N=4 Supersymmetric Yang Mills theory in soft collinear effective theory

(based on arXiv:*1011.6145*)

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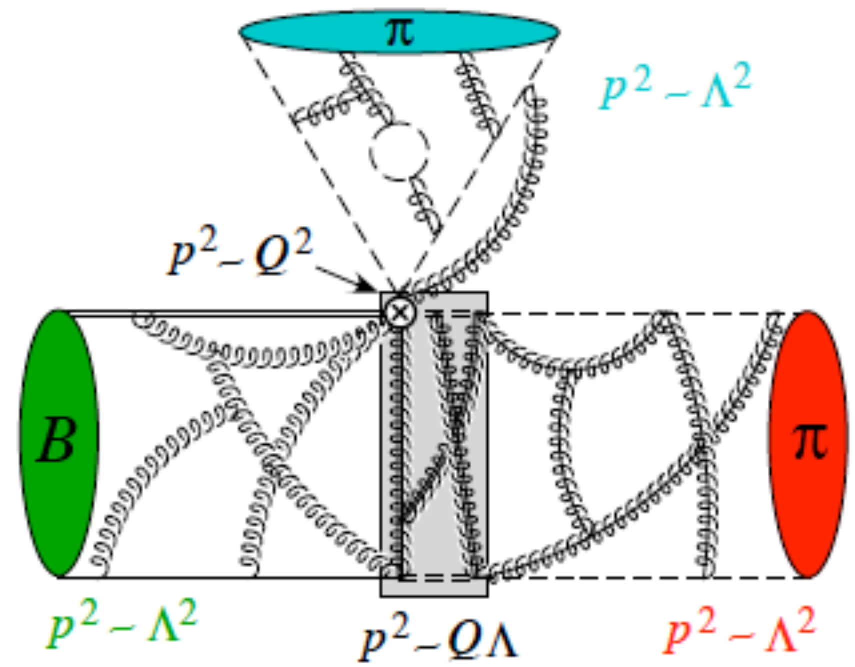
Pheno 2011
May 10, 2011

Outline

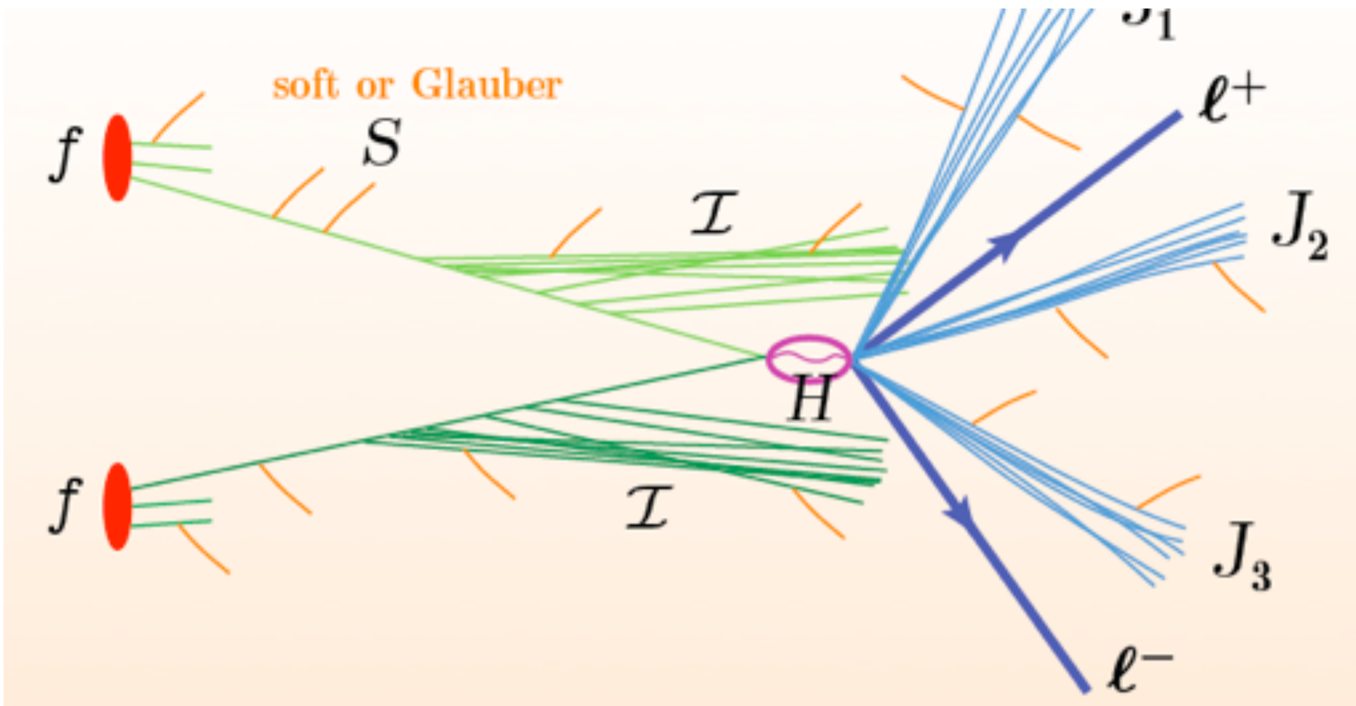
- Soft Collinear Effective Theory (SCET)
- *N=4* Supersymmetric Yang Mills theory (N=4 SYM)
- SCET for *N=4* SYM
- Collinear Wilson line and usoft Wilson line
- Outlook

Soft Collinear Effective Theory for "QCD"

B Physics



Collider Phenomenology



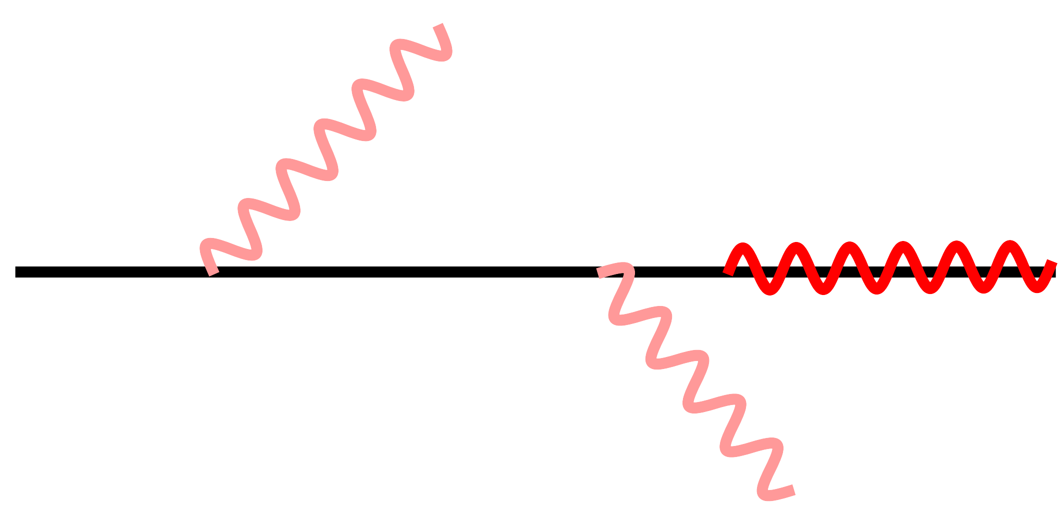
From Iain Stewart

IR singularities ?

On-shell amplitudes in theories with massless particles (such as gauge theories) have infrared (IR) divergences. They arise from configurations of soft and collinear loop momenta.

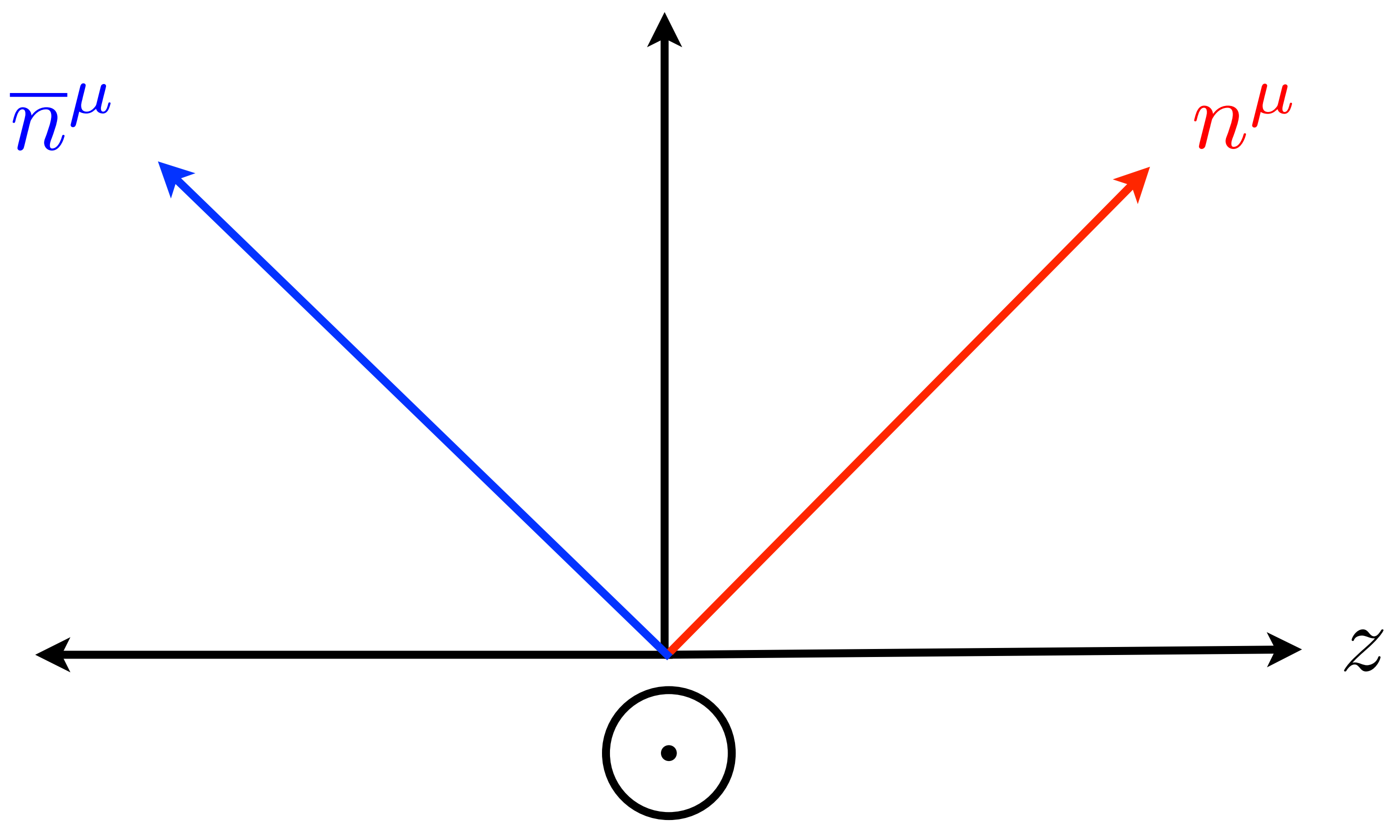
In physical observables, they cancel against real radiation contributions involving soft or collinear emissions.

To understand the structure of these singularities, we thus need to understand the theory in the soft or collinear limit.



Basic idea of SCET

$$p^\mu = \bar{n}\cdot p\frac{n^\mu}{2} + p_\perp^\mu + n\cdot p\frac{\bar{n}^\mu}{2} \sim \mathcal{O}(Q) + \mathcal{O}(Q\eta) + \mathcal{O}(Q\eta^2)$$

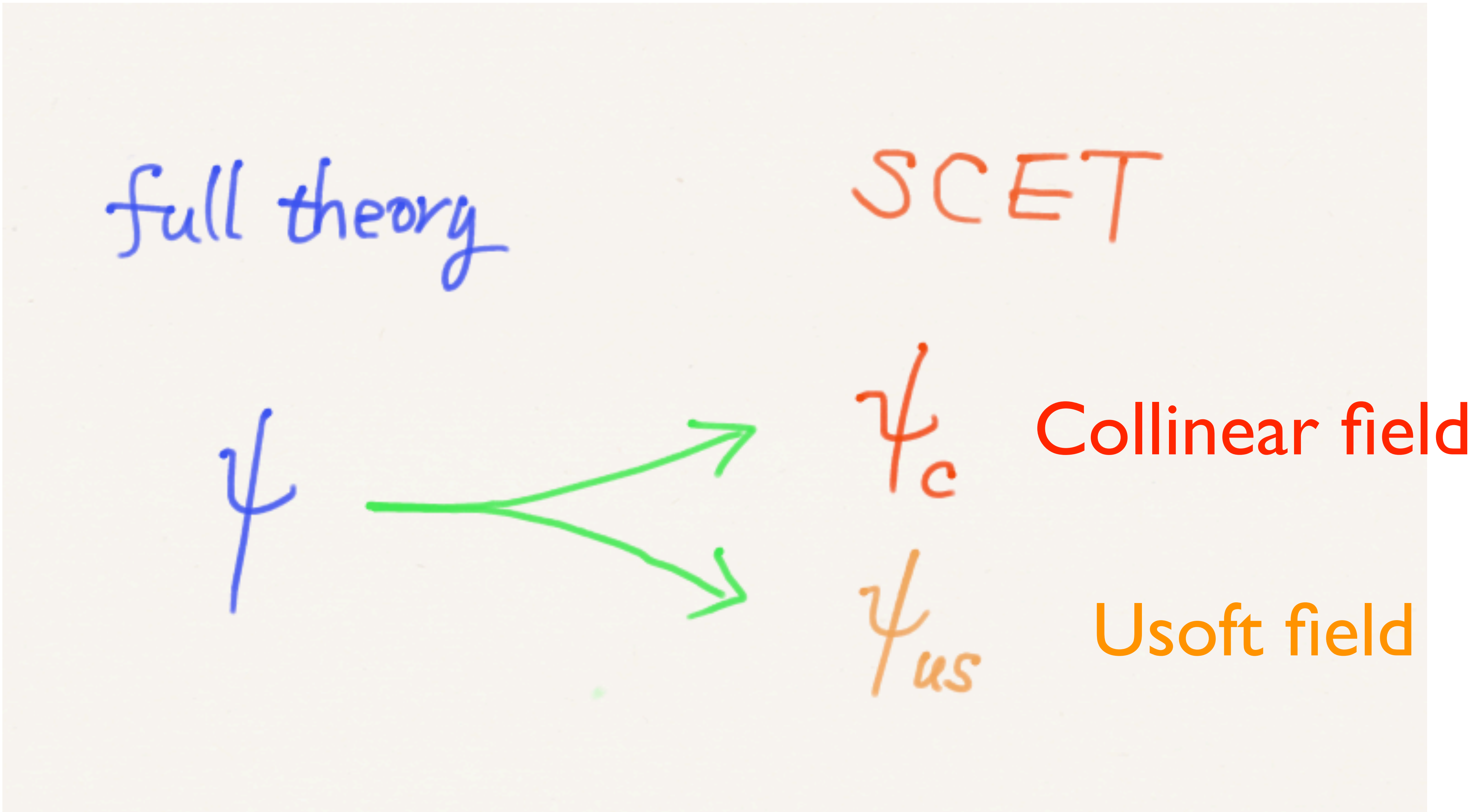


$$n^2 = \bar{n}^2 = 0,$$
$$n\cdot\bar{n} = 2,$$
$$\boxed{\eta = \frac{p_\perp}{\bar{n}\cdot p}.}$$

$$\bar{n}^\mu = (1,0,0,-1) \qquad n^\mu = (1,0,0,1)$$

Usoft momentum

$$p_{\rm us}^\mu = (\bar{n}\cdot p_{\rm us}, p_{{\rm us}\perp}^\mu, n\cdot p_{\rm us}) \sim Q(\eta^2,\eta^2,\eta^2).$$



When an usoft particle interacts w/ a collinear particle the momentum scaling behavior of a collinear particle is "unchanged."

N=4 SYM Lagrangian

$$\mathcal{L} = \mathrm{Tr}\Big(-\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \overline{\lambda}_i \overline{\sigma}^\mu i D_\mu \lambda_i + \frac{1}{2} D_\mu \phi_{ij} D^\mu \phi^{ij}$$
$$- ig\lambda_i[\lambda_j, \phi^{ij}] - ig\overline{\lambda}^i[\overline{\lambda}^j, \phi_{ij}] + \frac{g^2}{4}[\phi_{ij}, \phi_{kl}][\phi^{ij}, \phi^{kl}]\Big)$$

$$D_\mu\lambda = \partial_\mu\lambda - ig[A^\mu, \lambda], \quad \lambda = \lambda^a t^a, \phi = \phi^a t^a, A^\mu = A^{\mu a} t^a$$

$$i, j, k, l = 1, 2, 3, 4$$

Fermion

$\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$

Boson

A^μ

$\{\phi_{12}, \phi_{13}, \phi_{14}, \phi_{23}, \phi_{24}, \phi_{34}\}$

Features of N=4 SYM

- All fields are *massless*.
- (Quantum) superconformal symmetry
    - A single parameter g
    - Its β function vanishes up to *three loops* by direct calculations.

$$\mu \frac{dg}{d\mu} = \beta(g) = 0$$

    - There exist several arguments for the vanishing of the β function to all loops.
- AdS/CFT correspondence   '97 Maldacena

Symmetry of N=4 SYM

- Supersymmetry
- *SU(N)* gauge symmetry
- *SU(4)* global symmetry

field content	$SU(N)$	$SU(4)$
$\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$	adjoint.	4
A^μ	adjoint.	1
$\{\phi_{12}, \phi_{13}, \phi_{14}, \phi_{23}, \phi_{24}, \phi_{34}\}$	adjoint.	6

fundamental rep. $\longleftrightarrow$ $\boxed{\text{adjoint rep.}}$

λ *NxN matrix* $\boxed{\lambda \quad \textit{(N}^2\textit{-1) column}}$

A^μ *NxN matrix* $\boxed{\mathcal{A}^\mu \quad \textit{(N}^2\textit{-1)x(N}^2\textit{-1) matrix}}$

$$\mathcal{L}_\lambda = \mathrm{Tr}\big(\overline{\lambda}\overline{\sigma}^\mu i D_\mu \lambda\big) = \boxed{\frac{1}{2}\overline{\lambda}\overline{\sigma}^\mu i \mathcal{D}_\mu \lambda}$$

$$D_\mu \lambda = \partial_\mu \lambda - ig[A_\mu, \lambda], \qquad \mathcal{D}_\mu \lambda = \partial_\mu \lambda - ig\mathcal{A}_\mu \lambda$$

$$\begin{aligned} \mathcal{L}_{\rm int} &= \mathrm{Tr}\left(g\overline{\lambda}\overline{\sigma}^\mu [A_\mu, \lambda]\right) = g\overline{\lambda}_a \overline{\sigma}^\mu A_{\mu b}\lambda_c \mathrm{Tr}\left(t^a[t^b, t^c]\right) \\ &= igf^{abc}T_F \overline{\lambda}_a \overline{\sigma}\cdot A_b \lambda_c = \boxed{\frac{g}{2}\overline{\lambda}\overline{\sigma}\cdot \mathcal{A}\lambda,} \end{aligned}$$

where $T_F = 1/2$ for $SU(N)$, and $\mathcal{A}^\mu = A^{\mu b}T^b$ with the adjoint representation $(T^b)_{ac} = -if^{bac}$.

Collinear fermion

The fermion λ in the full theory can be written as

$$\lambda(x) = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x}\lambda_q(x),$$

where the label momentum $\tilde{p}^\mu = \overline{n}\cdot p n^\mu/2 + p_\perp^\mu$ is extracted. The field λ_q can be decomposed into $\lambda_q = \lambda_n + \lambda_{\bar{n}}$, where λ_n and $\lambda_{\bar{n}}$ are given by

$$\lambda_n = \frac{1}{4} n\cdot\sigma\overline{n}\cdot\overline{\sigma}\lambda_q, \;\; \lambda_{\bar{n}} = \frac{1}{4}\overline{n}\cdot\sigma n\cdot\overline{\sigma}\lambda_q.$$

These fields satisfy the relation $n\cdot\overline{\sigma}\lambda_n = 0$, $\overline{n}\cdot\overline{\sigma}\lambda_{\bar{n}} = 0$.

$$\boxed{\lambda \to \lambda_q \to \lambda_n + \lambda_{\bar{n}}}$$

Collinear fermion Lagrangian

fundamental rep.

$$\mathcal{L}_\lambda = \mathrm{Tr}\Bigg(\bar\lambda_n \frac{\bar n\cdot\bar\sigma}{2} n\cdot iD\lambda_n + \bar\lambda_n\bar\sigma\cdot(p_\perp + iD_\perp)\lambda_{\bar n}$$

$$+ \bar\lambda_{\bar n}\bar\sigma\cdot(p_\perp+iD_\perp)\lambda_n + \bar\lambda_{\bar n}\frac{n\cdot\bar\sigma}{2}(\bar n\cdot p + \bar n\cdot iD)\lambda_{\bar n}\Bigg).$$

$$\boxed{\lambda_{\bar n} = -\frac{\bar n\cdot\sigma}{2}\frac{1}{\bar n\cdot p+\bar n\cdot iD}\bar\sigma\cdot(p_\perp + iD_\perp)\lambda_n}$$

adjoint rep.

$$\mathcal{L}_\lambda = \frac{1}{2}\Bigg[\bar\lambda_n \frac{\bar n\cdot\bar\sigma}{2} in\cdot\mathcal{D}\lambda_n + \bar\lambda_n\bar\sigma\cdot(p_\perp + i\mathcal{D}_\perp)\lambda_{\bar n}$$

$$+ \bar\lambda_{\bar n}\bar\sigma\cdot(p_\perp+i\mathcal{D}_\perp)\lambda_n + \bar\lambda_{\bar n}\frac{n\cdot\bar\sigma}{2}(\bar n\cdot p + \bar n\cdot i\mathcal{D})\lambda_{\bar n}\Bigg].$$

$$\boxed{\lambda_{\bar n} = -\frac{\bar n\cdot\sigma}{2}\frac{1}{\bar n\cdot p+\bar n\cdot i\mathcal{D}}\bar\sigma\cdot(p_\perp + i\mathcal{D}_\perp)\lambda_n}$$

Collinear fermion Lagrangian

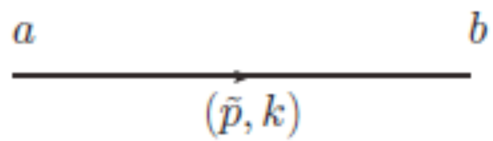
$$\mathcal{A}^\mu = \mathcal{A}_n^\mu + \mathcal{A}_{us}^\mu$$

$$\begin{aligned}\mathcal{A}_n^\mu &= (\overline{n}\cdot\mathcal{A}_n, \mathcal{A}_{n,\perp}^\mu, n\cdot\mathcal{A}) \sim Q(1,\eta,\eta^2),\\ \mathcal{A}_{\rm us}^\mu &= (\overline{n}\cdot\mathcal{A}_{\rm us}, \mathcal{A}_{{\rm us},\perp}^\mu, n\cdot\mathcal{A}_{\rm us}) \sim Q(\eta^2,\eta^2,\eta^2).\end{aligned}$$

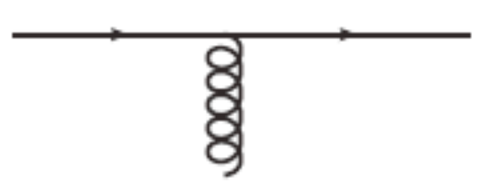
$$\boxed{\mathcal{L}_\lambda^{(0)} = \frac{1}{2}\overline{\lambda}_n \frac{\overline{n}\cdot\overline{\sigma}}{2}\Big(n\cdot i\mathcal{D} + \sigma\cdot\mathcal{D}_{c\perp}\frac{1}{\overline{n}\cdot\mathcal{P} + g\overline{n}\cdot\mathcal{A}_n}\overline{\sigma}\cdot\mathcal{D}_{c\perp}\Big)\lambda_n,}$$

where $n\cdot\mathcal{D} = n\cdot\partial - ig(n\cdot\mathcal{A}_n + n\cdot\mathcal{A}_{\rm us})$ includes the usoft gauge field because $n\cdot\mathcal{A}_{\rm us}$ has the same power counting $Q\eta^2$ as $n\cdot\mathcal{A}_n$, and $\mathcal{D}_{c\perp}^\mu = \mathcal{P}_\perp^\mu + g\mathcal{A}_{n\perp}^\mu$ is the collinear covariant derivative. The operators $\overline{n}\cdot\mathcal{P}$ and $\mathcal{P}_\perp^\mu$ extract the label momenta $\overline{n}\cdot p$ and $p_\perp^\mu$ respectively.

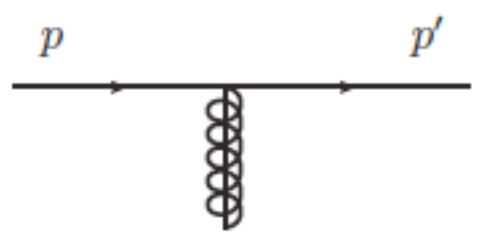
Feynman rules

(a) 

$$= i\delta_{ab}\frac{n\cdot\sigma}{2}\frac{\bar n\cdot p}{\bar n\cdot p\, n\cdot k + p_\perp^2 + i0} = -i\delta_{ab}\frac{n\cdot\bar\sigma}{2}\frac{\bar n\cdot p}{\bar n\cdot p\, n\cdot k + p_\perp^2 + i0}$$

(b) 

$$= ig n_\mu T^a\frac{\bar n\cdot\bar\sigma}{2}$$

(c) 

$$= igT^a\frac{\bar n\cdot\bar\sigma}{2}\left(n_\mu + \frac{\sigma_\mu^\perp\bar\sigma\cdot p_\perp}{\bar n\cdot p} + \frac{\sigma\cdot p'_\perp\bar\sigma_\mu^\perp}{\bar n\cdot p'} - \frac{\sigma\cdot p'_\perp\bar\sigma\cdot p_\perp}{\bar n\cdot p'\bar n\cdot p}\bar n_\mu\right)$$

FIG. 1. Feynman rules for $\mathcal{L}_\lambda^{(0)}$ to order g in SCET. (a) collinear fermion propagator with label momentum $\tilde p$ and residual momentum k, (b) collinear fermion interaction with a usoft gauge field, and (c) collinear fermion interaction with a collinear gauge field. Here $(T^a)_{bc} = -if^{abc}$ is the adjoint $SU(N)$ generators.

Collinear Wilson Line

The Lagrangian can be expressed in a form showing manifest collinear gauge invariance by introducing the collinear Wilson line

$$\mathcal{W}_n = \sum_{\text{perm.}} \exp\left[-g\frac{1}{\overline{n}\cdot\mathcal{P}}\overline{n}\cdot\mathcal{A}_n\right],$$

where the bracket implies that the operator $\overline{n}\cdot\mathcal{P}$ is applied only inside the bracket.

FIG. 2. Feynman diagram in which $\lambda_{\bar{n}}$ emits collinear gluons in the n direction. The intermediate states are off shell to be integrated out, and the result produces the collinear Wilson line $\mathcal{W}_n$.

Collinear Wilson Line

$$\mathcal{W}_n = \sum_{m=0} \sum_{\text{perm}} \frac{(-g)^m}{m!} \frac{\bar{n} \cdot A^{a_1}_{n,q_1} \cdots \bar{n} \cdot A^{a_m}_{n,q_m}}{\bar{n} \cdot q_1 \bar{n} \cdot (q_1 + q_2) \cdots \bar{n} \cdot \left(\sum_{i=1}^m q_i \right)} T^{a_m} \cdots T^{a_1}.$$

$$\mathcal{W}_n(x) = \mathrm{P} \exp\left(ig \int_{-\infty}^{x} ds \bar{n} \cdot A^a_n(ns) T^a \right).$$

$$\boxed{\frac{1}{\bar{n} \cdot \mathcal{P} + g \bar{n} \cdot \mathcal{A}_n} \Longrightarrow \mathcal{W}_n \frac{1}{\bar{n} \cdot \mathcal{P}} \mathcal{W}_n^\dagger}$$

Collinear gauge transformation

A collinear gauge transformation $\mathcal{U}_c(x) = \exp[i\alpha^a(x)T^a]$ is defined as the subset of gauge transformations where $\partial^\mu \mathcal{U}_c \sim Q(1, \eta, \eta^2)$. For a collinear gauge transformation $\mathcal{U}_c(x)$, we extract the large label momentum as was done for collinear fields,

$$\mathcal{U}(x) = \sum_P e^{-iP\cdot x}\mathcal{U}_{Pc}(x),$$

where $\partial^\mu \mathcal{U}_{Pc} \sim Q\eta^2$.

$$\mathcal{L}_\lambda^{(0)} = \frac{1}{2}\bar{\lambda}_n \frac{\bar{n}\cdot\bar{\sigma}}{2}\left(n\cdot i\mathcal{D} + \sigma\cdot\mathcal{D}_{c\perp}\mathcal{W}_n \frac{1}{\bar{\mathcal{P}}}\mathcal{W}_n^\dagger \bar{\sigma}\cdot\mathcal{D}_{c\perp}\right)\lambda_n.$$

The Lagrangian is manifestly invariant under the collinear gauge transformation $\lambda \to \mathcal{U}_c\lambda$, $\mathcal{W}_n \to \mathcal{U}_c\mathcal{W}_n$.

fundamental rep.

Using $\quad W_n t^a W_n^\dagger = \mathcal{W}_n^{ba} t^b \quad$ we get $\quad W_n = \sum_{\text{perm.}} \exp\left[-g\frac{1}{\bar{n}\cdot\mathcal{P}}\bar{n}\cdot A_n^a t^a\right]$

and $\quad \mathcal{L}_\lambda^{(0)} = \mathrm{Tr}\left[W_n^\dagger\left(\bar{\lambda}_n\frac{\bar{n}\cdot\bar{\sigma}}{2}\left(n\cdot iD + \sigma\cdot D_{c\perp}W_n\frac{1}{\bar{\mathcal{P}}}W_n^\dagger\bar{\sigma}\cdot D_{c\perp}\right)\lambda_n\right)W_n\right].$

Usoft Factorization

Collinear particles are decoupled form usoft interactions. This can be achieved by redefining the collinear fields in terms of the usoft Wilson line,

$$\lambda_n^a = \mathcal{Y}_n^{ab} \lambda_n^{(0)b},$$

$$\mathcal{Y}_n^{ab} = \delta^{ab} + \sum_{m=1}^{\infty} \sum_{\text{perm}} \frac{(ig)^m}{m!} \frac{n \cdot A_{\text{us}}^{a_1} \cdots n \cdot A_{\text{us}}^{a_m}}{n \cdot k_1 n \cdot (k_1 + k_2) \cdots n \cdot \left(\sum_{i=1}^m k_i\right)} f^{a_m a x_{m-1}} \cdots f^{a_2 x_2 x_1} f^{a_1 x_1 b}.$$

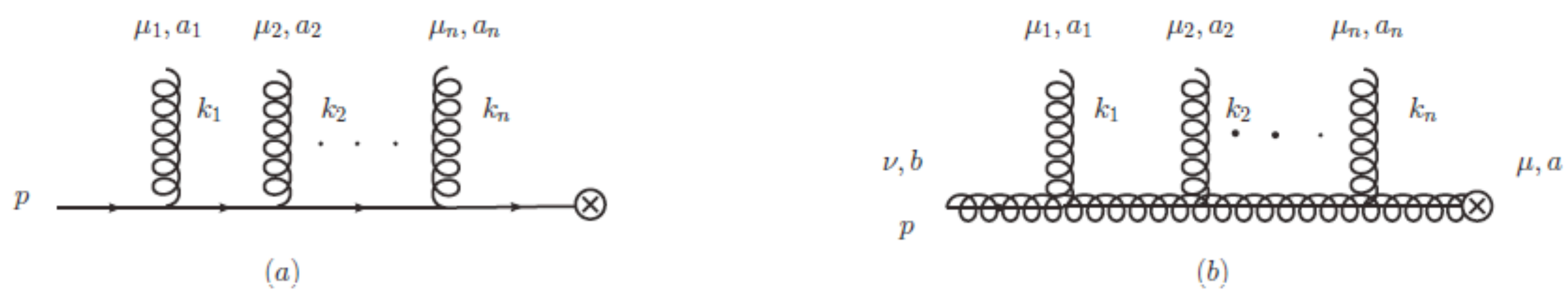


FIG. 3. Feynman diagrams in which usoft gauge particles are attached to (a) a collinear fermion and (b) a collinear gauge particle. The color factors both for the fermions, the scalars and the gauge particles are the same. The diagram with a collinear scalar particle is omitted.

Usoft Wilson Line

$$Y_n t^a Y_n^\dagger = \mathcal{Y}_n^{ba} t^b.$$

$$Y_n = 1 + \sum_{m=1}^{\infty} \sum_{\text{perm}} \frac{(-g)^m}{m!} \frac{n \cdot A_{\text{us}}^{a_1} \cdots n \cdot A_{\text{us}}^{a_m}}{n \cdot k_1 n \cdot (k_1 + k_2) \cdots n \cdot \left(\sum_{i=1}^m k_i\right)} t^{a_m} \cdots t^{a_1}.$$

$$W_n = \sum_{\text{perm}} \exp\left[-g \frac{1}{\overline{\mathcal{P}}} \overline{n} \cdot A_n\right] = \sum_{\text{perm}} \exp\left[-g \frac{1}{\overline{\mathcal{P}}} Y \overline{n} \cdot A_n^{(0)} Y^\dagger\right] = Y W_n^{(0)} Y^\dagger.$$

Usoft gauge transformations $\mathcal{U}_{\text{us}}(x) = \exp[i\beta_{\text{us}}^a(x) T^a]$ are the subset where $\partial^\mu \mathcal{U}_{\text{us}}(x) \sim Q(\eta^2, \eta^2, \eta^2)$.

Decoupling of usoft interaction

$$\mathcal{L}_\lambda^{(0)} \quad = \quad \mathrm{Tr}\left(\overline{\lambda}_n^{(0)} \frac{\overline{n}\cdot\overline{\sigma}}{2}\left[n\cdot iD_c^{(0)} + \sigma\cdot D_{c\perp}^{(0)} W_n^{(0)} \frac{1}{\overline{\mathcal{P}}} W_n^{(0)\dagger} \overline{\sigma}\cdot D_{c\perp}^{(0)}\right]\lambda_n^{(0)}\right),$$

where we use the facts that $\mathcal{P}_\perp^\mu$ commutes with Y and $Y^\dagger n\cdot D_{\mathrm{us}} Y = n\cdot\partial$ since $n\cdot D_{\mathrm{us}} Y = 0$. This is the final collinear Lagrangian in which the collinear fermion is decoupled from the usoft interaction.

TABLE I. Gauge transformations for the collinear, usoft fields and the Wilson lines. The label momenta are suppressed, which can be inserted with the label momentum conservation. For each field, the first (second) row corresponds to the fundamental (adjoint) representation.

Fields	Collinear transformation	Usoft transformation
λ_n	$U_c \lambda_n U_c^\dagger$	$U_{\rm us} \lambda_n U_{\rm us}^\dagger$
	$\mathcal{U}_c \lambda_n$	$\mathcal{U}_{\rm us} \lambda_n$
A_n^μ	$U_c A_n^\mu U_c^\dagger + \frac{1}{g} U_c [i\tilde{D}^\mu U_c^\dagger]$	$U_{\rm us} A_n^\mu U_{\rm us}^\dagger$
$\mathcal{A}_n^\mu$	$\mathcal{U}_c \mathcal{A}_n^\mu \mathcal{U}_c + \frac{1}{g} \mathcal{U}_c [i\tilde{\mathcal{D}}^\mu \mathcal{U}_c^\dagger]$	$\mathcal{U}_{\rm us} \mathcal{A}_n^\mu \mathcal{U}_{\rm us}^\dagger$
$\lambda_{\rm us}$	$\lambda_{\rm us}$	$U_{\rm us} \lambda_{\rm us} U_{\rm us}^\dagger$
	$\lambda_{\rm us}$	$\mathcal{U}_{\rm us} \lambda_{\rm us}$
$A_{\rm us}^\mu$	$A_{\rm us}^\mu$	$U_{\rm us} \left(A_{\rm us}^\mu + \frac{i}{g} \partial^\mu \right) U_{\rm us}^\dagger$
$\mathcal{A}_{\rm us}^\mu$	$\mathcal{A}_{\rm us}^\mu$	$\mathcal{U}_{\rm us} \left(\mathcal{A}_{\rm us}^\mu + \frac{i}{g} \partial^\mu \right) \mathcal{U}_{\rm us}^\dagger$
Wilson lines		
W	$U_c W$	$U_{\rm us} W U_{\rm us}^\dagger$
$\mathcal{W}$	$\mathcal{U}_c \mathcal{W}$	$\mathcal{U}_{\rm us} \mathcal{W} \mathcal{U}_{\rm us}^\dagger$
Y	Y	$U_{\rm us} Y$
$\mathcal{Y}$	$\mathcal{Y}$	$\mathcal{U}_{\rm us} \mathcal{Y}$

Outlook

QCD $\longrightarrow$ N=4 SYM

B physics

Collider physics

$\longrightarrow$

?

Outlook?

- Understand the behavior and the divergence

  structure of high-energy scattering amplitudes

- Duality between Wilson loops and gluon amplitudes

- Extension of the symmetry of the full theory

Back-up

An application

A back-to-back fermion vector current operator

$$J^\mu = \overline{\lambda}\overline{\sigma}^\mu \lambda \quad \Rightarrow \quad J_c^\mu = C(Q,\mu)\overline{\lambda}_{\overline{n}} \mathcal{W}_{\overline{n}} \overline{\sigma}^\mu \mathcal{W}_n^\dagger \lambda_n$$

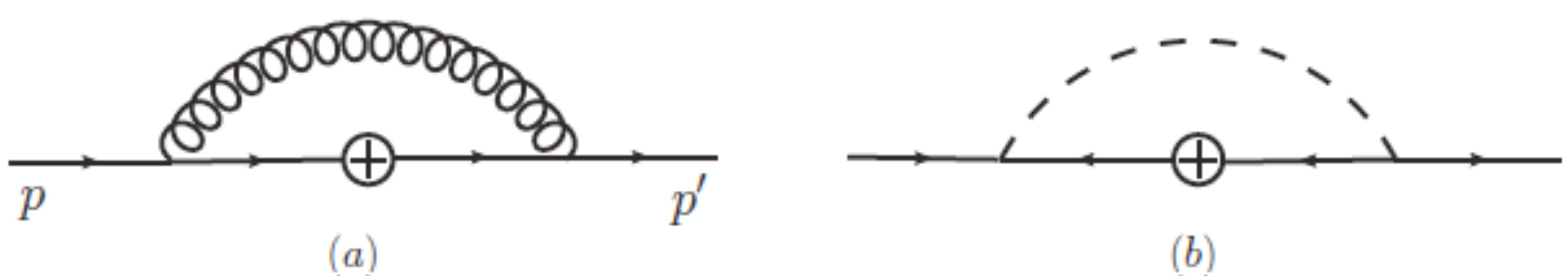


FIG. 4. Feynman diagrams for the vertex corrections at one loop in the full theory with the exchange of (a) a gauge particle, and (b) a scalar particle.

$$M_{\rm full} = -\frac{g^2 C_A}{16\pi^2}\overline{\sigma}^\mu \Big[2\ln\frac{-p^2}{Q^2}\ln\frac{-p'^2}{Q^2} + 2\ln\frac{-p^2}{Q^2} + 2\ln\frac{-p'^2}{Q^2} - 3 + \frac{2\pi^2}{3} + \ln\frac{Q^2}{\mu^2}\Big],$$

where $q^2 = (p-p')^2 = -2p\cdot p' = -\overline{n}\cdot p\, n\cdot p' = -Q^2$.

An application

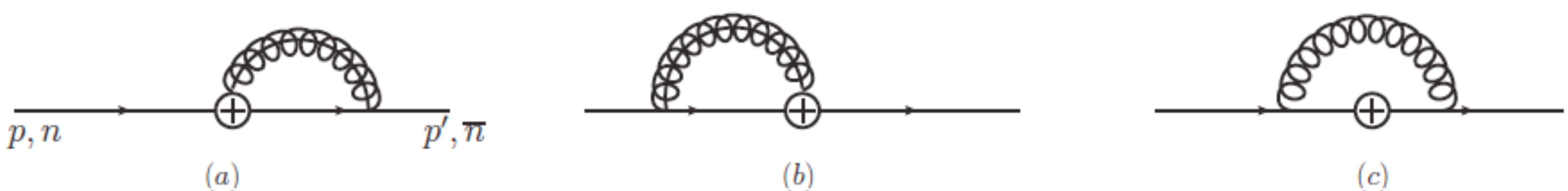


FIG. 5. Feynman diagrams for the vertex corrections at one loop in SCET with the exchange of (a) a $\overline{n}$-collinear gauge particle, (b) a n-collinear gauge particle and (c) an usoft gauge particle.

$$M = M_c + M_{\rm us} = -\frac{g^2 C_A}{16\pi^2}\overline{\sigma}^\mu \Big[-\frac{2}{\epsilon_{\rm uv}^2} - \frac{2}{\epsilon_{\rm uv}} \ln\frac{\mu^2}{Q^2} - \ln^2\frac{\mu^2}{Q^2} + 2\ln\frac{-p^2}{Q^2}\ln\frac{-p'^2}{Q^2} \Big].$$