Neutrino Physics: Some Recent Developments

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Compellings Evidence for $\nu-$Oscillations

$-\nu_{\text{atm}}$: SK UP-DOWN ASYMMETRY

θ_Z-, $L/E-$ **dependences of** $\mu-$**like events**

Dominant $\nu_\mu \longrightarrow \nu_\tau$ K2K, MINOS; CNGS (OPERA)

$-\nu_\odot$: **Homestake, Kamiokande, SAGE, GALLEX/GNO**

Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \longrightarrow \nu_{\mu,\tau}$ **BOREXINO; LowNu (?)**

$-$ LSND: **Dominant** $\bar{\nu}_\mu \longrightarrow \bar{\nu}_e$;

MiniBOONE 2010: $\nu_\mu \longrightarrow \nu_e$ incompatible, $\bar{\nu}_\mu \longrightarrow \bar{\nu}_e$ compatible (!?)

$$\nu_{l\mathrm{L}} = \sum_{j=1} U_{lj}\, \nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

The reference scheme: 3-ν mixing

$$\nu_{l\mathsf{L}} = \sum_{j=1}^{3} U_{lj}\,\nu_{j\mathsf{L}} \qquad l = e, \mu, \tau.$$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• $\nu_j -$ Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• $\nu_j -$ Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21}, α_{31} - the two Majorana CP-violation phases.

S.M. Bilenky, J. Hosek, S.T.P., 1980

- $\Delta m^2_\odot \equiv \Delta m^2_{21} \cong 7.59 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.318$, $\cos 2\theta_{12} \gtrsim 0.26$ (3σ),
- $|\Delta m^2_{\rm atm}| \equiv |\Delta m^2_{31}| \cong 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \cong 1$,
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} < 0.031$ (0.047) 2σ (3σ) (see further).

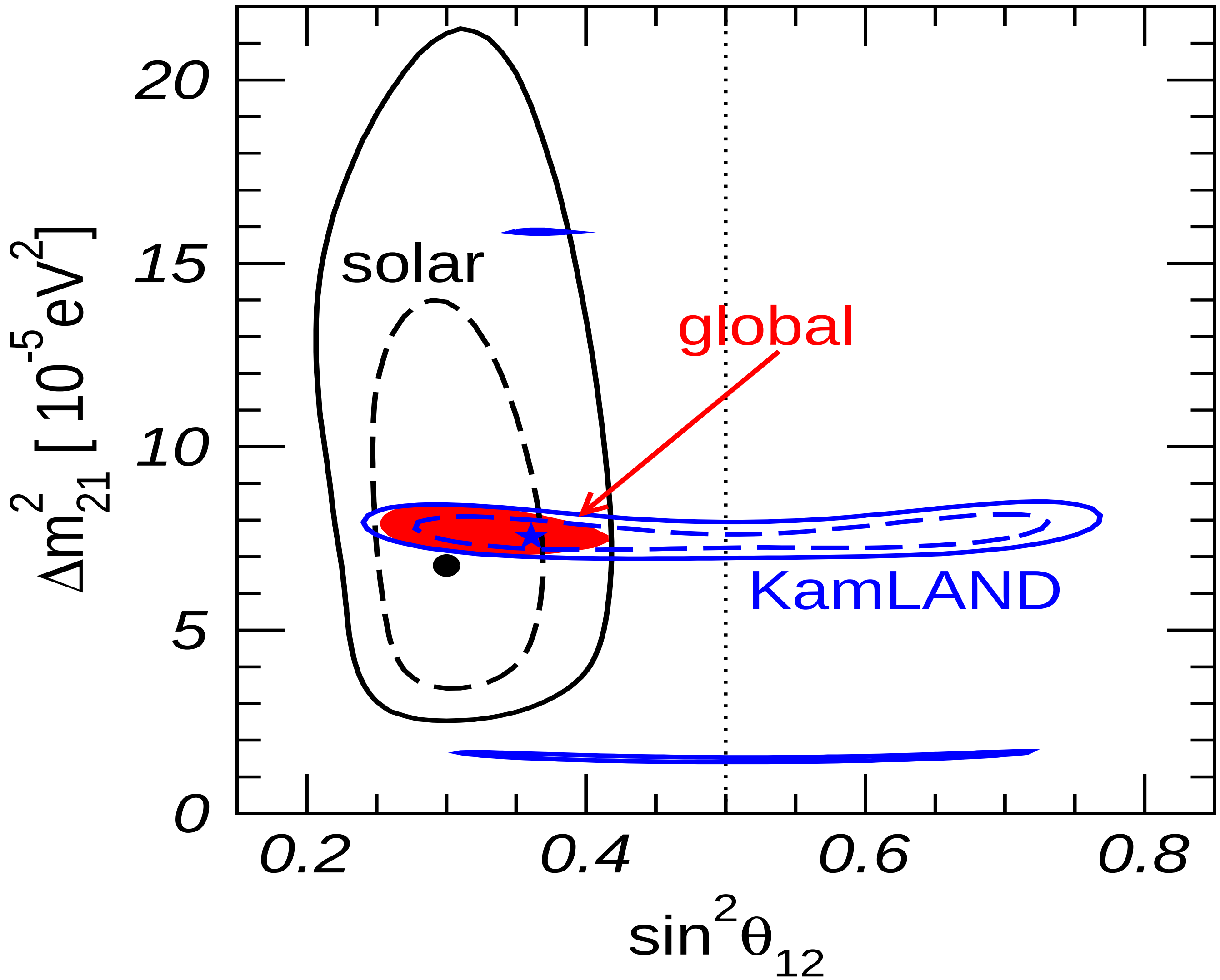
Mezzetto, Schwetz *et al.*, arXiv:1003.5800

$$\sin^2\theta_{13} = 0.016 \pm 0.010,\ \sin\theta_{13} = (0.077 - 0.161)\ ,\quad 1\sigma$$

A.B. Balantekin, D. Yilmaz, arXiv:0804.3345;

E. Lisi *et al.*, arXiv:0806.2649

"solar" parameters



T. Schwetz, arXiv:0710.5027[hep-ph]

- $\text{sgn}(\Delta m^2_{\text{atm}}) = \text{sgn}(\Delta m^2_{31})$ not determined

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering}$$

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering}$$

Convention: $m_1 < m_2 < m_3$ - NMO, $m_3 < m_1 < m_2$ - IMO

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \;\; m^2_{1,2,3} >> \Delta m^2_{\text{atm}}, \;\; \text{QD}; \;\; m_j \gtrsim 0.10 \text{ eV}.$$

- Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A^{(l,l')}_{\text{CP}} \propto J_{\text{CP}} \propto \sin\theta_{13} \sin\delta$
- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $\left|<m>\right|$ in $(\beta\beta)_{0\nu}-$decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\alpha_{21,31}$!

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ^{3}H β-decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \qquad (95\% \text{ C.L.})$$

It is expected that the following sensitivity will be reached:

$$\text{KATRIN}: \qquad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

$$\sum_j m_j \equiv \Sigma < (0.4 - 1.4) \text{ eV}$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \qquad \delta \cong 0.04 \text{ eV}.$$

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .

- Determination of sgn(Δm^2_{atm}), type of $\nu-$ mass spectrum

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\text{atm}},\ \text{QD};\ m_j \gtrsim 0.10\ \text{eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or min(m_j).

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, $\sin^2\theta_{13}$.

- High precision determination of $\Delta m^2_{\odot}$, $\theta_{\odot}$, Δm^2_{atm}, θ_{atm}.

- Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \to e + \gamma$, $\tau \to \mu + \gamma$, etc. decays.

• Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation. Includes understanding

– the origin of the observed patterns of ν-mixing and ν-masses ;

– the physical origin of CPV phases in $U_{\rm PMNS}$;

– Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

– Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12} + \theta_c = \pi/4$?

– Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

– Is there any correlation between the values of CPV phases and of mixing angles in $U_{\rm PMNS}$?

• Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

– Are the Majorana and/or Dirac CPVP in $U_{\rm PMNS}$ the leptogenesis CPV parameters at the origin of BAU?

"Some Recent Developments": a very subjective choice.

The main recent development: the "coming back" of sterile neutrinos.

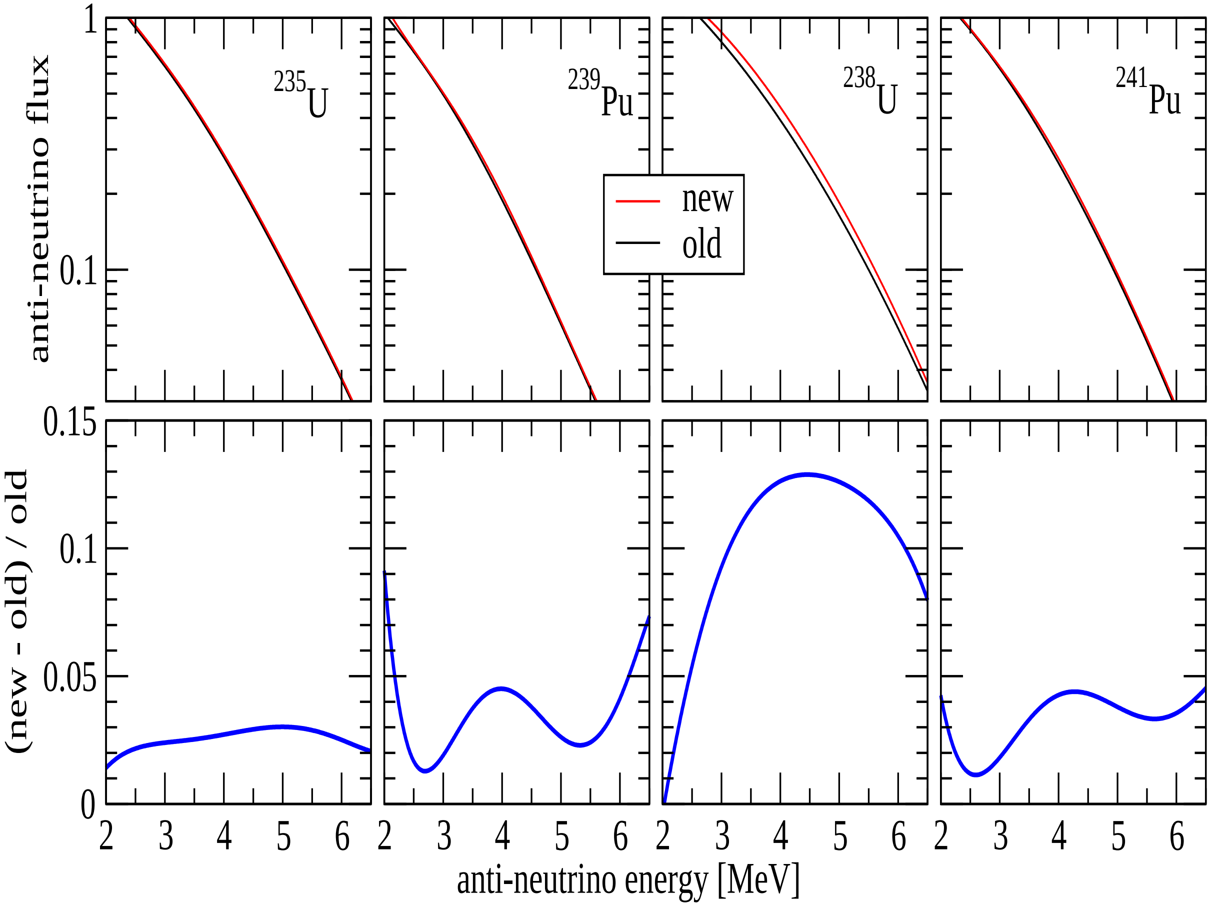
A number of hints at $\sim (2.0 - 2.5)\sigma$ (do not make an evidence, need to be tested experimentally).

- Reactor $\bar{\nu}_e$ flux and anomaly.
- LSND and MiniBooNE.
- Neutrino counting from cosmology.

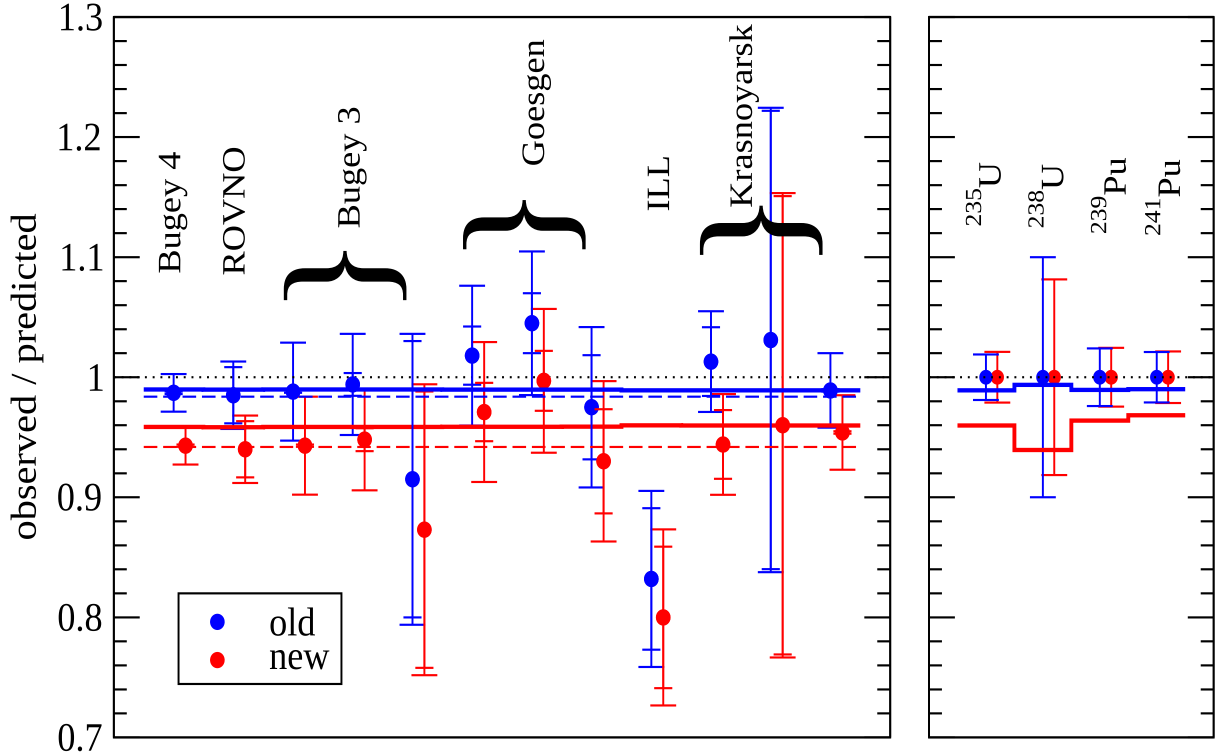
The possibilities of CPT violation and NSI are also being widely discussed.

New reactor $\bar{\nu}_e$ flux predictions

"new fluxes" Mueller et al., 1101.2663 compared to "old" fluxes Schreckenbach et al., '82, '85, '89.



New reactor $\bar{\nu}_e$ fluxes and SBL reactor data



Schwetz, Tortola, Valle, 1103.0734

- **Flux free analysis (dashed lines)**

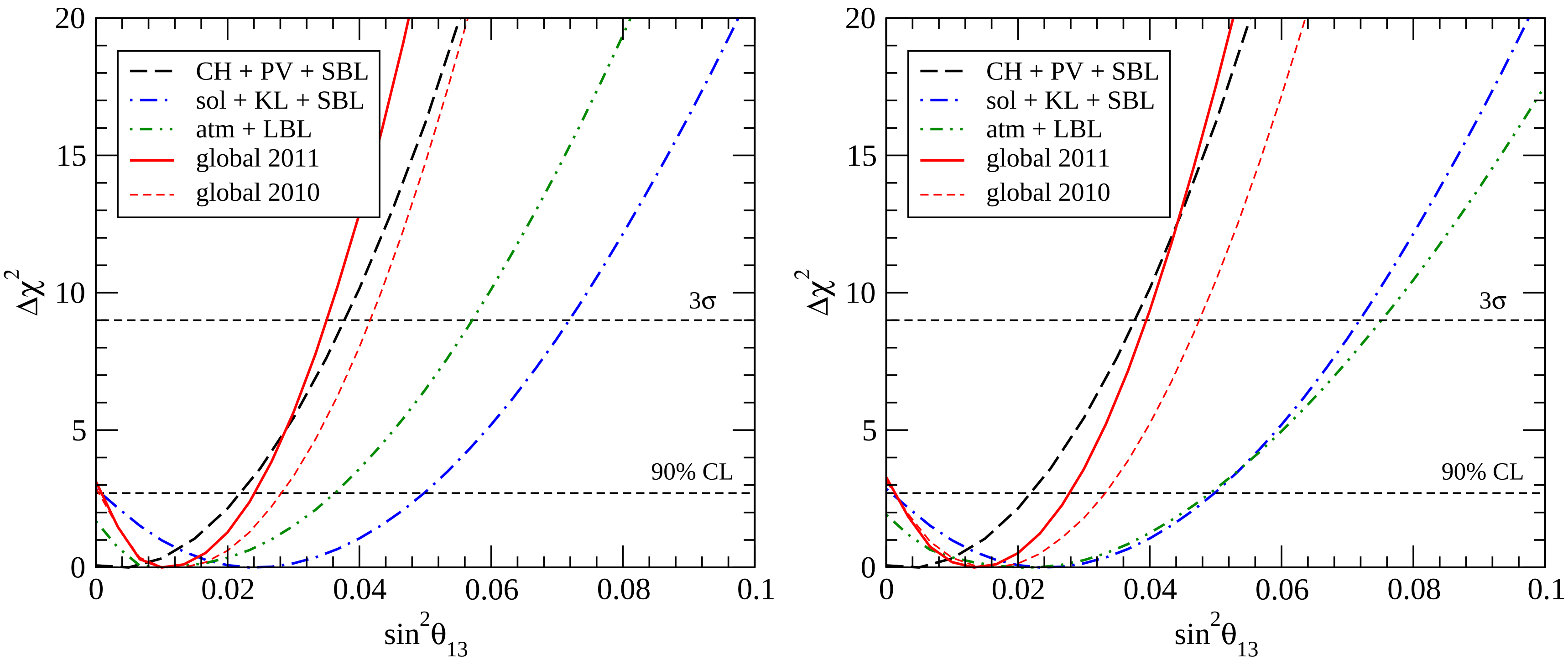
old flux best fit $f = 0.984$**,** $f = 1$ **within 1**σ**.**
new flux best fit $f = 0.942$**,** $f = 1$ **at 2.5**σ**:**
"reactor ν anomaly", Mention et al., 1101.2755.

$\theta_{13} \neq 0$**, or sterile neutrinos, or just a systematic error?**

- $\theta_{13} \sim 0.1$**, or sterile neutrino with** $\Delta m^2 \gtrsim 1$ eV^2, $\sin^2 2\theta \cong 0.12$

$\theta_{13} \neq 0$ **?**

3-ν analysis with the new reactor $\bar{\nu}_e$ fluxes



Schwetz, Tortola, Valle, 1103.0734

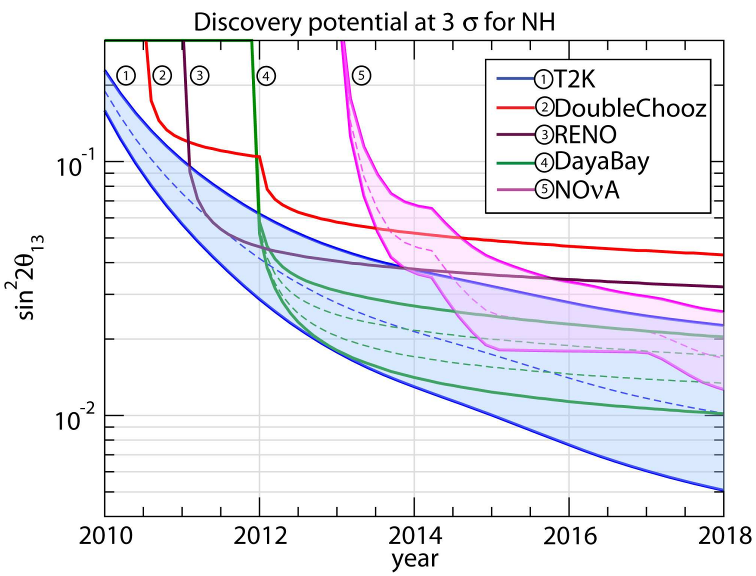
	$\sin^2\theta_{13}$	$\Delta\chi^2(\theta_{13}=0)$	3σ bound
NH	$0.010^{+0.009}_{-0.006}$	3.1 (1.8σ)	0.035
IH	$0.013^{+0.009}_{-0.007}$	3.3 (1.8σ)	0.039

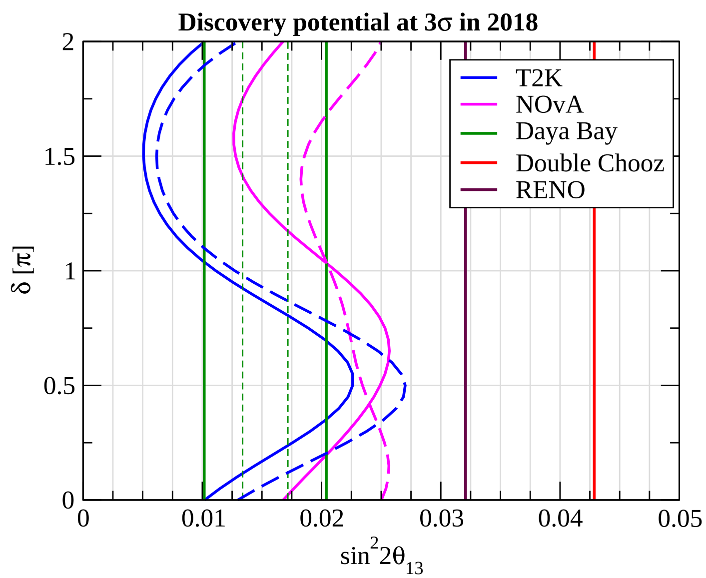
The results on θ_{13} depend on SBL reactor data treatment

	$\sin^2\theta_{13}$	$\Delta\chi^2(\theta_{13}=0)$	3σ bound
solar + KamLAND + SBL	$0.023^{+0.016}_{-0.013}$	2.9 (1.7σ)	0.072
Chooz + Palo Verde + SBL	$0.005^{+0.010}_{-0.020}$	0.07 (0.26σ)	0.038
atmospheric + MINOS	$0.010^{+0.016}_{-0.008}$	1.7 (1.3σ)	0.057
	$0.020^{+0.018}_{-0.015}$	1.9 (1.4σ)	0.075
atmospheric + MINOS + solar	$0.013^{+0.014}_{-0.009}$	2.3 (1.5σ)	0.053
	$0.020^{+0.015}_{-0.012}$	2.7 (1.6σ)	0.065
global with SBL	$0.010^{+0.009}_{-0.006}$	3.1 (1.8σ)	0.035
	$0.013^{+0.009}_{-0.007}$	3.3 (1.8σ)	0.039
global with SBL (free norm)	$0.007^{+0.009}_{-0.005}$	2.0 (1.4σ)	0.032
	$0.010^{+0.009}_{-0.007}$	1.9 (1.4σ)	0.037
global without SBL	$0.020^{+0.010}_{-0.008}$	7.0 (2.6σ)	0.048
	$0.027^{+0.009}_{-0.010}$	8.0 (2.8σ)	0.054
global without SBL (old fluxes)	$0.012^{+0.010}_{-0.007}$	2.9 (1.7σ)	0.042
	0.017 ± 0.010	3.2 (1.8σ)	0.048

NH: upper entries; IH: lower entries.

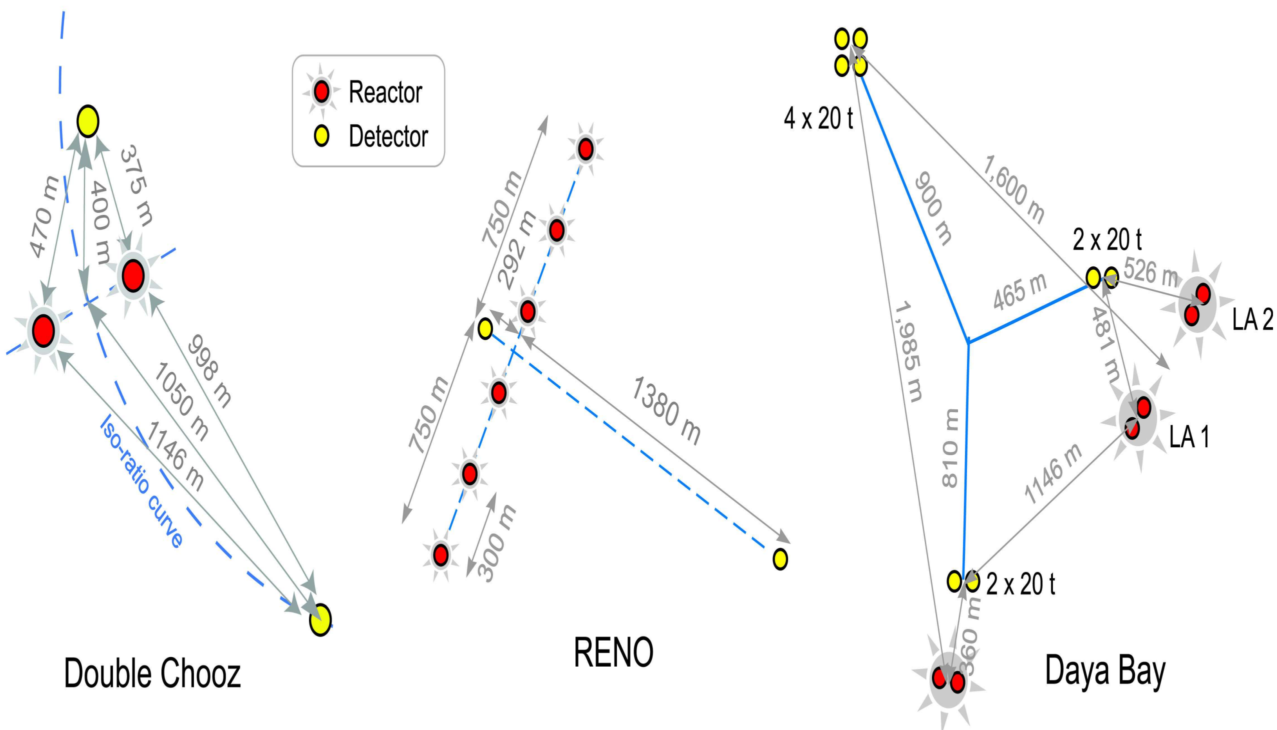
Prospects for θ_{13}





Width: experiments with ν beams - dependence on the CP phase; DayaBay - syst. uncert. $0.18\% - 0.6\%$

Mezzetto, Schwetz, 1003.5800



M. Mezzetto, T. Schwetz, arXiv:1003.5800[hep-ph]


Sendai
Super Kamiokande
295km
JAERI
(Tokai)
Japan
KEK
Tokyo
Gifu
Nagoya
Kawasaki
Kyoto
Yokohama
Osaka
(c) 2000 ESRI
~ 420.0 mi / 675.8 km across

T2K: First results

Observed 1 e^--event passing all cuts and having all the characteristics of being due to $\nu_\mu \to \nu_e$ oscillations. The estimated background is: 0.30 ± 0.07**(syst.) events;** 29% **probability to observe 1 event when the expected average is 0.3 events.**

NH: $\sin^2 2\theta_{13} < 0.44$; IH: $\sin^2 2\theta_{13} < 0.53$ (90% C.L.)

Assumed: $\Delta m^2_{23} = 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1$, $\delta = 0$.

Japan experienced very severe earthquake on March 11th 2011 at 14:46 JST. The J-PARC facility suffered some damages. Fortunately, the Tsunami did not hit the J-PARC.

"Our present priority is to restore life-supporting infrastructure such as electricity, water supply and gas at J-PARC. It may take some time, but we promise the full recovery of the J-PARC accelerator and T2K experiment in the near future. I thank you for the messages of solidarity and sympathy".
Director of the Institute of Particle and Nuclear Studies, KEK
Koichiro Nishikawa
Spokesperson of the T2K experiment
Takashi Kobayashi

We wish success to our colleagues from T2K, J-PARC and KEK in their reconstruction efforts.

Neutrino oscillation parameters summary.

parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	$7.59^{+0.20}_{-0.18}$	7.24–7.99	7.09–8.19
Δm^2_{31} $[10^{-3}$ $\mathrm{eV}^2]$	2.45 ± 0.09 $-(2.34^{+0.10}_{-0.09})$	$2.28 - 2.64$ $-(2.17 - 2.54)$	$2.18 - 2.73$ $-(2.08 - 2.64)$
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	0.28–0.35	0.27–0.36
$\sin^2\theta_{23}$	0.51 ± 0.06 0.52 ± 0.06	0.41–0.61 0.42–0.61	0.39–0.64
$\sin^2\theta_{13}$	$0.010^{+0.009}_{-0.006}$ $0.013^{+0.009}_{-0.007}$	≤ 0.027 ≤ 0.031	≤ 0.035 ≤ 0.039

Schwetz, Tortola, Valle, 1103.0734

New reactor $\bar{\nu}_e$ fluxes + SBL reactor $\bar{\nu}_e$ data used.
Δm^2_{31}, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$: upper (lower) row - NH (IH).

Sterile Neutrinos ?

Sterile neutrinos: oscillations with $\Delta m^2 \sim 1$ eV^2 can account for $\bar{\nu}_e$ disappearance at $L \lesssim$ 100 m

$\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ data at $E/L \sim 1$ eV^2

- LSND, $\bar{\nu}_\mu \to \bar{\nu}_e$, $87.9 \pm 22.4 \pm 6.0$ excess events
$P = (0.264 \pm 0.067 \pm 0.045)\%$ $\sim 3.8\sigma$ away from zero.

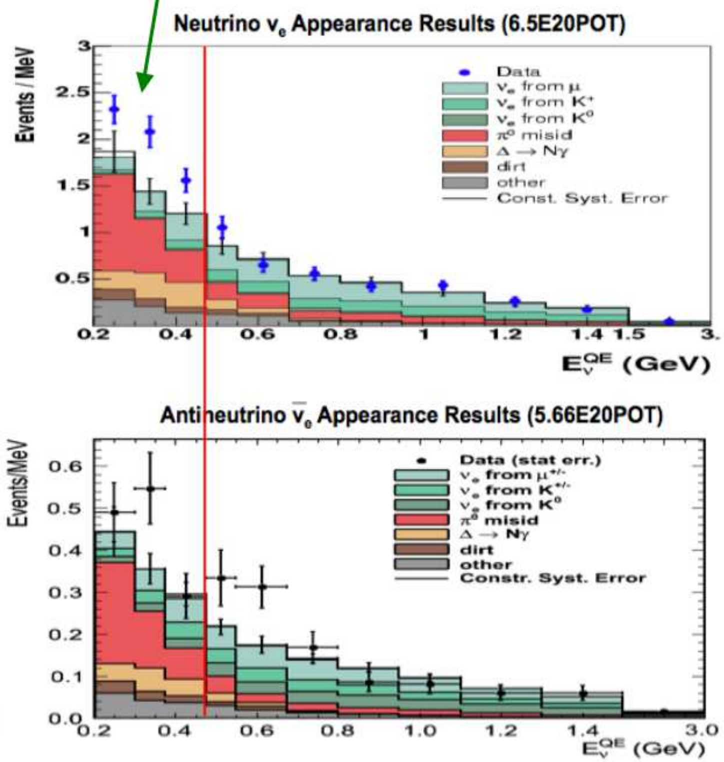
- MiniBooNE, $\bar{\nu}_\mu \to \bar{\nu}_e$, $\sim 2\sigma$ excess;
Consistent with LSND in a 2-ν mixing scheme.

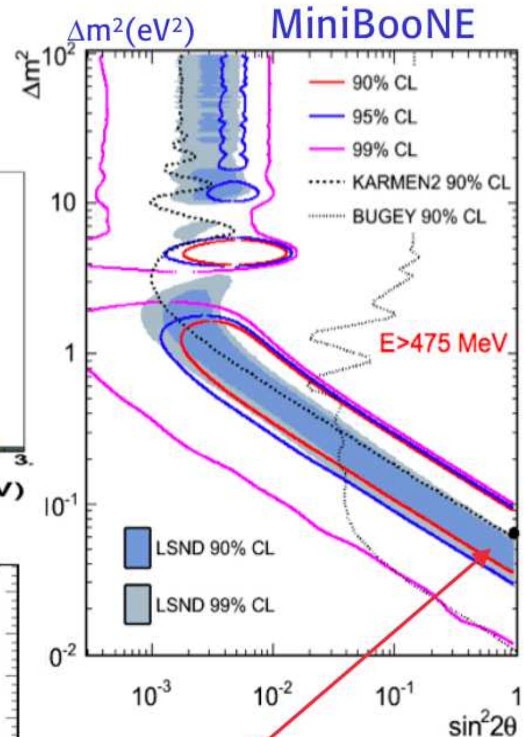
- MiniBooNE, $\nu_\mu \to \nu_e$,
$E > 475$ MeV: no excess; $E < 475$ MeV: $\sim 3\sigma$ excess.

- KARMEN, $\bar{\nu}_\mu \to \bar{\nu}_e$, tight constraint on the LSND region (slightly smaller L/E than at LSND).

Unidentified excess at low energy

Mills





Best fit point in Bugey excluded area

G. Mills, Talk given at NeuTel, Venice, March 2011

3 + 1 and 3 + 2 fits to the global SBL data

3 + 1 Oscillations:

$P_{ee} \cong 1 - 4|U_{e4}|^2(1-|U_{e4}|^2)\sin^2\frac{\Delta m^2_{41}\,L}{4E}$, reactor SBL data;

LSND, MiniBooNE, KARMEN data :

$P_{\mu e} \cong 4|U_{e4}|^2|U_{\mu 4}|^2 \sin^2\frac{\Delta m^2_{41}\,L}{4E}$,

effectively 2-ν oscillations, no CP violation.

$|U_{e4}|^2$ ($|U_{\mu 4}|^2$) constrained by the data on ν_e (ν_μ) disap-perance.

3 + 2 Oscillations:

LSND, MiniBooNE, KARMEN data:

$$P_{\mu e} \cong 4|U_{e4}|^2|U_{\mu 4}|^2 \sin^2\Delta_{41}$$

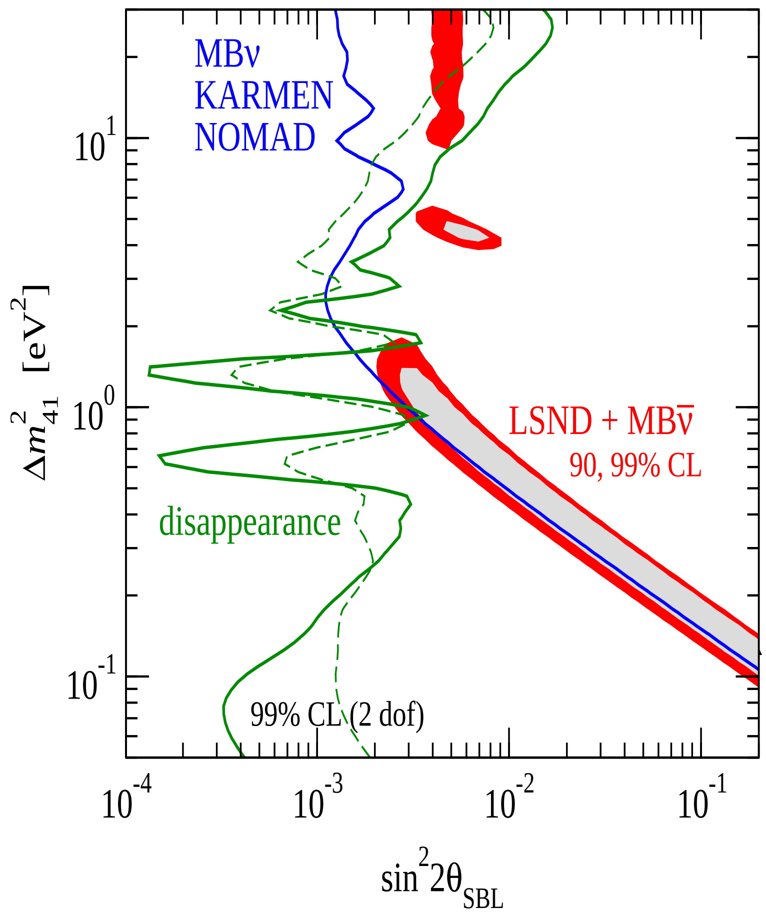
$$+4|U_{e5}|^2|U_{\mu 5}|^2 \sin^2\Delta_{51}$$

$$+8|U_{e4}U_{\mu 4}U_{e5}U_{\mu 5}| \sin\Delta_{41} \sin\Delta_{51} \cos(\Delta_{54} - \phi),$$

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}, \quad \phi \equiv \arg\left(U^*_{e4}U_{\mu 4}U_{e5}U^*_{\mu 5}\right).$$

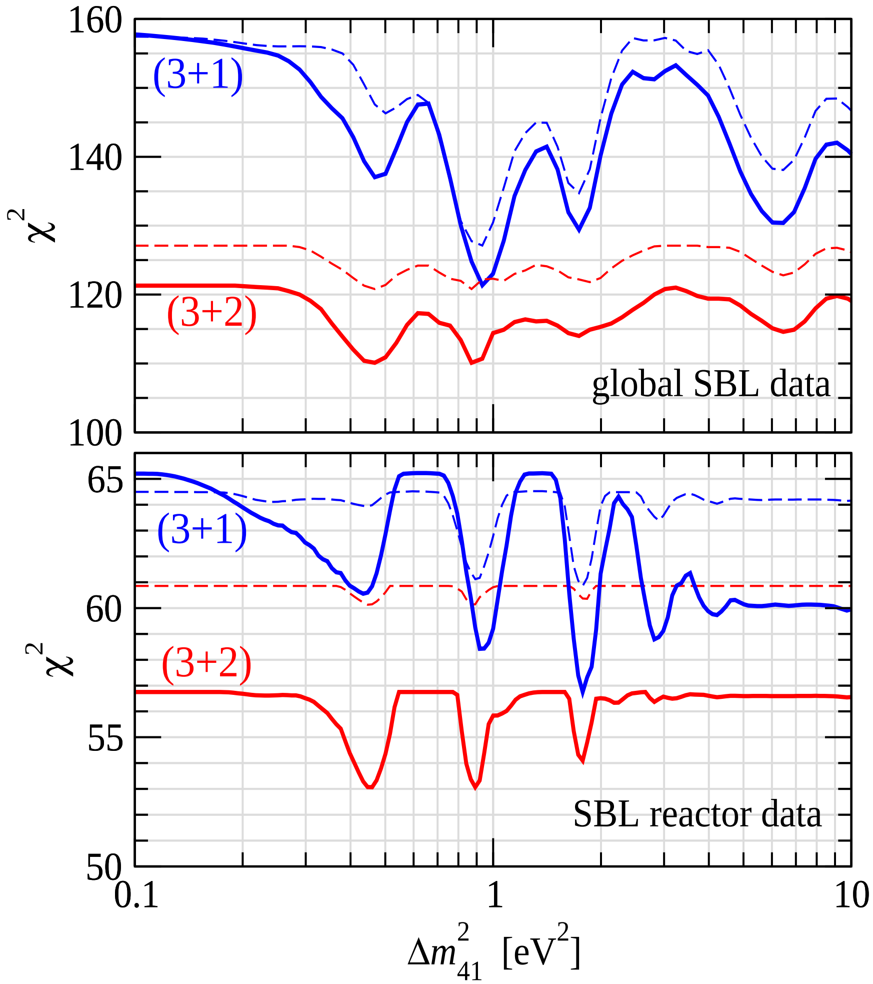
ϕ - CP violating phase (MiniBooNE).

Sorel et al., 2003; Maltoni and Schwetz, 2007; Karagiorgi, 2007



Kopp, Maltoni, Schwetz, 1103.4570

3 + 1 analysis (disappearance: CDHS + reactor SBL (solid (dashed) line - new (old) fluxes) + atmospheric ν data, 99% C.L.).



Kopp, Maltoni, Schwetz, 1103.4570

Dashed lines: old reactor $\bar{\nu}_e$ fluxes;
solid lines: new reactor $\bar{\nu}_e$ fluxes.

Results of 3 + 1 and 3 + 2 Fits

SBL Reactor Data

	Δm^2_{41} [eV^2]	$\lvert U_{e4}\rvert$	Δm^2_{51} [eV^2]	$\lvert U_{e5}\rvert$	χ^2/dof
3+1	1.78	0.151			50.1/67
3+2	0.46	0.108	0.89	0.124	46.5/65

Global SBL Data

	Δm^2_{41}	$\lvert U_{e4}\rvert$	$\lvert U_{\mu 4}\rvert$	Δm^2_{51}	$\lvert U_{e5}\rvert$	$\lvert U_{\mu 5}\rvert$	δ/π	χ^2/dof
3+2	0.47	0.128	0.165	0.87	0.138	0.148	1.64	110.1/130

$\Delta\chi^2$(old versus new fluxes)$\cong$ 10, 3+2.

$\Delta\chi^2$ (3+1 versus 3+2)$\cong$ 11.

Kopp, Maltoni, Schwetz, 1103.4570

Comments.

• 3 + 1 neutrino mixing scheme is strongly disfavored by the data (no CP violation, tension between the description of the "appearance" and "disappearance" data).

• 3 + 2 neutrino mixing scheme (with CP violation and two eV mass neutrinos) provides a good fit to the global SBL data.

• Only a relatively small active-sterile neutrino transition probability is allowed by the data.

• Theoretically, light sterile neutrinos are not particularly "needed", but can be incorporated in the neutrino mass models. If they are proven to exist, this will be a very important discovery and we will have to understand why "they are there".

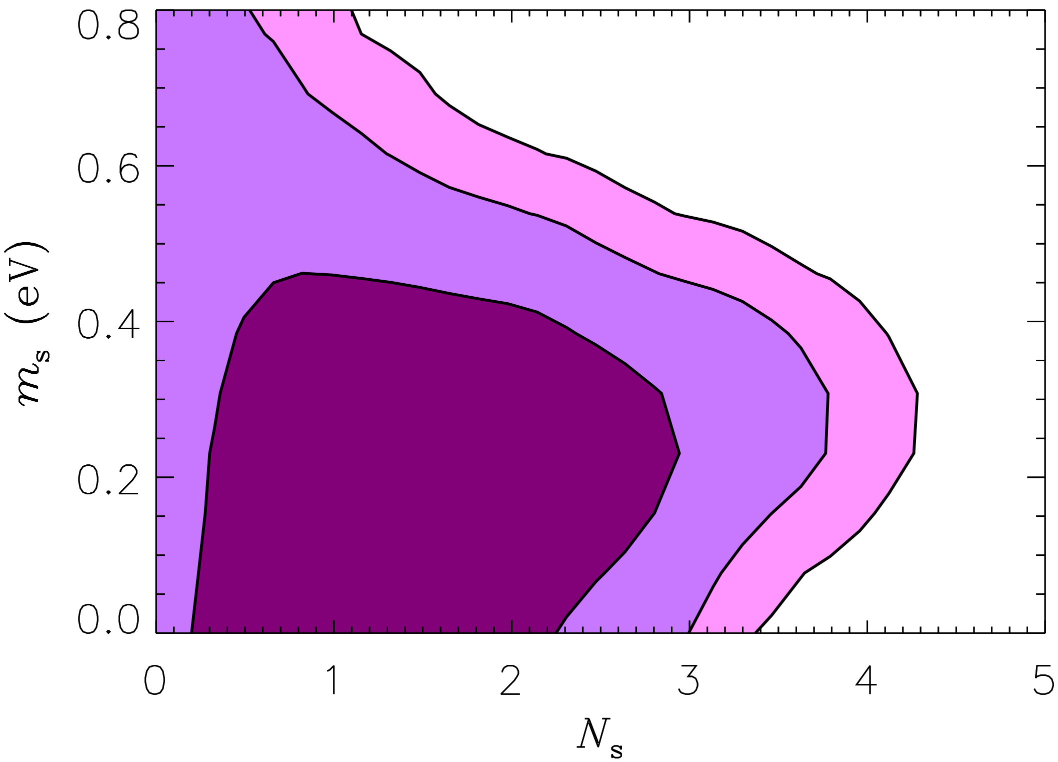
The notion of sterile neutrino: B. Pontecorvo, 1967.

Other possible indications of sterile neutrinos

- **Deficit in the observed rate due to a radioactive source with known intensity in the Gallium experiments (Giunti, Laveder, 2010).**

- **Cosmology**

CMB, SDSS, HST (68%, 95%, 99% C.L.)



Hamann et al., 1006.5276

Mangano, Serpico, 1103.1261

BBN: $N_s < 1.2$ (95% C.L.)

Prospects for sterile neutrinos

Experimental verification of the hints of existence of sterile neutrinos needed.

- New results (from 5.66 to 8.0 pot) from MiniBooNE are expected in the summer.

- C. Rubbia proposal: two argon detectors of 150 t and 600 t (ICARUS from LNGS) at $L \sim 150$m and 600 m at CERN, using the PS neutrino beam (can provide the best test of the LSND and MiniBooNE results).

- BOREXINO radioactive source experiment (Ianni et al., 1999).

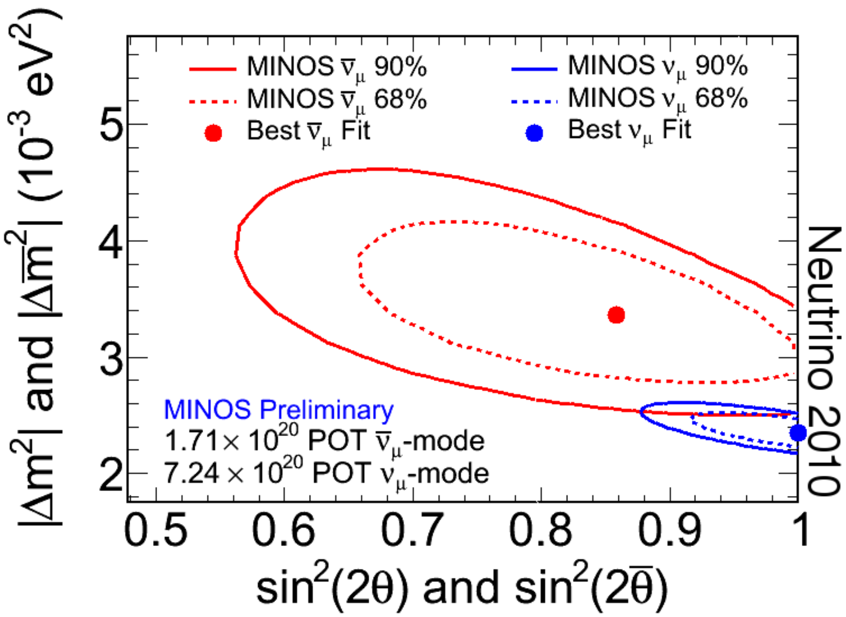
- New SAGE radioactive source experiment (Gorbachev et al., 2011).

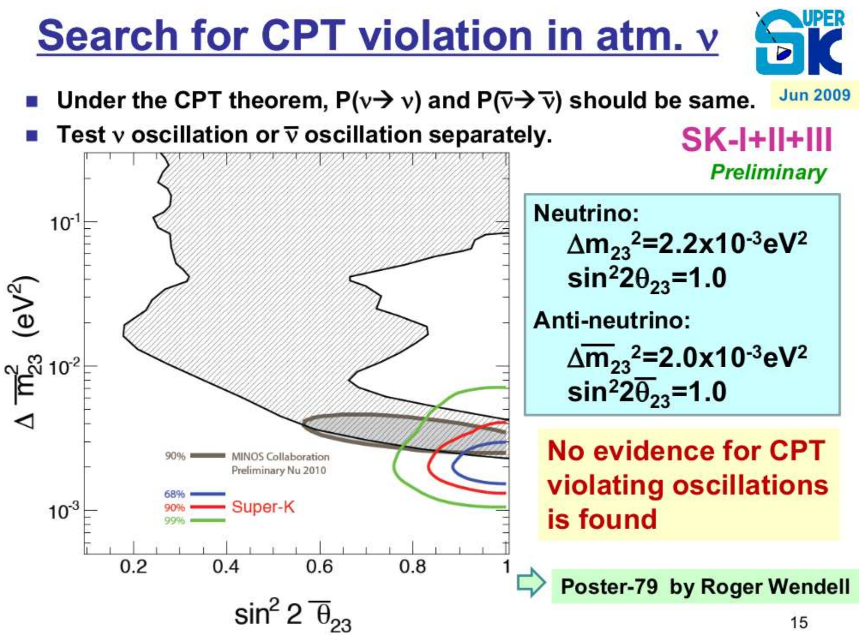
- NUCIFER in Saclay: OSIRIS (core size 57x57x60 cm^3), detector 1.2x07 m^2 (850 l), $\bar{L} = 7$ m (Lasserre et al., 2011).

- Many other proposals (move MiniBooNE closer to the source, MicroBooNE - 70 t Liquid Argon TPC, etc.).

CPT Violation in Neutrino Oscillations?

(I. Bigi, 1982; Murayama, Yanagida, 2001 (LSND))
(Barger, Marfatia, Whisnant (2003) (LSND: 4 ν's + CPT))





Expected further data from MINOS this summer.

The 3-neutrino mixing - still the reference framework.

3-ν oscillation parameters summary.

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$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	0.28–0.35	0.27–0.36
$\sin^2\theta_{23}$	0.51 ± 0.06 0.52 ± 0.06	0.41–0.61 0.42–0.61	0.39–0.64
$\sin^2\theta_{13}$	$0.010^{+0.009}_{-0.006}$ $0.013^{+0.009}_{-0.007}$	≤ 0.027 ≤ 0.031	≤ 0.035 ≤ 0.039

Schwetz, Tortola, Valle, 1103.0734

New reactor $\bar{\nu}_e$ fluxes + SBL reactor $\bar{\nu}_e$ data used.
Δm^2_{31}, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$: upper (lower) row - NH (IH).

Neutrino Mixing:

- $\theta_{12} = \theta_\odot \cong \frac{\pi}{5.4}$, $\qquad \theta_{23} = \theta_{atm} \cong \frac{\pi}{4}$, $\qquad \theta_{13} < \frac{\pi}{13}$

$$U_{PMNS} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} ;$$

Very different from the CKM-matrix!

- $\cos\theta_{12} \cong \cos(\frac{\pi}{4} - \frac{\pi}{12}) = \frac{1}{\sqrt{2}}(1+\lambda)$, $\quad \sin\theta_{12} \cong \frac{1}{\sqrt{2}}(1-\lambda)$,
- $\lambda \cong (0.20 - 0.25)$: $\quad \theta_\odot + \theta_c = (47 \pm 1.2)^{deg} = \pi/4$?
- U_{PMNS} **due to new approximate symmetry?**

A Natural Possibility:

$$U = U_{\rm lep}^{\dagger}(\lambda)\; U_{\rm bim(tri)}\; P(\alpha_{21}, \alpha_{31})\,,$$

with

$$U_{\rm tri} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}\;;\quad U_{\rm bim} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}\,.$$

- $U_{\rm lep}^{\dagger}(\lambda)$ - from diagonalization of the l^- mass matrix,

- $U_{\rm bim(tri)}P$ - from diagonalization of the $\nu-$mass matrix.

- $U_{\rm (tri)}$: $s_{12}^2 = 1/3$, $s_{23}^2 = 1/2$, $s_{13}^2 = 0$, in agreement with the current data.

Harrison, Perkins, Scott, 2002

$U_{\rm (tri)}$: Groups A4, S4,... (vast literature)

(Reviews: Altarelli, Feruglio, 1002.0211, Tanimoto et al., 1003.3552)

Typically, $\theta_{13} \sim (\lambda_c)^2$ is predicted.

• U_{bim}: $s^2_{12} = 1/2$, $s^2_{23} = 1/2$, $s^2_{13} = 0$; $s^2_{12} = 1/2$ must be corrected.

U_{bim}: Groups S4,...; typically $\theta_{13} \sim \lambda_c$.

• U_{bim}: alternatively $U(1)$,

$$L' = L_e - L_\mu - L_\tau \qquad (\Delta m^2_\odot \ll |\Delta m^2_{\text{atm}}|)$$

S.T.P., 1982

$U_{\text{bim(tri)}}$ requires a $\mu - \tau$ symmetry of M_ν

• Additional possibility: $\sin^2\theta_{12} = \frac{2}{5+\sqrt{5}} \cong 0.28$, "Golden ratio"; A_5.

Kajiyama et al., 2007; Everett, Stuart, 2008, 2011

$$U_{\text{GR}} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\frac{\sin\theta_{12}}{2} & \frac{\cos\theta_{12}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}}{2} & \frac{\cos\theta_{12}}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

Feruglio, 2011

Which is the correct approximate form of U_{PMNS}? Perhaps none of the above(?).

For $\sin^{\ell}\theta_{ij} \equiv \lambda_{ij}$ "small", $\lambda_{12} \gg \lambda_{13}$ (natural),

$\sin^2\theta_{12} = \frac{1}{2} - \sin\theta_{13}\,\cos\delta\,, \quad U_{\text{bim}}\,,$

δ is the Dirac CPV phase,

$\sin^2\theta_{12} = \frac{1}{3} - 2\,\frac{\sqrt{2}}{3}\,\sin\theta_{13}\,\cos\delta\,, \quad U_{\text{tri}}\,.$

P. Frampton, S.T.P., W. Rodejohann, 2004;
S. King, 2005; S. Antusch, S. King, 2005; I. Masina, 2006;
K. Hochmuth, S.T.P., W. Rodejohann, 2007

Can be tested experimentally.

S. Antusch, P. Huber, S. King, T. Schwetz, 2006

Neutrino Mixing within a Framework: A_4 Flavor Symmetry

A_4: the group of even permutations of 4 objects;
A_4 representations: **3, 1, 1', 1";**
related to the TETRAHEDRAL symmetry;
leads to $U_{\text{PMNS}} = U_{\text{TB}} P$.
Leptogenesis: see-saw mechanism; N_j - heavy RH ν's;
N_j, ν_k - Majorana particles.
Altarelli-Feruglio (AF) Type: $A_4 \times Z_3 \times U(1)_{\text{FN}} \times Z_2$;
Altarelli-Meloni (AM) Type: $A_4 \times Z_4 \times Z_2$.
3: ψ_l, ν_L^c; **1, 1', 1"**: e^c, τ^c, μ^c.

See-Saw (symmetries):

$$M_R = \begin{pmatrix} X+2Z & -Z & -Z \\ -Z & 2Z & X-Z \\ -Z & X-Z & 2Z \end{pmatrix}$$

$$M_D = y'_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} v_u$$

$$\text{diag}(M_1 e^{i\varphi_1}, M_2 e^{i\varphi_2}, M_3 e^{i\varphi_3}) = U_{TB}^T \, M_R \, U_{TB} \, ,$$

$$
\begin{aligned}
M_1 &= |X + 3Z| \;\equiv\; |X|\,|1 + \alpha e^{i\phi}|, \quad \varphi_1 = \arg(X + 3Z) \\
M_2 &= |X|, \quad \varphi_2 = \arg(X) \\
M_3 &= |X - 3Z| \;\equiv\; |X|\,|1 - \alpha e^{i\phi}|, \quad \varphi_3 = \arg(3Z - X)\,,
\end{aligned}
$$

$$
|X|, \quad \alpha \equiv |3Z/X|, \quad \phi \equiv \arg(Z) - \arg(X); \quad \varphi_i = \varphi_i(\alpha, \phi).
$$

A Majorana mass term for ν_{lL} (type I see-saw mechanism):

$$
m_\nu \;=\; -M_D^T\, M_R^{-1}\, M_D = U^* \mathrm{diag}\,(m_1, m_2, m_3)\, U^\dagger
$$

$$
U = i\, U_{TB}\, \mathrm{diag}\left(1, e^{i\varphi_2/2}, e^{i\varphi_3/2}\right) = i\, \mathrm{diag}\,(1, 1, -1)\; U_{\mathrm{PMNS}}\,,
$$

$$
U_{\mathrm{PMNS}} = V\,P\ : \qquad V = \mathrm{diag}\,(1, 1, -1)\; U_{TB}, \quad P = \mathrm{diag}\left(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}\right)\,,
$$

$$
\alpha_{21} = \varphi_2\,, \quad \alpha_{31} = \varphi_3 \;\; (s_{13} \sim \lambda^2 \sim 0.04)\,.
$$

$$
m_i \equiv \frac{(y'_\nu)^2 v_u^2}{M_i} \propto \frac{1}{M_i}\,, \quad i = 1, 2, 3
$$

The parameters α and ϕ are constrained by the ν- oscillation data:

$$r \equiv \frac{\Delta m^2_{\odot}}{|\Delta m^2_{\rm A}|} = \frac{(1+\alpha^2 - 2\,\alpha\,\cos\phi)(\alpha + 2\cos\phi)}{4\,|\cos\phi|} = 0.032 \pm 0.006\,,$$

$\Delta m^2_{\odot} = \Delta m^2_{21} \equiv m^2_2 - m^2_1 > 0$ and $|\Delta m^2_{\rm A}| = |\Delta m^2_{31}| \cong |\Delta m^2_{32}|$.

$\cos\phi > 0$ corresponds to light ν- mass spectrum with normal ordering (NO);
$\cos\phi < 0$ corresponds to ν-mass spectrum with inverted ordering (IO).

$$m_1^2 = \Delta m^2_{\rm A} r \left(\frac{1}{1+2\alpha^2} + \frac{2(1+\alpha^2)r}{(1+2\alpha^2)^3}\right)\,, \quad \text{NO spectrum}\,;$$

$$m_3^2 = |\Delta m^2_{\rm A}| \left(\frac{1}{2\alpha^2} + \frac{(1+\alpha^2)r}{\alpha^2(1+2\alpha^2)}\right)\,, \quad \text{IO spectrum}\,.$$

$$\tan\alpha_{21} = -\frac{\alpha\sin\phi}{1+\alpha\cos\phi}\,, \quad \tan\alpha_{31} = 2\,\frac{\alpha\sin\phi}{\alpha^2-1}\,.$$

The general sum rule holds (valid for both types of spectrum):

$$\frac{e^{i\alpha_{31}}}{m_3} = \frac{1}{m_1} - \frac{2e^{i\alpha_{21}}}{m_2}\,;$$

$$r = 0.032 \pm 0.006\,,$$

NO: $0.8 \lesssim \alpha \lesssim 1.2$;

$M_1 \cong 2M_2 \cong 10\, M_3$; $M_3 \sim 10^{12}$ GeV;

$\alpha_{21} \cong 0, \quad \pi \lesssim \alpha_{31} \lesssim 2\pi\,;$

3.8×10^{-3} eV $\lesssim m_1 \lesssim 6.9 \times 10^{-3}$ eV ;

6.25×10^{-2} eV $\lesssim m_1 + m_2 + m_3 \lesssim 6.76 \times 10^{-2}$ eV ;

$$|\!<\!m\!>\!| \;\cong \left|\tfrac{2}{3} m_1 + \tfrac{1}{3}\sqrt{m_1^2 + \Delta m^2_{\odot}}\right| \cong (6.5 - 7.5) \times 10^{-3}\ \text{eV}.$$

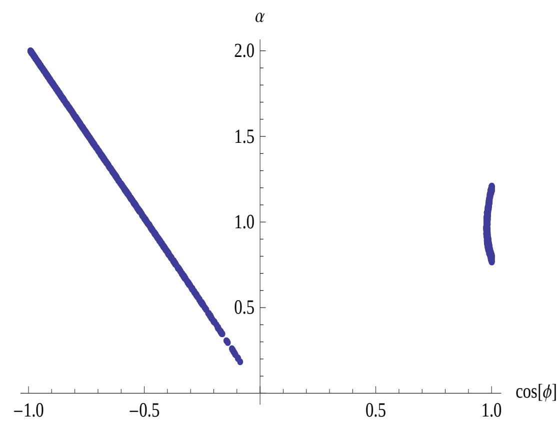
IO: $0.07 \lesssim \alpha \lesssim 2 \quad (0.07$: $m_{1,2,3} \leq 0.5$ eV);

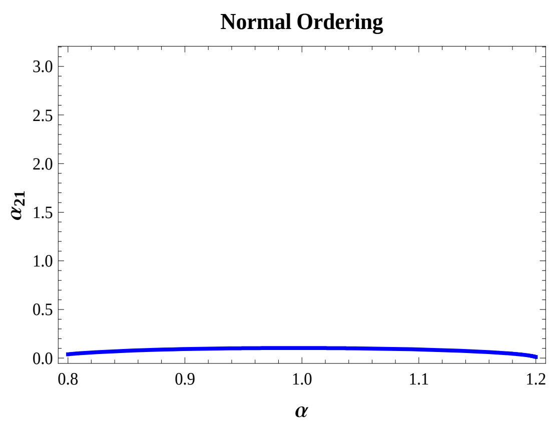
$M_1 \cong M_2 \cong M_3/3$; $M_1 \sim 10^{12}$ GeV;

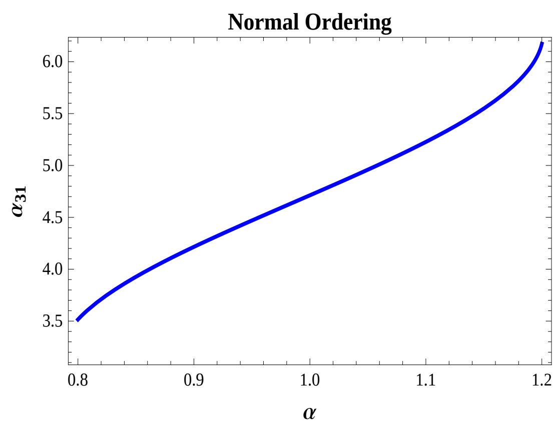
$0 \lesssim \alpha_{21} \lesssim \pi, \quad \pi \lesssim \alpha_{31} \lesssim 2\pi\,;$

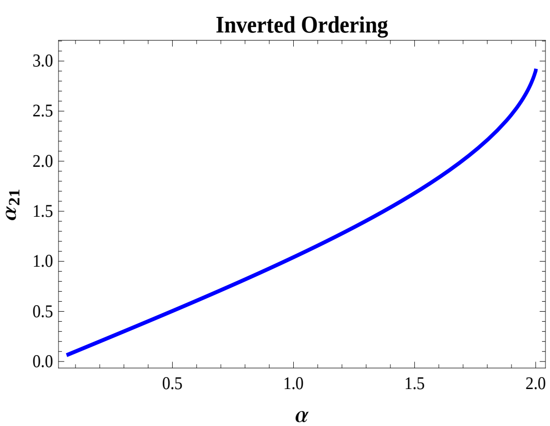
2×10^{-2} eV $\lesssim m_3 \lesssim 0.5$ eV; $\quad m_1 + m_2 + m_3 \gtrsim 0.125$ eV ;

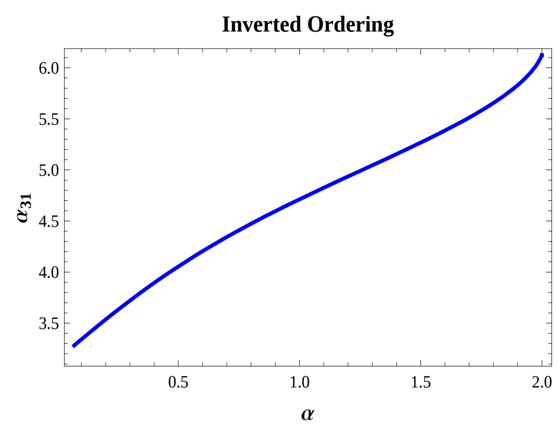
$$0.018\ \text{eV} \lesssim \frac{1}{3}\sqrt{m_3^2 + |\Delta m^2_{\text{A}}|} \lesssim |\!<\!m\!>\!| \;\lesssim \sqrt{m_3^2 + |\Delta m^2_{\text{A}}|}\,,\ \text{for}\ \ m_3 \gtrsim 0.02\ \text{eV}\,.$$


α
cos[ϕ]


Normal Ordering
α_{21}
α


Normal Ordering
α_{31}
α


Inverted Ordering
α_{21}
α


Inverted Ordering
α_{31}
α

Leptogenesis

Higher order corrections to $M_D M_D^\dagger$ must be taken into account.

$$Y_B \approx \sum_{i=1}^{3} Y_{Bi}\,, \qquad Y_{Bi} \;\equiv\; -1.48 \times 10^{-3} \epsilon_i \, \eta_{ii}$$

$$\epsilon_i \;=\; \frac{1}{8\pi v_u^2} \sum_{j \neq i} \frac{\mathrm{Im}[(\widehat{M}_D \widehat{M}_D^\dagger)_{ji}^2]}{(\widehat{M}_D \widehat{M}_D^\dagger)_{ii}} \, f\left(\frac{m_i}{m_j}\right) \;, \qquad \widehat{M}_D = U^\dagger M_D\,,$$

$$f(x) \;\equiv\; -x\left(\frac{2}{x^2-1} + \log\left(1+\frac{1}{x^2}\right)\right)$$

$$\eta_{ii} \;=\; \left(\frac{3.3 \times 10^{-3}\,\mathrm{eV}}{\widetilde{m}_i} + \left(\frac{\widetilde{m}_i}{0.55 \times 10^{-3}\,\mathrm{eV}}\right)^{1.16}\right)^{-1} \;,$$

$$\widetilde{m}_i \;\equiv\; \frac{(\widehat{M}_D \widehat{M}_D^\dagger)_{ii}}{M_i} = m_i(1 + \mathcal{O}(\lambda_c^2))$$

The Majorana phases α_{21} and α_{31} - leptogenesis CP violating parameters.

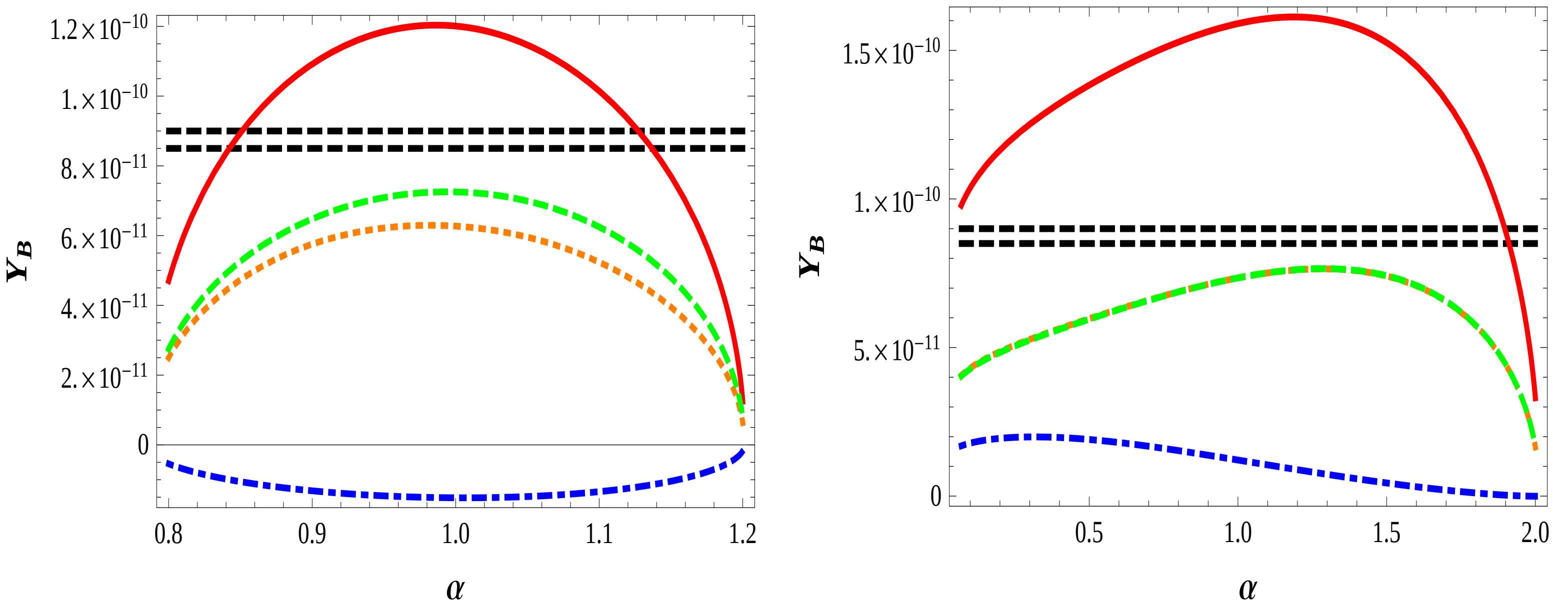
$$\epsilon_1 = -\frac{10^{-6}}{\pi}\left(6 f(m_1/m_2) \sin\alpha_{21}\, \mathrm{Re}(y_A)^2 + f(m_1/m_3) \sin\alpha_{31}\, \mathrm{Re}(y_B)^2\right) ,$$

$$\epsilon_2 = \frac{2\times 10^{-6}}{\pi}\left(3 f(m_2/m_1) \sin\alpha_{21}\, \mathrm{Re}(y_A)^2 + f(m_2/m_3) \sin(\alpha_{21}-\alpha_{31})\, \mathrm{Re}(y_B)^2\right)$$

$$\epsilon_3 = \frac{10^{-6}}{\pi}\left(2 f(m_3/m_2) \sin(\alpha_{31}-\alpha_{21}) + f(m_3/m_1) \sin\alpha_{31}\right) \mathrm{Re}(y_B)^2 .$$

Correct sign of Y_B: $\sin\phi < 0$.

C. Hagedorn, E. Molinaro, S.T.P., 2009



AF type model:Y_B(red continuous curve) versus α ($\sin\phi < 0$, Δm^2_{A} and r are fixed at their best fit values). Left panel: NO spectrum; right panel: IO spectrum. The horizontal dashed lines: the allowed range of the observed value of Y_B, $Y_B \in [8.5, 9] \times 10^{-11}$. Y_{B1}: green dashed curve; Y_{B2}: orange dotted curve; Y_{B3}: blue dot-dashed curve. On the right panel, the lines corresponding to Y_{B1} and Y_{B2} overlap.

Conclusions

New results on the reactor $\bar{\nu}_e$ flux: the flux is by $\sim 3\%$ larger than that obtained in the preceding calculations.

3-neutrino mixing: hint of $\theta_{13} \neq 0$ at (1.8 - 2.6)σ.

Intriguing accumulations of hints for the existence of eV-scale sterile neutrinos; the 3 + 2 model with two eV-scale neutrinos provides a good fit to the global (SBL) data.

These hints need further experimental verification.

We expect a wealth of new neutrino data from the $(\beta\beta)_{0\nu}$-decay, ^{3}H β-decay and oscillations experiments in the next few years.

A more clear picture of neutrino mixing is expected to emerge.