Coherent States in a Deformed Quantum Mechanics

Zack Lewis, Tatsu Takeuchi
Virginia Tech Physics Dept.
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Minimal Length Uncertainty Relation

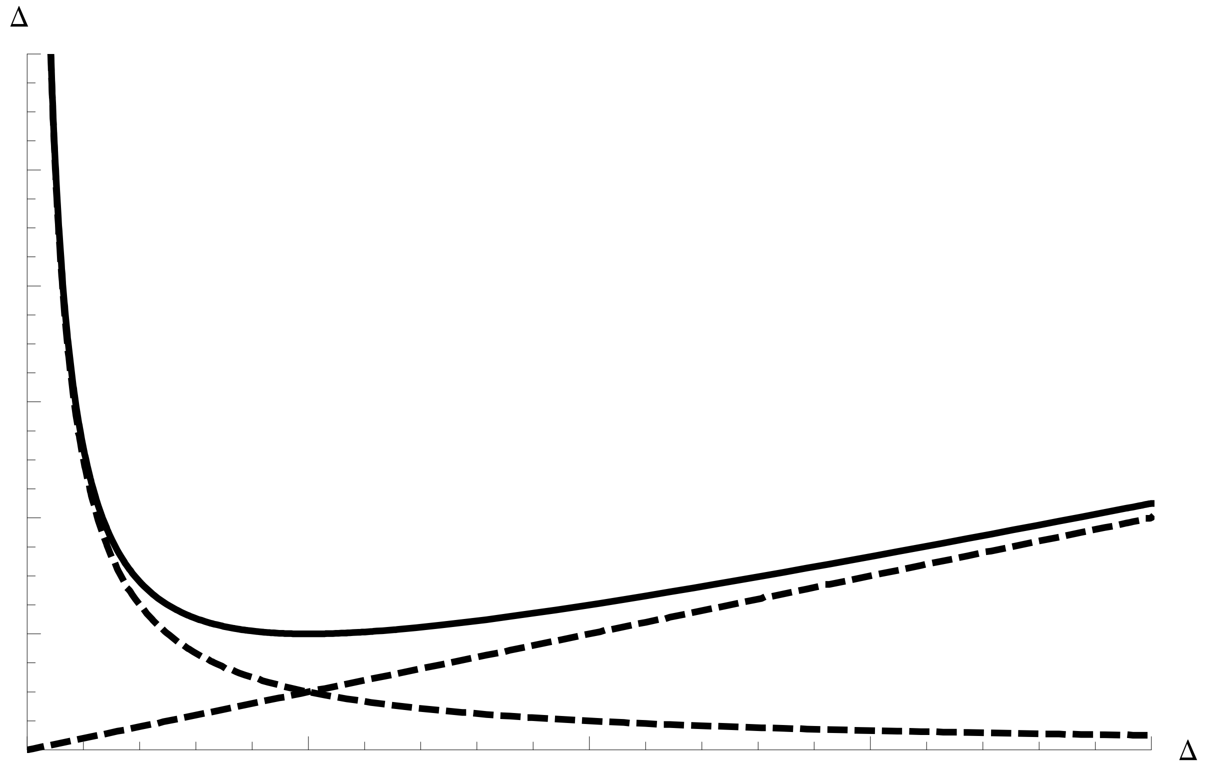
Start by deforming the commutator algebra

$$[\hat{x}, \hat{p}] = i\hbar(1 + \beta\, \hat{p}^2)$$

This results in a minimum in Δx.

$$\Delta x \Delta p \geq i\hbar(1 + \beta\, \Delta p^2) \quad \rightarrow \quad \Delta x \geq \hbar\sqrt{\beta}$$

Uncertainty graph



Test System: Harmonic Oscillator

Use standard Hamiltonian

$$\hat{H} = \frac{1}{2}\mu\omega^2\hat{x}^2 + \frac{1}{2\mu}\hat{p}^2$$

Choose representation and modify

$$\hat{p} = p, \quad \hat{x} = i\hbar(1+\beta\, p^2)\frac{\partial}{\partial p}$$

$$p = \frac{\tan(\sqrt{\beta}\rho)}{\sqrt{\beta}}, \quad (1+\beta\, p^2)\frac{\partial}{\partial p} = \frac{\partial}{\partial \rho}$$

Harmonic Oscillator (continued)

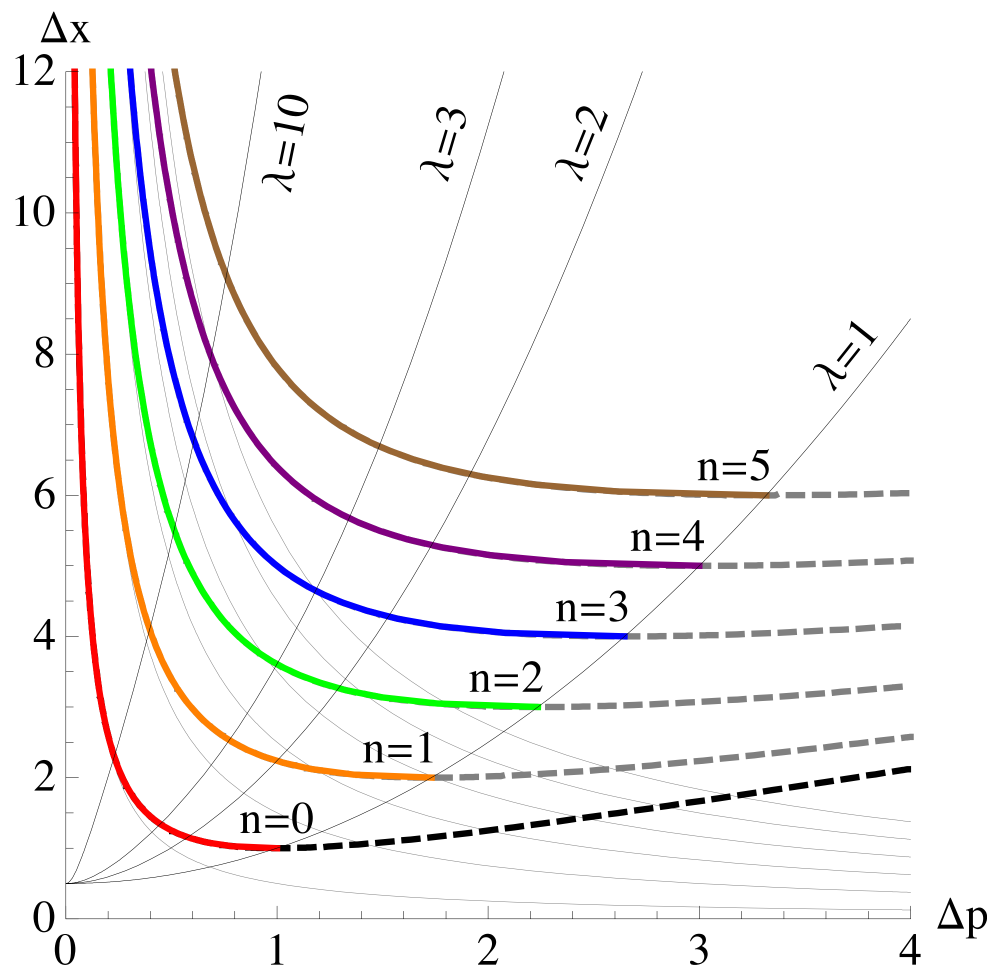
The energy eigenvalues and eigenvectors can be solved for exactly.

$$E_n = \hbar\omega\left((n+\tfrac{1}{2})\sqrt{1+\left(\frac{\mu\beta\hbar\omega}{2}\right)^2} + (n^2+n+\tfrac{1}{2})\frac{\mu\beta\hbar\omega}{2}\right)$$

$$\lambda = \frac{1}{2} + \sqrt{\frac{1}{2} + \frac{1}{(\mu\beta\hbar\omega)^2}}$$

$$\Psi_n(p) = \sqrt[4]{\beta}\, 2^\lambda \Gamma(\lambda)\sqrt{\frac{n!(n+\lambda)}{2\pi\,\Gamma(n+2\lambda)}}\cos^\lambda(\sqrt{\beta}\rho)\, C_n^\lambda(\sin(\sqrt{\beta}\rho))$$

Uncertainty graph II



Coherent States

Add impulse term to the Hamiltonian

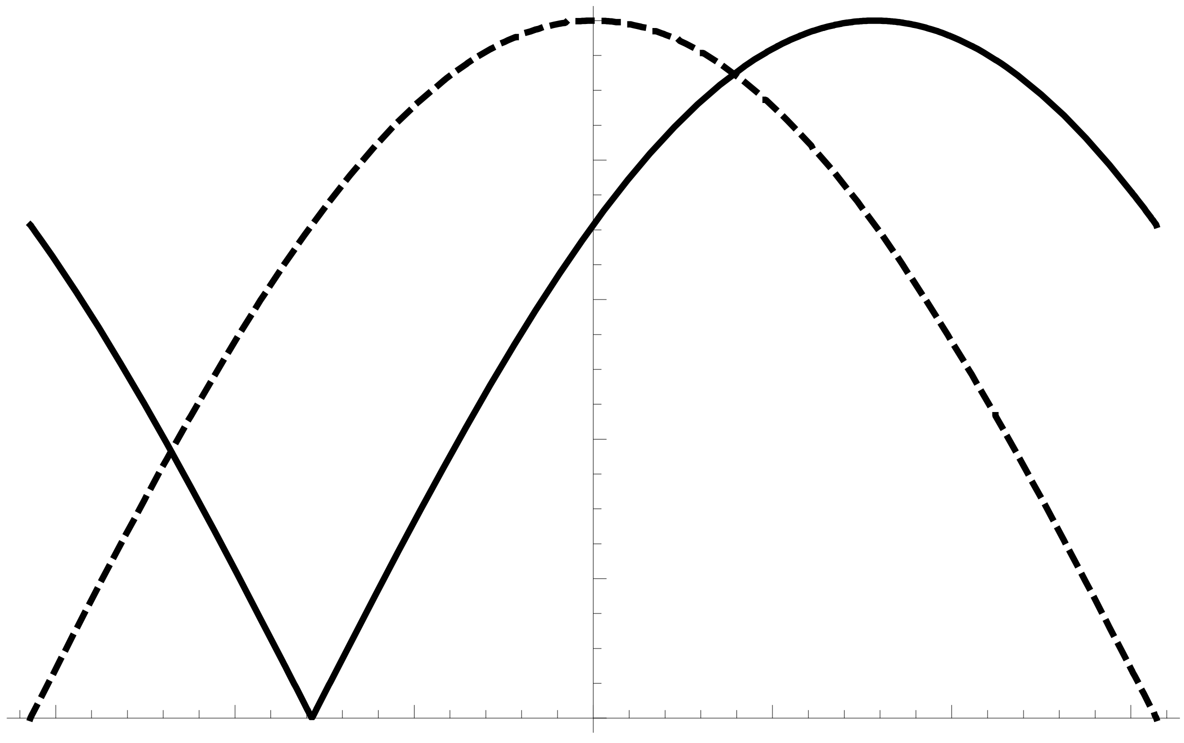
$$\hat{H} = \hat{H}_0 + \hat{H}_I = \frac{1}{2}\mu\omega^2\hat{x}^2 + \frac{1}{2\mu}\hat{p}^2 - \alpha\,\delta(t)\,\hat{x}$$

Time evolve the ground state

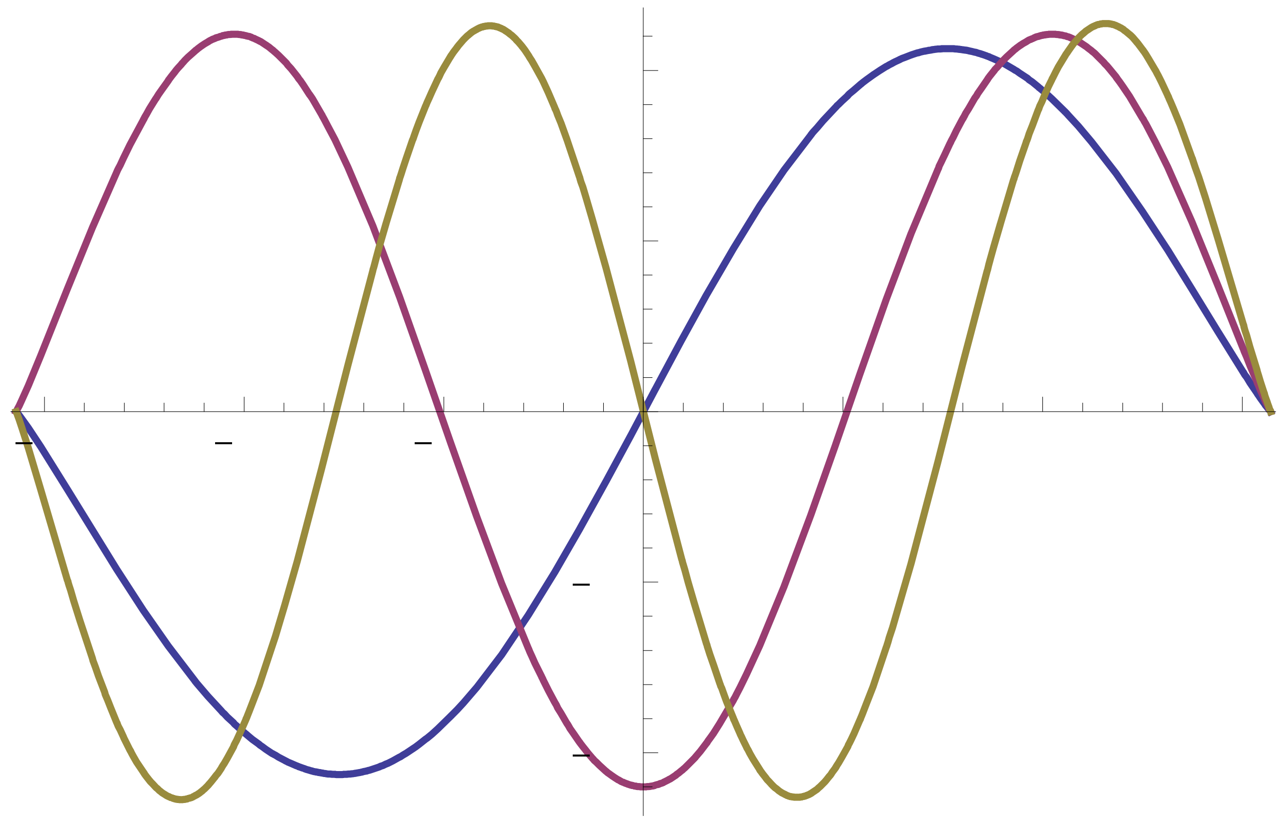
$$\hat{U}(t)\left|0\right\rangle = \hat{U}_0(t)\, e\wedge(-\alpha\frac{\square}{\square\rho})\left|0\right\rangle$$

State is shifted in ρ

Shifted Ground State



Energy Eigenstates



Resolution

To avoid divergent uncertainties, implement a cutoff.

$\Delta x_{min} \rightarrow \Delta p_{min}$: UV /IR

$\Delta x \Delta p \approx 1$ (?)