Two-Loop QED × QCD Virtual Corrections to Drell-Yan Production

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Motivations

This calculation is the first part of an effort to compute full QCD x EW corrections. We expect to apply this calculation to:

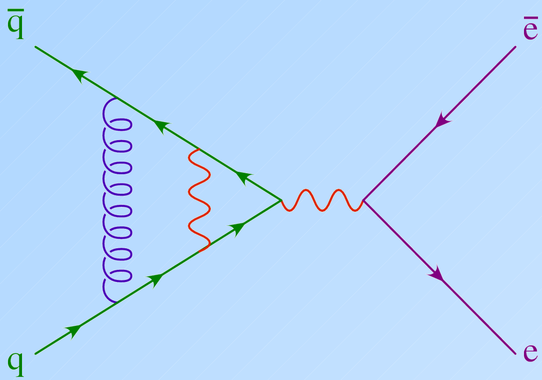
- The W boson mass measurement
- Precision Z measurements
- The search for new physics at the highest energies.

1-loop EW corrections to Drell-Yan exceed 10% for di-lepton masses of a few hundred GeV.

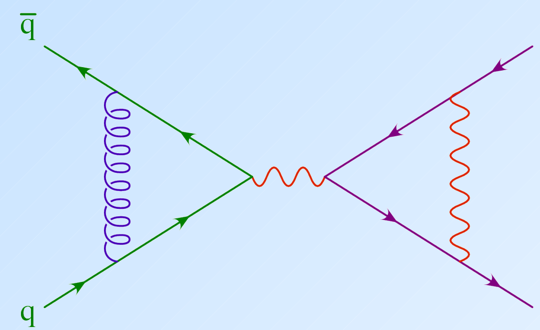
Classes of Contributions

There are four classes of contributions based upon photon interactions

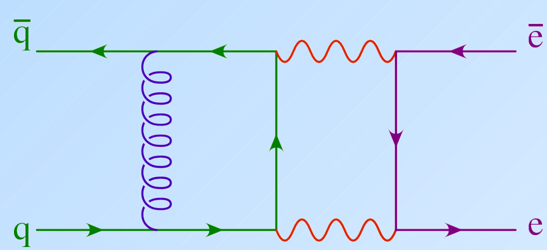
Initial State Photons



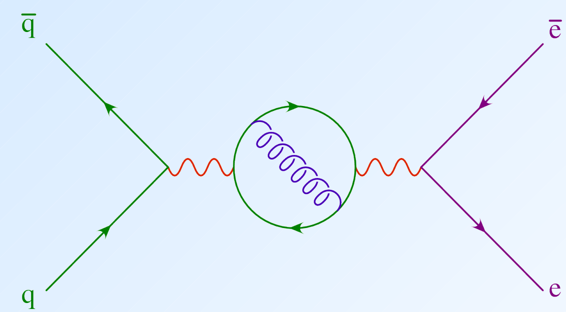
Final State Photons



Photons Connecting
Initial and Final States



Vacuum Polarization Corrections



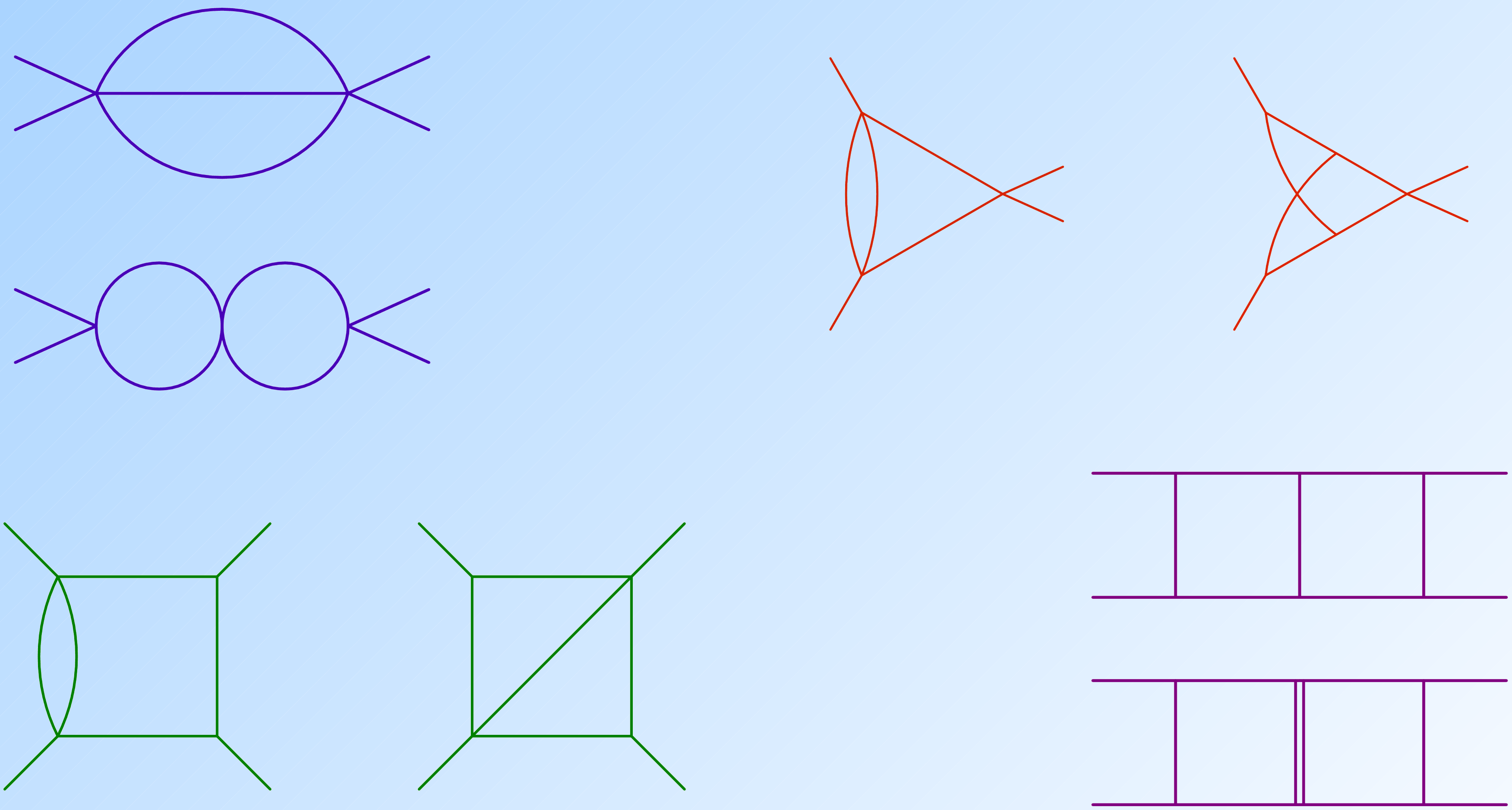
Method of Calculation

We pursue the following plan:

- Generate Feynman graphs with QGRAF.
- Implement Feynman rules in FORM.
- Interfere 2-loop amplitude with Tree-level to obtain scalar integrals.
- Use Integration-by-Parts identities to reduce to a set of Master Integrals and their coefficients.
- Evaluate Master Integrals.

Master Integrals

There are eight distinct master integrals.



Results

So far, we have only computed the virtual amplitude, so we cannot produce a physical result. We can, however, study the infrared structure of the amplitude.

The infrared singularities of QCD amplitudes are known to follow a universal structure. With minor modifications, we can apply that structure to QED x QCD amplitudes and show that our result agrees with expectations.

Infrared Structure of QCD

The infrared structure of QCD amplitudes is completely universal and can be predicted entirely in terms of the identities and momenta of the external states.

Sterman and Tejeda-Yeomans write an amplitude as the product of 3 factors: The Jet function, the Soft function and the Hard Scattering Function.

$$\left|\mathcal{M}_{\{i\}}\left(p_i,\alpha_s,\epsilon\right)\right\rangle_{QCD} = \mathcal{J}_{\{i\}}\left(\alpha_s,\epsilon\right)\mathbf{S}_{\{i\}}\left(p_i,\alpha_s,\epsilon\right)\left|\mathcal{H}_{\{i\}}\left(p_i,\alpha_s,\epsilon\right)\right\rangle$$

In color space, the Jet function is a scalar, the Hard Scattering function is a vector and the Soft function is a matrix.

The Jet Function

The Jet function describes the collinear evolution of the external partons. It is the product of individual jet functions for each leg. It is convenient to take only the poles from the square root of the Sudakov form factor.

$$\ln \mathcal{J}_i = \frac{\alpha_s}{\pi}\left[\frac{1}{8\,\epsilon^2}\gamma_K^{(1)} + \frac{1}{4\,\epsilon}\mathcal{G}_i^{(1)}(\epsilon)\right]$$

$$+\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{\beta_0}{8\,\epsilon^2}\left(\frac{3}{4\,\epsilon}\gamma_K^{(1)} + \mathcal{G}_i^{(1)}\right) - \frac{1}{8}\left(\frac{1}{4\,\epsilon^2}\gamma_K^{(2)} + \frac{1}{\epsilon}\mathcal{G}_i^{(2)}(\epsilon)\right)\right]$$

The Soft Function

The Soft function describes the soft exchanges between the external partons which alter the color flow of the interaction. Through two loops, it is built up from exchanges between pairs of partons.

$$\mathbf{S}_{\{i\}} = \mathrm{P}\, \exp\left\{ -\frac{1}{2}\int_0^{\mu^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\, \mathbf{\Gamma}\left(\frac{s_{ij}}{\mu^2}, \bar{\alpha}_s(\bar{\mu}^2), \epsilon\right)\right\}$$

$$= 1 + \frac{1}{2\,\epsilon}\frac{\alpha_s}{\pi}\mathbf{\Gamma}^{(1)} + \frac{1}{8\,\epsilon^2}\left(\frac{\alpha_s}{\pi}\right)^2\left[\left(\mathbf{\Gamma}^{(1)}\right)^2 - \frac{\beta_0}{2}\mathbf{\Gamma}^{(1)}\right]$$

$$+ \frac{1}{4\,\epsilon}\left(\frac{\alpha_s}{\pi}\right)^2\mathbf{\Gamma}^{(2)} + \dots$$

$$\mathbf{\Gamma}^{(1)} = \frac{1}{2}\sum_{j\in\{i\}}\sum_{k\neq j}\mathbf{T}_j\cdot\mathbf{T}_k\,\ln\left(\frac{\mu^2}{-s_{jk}}\right) \qquad \mathbf{\Gamma}^{(2)} = \frac{\gamma_K^{(2)}}{\gamma_K^{(1)}}\mathbf{\Gamma}^{(1)}$$

Infrared Structure of QED

QED amplitudes have the same factorization properties as QCD amplitudes.

The anomalous dimensions of QED Jet and Soft functions can be obtained from QCD simply by changing:

$$C_F \to Q_f^2\,, \qquad C_A \to 0\,, \qquad \frac{N_f}{2} \to \sum_{f \in \{i\}} Q_f^2\,, \qquad \mathbf{T}_f \to Q_f$$

$$\left|\mathcal{M}_{\{i\}}\left(p_i, \alpha, \epsilon\right)\right\rangle_{QED} = \mathcal{J}_{\{i\}}\left(\alpha, \epsilon\right) \mathbf{S}_{\{i\}}\left(p_i, \alpha, \epsilon\right) \left|\mathcal{H}_{\{i\}}\left(p_i, \alpha, \epsilon\right)\right\rangle$$

Infrared Structure of Mixed Amplitudes

An amplitude that is expanded in both QCD and QED receives most of its corrections from the product of the individual QCD and QED anomalous dimensions, but there are uniquely mixed two-loop contributions to the Jet function that contribute 1/ε poles.

$$\left|\mathcal{M}_{\{i\}}\left(p_i,\alpha,\alpha_s,\epsilon\right)\right\rangle = \mathcal{J}_{\{i\}}\left(\alpha,\alpha_s,\epsilon\right)\mathbf{S}_{\{i\}}\left(p_i,\alpha,\alpha_s,\epsilon\right)\left|\mathcal{H}_{\{i\}}\left(p_i,\alpha,\alpha_s,\epsilon\right)\right\rangle$$

We find that the mixed anomalous dimension has the same form its QCD and QED counterparts, but where the QCD term is ~ $(C_F)^2$ and the QED term is ~ $(Q_q^2)^2$, the mixed term is ~ $(C_F)(Q_q^2)$.

Conclusions

We have computed the two-loop QCD x QED virtual corrections to Drell-Yan production. This is part of a larger effort to compute the full QCD x EW corrections which could be important for the W boson mass measurement and searches for new physics.

We have also shown that the infrared structure of mixed amplitudes is follows the from same universal factorization structure that governs pure QCD and QED amplitudes and we have determined the value of the two-loop mixed anomalous dimension.