Evolution of Physics in the Extra Dimension

Lu-Xin Liu

National Institute for Theoretical Physics

University of the Witwatersrand Johanesburg

References

- Lu-Xin Liu, A. S. Cornell, arXiv:1105.1132 [hep-ph]

  Lu-Xin Liu, A. S. Cornell, Proceeding of Kruger2010: Workshop on Discovery Physics at the LHC, December 5-10, 2010, arXiv:1103.1527 [hep-ph]

  A. S. Cornell, Lu-Xin Liu, Phys. Rev. D.83, 033005, 2011

- G. Bhattacharyya, A. Datta, S. K. Majee and A. Raychaudhuri, Nucl. Phys. B 760, 117 (2007) [arXiv:hep-ph/0608208].
- K. R. Dienes, E. Dudas and T. Gherghetta, Nucl. Phys. B 537, 47 (1999) [arXiv:hep-ph/9806292].
- T. P. Cheng, E. Eichten and L. F. Li, Phys. Rev. D 9, 2259 (1974).

Standard Model Parameters

- The Yukawa matrices

$$L_{mass} = (\bar{d}_L)_i (Y_D)_{ij} (d_R)_j + (\bar{u}_L)_i (Y_U)_{ij} (u_R)_j + h.c.$$

- Quark flavor mixing matrix $s_{12} = \sin\theta_{12}$ $c_{12} = \cos\theta_{12}$ δ_{13}

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

- Scaling of Higgs Self-coupling λ

$$\frac{\lambda}{2}(\Phi^\dagger\Phi)^2$$

Renormalization Group Equation

- Renormalization group equation of Yukawa couplings

$$Y^o \bar{\psi}_L{}^o \psi_R{}^o \phi^o = Y^R Z_{couping} \bar{\psi}_L{}^R \psi_R{}^R \phi^R$$

$$\psi_L{}^o = Z_{\psi_L}{}^{1/2} \psi_L{}^R$$

$$Y^o = Z_{couping} Z_{\psi_L}{}^{-1/2} Z_{\psi_R}{}^{-1/2} Z_\phi{}^{-1/2} Y^R$$

$$\mu \frac{\partial}{\partial \mu} Y^o = 0$$

$$\mu \frac{\partial}{\partial \mu} \ln Y^R = \frac{1}{2} \mu \frac{\partial}{\partial \mu} \ln Z_{\psi_L} + \frac{1}{2} \mu \frac{\partial}{\partial \mu} \ln Z_{\psi_R} + \frac{1}{2} \mu \frac{\partial}{\partial \mu} \ln Z_\phi - \mu \frac{\partial}{\partial \mu} \ln Z_{couping}$$

- Renormalization Group Equation of Higgs Quartic Coupling λ

$$\frac{\lambda^0}{2}(\Phi^{0\dagger}\Phi^0)^2 = \frac{\lambda^R}{2} Z_{Vertex} (\Phi^{R\dagger}\Phi^R)^2$$

$$\Phi^o = Z_\Phi{}^{1/2}\Phi^R$$

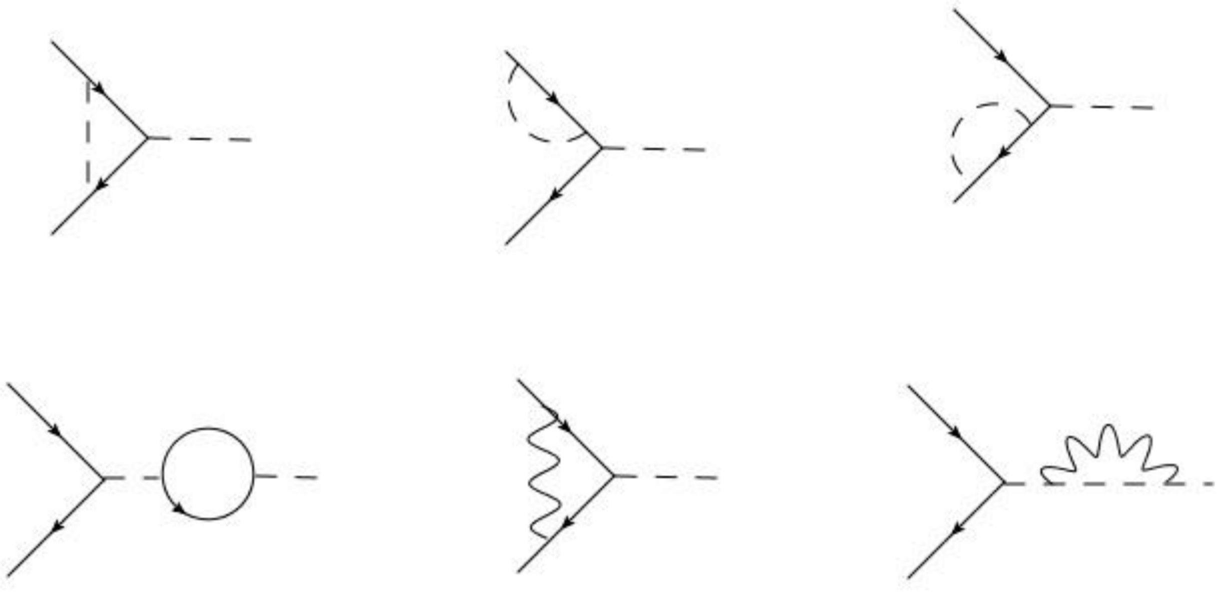
$$\lambda^o = Z_{Vertex}(Z_\Phi{}^{-1/2})^4 \lambda^R$$

$$\mu\frac{\partial}{\partial\mu}\lambda^o = 0$$

$$\mu\frac{\partial}{\partial\mu}\ln\lambda^R = \mu\frac{\partial}{\partial\mu}\ln Z_\Phi^2 - \mu\frac{\partial}{\partial\mu}\ln Z_{Vertex}$$

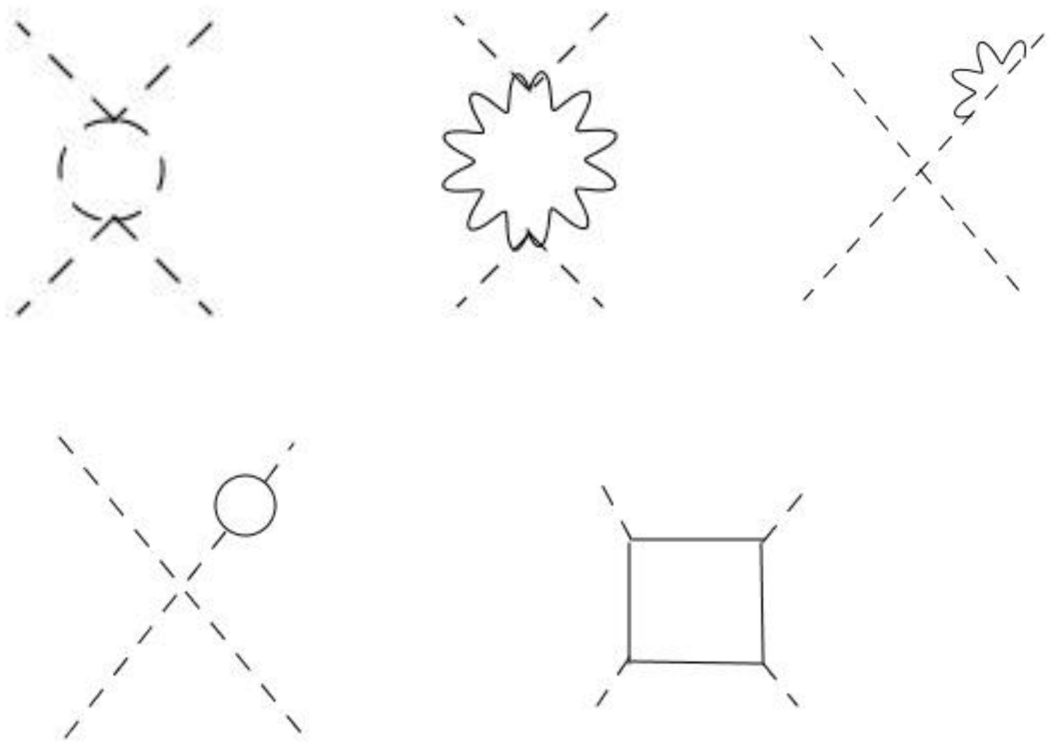
RGE of Yukawa Couplings in SM

- One-loop diagrams of Yukawa couplings (in Landau Gauge)



RGE of Higgs Quartic Coupling in SM

- One-loop diagrams of Higgs Quartic Coupling



Beta Functions of Yukawa Couplings in SM

- Up-type quark and down-type quark Yukawa couplings evolutions

$$16\pi^2 \frac{dY_U}{dt} = \beta_U^{SM}$$

$$16\pi^2 \frac{dY_D}{dt} = \beta_D^{SM}$$

- where

$$\beta_U^{SM} = Y_U\{-(8g_3^2 + \frac{9}{4}g_2^2 + \frac{17}{20}g_1^2) + \frac{3}{2}(Y_U^\dagger Y_U - Y_D^\dagger Y_D) + Tr[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E]\}$$

$$\beta_D^{SM} = Y_D\{-(8g_3^2 + \frac{9}{4}g_2^2 + \frac{1}{4}g_1^2) + \frac{3}{2}(Y_D^\dagger Y_D - Y_U^\dagger Y_U) + Tr[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E]\}$$

Beta Functions of Higgs Quartic Coupling in SM

- Higgs Quartic Coupling evolutions

$$16\pi^2 \frac{d\lambda}{dt} = \beta_\lambda^{SM}$$

- where

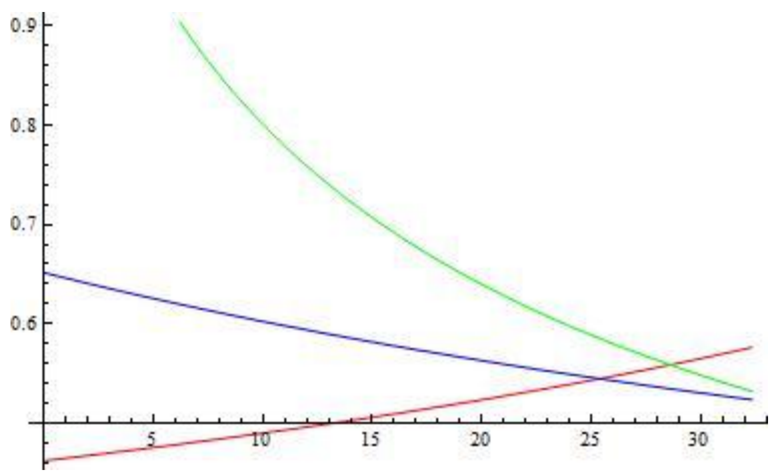
$$\beta_\lambda^{SM} = 12\lambda^2 - \left(\frac{9}{5}g_1^2 + 9g_2^2\right)\lambda + \frac{9}{4}\left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2 g_2^2 + g_2^4\right) + 4\lambda Tr[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E]$$

$$-4Tr[3(Y_U^\dagger Y_U)^2 + 3(Y_D^\dagger Y_D)^2 + (Y_E^\dagger Y_E)^2]$$

Gauge Coupling Evolutions in SM

- Gauge coupling evolutions of one-loop diagrams in SM

$$16\pi^2 \frac{dg_i}{dt} = b_i^{SM} g_i^{\,3} \qquad b_i^{SM} = \left(\frac{41}{6}, -\frac{19}{6}, -7\right)$$



Universal Extra Dimension Model

- Minimum UED model, places all SM particles in the bulk with one extra dimension of size R, compactified on an S_1 / Z_2 orbifold

$$Q(x,y) = \frac{1}{\sqrt{\pi R}}\{q_L(x) + \sqrt{2}\sum_{n=1}^{\infty}[Q_L^n(x)\cos(\frac{ny}{R}) + Q_R^n(x)\sin(\frac{ny}{R})]\}$$

$$u(x,y) = \frac{1}{\sqrt{\pi R}}\{u_R(x) + \sqrt{2}\sum_{n=1}^{\infty}[u_R^n(x)\cos(\frac{ny}{R}) + u_L^n(x)\sin(\frac{ny}{R})]\}$$

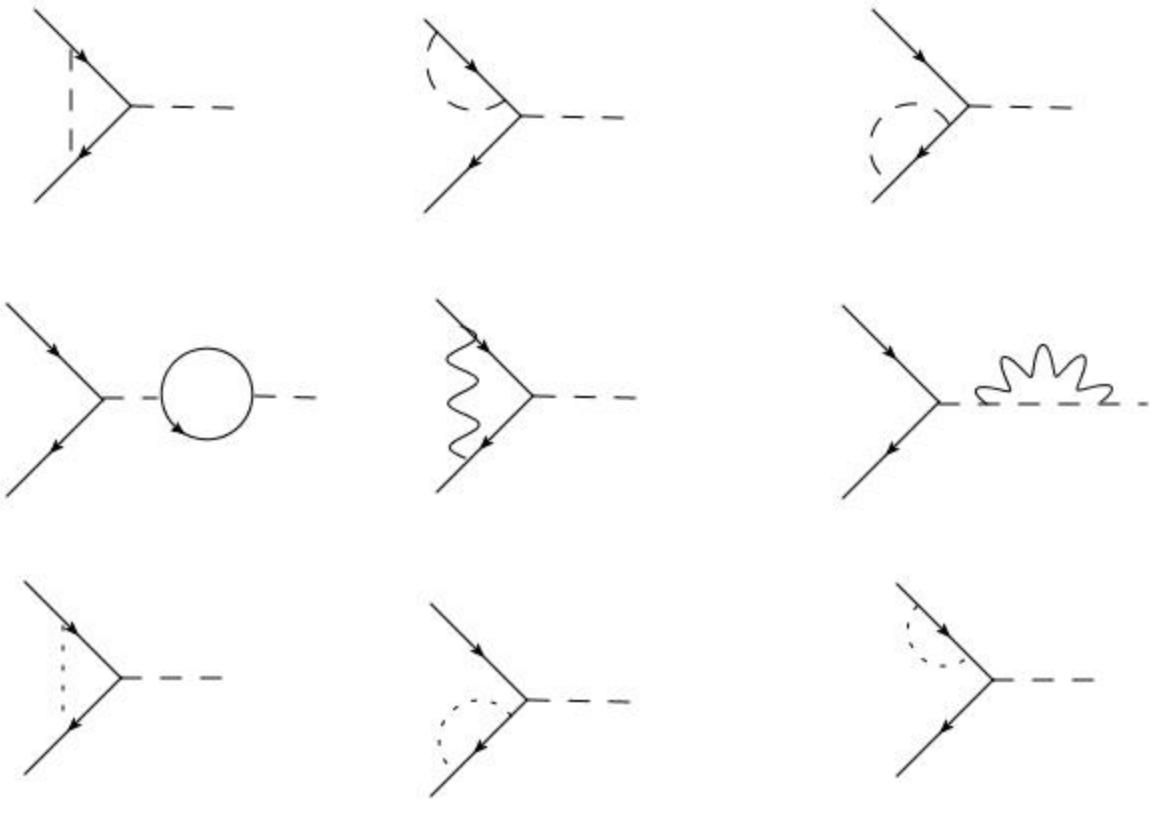
$$B_\mu(x,y) = \frac{1}{\sqrt{\pi R}}\{B_\mu^0(x) + \sqrt{2}\sum_{n=1}^{\infty}B_\mu^n(x)\cos(\frac{ny}{R})\}$$

$$B_5(x,y) = \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty}B_5^n(x)\sin(\frac{ny}{R})$$

$$\Phi(x,y) = \frac{1}{\sqrt{\pi R}}\{\Phi(x) + \sqrt{2}\sum_{n=1}^{\infty}\Phi_n(x)\cos(\frac{ny}{R})\}$$

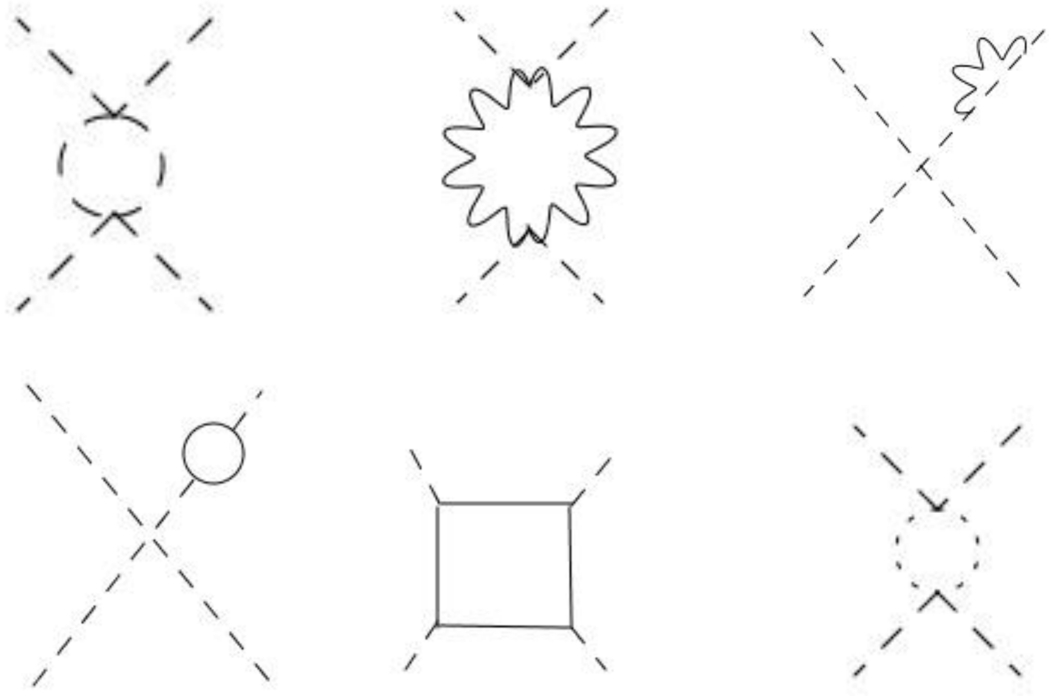
One-loop diagram contributions of UED model

- One-loop diagrams for Yukawa couplings at each Kaluza-Klein (KK) level



One-loop diagram contributions of UED model

- One-loop diagrams for Higgs Quartic Coupling at each Kaluza-Klein (KK) level



Renormalization Group Equations of Yukawa Couplings in UED model

- Between the scale R^{-1} where the first KK states are excited and the cutoff scale Λ, there are finite quantum corrections to the Yukawa and gauge couplings from the ΛR number of KK states.
- Once the KK states are excited, to a very good accuracy, the generic SM beta function is shown to have the power law evolution behavior.

$$\beta^{SM} \rightarrow \beta^{SM} + (S(\Lambda)-1)\tilde{\beta}$$

where $S(\Lambda)=\Lambda R$

Beta functions of Yukawa Couplings in UED model

- Up-type quark and down-type quark Yukawa couplings evolutions in UED model

$$16\pi^2 \frac{dY_U}{dt} = \beta_U^{SM} + \beta_U^{UED}$$

$$16\pi^2 \frac{dY_D}{dt} = \beta_D^{SM} + \beta_D^{UED}$$

where $\beta_U^{UED} = Y_U\{(S-1)[-(\frac{28}{3}g_3^2 + \frac{15}{8}g_2^2 + \frac{101}{120}g_1^2) + \frac{3}{2}(Y_U^\dagger Y_U - Y_D^\dagger Y_D) + $

$$+2(S-1)Tr[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E]\}$$

$$\beta_D^{UED} = Y_D\{(S-1)[-(\frac{28}{3}g_3^2 + \frac{15}{8}g_2^2 + \frac{17}{120}g_1^2) + \frac{3}{2}(Y_D^\dagger Y_D - Y_U^\dagger Y_U) +$$

$$+2(S-1)Tr[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E]\}$$

Renormalization Group Equations of CKM matrix

- Diagonalizing mass matrices by biunitary transformations

$$V_D{}^{+} M_D U_D = diag(m_d, m_s, m_b), \quad V_U{}^{+} M_U U_U = diag(m_u, m_c, m_t)$$

- The charged current

$$J_W = \bar{u}_L \gamma_\mu d_L = \bar{u}_L(mass)\gamma_\mu V_U{}^{+} V_D d_L(mass)$$

$$= \bar{u}_L(mass)\gamma_\mu V_{CKM} d_L(mass)$$

- CKM matrix

$$V_{CKM} = V_U{}^{+} V_D$$

Renormalization Group Equations of CKM matrix

- Diagonalizing mass matrices by biunitary transformations

$$16\pi^2 \frac{dV_{i\alpha}}{dt} = -\frac{3}{2} S[\sum_{\beta, j\neq i} \frac{f_i^2 + f_j^2}{f_i^2 - f_j^2} h_\beta^2 V_{i\beta} V_{j\beta}^* V_{j\alpha} + \sum_{j,\beta\neq\alpha} \frac{h_\alpha^2 + h_\beta^2}{h_\alpha^2 - h_\beta^2} f_j^2 V_{j\beta}^* V_{j\alpha} V_{i\beta}]$$

$$16\pi^2 \frac{d|V_{ij}|^2}{dt} = S\{3|V_{ij}|^2 (f_i^2 + h_j^2 - \sum_k f_k^2 |V_{kj}|^2 - \sum_k h_k^2 |V_{ik}|^2)$$

$$-3 f_i^2 \sum_{k\neq i} \frac{1}{f_i^2 - f_k^2} (2h_j^2 |V_{kj}|^2 |V_{ij}|^2 + h_l^2 V_{iklj}) - 3h_j^2 \sum_{l\neq j} \frac{1}{h_j^2 - h_l^2} (2 f_i^2 |V_{il}|^2 |V_{ij}|^2 + f_l^2 V_{iklj})\}$$

- Up and down type quark eigenvalues evolution

$$16\pi^2 \frac{df_i^2}{dt} = f_i^2 [2(2S-1)T - 2G_U + 3Sf_i^2 - 3S \sum_\alpha h_\alpha^2 |V_{i\alpha}|^2]$$

$$16\pi^2 \frac{dh_\alpha^2}{dt} = h_\alpha^2 [2(2S-1)T - 2G_D + 3Sh_\alpha^2 - 3S \sum_i f_i^2 |V_{i\alpha}|^2]$$

$$T = Tr[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E]$$

Beta functions of Higgs Quartic Coupling in UED model

- Higgs Quartic Coupling evolutions in UED model

$$16\pi^2 \frac{d\lambda}{dt} = \beta_\lambda^{SM} + \beta_\lambda^{UED}$$

where

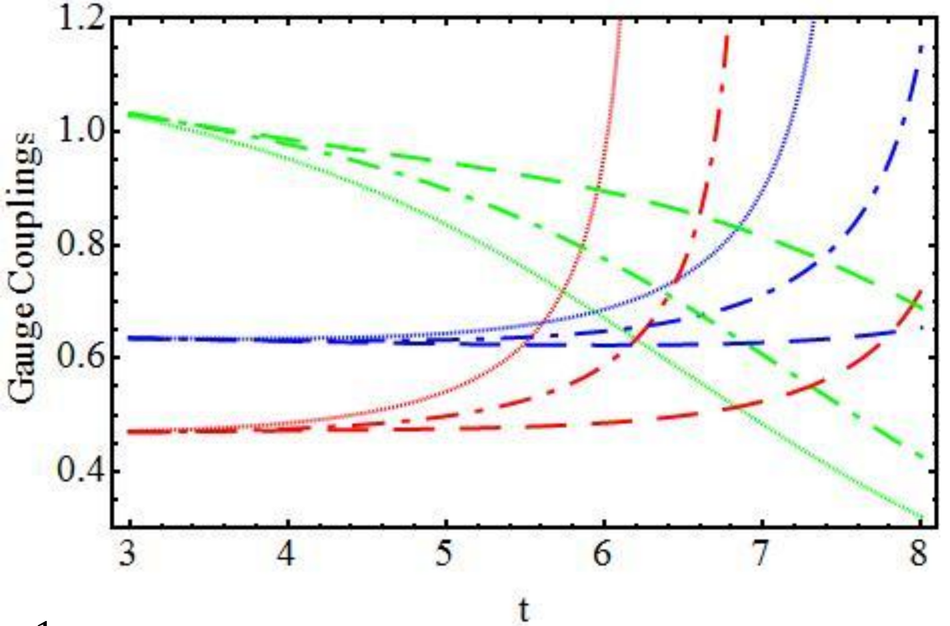
$$\beta_\lambda^{UED} = (S(t)-1)\left\{12\lambda^2 - 3\left(\frac{3}{5}g_1^2 + 3g_2^2\right)\lambda + \left(\frac{9}{25}g_1^4 + \frac{6}{5}g_1^2 g_2^2 + 3g_2^4\right)\right\}$$

$$+2(S(t)-1)\left\{4\lambda Tr[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E] - 4Tr[3(Y_U^\dagger Y_U)^2 + 3(Y_D^\dagger Y_D)^2 + (Y_E^\dagger Y_E)^2]\right\}$$

Evolution of Gauge Couplings in UED model

- Gauge coupling evolutions of one-loop diagrams in UED

$$16\pi^2 \frac{dg_i}{dt} = [b_i^{SM} + (S-1)\tilde{b}_i]g_i^3 \quad \tilde{b}_i = (\frac{81}{10}, \frac{7}{6}, -\frac{5}{2})$$

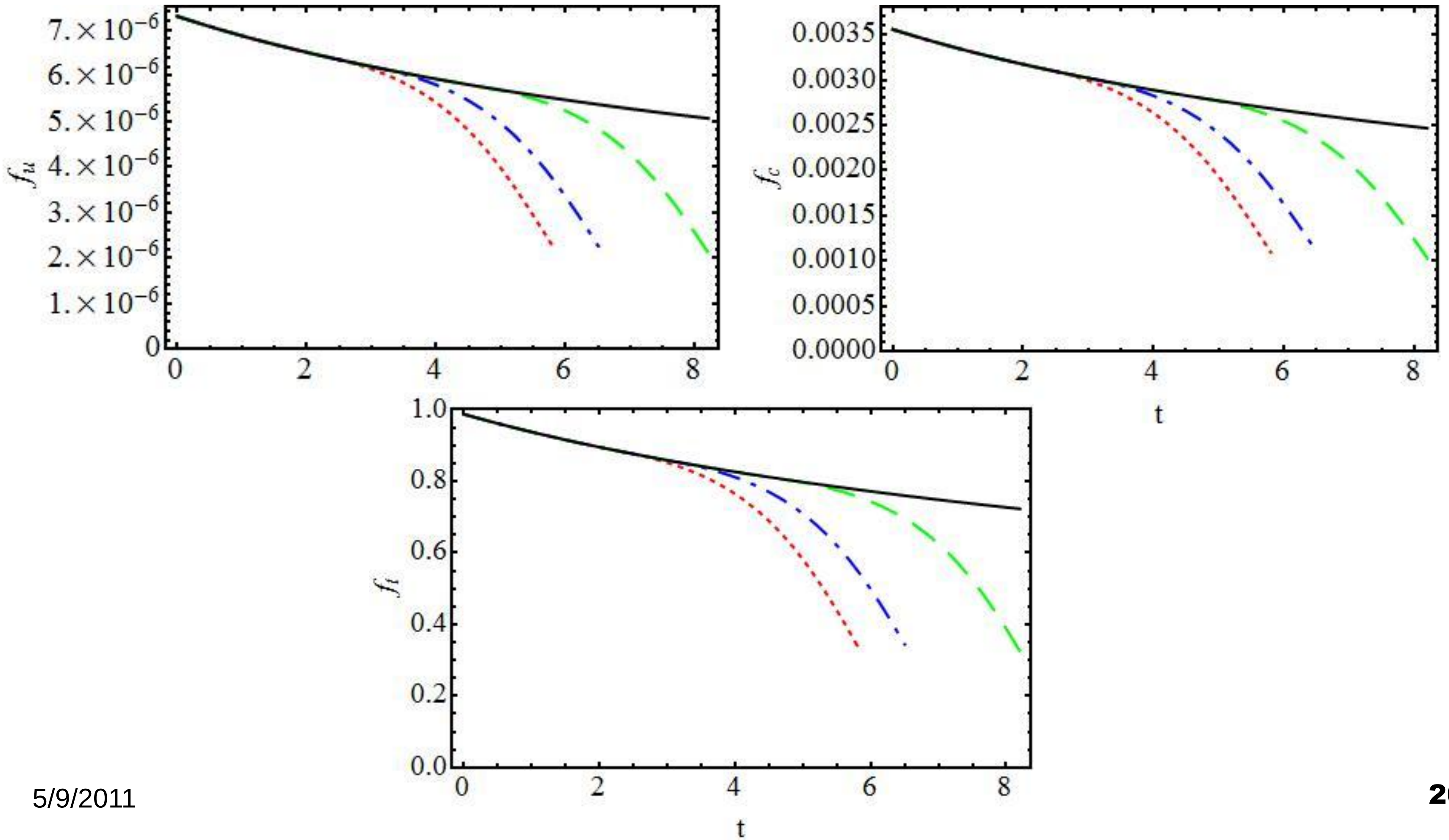


cut off scale 30, 60, 330 TeV

where R^{-1}=1, 2, 10 TeV

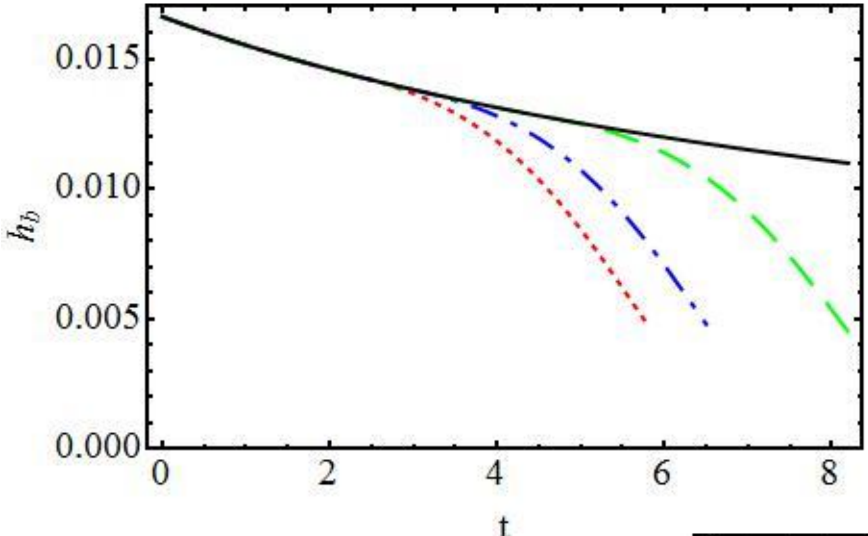
Evolution of Yukawa couplings

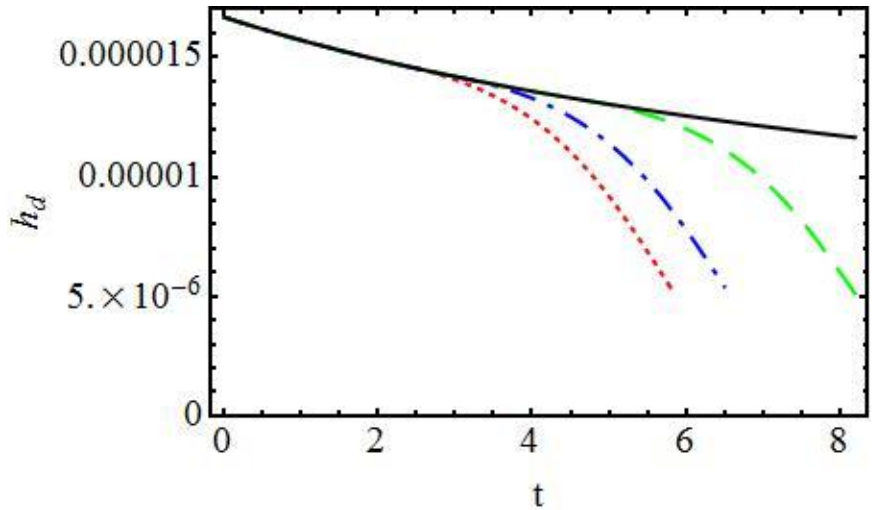
- Up-type quark Yukawa coupling evolution

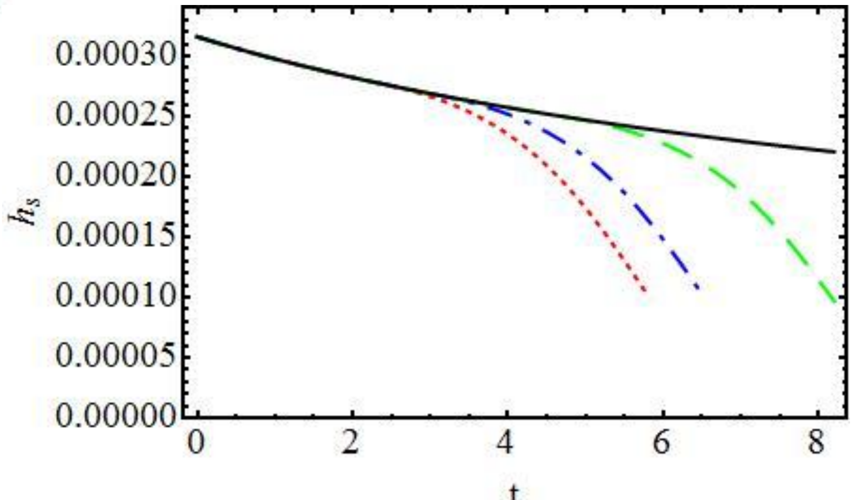


Evolution of Yukawa couplings—Cont.

- Down-type quark Yukawa coupling evolution

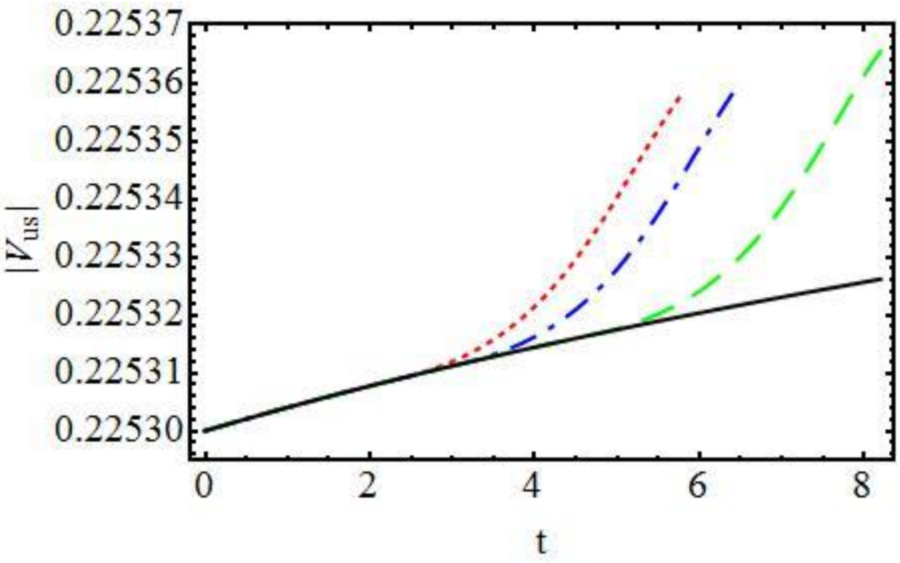


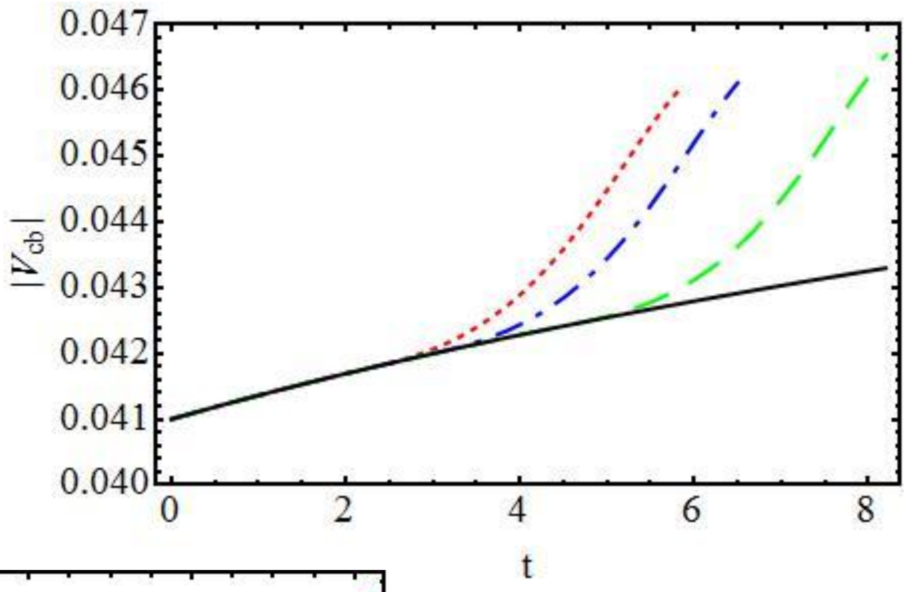


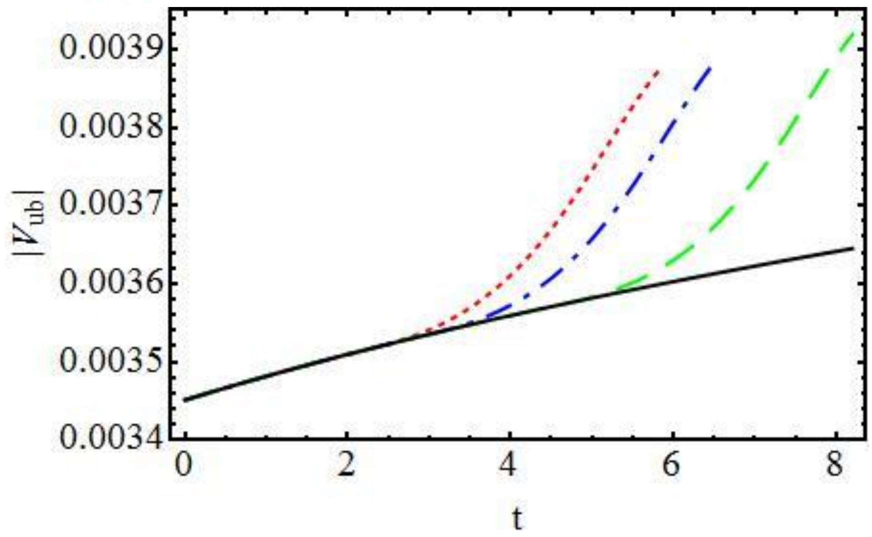


Evolution of CKM matrix elements

- Evolutions of three mixing angles $V_{ub} \approx \theta_{13} e^{-i\delta_{13}}$ $V_{cb} \approx \theta_{23}$ $V_{us} \approx \theta_{12}$

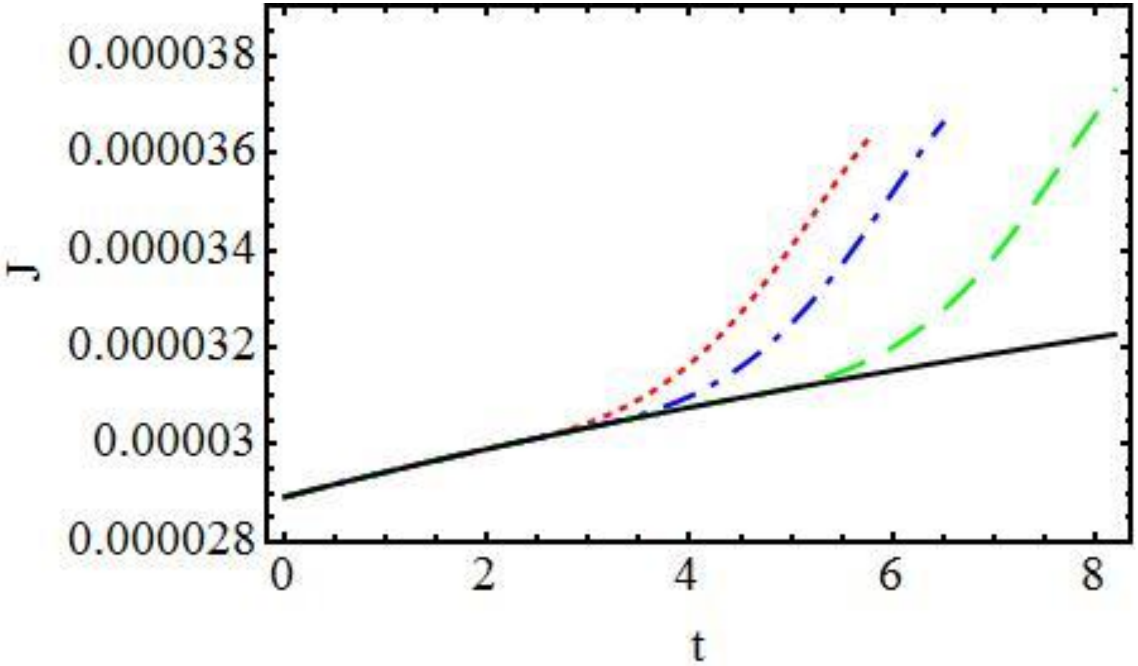






Evolution of CKM matrix elements—Cont.

- Evolutions of CP violation measure $J = \mathrm{Im} V_{cb} V_{us} V_{cs}^{*} V_{ub}^{*}$



Solution of Higgs Quartic Coupling Equation

- Recall the evolutions of λ

$$\frac{d\lambda}{dt}=\frac{1}{16\pi^2}\{12\cdot S(t)\lambda^2-S(t)\cdot\left(\frac{9}{5}g_1^2+9g_2^2\right)\lambda+(2S(t)-1)\cdot 4Tr[3Y_U^{\dagger}Y_U+3Y_D^{\dagger}Y_D+Y_E^{\dagger}Y_E]\lambda$$

$$+\frac{9}{4}\left(\frac{3}{25}g_1^4+\frac{2}{5}g_1^2g_2^2+g_2^4\right)+(S(t)-1)\cdot\left(\frac{9}{25}g_1^4+\frac{6}{5}g_1^2g_2^2+3g_2^4\right)-$$

$$(2S(t)-1)\cdot 4Tr[3(Y_U^{\dagger}Y_U)^2+3(Y_D^{\dagger}Y_D)^2+(Y_E^{\dagger}Y_E)^2]\}$$

$$=f_0(t)+f_1(t)\lambda+f_2(t)\lambda^2$$

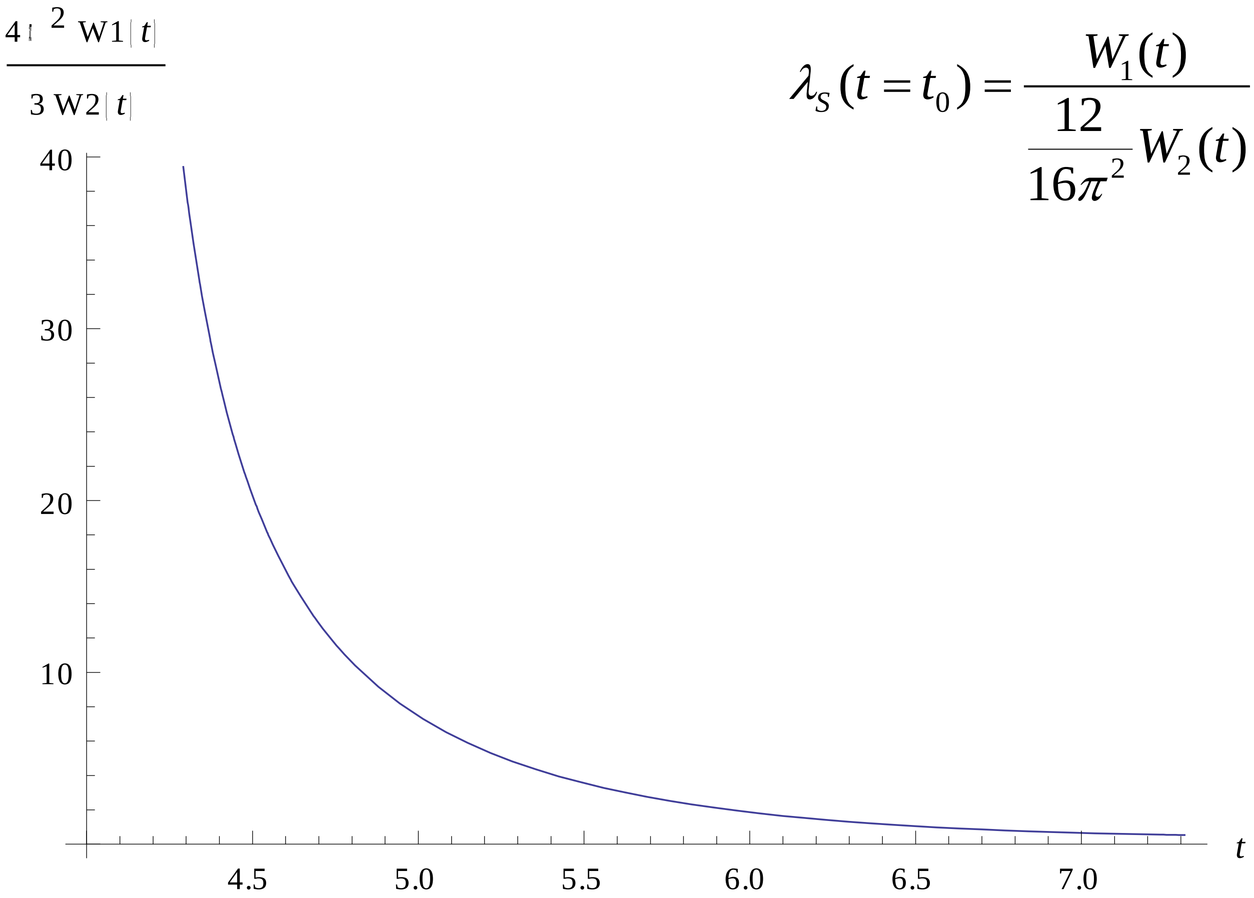
- Riccati type of differential equation

$$\lambda(t)=-\frac{16\pi^2}{12S(t)}\frac{W_1'(t)-\frac{12}{16\pi^2}S(t=t_0)\lambda(t=t_0)W_2'(t)}{W_1(t)-\frac{12}{16\pi^2}S(t=t_0)\lambda(t=t_0)W_2(t)}$$

$$W''-\left(\frac{f_2'(t)}{f_2(t)}+f_1(t)\right)W'+f_0(t)f_2(t)W=0$$

Singularity and Zero Values of Quartic Coupling λ

- Singularity of Quartic Coupling

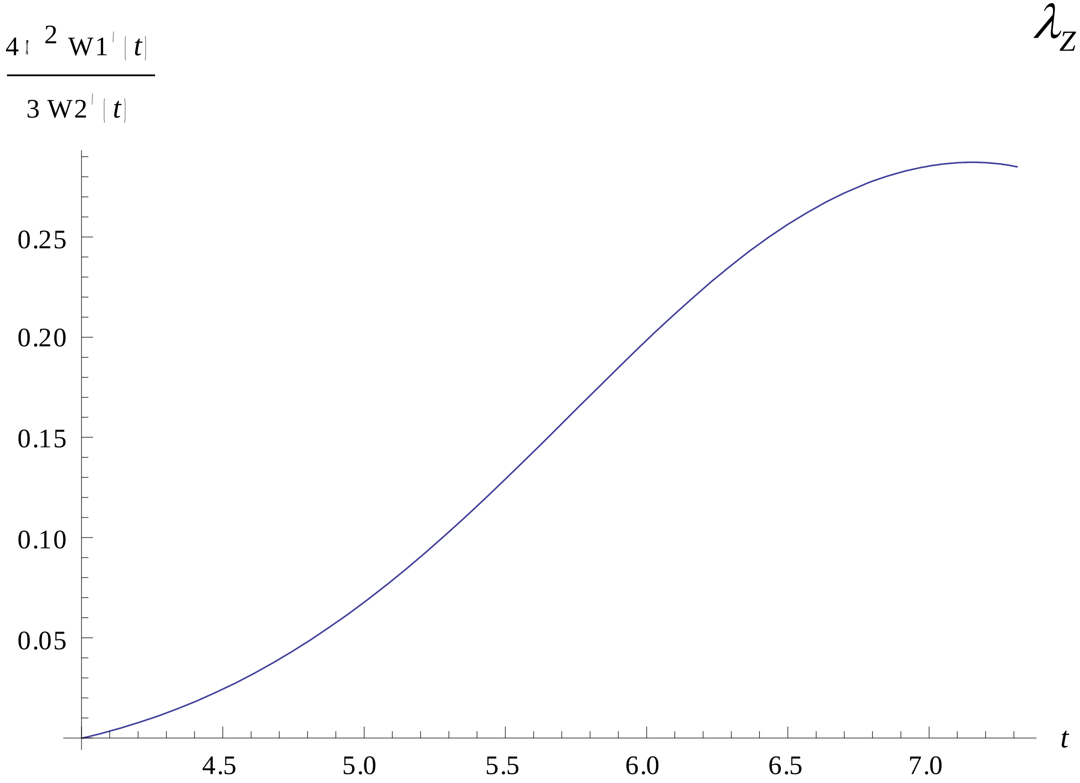


$$\lambda_S(t=t_0)=\frac{W_1(t)}{\frac{12}{16\pi^2}W_2(t)}=0.537$$

$$R^{-1}=5TeV$$

Singularity and Zero Values of Quartic Coupling λ

- Zero of Quartic Coupling

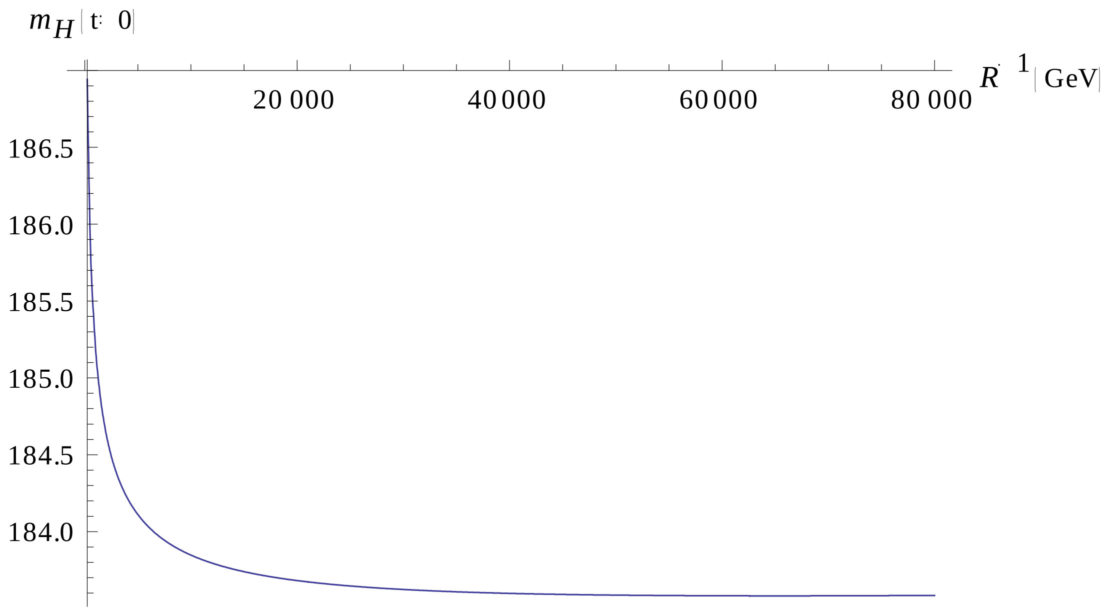


$$\lambda_Z(t=t_0) = \frac{W_1'(t)}{\frac{12}{16\pi^2} W_2'(t)} = 0.287$$

$$R^{-1} = 5TeV$$

Higgs Mass vs Compactification Scale R^{-1}

- Higgs Mass vs Compactification Scale from singularity condition

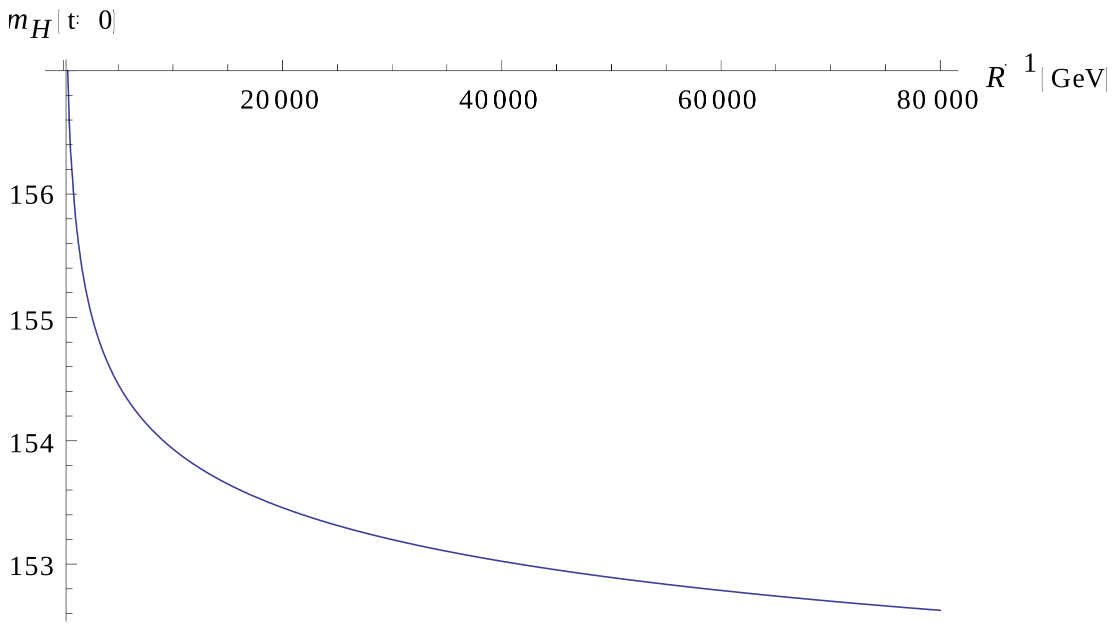


$$m_H = \sqrt{\lambda} v$$

$$v = 246 GeV$$

Higgs Mass vs Compactification Scale R^{-1}

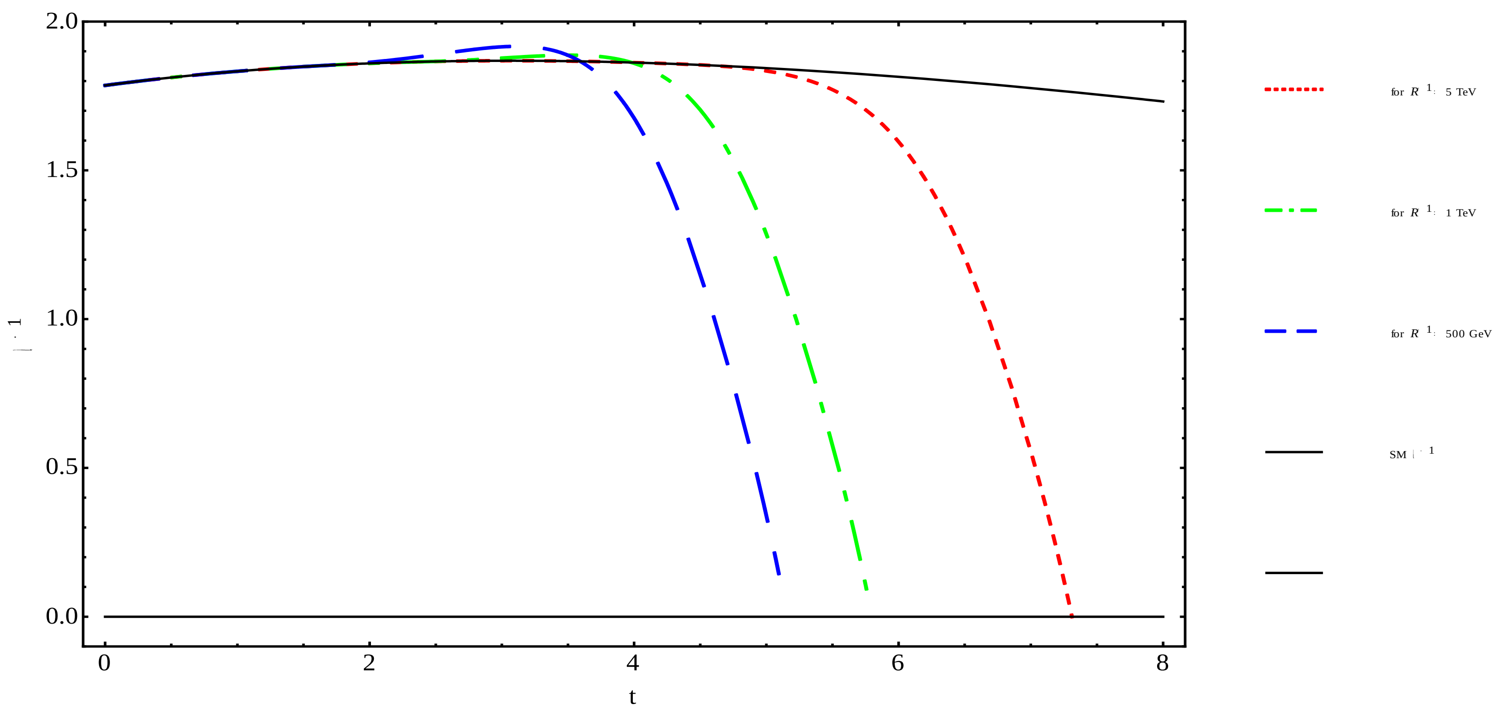
- Higgs Mass vs Compactification Scale from vacuum stability condition



Scaling of Higgs Coupling and the Bounds on Extra Dimension

- The upper bounds on compactification radius R^{-1} from singularity condition

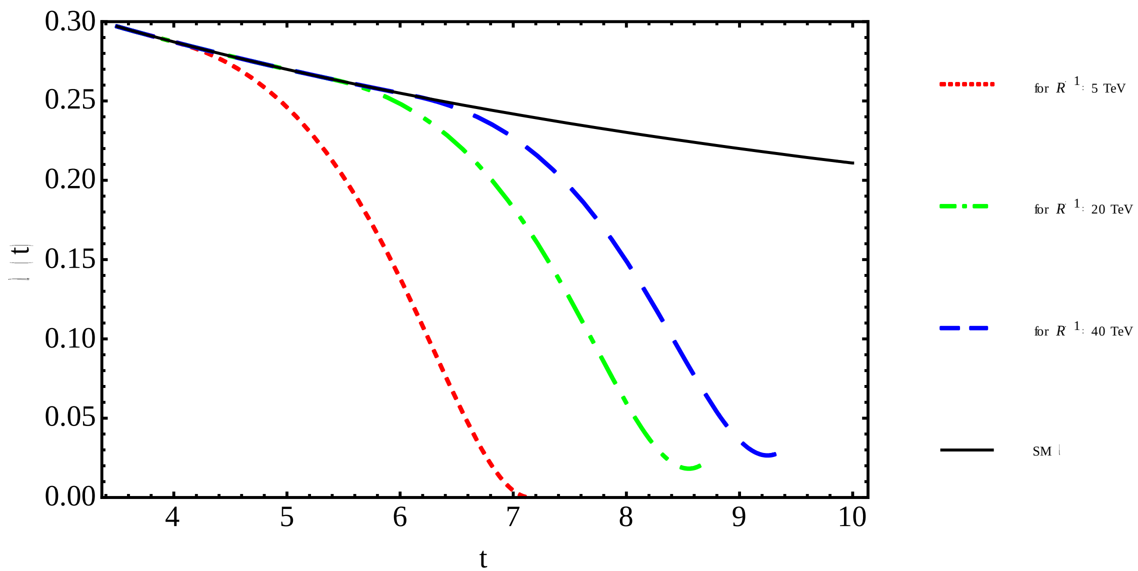
$$\lambda(M_Z) = 0.560$$



Scaling of Higgs Coupling and the Bounds on Extra Dimension

- The lower bounds on compactification radius R^{-1} from vacuum stability condition

$$\lambda(M_Z) = 0.394$$



Scaling of Higgs Coupling and the Bounds on Extra Dimension

- The evolution of the Higgs coupling for intermediate $\lambda(M_Z)$ for $250GeV \sim R^{-1} \sim 80TeV$