Transverse Momentum Distributions from Effective Field Theory

Ye Li
Northwestern University
In collaboration with Sonny Mantry, Frank Petriello

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and work in progress

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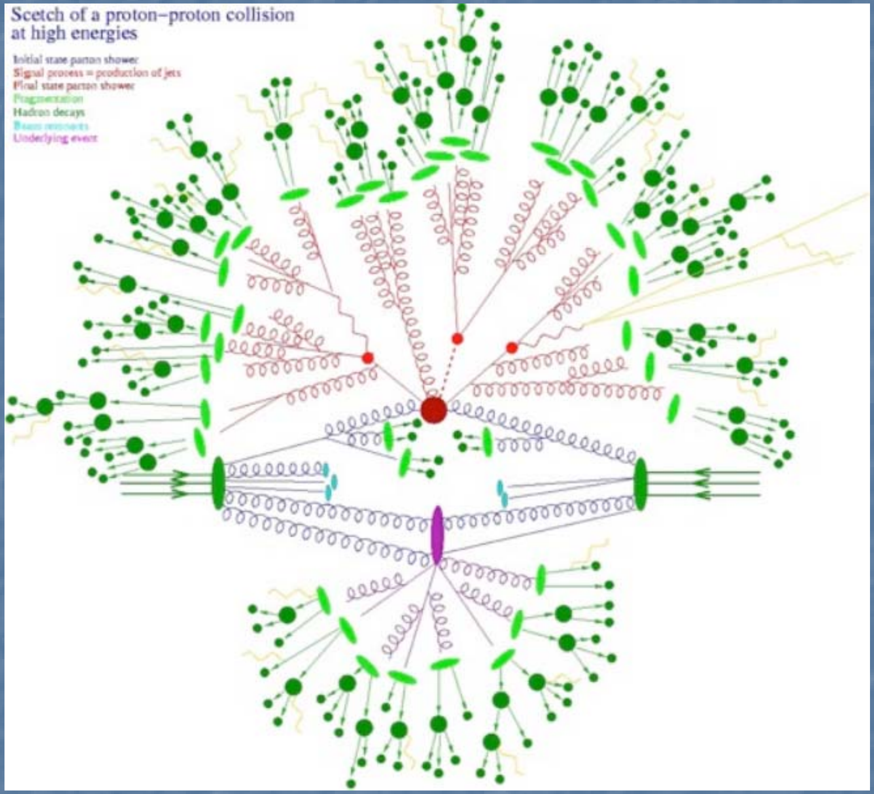
Outline

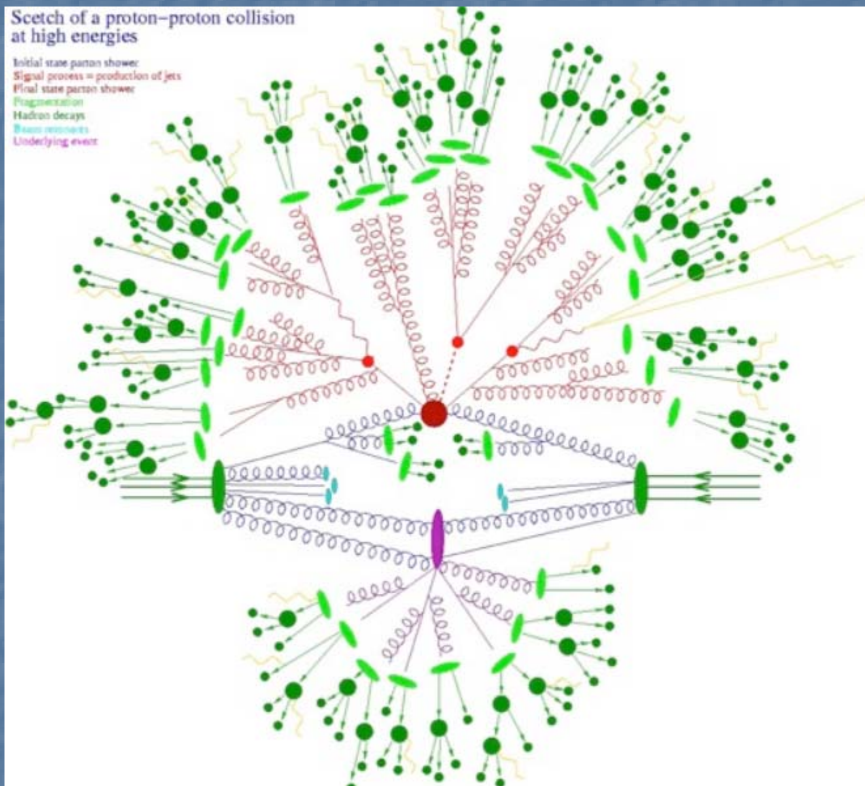
- Introduction
- The effective field theory approach
- Numerical results and comparison with data
- Higher orders

Factorization



- Physics of interest at hard scale M_H
- Parton shower evolution from M_H to Λ_{QCD}
- Final state hadronization at Λ_{QCD}
- Multiple parton interactions hadron decays, ...

Factorization



- How do we make sense of this gooey mess?

Factorization !

Factorization and Resummation

$$d\sigma = \sum_{i,j} d\sigma_{i,j}^{part} \otimes f_i(\xi_a) \otimes f_j(\xi_b)$$

Live at hard scale

Live at non-perturbative scale

Kinematic constraints

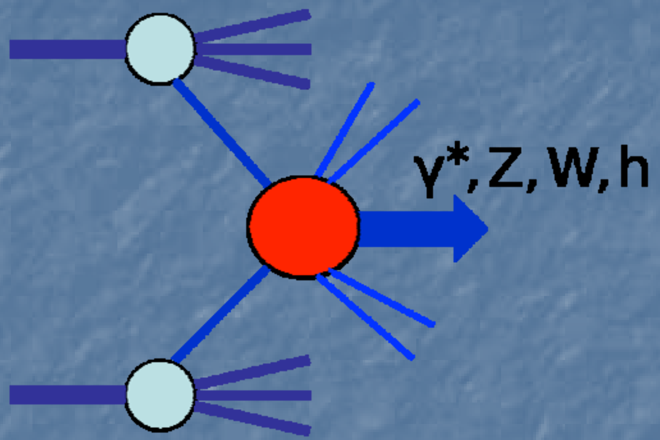
Multiple Disparate scales

RG evolve to hard scale

Additional factorization and resummation required

- Evolution of PDF allows us to resum the large logarithms of hard and non-perturbative scales
- Final state restriction introduces new scales
- Example: low transverse momentum distribution in Drell-Yan, Higgs production

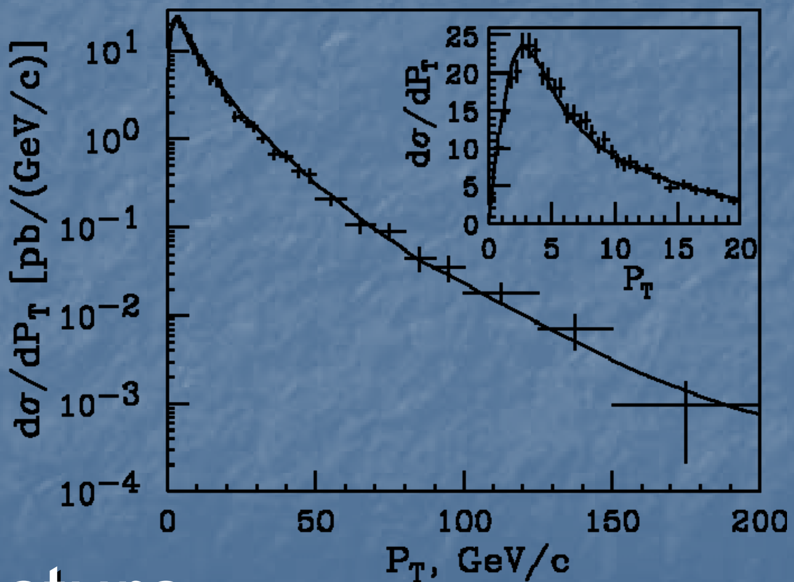
Transverse Momentum Spectrum



- Observable of interest

$$\frac{d\sigma}{dp_T^2}$$

- Restrict pT: new scale



CDF data for Z production: hep-ex/0001021

Motivations:

- Higgs boson searches
- W-mass measurement
- Tests of pQCD
- Transverse nucleon structure

Low pT region

- The schematic perturbative series for the pT distribution of pp → (V,h)+X

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} \simeq \frac{1}{p_T^2}\left[A_1\alpha_S \ln\frac{M^2}{p_T^2} + A_1\alpha_S^2 \ln^3\frac{M^2}{p_T^2} + \ldots + A_n\alpha_S^n \ln^{2n-1}\frac{M^2}{p_T^2} + \ldots\right]$$

Large logarithms spoil perturbative convergence ⇒ must be resummed

- Resummation has been studied in great detail in Collins-Soper-Sterman formalism

Davies, Stirling; Arnold, Kauffman; Qiu, Zhang; Berger, Qiu; Ellis, Veseli; Brock, Ladinsky Landry, Nadolsky; Yuan; Kulesza, Sterman, Vogelsang; Catani, Mangano, Nason,Trentadue; de Florian, Grazzini; Cherdnikov, Stefanis; Belitsky, Ji,....

- Resummation has been studied recently using Effective Field Theory approach

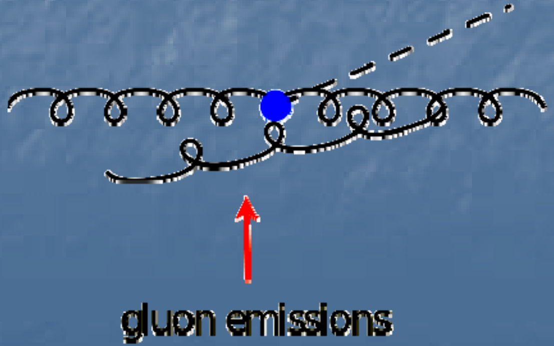
Idilbi, Ji, Juan; Gao, Li, Liu; Mantry, FP; Becher, Neubert; Chiu et al.

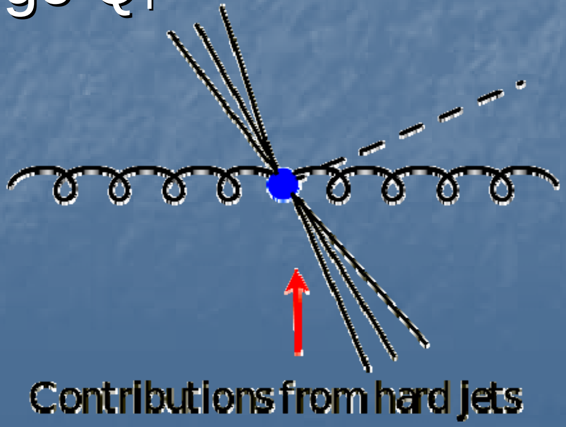
Collins-Soper-Sterman Formalism

$$\frac{d\sigma_{AB\to CX}}{dQ^2dydQ_T^2} = \frac{d\sigma_{AB\to CX}^{(resum)}}{dQ^2dydQ_T^2} + \frac{d\sigma_{AB\to CX}^{(Y)}}{dQ^2dydQ_T^2}$$

- Singular as at least Q_T^{-2} as $Q_T \to 0$
- Important in region of small Q_T
- Treated with resummation

- Obtained from fixed order calculation
- Less Singular terms
- Important in region of large Q_T





Collins-Soper-Sterman Formalism

$$\frac{d^2\sigma}{dp_T dY} = \sigma_0 \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-i\vec{p}_T \cdot \vec{b}_\perp} \sum_{a,b} [C_a \otimes f_{a/P}](x_A, b_0/b_\perp)[C_a \otimes f_{a/P}](x_B, b_0/b_\perp)$$

$$\times \exp\left\{ \int_{b_0^2/b_\perp^2}^{\hat{Q}^2} \frac{d\mu^2}{\mu^2} \left[\ln \frac{\hat{Q}^2}{\mu^2} A(\alpha_S(\mu^2)) + B(\alpha_S(\mu^2)) \right] \right\}$$

Sudakov Factor

Landau Pole

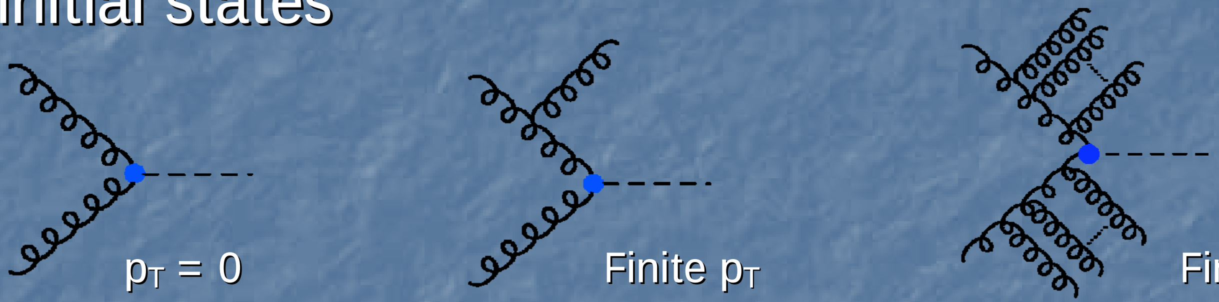
- Y term neglected for the purpose here
- A,B,C have well-defined perturbative expansions
- Integration of impact parameter $b_\perp$ introduce Landau pole: a treatment must work for *any* value of p_T
- Resummed exponent in $b_\perp$ space ⇒ difficult in matching to fixed order calculation in p_T space

EFT Framework

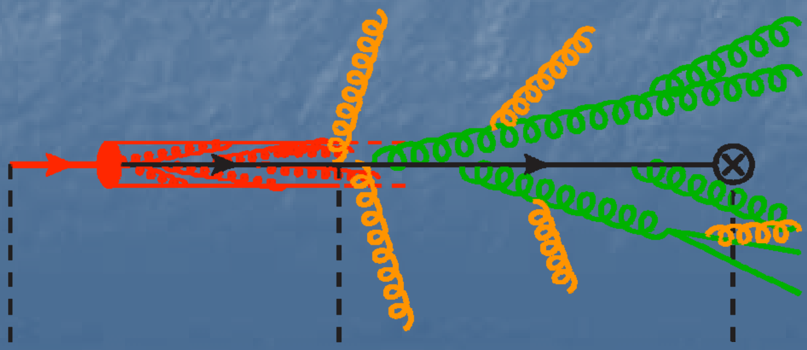
- Hierarchy of scales suggests EFT approach with well defined power counting

$$M_H \gg p_T \gg \Lambda_{QCD}$$

- Low p_T region dominated by soft and collinear emissions from initial states



- Colliding parton is part of initial state p_T radiation beam jet



EFT Framework

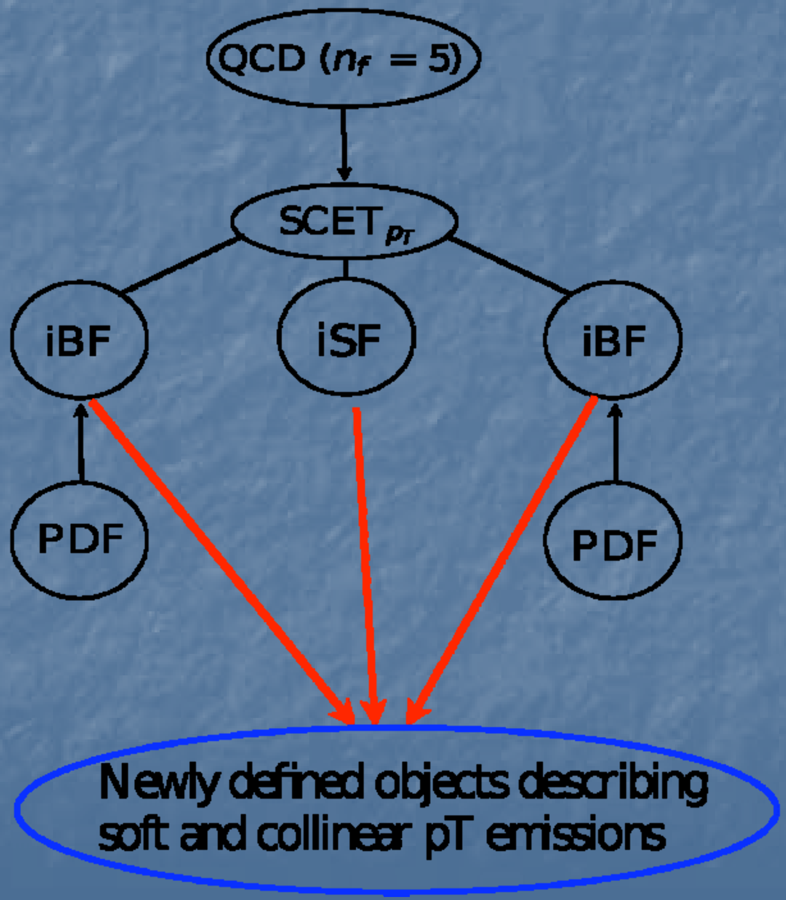
$$\mathrm{QCD}(n_f = 6) \to \mathrm{QCD}(n_f = 5) \to \mathrm{SCET}_{p_T} \to \mathrm{SCET}_{\Lambda_{QCD}}.$$

Top quark integrated out.

Matched onto SCET.

Soft-collinear factorization.

Matching onto PDFs.



$$\frac{d^2\sigma}{dp_T^2\, dY} \sim H \otimes G^{ij} \otimes f_i \otimes f_j$$

SCET Cross Section

- Schematic form of SCET cross-section:

$$\frac{d^2\sigma}{dp_T dY} \sim \int PS \, | C \otimes \langle O \rangle |^2$$

Wilson coefficient from hard matching

SCET matrix element

- Use soft collinear decoupling to factor out the soft sector

$$\frac{d^2\sigma}{dp_T dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1} \sim H \otimes G^{i,j} \otimes f_i \otimes f_j$$

Hard function

Impact-parameter Beam Functions (iBFs): collinear radiation

Soft function: soft emission

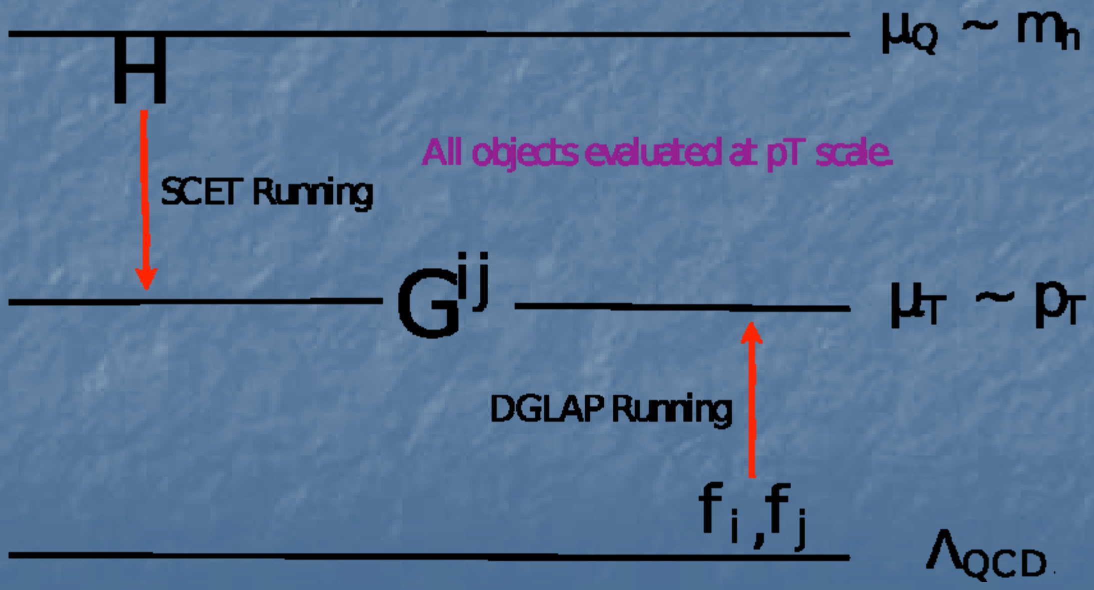
- Beam function is essentially unintegrated nucleon distribution function and can be matched to PDF

Running

- Factorization formula:

$$\frac{d^2\sigma}{dp_T dY} \sim H \otimes G^{i,j} \otimes f_i \otimes f_j$$

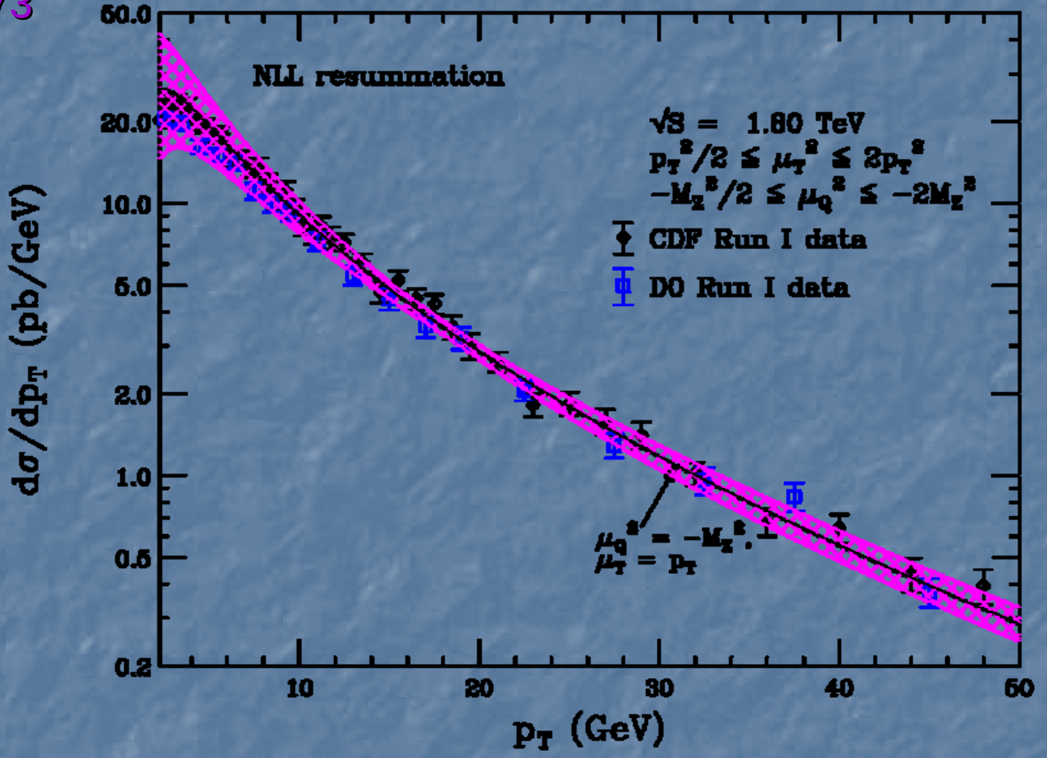
- Schematic picture of running:



Z-production: Comparison w/ Data

Phys.Rev.D83:053007,2011.

hep-ph/1007.3773



- Good agreement with data
- Theory curve determined completely by perturbative funcitons and standard PDFs.

Check to pQCD

- Expanded resummed formula to compare to fixed order

$$\frac{d^2\sigma_{Z,q\bar{q}}}{dp_T^2 dY} = \frac{4\pi^2}{3}\frac{\alpha}{\sin^2\theta_W} e_{q\bar{q}}^2 \frac{1}{s\,p_T^2} \sum_{m,n} \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^n {}_nD_m \ln^m \frac{M_Z^2}{p_T^2}$$

$$\begin{aligned} \text{leading logarithmic} &: \alpha_s^n L^{2n-1}, \\ \text{next-to-leading logarithmic} &: \alpha_s^n L^{2n-2}, \\ \text{next-to-next-to-leading logarithmic} &: \alpha_s^n L^{2n-3}. \end{aligned}$$

Leading Log

Next-to-Leading Log

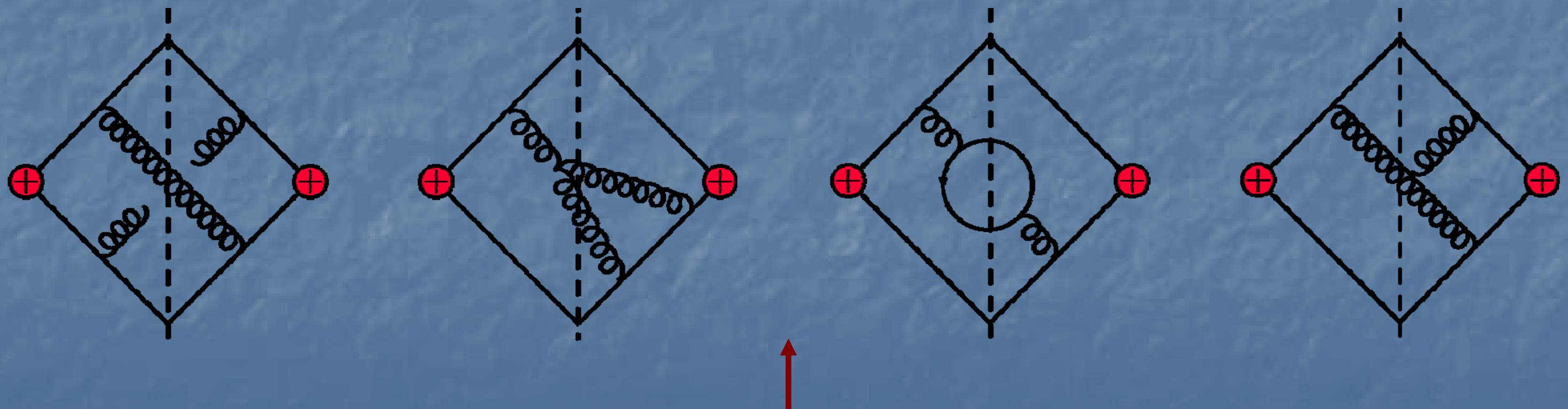
$$\begin{aligned}
{}_1D_1 &= A^{(1)} f_A f_B, \\
{}_1D_0 &= B^{(1)} f_A f_B + f_B \left(P_{qq}\otimes f\right)_A + f_A \left(P_{qq}\otimes f\right)_B, \\
{}_2D_3 &= -\frac{1}{2}\left[A^{(1)}\right]^2 f_A f_B, \\
{}_2D_2 &= -\frac{3}{2}A^{(1)}\left[f_B\left(P_{qq}\otimes f\right)_A + f_A\left(P_{qq}\otimes f\right)_B\right] - \left[\frac{3}{2}A^{(1)}B^{(1)} - \beta_0 A^{(1)}\right] f_A f_B, \\
{}_2D_1 &- \left\{-A^{(1)} f_B\left(P_{qq}\otimes f\right)_A \ln\frac{\mu_F^2}{M_Z^2} - 2B^{(1)} f_B \left(P_{qq}\otimes f\right)_A - \frac{1}{2}\left[B^{(1)}\right]^2 f_A f_B \right. \\
&\quad + \frac{\beta_0}{2}A^{(1)} f_A f_B \ln\frac{\mu_R^2}{M_Z^2} + \frac{\beta_0}{2}B^{(1)} f_A f_B - \left(P_{qq}\otimes f\right)_A \left(P_{qq}\otimes f\right)_B \\
&\quad \left. - f_B\left(P_{qq}\otimes P_{qq}\otimes f\right)_A + \beta_0 f_B \left(P_{qq}\otimes f\right)_A \right\} + \left[A \leftrightarrow B\right].
\end{aligned}$$

Agrees through NLL level

${}_2D_1$ requires NNLL

iBF and SF at NNLO

- Required for NNLL resummation
- Soft function has been worked out as our first step
  - Fully in momentum, position and impact-parameter space
  - Anomalous dimensions and renormalization obtained in momentum and impact-parameter space
- Impact-parameter beam function is still in progress



Two loop graphs for soft function

Soft Function at NNLO

- Anomalous dimensions in position and impact-parameter space
  - Old result confirmed: Belitsky (hep-ph/9808389)
  - New in impact-parameter space
- New renormalized soft function in full position and impact-parameter space

Define $L = -\frac{b^+ b^- \mu^2 e^{2\gamma_E}}{4}$

$$\gamma_s^{(1)}(b) = 2\frac{\alpha_s}{\pi} C_F \ln(L)$$

$$\gamma_s^{(2)}(b) = \left(\frac{\alpha_s}{\pi}\right)^2 \left\{ C_F N_F \left[-\frac{5}{9}\ln(L) + \frac{\pi^2}{36} - \frac{14}{27} \right] + C_F C_A \left[\left(-\frac{\pi^2}{6} + \frac{67}{18} \right) \ln(L) - \frac{7}{2}\zeta(3) - \frac{11\pi^2}{72} + \frac{101}{27} \right] \right\}$$

Define $L_{0,0} = \delta(q^-)\delta(q^+)$

and $L_{0,1} = \frac{1}{\mu}\left[\frac{\mu}{q^+}\right]_+ \delta(q^-) + \frac{1}{\mu}\left[\frac{\mu}{q^-}\right]_+ \delta(q^+)$

$$\gamma_s^{(1)}(q^-, q^+) = -2\frac{\alpha_s}{\pi} C_F L_{0,1}$$

$$\gamma_s^{(2)}(q^-, q^+) = \left(\frac{\alpha_s}{\pi}\right)^2 \left\{ C_F N_F \left[\frac{5}{9} L_{0,1} + \left(\frac{\pi^2}{36} - \frac{14}{27} \right) L_{0,0} \right] + C_F C_A \left[\left(\frac{\pi^2}{6} - \frac{67}{18} \right) L_{0,1} - \left(\frac{7}{2}\zeta(3) + \frac{11\pi^2}{72} - \frac{101}{27} \right) L_{0,0} \right] \right\}$$

Summary

- Factorization formula:

$$\frac{d^2\sigma}{dp_T dY} \sim H \otimes G^{i,j} \otimes f_i \otimes f_j$$

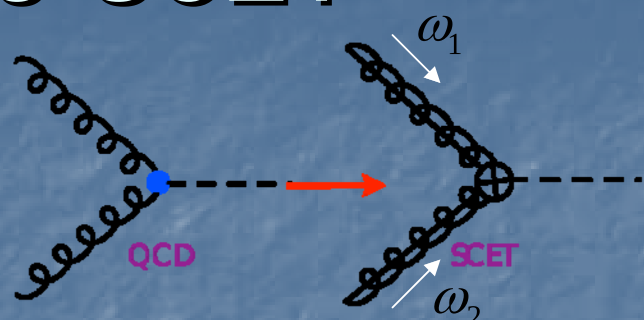
- Perturbative pT distribution given in terms of perturbatively calculable functions and the standard PDFs
- NLL resummation in good agreement with data
- Next step: NNLL resummation
  - Soft function done
  - Beam function in progress

Backup Slides

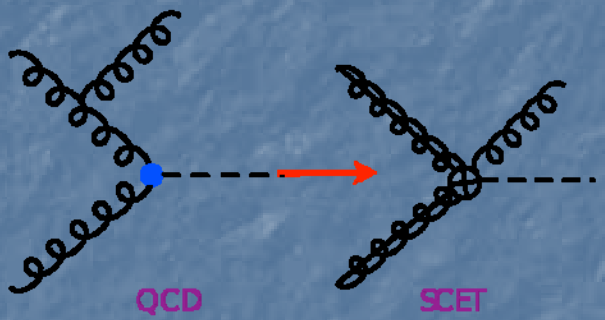
Match onto SCET

- Matching Equation:

$$O_{QCD} = \int d\omega_1 \int d\omega_2 C(\omega_1, \omega_2) O(\omega_1, \omega_2)$$

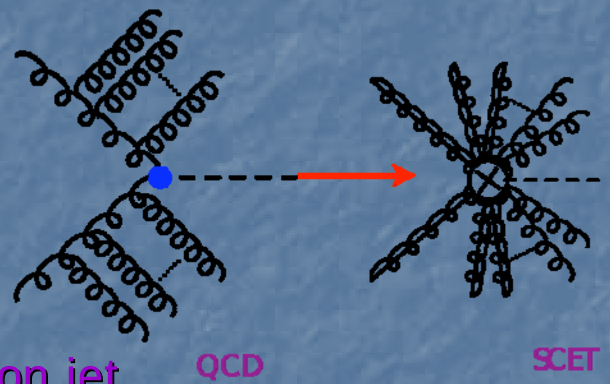


Tree level matching



Matching real emission graphs

Soft and Collinear emissions build into Wilson lines determined by soft and collinear gauge invariance of SCET.



- Effective SCET operator:

gluon jet

$$O(\omega_1, \omega_2) = g_{\mu\nu} \mathrm{T}\left\{ \mathrm{Tr}\left[S_n (gB_\perp^\mu)_{\omega_1} S_n^\dagger S_{\bar{n}} (gB_\perp^\nu)_{\omega_2} S_{\bar{n}}^\dagger \right] \right\}$$

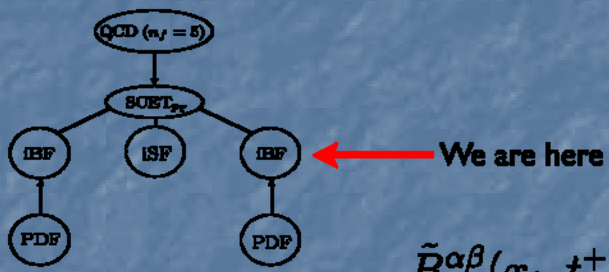
$$O^\mu(\omega_1, \omega_2) = (\bar{\xi} W) \mathrm{T}\left[S_{\bar{n}} \Gamma^\mu S_n^\dagger \right] (W^\dagger \xi)$$

quark jet

S = Soft Wilson line

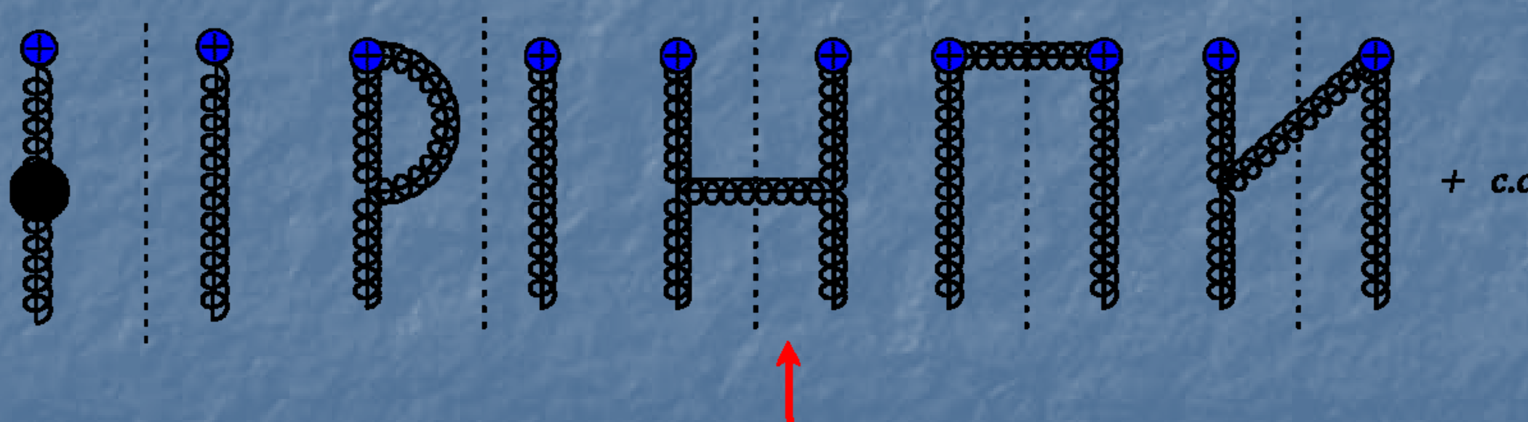
W = Collinear Wilson line

Impact-parameter Beam Function



- Unintegrated nucleon function
- Definition:

$$\tilde{B}_n^{\alpha\beta}(x_1, t_n^+, b_\perp, \mu) = \int \frac{db^-}{4\pi} e^{\frac{i}{2}\frac{t_n^+ b^-}{Q}} \sum_{\text{initial pols.}} \sum_{X_n} \langle p_1 | \left[g B^A_{1n\perp\beta}(b^-, b_\perp) | X_n \rangle \right.$$
$$\left. \times \; \langle X_n | \delta(\bar{\mathcal{P}} - x_1 \bar{n} \cdot p_1) g B^A_{1n\perp\alpha}(0) \right] | p_1 \rangle,$$

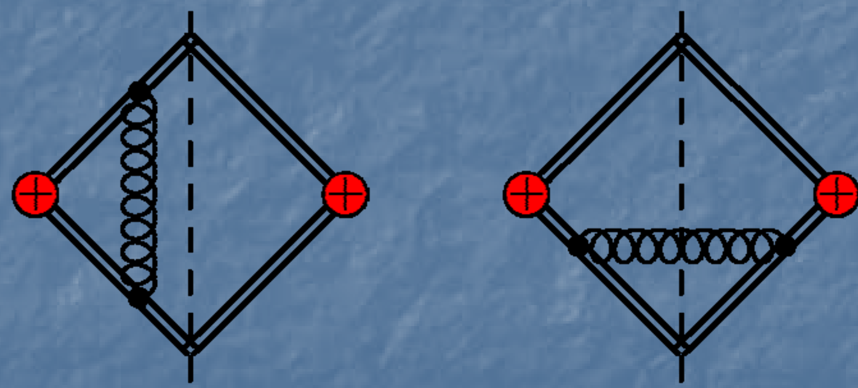


One loop graphs

Soft Function

- Definition:

$$S(z) \;=\; \langle 0|\mathrm{Tr}\Big(\bar{T}\{S_{\bar{n}}T^{D}S^{\dagger}_{\bar{n}}S_{n}T^{C}S^{\dagger}_{n}\}\Big)(z)\mathrm{Tr}\Big(T\{S_{n}T^{C}S^{\dagger}_{n}S_{\bar{n}}T^{D}S^{\dagger}_{\bar{n}}\}\Big)(0)|0\rangle$$



One loop graphs

Zero-bin and Soft Subtraction

- Naïve calculation of iBF double counts the soft region: zero-bin subtraction required

$$B_{n,\bar{n}} = \tilde{B}_{n,\bar{n}} - B_{n0,\bar{n}0}$$

Naïve iBF Zero-bin (soft region)

- Equivalence btw zero-bin and soft subtraction:

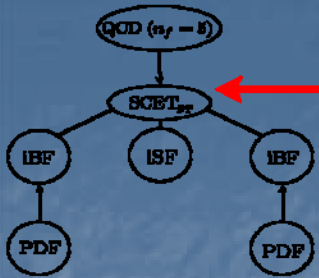
$$\frac{d^2\sigma}{dp_T dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$

- iBF can be matched onto PDFs to separate perturbative and non-perturbative scales

$$\tilde{B}_{n,\bar{n}} = I_{(n,\bar{n}),i} \otimes f_i$$

- Define Transverse momentum Function:

$$G^{i,j} \sim I_{n,i} \otimes I_{\bar{n},j} \otimes S^{-1} \Rightarrow \frac{d^2\sigma}{dp_T dY} = H \otimes G^{i,j} \otimes f_i \otimes f_j$$



We are here

SCET Cross Section

- **Formula in detail:**

$$\frac{d^2\sigma}{du\,dt} = \frac{(2\pi)}{(N_c^2-1)^2 8Q^2}\int dp_h^+ dp_h^- \int d^2k_h^\perp \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-i\vec{k}_h^\perp\cdot\vec{b}_\perp}$$

$$\times\, \delta\left[u - m_h^2 + Qp_h^-\right]\delta\left[t-m_h^2+Qp_h^+\right]\delta\left[p_h^+p_h^- - \vec{k}_{h\perp}^2 - m_h^2\right]\int d\omega_1 d\omega_2 \overbrace{|C(\omega_1,\omega_2,\mu)|^2}^{\text{Hard}}$$

$$\times \int dk_n^+ dk_{\bar n}^- \underbrace{B_n^{\alpha\beta}(\omega_1,k_n^+,b_\perp,\mu)}_{\substack{\text{n-collinear}\\ \text{iBF}}} \underbrace{B_{n\alpha\beta}(\omega_2,k_n^-,b_\perp,\mu)}_{\substack{\text{bn-collinear}\\ \text{iBF}}} \underbrace{\mathcal{S}(\omega_1 - p_h^- - k_n^-, \omega_2 - p_h^+ - k_n^+, b_\perp,\mu)}_{\text{Soft}}$$

- **iBFs and soft functions field theoretically defined as the fourier transform of:**

$$J_n^{\alpha\beta}(\omega_1, x^-, x_\perp, \mu) = \sum_{\text{initial pols.}} \langle p_1 | \left[g B^A_{1n\perp\beta}(x^-, x_\perp)\delta(\bar{\mathcal{P}} - \omega_1) g B^A_{1n\perp\alpha}(0)\right] | p_1\rangle$$

$$J_{\bar n}^{\alpha\beta}(\omega_1, y^+, y_\perp, \mu) = \sum_{\text{initial pols.}} \langle p_2 | \left[g B^A_{1n\perp\beta}(y^+, y_\perp)\delta(\bar{\mathcal{P}} - \omega_2) g B^A_{1n\perp\alpha}(0)\right] | p_2\rangle$$

$$S(z,\mu) = \langle 0 | \bar{T}\left[\mathrm{Tr}\left(S_{\bar n} T^D S^\dagger_{\bar n} S_n T^C S^\dagger_n\right)(z)\right] T\left[\mathrm{Tr}\left(S_n T^C S^\dagger_n S_{\bar n} T^D S^\dagger_{\bar n}\right)(0)\right] |0\rangle.$$

Factorization Formula

- Factorization formula in full detail:

$$\frac{d^2\sigma}{d\mathrm{p}_T^2\,dY} = \frac{\pi^2}{4(N_c^2-1)^2Q^2}\int_0^1 \frac{dx_1}{x_1}\int_0^1 \frac{dx_2}{x_2}\int_{x_1}^1 \frac{dx_1'}{x_1'}\int_{x_2}^1 \frac{dx_2'}{x_2'}$$

$$\times\ \underbrace{H(x_1,x_2,\mu_Q;\mu_T)}_{\text{Hard function.}}\,\underbrace{\mathcal{G}^{ij}(x_1,x_1',x_2,x_2',p_T,Y,\mu_T)}_{\text{Transverse momentum function.}}\,\underbrace{f_{i/P}(x_1',\mu_T)\,f_{j/P}(x_2',\mu_T)}_{\text{PDFs.}}$$

- The transverse momentum function is a convolution of the iBF matching coefficient and the inverse soft function

$$\mathcal{G}^{ij}(x_1,x_1',x_2,x_2',p_T,Y,\mu_T) = \int dt_n^+\int dt_{\bar n}^-\int \frac{d^2b_\perp}{(2\pi)^2}J_0(|\vec b_\perp|p_T)$$

Collinear pT emissions $\longrightarrow \times\ \mathcal{I}^{\beta\alpha}_{n;g,i}\Big(\frac{x_1}{x_1'},t_n^+,b_\perp,\mu_T\Big)\,\mathcal{I}^{\beta\alpha}_{\bar n;g,j}\Big(\frac{x_2}{x_2'},t_{\bar n}^-,b_\perp,\mu_T\Big)$

Soft pT emissions $\longrightarrow \times\ \mathcal{S}^{-1}\Big(x_1Q-e^{Y}\sqrt{\mathrm{p}_T^2+m_h^2}-\frac{t_{\bar n}^-}{Q},\,x_2Q-e^{-Y}\sqrt{\mathrm{p}_T^2+m_h^2}-\frac{t_n^+}{Q},\,b_\perp,\mu_T\Big)$