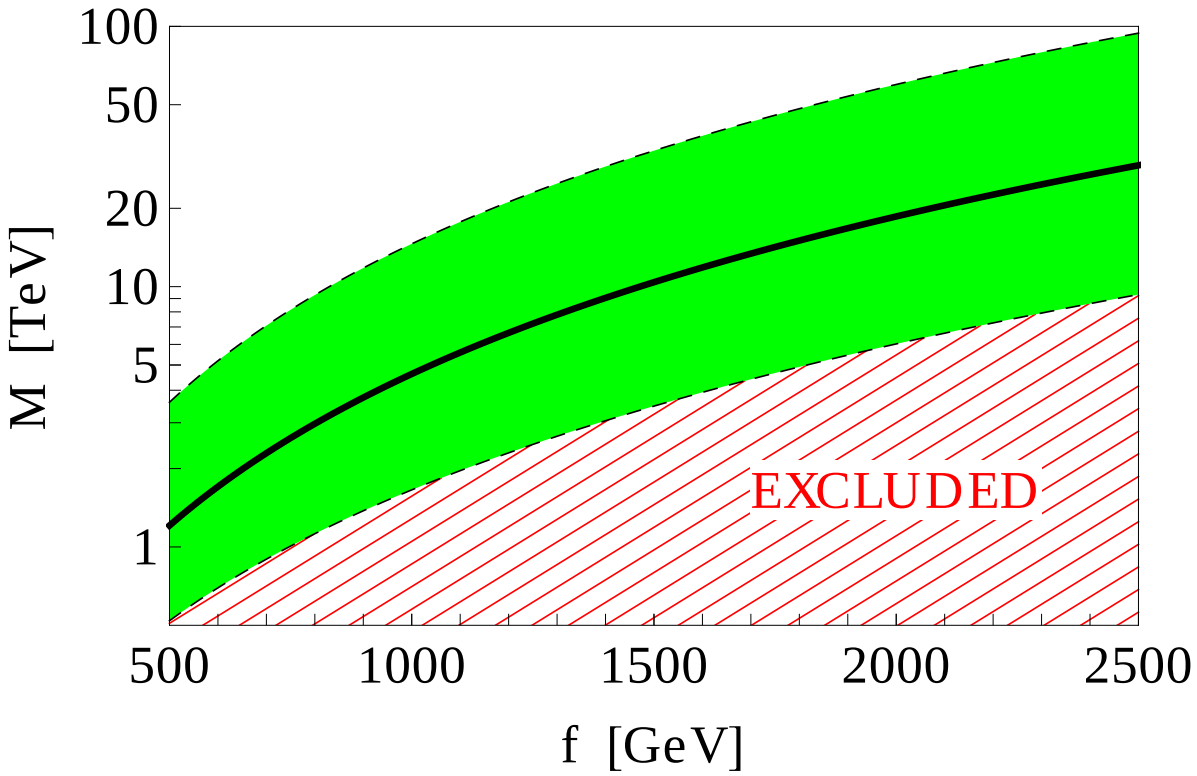
Griest, Kamionkowski (1990)

$\rightarrow$ a very natural choice of parameters!

Relic density of skyrmions

Implementing an effective skyrmion-to-matter coupling in microMEGAs:

Bélanger *et al.* (2011) Belyaev *et al.* (2006)



Coupling the skyrmion both to the scalar sector and to the fermions yield comparable bounds.

Conclusions and outlook

Skyrmions are present in many BSM models where the Higgs is a pseudo-Goldstone boson, even when the global symmetry is explicitly broken by gauge interactions.

The littlest skyrmion is expected to be heavy, above the symmetry breaking scale f, but is still a surprisingly good dark matter candidate due to its small cross-section

The quantization of the model remains to be performed in order to study the skyrmion physical properties:

- exact mass including quantum loop corrections,
- spin statistics and charge under the EW gauge group,
- interactions with standard matter,
- possible collider signature at the LHC?