Flavour Physics

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Lecture 1

Outline

- what is flavour physics
- CKM matrix and Unitarity Triangle
- CP violation in the Standard Model
- CPV in B physics

What is flavour physics?

WIKIPEDIA
The Free Encyclopedia

Flavour (particle physics)

From Wikipedia, the free encyclopedia

In particle physics, **flavour** or **flavor** is a quantum number of elementary particles. In quantum chromodynamics, flavour is a global symmetry. In the electroweak theory, on the other hand, this symmetry is broken, and flavour-changing processes exist, such as quark decay or neutrino oscillations.

Flavour in particle physics

Flavour quantum numbers:

- Baryon number: B
- Lepton number: L
- Strangeness: S
- Charm: C
- Bottomness: B'
- Topness: T
- Isospin: **I** or I_3
- Weak isospin: **T** or T_3
- Electric charge: Q
- X-charge: X

Combinations:

- Hypercharge: Y
  - $Y = (B + S + C + B' + T)$
  - $Y = 2\,(Q - I_3)$
- Weak hypercharge: Y_W
  - $Y_W = 2\,(Q - T_3)$
  - $X + 2Y_W = 5\,(B - L)$

Flavour mixing

- CKM matrix
- PMNS matrix
- Flavour complementarity

“The term flavor was first used in particle physics in the context of the quark model of hadrons. It was coined in 1971 by Murray Gell-Mann and his student at the time, Harald Fritzsch, at a Baskin-Robbins ice-cream store in Pasadena. Just as ice cream has both color and flavor so do quarks.”

RMP 81 (2009) 1887

What is flavour physics?

- ◉ 3 gauge couplings + QCD vacuum angle
- ◉ 2 Higgs parameters
- ◉ 6 quark masses
- ◉ 3 quark mixing angles + 1 phase
- ◉ 3 (+3) lepton masses
- ◉ (3 lepton mixing angles + 1 phase)

Cabibbo–Kobayashi–Maskawa

CKM matrix

PMNS matrix

Pontecorvo–Maki–Nakagawa–Sakata

flavour parameters

( ) = with Dirac neutrino masses

What is flavour physics?

$m_u \approx 3$ MeV
$m_d \approx 5$ MeV
$m_s \approx 100$ MeV
$m_c \approx 1300$ MeV
$m_b \approx 4200$ MeV
$m_t \approx 170000$ MeV

$m_{\nu 1} \leq 10^{-6}$ MeV
$m_{\nu 2} \leq 10^{-6}$ MeV
$m_{\nu 3} \leq 10^{-6}$ MeV
$m_e \approx 0.5$ MeV
$m_\mu \approx 100$ MeV
$m_\tau \approx 1800$ MeV

The neutrinos have their own phenomenology

Studies of the u and d quarks are the realm of nuclear physics

Rare decays of kaons provide sensitive tests of the SM

Studies of electric and magnetic dipole moments of the leptons test the Standard Model

Searches for lepton flavour violation are another hot topic

The top quark has its own phenomenology (since it does not hadronise)

Heavy quark flavour physics

The focus in these lectures will be on:

◎ CKM matrix as source of CP violation in the Standard Model

Hence specifically

◎ flavour-changing interactions of beauty quarks

- ◉ charm is also very interesting and I will mention it very briefly

But quarks feel the strong interaction and hence hadronise:

◎ various different charmed and beauty hadrons

- ◉ many, many possible decays to different final states
- ◉ hadronisation greatly increases the observability of CP violation

Why is heavy flavour physics interesting?

- ◉ Hope to learn something about the mysteries of the flavour structure of the Standard Model
- ◉ CP violation and its connection to the matter–antimatter asymmetry of the Universe
- ◉ Discovery potential far beyond the energy frontier via searches for rare or SM forbidden processes

Flavour for new physics discoveries

A lesson from history:

- ◎ New physics showed up at precision frontier before energy frontier
  - ◉ GIM mechanism before discovery of charm
  - ◉ CP violation / CKM before discovery of bottom & top
  - ◉ Neutral currents before discovery of Z
- ◎ Particularly sensitive – loop processes
  - ◉ Standard Model contributions suppressed / absent
  - ◉ flavour changing neutral currents (rare decays)
  - ◉ CP violation
  - ◉ lepton flavour / number violation / lepton universality

FCNC suppressed
$\Delta S=2$ suppressed wrt $\Delta S=1$

NP scale analysis from $\Delta S=2$ processes

What breaks the flavour symmetries?

◎ In the Standard Model, the vacuum expectation value of the Higgs field breaks the electroweak symmetry

◎ Fermion masses arise from the Yukawa couplings of the quarks and charged leptons to the Higgs field (taking m_n=0)

◎ The CKM matrix arises from the relative misalignment of the Yukawa matrices for the up- and down-type quarks

◎ Consequently, the only flavour-changing interactions are the charged current weak interactions

- ◉ no flavour-changing neutral currents (GIM mechanism)
- ◉ not generically true in most extensions of the SM
- ◉ flavour-changing processes provide sensitive tests

CP violation source in the Standard Model

- The CKM matrix arises from the relative misalignment of the Yukawa matrices for the up- and down-type quarks:
  - It is a 3x3 complex unitary matrix described by 4 (real) parameters:
    - 3 can be expressed as (Euler) mixing angles
    - the fourth makes the CKM matrix complex (i.e. gives it a phase)
      - weak interaction couplings differ for quarks and antiquarks
      - CP violation

We need more CP violation

To create a larger asymmetry, require:

- ▣ new sources of CP violation
  - ▻ that occur at high energy scales

Where might we find it?

- ▣ lepton sector: CP violation in neutrino oscillations
- ▣ quark sector: discrepancies with KM predictions
- ▣ gauge sector, extra dimensions, other new physics:
  - ▻ precision measurements of flavour observables are generically sensitive to additions to the Standard Model

CKM quark-mixing matrix

CK @ CKM2006 in Nagoya

CKM matrix in the Standard Model: quark mixing

The charged current interactions get a flavour structure encoded in the Cabibbo-Kobayashi-Maskawa (CKM) matrix V:

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}}\left(\bar{\tilde{U}}_L \gamma^\mu W_\mu^+ V \tilde{D}_L + \bar{\tilde{D}}_L \gamma^\mu W_\mu^- V^\dagger \tilde{U}_L\right).$$

V_{ij} connects left-handed up-type quark of the *i*th generation to left-handed down-type quark of *j*th generation. Intuitive labelling by flavour:

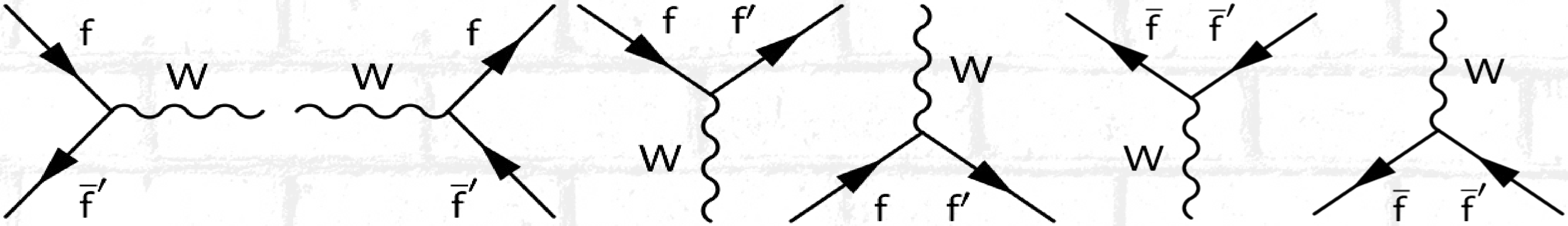
$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \quad V_{13} = V_{ub} \;\; etc$$

Matrix V is unitary by construction

Via W exchange is the only way to change flavour in the SM.

CKM matrix in the Standard Model: quark mixing

Quarks change type in weak interactions.
We parameterise the couplings V_{ij} in the CKM matrix.
All the possible weak interaction involving a W are combinations of:



Where f = e, μ, τ, d, s, b
f' = ν_e, ν_μ, ν_τ, u, c, t

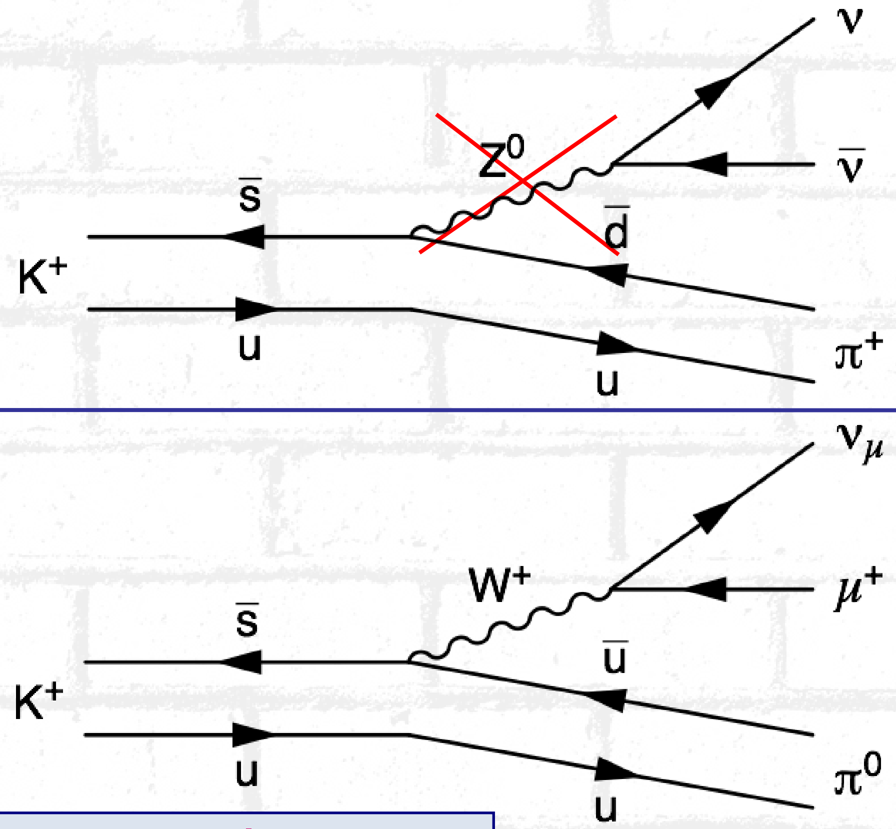
The $W^{\pm}$ is flavour changing i.e. u → d, s → u etc

No Flavour Changing Neutral Currents

All observed neutral currents are found to obey $\Delta S = 0$

Measure ratio:

$$\frac{K^+ \rightarrow \pi^+ + \nu + \bar{\nu}}{K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu} = \frac{\text{[top diagram]}}{\text{[bottom diagram]}} < 10^{-5}$$



There are no flavour changing neutral currents

CKM matrix in the Standard Model

With u, d, s, c quarks the weak charged current is given by:

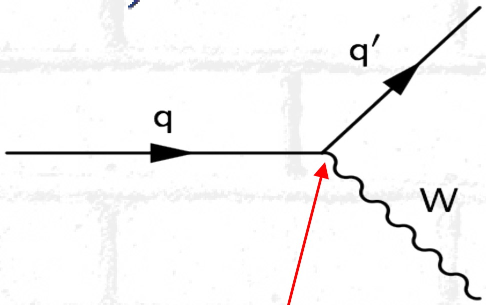
$$J^- = (\bar{u}, \bar{c}) \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Normal coordinate rotation matrix

$$\begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}$$

With u, d, s, c, b, t quarks this becomes:

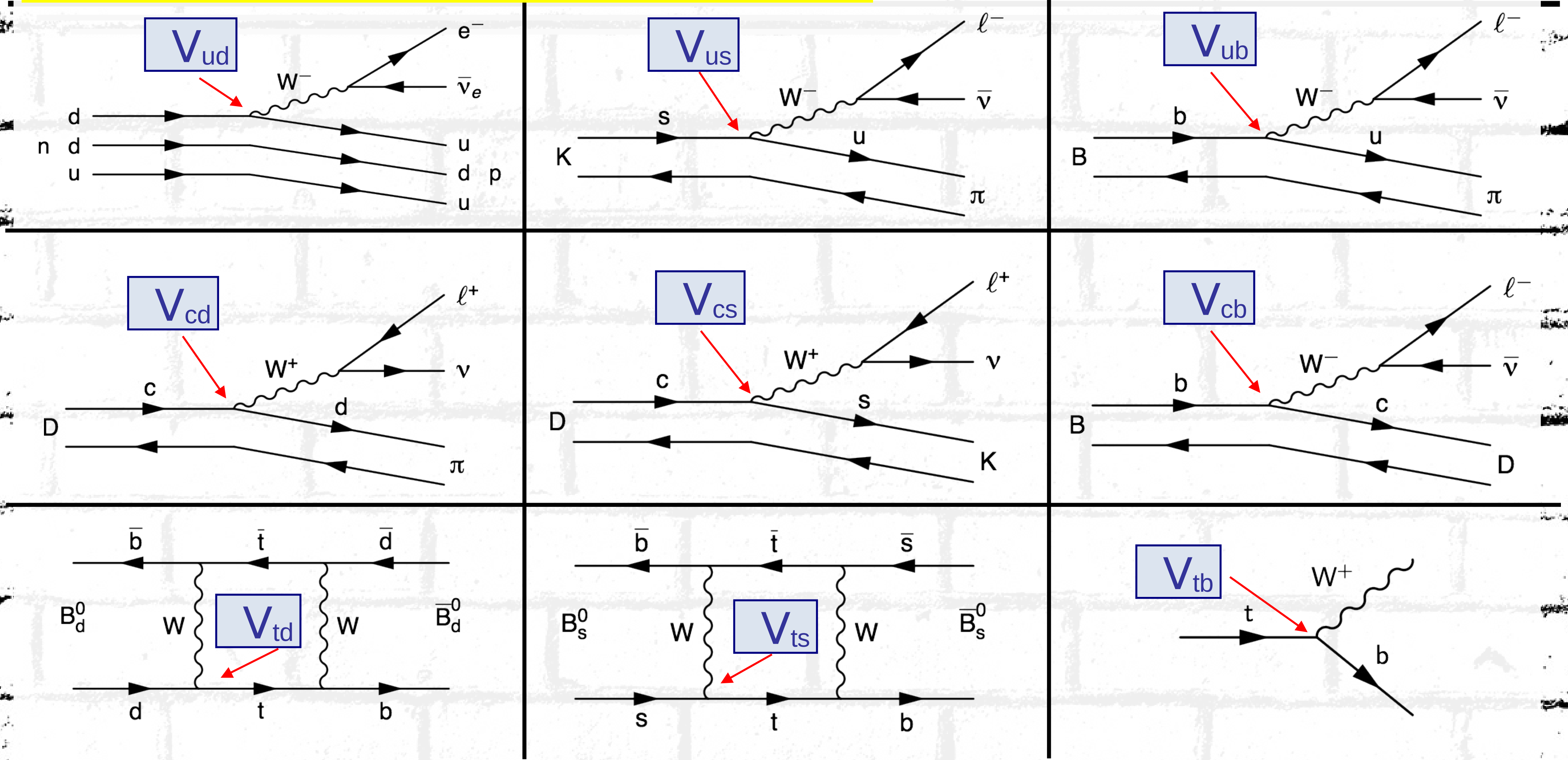
$$J^- = (\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



The Cabibbo, Kobayashi, Maskawa (CKM) Matrix

Put $V_{qq'}$ in amplitude

CKM matrix in the Standard Model



CKM matrix: rotation decomposition

The CKM matrix can be seen as the product of three rotation matrices and each rotation involves two of the three families:

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

which gives the classic exact parameterisation that can be found for example on the PDG:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

with $c_{ij}=\cos\theta_{ij}$ and $s_{ij}=\sin\theta_{ij}$, and i,j=1,2,3. δ is the CP violating phase

Unitary matrix independent parameters

In general, an n x n unitary matrix has n^2 real and independent parameters:

- ► a n x n matrix would have $2n^2$ parameters
- ► the unitary condition imposes n normalization constraints
- ► n(n - 1) conditions from the orthogonality between each pair of columns:

thus $2n^2 - n - n(n - 1) = n^2$.

In the CKM matrix, not all of these parameters have a physical meaning:

- ► given n quark generations, 2n - 1 phases can be absorbed by selecting the phases of quark fields
  - ▻ Each u, c or t phase allows for multiplying a row of the CKM matrix by a phase, while each d, s or b phase allows for multiplying a column by a phase.

thus: $n^2 - (2n - 1) = (n - 1)^2$.

Among the n^2 real independent parameters of a generic unitary matrix:

- ► ½ n(n - 1) of these parameters can be associated to real rotation angles, so the number of independent phases in the CKM matrix case is:

$n^2 - ½\, n(n- 1) - (2n - 1) = ½\, (n - 1)(n - 2)$

n(families)	Total indep. params. $(n-1)^2$	Real rot. angles $\frac{1}{2}n(n-1)$	Complex phase factors $\frac{1}{2}(n-1)(n-2)$
2	1	1	0
3	4	3	1
4	9	6	3

CKM matrix: Wolfenstein parameterisation

From measurements, V results hierarchical → $\theta_{13} \ll \theta_{23} \ll \theta_{12}$

We can see this hierarchy via the Wolfenstein parameterisation:
→ the CKM matrix elements are expanded in order of sin θ_{12} historically called Cabibbo angle θ_C:

→ Wolfenstein parameter $\lambda = \sin\theta_{12} \sim 0.22$

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + (\lambda^4)$$

→ Wolfenstein parameters: $\lambda \sim 0.22$, $A \sim 0.83$, $\rho \sim 0.15$, $\eta \sim 0.35$

CKM matrix: Wolfenstein parameterisation

From the Wolfenstein parameter $\lambda = \sin\theta_{12} \sim 0.22$, we can get an idea on the sizes of the various CKM matrix elements:

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + (\lambda^4)$$

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At λ^2 order, the third generation decouples

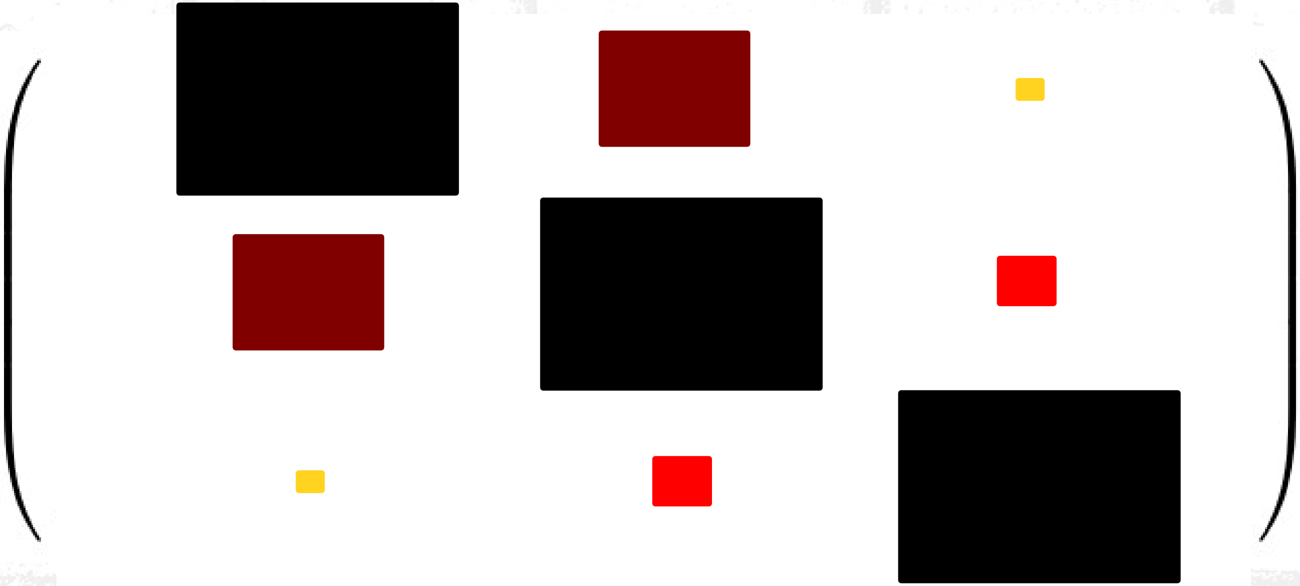
$\eta \neq 0$ signals CP violation

→ imaginary part of the V_{ub} and V_{td} elements ($1^{st} \rightleftarrows 3^{rd}$ family)

CKM matrix: Wolfenstein parameterisation

From the Wolfenstein parameter $\lambda = \sin\boldsymbol{\theta}_{12} \sim 0.22$, we can get an idea on the sizes of the various CKM matrix elements:

V_{CKM} =



$+ (\lambda^4)$

So the preferred decays are t → b → c → s → u

Unitarity relations

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

multiply with its hermitian conjugate (complex conjugate + transpose)
$VV^\dagger = V^\dagger V = \mathbf{1}$

$$\Sigma_i V_{ij} V^*_{ik} = \delta_{jk}$$ column orthogonality

$$\Sigma_j V_{ij} V^*_{kj} = \delta_{ik}$$ row orthogonality

The six vanishing combinations can be represented as triangles in a complex plane

Unitarity relations

The triangles obtained by taking scalar products of neighboring rows or columns are nearly degenerate. However, the areas of all triangles are the same, half of the Jarlskog invariant J.

$1^{st} \rightleftarrows 2^{nd}$ family

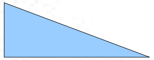
$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* \simeq \mathcal{O}(\lambda) + \mathcal{O}(\lambda) + \mathcal{O}(\lambda^5) = 0$$

$2^{nd} \rightleftarrows 3^{rd}$ family

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* \simeq \mathcal{O}(\lambda^4) + \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^2) = 0$$

triangles
not to scale

$1^{st} \rightleftarrows 3^{rd}$ family



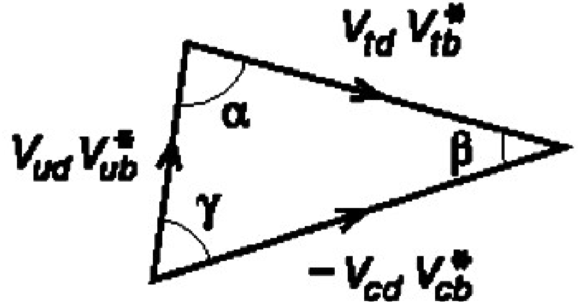
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \simeq \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) = 0$$

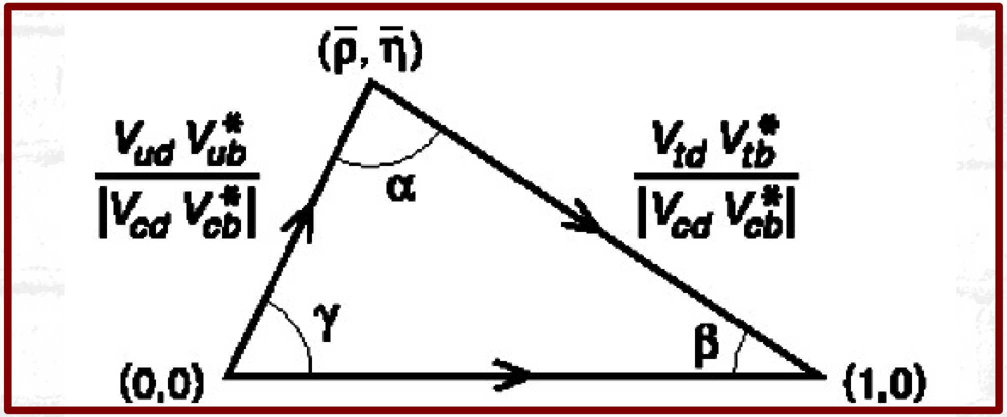
Third unitarity relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \simeq \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) = 0$$

$V_{id}V^*_{ib} = 0$ represents the orthogonality condition between the first and the third column of the CKM matrix (the orientation depends on the phase convention)

re-scaled version where sides have been divided by $|V_{cd}V^*_{cb}|$

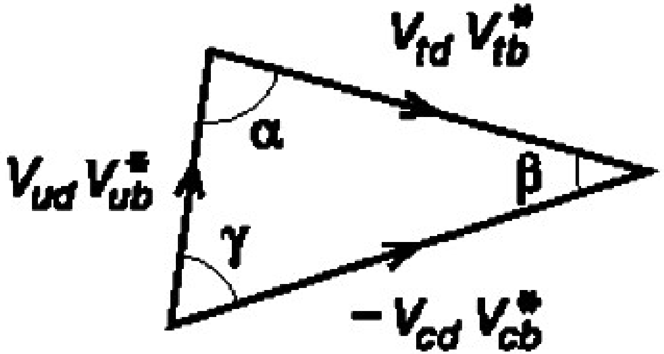




In terms of the Wolfenstein parameterization, the coordinates of this triangle are (0, 0), (1, 0) and ($\bar\rho$, $\bar\eta$):
the two sides are ($\bar\rho + i\bar\eta$) and ($1 - \bar\rho - i\bar\eta$).

The Unitarity Triangle

$$V_{ud}V_{ub}^{*}+V_{cd}V_{cb}^{*}+V_{td}V_{tb}^{*} \simeq \mathcal{O}(\lambda^3)+\mathcal{O}(\lambda^3)+\mathcal{O}(\lambda^3) = 0$$

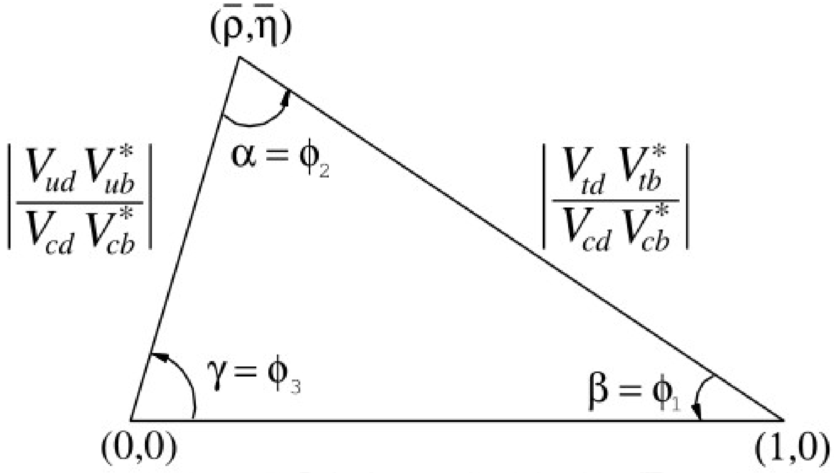


The angles can be written in terms of CKM matrix elements as:

$$\alpha \equiv \arg[-V_{td}V_{tb}^{*}/V_{ud}V_{ub}^{*}]$$
$$\beta \equiv \arg[-V_{cd}V_{cb}^{*}/V_{td}V_{tb}^{*}]$$
$$\gamma \equiv \arg[-V_{ud}V_{ub}^{*}/V_{cd}V_{cb}^{*}]$$

The Unitarity Triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \simeq \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) = 0$$



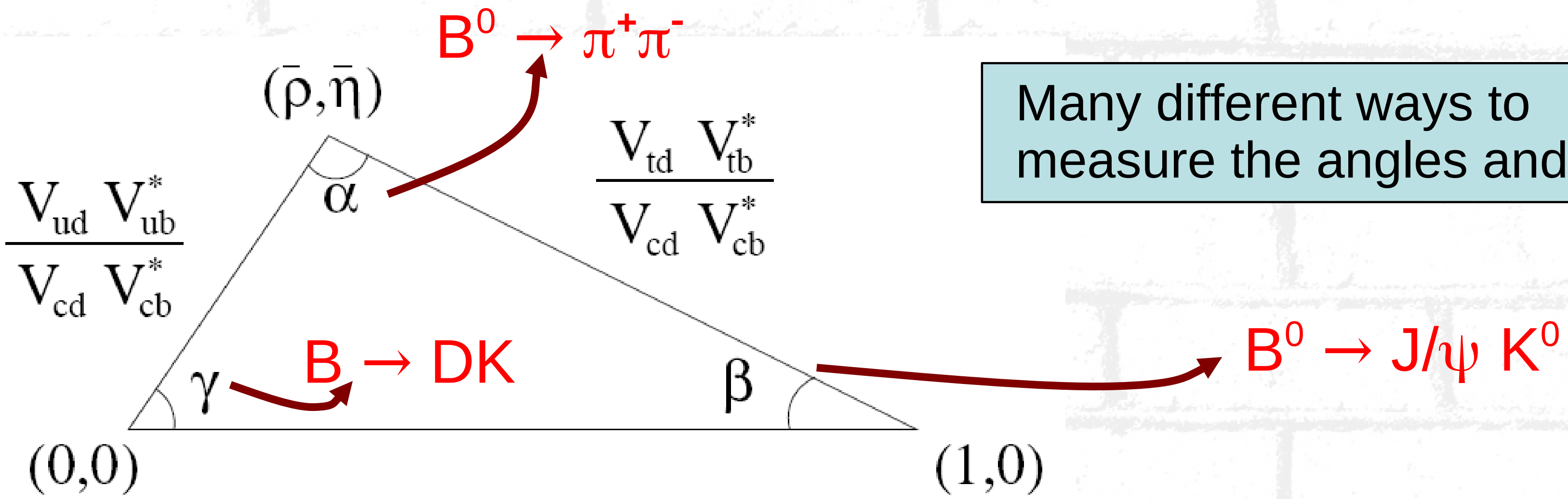
The angles can be written in terms of CKM matrix elements as:

$$\alpha \equiv \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*]$$
$$\beta \equiv \arg[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*]$$
$$\gamma \equiv \arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]$$

In the Wolfenstein parameterisation:

- the β/ϕ_1 angle corresponds to the phase of V_{td}
- the γ/ϕ_3 angle corresponds to the phase of V_{ub}
- the α/ϕ_2 angle can be obtained with $\pi - \beta - \gamma$ (assumes unitarity)

Probing the structure of the CKM mechanism



Many different ways to measure the angles and sides.

- We need to measure the angles and sides to over-constrain this triangle, and test that it closes.
- Need to define observables and experiments to measure these quantities

The Unitarity Triangle

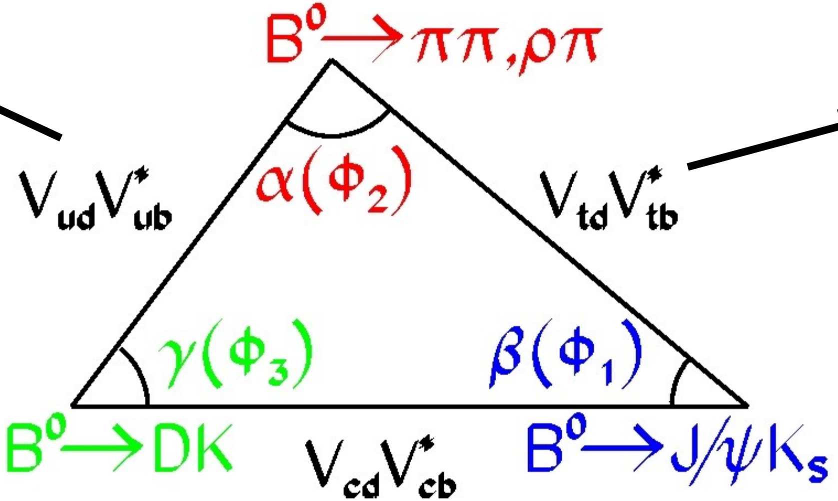
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

many observables functions of $\bar{\rho}$ and $\bar{\eta}$: overconstraining

$$\alpha = \pi - \beta - \gamma$$

normalised:
$\bar{\rho} + i\bar{\eta}$

normalised:
$1 - \bar{\rho} - i\bar{\eta}$

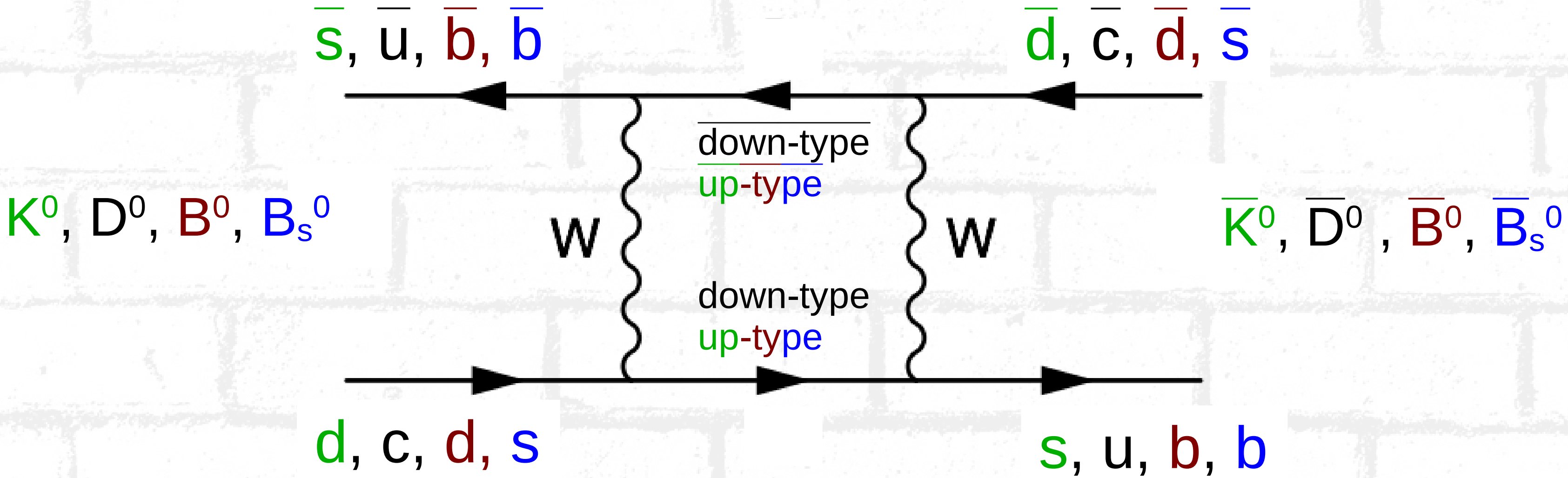


$$\gamma = atan\left(\frac{\bar{\eta}}{\bar{\rho}}\right)$$

$$\beta = atan\left(\frac{\bar{\eta}}{(1-\bar{\rho})}\right)$$

Neutral Meson Systems

The amazing case of neutral non-flavourless meson systems
→ considering neutral mesons $u\bar{u}'$ where u has a different flavour with respect to u' → so not applicable to $c\bar{c}$ for example
These systems are:
→ K^0-$\bar{K}^0$ ($d\bar{s}$), D^0-$\bar{D}^0$ ($c\bar{u}$), B^0-$\bar{B}^0$ ($d\bar{b}$), B_s^0-$\bar{B}_s^0$ ($s\bar{b}$)
they are subject to the mixing phenomenon via box diagrams:



Neutral Meson Systems

These systems are:
→ K^0-$\overline{K}^0$ ($d\overline{s}$), D^0-$\overline{D}^0$ ($c\overline{u}$), B^0-$\overline{B}^0$ ($d\overline{b}$), B_s^0-$\overline{B}_s^0$ ($s\overline{b}$)

The neutral meson mixing corresponds to another case of misallignment between two sets of eigenstates:

Flavour eigenstates → defined flavour content:

M^0 and $\overline{M}^0$

Mass eigenstates → defined masses $m_{1,2}$ and decay width $\Gamma_{1,2}$:

$pM^0 \pm q\overline{M}^0$

p & q complex coefficients that satisfy $|p|^2 + |q|^2 = 1$

In the famous case of kaons: $K_{S,L} \sim (1+\varepsilon)K^0 \pm (1-\varepsilon)\overline{K}^0$

In the formalism for the B mesons: $B_{L,H} \sim pB^0 \pm q\overline{B}^0$

Types of CP Violation in the SM

Three Types of CP Violation

Need more than one amplitude to have a non-zero CP violation: *interference*

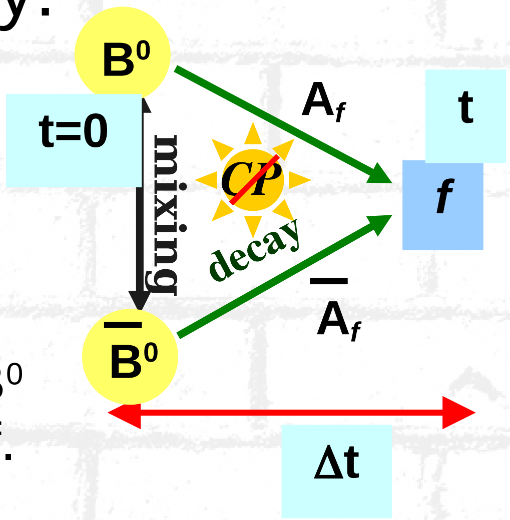
1. Indirect CP violation, or CPV in mixing:

$$P(B^0 \rightarrow \overline{B}^0) \neq P(\overline{B}^0 \rightarrow B^0)$$

2. Direct CP violation, or CPV in the decay:

$$P(B^0 \rightarrow f) \neq P(\overline{B}^0 \rightarrow \bar{f})$$

3. CPV in the interference between mixing and decay.



Cartoon shows the decay of a B^0 or $\overline{B}^0$ into a common final state f.

CPV Types for the B Meson System

◎ Define the quantity λ:

$$\lambda_{f_{CP}} = \frac{q}{p} \cdot \frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}}$$

1. Indirect CP violation, or CPV in the mixing:
   $| q/p | \neq 1$
2. Direct CP violation, or CPV in the decays:
   $|\bar{A}/A | \neq 1$ ← both neutral and charged B
3. CP violation in interference between mixing and decay: $\mathrm{Im}\lambda \neq 0$ ← neutral B

Time evolution and CP violation

◎ Consider a B meson which is known to be a B (or $\overline{B}$) at t=0
◎ at t>0, the physical state has evolved in time with the amplitutes:

$$B^0_{phys}(t) = e^{-iMt} e^{-\Gamma t/2} \cos(\Delta m\ t/2)\ B^0 + i(q/p)\ e^{-iMt} e^{-\Gamma t/2} \sin(\Delta m\ t/2)\ \overline{B}^0$$

$$\overline{B}^0_{phys}(t) = i(q/p)\ e^{-iMt} e^{-\Gamma t/2} \sin(\Delta m\ t/2)\ B^0 + e^{-iMt} e^{-\Gamma t/2} \cos(\Delta m\ t/2)\ \overline{B}^0$$

mixing parameters

Δm = mass difference between the two mass eigenstates

→ in case of the B^0 mesons, difference between the heavy and light states

→ $\Delta m_d = 0.507 \pm 0.005$ ħ/ps

Time evolution and CP violation

◎ If we consider that both B^0 and $\overline{B}^0$ can decay to the same final state and considering here a final state that is a CP eigenstate, then the time evolution of the physical system becomes:

$$f(B^0_{phys} \to f_{CP}, \Delta t) = \frac{\Gamma}{4} e^{-\Gamma|\Delta t|} \left[1 - S_{f_{CP}} \sin(\Delta m_d \Delta t) + C_{f_{CP}} \cos(\Delta m_d \Delta t)\right]$$

$$f(\bar{B}^0_{phys} \to f_{CP}, \Delta t) = \frac{\Gamma}{4} e^{-\Gamma|\Delta t|} \left[1 + S_{f_{CP}} \sin(\Delta m_d \Delta t) - C_{f_{CP}} \cos(\Delta m_d \Delta t)\right]$$

- ◉ direct CP violation $C \neq 0$ $$C_f (= -A_f) = \frac{1 - |\lambda_{f_{CP}}|^2}{1 + |\lambda_{f_{CP}}|^2}$$
- ◉ CP violation in interference $S \neq 0$ $$S_f = \frac{2 Im \lambda_{f_{CP}}}{1 + |\lambda_{f_{CP}}|^2}$$

Time-dependent CP asymmetries

- Ingredients of a time-dependent CP asymmetry measurement:
  - Isolate interesting signal B decay: B_{RECO}.
  - Identify the flavour of the non-signal B meson (B_{TAG}) at the time it decays.
  - Measure the spatial separation between the decay vertices of both B mesons: convert to a proper time difference $\Delta t = \Delta z / \beta\gamma c$;
  - fit for S and C.

- The time evolution of $B_{TAG} = \overline{B}^0 (B^0)$ is

$$f_{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \left\{ 1 \pm \left[-\eta_f S \sin(\Delta m_d \Delta t) - C \cos(\Delta m_d \Delta t) \right] \right\}.$$

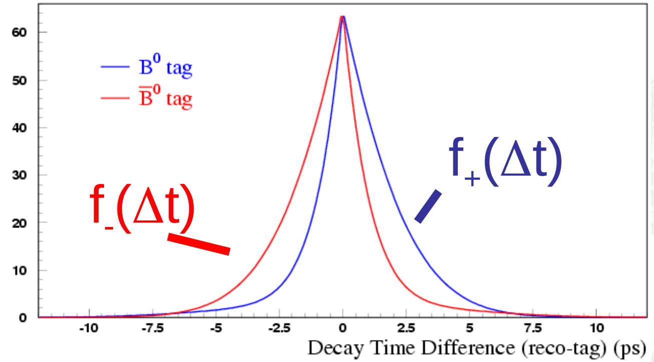
→ B^0 lifetime $\tau_B = 1.530 \pm 0.009$ ps

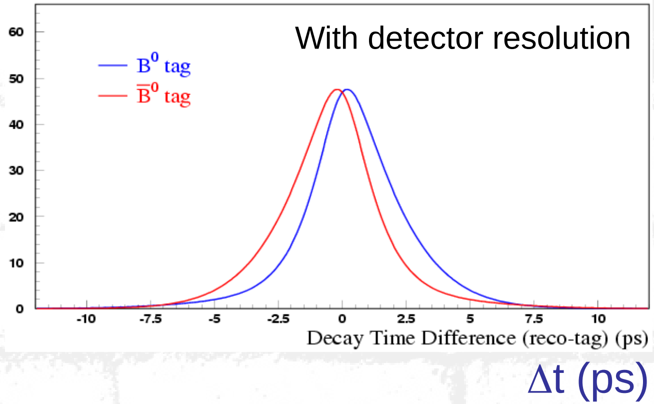
Time-dependent CP asymmetries

◉ Construct an asymmetry as a function of Δt:

$$\mathcal{A}(\Delta t) = \frac{\Gamma(\Delta t) - \overline{\Gamma}(\Delta t)}{\Gamma(\Delta t) + \overline{\Gamma}(\Delta t)}$$

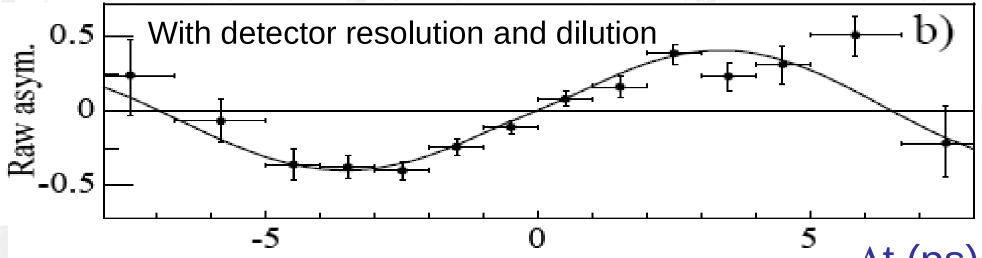
$$\mathcal{A}(\Delta t) = S\sin(\Delta m_d \Delta t) - C\cos(\Delta m_d \Delta t)$$





Experimental effects we need to include:

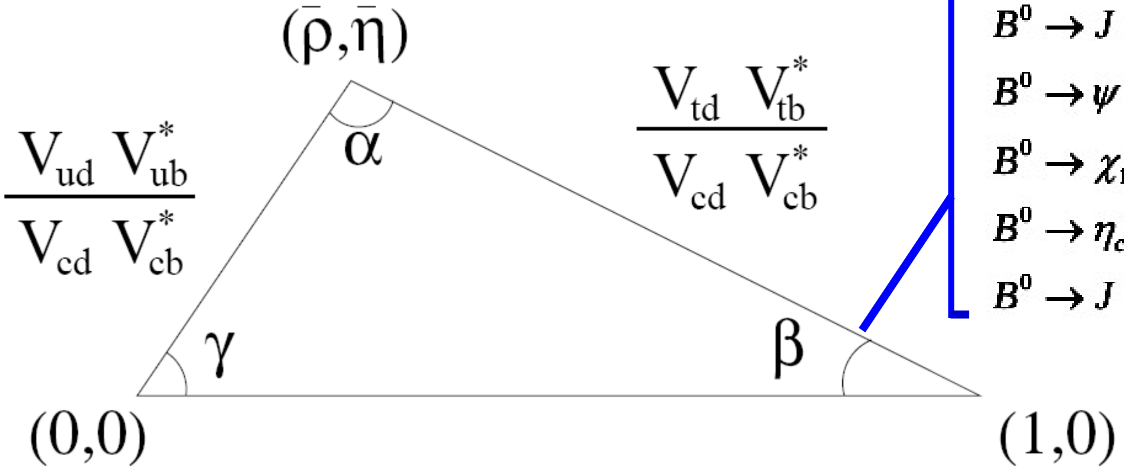
- ◎ Detector resolution on Δt.
- ◎ Dilution from flavour tagging



β/ϕ_1 angle

Theoretically cleaner (SM uncertainties ~10^{-2} to 10^{-3})
→ tree dominated decays to Charmonium + K^0 final states.

$$\beta \equiv \arg\left[-V_{cd}V_{cb}^{*}/V_{td}V_{tb}^{*}\right]$$

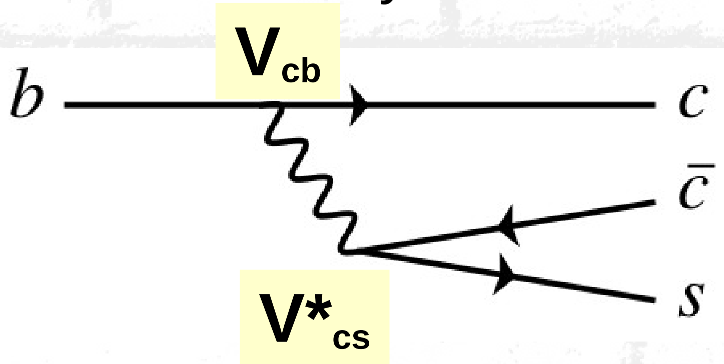


$b \to c\bar{c}s$
- $B^0 \to J/\psi K_L^0$
- $B^0 \to J/\psi K_S^0$
- $B^0 \to \psi(2S) K_S^0$
- $B^0 \to \chi_{1c} K_S^0$
- $B^0 \to \eta_c K_S^0$
- $B^0 \to J/\psi K^{*0}$

- $B \to J/\psi \pi^0$
- $B \to D^{(*)+}D^{(*)-}$
- $B \to \eta' K^0$
- $B \to \rho K^0$
- $B \to \omega K^0$
- $B \to \pi^0 K^0$
- $B \to \phi K^{(*)0}$
- $B \to KKK^0$
- $B \to f^0(980) K^0$

$\sin 2\beta$ in golden $b \to c\bar{c}s$ modes

Leading-order tree decays to $c\bar{c}s$ final states

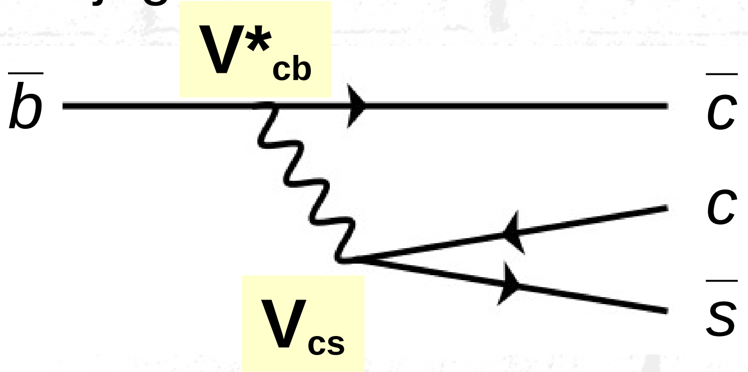


+$\bar{d}$ $\bar{K}^0 \to K_{S,L}$

$\bar{B}^0 \to J/\psi K_{S,L}$

Here the CKM elements contributing are $V_{cb}V^*_{cs}$ that have no phase.

The CP conjugated case is also leading to (about) the same final state:



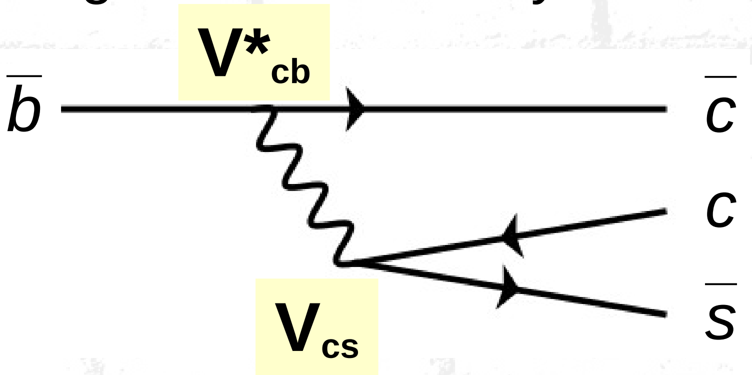
+d $K^0 \to K_{S,L}$

$B^0 \to J/\psi K_{S,L}$

sin2β in golden $b \to c\bar{c}\bar{s}$ modes

$B^0 \to J/\psi K_{S,L}$

Leading-order tree decays to $c\bar{c}s$ final states



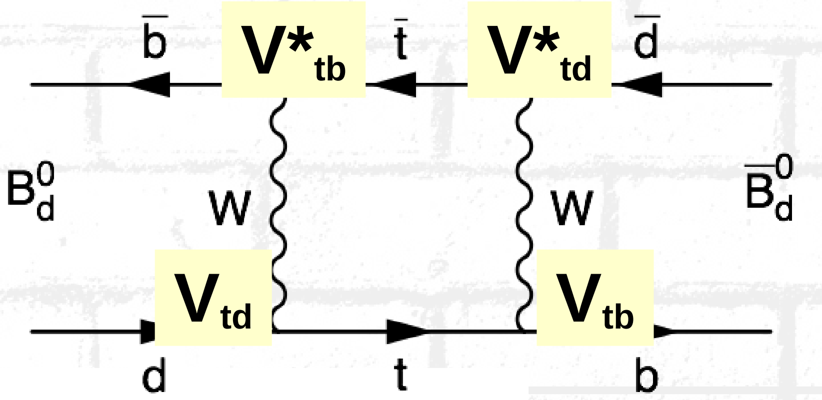
+d $K^0 \to K_{S,L}$

tree diagram

$$\frac{\bar{A}}{A} = \frac{V_{cb}V^*_{cs}}{V^*_{cb}V_{cs}} \frac{V_{cs}V^*_{cd}}{V^*_{cs}V_{cd}}$$

K mixing

because both B and $\bar{B}$ can decay in this common final state, this can interfere with the oscillation diagram:



$$\lambda = \frac{q}{p}\frac{A(\bar{B} \to f)}{A(B \to f)} = \frac{V^*_{td}V_{tb}}{V_{td}V^*_{tb}}\frac{\bar{A}}{A} \sim e^{-i2\beta}\frac{\bar{A}}{A}$$

$\sin 2\beta$ in golden $b \to c\bar{c}s$ modes

$B^0 \to J/\psi K_{S,L}$

no possibility to generate this way direct or indirect CPV

$$\lambda_{CP} = \eta_{CP} \frac{q}{p} \frac{\bar{A}}{A} = \eta_{CP} \underbrace{\frac{V_{td}V^*_{tb}}{V^*_{td}V_{tb}}}_{e^{-i2\beta}} \frac{V_{cb}V^*_{cd}}{V^*_{cb}V_{cd}}$$

$|\lambda_{CP}| = 1$ → $C_{f_{CP}} = 0$

$\text{Im}\,\lambda_{CP} = -\eta_{CP}\sin 2\beta$ → $S_{f_{CP}} = -\eta_{CP}\sin 2\beta$

$J/\psi\ (c\bar{c}) \to J^{PC} = 1^{--}$

$K_S \sim K_1 \to \eta_{CP} = +1$

$L=1 \to P = (-1)^L$

$\eta_{CP}(J/\psi K_S) = -1$

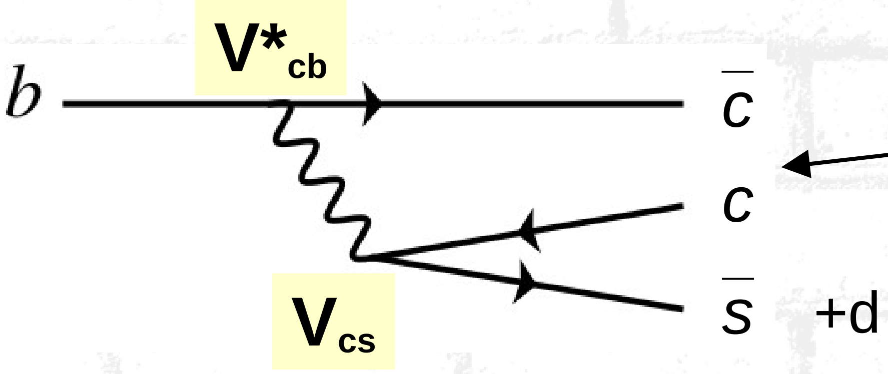
$\eta_{CP}(J/\psi K_L) = +1$

CPV in interference between mixing and decay

$\sin 2\beta$ in golden $b \to c\bar{c}s$ modes

$B^0 \to J/\psi K_{S,L}$

Leading-order tree decays to $c\bar{c}s$ final states



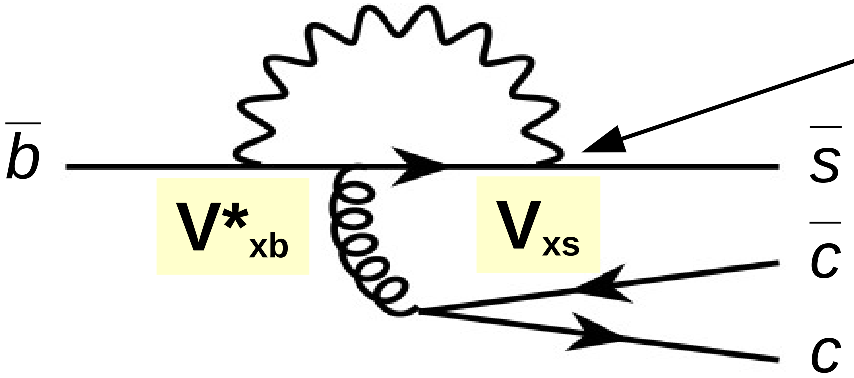
$K^0 \to K_{S,L}$

tree diagram

$$\frac{\bar{A}}{A} = \frac{V_{cb}V^*_{cs}}{V^*_{cb}V_{cs}} \; \frac{V_{cs}V^*_{cd}}{V^*_{cs}V_{cd}}$$

K mixing

possible penguin contributions:

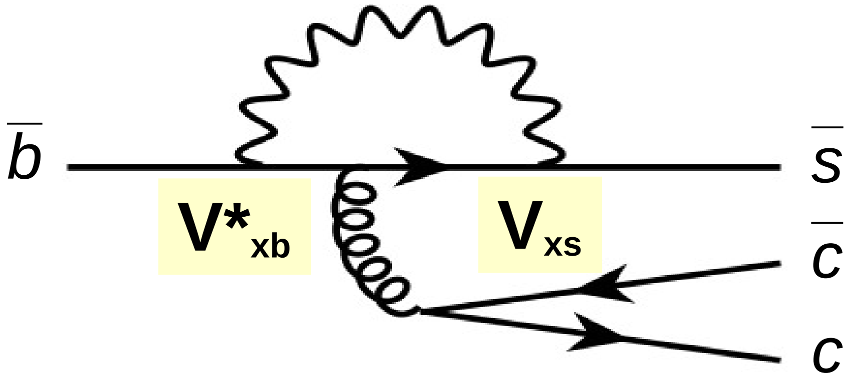


where x can be any up-type quark hence this counts for three penguin diagrams

can this be a problem?

$\sin 2\beta$ in golden $b \to c\bar{c}s$ modes

$B^0 \to J/\psi K_{S,L}$



$$\begin{cases} x=u \to P^u \sim V_{ub}V^*_{us} \\ x=c \to P^c \sim V_{cb}V^*_{cs} \\ x=t \to P^t \sim V_{tb}V^*_{ts} \end{cases}$$

using this unitary condition (2nd ⇄ 3rd family), we eliminate $V_{tb}V^*_{ts}$

$$V_{ub}V^*_{us} + V_{cb}V^*_{cs} + V_{tb}V^*_{ts} = 0 \quad \rightarrow \quad V_{tb}V^*_{ts} = -V_{ub}V^*_{us} - V_{cb}V^*_{cs}$$

thus the amplitude is:

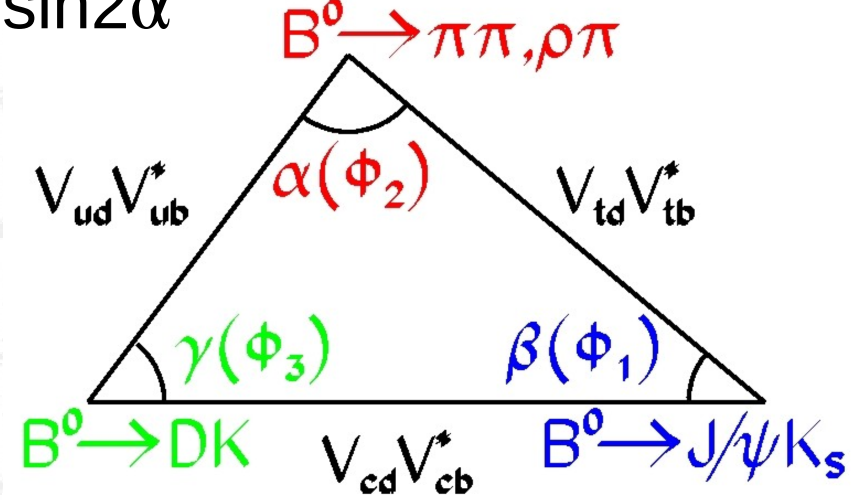
$$A_{ccs} \sim \underbrace{V_{cb}V^*_{cs}}_{\mathcal{O}(\lambda^2)}(T + P^c - P^t) + \underbrace{V_{ub}V^*_{us}}_{\mathcal{O}(\lambda^4)}(P^u - P^t)$$

CKM-suppressed pollution by penguins

CP Violation in the B Meson System

Time-dependent analysis
CP violation in interference
Less clean channel due to big penguin contributions

$S_{f_{CP}} \propto \sin 2\alpha$



Time-dependent analysis:
CP violation in interference

$S_{f_{CP}} = -\eta_{CP}\sin 2\beta$

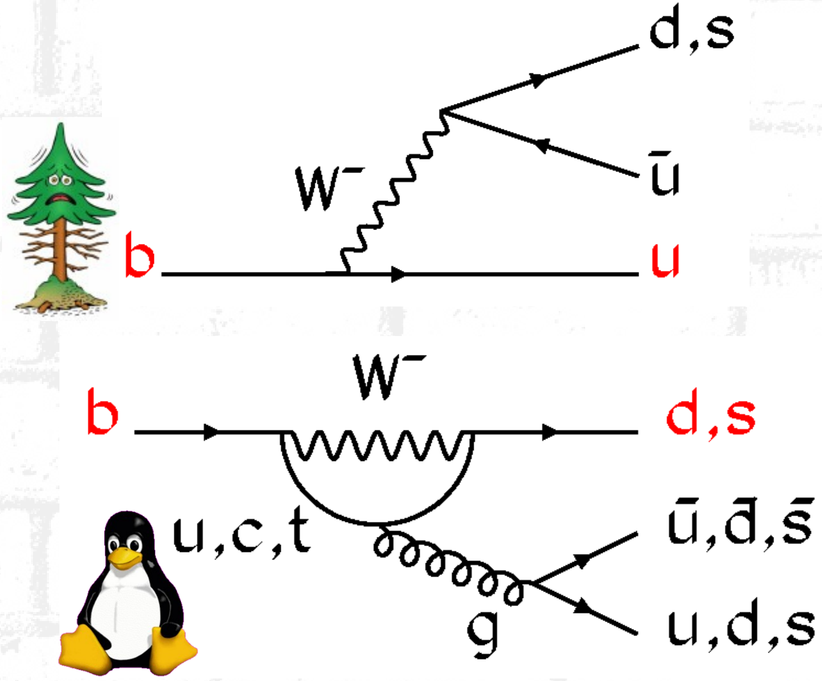
Direct CP violation
Interference of two tree diagrams

α (ϕ_2) from $\pi\pi$, $\rho\rho$, $\pi\rho$ decays with Isospin analysis

Interference between box mixing and tree diagrams results in an asymmetry that is sensitive to α in $B \rightarrow hh$ decays: $h = \pi, \rho$

Unlike for β, loop (penguin diagrams) corrections are not negligible for α

Need Isospin analysis including all modes (B of all charges and flavours) to obtain the α estimate

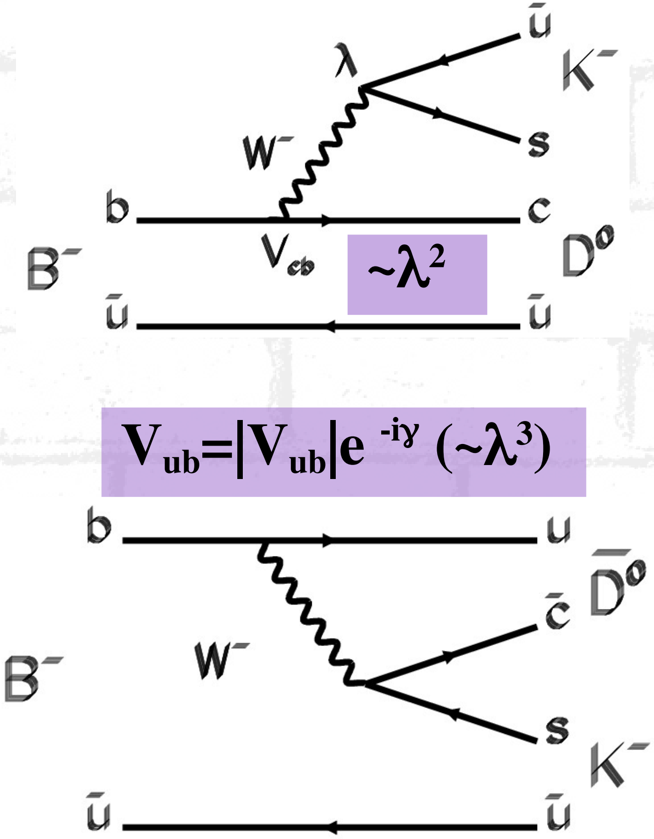


γ (ϕ_3) from B decays in DK

B to $D^{(*)}K^{(*)}$ decays: from BRs and BR ratios, no time-dependent analysis, just rates.

The phase γ is measured exploiting interferences between $b \rightarrow c$ and $b \rightarrow u$ transitions: two amplitudes leading to the same final states

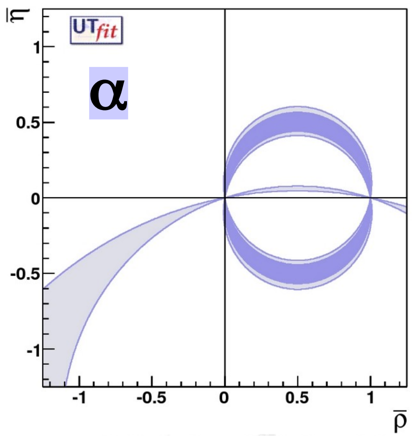
Some rates can be really small: $\sim 10^{-7}$
need to combine all the possible modes and analysis methods.

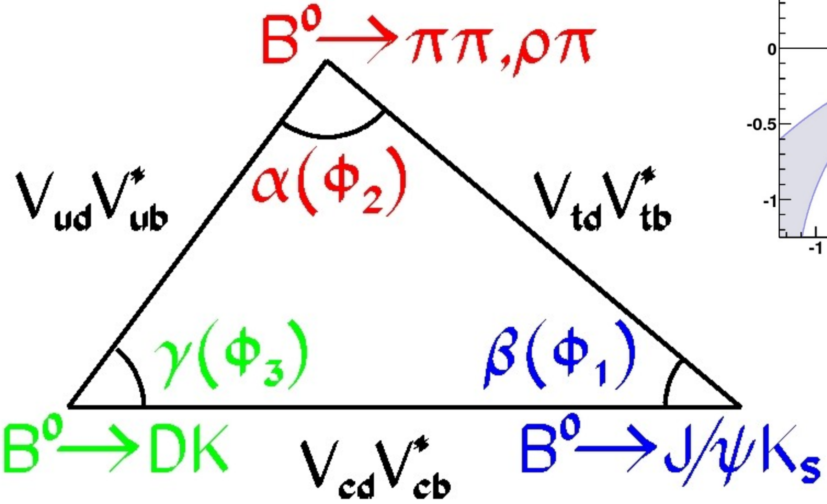


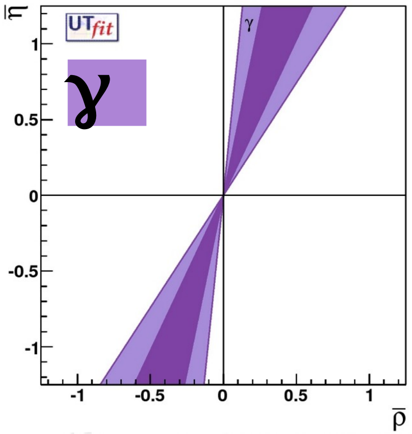
$\sim\lambda^2$

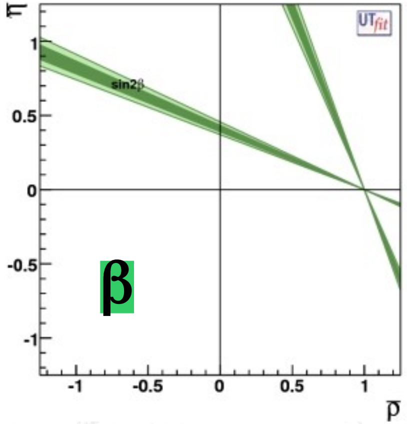
$V_{ub}=|V_{ub}|e^{-i\gamma}$ $(\sim\lambda^3)$

CP Violation in the B Meson System

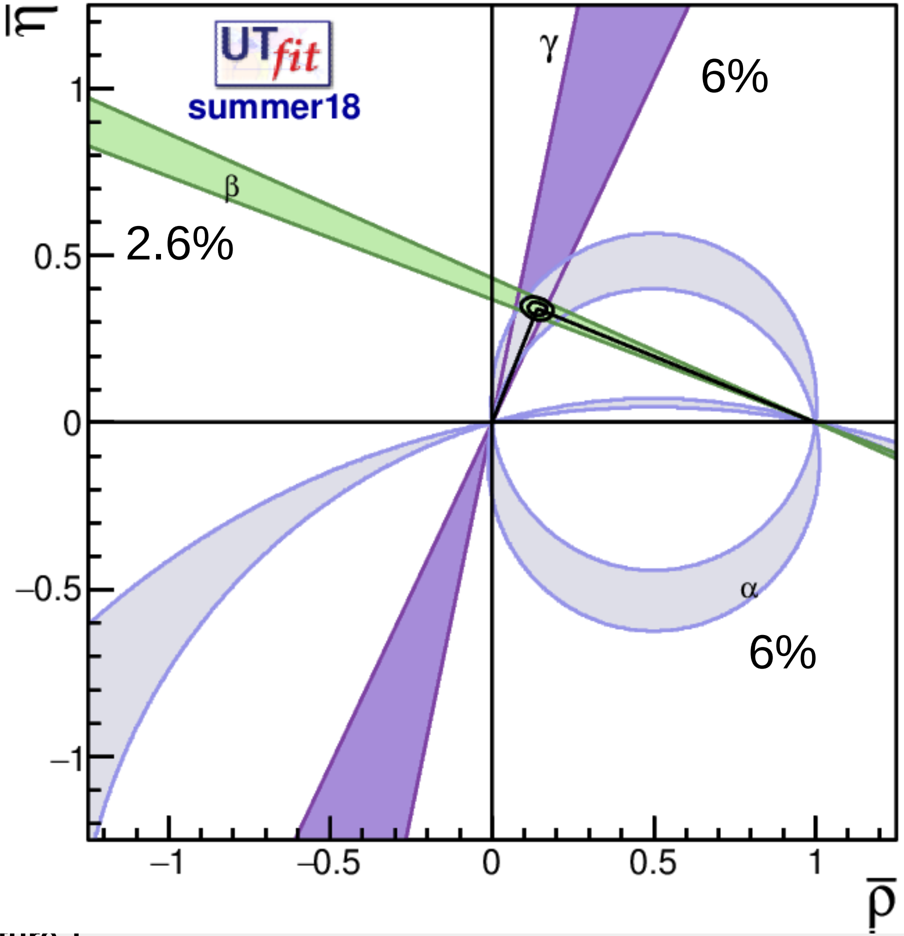








CP Violation in the B Meson System as Unitary Triangle



Back-up Slides

Jarlskog invariant J

J → determinant of the commutator of the mass matrices for the up-type quarks and the down-type quarks:

⇨ the commutator tells us if the two matrices can be simultaneously diagonalised or not

⇨ this specific determinant vanishes if and only if there are no CP violating terms

It is a phase-convention-independent measure of CP violation and it defined as:

$$\mathrm{Im}[V_{ij}V_{kl}V_{il}^{*}V_{kj}^{*}] = J \sum_{m,n} \varepsilon_{ikm}\varepsilon_{jln}$$

in the PDG parameterisation of the CKM matrix:

$$J = s_{12}s_{13}s_{23}c_{12}c_{13}^{2}c_{23}\sin\delta.$$

This is twice the area of the unitarity triangles

Discrete Symmetries

What do we mean by conservation/violation of a symmetry?

- Define a quantum mechanical operator O.
- If O describes a good symmetry:

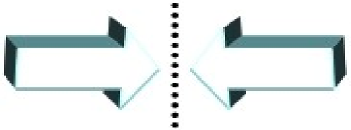
Physics ‘looks the’ same before and after applying the symmetry i.e. the observed quantity associated with O is conserved (same before and after the operator is applied). e.g. conservation of energy-momentum etc.

- If this condition is not met – the symmetry is broken.
  - That is, the symmetry is not respected by nature. So O is (at best) a mathematical tool used to help our understanding of nature.
  - Slightly broken symmetries (like isospin in EW interactions) can be very useful)!

e.g. Isospin symmetry assumes that $m_u=m_d$. In doing so we can estimate branching fractions where the final state differs by a π^0 vs a $\pi^\pm$ etc. The difference comes from a Clebsch-Gordan coefficient.

Parity P

- Reflection through a mirror, followed by a rotation of π around an axis defined by the mirror plane.
  - Space is isotropic, so we care if physics is invariant under a mirror reflection.



$$\mathbf{r} \to -\mathbf{r}$$

$$\mathbf{p} \to -\mathbf{p}$$

$$\mathbf{L} \to \mathbf{L}$$

- P is maximally violated in weak interactions:

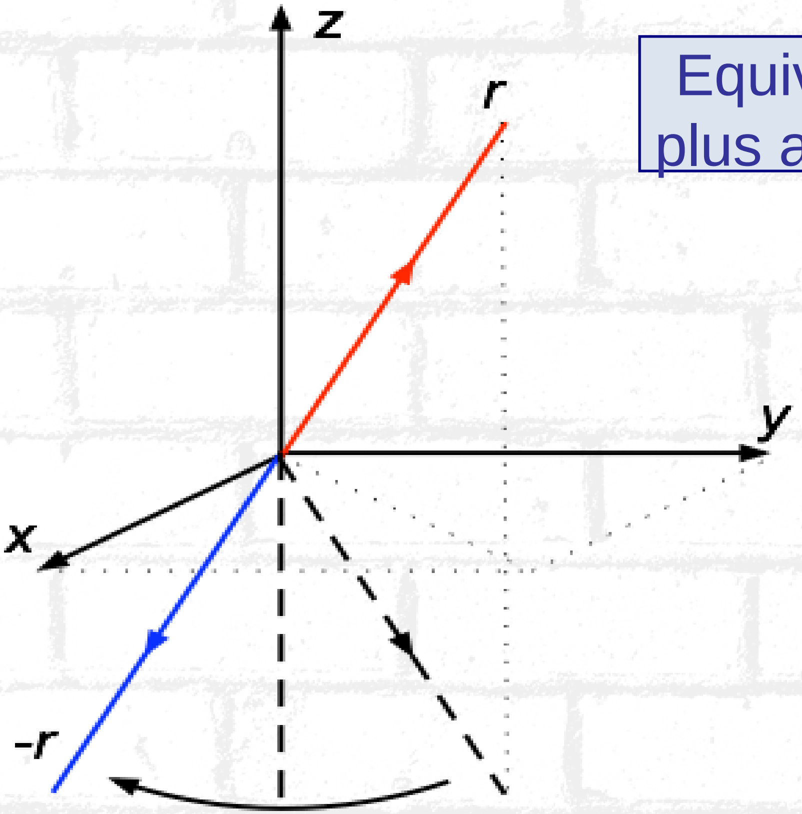
$$[\mathcal{P}, \mathcal{H}_W] \neq 0$$

- Vectors change sign under a P transformation, pseudo-vectors or axial-vectors do not.

- P is a unitary operator: $P^2=1$.

T. D. Lee & G. C. Wick Phys. Rev. **148** p1385 (1966) showed that there is no operator P that adequately represents the parity operator in QM.

Parity P



Equivalent to a reflection in an x, y mirror plus a rotation through 180° about the z axis

Parity P

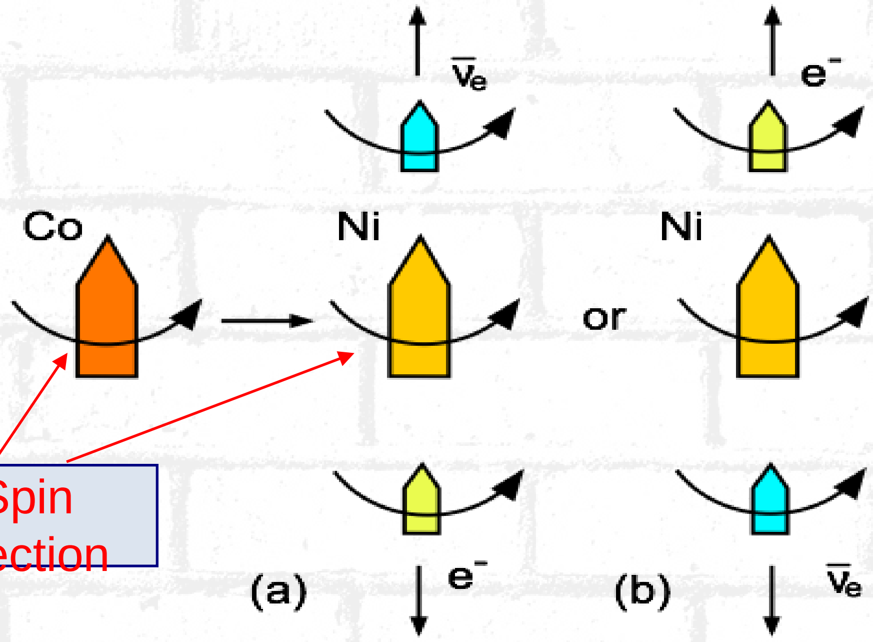
In 1956 if was found that β decay violates parity conservation. It was subsequently found that <u>all</u> weak decays violate parity conservation

In the decay of nuclei with spins aligned in a strong magnetic field and cooled to 0.01° K

$$^{60}Co \longrightarrow {}^{60}Ni + e^{-} + \bar{\nu}_e$$

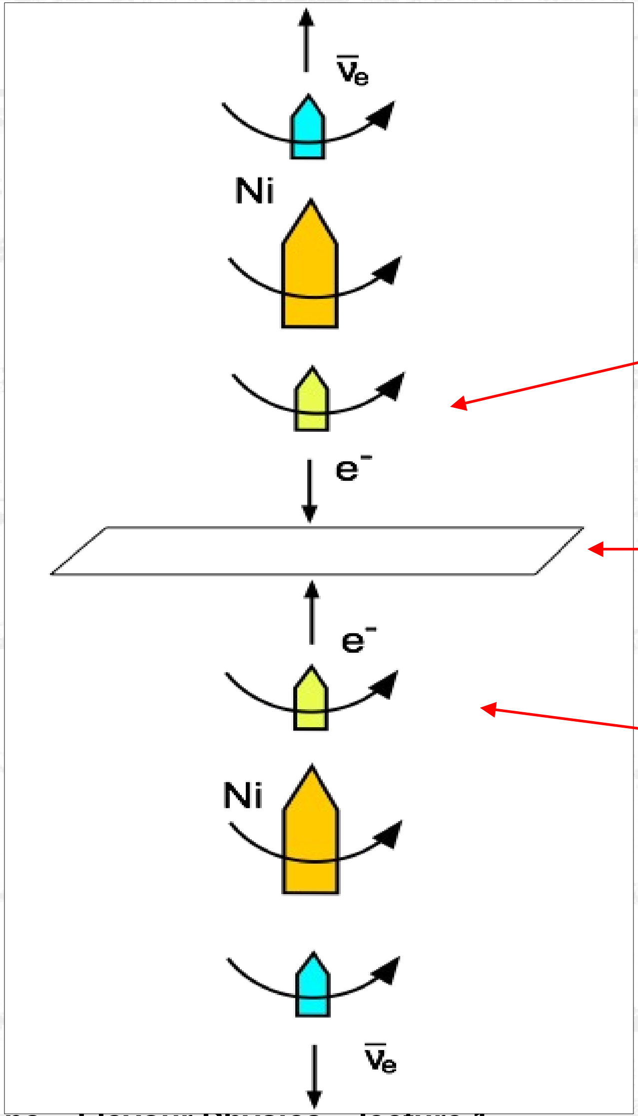
J=5 J=4 J=½ J=½

Spin direction



It was found that electrons were emitted predominantly in configuration (a).
If parity were conserved one would expect (a) and (b) equally

Parity P



Spin and momentum of electron are opposite directions

Mirror

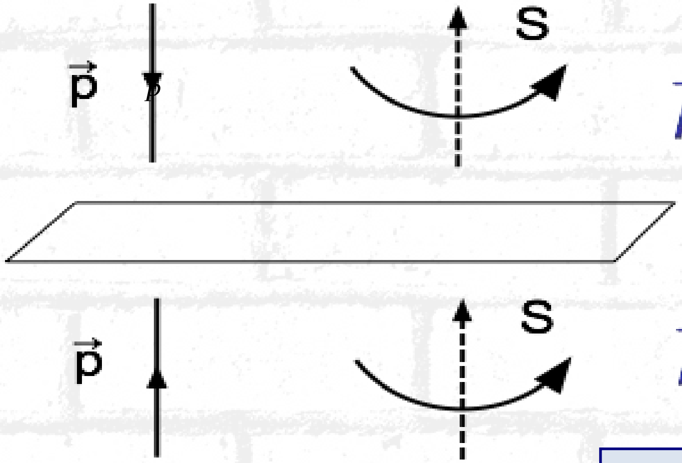
Spin and momentum of electron are the same direction

Parity P

This is understood as interference between the Vector (V) and Axial Vector (A) parts of the Weak Interaction

$\vec{p}$, $\vec{r}$ etc are Vectors $\vec{p} \rightarrow -\vec{p}$ under mirror transformations

Spin $\vec{s}$ is an Axial Vector $\vec{s} \rightarrow \vec{s}$ under mirror transformations



$\vec{p} \cdot \vec{s} = -|\vec{p}|\,|\vec{s}|$

$\vec{p} \cdot \vec{s} = +|\vec{p}|\,|\vec{s}|$

Any law depending on (Vector) × (Axial Vector) will not conserve parity

The Fermi Matrix Element $M_F \rightarrow M_V - M_A$

$$|M_F|^2 = |M_V|^2 + |M_A|^2 - 2M_V \cdot M_A$$

Interference term gives parity violation

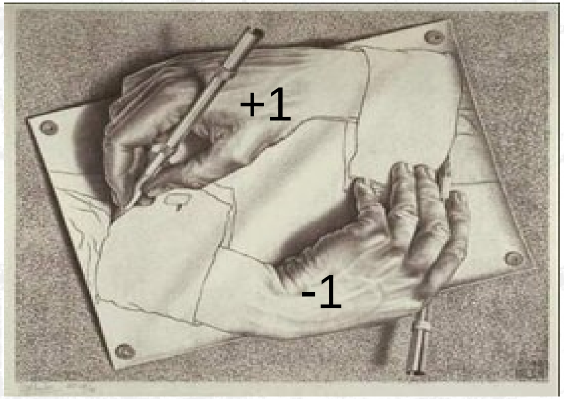
Aside: helicity

- Signed projection of a particle's spin along the direction of its momentum:

$$h = \frac{\bar{s} \cdot \bar{p}}{|\bar{p}|}$$

$$h \in \{-1, +1\}$$

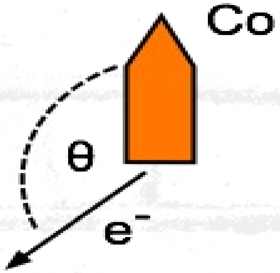
$$\mathcal{P}(h) = -h$$
$$\mathcal{C}(h) = h$$
$$\mathcal{T}(h) = h$$



Aside: helicity

The intensity of emitted electrons from the ^{60}Co was found to be consistent with a distribution:

$$I(\theta) = 1 + \alpha \frac{(\vec{s} \cdot \vec{p})}{E} = 1 + \alpha \frac{v}{c} \cos\theta$$



The polarisation or Helicity is defined as:

$$H = \frac{I_+ - I_-}{I_+ + I_-} = \alpha \frac{v}{c}$$

Where I_+, I_- represent the intensities for $\vec{s}$ and $\vec{p}$ parallel ($\cos\theta = +1$) and for $\vec{s}$ and $\vec{p}$ antiparallel ($\cos\theta = -1$)

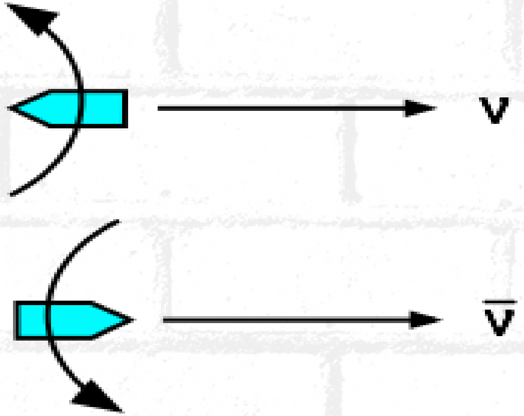
Aside: helicity

Experimentally we find:

$\alpha = +1$ for e^+ → $H = +v/c$

$\alpha = -1$ for e^- → $H = -v/c$

Neutrinos (assuming $m_\nu = 0$ → $v = c$) are fully polarised with $H = +1$ or -1



Find neutrinos are always $H = -1$

→ 'Left Handed'

Antineutrinos have $H = +1$

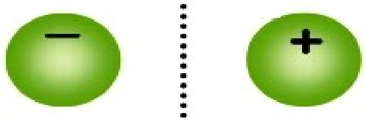
Aside: helicity and chirality

- helicity:
  - projection of the particle spin $\bar{s}$ along the direction of motion $\bar{p}$
    - ◈ $\bar{s} \cdot \bar{p} \Rightarrow$ $\bar{s} \uparrow\downarrow \bar{p}$ negative, left helicity
    - $\bar{s} \uparrow\uparrow \bar{p}$ positive, right helicity
  - for massive particles (m>0):
    - the sign of the helicity depends on the frame of reference

- chirality or handedness:
  - Lorentz invariant analogue of helicity
    - two states: left-handed (LH) and right-handed (RH)
      - massless particles: either pure RH or LH
      - massive particles: both LH+RH components
    - helicity eigenstate is a combination of handedness states

Charge Conjugation: C

- Change a quantum field ϕ into $\phi^\dagger$, where $\phi^\dagger$ has opposite U(1) charges:
  - *baryon number, electric charge, lepton number, flavour quantum numbers like strangeness & beauty etc.*
- Change particle into antiparticle.
  - *the choice of particle and antiparticle is just a convention.*



$$e^- \rightarrow e^+$$

$$\gamma \rightarrow \gamma$$

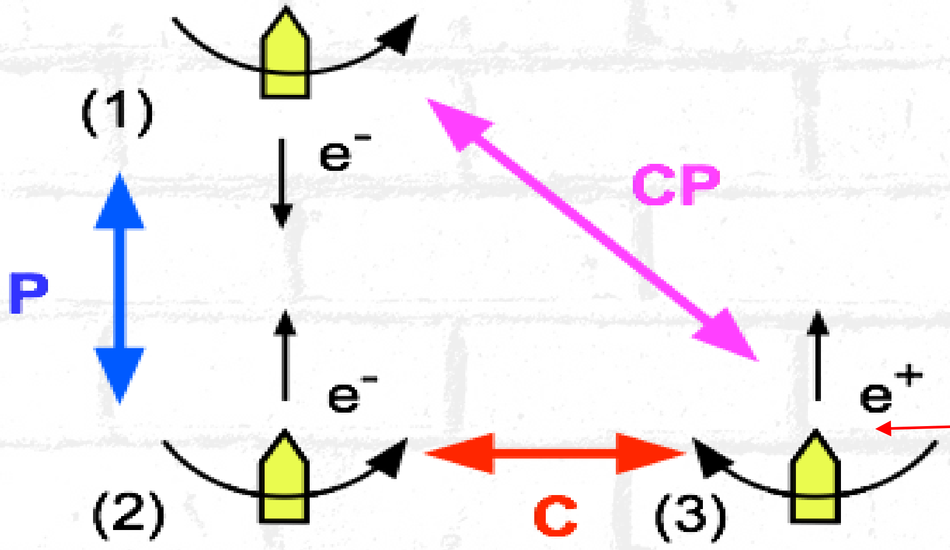
- C is violated in weak interactions, so matter and antimatter behave differently, and:

$$[\mathcal{C}, \mathcal{H}_W] \neq 0$$

- C is a unitary operator: $C^2=1$.

Parity and Charge Conjugation: CP

Note that Charge Conjugation C is also violated but CP is (usually) conserved



Charge Conjugation C changes particle into antiparticle

(3) is again a favourite configuration from the point of view of weak interaction, just like (1) was.

Parity and Charge Conjugation: CP

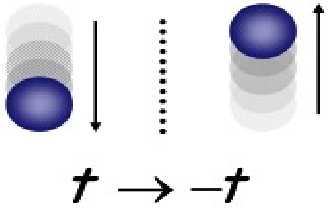
The fundamental point is that CP symmetry is broken in any theory that has complex coupling constants in the Lagrangian which cannot be removed by any choice of phase redefinition of the fields in the theory.

- Weak interactions are left-right asymmetric.
  - *It is not sufficient to consider C and P violation separately in order to distinguish between matter and antimatter.*
  - *i.e. if helicity is negative (left) or positive (right).*
- CPs a unitary operator: $CP^2=1$

Time reversal: T

Not to be confused with the classical consideration of the entropy of a macroscopic system.

- ‘Flips the arrow of time’
  - *Reverse all time dependent quantities of a particle (momentum/spin).*
  - *Complex scalars (couplings) transform to their complex conjugate.*
  - *It is believed that weak decays violate T, but EM interactions do not.*
- T is an anti-unitary operator: T^2=-1.



Summary on discrete symmetries

Three discrete operations are potential symmetries of a field theory Lagrangian.

Two of them, parity and time reversal are space-time symmetries.

- Parity sends $(t; x) \rightarrow (t; -x)$, reversing the handedness of space.
- Time reversal sends $(t; x) \rightarrow (-t; x)$, interchanging the forward and backward light-cones.

A third (non-space-time) discrete operation is:

- Charge conjugation: it interchanges particles and anti-particles.

The operators associated to these symmetries have different properties:

- *P and C operators are:*
  - → unitary and thus they satisfy the relation $U^T = U^{-1}$
  - → linear and thus $U(\alpha|a\rangle + \beta|b\rangle) = \alpha\ U|a\rangle + \beta\ U|b\rangle$.
- *T operator is anti-unitary, that means that is satisfies*
  - → the unitary relation: $A^T = A^{-1}$
  - → but it is anti-linear: $A(\alpha|a\rangle + \beta|b\rangle) = \alpha^*\ A|a\rangle + \beta^*\ A|b\rangle$.

CPT

- all locally invariant Quantum Field Theories conserve CPT[1].
- CPT is anti-unitary: $CPT^2=-1$.
- CPT can be violated by non-local theories like quantum gravity. These are hard to construct.
  - *see work by Mavromatos, Ellis, Kostelecky etc. for more detail.*
- If CPT is conserved, a particle and its antiparticle will have
  - *The same mass and lifetime .*
  - *Symmetric electric charges.*
  - *Opposite magnetic dipole moments (or gyromagnetic ratio for point-like leptons).*

[1]See Weinberg volume I and references therein (Lueders 1954) for a proof of this.

Broken symmetries

- The symmetry CPT is conserved in the Standard Model.
- The other symmetries introduced here are broken by some amount.
- CP violation has been seen in kaon and B meson decays.
- These symmetries are broken for weak interactions only!
  - *They are conserved (as far as we know) in strong and electromagnetic interactions.*

Summary on CP and CPT

The combination CPT is an exact symmetry in any local Lagrangian field theory:

- ⇨ The CPT theorem is based on general assumptions of field theory and relativity and states that every Hamiltonian that is Lorentz invariant is also invariant under combined application of CPT, even if it is not invariant under C, P and T separately.
  - ▣ One of the consequences of this theorem is that particles and anti-particles should have exactly the same mass and lifetime.

- ⇨ From experiment, it is observed that electromagnetic and strong interactions are symmetric with respect to C, P and T.
- ⇨ The weak interactions violate C and P separately, but preserve CP and T to a good approximation. Only certain rare processes have been observed to exhibit CP violation.

  - ▣ All these observations are consistent with exact CPT symmetry.

Examples

$$CP\,|\,u\,\rangle = |\,\bar{u}\,\rangle$$

The u quark has $J^P = \frac{1}{2}^+$, so the P operator acting on u has an eigenvalue of +1. The C operator changes particle to antiparticle.

$$CP\,|\,\pi^0\,\rangle = -\,|\,\pi^0\,\rangle$$

The π^0 has $J^{PC} = 0^{-\,+}$, so the minus sign comes from the parity operator acting on the π^0 meson. The C operator changes particle to antiparticle. A π^0 is its own antiparticle.

$$CP\,|\,\pi^\pm\,\rangle = -\,|\,\pi^\mp\,\rangle$$

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History of Mixing, CP violation, B factories and Nobel prizes

- **1963:** Cabibbo introduced his angle for the quark mixing with 2 families
- **1964:** Christensen, Cronin, Fitch and Turlay discover CP violation in the K^0 system.
- **1967:** A. Sakharov: 3 conditions required to generate a baryon asymmetry:
    - Period of departure from thermal equilibrium in the early universe.
    - Baryon number violation.
    - C and CP violation.

- **1973:** Kobayashi and Maskawa propose 3 generations
- **1980:** Nobel Prize to Cronin and Fitch
- **1981:** I. Bigi and A. Sanda propose measuring CP violation in $B \rightarrow J/\psi K^0$ decays.
- **1987:** P. Oddone realizes how to measure CP violation: convert the PEP ring into an asymmetric energy e^+e^- collider.
- **1999:** BaBar and Belle start to take data. By 2001 CP violation has been established (and confirmed) by measuring $\sin 2\beta \neq 0$ in $B \rightarrow J/\psi K^0$ decays.
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Dynamic generation of baryon asymmetry

Suppose equal amounts of matter (X) and antimatter ($\bar{X}$):

- X decays to
  - A (baryon number N_A) with probability p
  - B (baryon number N_B) with probability $(1 - p)$
- $\bar{X}$ decays to
  - $\bar{A}$ (baryon number $-N_A$) with probability $\bar{p}$
  - $\bar{B}$ (baryon number $-N_B$) with probability $(1 - \bar{p})$
- Generated baryon asymmetry:
  - $\Delta N_{TOT} = N_A p + N_B (1 - p) - N_A \bar{p} - N_B (1 - \bar{p}) = (p - \bar{p}) (N_A - N_B)$
  - $\Delta N_{TOT} \neq 0$ requires $p \neq \bar{p}$ & $N_A \neq N_B$

CP violation and the baryon asymmetry

We can estimate the magnitude of the baryon asymmetry of the Universe caused by KM CP violation

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \sim \frac{J \times P_u \times P_d}{M^{12}}$$

$$J = \cos(\theta_{12})\cos(\theta_{23})\cos^2(\theta_{13})\sin(\theta_{12})\sin(\theta_{23})\sin(\theta_{13})\sin(\delta)$$
$$P_u = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$$
$$P_d = (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$$

- The Jarlskog parameter J is a parametrization invariant measure of CP violation in the quark sector: J ~ O(10^{-5})
- The mass scale M can be taken to be the electroweak scale O(100 GeV)
- This gives an asymmetry O(10^{-17}):

much much below the observed value of O(10^{-10})