

# **Neutrino Oscillation and New Physics in Future Experiments**

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# Outline

**Introduction**

**Neutrino oscillation beyond the SM**

**Neutrino states for NSI, entalgement**

**Conclusions**

## Introduction

Currently, the vSM is used for the analysis of neutrino oscillation data,

where

- ✿ **neutrinos interact through a left-handed currents,**
- ✿ **mixing between the flavour and the mass of neutrino states is unitary,**
- ✿ **only extremely relativistic neutrino are produced and detected,**

so

In the processes of production and detection of neutrinos, lepton flavour number violation (**LFV**) can be safely ignored

Then

There is no entanglement between internal neutrinos degree of freedom (**spin, mass, flavour**) in the production and detection processes, and neutrinos (antineutrinos) states are pure quantum mechanical states

$$|\nu_\alpha \downarrow\rangle = \sum_i U_{\alpha i}^* |\nu_i \downarrow\rangle$$

$$|\bar{\nu}_\beta \uparrow\rangle = \sum_i U_{\beta i} |\bar{\nu}_i \uparrow\rangle$$

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To observe neutrino oscillation

Kinematical disentanglement is necessary to have coherence for neutrino production and detection processes,

(Neutrino energy and momentum states in the production and detection processes are not entangled with energy and momentum states of other accompanying particles)

# But, NSI can change the neutrino propagation in a matter

L. Wolfenstein, Phys. Rev. D17, 2369 (1978).

## NSI can change also the neutrinos production and detection states

Y. Grossman, Phys. Lett. B359, 141 (1995).  
M. Blennow, T. Ohlsson and J. Skrotzki, Phys. Lett. B660, 522 (2008).  
T. Ohlsson and H. Zhang, Phys. Lett. B671, 99 (2009).  
M. Blennow, T. Ohlsson and W. Winter, Eur. Phys. J. C 49, 1023 (2007).  
M. Ochman, R. Szafron and M. Z., J.Phys. G35, 065003 (2008).

### Generally NSI can:

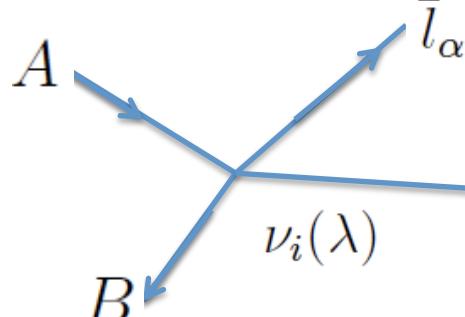
- ✿ In addition to **the left-handed**, also **the right-handed currents** can describe the processes of neutrino production and detection,
- ✿ Neutrino mixing matrix for light neutrinos can not be **unitary**, **sterile neutrinos** can exist,
- ✿ Open **LFV** and / or **LVN** (Lepton Number Violation) interaction exist

T. Ohlsson, T. Schwetz, and H. Zhang, Phys.Lett.B 681, 269 (2009).  
N. A. Ky and N. T. H. Van, Phys.Rev.D72 , 115017 (2005).  
P. Duka, J. Gluza, and M. Z., Annals Phys. 280, 336 (2000).  
R. Barbier et al., Phys. Rept. 420, 1 (2005).

# Neutrino oscillation beyond the SM

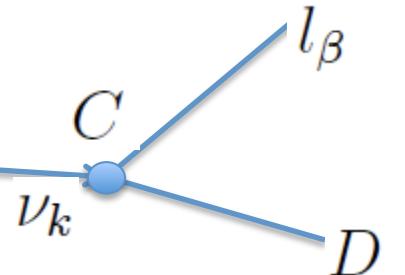
M.Ochman, R.Szafron, M. Z.,  
[arXiv:1012.4123 \[hep-ph\]](https://arxiv.org/abs/1012.4123)

$$A \rightarrow B + \bar{l}_\alpha + \nu_i(\lambda)$$



$$M_{i,\lambda}^{P,\alpha}([\bar{p}], [\bar{\lambda}])$$

$$\nu_k(\mu) + C \rightarrow l_\beta + D$$



$$M_{k,\mu}^{D,\beta}([\bar{q}][\bar{\mu}])$$

$$iA_{\alpha\beta}(\lambda, \mu; [\bar{p}], [\bar{\lambda}], [\bar{q}], [\bar{\mu}]) =$$

$$\Theta(T) \sum_{k=1,2,3} M_{k,\lambda}^{P,\alpha}([\bar{p}], [\bar{\lambda}]) M_{k,\mu}^{D,\beta}([\bar{q}], [\bar{\mu}]) \int \frac{d^3 p}{(2\pi)^3} F_{P,k}(E_k(\vec{p}), \vec{p}) F_{D,k}(E_k(\vec{p}), \vec{p}) e^{-iE_k(\vec{p})T + i\vec{p}\vec{L}}$$

E. Kh. Akhmedov and J. Kopp,  
JHEP 04, 008 (2010)

$$F_{P,k}(E_k(\vec{p}), \vec{p}) \quad F_{D,k}(E_k(\vec{p}), \vec{p})$$

Functions which describe the momenta distributions for all particles in the production and detection processes

The neutrino states are calculated from the production amplitudes

$$\varrho_{\lambda,i;\eta,k}^{\alpha}(E,\theta,\varphi) = \frac{1}{N_{\alpha}} \sum_{\lambda_A, \lambda_{A'}, \lambda_B, \lambda_l} \int \overline{dLips} M_{i,\lambda}^{P,\alpha}(\lambda_A; \lambda_B, \lambda_l; E, \theta, \varphi) \varrho_{\lambda_A, \lambda_{A'}} M_{k,\eta}^{P,\alpha*}(\lambda_{A'}; \lambda_B, \lambda_l; E, \theta, \varphi),$$

The final cross section is calculated from the detection amplitudes and the initial density matrix

$$\sigma_{\alpha \rightarrow \beta}(E_b, L) = \frac{1}{32\pi s} \frac{|\vec{p}_f|}{|\vec{p}_i|} \frac{1}{2s_C + 1} \sum_{i,k;\lambda,\eta;[\bar{\mu}]} \int dLips M_{i,\lambda}^{D,\beta}(\vec{p}_f, [\bar{\mu}]) \varrho_{i,\lambda;k,\eta}^{\alpha}(E_b) e^{i \frac{(m_k^2 - m_i^2)L}{2E_b}} M_{k,\eta}^{D,\beta*}(\vec{p}_f, [\bar{\mu}])$$

# Neutrino states for NSI

R. Szafron and M. Z.  
[arXiv:1010.6034 \[hep-ph\]](https://arxiv.org/abs/1010.6034)

The decay of pions, nuclei, or muons are in the lowest order described by the d=6 effective Lagrangian

$$\mathcal{L}_{\text{NSI}} = -2\sqrt{2}G_F \sum_{\substack{\Delta=S,V,T \\ \varepsilon,\eta=L,R}} \varepsilon_{\varepsilon,\eta}^{\Delta} (\bar{\psi}_a \Gamma^{\Delta} P_{\varepsilon} \psi_b) (\bar{\psi}_c \Gamma_{\Delta} P_{\eta} \psi_d) + h.c.$$

where the field  $\Psi_a$  ( $\Psi_b$ ) describes the produced neutrino (antineutrino).

If only left-handed neutrino fields  $\Psi_a$ , are present, five couplings are not vanishing:  $\varepsilon_{R,R(L)}^S$ ,  $\varepsilon_{L,R(L)}^V$  and  $\varepsilon_{R,L}^T$  and

If two left-handed neutrinos appear (e. g., in addition  $d = v$ , such as in muon decay) then only:  $\varepsilon_{R,L}^S, \varepsilon_{L,L}^V \neq 0$

NSI described by the effective Lagrangian for two left-handed neutrinos:

$$\mathcal{L}_{\text{NSI}} = -2\sqrt{2}G_F \sum_{\substack{\alpha, \beta = e, \mu, \tau \\ C = L, R}} \varepsilon_{\alpha\beta}^{ff'C} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta) (\bar{f} \gamma_\mu P_C f') + h.c.$$

As an example of neutrino production let us consider the muon decay process

$$\mu^- \rightarrow e^- \nu_i \bar{\nu}_j$$

After Fierz rearrangement

$$\mathcal{L}_\mu = -2\sqrt{2}G_F [g_{ij}^S (\bar{\nu}_i P_R \mu) (\bar{e} P_L \nu_j) + g_{ij}^V (\bar{\nu}_i \gamma^\alpha P_L \mu) (\bar{e} \gamma_\alpha P_L \nu_j)] + h.c.$$

With the SM part

$$\varepsilon_{\alpha\beta}^{e\mu L} \Rightarrow G_{\alpha\beta} = \delta_{\alpha\mu}\delta_{e\beta} + \varepsilon_{\alpha\beta}^{e\mu L}$$

$$g_{ij}^V = - \sum_{\alpha,\beta} U_{\alpha i}^* \varepsilon_{\alpha\beta}^{e\mu L} U_{\beta j} - U_{\mu i}^* U_{e j}$$

$$g_{ij}^S = 2 \sum_{\alpha,\beta} U_{\alpha i}^* \varepsilon_{\alpha\beta}^{e\mu R} U_{\beta j}$$

$$1) \quad \varepsilon_{\alpha\beta}^{e\mu L} \neq 0 \quad \varepsilon_{\alpha\beta}^{e\mu R} = 0$$

We can calculate the production two neutrino density matrix, only L neutrinos

$$\varrho(i,j;k,l) = \chi_{i,j}^* \chi_{k,l}$$

$$\chi = -\frac{W}{N}, \quad W = U^+ G U, \quad N = \sqrt{\text{Tr}(WW^+)}$$

State of two neutrinos is pure Quantum Mechanical state

$$\boxed{\text{Tr}[\rho^2] = 1} \xrightarrow{\text{blue arrow}} \boxed{|\nu\bar{\nu}'\rangle = \sum_{i,j} (\chi_{i,j})^* |\nu_i\bar{\nu}_j\rangle = \frac{1}{N} (|\nu_\mu\bar{\nu}_e\rangle + \sum_{\alpha,\beta} (\varepsilon_{\alpha\beta}^{e\mu L})^* |\nu_\alpha\bar{\nu}_\beta\rangle)}$$

To get one particle states

$$\varrho^\nu(i; k) = \sum_j \varrho(i, j; k, j)$$

$$\varrho^{\bar{\nu}}(j; l) = \sum_i \varrho(i, j; i, l)$$

$$\varrho^\nu(i; k) = \frac{(W^\dagger W)_{ik}^*}{\text{Tr}(WW^\dagger)}, \quad \varrho^{\bar{\nu}}(j; l) = \frac{(W^\dagger W)_{jl}}{\text{Tr}(W^\dagger W)},$$

Now in both cases, for neutrino and for antineutrino

$$Tr(\varrho^2) = \frac{Tr(A^2)}{(Tr(A))^2} \neq 1 , \quad A = WW^\dagger$$

Neutrino and antineutrino states are pure, if

$$Tr(A^2) = (TrA)^2$$

Independently whether the matrix U is unitary or not,  
the quantum states of the neutrinos are pure, if

$$G_{\alpha\beta} = \delta_{\alpha\mu}\delta_{e\beta} + \varepsilon_{\alpha\beta}^{e\mu L}$$

$\varepsilon_{\alpha\beta}^{e\mu L} = 0$   
 $\varepsilon_{\alpha\beta}^{e\mu L} \sim \delta_{\beta e}\varepsilon_\alpha$   
 $\varepsilon_{\alpha\beta}^{e\mu L} \sim \delta_{\alpha\mu}\varepsilon_\beta$

# In models with two left-handed neutrinos

- 1) In models with heavy neutrinos, U matrix is not unitary – standard approach is correct, if there are no new LFV interaction
- 2) In models with an open LFV – standard approach is correct, if

Parametrization of G matrix

$$\begin{pmatrix} G_{ee} & G_{e\mu} & G_{e\tau} \\ G_{\mu e} & G_{\mu\mu} & G_{\mu\tau} \\ G_{\tau e} & G_{\tau\mu} & G_{\tau\tau} \end{pmatrix}$$

Only one column, e.g.

$$G_{\alpha\beta} = \epsilon_\alpha \delta_{\beta e}$$

$$\begin{pmatrix} \times & 0 & 0 \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

Only one row, e.g.

$$G_{\alpha\beta} = \delta_{\alpha\mu} \epsilon_\beta$$

$$\begin{pmatrix} 0 & 0 & 0 \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}$$

Standard approach must be modified, if two rows and two columns have not zero elements e.g.

$$\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- A. Zee, Phys. Lett. B93, 389 (1980).
- A. Zee, Phys. Lett. B161, 141 (1985).
- A. Zee, Nucl. Phys. B264, 99 (1986).
- K. S. Babu, Phys. Lett. B203, 132 (1988).

$$2) \quad \varepsilon_{\alpha\beta}^{e\mu L} \neq 0, \quad \varepsilon_{\alpha\beta}^{e\mu R} \neq 0$$

In models with charged scalar boson e.g.

H. E. Logan and D. MacLennan  
[Phys.Rev.D79, 115022 \(2009\).](#)

$$\varrho = \frac{1}{\overline{N}} (Bf(E) + Cg(E) - 2Re[(g^V)^T (g^S)^*]h(E))$$

$$B = (g^V)^T (g^V)^*$$

$$C = (g^S)^T (g^S)^*$$

$$f(E) = 6M - 8E + O(\frac{m_e}{E})$$

$$g(E) = 3(M - 2E) + O(\frac{m_e}{E})$$

$$h(E) = O(\frac{m_e}{E})$$

$$\overline{N} = f(E)TrB + g(E)TrC$$

$$Tr(\varrho^2) = \frac{1}{\overline{N}^2} (f^2(E)Tr(B^2) + g^2(E)Tr(C^2) + 2f(E)g(E)Tr(BC))$$

Then the neutrino state is pure ( $Tr(\rho^2) = 1$ ) , if

$$Tr(B^2) = (TrB)^2$$

$$Tr(C^2) = (TrC)^2$$

$$Tr(BC) = Tr(B)Tr(C)$$

The criteria are very similar to those that were previously considered

### 3) One left-handed neutrino (pion decay, nuclear beta decay)

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$A \rightarrow B + e^+ + \nu_e$$

Generally 5 parameters:

$$\mathcal{E}_{R,R(L)}^S, \mathcal{E}_{R,R(L)}^V, \text{ and } \mathcal{E}_{R,L}^T$$

Without tensor operators for antineutron beta decay

$$\bar{n} \rightarrow \bar{p} + e^+ + \nu_e \Leftrightarrow p \rightarrow n + e^+ + \nu_e$$

$$\mathcal{L}_\mu = -2\sqrt{2}G_F \left[ g_i^{S_{L,R}} (\bar{\nu}_i P_R e) (\bar{n} P_{L,R} p) + g_i^{V_{L,R}} (\bar{\nu}_i \gamma^\alpha P_L e) (\bar{n} \gamma_\alpha P_{L,R} p) \right] + h.c.$$

If the couplings have a similar structure as before, e.g.,

$$g_i^{S_L} = \sum_\alpha g_\alpha^{S_L} U_{\alpha i}^*$$

The decay amplitudes:

$$f(\lambda_n, \lambda_p, \lambda_e, \nu_i) = \sum_{\alpha, \Delta} U_{\alpha i}^* g_{\alpha}^{\Delta} f_{\Delta}(\lambda_n, \lambda_p, \lambda_e)$$

The left-handed neutrino density matrix:

$$\rho_{i,k} = \frac{1}{N} \sum_{\alpha\beta} U_{i\alpha}^+ U_{k\beta}^T A_{\alpha\beta} = \frac{1}{N} (U^+ A U)_{ik}$$

$$A_{\alpha\beta} = \sum_{\lambda, \Delta, \Omega} g_{\alpha}^{\Delta} g_{\beta}^{\Omega*} f_{\Delta}(\lambda) f_{\Omega}^*(\lambda)$$

where:

$$N = \text{Tr}(U^+ A U)$$

Neutrino state is pure QM state, if

$$Tr(\rho^2) = \frac{1}{N^2} Tr(U^+ A U U^+ A U) = 1$$

so

$$N^2 \equiv (Tr(W))^2 = Tr(W^2)$$

where

$$W = U^+ A U$$

If there is only one spin amplitude, like in pion decay

$$A_{\alpha, \beta} = c_\alpha c_\beta^*$$

where

$$c_\alpha = \sum_\Delta g_\alpha^\Delta f_\Delta$$

then

$$Tr(\rho^2) = 1$$

For a richer structure of spin amplitudes (as in nuclear beta decay) without explicit lepton flavour number violation

$$g_{\alpha}^{\Delta} = \delta_{\alpha e} \epsilon^{\Delta}$$

Then

$$A_{\alpha\beta} = a \delta_{\alpha e} \delta_{\beta e}$$

And, once again

$$Tr(\rho^2) = 1$$

## Conclusions

1. Without NSI (SM interaction and massive neutrinos) intrinsically (mass, flavour, spin) particles and neutrinos states are not entangled, and for this reason, neutrino states are pure
2. If NSI describes the processes of production and detection, neutrinos and particles accompanying them become entangled, then the neutrino states are mixed
  - a. If there is only one helicity amplitude which describes the neutrino production process (as in a pion decay) neutrinos are always in pure QM state (their states are not entangled),
  - b. Entanglement between the states of massive neutrinos and other particles is a consequence of the explicit LFN violation.
  - c. The LFV violation caused by the coupling with sterile neutrinos does not cause an entanglement

3. In the future neutrino oscillation experiments (neutrino factory - muon decay or beta beam - neutrino from nuclear beta decay) we must remember that in the case of NSI the known approach is only the first approximation.