Threshold resummation for top-quark production

(and other coloured particles)

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(Based on M.Beneke, P.Falgari, CS,arXiv:1007.5414 [hep-ph]

M.Beneke, P.Falgari, S. Klein, CS, in progress)

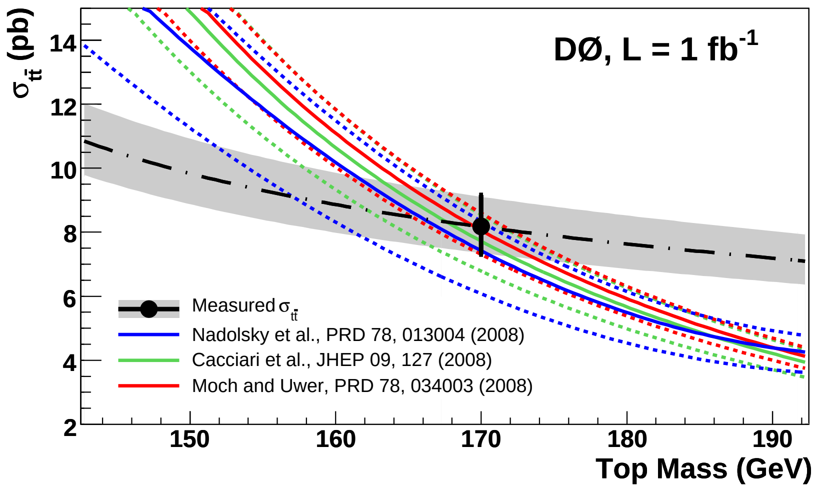
Pair production of heavy coloured particles at Tevatron/LHC

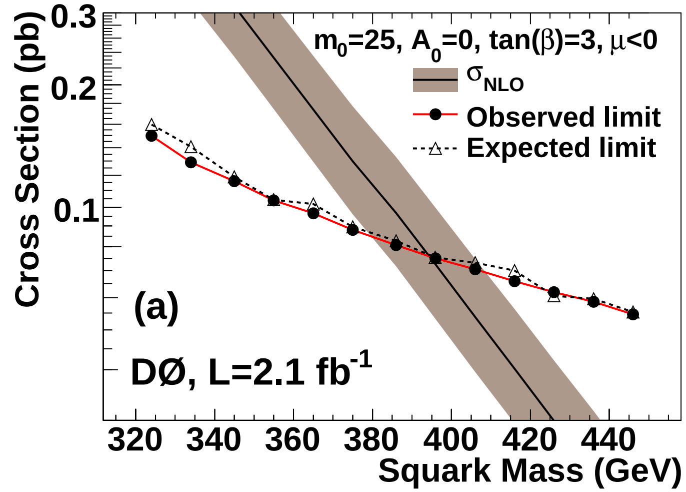
$$NN' \to HH' + X$$

- N, N': pp, $p\bar{p}$; HH': top-quark, squark, gluino... pairs

Precise knowledge of total cross sections:

- top-quarks: sensitivity on mass, constraining gluon PDFs
- new particles: Exclusion bounds, model discrimination,...

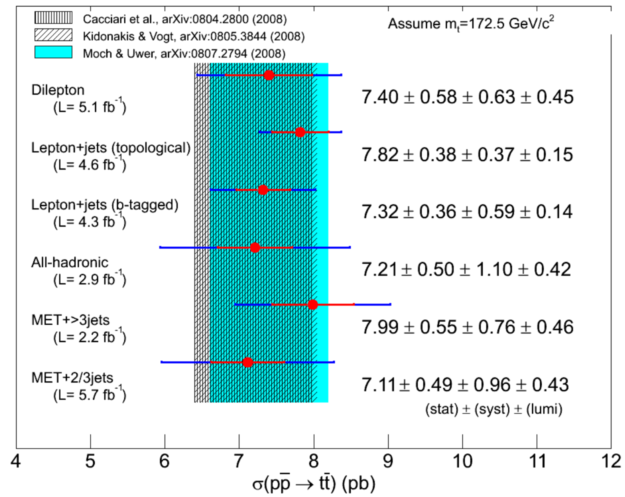




Experimental knowledge of $t\bar{t}$ cross section:

Tevatron: $\Delta\sigma_{t\bar{t}} = 6.8\%$;

LHC Goal: $\Delta\sigma_{t\bar{t}} \approx 5\%$



Theory status:

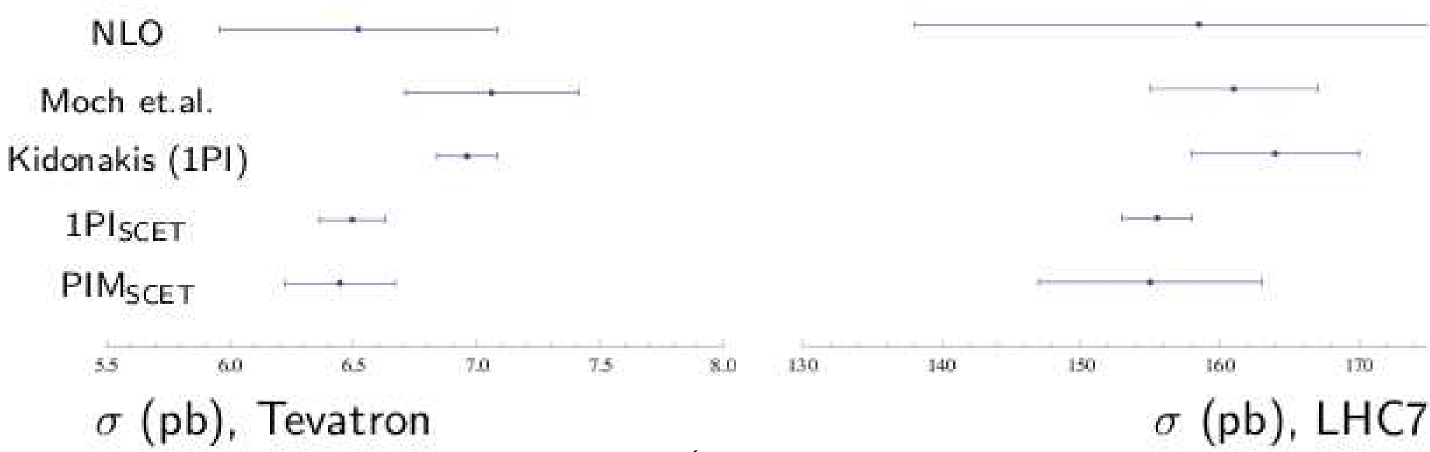
NLO + higher-order soft gluons

$\Rightarrow \Delta\sigma_{t\bar{t}} \approx 10\%$

NNLO:

in progress (Czakon et.al., Bonciani et.al.)

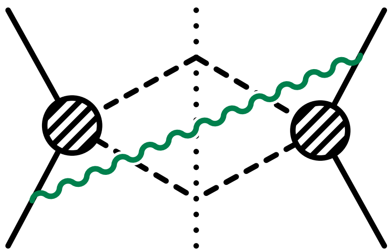
NNLO$_{\text{app.}}$ /NNLL (Moch/Uwer 08, Beneke et.al.; Ahrens et.al. 09-11, Kidonakis 10)



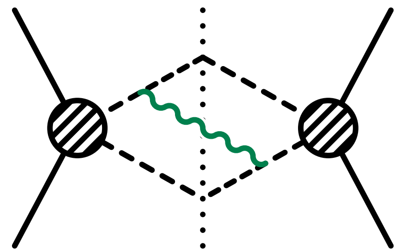
(B.Pecjak, Helmholtz Top-workshop Wuppertal 11)

Threshold resummation

Soft corrections: (Resummation in Mellin space: Sterman 87; Catani, Trentadue 89, Kidonakis,Sterman 97, Bonciani et.al. 98, ...)



$\Rightarrow \alpha_s \ \log^2(8\beta^2)$



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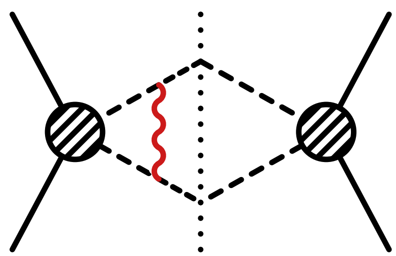
Coulomb gluon corrections (Fadin, Khoze 87; Peskin, Strassler 90, NRQCD,...)

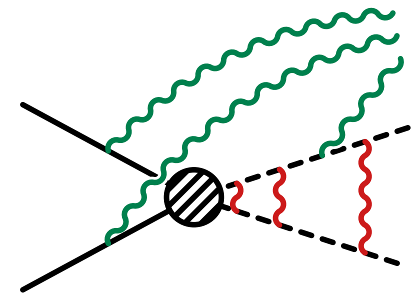


$\Rightarrow \alpha_s \ \frac{1}{\beta}$

Counting of threshold corrections:

$$\hat{\sigma}_{pp'} \quad \propto \quad \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \dots\Big]$$

$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \{1\,\text{(LL,NLL)}; \alpha_s, \beta\,\text{(NNLL)}; \dots\} :$$



Combination of Coulomb- and soft effects?

Heavy particles nonrelativistic near threshold:

$$E \sim m\beta^2 \,, \qquad |\vec{p}| \sim m\beta$$

soft gluon momenta of same order: $q_s \sim m\beta^2 \sim E$

$\Rightarrow$ heavy particles "feel" soft radiation

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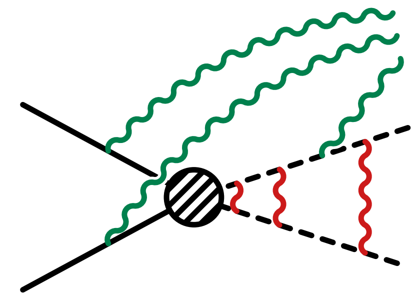
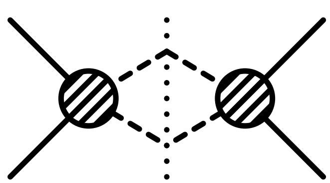
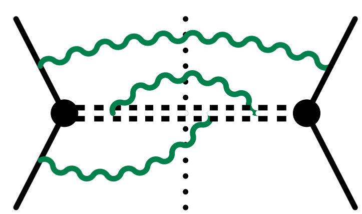
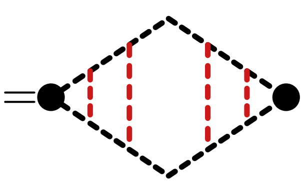


Factorization of cross section (Beneke, Falgari, CS 09/10)

$$\hat{\sigma}_{pp'\to HH'}|_{\hat{s}\to 4M^2} = \sum_{R,i} H_i W_i^R \otimes J^R$$

Hard, soft and Coulomb functions:

$H_i =$  , $W_i{}^R =$  , $J^R =$ 

Soft radiation "sees" only total colour charge R of heavy particles

(Singlet, octet,...)

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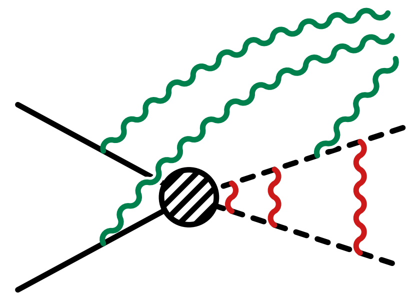

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$\Rightarrow$ heavy particles "feel" soft radiation

Factorization of cross section (Beneke, Falgari, CS 09/10)

$$\hat{\sigma}_{pp' \to HH'}|_{\hat{s} \to 4M^2} = \sum_{R,i} H_i W_i^R \otimes J^R$$

- disentangles hard, soft and Coulomb contribution

  (for S-wave production and up to NNLL)

- can perform simultaneous summation of threshold Logs and Coulomb corrections

Factorization scale dependence of H, W cancels against PDFs:

$$\frac{d\sigma}{d\mu} = \frac{d}{d\mu}\left(f_1 \otimes f_2 \otimes H \otimes W \otimes J\right) = 0$$

- $\frac{df_i}{d\mu} \Rightarrow$ Altarelli-Parisi equation (3-loop: Moch/Vermaseren/Vogt 04/05)
- $\frac{dH_i}{d\mu} \Rightarrow$ related to IR singlarities (2-loop: Becher, Neubert; Ferroglia et.al. 09)

$\Rightarrow$ RGE for soft function (NNLL: Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

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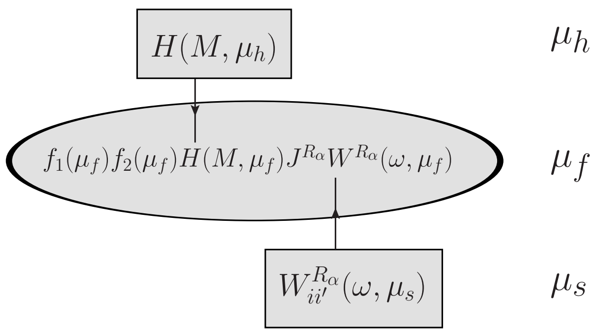
Resummation:

- evolve hard function from $\mu_h \sim 2m_t$ to μ_f
- evolve soft function from μ_s to μ_f (Mellin space: Korchemsky/Marchesini 92 momentum space: Becher/Neubert 06)



- (N)LO Coulomb-Green function (Fadin/Khoze 87; Beneke/Signer/Smirnov 99,. . .)

Estimate of uncertainties

- NNLL$_3$: Expand NNLL to $\mathcal{O}(\alpha_s^3)$
- Estimate of $\mathcal{O}(\alpha_s^2)$ constant in threshold expansion $C^{(2)} \approx (C^{(1)})^2$
- Ambiguity $E = \sqrt{\hat{s}} - 2m_t \approx m_t\beta^2$

Choice of soft scale Introduce β_{cut}

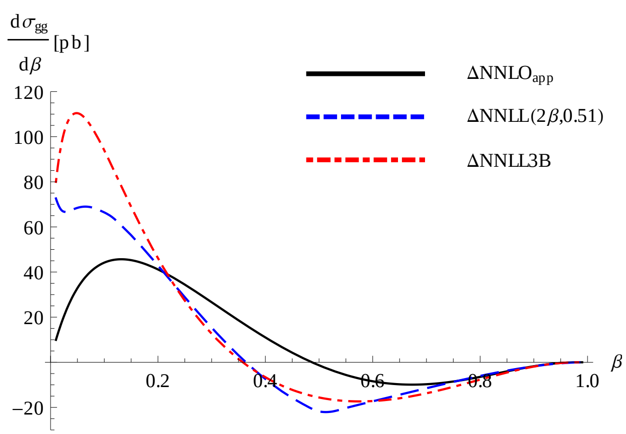
- allow for different implementations

  $\beta < \beta_{\text{cut}}$: NNLL $(\mu_s = k_s m_t \beta_{\text{cut}}^2)$ with/without constant at $\mathcal{O}(\alpha_s^2)$

  $\beta > \beta_{\text{cut}}$: NNLL $(\mu_s = k_s m_t \beta^2)$; NNLO$_{\text{approx}}$; NNNL$_3$

- Choose β_{cut} so that not too sensitive to
  - ambiguities for $\beta \to 1$
  - breakedown of perturbation theory for $\beta \to 0$

(E.g. LHC7: $\mu_s = 2m_t\beta^2$, $\beta_{\text{cut}} = 0.51$)



Results for $t\bar{t}$ production

(Beneke, Falgari, Klein, CS preliminary)

$\sigma_{t\bar{t}}$(pb)	Tevatron	LHC7
NLO	$6.50^{+0.32+0.33}_{-0.70-0.24}$	150^{+18+8}_{-19-8}
$\text{NNLO}_{\text{app}}(\beta)$	$7.10^{+0.0+0.36}_{-0.26,-0.26}$	162^{+2+9}_{-3-9}
NNLL	$7.24^{+0.48+0.37}_{-0.57-0.27}$	161^{+9+9}_{-9-9}

($m_t = 173.1$ GeV, $\mu_f = mt$, MSTW08NNLO)

Error breakedown for NNLL (added in quadrature above):

- Scale variation (μ_f, μ_h, μ_C):

$$\Delta_\mu \sigma_{\text{NNLL}}(\text{TeV}) = \begin{matrix} +0.21 \\ -0.36 \end{matrix}, \quad \Delta_\mu \sigma_{\text{NNLL}}(\text{LHC7}) = \begin{matrix} +1 \\ -1 \end{matrix}$$

- Uncertainty in resummation procedure:
  (vary β_{cut} by 20%, envelope of various approximations, ambiguity $E \leftrightarrow m_t\beta^2$)

$$\Delta_{\text{Res}} \sigma_{\text{NNLL}}(\text{TeV}) = \begin{matrix} +0.20 \\ -0.21 \end{matrix}, \quad \Delta_{\text{Res}} \sigma_{\text{NNLL}}(\text{LHC7}) = \begin{matrix} +4 \\ -5 \end{matrix}$$

- Estimate of missing constant at $\mathcal{O}(\alpha_s^2)$

$$\Delta_{\text{Const}} \sigma_{\text{NNLL}}(\text{TeV}) = \pm 0.38\,, \quad \Delta_{\text{Const}} \sigma_{\text{NNLL}}(\text{LHC7}) = \pm 7$$

Squark -antisquarks at LHC

- Two production channels:

$$q_i \bar{q}_j \longrightarrow \tilde{q}_k \overline{\tilde{q}_l} \quad , \qquad gg \longrightarrow \tilde{q}_k \overline{\tilde{q}_l}$$

- Simplified setup: equal squark masses, no stop
- Matching to NLO result (Beenakker et.al. 96, PROSPINO)

Resummed Results:

NLL: full Coulomb $\otimes$ res. soft

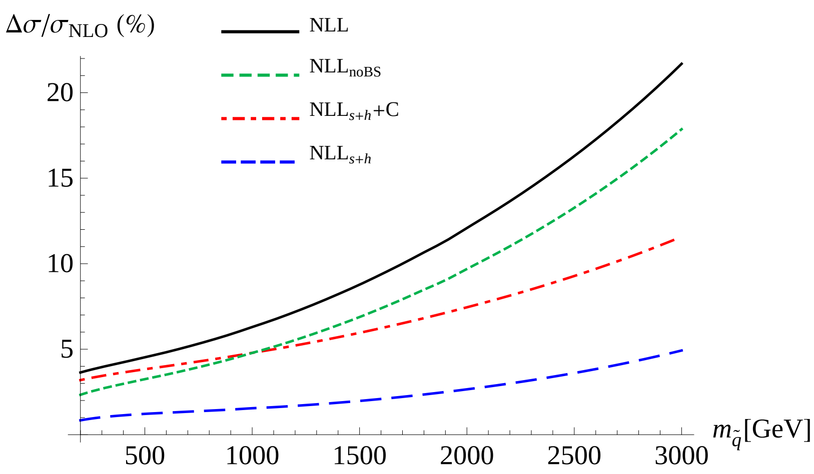
noBS:

NLL without bound states

NLL$_{\mathbf{s+h}}$:

resummation of H and W

C: Coulomb resummation



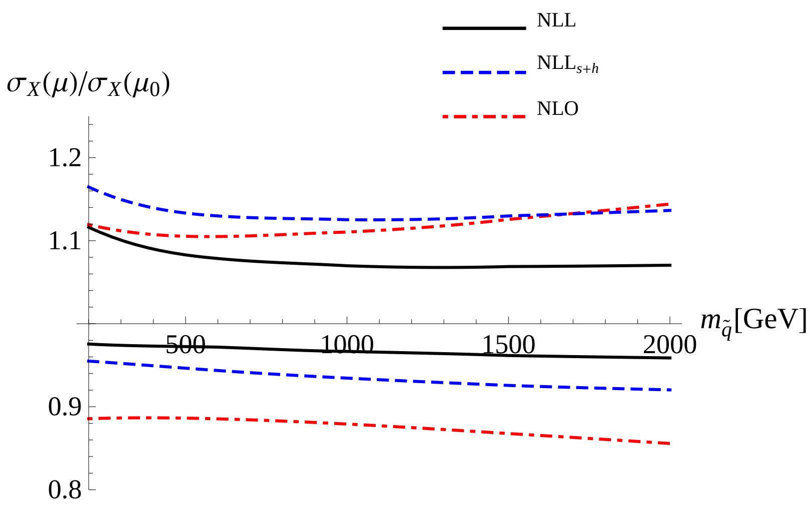
($\sqrt{s} = 14$ TeV, $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$MSTW08NLO)

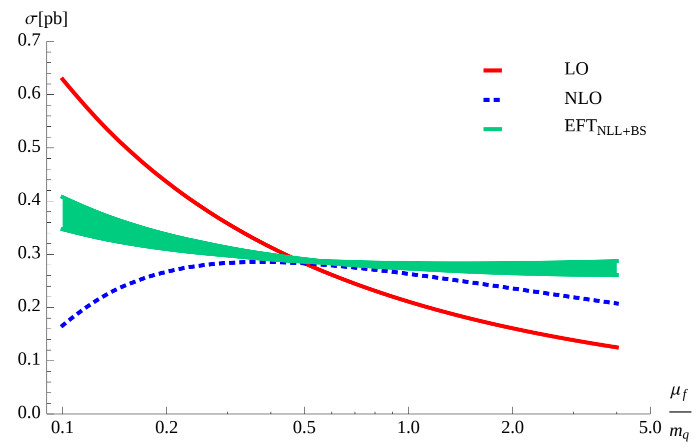
Scale uncertainty reduced by combined resummation

NLO $\frac{m_{\tilde{q}}}{2} < \mu_f < m_{\tilde{q}}$

NLL: vary all scales $\frac{\tilde{\mu}_i}{2} < \mu_i < 2\tilde{\mu}_i$, add in quadrature

$\Rightarrow$ significant reduction for combined resummation!





$(\sqrt{s} = 14$ TeV, MSTW08NLO, $m_{\tilde{g}}/m_{\tilde{q}} = 1.25)$

$(m_{\tilde{q}} = 1$ TeV, $\mu_s^0/2 < \mu_s < 2\mu_s^0)$

Threshold corrections $\sim \log^n \beta$, $\frac{1}{\beta^n}$

- Factorization of soft and Coulomb corrections
- combined Soft and Coulomb resummation possible
- theoretical progress: now NNLL resummation feasible

NNLL resummation for $t\bar{t}$

- Estimate of residual uncertainty

  approx. N3LO terms, kinematic ambiguities, uncertainties in resummation procedure
- discrepancy to (Ahrens et.al. 10) remains for central values, but results now marginally consistent

Squark-antisquark production

- total corrections $4-10\%$ for $m_{\tilde{q}} = 300$ GeV-2 TeV
- reduced μ_f-dependence for combined soft/Coulomb resummation

Hadron collider cross sections from QCD factorization

(Collins, Soper, Sterman)

$$\sigma_{NN'}(s) = \sum_{pp'} \int dx_1 dx_2 \; f_{N/p}(x_1,\mu_f) f_{N'/p'}(x_2,\mu_f)\, \hat{\sigma}_{pp'}(sx_1x_2,\mu_f)$$

- $\hat{\sigma}_{pp'}$: partonic cross section: compute in perturbation theory
- $f_{p/N}(x)$: Parton distribution function for parton p in hadron N : fitted to experiment

PDF uncertainties for top: (e.g. Guffanti/Rojo arXiv:1008.4671 [hep-ph])

	CTEQ6.6	MSTW2008	NNPDF2.0	ABKM09	HERAPDF1.0
$\sigma_{t\bar{t}}^{\text{NLO}}(7\text{TeV})[pb]$	147.7 ± 6.4	159.0 ± 4.7	160.0 ± 5.9	131.9 ± 4.8	136.4 ± 4.7

- Different α_s values
- Differences in gluon pdf at large x (impact of Tevatron jet-data)

NNLO$_{\text{approx}}$

Approximations for different observables

- total partonic cross section (Bonciani et.al. 98, Moch/Uwer/Langenfeld)

$$\hat{\sigma}(t\bar{t})(\hat{s}) \quad \Rightarrow \log^n \beta \, , \frac{1}{\beta^m} \, , \, \beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}}$$

- Pair invariant mass cross sections(Kidonakis,Sterman 97, Ahrens et.al. 10)

$$\frac{d\hat{\sigma}(t\bar{t})}{dM_{t\bar{t}}} \quad \Rightarrow \left[\frac{\log^n(1-z)}{1-z}\right]_+ \, , \, z = \frac{M_{t\bar{t}}^2}{\hat{s}} \, ,$$

$$\text{PIM}_{\text{SCET}} : \log\left(\frac{1-z}{\sqrt{z}}\right)$$

- One particle inclusive cross sections:

(Laenen et.al. 98, Kidonakis 10, Ahrens et.al. 11)

$$\frac{d\hat{\sigma}(t+X)}{ds_4} \quad \Rightarrow \left[\frac{\log^n(s_4/m^2)}{s_4}\right]_+ \quad , \, s_4 = p_X^2 - m_t^2 \, ,$$

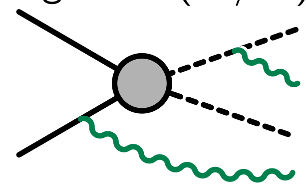
$$1\text{PI}_{\text{SCET}} : \log\left(s_4/\sqrt{m^2+s_4}\right)$$

All threshold enhanced $\mathcal{O}(\alpha_s^2)$ terms (Beneke, Czakon, Falgari, Mitov, CS 09

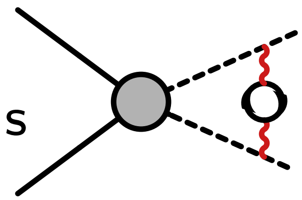
Implemented in HATHOR, Aliev et.al. 10)

Pure soft corrections: (also Moch/Uwer+Langenfeld (08/09))

$$\Delta\sigma_{\mathsf{s}}^{(2)} \sim \alpha_s^2(c_{\mathsf{LL}}^{(2)} \ln^4\beta + c_{\mathsf{NLL}}^{(2)}\ln^3\beta + c_{\mathsf{NNLL,2}}^{(2)} \ln^2\beta + \underbrace{c_{\mathsf{NNLL,1}}^{(2)} \ln\beta}_{\text{2-loop } \gamma_{H,s}})$$



Potential corrections: 2nd Coulomb, NLO potentials

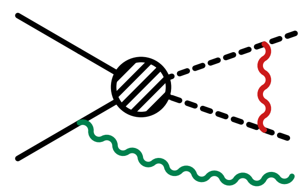


$$\Delta\sigma_{\mathsf{p}}^{(2)} \sim \alpha_s^2\,(\tfrac{c_{C2}}{\beta^2} + \tfrac{1}{\beta}(c_{\mathsf{C,0}}^{(2)} + c_{\mathsf{C,1}}^{(2)} \log\beta) + \underbrace{c_{\mathsf{n\text{-}C}}^{(2)} \ln\beta}_{\text{spin-dependent}}\,)$$

(using Beneke, Signer, Smirnov 99, Czarnecki/Melnikov 97/01)

mixed Coulomb/soft, hard corrections:

$$\Delta\sigma_{\mathsf{p}\otimes\,\mathsf{sh}}^{(2)} \sim\sim \tfrac{\alpha_s}{\beta}\alpha_s(c_{\mathsf{LL}}^{(1)}\ln\beta^2 + c_{\mathsf{NLL}}^{(1)}\ln\beta + c + \underbrace{H^{(1)}}_{\text{process dependent}}\,)$$



Soft scale choice in momentum-space resummation

Structure of resummation formula (NLL, no Coulomb summation)

$$\hat{\sigma}^{\text{NLL}} = \sum_{i,R_\alpha} \hat{\sigma}^{i,(0)} U_i^{R_\alpha}(\mu_s,\mu_f,\mu_h,m_t) \left[1-\frac{4m_t^2}{\hat{s}}\right]_*^{2\eta} \frac{\sqrt{\pi}e^{-2\eta\gamma_E}}{2\Gamma(2\eta+\frac{3}{2})}$$

$$U_i^{R_\alpha} = e^{-\frac{\alpha_s\Gamma^{(0)}_{\text{cusp}}}{2\pi}\log^2(\frac{\mu_s}{\mu_f})+\dots},$$

$$\eta = -2\int_{\alpha_s(\mu_f)}^{\alpha_s(\mu_s)} d\alpha_s \frac{\Gamma_{\text{cusp}}(\alpha_s)}{\beta(\alpha_s)} = \frac{\alpha_s\Gamma^{(0)}_{\text{cusp}}}{2\pi}\log(\frac{\mu_s}{\mu_f}) + \dots$$

Expansion in α_s generates all logs in $\hat{\sigma}$ for $\mu_s \sim m_t\beta^2$

Resummation: Cannot convolute $\exp\left[+c\log(1-\frac{4m_t^2}{\hat{s}})\right]$ with PDFs

- multiply with PDFs in Mellin space (Catani et.al. 96)
- Introduce cutoffs (Berger/Contopagnaos 96; Bonvini/Forte/Ridolfi 10)

EFT approach: fixed μ_s that minimizes soft corrections to hadronic σ (Becher, Neubert, Xu 07)

Expand NNLL to $\mathcal{O}(\alpha_s^3)$, e.g.

$$\Delta\sigma^{(3)}_{qq,\text{NNLL}} = 12945.4\log^6\beta - 37369.1\log^5\beta + 27721.4\log^4\beta + 41839.4\log^3\beta + \frac{1}{\beta}\left(-6278.5\log\beta + 3862.5\log^2\beta + 2804.7\log^3\beta - 2994.5\log^4\beta\right) + \frac{153.9\log^2\beta + 122.9\log\beta - 145}{\beta^2} + \underbrace{\left\{\log\beta^{1,2}, 1/\beta, C^{(3)}\right\}}_{\text{not known exactly}} + \text{scale dep.}$$

NNLL$_3$A: keep all terms, including k-dependence and constants

NNLL$_3$B: only keep terms known exactly