A Slant on Warped Extra Dimensions

Daniel Hernandez and MS, JHEP last month

or

How can studies of the NH_3 spectrum in radio quasars constrain particle physics models of warped extra dimensions?

Warped Extra Dimensions

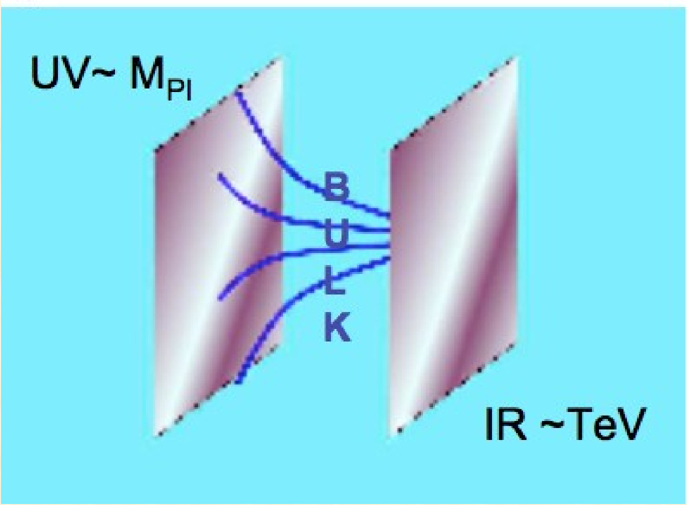
The RS model: An extra dimension is compactified on an orbifold, so there are two 4-D branes separated by a distance R in the fifth dimension.

One assumes that the space in the fifth dimension has constant negative curvature, or AdS_5.

It is easy to solve the Einstein equations and get the metric, **assuming** 4D Lorentz invariance. It is

$$ds^2 = e^{-2kR|\phi|}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + R^2 d\phi^2$$

where k is the curvature and ϕ goes from 0 to π.



The vacuum value of the Higgs field is then $v = v_0\, e^{-\pi kR}$

where v_0 is of the order of terms in the Lagrangian. For

$kR = 11\text{-}12$, this solves the hierarchy problem.

One can put the gauge fields and fermions in the bulk, and this can explain the hierarchy of fermion masses in terms of O(1) coefficients. This will be only discussed briefly here and won't substantially change our results.

Question: It is always assumed in the warped extra dimension model that the two branes are **exactly** parallel. What if they aren't?

In other words, suppose the distance between the branes, R, varies with location. What is the upper bound on the angle between them?

Since they can't cross within our horizon, the angle must smaller than an inverse TeV divided by the Hubble radius, which is around 10^{-44}.

What experimental result gives the strongest limits?

The model: We make an ansatz for the metric ($\phi = y/R$)

$$ds^2 = e^{-2C\phi+2ax\phi}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + (R^2 + Dax)\delta\phi^2$$

where x is a spatial direction (which is arbitrary) and "a" parametrizing the tilt. a=0 is the parallel case. NOTE: In a moment, a much more general ansatz will be considered.

We then solve the Einstein equations. Requiring the stress-tensor to be diagonal yields $CD = -2R^2$. From the diagonal elements, we find, to first order in a, that the cosmological constant is independent of x (true AdS_5 space).

Plugging into the Lagrangian, one finds the main effect is a variation in the vacuum value of the Higgs, i.e.

$$v = v_0\,(1 + \pi a x)$$

This is the main result. We now ask how this variation will show up in astrophysical and/or cosmological measurements.

What are the main effects of changing the vacuum value?

$$v = v_0 \, (1 + \pi \, a \, x)$$

The electron mass is proportional to v, so m_e varies as v/v_0

The proton mass primarily comes from QCD, which changes slightly due to thresholds. Estimates give a variation of $(v/v_o)^{0.3}$

Existence of supernovae: If the electron, quark and pion masses all change, then the triple-alpha process won't occur since the resonance will shift. Estimates (Jeltema and Sher, 1999) give that v must be at least 90% of v_0. Gives a very weak bound of roughly $\pi \, a < 0.1/ \, (3 \text{ Gpc})$.

QuickTime™ and a
decompressor
are needed to see this picture.

Cosmic microwave background: Certainly sensitive to the electron mass. The temperature depends on the binding energy of hydrogen, which scales linearly with the electron mass. Thus, one would expect to see a dipole distribution in the temperature. Unfortunately, there already is one from the Earth's motion. There seems to be no easy way to separate them out. If there is no fine-tuning, one would get (from $\Delta T/T = 0.0012$ for the dipole), $\pi a < 0.0012/R$, where R is the Hubble radius, but that can be eliminated by fine-tuning.

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Spectral lines of distant objects: A shift in the electron mass can't be distinguished from a red-shift or a blue-shift.

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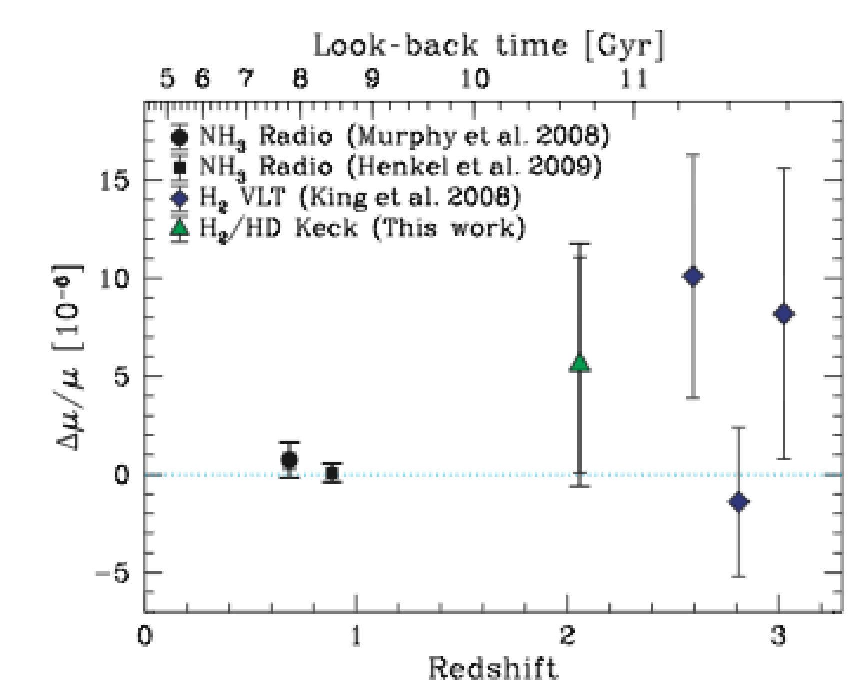
The strongest bound, by far, comes from spectral measurements of the electron-proton mass ratio, μ. The shift, $\delta\mu/\mu$ will scale as 0.7 π a x. So we look for bounds on variation of the ratio from distant objects.

Fortunately, others have already searched for variations in this ratio, although the motivation was always to look for a time dependence. We can use their results.

Malec et al. find



Expt.	authors	Object	Redshift	Dist.	bound ($\times 10^{-6}$)
H_2/HDKeck	Malec[15]	J2123-0050	2.059	5.26	5.6 ± 6.2
H_2/VLT	King [16]	Q0405-443	2.595	5.94	10.1 ± 6.2
		Q-0347-383	3.025	6.38	8.2 ± 7.4
		Q-0528-250	2.811	6.17	-1.4 ± 3.9
NH_3	Murphy [17]	B0218+357	0.68	2.45	0.74 ± 0.89
NH_3	Henkel [18]	PKS1830-211	0.89	3.03	0.08 ± 0.47

Table 1: Bounds on the electron-proton mass difference from six observations of distant spectra. The experiments are given, along with the object, the redshift, the distance away in comoving coordinates in units of gigaparsecs, and the bound.

Alas, two of these, the radio quasars, are very precise, but one needs three to have any bound at all, given that the direction of the tilt is arbitrary.

We find a 2σ upper bound of $a < 7.5 \times 10^{-7}$ Gpc^{-1}

Measurement of a third radio quasar spectrum would improve this by close to an order of magnitude.

This value can be written as 5×10^{-51} TeV. In many models of Lorentz violation, effects vary as the weak scale times 10^{-17n} where n is an integer (the dimensionality of the higher-dimension operator). Thus, current bounds are very close to n=3.

Measurements of the NH_3 spectrum in a radio quasar could thus give stronger bounds on models of warped extra dimensions.

Adding matter in the bulk has a small effect since the masses also depend on the Yukawa couplings. The size of the effect depends on the ratio of the left and right handed 5D mass terms, and tightens the bounds by between 15% and 30%.

Finally, we look at a much more general ansatz.

One wishes to solve

$$G_{MN} = - (\Lambda(x)/4M^3)\, g_{MN}$$

which allows for an x-dependent cosmological “constant”

Try $ds^2 = e^{2k(x,|\phi|)}\eta_{\mu\nu}\, dx^\mu\, dx^\nu + R^2\, \dot{k}^2(x,|\phi|)\, d\phi^2$

where $\dot{k} = dk/d\phi$ and $k' = dk/dx$

This automatically gives a diagonal Einstein tensor.

Plugging into the Einstein equations gives (for the 55 component)

$\Lambda(x) = -24M^3/R^2 - 12\, M^3\, e^{-2k}\, (k'^2 + k'')$

Expanding k in a power series in ax, one finds that $\Lambda(x)$ is independent of x up to order a, which explains why the linear approximation gave pure AdS.

Solving the other Einstein equations give differential equations which we have analytically solved, and the solutions can be expanded. We find brane tensions similar to the standard RS model.

Summary

In RS models, it is always assumed that two branes are parallel.

Here, we simply ask “What if there is a small tilt”?

The strongest bounds come from measurements of the electron-proton mass ratio in quasar spectra.   The tightest constraints come from the ammonia spectra of radio quasars and from the H_2 spectra of optical quasars.

The upper bound on the tilt is approximately 5×10^{-51} TeV.   Only two radio spectra have been measured at high red shift, and a third would improve this by an order of magnitude.