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The Charge Radius of the Proton

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Richard J. Hill, GP PRD **82** 113005 (2010)

Richard J. Hill, GP [arXiv:1103.4617]

Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle p(p_f)| \sum_q e_q \, \bar{q}\gamma^\mu q |p(p_i)\rangle = \bar{u}(p_f)\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q_\nu\right] u(p_i)$$

- Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E^p(0) = 1 \qquad G_M(0) = \mu_p \approx 2.793$$

- The slope of G_E^p

$$\langle r^2\rangle_E^p = 6 \frac{dG_E^p}{dq^2}\bigg|_{q^2=0} \quad \text{or} \quad G_E^p(q^2) = 1 + \frac{q^2}{6}\langle r^2\rangle_E^p + \ldots,$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2\rangle_E^p}$

Charge radius from atomic physics

$$\langle p(p_f)|\sum_q e_q\,\bar{q}\gamma^\mu q|p(p_i)\rangle = \bar{u}(p_f)\left[\gamma_\mu F_1^p(q^2) + \frac{i\sigma_{\mu\nu}}{2m}F_2^p(q^2)q_\nu\right]u(p_i)$$

- For a point particle amplitude for $p+\ell \to p+\ell$

$$\mathcal{M} \propto \frac{1}{q^2} \quad \Rightarrow \quad U(r) = -\frac{Z\alpha}{r}$$

- Including q^2 corrections from proton structure

$$\mathcal{M} \propto \frac{1}{q^2}q^2 = 1 \quad \Rightarrow \quad U(r) = \frac{4\pi Z\alpha}{6}\delta^3(r)(r_E^p)^2$$

- Proton structure corrections $\left(m_r = m_\ell m_p/(m_\ell + m_p) \approx m_\ell\right)$

$$\Delta E_{r_E^p} \;=\; \frac{2(Z\alpha)^4}{3n^3}m_r^3(r_E^p)^2\delta_{\ell 0}$$

- **Muonic hydrogen can give the best measurement of $\mathbf{r_E^p}$!**

Charge radius from atomic physics

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
- CODATA value [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.8768(69)$ fm
  extracted mainly from (electronic) hydrogen
- **5σ discrepancy!**
- We can also extract it from electron-proton scattering data

The recent discrepancy

- [Hill, GP PRD **82** 113005 (2010)] showed
  previous extractions are model dependent
  underestimated the error by a factor of 2 or more

- Based on a model-independent approach
  using scattering data from proton, neutron and $\pi\pi$
  [Hill, GP PRD **82** 113005 (2010)]
  $r_E^p = 0.871(11)$ fm

- CODATA value (extracted mainly from electronic hydrogen)
  [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.8768(69)$ fm

- Lamb shift in muonic hydrogen
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  $r_E^p = 0.84184(67)$ fm

Lamb shift in muonic hydrogen

- CREMA measured [Pohl et al. Nature **466**, 213 (2010)]

$$\Delta E = 206.2949 \pm 0.0032 \text{ meV}$$

- Comparing to the theoretical expression
  [Pachucki PRA **60**, 3593 (1999), Borie PRA **71**(3), 032508 (2005)]

$$\Delta E = 209.9779(49) - 5.2262(r_E^p)^2 + 0.0347(r_E^p)^3 \text{ meV}$$

- They got

$$r_E^p = 0.84184(67) \text{ fm}$$

The Theoretical Prediction

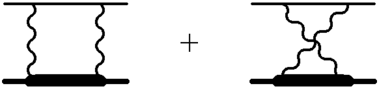
- Is there a problem with the theoretical prediction?

[Pachucki PRA **60**, 3593 (1999), Borie PRA **71**(3), 032508 (2005)]

$$\Delta E = \quad 209.9779(49) \quad - \quad 5.2262(r_E^p)^2 \quad + \quad 0.0347(r_E^p)^3 \text{ meV}$$

↑	↑	↑
mostly μ QED	already discussed	where does this term come from?

Two-photon amplitude: “standard” calculation



- “standard” calculation: separate to proton and non-proton
  - non-proton $\leftrightarrow$ DIS
- For proton
  - Insert form factors into vertices

$$\mathcal{M} = \int_0^\infty dq^2\, f(G_E, G_M)$$

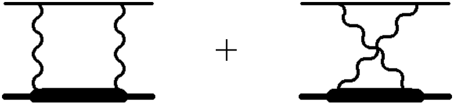
  - Using a “dipole form factor”

$$G_i(q^2) \approx G_i(q^2)/G_i(0) \approx [1 - q^2/\Lambda^2]^{-2}$$

  - $\mathcal{M}$ is a function of $\Lambda \Rightarrow (r_E^p)^3$ term

- Using, $\Lambda^2 = 0.71\,\mathrm{GeV}^2 \Rightarrow \Delta E \approx 0.018$ meV
  [K. Pachucki, PRA **53**, 2092 (1996)]

Two-photon amplitude: “standard” calculation



- Is insertion of form factors in vertices valid?

- Even if it is, result looks funny
  two-photon amplitude $\Leftrightarrow$ the charge radius
  only for one parameter model for G_E and G_M

- In "standard approach" two-photon $\Rightarrow \Delta E \approx 0.018$ meV
  Need $0.258(90)\,\mathrm{meV}$ (scattering) or $0.311(63)\,\mathrm{meV}$ (spec.)
  to explain discrepancy

NRQED

- Model Independent approach: use NRQED
  [Caswell, Lepage PLB **167**, 437 (1986); Kinoshita Nio PRD **53**, 4909 (1996); Manohar PRD **56**, 230 (1997)]

$$
\begin{aligned}
\mathcal{L}_e \;=\; & \psi_e^\dagger \Big\{ iD_t + \frac{\mathbf{D}^2}{2m_e} + \frac{\mathbf{D}^4}{8m_e^3} + c_F e \frac{\boldsymbol{\sigma}\cdot\mathbf{B}}{2m_e} + c_D e \frac{[\partial\cdot\mathbf{E}]}{8m_e^2} \\
& + i c_S e \frac{\boldsymbol{\sigma}\cdot(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})}{8m_e^2} + c_{W1} e \frac{\{\mathbf{D}^2,\boldsymbol{\sigma}\cdot\mathbf{B}\}}{8m_e^3} \\
& - c_{W2} e \frac{D^i \boldsymbol{\sigma}\cdot\mathbf{B} D^i}{4m_e^3} + c_{p'p} e \frac{\boldsymbol{\sigma}\cdot\mathbf{D}\mathbf{B}\cdot\mathbf{D} + \mathbf{D}\cdot\mathbf{B}\boldsymbol{\sigma}\cdot\mathbf{D}}{8m_e^3} \\
& + i c_M e \frac{\{\mathbf{D}^i,[\partial\times\mathbf{B}]^i\}}{8m_e^3} + c_{A1} e^2 \frac{\mathbf{B}^2-\mathbf{E}^2}{8m_e^3} - c_{A2} e^2 \frac{\mathbf{E}^2}{16m_e^3} + \ldots \Big\} \psi_e
\end{aligned}
$$

- Need also

$$
\mathcal{L}_{\rm contact} = d_1 \frac{\psi_p^\dagger \boldsymbol{\sigma}\psi_p \cdot \psi_e^\dagger \boldsymbol{\sigma}\psi_e}{m_e m_p} + d_2 \frac{\psi_p^\dagger \psi_p \psi_e^\dagger \psi_e}{m_e m_p}
$$

NRQED

- From c_i and d_i determine proton structure correction, e.g.

$$\delta E(n,\ell) = -\delta_{\ell 0} \frac{m_r^3 (Z\alpha)^3}{\pi n^3} \frac{d_2}{m_e m_p}$$

- Matching

- Operators with one photon coupling:
  c_i given by $F_i^{(n)}(0)$

- Operators with only two photon couplings:
  c_{A_i} given by forward and backward Compton scattering

- d_i from two-photon amplitude

Two-photon amplitude: matching

$$\frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \mathbf{k}, s|\, T\{J^\mu_{\rm e.m.}(x) J^\nu_{\rm e.m.}(0)\}|\mathbf{k}, s\rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1 + \left(k^\mu - \frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu - \frac{k\cdot q\, q^\nu}{q^2}\right) W_2$$

- Matching

$$\frac{4\pi m_r}{\lambda^3} - \frac{\pi m_r}{2m_e m_p \lambda} - \frac{2\pi m_r}{m_p^2\lambda}\left[F_2(0) + 4m_p^2 F_1'(0)\right]$$

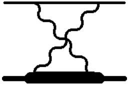
$$-\frac{2}{m_e m_p}\left[\frac{2}{3} + \frac{1}{m_p^2 - m_e^2}\left(m_e^2 \log\frac{m_p}{\lambda} - m_p^2\log\frac{m_e}{\lambda}\right)\right] + \frac{d_2(Z\alpha)^{-2}}{m_e m_p}$$

$$= -\frac{m_e}{m_p}\int_{-1}^{1} dx\sqrt{1-x^2}\int_0^\infty dQ\, \frac{Q^3}{(Q^2+\lambda^2)^2(Q^2+4m_e^2x^2)}$$

$$\times\left[(1+2x^2) W_1(2im_pQx, Q^2) - (1-x^2) m_p^2 W_2(2im_pQx, Q^2)\right]$$

d_2

- In order to determine d_2 need to know W_i

- Im  $+$  $\sim$ Im W_i

  can be extracted from on-shell quantities:

  Proton form factors and Inelastic structure functions

- To find W_i from Im W_i, need dispersion relations

Dispersion relation

- Dispersion relations ($\nu = 2k \cdot q,\ Q^2 = -q^2$)

$$W_1(\nu, Q^2) = W_1(0, Q^2) + \frac{\nu^2}{\pi} \int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2 \frac{{\rm Im}\, W_1(\nu', Q^2)}{\nu'^2(\nu'^2 - \nu^2)}$$

$$W_2(\nu, Q^2) = \frac{1}{\pi} \int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2 \frac{{\rm Im}\, W_2(\nu', Q^2)}{\nu'^2 - \nu^2}$$

- W_1 requires subtraction...

- ${\rm Im}\, W_i^p$ from form factors
- ${\rm Im}\, W_i^c$ from DIS
- What about $W_1(0, Q^2)$?

$W_1(0, Q^2)$

- Can calculate in two limits:

- $Q^2 \ll m_p^2$
  The photon sees the proton "almost" like an elementary particle
  Use NRQED to calculate $W_1(0, Q^2)$ upto $\mathcal{O}(Q^2)$ (including)

$$W_1(0, Q^2) = 2(c_F^2 - 1) + 2\frac{Q^2}{4m_p^2}\left(c_{A_1} + c_F^2 - 2c_F c_{W1} + 2c_M\right)$$

- $Q^2 \gg m_p^2$
  The photon sees the quarks inside the proton
  Use OPE to find $W_1(0, Q^2) \sim 1/Q^2$ for large Q^2

- In between you will have to model!
  Current calculation **pretends** there is no model dependence
  How big is the model dependence?

Bound State Energy

1) Proton: $\mathrm{Im}\, W_i^p$ using dipole form factor

$$\Delta E = -0.016 \text{ meV}$$

2) Continuum: $\mathrm{Im}\, W_i^c$ [Carlson, Vanderhaeghen arXiv:1101.5965]

$$\Delta E = 0.0127(5) \text{ meV}$$

3) What about $W_1(0, Q^2)$?

"Sticking In Form Factors" (SIFF) model

$$W_1^{\mathrm{SIFF}}(0, Q^2) = 2F_2(2F_1 + F_2) \quad F_i \equiv F_i(Q^2)$$

SIFF

- "Sticking In Form Factors" (SIFF) model

$$W_1^{\rm SIFF}(0, Q^2) = 2F_2(2F_1 + F_2) \quad F_i \equiv F_i(Q^2)$$

Notice that for large Q^2, $W_1^{\rm SIFF}(0, Q^2) \propto 1/Q^8$
In contradiction to OPE

- There is **no** local Lagrangian that has a Feynman rule

$$\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q_\nu$$

- Numerically using the dipole form factor

$$\Delta E^{\rm SIFF} = 0.034 \text{ meV}$$

Model Dependence

- How big is the model dependence?

$$0.018\,\text{meV} = \underset{\substack{\uparrow\\ \text{Model independent}}}{-0.016\,\text{meV}} + \underset{\substack{\uparrow\\ \text{Model dependent}}}{0.034\,\text{meV}}$$

- The model dependent piece is the dominant one!
- Experimental discrepancy $\sim$ 0.3 meV
- Can we find a model that explains (or reduces) the discrepancy?

Conclusions

- Recent discrepancy in the extraction the proton charge radius between muonic and regular hydrogen

Conclusions

- Recent discrepancy in the extraction the proton charge radius between muonic and regular hydrogen
- From **model independent** extraction of the charge radius from $e-p$ scattering data: $r_E^p = 0.871(11)\,\mathrm{fm}$
- Previous extractions have underestimated the error
- Results are compatible with CODATA value of $r_E^p = 0.8768(69)$ fm

Conclusions

- Analyzed Proton structure effects in hydrogenic bound states
  Using NRQED

- Isolated model-**dependent** assumptions in previous analyses:
  $W_1(0, Q^2)$ was calculated by "Sticking In Form Factors" model

- Model **independent** calculation of $W_1(0, Q^2)$:
  low Q^2 via NRQED, high Q^2 via OPE
  In between one has to model

- Possibility for a significant new effects in the two-photon amplitude

- NRQED predicts a universal shift for spin-independent
  energy splittings in muonic hydrogen.

Future Directions

- Analyze spin dependent effects
- Application to deuterium
- Resolution of the discrepancy?