## Many Leptons at the LHC from the NMSSM

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#### The $\mu$ problem

The MSSM superpotential is given by

$$W_{\rm MSSM} = {}_{\boldsymbol{\mu}}\hat{H}_{\boldsymbol{u}}\hat{H}_{\boldsymbol{d}} + \hat{\boldsymbol{u}}\boldsymbol{y}_{\boldsymbol{u}}\hat{\boldsymbol{Q}}\hat{H}_{\boldsymbol{u}} - \hat{\boldsymbol{d}}\boldsymbol{y}_{\boldsymbol{d}}\hat{\boldsymbol{Q}}\hat{H}_{\boldsymbol{d}} - \hat{\boldsymbol{e}}\boldsymbol{y}_{\boldsymbol{e}}\hat{\boldsymbol{L}}\hat{H}_{\boldsymbol{d}},$$

This provides part of the MSSM Higgs potential

$$V = V_F + V_D + V_{\text{soft}}$$

where

$$V_F = |\mu|^2 (|H_d|^2 + |H_u|^2)$$

$$V_D = \frac{g_1^2 + g_2^2}{8} (|H_d|^2 - |H_u|^2)^2 + \frac{g_2^2}{2} (|H_d|^2 |H_u|^2 - |H_u \cdot H_d|^2)$$

$$V_{\text{soft}} = m_d^2 |H_d|^2 + m_u^2 |H_u|^2 + (BH_u \cdot H_d + \text{h.c.})$$

Minimizing this potential gives the two Higgs VEVs

$$\langle \hat{H}_{u}^{0} \rangle = \frac{v_{u}}{\sqrt{2}}$$
  
 $\langle \hat{H}_{d}^{0} \rangle = \frac{v_{d}}{\sqrt{2}}$ 

where

$$v_{\rm SM} = \sqrt{v_u^2 + v_d^2} \approx 246 \text{ GeV}$$

Note that

 $\mu \rightarrow \text{supersymmetry conserving}$  $m_{\mu}^2, m_{e}^2, B \rightarrow \text{supersymmetry breaking } (O(\text{TeV}))$ 

- Naturalness problem:  $\mu$  must be O(TeV) to avoid fine-tuning.
- This is the  $\mu$  problem.

#### **NMSSM**

- One possible solution is to generate the  $\mu$  term dynamically.
- Relate μ to the VEV of a new field:

$$\lambda \hat{S} \hat{H}_u \cdot \hat{H}_d \rightarrow \lambda \langle S \rangle \hat{H}_u \cdot \hat{H}_d = \mu_{\text{eff}} \hat{H}_u \cdot \hat{H}_d$$

- Here  $\hat{S}$  is a gauge-singlet, chiral superfield and  $\lambda$  is a dimensionless  $\mathcal{O}(1)$  parameter.
- Next-to-Minimal Supersymmetric Standard Model (NMSSM) is characterized by the superpotential

$$W_{\text{NMSSM}} = W_{\text{MSSM}}|_{\mu \to 0} + \lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + \frac{1}{3} \kappa \hat{S}^3$$

- κ is a dimensionless O(1) parameter
- The  $\kappa$  term forbids a global  $U(1)_{PQ}$  symmetry (but leaves a discrete  $Z_3$ )

- The chiral superfield Ŝ contains both a complex scalar boson state and a fermion state.
- These mix with the other neutral states, providing two new Higgs bosons and one additional neutralino

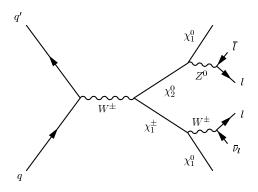
|                              | MSSM                  | NMSSM                   |
|------------------------------|-----------------------|-------------------------|
| CP-Even Higgs H <sub>i</sub> | <i>H</i> <sub>1</sub> | H <sub>1</sub>          |
|                              | $H_2$                 | $H_2$                   |
|                              |                       | $H_3$                   |
| CP-Odd Higgs A <sub>i</sub>  | A <sub>1</sub>        | A <sub>1</sub>          |
|                              |                       | $A_2$                   |
| Neutralinos $\chi_i^0$       | $\chi_1^0 - \chi_4^0$ | $\chi_1^0$ - $\chi_4^0$ |
|                              |                       | $\chi_5^0$              |

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|                              |                       | $H_3$                 |
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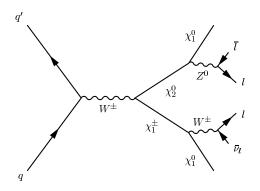
How do these new states affect the collider phenomenology?

• In the MSSM, one possible source of a multi-lepton signal is



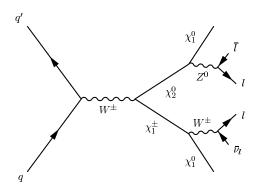
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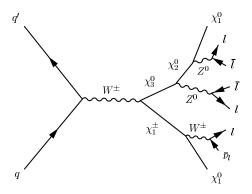
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- How does this change for the NMSSM?

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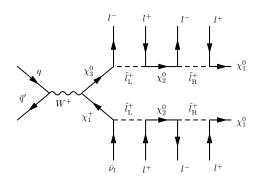


- Important if squarks and gluinos are heavy
- How does this change for the NMSSM?
- What if mixing is small and  $\chi_1^0 \sim \tilde{S}$  is light?

• For example, an extended decay in the NMSSM could be



We will actually be looking at the following decay



- This would give a signal with 7 leptons and 0 jets!
- What would make this have a large cross section?
  - \* Large  $W^{\pm}\chi^{\mp}\chi^{0}_{3}$  coupling
  - \* Large BR( $\chi_i^0 \rightarrow \tilde{l}_{LR}^{\pm} l^{\pm}$ )
  - \* Large BR( $\tilde{I}_{L}^{\pm} \rightarrow I^{\pm} \chi_{2}^{0}$ )

- Large  $W^{\pm}\chi^{\mp}\chi^0_3$  coupling:
  - $^*~W^\pm$  boson couples to  $ilde{W}^3$  and  $ilde{H_d}, ilde{H_u}$
  - \* Therefore, maximize the Wino and Higgsino components of  $\chi_3^0$
- Large BR( $\chi_i^0 o \tilde{\it I}_{L,R}^\pm \it I^\pm$ )
  - \* Usually the case when this decay mode is available on-shell
  - \* Therefore, require  $M_{\chi_3^0}, M_{\chi_1^\pm} > M_{l_L^\pm} > M_{\chi_2^0} > M_{l_R^\pm} > M_{\chi_1^0}$
- Large BR( $\tilde{\it I}_{\it L}^{\pm} 
  ightarrow {\it I}^{\pm}\chi_{2}^{0}$ )
  - \* Under most circumstances the decay to  $\chi_1^0$  is larger
  - \* BUT, that branching ratio is suppressed if  $\chi_1^0 \sim \tilde{S}$
  - Therefore, maximize the Singlino component of  $\chi_1^0$

## Can we actually do this?

- All the mixing parameters will be determined by the neutralino and chargino mass matrices
- In the basis  $(\tilde{B}, \tilde{W}^3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S})$

$$\mathbf{M}_{\chi^0} = \left( \begin{array}{cccc} M_1 & 0 & -g_1 v_d/2 & g_1 v_u/2 & 0 \\ 0 & M_2 & g_2 v_d/2 & -g_2 v_u/2 & 0 \\ -g_1 v_d/2 & g_2 v_d/2 & 0 & -\mu_{\mathrm{eff}} & -\mu_{\mathrm{eff}} v_u/s \\ g_1 v_u/2 & -g_2 v_u/2 & -\mu_{\mathrm{eff}} & 0 & -\mu_{\mathrm{eff}} v_d/s \\ 0 & 0 & -\mu_{\mathrm{eff}} v_u/s & -\mu_{\mathrm{eff}} v_d/s & \sqrt{2}\kappa s \end{array} \right)$$

• 1

\* 
$$s \gg v_u, v_d$$
  
\*  $\sqrt{2}\kappa s < \max[M_1, M_2, \mu_{\text{eff}}]$   
then  $\gamma_0^4 \approx \tilde{S}$ 

- If the above conditions are met and
  - \*  $M_1 < M_2$ ,  $\mu_{\rm eff}$ then  $\chi_3^0$  may have large  $\tilde{W}$  and  $\tilde{H}_d^0$ ,  $\tilde{H}_u^0$  components

#### Parameter Scan

- Now we need to find a benchmark that satisfies:
  - \* our requirements for large cross section and
  - \* all relevant experimental constraints
- To calculate cross sections we
  - \* Implement the NMSSM in MadGraph: Calculate  $pp o W^{*\pm} o \chi_1^\pm \chi_3^0$
  - \* Use BRIDGE to calculate all branching fractions
  - \* Calculate total branching fractions to multi-lepton final states
- To verify experimental constraints we NMSSMtools:
  - \* This program calculates the predicted relic density and compares it to observed value  $0.094 < \Omega h^2 < 0.136$  (we only take upper bound).
  - \* It also checks collider constraints such as LEP mass limit and limits from  $(g-2)_\mu$  and  ${\sf BR}(b\to s\gamma)$

• The NMSSM-specific parameters are:

$$s, \kappa, A_{\kappa}, A_{s}$$

and the parameters shared with the MSSM are

$$\mu_{\rm eff}$$
,  $\tan \beta$ ,  $A_t$ ,  $A_b$ ,  $A_{\tau}$ ,  $M_1$ ,  $M_2$ ,  $M_3$ ,  $M_{Q_i}$ ,  $M_{U_i}$ ,  $M_{D_i}$ ,  $M_{L_i}$ ,  $M_{E_i}$ 

- We can simplify things by making a few assumptions:
  - Gaugino Mass Unification:

$$M_1 = \frac{1}{2}M_2 = \frac{1}{6}M_3$$

\* Family-Universal Sfermion Mass Parameters:

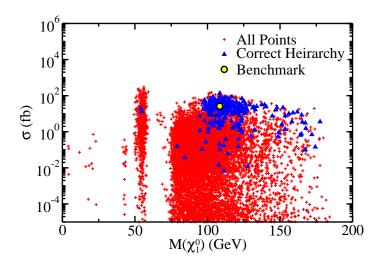
$$M_{L_1} = M_{L_2} = M_{L_3} = M_L$$
 etc.

\* Heavy Squarks:

$$M_Q = M_U = M_D = 2 \text{ TeV}$$

\* Light sleptons:

$$M_I, M_F \lesssim 200 \text{ GeV}$$



$$pp \rightarrow W^+ \rightarrow \chi_3^0 \chi_1^+ \rightarrow 5 \text{ leptons} + 0 \text{ jets}$$

#### **Event Generation and Detector Simulation**

- To simulate LHC detection we
  - Generate signal events with MadGraph + BRIDGE (decaying to all final states)
  - \* Generate background events with ALPGEN
- These events are then subject to energy smearing as

$$\frac{\Delta E}{E} = \begin{cases} \frac{0.5}{\sqrt{E/\text{GeV}}} \oplus 0.03 \text{ for jets} \\ \frac{0.1}{\sqrt{E/\text{GeV}}} \oplus 0.007 \text{ for leptons} \end{cases}$$

and the following  $p_T$ ,  $\eta$ , and  $\Delta R = \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2}$  cuts:

$$p_{T} > \begin{cases} 20 \text{ GeV for the hardest two leptons } (e,\mu) \\ 7 \text{ GeV for all other light leptons} \\ 15 \text{ GeV for } \tau \text{ leptons} \\ 20 \text{ GeV for jets} \end{cases}$$

$$|\eta| < \left\{ \begin{array}{l} 2.4 \text{ for electrons} \\ 2.1 \text{ for muons} \\ 2.5 \text{ for } \tau\text{-leptons and jets} \end{array} \right.$$

$$\Delta R > \begin{cases} 0.2 \text{ for light leptons} \\ 0.4 \text{ for all others} \end{cases}$$

- We also include
  - basic tagging efficiencies
  - \* isolated leptons from heavy quark decay ( $P \sim 1/200$ )
  - \* a jet veto

| Background Cross Sections (fb)  N leptons |     |     |      |      |       |       |       |     |     |       |     |       |
|---|-----|-----|------|------|-------|-------|-------|-----|-----|-------|-----|-------|
|   | WZ  | ZZ  | WWW  | WWZ  | WZZ   | ZZZ   | Wtī   | Zcē | Zbb | Ztī   | tīt | TOTAL |
| $\sqrt{s} = 7 \text{ TeV}$                |     |     |      |      |       |       |       |     |     |       |     |       |
| 3/  | 70  | 7.2 | 0.22 | 0.26 | 0.13  | 0.012 | 1.3   | 5.5 | 5.3 | 1.2   | 7.4 | 99    |
| w/ jet veto                               | 70  | 7.0 | 0.22 | 0.07 | 0.045 | 0.002 | 0.007 | -   | -   | 0.005 | 1.8 | 80    |
| 41  | -   | 7.2 | -    | 0.07 | 0.005 | 0.020 | 0.003 | -   | -   | 0.12  | -   | 7.4   |
| w/ jet veto                               | -   | 7.2 | -    | 0.06 | 0.003 | 0.003 | -     | -   | -   | 0.002 | -   | 7.3   |
| 5/  | -   | -   | -    | -    | -     | -     | -     | -   | -   | 0.002 | -   | 0.002 |
| w/ jet veto                               | -   | -   | -    | -    | -     | -     | -     | -   | -   | -     | -   | -     |
| $\sqrt{s} = 14 \text{ TeV}$               |     |     |      |      |       |       |       |     |     |       |     |       |
| 3/  | 140 | 18  | 0.54 | 1.5  | 0.33  | 0.04  | 3.6   | 19  | 7.5 | 7.7   | 36  | 240   |
| w/ jet veto                               | 140 | 17  | 0.54 | 0.12 | 0.087 | 0.01  | 0.04  | 1.5 | -   | 0.02  | 3.9 | 170   |
| 41  | -   | 19  | -    | 0.12 | 0.027 | 0.01  | 0.01  | -   | -   | 0.84  | -   | 20    |
| w/ jet veto                               | -   | 19  | -    | 0.12 | 0.027 | 0.01  | -     | -   | _   | 0.013 | -   | 19    |
| 5/  | -   | _   | -    | -    | 0.003 | _     | -     | -   | _   | 0.005 | -   | 0.008 |
| w/ jet veto                               | -   | _   | -    | -    | 0.003 | _     | -     | -   | _   | 0.003 | -   | 0.006 |

| Signal Cross Sections (fb)  N leptons + 0 jets |      |      |      |      |      |  |
|--|------|------|------|------|------|--|
| $\sqrt{s}$                                     | 3/   | 41   | 5/   | 6/   | 71   |  |
| 7 TeV  | 25.6 | 4.91 | 2.31 | 0.09 | 0.03 |  |
| 14 TeV   | 68.7 | 13.3 | 6.09 | 0.29 | 0.06 |  |

|        | NMSSM Signal       |
|--------|--------------------|
| $\geq$ | N leptons + 0 jets |

|  | ≥ 3 /             |                     | ≥ 5 /               |            |
|--|-------------------|---------------------|---------------------|------------|
|  | Signal            | Background          | Signal              | Background |
| $\sqrt{s} = 7 \text{ TeV}$                   |                   |                     |                     |            |
| Cross section (fb)                           | 33                | 87                  | 2.4                 | $\sim 0.0$ |
| Luminosity for $3\sigma$ (fb <sup>-1</sup> ) |                   | 1.0                 |                     | 3.7        |
| Luminosity for $5\sigma$ (fb <sup>-1</sup> ) |                   | 2.8                 |                     | 10         |
| $\sqrt{s} = 14 \text{ TeV}$                  |                   |                     |                     |            |
| Cross section (fb)                           | 88.4              | 187                 | 6.44                | 0.006      |
| $N_{\rm events}$ (600 fb <sup>-1</sup> )     | $5.3 \times 10^4$ | $1.1 \times 10^{5}$ | $3.9 \times 10^{3}$ | 4          |
| Luminosity for $3\sigma$ (fb <sup>-1</sup> ) |                   | 0.32                |                     | 1.4        |
| Luminosity for $5\sigma$ (fb <sup>-1</sup> ) |                   | 0.88                |                     | 3.9        |

#### This signal could be discovered with

- \*  $\sim 3 \text{ fb}^{-1}$  of data at  $\sqrt{s} = 7 \text{ TeV}$
- \* < 1 fb<sup>-1</sup> of data at  $\sqrt{s}$  = 14 TeV

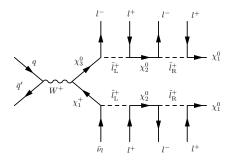
|        | NMSSM    | Sign  | al   |
|--------|----------|-------|------|
| $\geq$ | N lepton | s + 0 | jets |

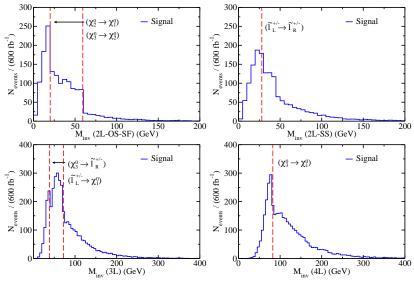
|  |                     | ≥ 3 /               | ≥ 5 <i>I</i>        |            |  |
|--|---------------------|---------------------|---------------------|------------|--|
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- \*  $< 1 \text{ fb}^{-1}$  of data at  $\sqrt{s} = 14 \text{ TeV}$
- Can also use kinematic edges to estimate mass differences

| Process  | $\Delta m$ (GeV) | $M_{max}$ (GeV) | Distribution |
|--|------------------|-----------------|--------------|
| $\chi_3^0 	o \chi_2^0$   | 62.4             | 59.3            | 2L-OS-SF     |
| $\chi_3^0 \to \chi_1^0$  | 82.5             | 82.5            | 4L           |
| $\chi_3^0 	o 	ilde{\it l}_R^\pm$                                 | 72.6             | 72.4            | 3L           |
| $	ilde{	ilde{I}_{L}^{\pm}}  ightarrow 	ilde{	ilde{I}_{R}^{\pm}}$ | 28.9             | 28.0            | 2L           |
| $	ilde{\it I}_L^\pm  ightarrow \chi_1^0$                         | 38.8             | 38.8            | 3L           |
| $\chi_2^0 	o \chi_1^0$   | 20.1             | 20.1            | 2L-OS-SF     |





 $\geq$ 5 leptons + 0 jets, 600 fb<sup>-1</sup>

#### **Conclusions**

- The NMSSM can have large cross sections for signals with  $\geq$  3 leptons + 0 jets and  $\geq$  5 leptons + 0 jets.
- Our benchmark point could be detected at the LHC with
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## THANK YOU!

## **EXTRA SLIDES**

• Our benchmark point is defined by the following parameters:

|                  | NMSSM Benchmark Model Parameters |                       |       |       |              |       |                |            |
|------------------|----------------------------------|-----------------------|-------|-------|--------------|-------|----------------|------------|
| tan β            | h <sub>s</sub>                   | As                    | μ     | к     | $A_{\kappa}$ | $A_t$ | A <sub>b</sub> | $A_{\tau}$ |
| 7.55             | 0.056                            | 488                   | 199   | 0.015 | -39.6        | -1170 | 1886           | -143       |
| $\overline{M_1}$ | <i>M</i> <sub>2</sub>            | <i>M</i> <sub>3</sub> | $M_Q$ | Mυ    | $M_D$        | $M_L$ | ME             |            |
| 149              | 297                              | 891                   | 2000  | 2000  | 2000         | 140   | 110            |            |

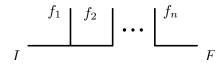
| Sparticle Mass Spectrum (GeV) |     |                                  |     |  |  |  |
|-------------------------------|-----|----------------------------------|-----|--|--|--|
| $\chi_1^0$ :                  | 109 | $	ilde{\it I}_{\it L}^{\pm}$ :   | 147 |  |  |  |
| $\chi_2^0$ :                  | 129 | $	ilde{\it I}_{R}^{\pm}$ :       | 118 |  |  |  |
| $\chi_3^0$ :                  | 191 | $	ilde{	au}_{1}^{\pm}$ :         | 114 |  |  |  |
| $\chi_4^0$ :                  | 206 | $	ilde{	au}_{	extsf{2}}^{\pm}$ : | 150 |  |  |  |
| $\chi_5^0$ :                  | 333 | $	ilde{ u}_I$ :                  | 125 |  |  |  |
| $\chi_1^\pm$ :                | 173 | $\tilde{\nu}_{\tau}$ :           | 125 |  |  |  |
| $\chi_2^\pm$ :                | 333 |                                  |     |  |  |  |

| Neutralino Composition |      |        |                 |                                |        |  |  |
|------------------------|------|--------|-----------------|--------------------------------|--------|--|--|
|                        | Ã    | Ŵ      | $\tilde{H}_{u}$ | $	ilde{	extit{H}}_{	extit{d}}$ | Š      |  |  |
| $\chi_1^0$ :           | 0.02 | < 0.01 | 0.01            | 0.01                           | 0.95   |  |  |
| $\chi_2^0$ :           | 0.64 | 0.03   | 0.20            | 0.09                           | 0.04   |  |  |
| $\chi_3^0$ :           | 0.33 | 0.17   | 0.26            | 0.24                           | < 0.01 |  |  |
| $\chi_4^0$ :           | 0.01 | 0.01   | 0.47            | 0.51                           | < 0.01 |  |  |
| $\chi_5^0$ :           | 0.01 | 0.79   | 0.06            | 0.14                           | < 0.01 |  |  |

| Dominant Leptonic Branching Fractions |                                      |      |
|---------------------------------------|--------------------------------------|------|
| $\chi_3^0  ightarrow$                 | $I^{\pm}\widetilde{I}_{ m R}^{\mp}$  | 0.40 |
|                                       | $I^{\pm}\widetilde{I}_{ m L}^{\mp}$  | 0.12 |
|                                       | $v_I \tilde{v}_I$                    | 0.01 |
| $\chi_1^\pm 	o$                       | $I^{\pm} \widetilde{\mathfrak{V}}_I$ | 0.53 |
|                                       | $ u_I 	ilde{I}_{ m L}^\pm$           | 0.08 |
| $	ilde{\it I}_L^\pm  ightarrow$       | $I^{\pm}\chi_2^0$                    | 0.97 |
|                                       | $I^\pm\chi_1^0$                      | 0.03 |
| $\chi_2^0  ightarrow$                 | $I^{\pm}\widetilde{I}_{R}^{\mp}$     | 0.48 |
|                                       | $v_I \tilde{v}_I$                    | 0.04 |
| $\tilde{\nu}_I \rightarrow$           | $v_1\chi_1^0$                        | 1.00 |
| $	ilde{\it I}_{R}^{\pm}  ightarrow$   | $I^{\pm}\chi_1^0$                    | 1.00 |
|                                       |                                      |      |

# Kinematic Edges

• Consider the decay  $I \rightarrow f_1 f_2 ... f_n F$ 



#### where

- \* I, F are massive particles
- \* fi are n massless final-state fermions
- The total mass of the system can be related to invariant mass of the n
  fermions

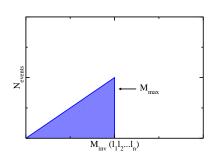
$$m_I^2 = (p_1 + p_2 + ... + p_n + p_F)^2$$
  
 $m_I^2 = m^2(f_1...f_n) + 2p_I \cdot p_F - m_F^2$ 

In the rest-frame of I this gives

$$m^2(f_1...f_n) = m_I^2 + m_F^2 - 2m_I E_F$$
  
 $m^2(f_1...f_n) \le (m_I - m_F)^2$ 

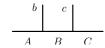
with the maximum reached when F is at rest in the rest-frame of I.

- This shows up as an edge in an invariant mass plot
- However, it is not always possible to reach this kinematic limit.



## 2 fermions

• For the process  $A \rightarrow bB \rightarrow bcC$ 



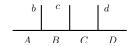
the kinematic upper limit will be given by

$$M_{max}^2(bc) = rac{(m_A^2 - m_B^2)(m_B^2 - m_C^2)}{m_B^2}$$

(This will occur when particles b and c are emitted back-to-back)

## 3 fermions

• For the process  $A \rightarrow bB \rightarrow bcC \rightarrow bcdD$ 



the kinematic upper limit will be given by

$$M_{\max}^{2}(\textit{bcd}) = \begin{cases} \frac{(m_{A}^{2} - m_{B}^{2})(m_{B}^{2} - m_{D}^{2})}{m_{B}^{2}} & \text{iff} & \frac{m_{A}}{m_{D}} > \frac{m_{B}^{2}}{m_{D}^{2}} \\ \frac{(m_{A}^{2} m_{C}^{2} - m_{B}^{2} m_{D}^{2})(m_{B}^{2} - m_{C}^{2})}{m_{B}^{2} m_{C}^{2}} & \text{iff} & \frac{m_{A}}{m_{D}} < \frac{m_{B}^{2}}{m_{C}^{2}} \\ \frac{(m_{A}^{2} - m_{C}^{2})(m_{C}^{2} - m_{D}^{2})}{m_{C}^{2}} & \text{iff} & \frac{m_{A}}{m_{D}} < \frac{m_{C}^{2}}{m_{D}^{2}} \end{cases}$$

$$(m_{A} - m_{D})^{2} \quad \text{otherwise}$$

## 4 fermions

• For the process  $A \rightarrow bB \rightarrow bcC \rightarrow bcdD \rightarrow bcdeE$ 



the kinematic limit will be given by

$$M_{\max}^{2}(bcde) = \begin{cases} \frac{(m_{A}^{2} - m_{B}^{2})(m_{B}^{2} - m_{E}^{2})}{m_{B}^{2}} & \text{iff} & \frac{m_{A}}{m_{E}} > \frac{m_{B}^{2}}{m_{E}^{2}} \\ \frac{(m_{A}^{2} m_{C}^{2} - m_{B}^{2} m_{E}^{2})(m_{B}^{2} - m_{C}^{2})}{m_{B}^{2} m_{C}^{2}} & \text{iff} & \frac{m_{A}}{m_{E}} < \frac{m_{B}^{2}}{m_{C}^{2}} \\ \frac{(m_{A}^{2} m_{D}^{2} - m_{C}^{2} m_{E}^{2})(m_{C}^{2} - m_{D}^{2})}{m_{C}^{2} m_{D}^{2}} & \text{iff} & \frac{m_{A}}{m_{E}} < \frac{m_{C}^{2}}{m_{E}^{2}} \\ \frac{(m_{A}^{2} - m_{D}^{2})(m_{D}^{2} - m_{E}^{2})}{m_{D}^{2}} & \text{iff} & \frac{m_{A}}{m_{E}} < \frac{m_{C}^{2}}{m_{E}^{2}} \\ (m_{A} - m_{E})^{2} & \text{otherwise} \end{cases}$$

It is interesting to note that these may all be written as

$$M_{max}^2(f_1...f_n) = (m_l - m_F)^2 - (m_l m_F) \left(x + \frac{1}{x} - 2\right)$$

where  $x \ge 1$  and depends on a product of mass ratios.

• If  $m_l \approx m_F$  then  $x \approx 1$  and

$$M_{max}^2(f_1...f_n) \approx (m_I - m_F)^2$$

 Therefore, we can think about these kinematics edges as mass differences if the differences are not too large.