

# NNLO Penguin Correction in B to light mesons decays - Imaginary part

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A decorative graphic consisting of several horizontal lines of varying lengths and colors (teal, white, and light blue) extending from the right side of the slide towards the center.

# Motivation

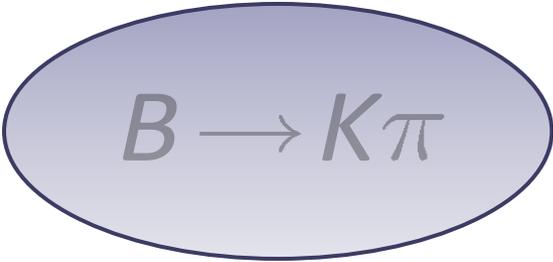
- B physics has fruitful phenomenology.
- CP violating decays and many rare decays are possible windows to see the New Physics.
- NNLO calculation is required for giving more clean theoretical estimation.

A.J. Buras 1102.5650

*Climbing NLO and NNLO Summits of Weak Decays*



# Motivation


$$B \rightarrow K\pi$$

Rare decays :  $\text{Br} \sim 10^{-5}$

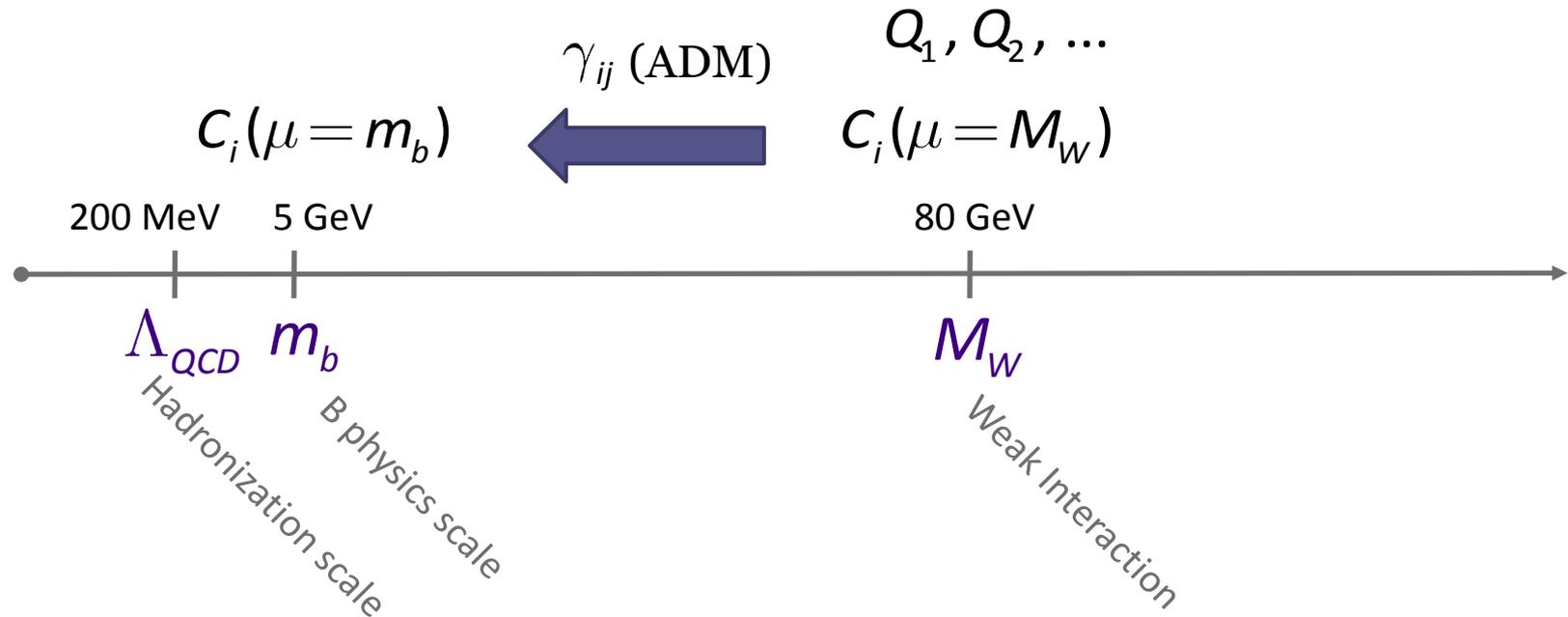
Penguin Dominant decays

They are CP violating decays

K pi puzzle

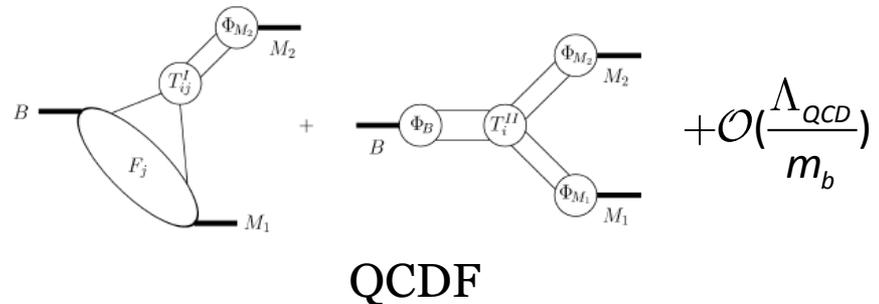
Can be handled by QCDF

# Formalism



$$A = G_F \sum C_i \times \langle P_1 P_2 | Q_i | B \rangle$$

$$\langle P_1 P_2 | Q_i | B \rangle = \int F * \phi * T$$



# Status for NNLO

(Form factor Term)

Tree amplitude:



$$T = e^{i\gamma} [(t_0 + t_1 \alpha_s + t_2 \alpha_s^2 + \dots) + (\tilde{t}_0 + \tilde{t}_1 \alpha_s + \tilde{t}_2 \alpha_s^2 + \dots) I]$$

0
0

*G. Bell (2009)*
*G. Bell (2007)*

$$= |T| e^{i\gamma} e^{i\delta_T}$$

(a)

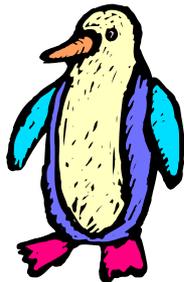
(b)

(c)

(d)

Fig. 6. "Non-factorizable" vertex corrections.

Penguin amplitude:



$$P = [(p_0 + p_1 \alpha_s + p_2 \alpha_s^2 + \dots) + (\tilde{p}_0 + \tilde{p}_1 \alpha_s + \tilde{p}_2 \alpha_s^2 + \dots) I]$$

0
0

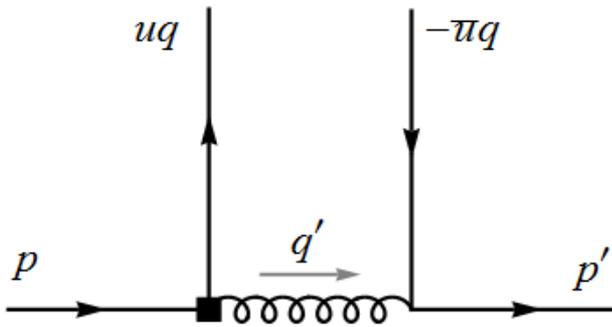
$$= |P| e^{i\delta_P}$$

(e)

(f)

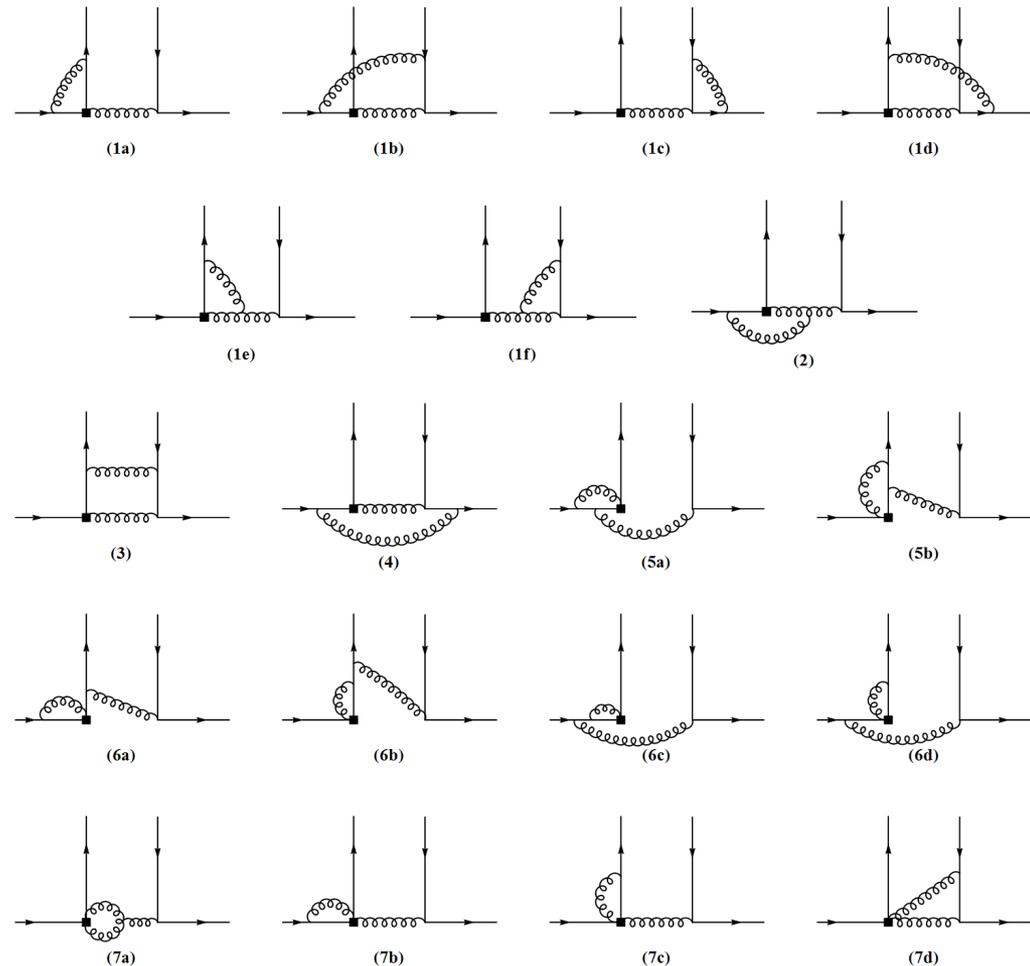
# NNLO for Penguin

## ● Part 1 - Magnetic Penguin



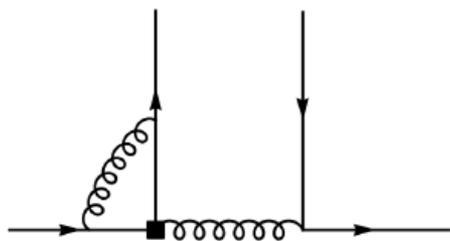
$$Q_{8g} = -\frac{g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) G^{\mu\nu} b$$

Insertion

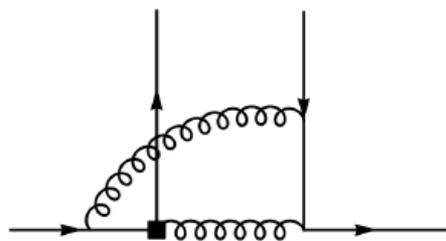


# IR cancelation

## Soft divergence cancelation



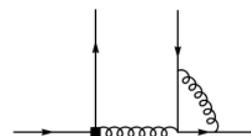
(1a)



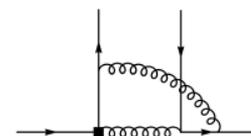
(1b)

$$(1a) + (1b) = \frac{1}{l^2((l+p)^2 - m^2)} \left( \frac{[\gamma_\nu(l+uq)\Gamma_{q'}^\mu(l+\not{p}+m)\gamma_\nu][\gamma_\mu]}{q'^2(l+uq)^2} - \frac{[\Gamma_{l+q'}^\mu(l+\not{p}+m)\gamma_\nu][\gamma_\mu(l+\bar{u}q)\gamma_\nu]}{(l+q')^2(l+\bar{u}q)^2} \right)$$

Similarly, (1c)+(1d) are canceled



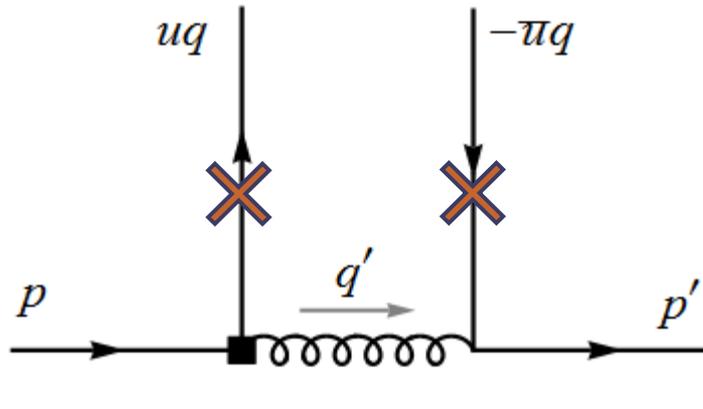
(1c)



(1d)

# IR cancelation

Collinear divergence  $1 - l \parallel q$



$$l = \alpha q + \beta \bar{q} + l_{\perp}^2$$

$$\alpha \sim 1, \beta \sim \lambda^2, l_{\perp}^2 \sim \lambda^2 m_b^2$$

$$dl^4 \sim \lambda^4 m_b^4$$

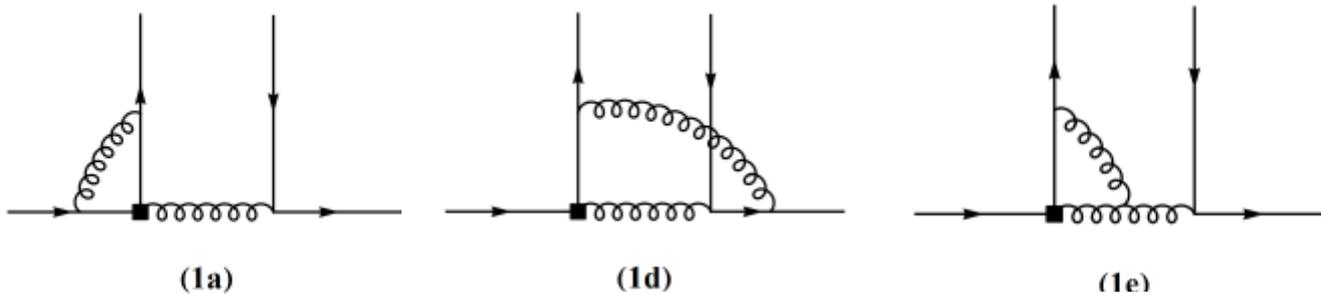
where  $\lambda \sim \Lambda_{\text{QCD}}$

$$\mathcal{T}_{\text{NLO}} = -2 \left( \frac{\alpha_s}{4\pi} \right) C_F P(u)$$

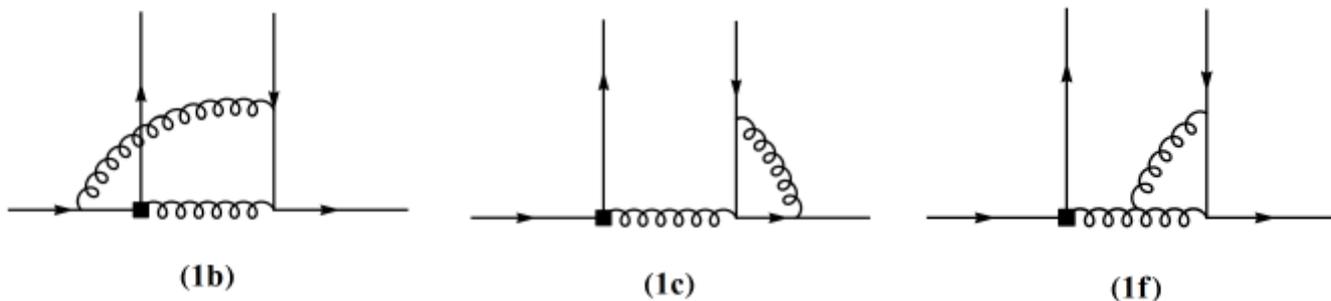
→ 7 diagrams out of 19

# IR cancelation

Collinear divergence  $1 - l \parallel q$



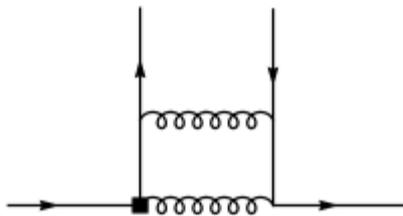
$$X_{(1a)+(1d)+(1e)} = C_F \frac{2}{l^2(l+uq)^2} \left( \frac{\alpha+u}{\alpha} \right) (P(u) - P(u+\alpha))$$



$$X_{(1b)+(1c)+(1f)} = C_F \frac{2}{l^2(l+uq)^2} \left( \frac{\alpha+\bar{u}}{\alpha} \right) (P(u) - P(u-\alpha))$$

# IR cancelation

Collinear divergence  $1 - l \parallel q$



(3)

$$X_{(3)} = C_F \frac{2l_{\perp}^2}{l^2(l+uq)^2(1-\bar{u}q)^2} P(u+\alpha)$$

Total Collinear Divergence is

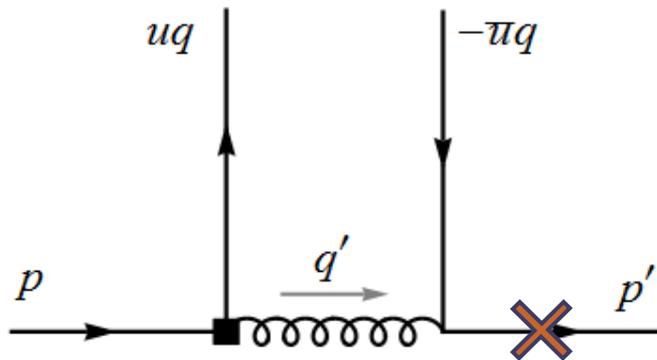
$$X_{\text{tot}} = C_F \left( \frac{2(\alpha+u)}{\alpha} \frac{1}{l^2(l+uq)^2} - \frac{2(\bar{u}-\alpha)}{\alpha} \frac{1}{l^2(l-\bar{u}q)^2} \right) (P(u) - P(u+\alpha)) \\ - C_F \frac{l_{\perp}^2}{q \cdot l} \left( \frac{1}{l^2(l+uq)^2} - \frac{1}{l^2(l-\bar{u}q)^2} \right) P(u+\alpha)$$

$$X_{\text{tot}} = C_F \frac{\alpha_s}{\pi} \ln \frac{\mu_{\text{UV}}}{\mu_{\text{IR}}} \int_0^1 dw P(w) V(w, u)$$

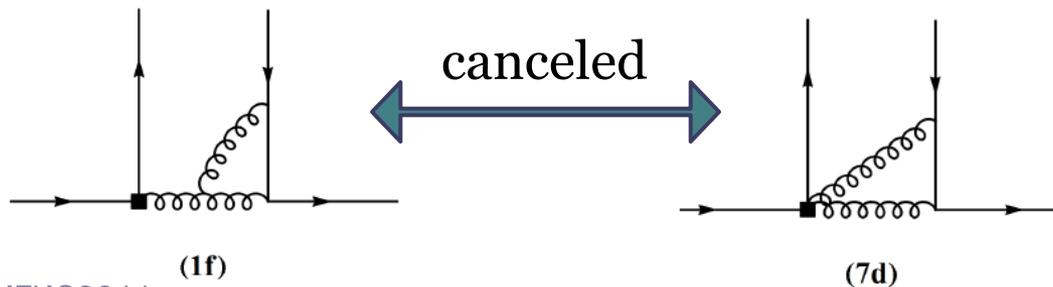
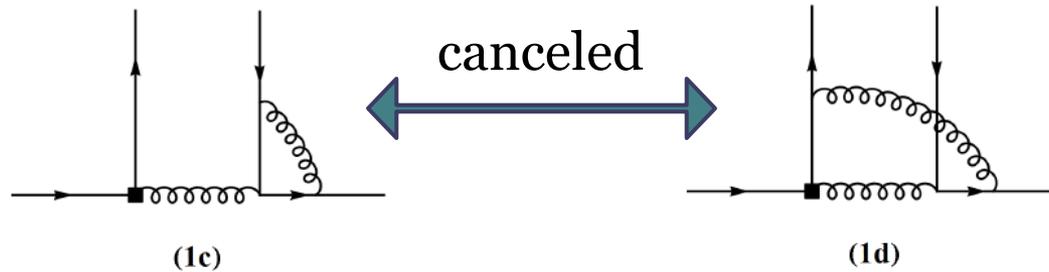
BBNS, NPB, (2000)

Exactly canceled by  $F^{(0)} T^{(1)} \Phi^{(1)}$

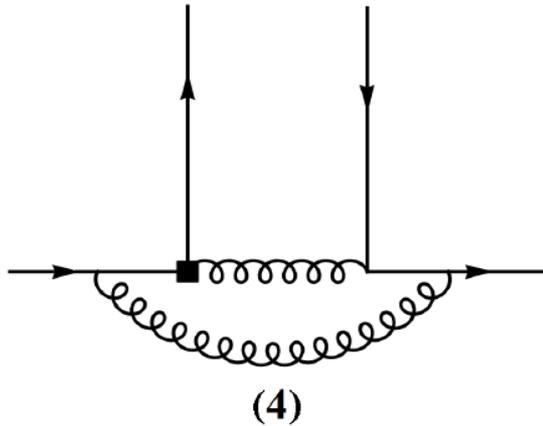
# Collinear divergence 2 - $l \parallel p'$



→ 4 diagrams out of 19



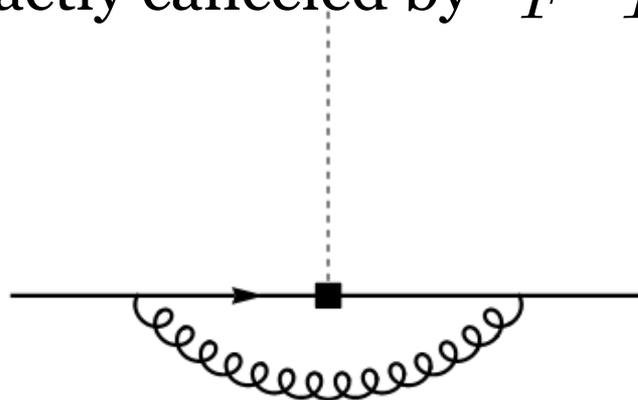
# Form-factor correction



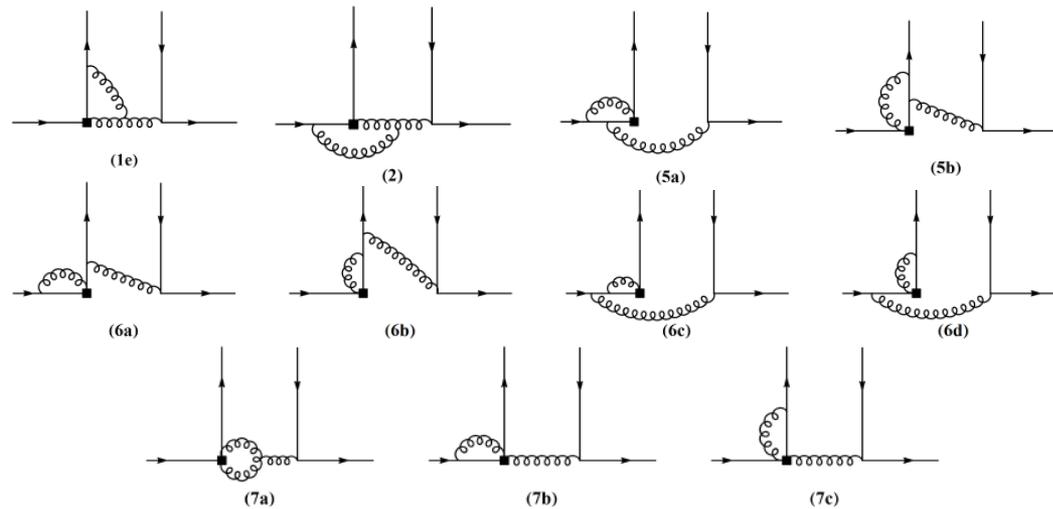
→ Collinear (with  $p'$ ) and Soft

$$-C_F \frac{2}{l^2(l-p')^2} \left( \frac{1-\alpha}{\alpha} \right) P(u)$$

IR div.s are Exactly canceled by  $F^{(1)}T^{(1)}\Phi^{(0)}$  :



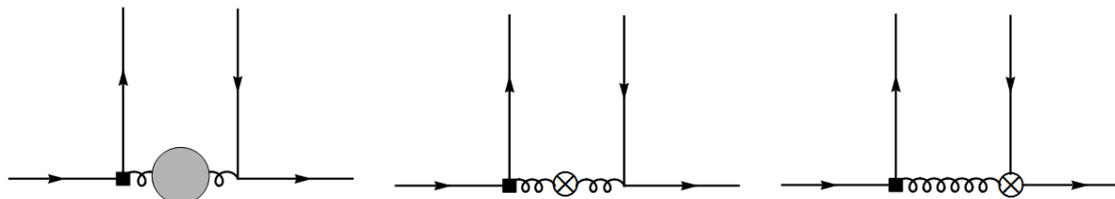
# Renormalization



→ UV div. same with ADM  $\gamma_{88} = 16C_F - 4N$

$$\langle Q_{8g} \rangle_{\text{ren}} = Z_m Z_q Z_G^{1/2} Z_{88} \langle Q_{8g} \rangle_0$$

We add following counter term diagrams



# Final Result

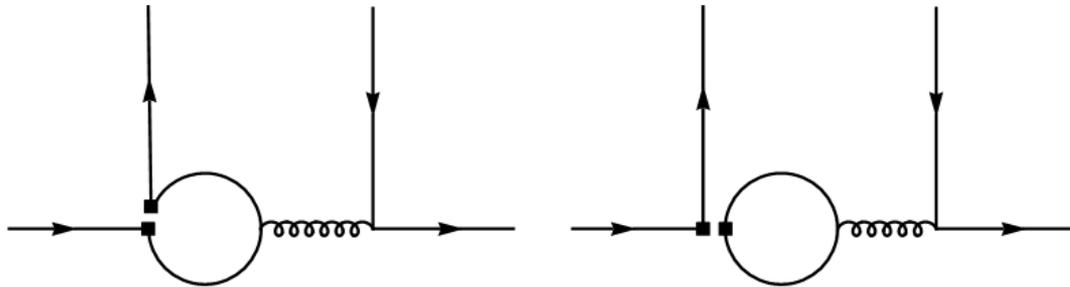
$$\mathcal{T}_{MP}^{(1)} = \frac{2}{\bar{u}}$$

$$\begin{aligned} u\bar{u}\mathcal{T}_{MP}^{(2)} = & \log\left(\frac{\mu^2}{m^2}\right) \left( C_F \left( \frac{2u}{3} - 4u \log(\bar{u}) \right) - \frac{4un_f}{3} + \frac{34u}{3N} \right) \\ & + i\pi \left( C_F \left( -\frac{4 \log(\bar{u})}{u^2} - 16\bar{u} - \frac{16\bar{u}^2 \log(\bar{u})}{u} + 20u \log(\bar{u}) + \frac{20 \log(\bar{u})}{u} - 32 \log(\bar{u}) \right. \right. \\ & \left. \left. - \frac{56u}{3} - \frac{4}{u} + 18 \right) \right) + \frac{1}{N} \left( -\frac{2 \log(\bar{u})}{u^2} - \bar{u} + 2u \log(\bar{u}) - \frac{16u}{3} - \frac{2}{u} - 2u \log(u) \right) \\ & + C_F \left( -\frac{2 \log^2(\bar{u})}{u^2} - \frac{8\bar{u}^2 \log^2(\bar{u})}{u} + 10u \log^2(\bar{u}) + \frac{10 \log^2(\bar{u})}{u} - 16 \log^2(\bar{u}) \right. \\ & \left. - \frac{62}{3} u \log(\bar{u}) - 18\bar{u} \log(\bar{u}) - \frac{4 \log(\bar{u})}{u} + 16 \log(\bar{u}) + \frac{88u}{9} - 2 \right) \\ & + \sum_f \left( -\frac{8uz_f J_1\left(\frac{\bar{u}}{z_f}\right)}{3\bar{u}} - \frac{4}{3} u J_1\left(\frac{\bar{u}}{z_f}\right) - \frac{16uz_f}{3\bar{u}} + \frac{4}{3} u \log(z_f) - \frac{20u}{9} \right) \\ & + \frac{1}{N} \left( 6u \text{Li}_2(\bar{u}) - \frac{\log^2(\bar{u})}{u^2} + \frac{2u^2 \log(u)}{\bar{u}} + 3u J_1(\bar{u}) - J_1(\bar{u}) - \frac{2y(\bar{u})}{u} - u \log^2(\bar{u}) \right. \\ & \left. - 4 \log^2(\bar{u}) - \frac{4\bar{u} \log^2(\bar{u})}{u} + \frac{4 \log^2(\bar{u})}{u} - \frac{2u \log(u)}{\bar{u}} + 4u \log(u) \log(\bar{u}) \right. \\ & \left. - \frac{16}{3} u \log(\bar{u}) + \bar{u} \log(\bar{u}) - \frac{2 \log(\bar{u})}{u} + J_2(\bar{u}) \left( \frac{2}{u} + 2 \right) + 2u \text{Li}_2(u) \right. \\ & \left. - \frac{2\pi^2 u}{3} + \frac{242u}{9} + u \log^2(u) \right) \end{aligned}$$

$$J_1(\bar{u}) = \frac{1 + y(\bar{u})}{1 - y(\bar{u})} \log(y(\bar{u})), \quad y(\bar{u}) = \frac{\sqrt{4 - \bar{u}} - \sqrt{-\bar{u}}}{\sqrt{4 - \bar{u}} + \sqrt{-\bar{u}}}, \quad J_2(\bar{u}) = \frac{\text{Li}_2(\bar{u})}{u} - \frac{\pi^2}{6u} - \int_0^1 d\xi \frac{\log(1 - \bar{u}\xi(1 - \xi))}{(1 - \xi)(1 - \bar{u}\xi)}$$

# NNLO for Penguin

- Part 2 - Penguin (Im) - *Working in Progress*



We use CMM basis  
to avoid trace with  $\gamma_5$

Chetyrkin, Misiak, Muns, NPB, (1998)

$$Q_1 = (\bar{s}_L \gamma_\mu T^a c_L) (\bar{c}_L \gamma^\mu T^a b_L),$$

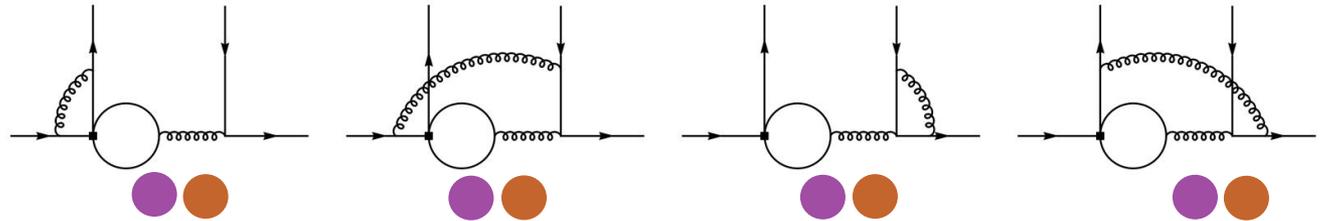
$$Q_2 = (\bar{s}_L \gamma_\mu c_L) (\bar{c}_L \gamma^\mu b_L),$$

$$Q_3 = (\bar{s}_L \gamma_\mu b_L) \sum_q (\bar{q} \gamma^\mu q),$$

$$Q_4 = (\bar{s}_L \gamma_\mu T^a b_L) \sum_q (\bar{q} \gamma^\mu T^a q),$$

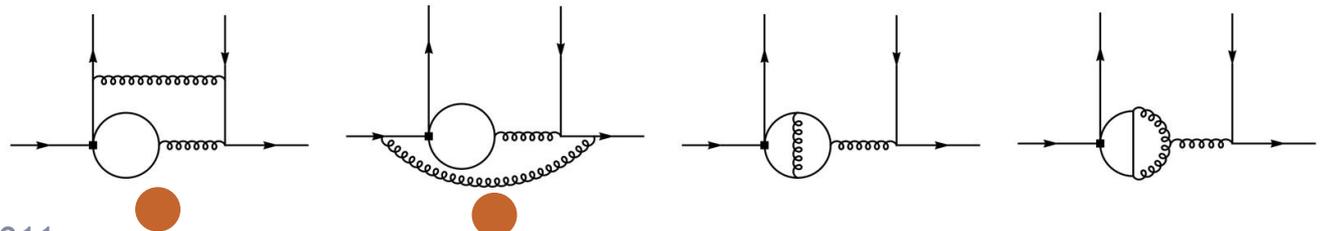
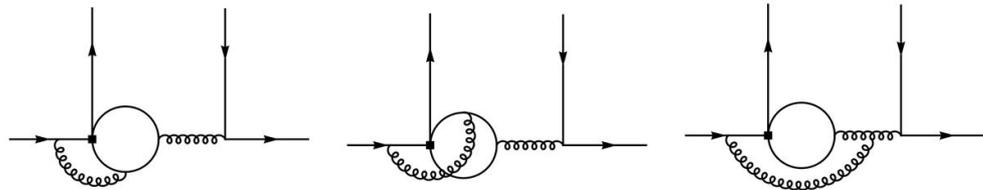
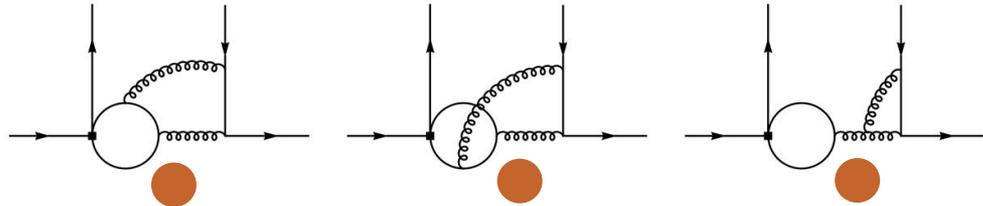
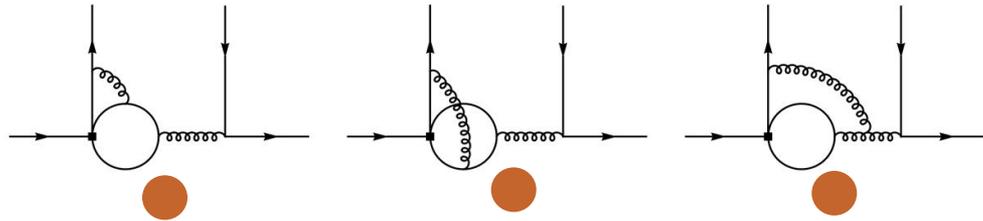
$$Q_5 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L) \sum_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\rho q),$$

$$Q_6 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho T^a b_L) \sum_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\rho T^a q),$$



Similarly, we confirmed  
all the soft, collinear div.  
are canceled.

- Soft div
- Collinear div



# Technical comments

- We follow typical method:



→ 32 Master Integrals are collected

- Imaginary part is 1 order simple :

$$\begin{array}{c}
 \text{---} q^2 \text{---} \bigcirc \text{---} \\
 \text{---} m \text{---} \\
 \text{---} m \text{---}
 \end{array}
 = - \int_0^1 \log(1 + x\xi(1 - \xi))
 \begin{array}{l}
 = 2 + \frac{1 + y(x)}{1 - y(x)} \ln(y(x)) \\
 \text{Im} \searrow \\
 = -i\pi \sqrt{\frac{4+x}{x}}
 \end{array}$$

# Summary

- We have computed NNLO penguin diagrams in B to light mesons decays.
  - Magnetic Penguin Op. : finished
  - Penguin Op. (Im): in progress
- The IR divergence cancelation is explicitly shown → validation of QCDF framework.
- This work would provide more stringent test for SM in current and future B factories.