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Effective Potential Analysis for Unified Symmetry Breaking

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Outline

- Unified Symmetry Breaking
- Effective Potential theory
- Effective Potential of SU(5)
- Analysis for Symmetry Breaking in SU(5)
- Summary

Unified Symmetry Breaking(Li)

Adjoint representation Φ in SU(n) can be taken as a diagonal form

$$\Phi = \{\phi_1, \phi_2, \ldots, \phi_n\}$$

with $\sum_{i=1}^{n} \phi_i = 0$

It has invariant potential

$$V = -\frac{1}{2}\mu^2 Tr\Phi^2 + \frac{\lambda_1}{4}(Tr\Phi^2)^2 + \frac{\lambda_2}{4}(Tr\Phi^4)$$

Li proved that the possible symmetry-breaking patterns

$$\begin{cases} \lambda_2 > 0 : & SU(n) \rightarrow SU(n_1) \times SU(n-n_1) \\ \lambda_2 < 0 : & SU(n) \rightarrow SU(n-1) \end{cases} \quad \text{where} \begin{cases} n_1 = \frac{1}{2}n & n \text{ even} \\ n_1 = \frac{1}{2}(n+1) & n \text{ odd} \end{cases}$$

Unified Symmetry Breaking(Ruegg)

No extra symmetry $\Phi \rightarrow -\Phi$

one more term in potential

$$+ d(Tr\Phi^3)$$

Ruegg proved the lemma: V admits extrema only if at most two eigenvalues a_i are different

at tree level:

$$SU(5) \longrightarrow SU(2) \times SU(3)$$

Question?

Possible improvement: radiative corrections (effective potential)

Effective Potential theory

Lagrangian + **c**oupling to external sources

$$\mathcal{L}(\phi_i, \partial^\mu \phi_i) \longrightarrow \mathcal{L}(\phi_i, \partial^\mu \phi_i) + \sum \phi_i(x) J_i(x)$$

ϕ_i represents all the fields in the theory, whatever their spin.

1, The effective action

$$\Gamma[\Phi_c] = \sum_{n_1,\ldots,n_i,\ldots,n_k} \frac{1}{n_1!\ldots n_i!\ldots n_k!} \int (d^4x_1 \ldots d^4x_{n_1}) \ldots (d^4w_1 \ldots d^4w_{n_k})$$
$$\times \Gamma^{(n_1,\ldots,n_i,\ldots,n_k)}(x_1,\ldots,x_{n_1},\ldots,w_1,\ldots,w_{n_k})$$
$$\times [\phi_{1c}(x_1)\ldots\phi_{1c}(x_{n_1})]\ldots[\phi_{kc}(w_1)\ldots\phi_{kc}(w_{n_k})]$$

$\Gamma^{(n_1,\ldots,n_i,\ldots,n_k)}(x_1,\ldots,x_{n_1},\ldots,w_1,\ldots,w_{n_k})$ 1PI Green's functions , no propagators on the external lines

2, Taylor series expansion about the constant value φ_{ic} of the classical field ϕ_{ic}

$$\Gamma[\phi_c] = \int (d^4x)\{-V(\varphi_c) + \frac{1}{2}\partial_\mu\phi_c(x)\partial^\mu\phi_c(x)Z(\varphi_c)$$
$$+ \partial_\mu\phi_c(x)A_c^\mu(x)\phi_c(x)G(\varphi_c) + [\phi_c(x)]^2[A_c^\mu(x)]^2 H(\varphi_c) + \ldots\}$$

So the effective potential is $V(\varphi_c)$:the sum of all Feynman diagrams with only external scalar lines and with vanishing external momentum.

Effective Potential theory

How to calculate the effective potential

1st order: classical potential

2nd order: Sum of all 1PI diagram with a single loop and with zero external momenta

Eg: A single, real massless scalar field

$$V^{(0)} = g\frac{\phi_c^n}{n!}$$

$$V^{(1)} = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} \ln\left(1 + \frac{V^{(0)''}(\phi_c)}{k^2}\right)$$

divergent, cut at $k^2 = \Lambda^2$

$$V^{(1)} = \frac{1}{64\pi^2}(V^{(0)''})^2 \ln\left(\frac{V^{(0)''}}{\Lambda^2}\right) + \frac{\Lambda^2}{32\pi^2}V^{(0)''}$$

Effective Potential theory

Generally

Contribution from the scalar particles:

$$V_S^{(1)}(\overline{\varphi}) = \frac{1}{64\pi^2} Tr\left(W^2 \ln \frac{W}{\Lambda^2} \right)$$

Where $W^{ij} = \frac{\partial^2 V^{(0)}}{\partial \phi_i \partial \phi_j}\Big|_{\phi=\phi_c}$

Contribution from gauge bosons:

$$V_g^{(1)}(\overline{\varphi}) = \frac{3}{64\pi^2} Tr\left(M^4 \ln \frac{M^2}{\Lambda^2} \right)$$

where $M^2_{\alpha\beta} = g_\alpha g_\beta (T_a \phi_i, T_\beta \phi_i)$

Contribution from fermions:

Yukawa coupling $\sum_{a,b,i} \overline{\psi}^a \Gamma^i_{ab} \phi_i \psi^b$

$$V_f^{(1)}(\phi) = -\frac{4}{64\pi^2} Tr_f\left[(\phi\Gamma)^4 \ln \frac{(\phi\Gamma)^2}{\Lambda^2} \right]$$

Effective Potential of SU(5)

Adjoint represention in SU(5)

$$\Phi = \Phi^{\dagger}$$

Classical potential

$$V_0 = -\mu^2 Tr\Phi^2 + dTr\Phi^3 + \lambda_1 Tr\Phi^4 + \lambda_2 (Tr\Phi^2)^2$$

we write it as diagonal form

$$\Phi = \{\phi_1, \phi_2, \phi_3, \phi_4, \phi_5\}$$

So the contribution from the scalar particles

$$W_{i,j}(\overrightarrow{\varphi_c}) = \frac{\partial^2 V_0}{\partial \phi_i \partial \phi_j}$$

a,b=1,...,5

$$V_S^{(1)}(\overline{\varphi}) = \frac{1}{64\pi^2} Tr\left(W^2 \ln \frac{W}{\Lambda^2} \right)$$

Effective Potential of SU(5)

Contribution from gauge bosons:

n^2-1 vector gauge bosons W_μ^a

The interaction comes from $D_\mu \Phi^\dagger D_\mu \Phi$

under SU(n) $D_\mu \Phi = \partial_\mu \Phi - ig[W_\mu, \Phi]$

Then $M^2(\Phi)_{ab} = -2g^2 Tr([\Phi, T_a][\Phi, T_b])$

$M^2(\Phi)_{ab}$ is diagonalized matrix $\ln[M^2(\Phi)_{ab}] = [\ln M_{ab}^2]$

$$V_g = \frac{3}{64\pi^2} 16 g^4 \left((\phi_1 - \phi_2)^4 \ln \frac{(\phi_1 - \phi_2)^2}{\frac{M^2}{4g^2}} + \text{cyclic permutations} \right)$$

And

$$(\phi_1 - \phi_2)^4 + \text{cyclic permutations} = 5 Tr\Phi^4 + 3(Tr\Phi^2)^2$$

$$V_g \propto \{(\phi_1 - \phi_2)^4 \ln(\phi_1 - \phi_2)^2 + \text{cyclic permutations}\}$$

Effective Potential of SU(5)

Contribution from fermions

In Yukawa interaction $\overline{m}(\Phi) = g'\Phi$

$$\Rightarrow \quad \overline{m}(\Phi)\overline{m}(\Phi)^\dagger = g'^2\Phi\Phi^\dagger, (\overline{m}(\Phi)\overline{m}(\Phi)^\dagger)^2 = g'^4(\Phi\Phi^\dagger)^2$$

$$V_f \propto \phi_1^4 \ln \phi_1^2 + \phi_2^4 \ln \phi_2^2 + \phi_3^4 \ln \phi_3^2 + \phi_4^4 \ln \phi_4^2 + \phi_5^4 \ln \phi_5^2$$

Total 1-loop effective potential

$$\begin{aligned}
V &= V_0 + V_s^{(1)} + V_f^{(1)} + V_g^{(1)} + V_c \\
&= -\mu^2 Tr\Phi^2 + dTr\Phi^3 + \lambda_1 Tr\Phi^4 + \lambda_2 (Tr\Phi^2)^2 \\
&\quad + \frac{1}{64\pi^2} Tr\left(W^2 \ln \frac{W}{\Lambda^2}\right) \\
&\quad + g_V\left[(\phi_1 - \phi_2)^4 \ln(\phi_1 - \phi_2)^2 + \text{cyclic permutations}\right] \\
&\quad + g_f(\phi_i^4 \ln \phi_i^2) \\
&\quad + \ldots
\end{aligned}$$

Analysis for Symmetry Breaking in SU(5)

For simplify, only $V_g^{(1)}, V_f^{(1)}$

Perturbation about results form tree level,

$$\phi_1 = \phi_2 = \phi_3, \phi_4 = \phi_5$$

Min condition:

$$\frac{\partial V}{\partial \phi_i} = 0$$

$$\frac{\partial^2 V}{\partial \phi_i^2} > 0, \det\left(\frac{\partial^2 V}{\partial \phi_i \partial \phi_j}\right) > 0$$

Connecting with tree level $V = V_0 + V^{(1)}$

V_0 already satisfies $\frac{\partial V_0}{\partial \phi_i} = 0$

$$\frac{\partial^2 V}{\partial \phi_i^2} > 0, \det\left(\frac{\partial^2 V}{\partial \phi_i \partial \phi_j}\right) > 0$$

Analysis for Symmetry Breaking in SU(5)

- Extreme condition $\frac{\partial V_0}{\partial \phi_i} + \frac{\partial V^{(1)}}{\partial \phi_i} = 0$

$$\frac{\partial V_0}{\partial \phi_1} + 2g_V(4(\phi_1 - \phi_4)^3 \ln(\phi_1 - \phi_4)^2 + 2(\phi_1 - \phi_4)^3) + g_f(4\phi_1^3 \ln \phi_1^2 + 2\phi_1^3) = 0$$

$$\frac{\partial V_0}{\partial \phi_4} - 3g_V(4(\phi_1 - \phi_4)^3 \ln(\phi_1 - \phi_4)^2 - 2(\phi_1 - \phi_4)^3) + g_f(4\phi_4^3 \ln \phi_4^2 + 2\phi_4^3) = 0$$

$$g_f(4\phi_1^3 \ln \phi_1^2 - 4\phi_4^3 \ln \phi_4^2) = g_f\left(4\phi_1^3 \ln \frac{\phi_1^2}{\phi_4^2} + 4(\phi_1^3 - 4\phi_4^3) \ln \phi_4^2\right)$$

$$(\phi_1 - \phi_4)[\ldots] - g_f\left(4\phi_1^3 \ln \frac{\phi_1^2}{\phi_4^2}\right) = 0$$

3 solutions are possible now!

- Min condition $\det\left(\frac{\partial^2 V}{\partial \phi_i \partial \phi_j}\right) > 0$

$$\frac{\partial^2 V^0}{\partial \phi_i \partial \phi_j} - g_V\left(6(\phi_i - \phi_j)^2 \ln(\phi_i - \phi_j)^2 + 7(\phi_i - \phi_j)^2\right) ?\ 0$$

$$\frac{\partial^2 V^0}{\partial \phi_i \partial \phi_i} + g_V n\left(12(\phi_i - \phi_j)^2 \ln(\phi_i - \phi_j)^2 + 14(\phi_i - \phi_j)^2\right) - g_f(12\phi_i^2 \ln \phi_i^2 + 14\phi_i^2) ?0$$

Summary

Directly broken is quite possible with 1-loop effective potential!

$$\begin{aligned} SU(5) &\rightarrow SU(3) \times U(1) \times U(1) \\ &\rightarrow SU(2) \times SU(2) \times U(1) \end{aligned}$$

Improvement: finite temperature