Non-Static Extra Dimensions

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Introduction

- Extra dimensions:
  (1) Large (ADD)
  (2) Warped (RS)
  (3) Infinite: (DGP) or (RS II)

- What if the compactified extra dimension is non-static?

What does it mean by "Non-Static"?

- Consider the metric:

$$d\tau^2 = \eta_{ij} dx^i dx^j + dt^2 + 2\, g_0\, dt\, du - h_0\, du^2$$

The extra dimension is compactified on S^1 with $L = 2\,\pi\,R$
$\Rightarrow\ u \sim u + L$

- Take Det [metric tensor] = $g_0^2 + h_0 = 1$
- Take $h_0 > 0$ to ensure that u is spacelike
$\Rightarrow 0 < h_0 < 1$ and $|g_0| < 1$

- Without loss of generality, assume $0 < g_0 < 1$

Number and Size of the Extra Dimensions?

- Cullen and Perelstein (PRL83,268,1999):

  Most energy from core-collapsed supernova is carried away by neutrinos. Emission of KK gravitons may lead to excessive cooling of the hot core

- Agreement with neutrino observations implies the bounds:

$$\begin{array}{lll} n = 2 & R \lesssim 10^{-7}\, m & M \gtrsim 50\, TeV \\ n = 3 & R \lesssim 10^{-10}\, m & M \gtrsim 4\, TeV \\ n = 4 & R \lesssim 10^{-11}\, m & M \gtrsim 1\, TeV \end{array}$$

- Add one or more flat and static extra dimensions:

$$\begin{aligned} d\tau^2 &= \eta_{ij} dx^i dx^j + dt^2 + 2\, g_0\, dt\, du - h_0\, du^2 \\ &\quad -(dx^6)^2 - (dx^7)^2 - \cdots \end{aligned}$$

Without affecting the discussions in the rest of this talk

The Question

- What would particle physics look like in this spacetime?

- Add the following 5D renormalizable Lagrangian density to standard model:

$$\mathcal{L}^{(5D)} = \frac{G^{AB}}{2}\,\partial_A\,\phi\,\partial_B\,\phi - \frac{m^2}{2}\,\phi^2 - \lambda_1\,\phi - \lambda_3\,\phi^3 \\ - \alpha\,\phi\,H^\dagger\,H\,\delta(u) - L\,\lambda\,\phi^2\,H^\dagger\,H\,\delta(u)$$

G^{AB} = inverse metric A, B = 0,1,2,3,5
ϕ = scalar singlet H = SM Higgs doublet
SM particles are confined to the brane $\Rightarrow \delta(u)$

- ϕ^4 is non-renormalizable in 5D and so neglected

Imposing Z_2 Symmetry

- Imposing the symmetry $\phi \leftrightarrow -\phi$ removes all terms odd in ϕ from $\mathcal{L}^{(5D)}$:

$$\mathcal{L}^{(5D)} = \frac{G^{AB}}{2} \, \partial_A \, \phi \, \partial_B \, \phi - \frac{m^2}{2} \, \phi^2 - L \, \lambda \, \phi^2 \, H^\dagger \, H \, \delta(u)$$

- Since ϕ is a gauge singlet and stable, in principle, it could be a dark matter candidate

  What would be the dark matter candidate in $\mathcal{L}^{(5D)}$? zeroth KK mode?

- We obtain:

$$
\begin{aligned}
\mathcal{L}^{(4D)} \;=\; & \frac{1}{2}\sum_n \Big\{ h_0\,\partial_0\,\phi_n\,\partial_0\,\phi^*_{-n} - \nabla\,\phi_n \cdot \nabla\,\phi^*_n \\
& - \; (\, m^2 + \frac{n^2}{R^2} + \lambda\, v^2\,)\,\phi_n\,\phi^*_n - 2\,i\,g_0 \left(\frac{n}{R}\right) (\,\partial_0\,\phi_n\,)\,\phi^*_n \Big\} \\
& -\frac{1}{2}\,\lambda\, v^2 \left(\sum_{n_1, n_2 \neq n_1} \phi_{n_1}\phi^*_{n_2} \right) \\
& -\lambda\, v\, h \left(\sum_n \phi_n\,\phi_{-n} + \sum_{n_1, n_2 \neq n_1} \phi_{n_1}\phi_{-n_2} \right) + \ldots\ldots
\end{aligned}
$$

$\delta(u)$ breaks translational invariance in the fifth dimension and so KK numbers need not be conserved

$\Rightarrow \; h\,\phi_{n_1}\;\phi_{-n_2}$

Dispersion Relation?

- Recall:

$$\mathcal{L}^{(4D)}_{\text{free}} = \frac{1}{2} \sum_n \Bigg\{ h_0\, \partial_0\, \phi_n\, \partial_0\, \phi_n^* - \nabla\, \phi_n \cdot \nabla\, \phi_n^* - \left(m^2 + \frac{n^2}{R^2} \right) \phi_n\, \phi_n^* - 2\, i\, g_0 \left(\frac{n}{R} \right) (\, \partial_0\, \phi_n\,)\, \phi_n^* \Bigg\}$$

- Calculate Euler-Lagrange equation for ϕ_n^* and apply the Fourier transform $\phi_n(x^\mu) = \int d^4 p\, \Phi_n(p^\mu)\, e^{-i\, E\, t}\, e^{i\, \vec{p} \cdot \vec{r}}$, and solve for E:

$$E = \frac{-g_0\, \frac{n}{R} + \sqrt{h_0\, (p^2 + m^2) + \frac{n^2}{R^2}}}{h_0}$$

Negative root deleted

What's so Special about E?

- Recall that $0 < g_0, h_0 < 1$:

$$E = \frac{-g_0 \frac{n}{R} + \sqrt{h_0 \left(p^2 + m^2\right) + \frac{n^2}{R^2}}}{h_0}$$

The metric breaks reflection symmetry in u
$\Rightarrow$ E not symmetric in n

- Zeroth mode energy:

$$E_{n=0} = \frac{\sqrt{p^2 + m^2}}{\sqrt{h_0}}$$

- The term $-g_0 \frac{n}{R}$ gives a negative contribution for $n > 0$, so some $n > 0$ modes may have energy:

$$E_{n>0} < E_{n=0} \quad !!!$$

What is the Minimum of E?

- Recall that $0 < g_0, h_0 < 1$:

$$E = \frac{-g_0 \frac{n}{R} + \sqrt{h_0 (p^2 + m^2) + \frac{n^2}{R^2}}}{h_0}$$

- For a fixed momentum p, minimum E occurs at:

$$n_\star = g_0 \sqrt{p^2 + m^2} \, R > 0$$

For $\sqrt{p^2 + m^2} \sim 10$ GeV and $R \sim 10^{-10}$m, $n_\star \sim 10^7$

- Minimum E:

$$E_{n_\star} = \sqrt{p^2 + m^2}$$

What is $n_\star$ in our Model?

- Consider $h \rightarrow \phi_{n_\star}\, \phi_{n_\star}$. Energy conservation in the center-of-mass frame requires:

$$M_h = 2E_{n_\star} = 2\sqrt{p^2 + m^2}$$

- Recall: $n_\star = g_0\, \sqrt{p^2 + m^2}\; R$

  $\Rightarrow\;\; n_\star = \frac{1}{2}\, g_0\, M_h\, R$, but this is generally not true!

- The $n_\star$ mode may not be kinematically allowed

- If $n_\star = \frac{1}{2}\, g_0\, M_h\, R$ happens to hold, it will be kinematically allowed
- OR maybe the $n_\star$ mode is kinematically allowed by some other models which produce it (not by SM Higgs decay)
- $n_\star \gg 1$ mode as an unconventional DM candidate, in contrast to:

  (1) $n = 0$ mode DM in the ADD scenario
  (2) $n = 1$ mode DM in UED theories with KK parity $(-1)^n$

- KK number conserving process:

$$\Gamma_{h\to\phi_n\,\phi_{-n}} = \frac{\lambda\, v^2}{16\,\pi\, M_h}\,\frac{1}{\sqrt{h_0}}\,\sqrt{1-\frac{4\,\bar{M}_n^2}{h_0^2\, M_h^2}}$$

$M_h =$ Higgs mass $\qquad \bar{M}_n^2 = h_0\, m^2 + \frac{n^2}{R^2}$

- KK number violating process:

$$\Gamma_{h\to\phi_n\,\phi_n} = \frac{\lambda^2\, v^2}{8\,\pi\, M_h}\,\frac{1}{\sqrt{h_0}}\,\sqrt{1-\frac{4\,\bar{M}_n^2}{\left(h_0\, M_h + \frac{2\,g_0\,n}{R}\right)^2}}$$

In the limit $g_0 \to 0$ and $h_0 \to 1$, one gets back the usual results for flat and static extra dimensions

A Caveat in Calculating Decay Rates

- For flat and static spacetime, the calculation for Γ involves the Lorentz invariant integral:

$$\int d^4p\,\delta(\,G^{\mu\nu}\,p_\mu\,p_\nu - m^2\,)\,\theta(p_0) = \int \frac{d^3\vec{p}}{2\,E}$$

- For non-static spacetime, this must be promoted to a generally covariant integral:

$$\int \sqrt{|\mathrm{Det}(\mathrm{G_{AB}})|}\,d^4p\,\delta(\,G^{AB}\,p_A\,p_B - m^2\,)\,\theta(p_0)$$
$$= \int \frac{d^3\vec{p}}{2\,(\,h_0\,E_n + \frac{g_0\,n}{R}\,)}$$

- Higgs decaying into a pair of ϕ_0:

$$\Gamma_{h\rightarrow\phi_0\,\phi_0} = \frac{\lambda^2\,v^2}{8\,\pi\,M_h}\,\frac{1}{\sqrt{h_0}}\,\sqrt{1-\frac{4\,m^2}{h_0\,M_h^2}}$$

- This is possible only if:

$$M_h > \frac{2\,m}{\sqrt{h_0}}$$

$\Rightarrow$ Whether SM Higgs can decay into a pair of ϕ_0 depends on the configuration of the extra dimension!

- If Higgs is light, say $M_h \sim 120$ GeV, then even if $m \sim 10$ GeV and $\sqrt{h_0} \sim 0.1$ would forbid this process

Consider $h \to \phi_{n_\star}\,\phi_{n_\star}$

- Higgs decaying into a pair of $\phi_{n_\star}$:

$$\Gamma_{h\to\phi_{n_\star}\,\phi_{n_\star}} = \frac{\lambda^2\,v^2}{8\,\pi\,M_h}\sqrt{1-\frac{4\,m^2}{M_h^2}}$$

- Recall, for a flat and static extra dimension:
$E^{\rm static} = \sqrt{p^2 + m^2 + \frac{n^2}{R^2}}$

$$E_{n_\star}^{\rm non\text{-}static} = E_{n=0}^{\rm static} = \sqrt{p^2 + m^2}$$

$$\Gamma_{h\to\phi_{n_\star}\,\phi_{n_\star}}^{\rm non\text{-}static} = \Gamma_{h\to\phi_0\,\phi_0}^{\rm static}$$

- Need to find another way to distinguish this case?

Are $h \to \phi_0\, \phi_0$ and $h \to \phi_{n_\star}\, \phi_{n_\star}$ subdominant?

- Comparison to $\Gamma_{h\to\tau^+\tau^-}$:

$$\frac{\Gamma_{h\to\phi_0\,\phi_0}}{\Gamma_{h\to\tau^+\tau^-}} \sim \frac{\lambda^2\, v^4}{M_h^2\, m_\tau^2}\, \frac{1}{\sqrt{h_0}}\, \sqrt{1 - \frac{4\, m^2}{h_0\, M_h^2}}$$

$$\frac{\Gamma_{h\to\phi_{n_\star}\,\phi_{n_\star}}}{\Gamma_{h\to\tau^+\tau^-}} \sim \frac{\lambda^2\, v^4}{M_h^2\, m_\tau^2}\, \sqrt{1 - \frac{4\, m^2}{M_h^2}}$$

Neglected terms of order $(\frac{m_\tau}{M_h})^2$

- For $M_h > \frac{2\,m}{\sqrt{h_0}}$, both ratios could be large even for perturbatively small λ

Some Other (Maybe) Interesting Questions

- KK gravitons in non-static extra dimensions?
- Implications of the $n_\star$ mode dark matter candidate?
- LHC signatures?
- Other interesting phenomenology of non-static extra dimensions?

Conclusions

- Particles carry an unconventional energy dispersion
- Positive KK modes can have lower energy than the zeroth mode
- $n_\star \gg 1$ mode can be the dark matter candidate

- Yes, we could:

$$d\tau^2 = \eta_{ij}\, dx^i dx^j + d\bar{t}^2 - d\bar{u}^2$$

$$d\bar{t} \equiv dt + g_0\, du \qquad d\bar{u} \equiv \sqrt{g_0^2 + h_0}\, du$$

- $\bar{u} = u$ but $\bar{t}$ is pathological:

$$\bar{t} = t + g_0\, u$$

- For fixed u, $\bar{t}$ is smooth and continuous, with domain $\bar{t} \in [-\infty, +\infty]$
- For fixed t, $u \sim u + L$ implies $\bar{t} \sim \bar{t} + g_0\, L$, with domain $\bar{t} \in [0, g_0\, L]$

- The metric for a spinning cosmic string:

$$d\tau^2 = (dt + 4\,G\,J\,d\theta)^2 - dr^2 - (1 - 4\,G\,M)^2\,r^2\,d\theta^2 - dz^2$$

G = Newton's constant J = angular momentum per unit length M = mass per unit length

- Define $\tilde{t} = t + 4\,G\,J\,\theta$ and $\varphi = (1 - 4\,G\,M)\,\theta$:

$$d\tau^2 = d\tilde{t}^2 - dr^2 - r^2\,d\varphi^2 - dz^2$$

- Deser, Jackiw and 't Hooft (Ann.Phys.152,220,1984):
  $\tilde{t} = t + 4\,G\,J\,\theta$ is pathological because it is BOTH continuous and compactified