THE COSMOLOGICAL CONSTANT SEE-SAW MECHANISM

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WHY DOES THE VACUUM WEIGH SO LITTLE $\left((10^{-3}\,eV)^4\right)$

(WHAT IS THE VACUUM ? HOW DO WE WEIGH ?) $\quad \| \quad \epsilon^4$

THE VACUUM ENERGY RELATES UV & IR

IR → $S \sim \Lambda \int \sqrt{-g}$

$\Lambda \rightsquigarrow$ SIZE OF THE UNIVERSE R

$\Lambda \sim \frac{1}{R^2}$

UV → # OF D.O.F $\quad \sum_p \frac{1}{2}\hbar\omega_p$

$\Leftarrow$

UV/IR NON-DECOUPLING

(EFFECTIVE FIELD THEORY (EFT) FAILS!

YET EFT WORKS IN THE IR)

A TRUE QUANTUM GRAVITY (QG) PROBLEM

WE HAVE IN MIND STRING THEORY AS A QG (NOT AN EFT BUT CONSISTENT WITH EFT IN THE IR)

NUMEROLOGY $\epsilon \sim \frac{M_{SM}^2}{M_P}$; $M_{SM} \sim 1\,TeV$, $M_P \sim 10^{19}\,GeV$

THIS SUGGESTS A SEE-SAW MECHANISM ($M_P \to \infty \Rightarrow \epsilon \to 0 \Rightarrow$ NATURALNESS)

(UNLIKE NEUTRINO MASS SEE-SAW, WHICH IS BASED ON EFT, COSMOLOGICAL CONSTANT SEE-SAW SHOULD BE A QG EFFECT)

LOOK AT $\int^{\epsilon} d^3p \, \frac{1}{2} \hbar \omega_p \sim \epsilon^4$ ($\omega_p \sim p$ FOR MASSLESS FIELDS)

$\Downarrow$ WANT

$\Leftarrow \int^{\epsilon} d^3p \, \mathcal{F}(p^2) \, \frac{1}{2} \hbar \omega_p$

a) CHANGE THE WEIGHT $\mathcal{F}(p^2)$ (PHASE-SPACE)

b) CHANGE ϵ (MAKE IT $\epsilon \sim \frac{M_{SM}^2}{M_P}$)

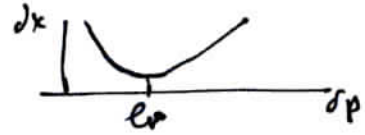
SIMILAR TO PLANCK'S SOLUTION OF THE BLACK-BODY PROBLEM

($\sum \frac{1}{2} kT \sim T^4$) $\Rightarrow$ $\mathcal{F}(\frac{\omega}{T}) \sim \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1}$ (PHASE-SPACE QUANTIZED, COUNTING CHANGED)

ONE WAY TO GET $\mathcal{F}(p^2)$: EFFECTIVE $\not{x}(p^2) = \not{x} A(p^2)$

$\Rightarrow \frac{1}{i\hbar} [x_a, p_a] = A(p^2) \delta_{ij} + \cdots$

STRING PERTURBATION THEORY (STRING-STRING SCATTERING)



$\delta x \, \delta p \sim \frac{\hbar}{2} (1 + \beta \, \delta p^2)$

IF $\frac{1}{i\hbar}[x_a, p_b] = \underbrace{A(p^2)}_{1+\beta p^2} + \underbrace{B(p^2)}_{\beta'} p_i p_j$ AND $[p_a, p_b] = 0$

JACOBI $\Rightarrow$ $\frac{1}{i\hbar}[x_a, x_b] = -\left[2(A+Bp^2)\frac{dA}{dp^2} - AB\right] L_{ij}$; $L_{ij} = \frac{x_i p_j - x_j p_i}{A}$; $[x_i, x_j] \neq 0$!!

CLASSICAL LIMIT : $\frac{1}{i\hbar}[\,,\,] \Rightarrow \{\,,\,\}$ (Poisson) THEN CLASSICAL PHASE-SPACE VOLUME: $\frac{d^D x\, d^D p}{A^{D-1}(A+Bp^2)}$

SO (B=0) $\int d^3p\, f \Rightarrow \int \frac{d^3p}{A^3(p^2)} f = 4\pi \underbrace{\int dp \frac{p^3}{A^3(p^2)}}_{\mathcal{F}(p^2) = A^{-3}(p^2)}$

EVEN IF $A^{-3}(p^2) \sim \frac{p^4}{M_p^4} \Rightarrow \Delta t(p^2) = \Delta t\, A(p^2)$ BLOWS UP IN THE IR !

NEED SOMETHING ELSE ! $[p_i, p_j] = 0 \Rightarrow [p_i, p_j] \neq 0$!!

(STILL WANT $\delta x\, \delta p \sim \frac{\hbar}{2}(1+\beta \delta p^2)$ BECAUSE OF UV/IR $\delta x \sim \frac{\hbar}{2}\beta \delta p$!! (DEEP UV))

HEURISTICALLY, $ds^2 = g_{ab}\,dx^a dx^b \Longrightarrow \delta s_x^2 = g_{ab}\,\delta x^a \delta x^b$

DYNAMICAL SPACE-TIME (GR)

(δx ~ QUANTUM FLUCTUATION!)

NOW, IF $\delta x \sim \star\, p\, \delta p \Rightarrow \delta s_p^2 = G_{ab}\,\delta p^a \delta p^b \rightsquigarrow$ DYNAMICAL ENERGY-MOMENTUM SPACE!

(δp ~ quantum fluctuation) (QG)

FLUCTUATING ENERGY-MOMENTUM SPACE (i.e. "GRAVITY" IN ENERGY-MOMENTUM SPACE)

(FOUNDATIONAL ISSUES REGARDING quantum theory ...)

USUALLY WE EXPECT "FOAMY" SPACE-TIME IN QG.

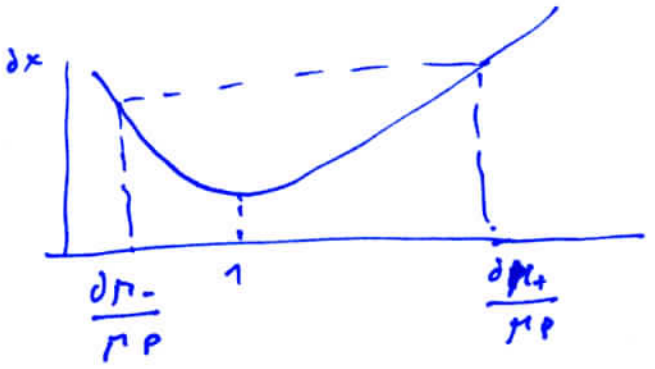
NOW, NOT ONLY SPACE-TIME "FOAM" BUT ENERGY-MOMENTUM "FOAM" AS WELL

GENERICALLY IN QG EXPECT HOLOGRAPHY: $\left(\frac{\ell}{\ell_p}\right)^2$ BH SCALING ($S \sim \frac{A}{4G\hbar}$)

OF SPACE-TIME FOAM CELLS (in D=4!) $\left(\frac{\ell^4}{c}\right) \Big/ \left(\frac{\ell^2}{\ell_p}\right) = \frac{\ell^2 \ell_p^2}{c} \sim \delta\ell^4$

$\Rightarrow \delta\ell \sim \ell^{1/2} \ell_p^{1/2}$ (BROWNIAN SCALING; VON DAM)

USE THIS REASONING IN ENERGY-MOMENTUM SPACE: $\boxed{\delta\mu \sim \mu^{1/2} \mu_p^{1/2}}$



$\delta\mu_+$ - trans-Planckian (not EFT!)

$\delta\mu_-$ - sub-Planckian (EFT!)

$\delta\mu_\pm \sim \mu_\pm^{1/2}\,\mu_P^{1/2}$ (energy-mismatch form)

$(\delta\mu_\pm^2 \sim \mu_\pm \mu_P) \Rightarrow \underbrace{\mu_-\,\delta\mu_+^2 = \mu_+\,\delta\mu_-^2 \sim \mu_P^3}_{\text{see-saw}}$

This EFT sees: $\mu_- = \dfrac{\delta\mu_-^2}{\mu_P} \Rightarrow \epsilon^4 \sim \mu_-^4 = \dfrac{\delta\mu_-^8}{\mu_P^4}$

If $\delta\mu_- \sim M_{SM}$ (RELATION TO THE HIERARCHY PROBLEM?) $\Rightarrow$ ✓✓ $(\epsilon^4 \sim (10^{-3}\,\text{eV})^4)$

PHYSICS: JAMMING in NON-EQUILIBRIUM STAT PHYSICS

(OR "FREEZING BY HEATING" $\rightsquigarrow$ EXAMPLE TRAFFIC JAM)

$\Rightarrow$ INTEGRATE IN ("HEAT UP") $\rightsquigarrow$ FREEZE VACUUM ENERGY (low ϵ!!)

NON-DECOUPLING OF UV & IR (FERMICAL HOLOMETER?)

(VACUUM ENERGY REVEALS TRANS-PLANCKIAN D.O.F JUST AS NEUTRINO SEE-SAW REVEALS GUT D.O.F)

APPLICATION TO COLLIDER PHYSICS (WORK IN PROGRESS)