Neutrino Physics

History, electroweak interactions and neutrino scattering

NExT PhD Workshop 2023

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Resources

- Giunti & Kim: “Fundamentals of Neutrino Physics and Astrophysics” available in the library

- Pascoli: “Neutrino Physics” available Neutrino Physics

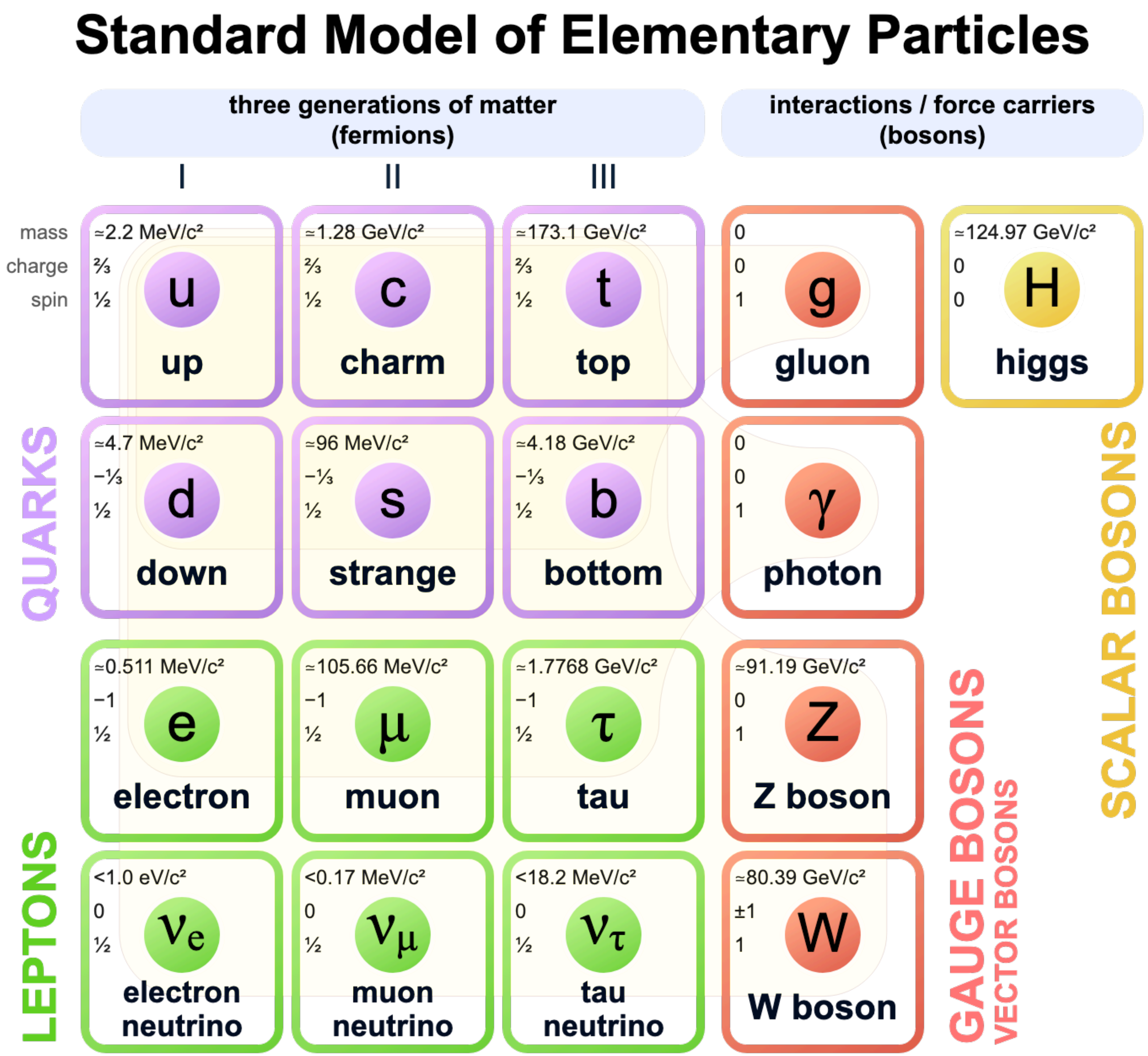
- Hernandez: “Neutrino Physics ” available hep-ph/1708.01046.pdf

- de Gouvea: “TASI lectures on Neutrino ” available hep-ph/0411274.pdf

NeutrinoScope 4+
Bring neutrinos alive with AR!
Cambridge Consultants
★★★★★ 5.0, 7 Ratings
Free

Neutrinoscope is a free App for iPhone and iPad developed by Cambridge Consultants and Durham University. It allows to visualise the neutrinos around us.

Neutrinos in the Standard Model

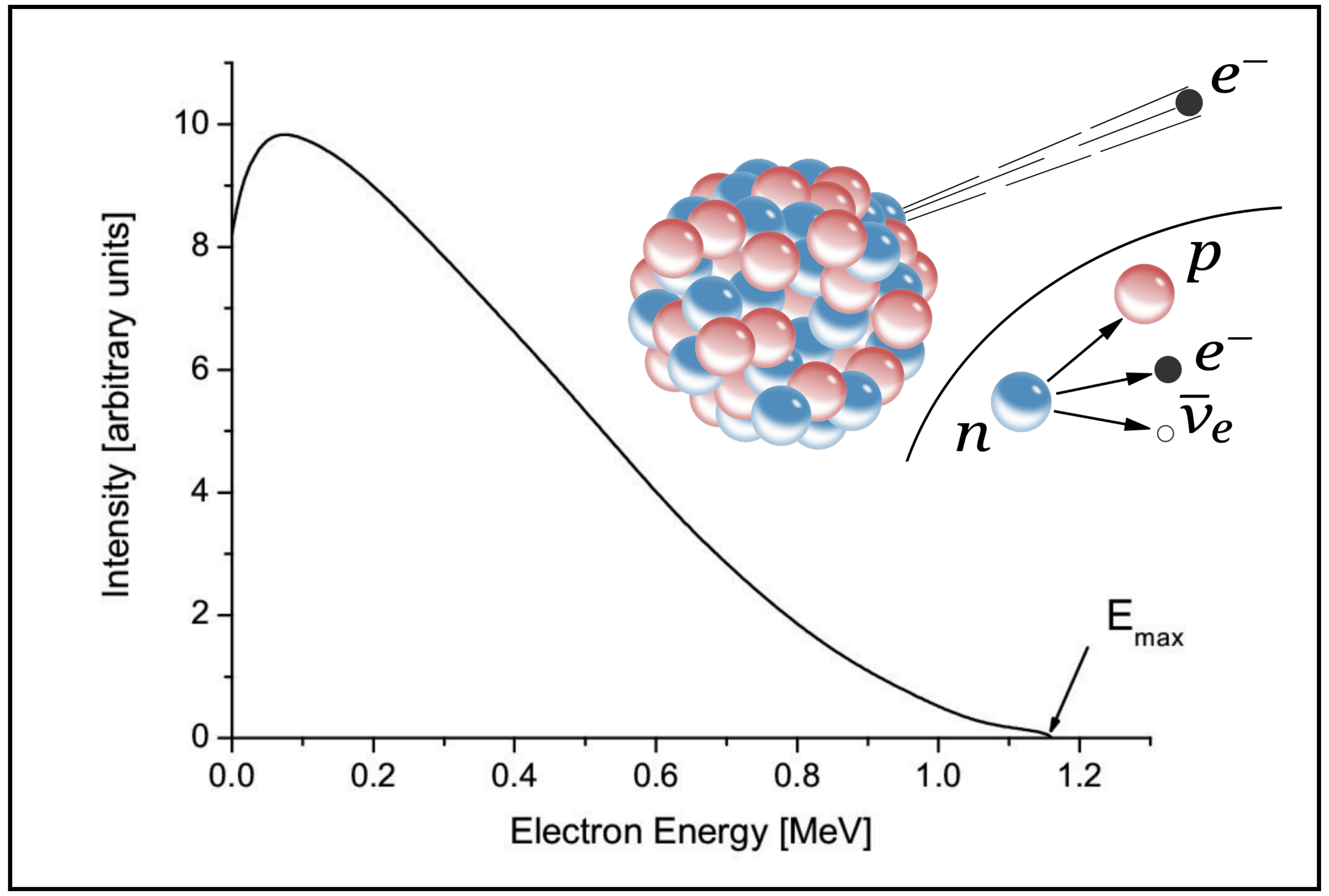


- Neutrinos are **electrically neutral** fermions & part of $SU(2)_L$ doublet
- Neutrinos undergo **weak interactions** with the W and Z boson $(m(W^{\pm}) \sim 80\,\mathrm{GeV}, m(Z) \sim 91\,\mathrm{GeV})$
- Neutrinos are very light $m_\nu \lesssim 1\,\mathrm{eV}$

Discoveries of the Neutrino

- 1800s was an extraordinary time for radioactivity discovery: $\boldsymbol{\alpha}$, β, ɣ all identified
- $\boldsymbol{\alpha}$ - Helium nucleus, discovered by Rutherford 1899
- ɣ - Penetrating form of electromagnetic radiation arising from the radioactive decay of atomic nuclei 1900
- β - electron emitted by radioactive nuclei, discovered by Rutherford 1899

$$(A, Z) \to (A, Z+1) + e^- \implies E_e = M(A, Z+1) - M(A, Z)$$



From energy conservation, the electron should have had a fixed energy, not a spectrum which was what was observed.

Discoveries of the Neutrino

- Some scientist thought that energy conservation principle must be violated.

- In his famous letter to “Radioactive Ladies and Gentlemen” Pauli (1930) proposed the existence of a new and yet undiscovered electrically neutral particle which would explain the continuous spectrum observed in beta decay.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Des. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

$$(A, Z) \rightarrow (A, Z+1) + e^- + \overline{\nu}_e$$

- Fermi proposed the new particle should be light, spin 1/2 and electrically neutral. He dubbed this new particle the “neutrino”.

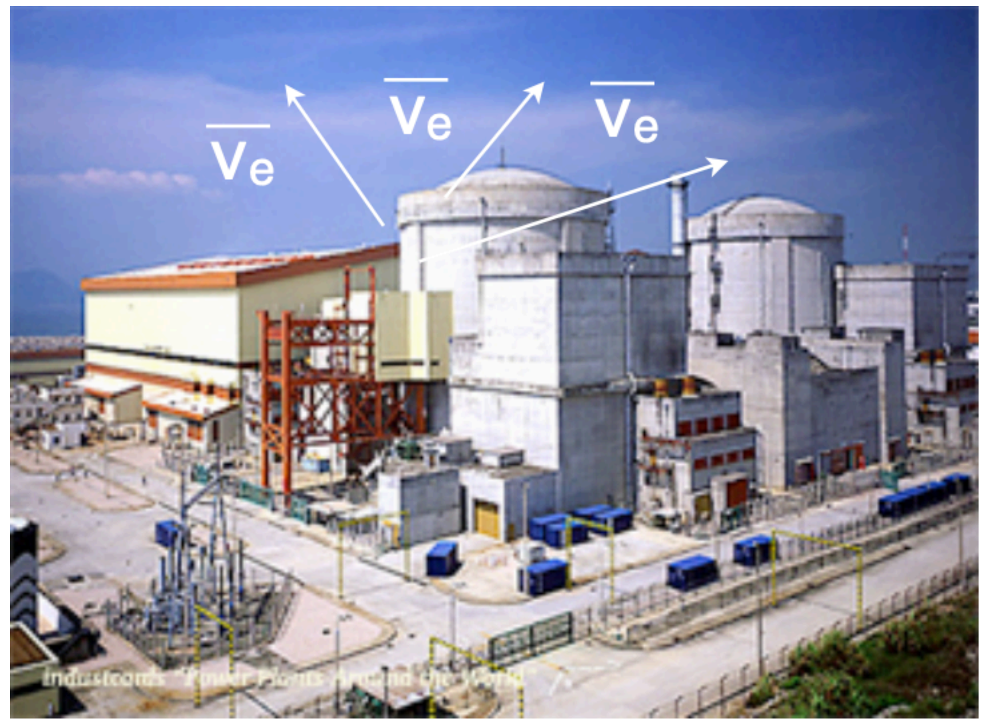
Discoveries of the Neutrino

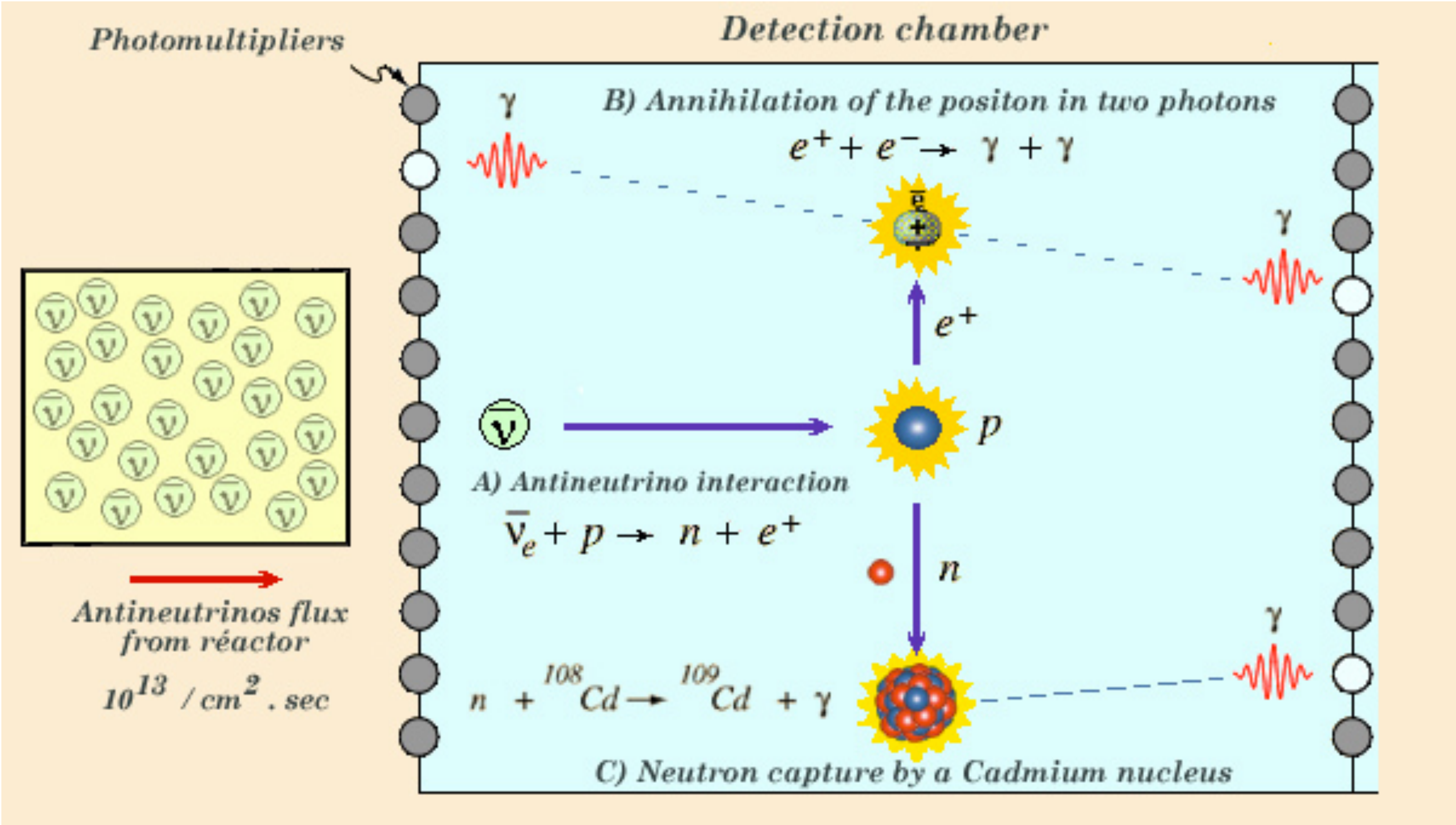
- 1934: Bethe and Peierls showed the interaction cross section of the neutrino should be extremely small → a neutrino can traverse the Earth without deviation

> *"Today I have done something which no theoretical physicist should ever do in his life: I have predicted something which shall never be detected experimentally!"*

- Walter Baade, had great faith in his experiment colleagues and bet Pauli the neutrino would be discovered. The bet was a crate of champagne.

- 1956: Reines and Cowan placed neutrino detector (vat of water combined with cadmium chloride) in front a fission reactor.





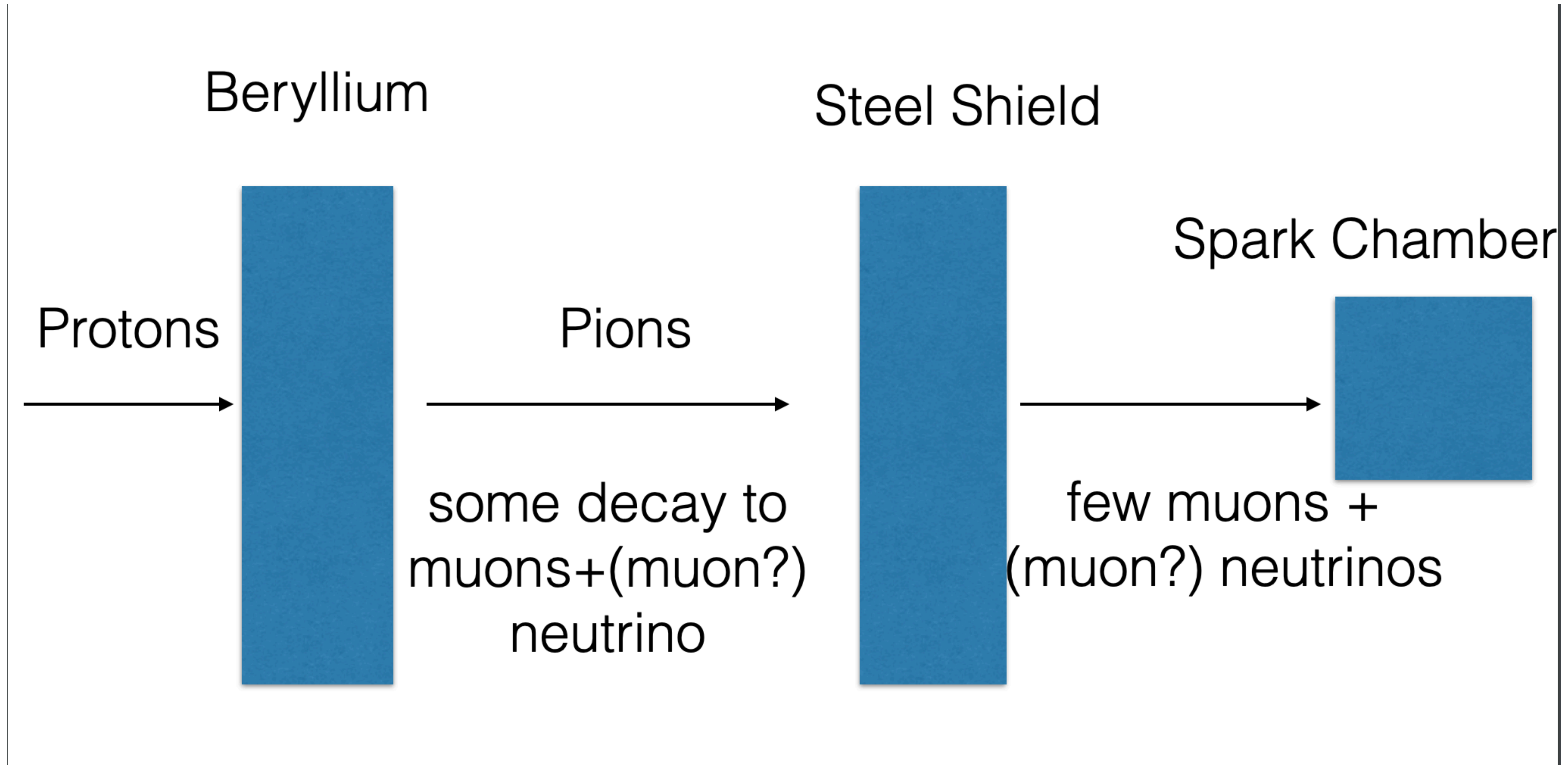
15 msec delay from photon emitted by neutron capture

Discoveries of the Neutrino

- $\overline{\nu_e}$ discovered from inverse beta decay where e^- was in final state

- μ discovered (cosmic ray showers) in 1937. Was there an associated ν_μ?

- 1959 Gaillard, Lederman, Schwartz and Steinberger built a spark chamber (chambers metal plates placed in a sealed box filled with a gas such as helium, neon)
to detect muon neutrino coming from pion decays



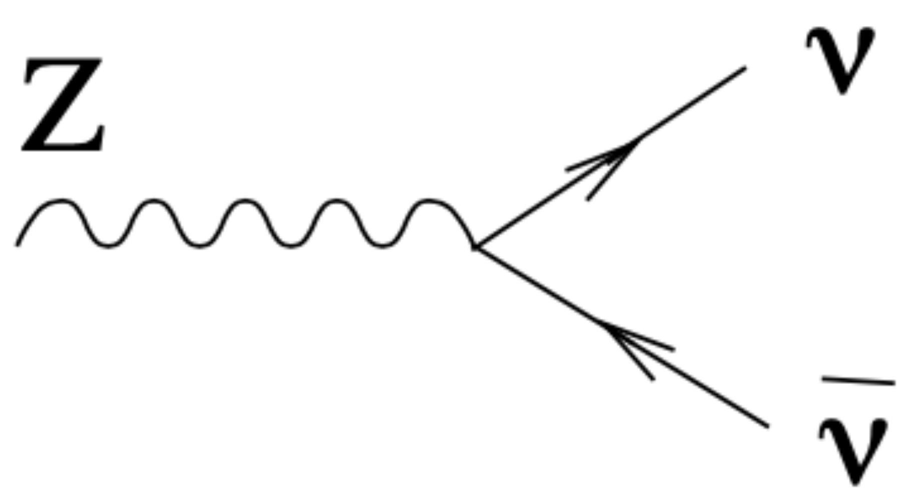
$$\nu_e \equiv \nu_\mu \implies N(\mu^-) = N(e^-)$$

$$36\,\mu^-, 4\,e^-$$

$$36\,\mu^-, 4\,e^- \implies \nu_\mu \not\equiv \nu_e$$

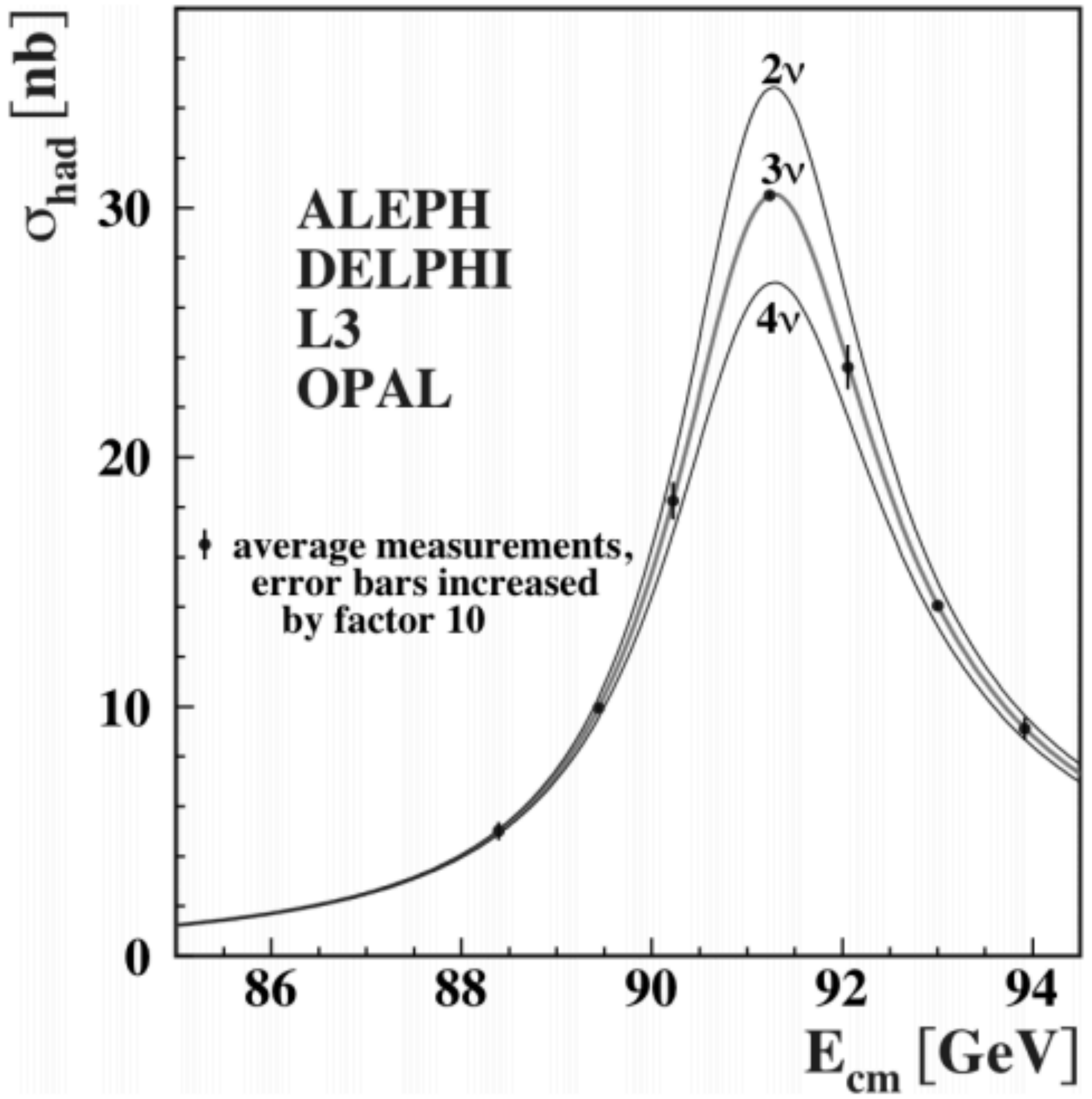
Discoveries of the Neutrino

- neutrinos couple to weak gauge bosons and will modify their decay width
  Z boson width measurement by LEP, DELPHI, L3 and OPAL 1989 confirmed 3 generations of neutrinos



$$N_\nu = \frac{\Gamma_{\rm inv}}{\Gamma_\nu} = \frac{\Gamma_{\rm total} - \Gamma_{\rm visible}}{\Gamma_\nu} = \frac{\Gamma_{\rm total} - \Gamma_{\rm had} - 3\Gamma_{\rm lep}}{\Gamma_\nu}$$

Requires $N_\nu = 2.98 \pm 0.082$

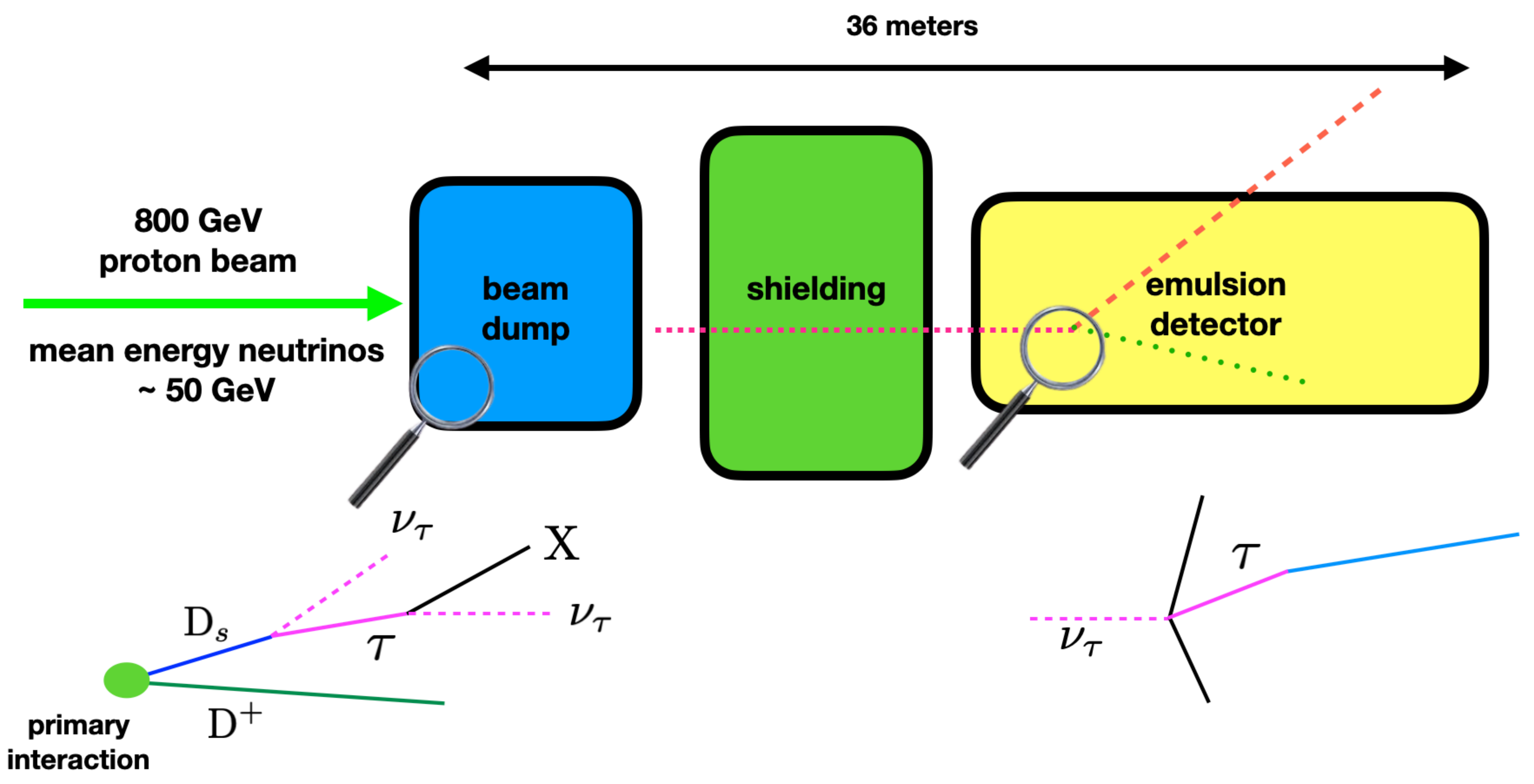


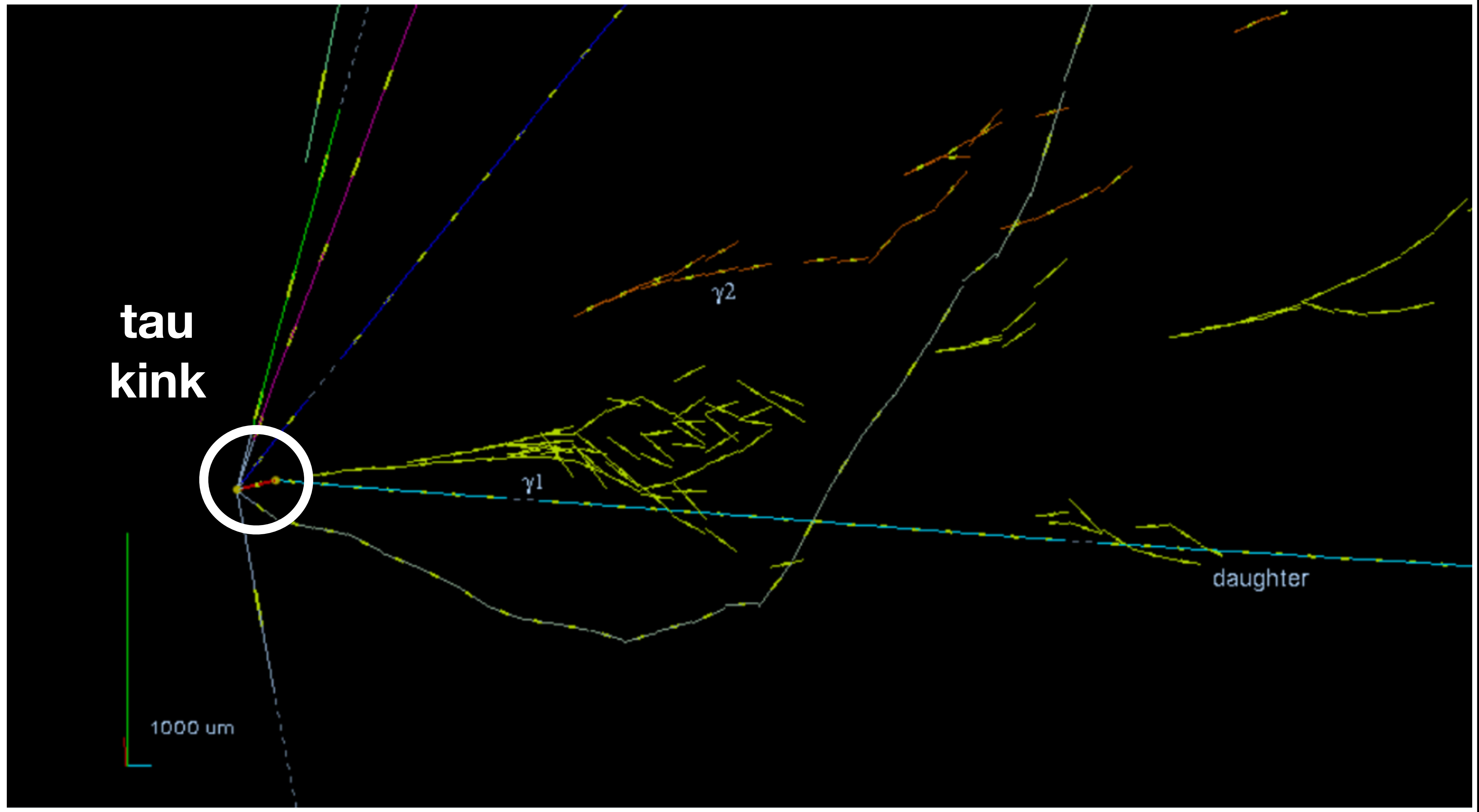
Discoveries of the Neutrino

- τ discovered in 1975 by Perls and colleague

$$e^+ + e^- \to \tau^+ + \tau^- \to e^\pm + \mu^\pm + 4\nu$$

- Was there ν_τ associated to this new heavy lepton?
- Confirmed in 1997 by the DONUT experiment: they observed 4 tau neutrinos





- OPERA experiment observed about 10 tau neutrinos. These are the least well measured Standard Model particles!!

- The Standard Model (SM) gauge group based on the following symmetry:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow SU(3)_C \times U(1)_{EM}$$

- Spin-0 particle ϕ: $(1, 2, \frac{1}{2})$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow{SSB} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

- We have 3 generations of fermions: generations with identical gauge quantum number but different masses

$$Q_{EM} = T_{L_3} + Y$$

$(1, 2, -\frac{1}{2})$ L_L	$(3, 2, \frac{1}{6})$ Q^i_L	$(1, 1, -1)$ E_R	$(3, 1, \frac{2}{3})$ U^i_R	$(3, 1, -\frac{1}{3})$ D^i_R
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t^i_R	b^i_R

T_{L3} **weak component of isospin** is quantum number related to weak interactions

Neutrinos T_{L3} = 1/2 component of doublet

Neutrinos have no strong or EM interactions

No right handed neutrinos in the SM
only LH-neutrinos and RH antineutrinos

Board

Recap on Electroweak Theory for neutrinos

In weak interactions, neutrinos and charged leptons come in a $SU(2)_L$ doublet always interact via their **left-handed** components:

$$\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

Right-handed components are $SU(2)_L$ singlets: e_R, ν_R

Write the weak interaction Lagrangian in terms of a single flavour (e) but this can be generalised easily. We begin with **kinetic terms**:

$$\boxed{\mathcal{L} = i\bar{\psi}_L \partial \psi_L + i\bar{e}_R \partial e_R + i\bar{\upsilon}_R \partial \nu_R}$$

We want to add gauge interactions (invariant under local gauge group $SU(2)_L \times U(1)_Y$)

So promote derivative to covariant derivative: $\partial^\mu \rightarrow D^\mu$

$$\boxed{D^\mu = \partial^\mu + i\frac{g}{2}\vec{\tau} \cdot \vec{W}^\mu - i\frac{g'}{2} Y B^\mu}$$

Electroweak Gauge Interactions

$$\vec{\tau}\cdot\vec{W}^{\mu}=\sum_{i=1}^{3}\tau^{i}W^{i\mu}=\begin{pmatrix} W^{3\mu} & W^{1\mu}-iW^{2\mu} \\ W^{1\mu}+iW^{2\mu} & -W^{3\mu}\end{pmatrix}$$

τ - Pauli matrices $SU(2)_L$ generators

$$\mathcal{L}=i\bar{\psi}_L \not{D}\psi_L+i\bar{e}_R\not{D}e_R+i\bar{v}_R\not{D}v_R-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}-\frac{1}{4}G^{i}_{\mu\nu}G^{\mu\nu}_{i}$$

$$D^{\mu}\psi_L=\left(\partial^{\mu}+\frac{ig}{2}\vec{\tau}\cdot\vec{W}^{\mu}-\frac{ig'}{2}B^{\mu}\right)\psi_L$$

$$D^{\mu}e_R=\left(\partial^{\mu}-ig'B^{\mu}\right)e_R$$

$$D^{\mu}\nu_R=\partial^{\mu}\nu_R$$

$$B_{\mu\nu}=\partial_{\mu}B_{\nu}-\partial_{\nu}B_{\mu}$$

$$G^{i}_{\mu\nu}=\left(\partial_{\mu}W^{i}_{v}-\partial_{\nu}W^{i}_{\mu}\right)+g\epsilon_{ijk}W^{j}_{\mu}W^{k}_{v}$$

$$\mathcal{L}=i\bar{\psi}_L\not{\partial}\psi_L-\frac{g}{2}\bar{\psi}_L\gamma_{\mu}\begin{pmatrix} W^{3\mu} & W^{1\mu}-iW^{2\mu} \\ W^{1\mu}+iW^{2\mu} & -W^{3\mu}\end{pmatrix}\psi_L+\frac{g'}{2}\bar{\psi}_L\gamma_{\mu}B^{\mu}\psi_L+i\bar{e}_R\not{\partial}e_R+i\bar{\nu}_R\not{\partial}\nu_R+g'\bar{e}_R\gamma_{\mu}B^{\mu}e_R$$

$$=i\bar{\psi}_L\not{\partial}\psi_L+i\bar{e}_R\not{\partial}e_R+i\bar{\nu}_R\not{\partial}\nu_R-\frac{g}{\sqrt{2}}\left(\bar{\nu}_L\gamma_{\mu}W^{\mu+}e_L+\bar{e}_L\gamma_{\mu}W^{\mu-}\nu_L\right)-\frac{1}{2}\bar{\nu}_L\gamma_{\mu}\left(gW^{3\mu}-g'B^{\mu}\right)\nu_L$$

$$+\frac{1}{2}\bar{e}_L\gamma_{\mu}\left(gW^{3\mu}+g'B^{\mu}\right)e_L+g'\bar{e}_R\gamma_{\mu}B^{\mu}e_R$$

$$W^{\pm}_{\mu}=\frac{W^{1}_{\mu}\mp iW^{2}_{\mu}}{\sqrt{2}}$$

Weak Charged Current and Electromagnetic Interactions

$$\mathcal{L} = i\bar{\psi}_L\gamma^\mu\left(\partial_\mu + \frac{ig}{2}\vec{\tau}\cdot\vec{W}_\mu - \frac{ig'}{2}B_\mu\right)\psi_L + i\bar{e}_R\gamma^\mu\left(\partial_\mu - ig'B_\mu\right)e_R + i\bar{\nu}_R\gamma_\mu\partial^\mu\nu_R$$

$$\begin{aligned}
\mathcal{L}_{\text{int}} =& \bar{\psi}_L\gamma^\mu\frac{1}{2}\left(g'B_\mu - g\vec{\tau}\cdot\vec{W}_\mu\right)\psi_L + g'\bar{e}_R\gamma^\mu B_\mu e_R \\
=& \left(\bar{\nu}_L \quad \bar{e}_L\right)\gamma_\mu\frac{1}{2}\begin{pmatrix} g'B_\mu - gW_\mu^3 & g\left(-W_\mu^1 + iW_\mu^2\right) \\ -g\left(W_\mu^1 + iW_\mu^2\right) & g'B_\mu + gW_\mu^3 \end{pmatrix}\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + g'\bar{e}_R\gamma_\mu B_\mu e_R \\
=& -\frac{g}{\sqrt{2}}\bar{\nu}_L\gamma_\mu W_\mu^+ e_L + \frac{1}{2}\bar{\nu}_L\gamma^\mu\left(g'B_\mu - gW_\mu^3\right)\nu_L - \frac{g}{\sqrt{2}}\bar{e}_L\gamma_\mu W_\mu^- \nu_L \\
&+ \frac{1}{2}\bar{e}_L\gamma^\mu\left(g'B_\mu + gW_\mu^3\right)e_L + g'\bar{e}_R\gamma_\mu B_\mu e_R \\
=& -\frac{g}{\sqrt{2}}\bar{\nu}_L\gamma_\mu W_\mu^+ e_L - \frac{g}{\sqrt{2}}\bar{e}_L\gamma_\mu W_\mu^- \nu_L + \frac{g'}{2}\left(\bar{\nu}_L\gamma_\mu B_\mu\nu_L + \bar{e}_L\gamma_\mu B_\mu e_L + 2\bar{e}_R\gamma_\mu B_\mu e_R\right) \\
&+ \frac{g}{2}\left(\bar{e}_L\gamma_\mu W_\mu^3 e_L - \bar{\nu}_L\gamma_\mu W_\mu^3\nu_L\right).
\end{aligned}$$

Charged Current Interactions

$$\mathcal{L}_{\rm int} = -\frac{g}{2\sqrt{2}}\left(\bar{\nu}_e\gamma^\mu\left(1-\gamma_5\right)eW_\mu^+ + \bar{e}\gamma^\mu\left(1-\gamma_5\right)\nu_e W_\mu^-\right) - \frac{\sqrt{g^2+g'^2}}{2}\bar{\nu}_L\gamma^\mu\nu_L Z_\mu$$
$$+ \frac{gg'}{\sqrt{g^2+g'^2}}\bar{e}\gamma^\mu e A_\mu + \frac{Z_\mu}{\sqrt{g^2+g'^2}}\left[-g'^2\bar{e}_R\gamma^\mu e_R + \frac{g^2-g'^2}{2}\bar{e}_L\gamma^\mu e_L\right]$$

$$Z_\mu = \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2+g'^2}} \qquad A_\mu = \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2+g'^2}}$$

$$\mathcal{L}_{\rm int}^{\rm CC} = -\frac{g}{2\sqrt{2}}\bar{\nu}_e\gamma^\mu\left(1-\gamma_5\right)eW_\mu^+ + \text{h.c.}$$

$$\left(\frac{g}{2\sqrt{2}}\right)^2\frac{1}{M_W^2} = \frac{G_F}{\sqrt{2}} \qquad \text{i.e.} \qquad g^2 = 4\sqrt{2}M_W^2 G_F$$

Neutral Current Interactions

$$\mathcal{L}_{\text{int}} = -\frac{g}{2\sqrt{2}}\left(\bar{\nu}_e\gamma^\mu\left(1-\gamma_5\right)eW_\mu^+ + \bar{e}\gamma^\mu\left(1-\gamma_5\right)\nu_e W_\mu^-\right) - \frac{\sqrt{g^2+g'^2}}{2}\bar{\nu}_L\gamma^\mu\nu_L Z_\mu$$

$$+\frac{gg'}{\sqrt{g^2+g'^2}}\bar{e}\gamma^\mu e A_\mu + \frac{Z_\mu}{\sqrt{g^2+g'^2}}\left[-g'^2\bar{e}_R\gamma^\mu e_R + \frac{g^2-g'^2}{2}\bar{e}_L\gamma^\mu e_L\right] \quad \text{Eq.(1)}$$

$$Z_\mu = \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2+g'^2}} \qquad A_\mu = \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2+g'^2}}$$

$$\mathcal{L}_{\text{NC}}^{\nu} = -\frac{\sqrt{g^2+g'^2}}{2}\bar{\nu}_L\gamma^\mu\nu_L Z_\mu$$

$$\mathcal{L}_{\text{int}} = \underbrace{\frac{gg'}{\sqrt{g^2+g'^2}}}_{e}\bar{e}\gamma^\mu e A_\mu$$

We just derived the interaction of electroweak theory (developed by Glashow, Weinberg and Salam in the 1970's) Nobel Prize 1979.

Electroweak Theory

$$Z_\mu = \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2 + g'^2}} \qquad A_\mu = \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2 + g'^2}}$$

Z and photon are different linear combinations of W^3 and B weighted by different factors. We can conveniently parametrise this in terms of the “weak mixing angle” θ_W

$$Z_\mu = \cos\theta_W W_{3\mu} - \sin\theta_W B_\mu$$
$$A_\mu = \sin\theta_W W_{3\mu} + \cos\theta_W B_\mu$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \qquad M_W = M_Z \cos\theta_W$$

Ex: rewrite $\mathrm{Eq.}(1)$ in terms of weak mixing angle:

$$\begin{aligned}\mathcal{L}_{\text{int}} =& -\frac{g}{2\sqrt{2}}\left(\bar{\nu}_e\gamma^\mu(1-\gamma_5)eW_\mu^+ + \bar{e}\gamma^\mu(1-\gamma_5)\nu_e W_\mu^-\right) - \frac{\sqrt{g^2+g'^2}}{2}\bar{\nu}_L\gamma^\mu\nu_L Z_\mu \\ &+ \frac{gg'}{\sqrt{g^2+g'^2}}\bar{e}\gamma^\mu e A_\mu + \frac{Z_\mu}{\sqrt{g^2+g'^2}}\left[-g'^2\bar{e}_R\gamma^\mu e_R + \frac{g^2-g'^2}{2}\bar{e}_L\gamma^\mu e_L\right]\end{aligned}$$

$\mathrm{Eq.}(1)$

Electroweak Theory

$$\mathcal{L} = \mathcal{L}^{\mathrm{em}} + \mathcal{L}^{\mathrm{CC}} + \mathcal{L}^{\mathrm{NC}}$$

Vector coupling of electron to Z

Axial coupling Electron to Z

$$\mathcal{L}^{\mathrm{em}} = e\bar{e}\gamma^{\mu} e A_{\mu}$$

$$\mathcal{L}^{\mathrm{CC}} = -\frac{g}{2\sqrt{2}}\left[\bar{\nu}_e\gamma^{\mu}\left(1-\gamma_5\right) e W_{\mu}^{+} + \mathrm{h.c.}\ \right]$$

$$\mathcal{L}^{\mathrm{NC}} = -\frac{g}{4\cos\theta_W}\left[\bar{\nu}_e\gamma^{\mu}\left(1-\gamma_5\right)\nu_e + \bar{e}\gamma^{\mu}\left(g_{\nu}^{e} - g_{A}^{e}\gamma_5\right) e\right] Z_{\mu}$$

Where we used

$$g_V^e = 4\sin^2\theta_W\,, \qquad g_A^e = -1,$$

$$\frac{g}{2\sqrt{2}} = \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}}, \quad \frac{g}{4\cos\theta_W} = \frac{1}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{\frac{1}{2}}$$

Neutrino Interactions

$$\nu_e(k) + e^-(p) \rightarrow \nu_e(k') + e^-(p')$$

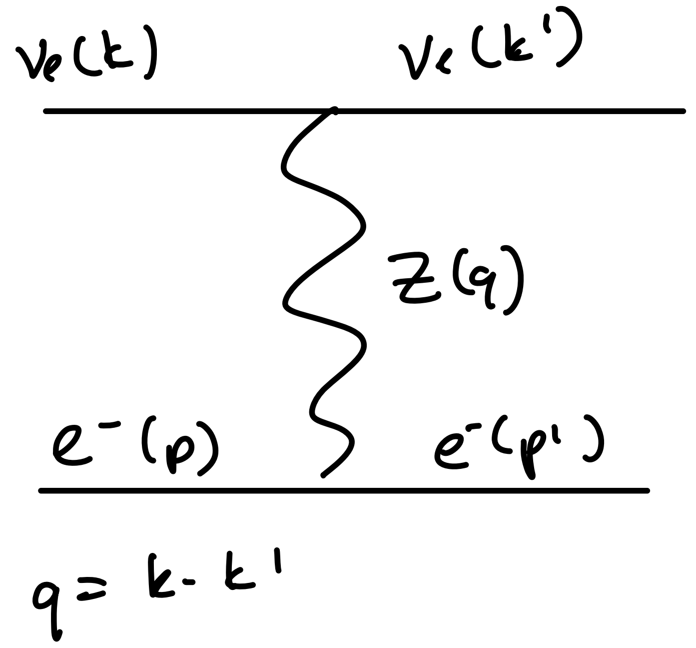
Let's calculate the
cross-section for this scattering process

$$\mathcal{L}_{\nu_e e^- W^+} = \frac{-g}{2\sqrt{2}} \bar{\psi}(p') \gamma_\mu (1-\gamma_5) \psi(k) W^{+\mu}$$

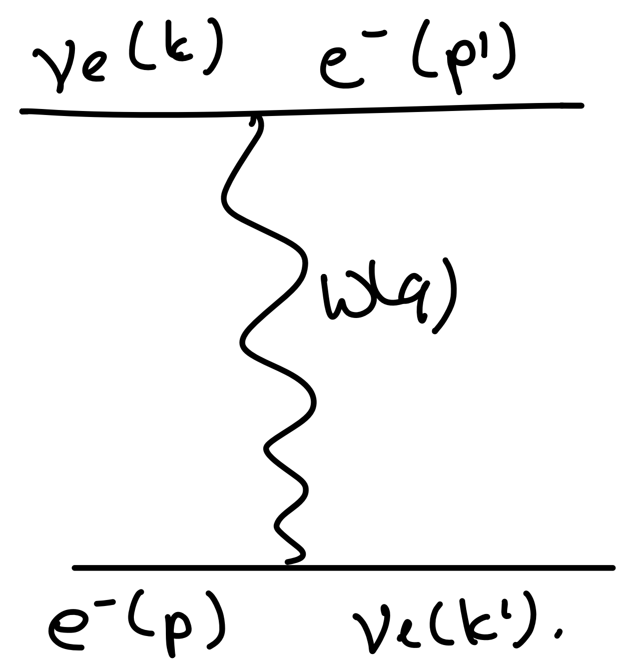
$$L^{\nu\nu Z} = \frac{-g}{4\cos\theta_W} \bar{\psi}(k') \gamma_\mu \left(1-\gamma^5\right) \psi(k) Z^\mu$$

$$L^{eeZ} = \frac{-g}{2\cos\theta_W} \bar{\psi}(p') \gamma_\mu \left(\tilde{g}_V^e - \tilde{g}_A^e \gamma^5\right) \psi(p) Z^\mu$$

$$\tilde{g}_V^e = -\frac{1}{2} + 2\sin^2\theta_W \qquad \tilde{g}_A^e = \frac{1}{2}$$



Neutral Current Interactions



Charged Current Interactions

$$-i\mathcal{M}^{\mathrm{CC}} = \left[\bar{u}(p') \frac{-ig}{2\sqrt{2}} \gamma_\mu (1-\gamma_5) u(k)\right] \left(-\frac{ig^{\mu v}}{M_W^2}\right) \left[\bar{u}(k') \frac{-ig}{2\sqrt{2}} \gamma_v (1-\gamma_5) u(p)\right],$$

$$\Rightarrow \mathcal{M}^{\mathrm{CC}} = \frac{G_F}{\sqrt{2}} \left[\bar{u}(p') \gamma_\mu (1-\gamma_5) u(k)\right] \cdot \left[\bar{u}(k') \gamma^\mu (1-\gamma_5) u(p)\right],$$

Where we used the replacement:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

Neutrino Interactions

$$-i\mathcal{M}_{\rm NC} = \left[\bar{u}\left(k'\right)\frac{-ig}{4\cos\theta_W}\gamma_\mu\left(1-\gamma_5\right)u(k)\right]\times\left(-\frac{ig^{\mu\nu}}{M_Z^2}\right)$$
$$\times\left[\bar{u}\left(p'\right)\frac{-ig}{2\cos\theta_W}\gamma_v\left(\tilde{g}_V^e-\tilde{g}_A^e\gamma_5\right)u(p)\right],$$
$$\Rightarrow \mathcal{M}_{\rm NC} = \frac{G_F}{\sqrt{2}}\left[\bar{u}\left(k'\right)\gamma_\mu\left(1-\gamma_5\right)u(k)\right]\cdot\left[\bar{u}\left(p'\right)\gamma^\mu\left(\tilde{g}_V^e-\tilde{g}_A^e\gamma_5\right)u(p)\right]$$

The following Fierz arrangement will be used:

$$(\overline{\Psi}_1\gamma_\mu P_L\Psi_2)(\overline{\Psi}_3\gamma^\mu P_L\Psi_4) = (\overline{\Psi}_1\gamma_\mu P_L\Psi_4)(\overline{\Psi}_3\gamma^\mu P_L\Psi_2)$$

$$\mathcal{M} = \mathcal{M}_{\rm CC}+\mathcal{M}_{\rm NC} = \frac{G_F}{\sqrt{2}}\left[\left[\bar{u}\left(p'\right)\gamma_\mu\left(1-\gamma_5\right)u(k)\right]\cdot\left[\bar{u}\left(k'\right)\gamma^\mu\left(1-\gamma_5\right)u(p)\right]\right.$$
$$\left.+\left[\bar{u}\left(k'\right)\gamma_\mu\left(1-\gamma_5\right)u(k)\right]\cdot\left[\bar{u}\left(p'\right)\gamma^\mu\left(\tilde{g}_V^e-\tilde{g}_A^e\gamma_5\right)u(p)\right]\right].$$

Neutrino Interactions

The matrix element squared is:

$$\overline{\sum_i}\sum_f |\mathcal{M}|^2 = \overline{\sum_i}\sum_f \left(|\mathcal{M}_{\mathrm{CC}}|^2 + \mathcal{M}_{\mathrm{CC}}\mathcal{M}^*_{\mathrm{NC}} + \mathcal{M}_{\mathrm{NC}}\mathcal{M}^*_{\mathrm{CC}} + |\mathcal{M}_{\mathrm{NC}}|^2\right)$$

$$= 16\mathrm{G}_F^2\left[\left(g'_V + g'_A\right)^2 (k'\cdot p')(k\cdot p) + \left(g'_V - g'_A\right)^2 (k'\cdot p)(k\cdot p') - m_e^2\left(g'^2_V - g'^2_A\right)(k\cdot k')\right]$$

Note that we average over the spins of the incoming states, sum of the spins of the outgoing states $\implies 1/2 \times 1/2$ factor. The scalar products are a result of applying the completeness relation and tracing over spinors and Gamma matrices. We also redefined

$$g'_V = \tilde{g}^e_V + 1 \quad g'_A = \tilde{g}^e_A + 1$$

This expression for the matrix element squared can be written for $\nu_\mu e^- \to \nu_\mu e^-, \bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-, \bar{\nu}_e e^- \to \bar{\nu}_e e^-$ as

$$\overline{\sum_i}\sum_f |\mathcal{M}|^2 = 16G_F^2\left[\alpha\,(k'\cdot p')(k\cdot p) + \beta\,(k'\cdot p)(k\cdot p') - \gamma m_e^2\,(k\cdot k')\right] \quad \text{Eq. (2)}$$

Where α, β, γ depends on the various couplings of neutrinos/antineutrino to leptons.

Neutrino Interactions

We have the following expression for the matrix element squared but to find Differential cross-section we still need to integrate over phase space of the outgoing states. In the centre-of-mass frame:

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{4\pi^2 s} G_F^2 \left[(g'_V + g'_A)^2 \left(\frac{s - m_e^2}{2}\right)^2 + (g'_V - g'_A)^2 \left(\frac{u - m_e^2}{2}\right)^2 + \frac{m_e^2}{2} \left\{ (g'_V)^2 - (g'_A)^2 \right\} t \right] \text{ Eq. (3)}$$

The centre-of-mass frame is defined as and we use the Mandelstam variables:

$$s = (k+p)^2 = (k'+p')^2, \quad t = (k-k')^2 = (p'-p)^2, \quad u = (k-p')^2 = (k'-p)^2$$

To go from Eq. (2) to Eq. (3), integrate over the momentum of the outgoing fermions and replace the scalar products of four-momenta in the following way:

$$s = (k+p)^2 = k^2 + p^2 + 2(k.p) = m_e^2 + 2(k.p) \implies (k.p) = \frac{s - m_e^2}{2}$$

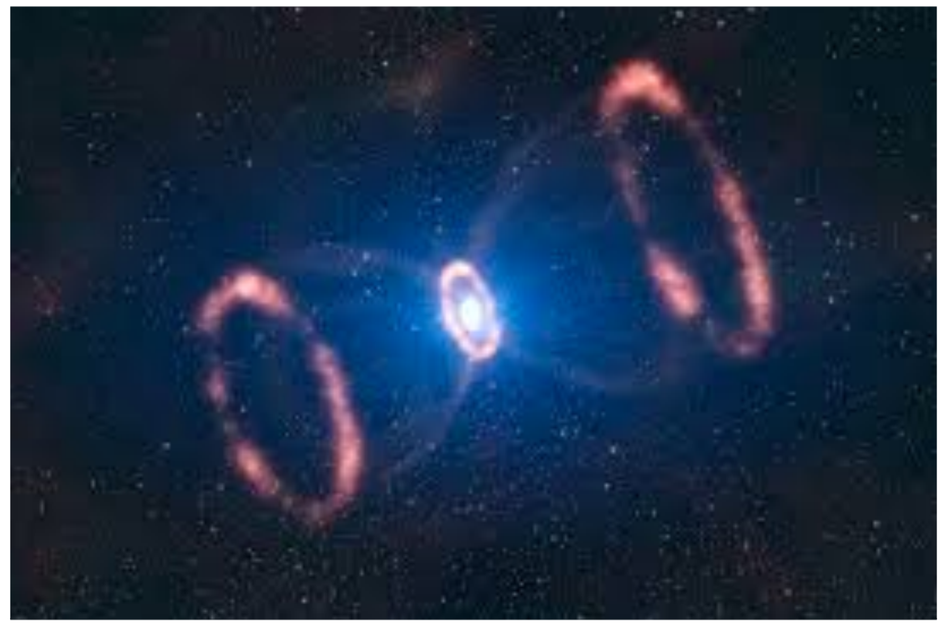
Neutrino Interactions

- By 1990s we knew there were three neutrinos: extremely light, electrically neutral and very weakly interacting → least well understood particle of the SM

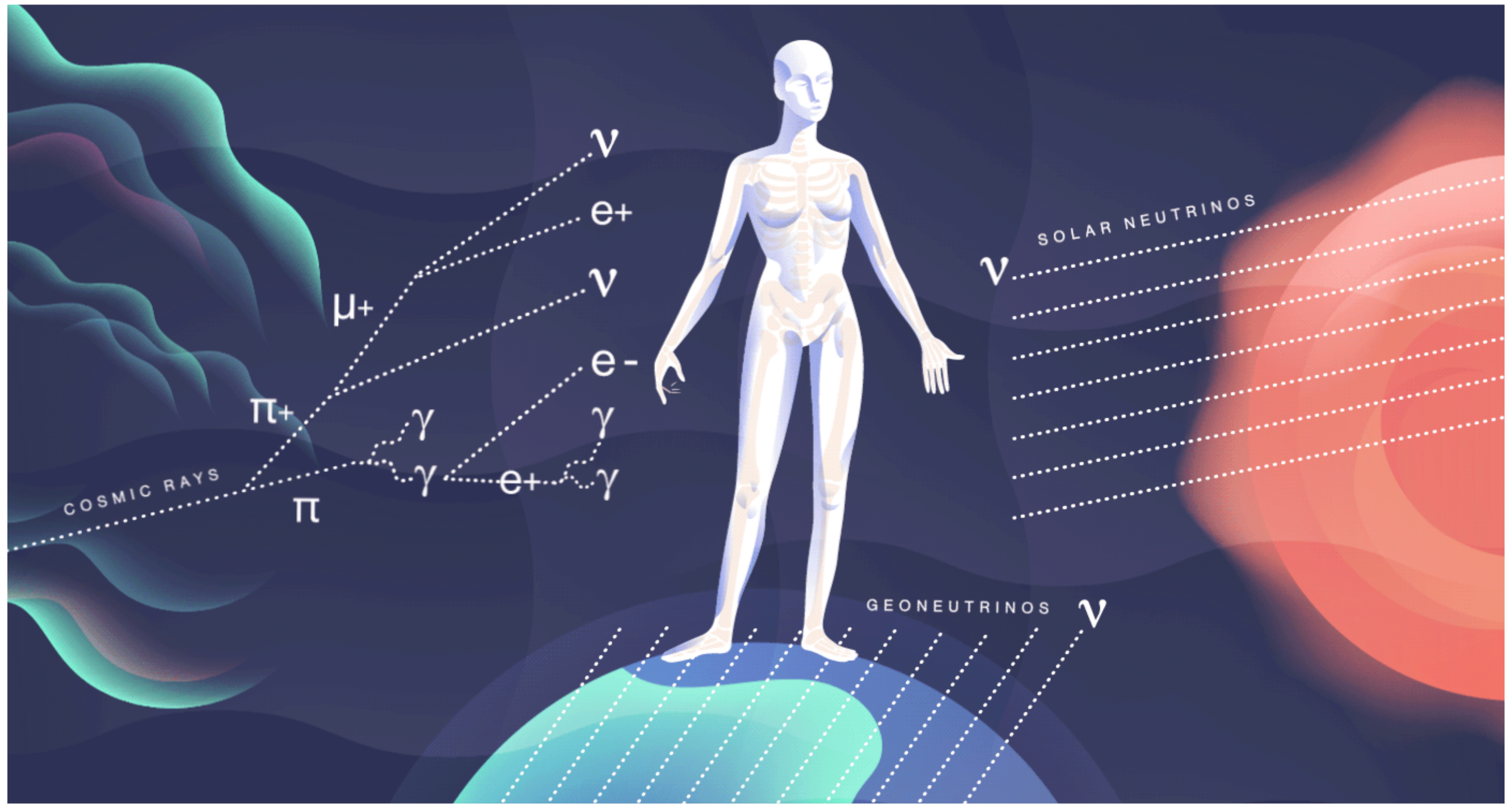
BB: $E_\nu \sim 10^{-4}\,\mathrm{eV}$ $\rho_\nu \sim 330/\mathrm{cm}^3$

Human:
$\Phi_\nu \sim 360 \times 10^6 \nu/\mathrm{day}$

SN1986: $E_\nu \sim \mathrm{MeV}$



atmospheric neutrino:
$E_\nu \sim \mathrm{MeV} - \mathrm{EeV}$
$\Phi_\nu \sim 1\,\nu/\mathrm{cm}^2\mathrm{s}$



Earth: $E_\nu \sim \mathrm{MeV}$

$\Phi_\nu \sim 6 \times 10^6 \nu/\mathrm{cm}^2\mathrm{s}$

$E_\nu \sim \mathrm{MeV}$

$E_\nu \sim \mathrm{GeV}$

Neutrino Interactions

Consider atmospheric muon neutrinos with mean energy of a GeV. The interaction cross-section with a proton is

$$\sigma_{\nu p} \sim 10^{-38} \mathrm{cm}^2 \frac{\mathrm{E}_\nu}{\mathrm{GeV}}$$

How many atmospheric muon neutrinos would interact with you in your lifetime?

$$\Phi = \frac{1\,\nu}{\mathrm{cm}^2 \mathrm{sec}} \qquad \langle E_\nu \rangle \sim \mathrm{GeV} \qquad N_{\mathrm{events}} = \sigma \times \Phi \times N_{\mathrm{target}} \times \mathrm{Time}$$

$$1 \text{ kg water } \Longrightarrow 3.3 \times 10^{25} \mathrm{H_2O} \Longrightarrow 1 \text{ kg water } \Longrightarrow 3.3 \times 10^{26} \text{ protons}$$

Say you’re 60 kg and live to a ripe old age

$$N_{\mathrm{target}} \sim 2 \times 10^{28} \text{ protons} \quad \mathrm{Time} = 80 \text{ years} = 2 \times 10^{9} \text{ secs}$$

$$N_{\mathrm{events}} \sim 2 \times 10^{28} \times 2 \times 10^{9} \times 10^{-38} \sim 1!$$

It’s likely you will have 1 CC interaction from atmospheric neutrinos your whole life! Not sure why cinema paints neutrinos in such a menacing light (see Alien Covenant and The Core) **Physics point: need huge detectors with long exposures!**