# Neutrino Oscillations

## Two neutrino oscillation

$$\hat{H}\psi = i\frac{d\psi}{dt} = \underset{\uparrow\,\text{eigenvalue}}{E}\psi$$

Time evolution of the eigenstate :

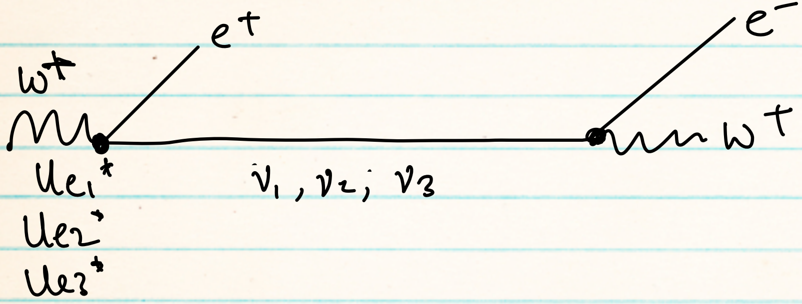
$$\psi(x,t) = \underbrace{\phi(x)}_{\text{amplitude}}\ \underbrace{e^{iEt}}_{\text{phase}}$$

massive states : $\nu_1$ , $\nu_2$, $\nu_3$

interaction states : $\nu_e$, $\nu_\mu$, $\nu_\tau$

# Neutrino Oscillations

charged current interaction $W e^+ \nu_e$




we can't know which eigenstate was produced
$\Rightarrow$ system is described by a linear superposition of $\nu_1, \nu_2, \nu_3$

$$\mathcal{L}_{cc} \supset -\frac{ig}{\sqrt{2}} \bar{l}_\alpha \gamma^\mu P_L \nu_\alpha = -\frac{ig}{\sqrt{2}} \bar{l}_\alpha \gamma^\mu P_L U_{\alpha i} \nu_i$$

flavor index
$\alpha = e, \mu, \tau$

mass index
$i = 1, 2, 3$

# Neutrino Oscillations

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}}_{\text{PMNS matrix}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$|\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle$$

- $|\nu_e\rangle$ propagates as a linear superposition until it is measured. then wavefunction collapses ⇒ neutrinos are measured as a flavour state.
- the wavefunction evolves in time due to the phase shift and this allows for $\nu$-oscillations ie the phenomenon of neutrino flavour transformation.

# Neutrino Oscillations

## Two neutrino mixing

- $(\nu_e, \nu_\mu) \qquad (\nu_1, \nu_2)$

$$|\nu_1(t)\rangle = |\nu_1\rangle e^{i(p_1 \cdot x - E_1 \cdot t)}$$
$$|\nu_2(t)\rangle = |\nu_2\rangle e^{i(p_2 \cdot x - E_2 \cdot t)}$$

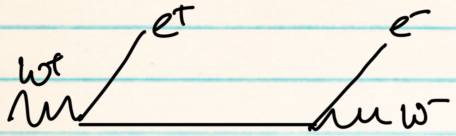
$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$c_\theta \equiv \cos\theta$

$s_\theta \equiv \sin\theta$

**Ex** show from $U^\dagger U = \mathbb{1}_{2\times 2}$ that $U = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}$

# Neutrino Oscillations




In the two-neutrino case consider that a $\nu_e$ is created at time $t=0$:

$$|\psi(0)\rangle = |\nu_e\rangle = c_\theta|\nu_1\rangle + s_\theta|\nu_2\rangle$$

at time $t$:

$$|\psi(L,t)\rangle = c_\theta|\nu_1\rangle e^{-i\phi_1} + s_\theta|\nu_2\rangle e^{-i\phi_2}$$

$$\phi_i = -\vec{p}_i L + E_i t$$

# Neutrino Oscillations

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

rewrite massive states in terms of flavor states.

$$|\psi(L,t)\rangle = c\theta\left[c\theta|\nu_e\rangle - s\theta|\nu_\mu\rangle\right]e^{-i\Phi_1} + s\theta\left[s\theta|\nu_e\rangle + c\theta|\nu_\mu\rangle\right]e^{-i\Phi_2}$$

$$= \left[c^2\theta e^{-i\Phi_1} + s^2\theta e^{-i\Phi_2}\right]|\nu_e\rangle + \left[-s\theta c\theta e^{-i\Phi_1} + s\theta c\theta e^{-i\Phi_2}\right]|\nu_\mu\rangle$$

$$\boxed{|\psi(L,t)\rangle = e^{-i\Phi_1}\left[c^2\theta + s^2\theta e^{i\Delta\Phi_{12}}\right]|\nu_e\rangle + c\theta s\theta\left[e^{-i\Delta\Phi_{12}} \;\; 1\right]|\nu_\mu\rangle}$$

$$\Delta\Phi_{12} = \Phi_1 - \Phi_2$$

$$\Delta\Phi_{12} = 0$$

$$|\psi(L,t)\rangle = e^{i\Phi_1}(\underbrace{c^2\theta + s^2\theta}_{1})|\nu_e\rangle$$

$$= e^{i\Phi_1}|\nu_e\rangle$$ $\Rightarrow$ no phase difference, no oscillation so $|\nu_e\rangle$ stays a $|\nu_e\rangle$

# Neutrino Oscillations

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \psi(L,t) \rangle|^2$$

$$\langle \nu_\mu | \nu_e \rangle = 0 \qquad \langle \nu_\mu | \nu_\mu \rangle = 1$$

$$P(\nu_e \to \nu_\mu) = c_\theta^2 s_\theta^2 (e^{-i\Delta\phi_{12}} - 1)(e^{i\Delta\phi_{12}} - 1)$$

use $2\cos\theta \sin\theta = \sin 2\theta$

$$P(\nu_e \to \nu_\mu) = \frac{\sin^2 2\theta}{4} (1 - e^{-i\Delta\phi_{12}})(1 + e^{i\Delta\phi_{12}})$$

use $e^{ix} = \cos x + i \sin x$ to rewrite:

$$\boxed{P(\nu_e \to \nu_\mu) = \sin^2 2\theta \, \sin^2\left(\frac{\Delta\phi_{12}}{2}\right)}$$

# Neutrino Oscillations

1. assume that $\vec{p}_1 = \vec{p}_2 = \vec{p}$ ie 3-momentum of $\nu_1$ & $\nu_2$ are approximately equal.

$$\Rightarrow \Delta\phi_{12} = (E_1 - E_2)t - (\vec{p}_1 - \vec{p}_2)x.$$

$$= \left(\sqrt{\vec{p}_1^{\,2} + m_1^2} - \sqrt{p_1 + m_2^2}\right) t - (\vec{p}_1 - \vec{p}_2)\, x$$

$$= |\vec{p}| \left( \left(1 + \frac{m_1^2}{|\vec{p}|^2}\right)^{1/2} - \left(1 - \frac{m_2^2}{|\vec{p}|^2}\right)^{1/2} \right)$$

$$= |\vec{p}| \left( \frac{m_1^2 - m_2^2}{2|\vec{p}|^2} \right) = \frac{m_1^2 - m_2^2}{2|\vec{p}|}$$

Since neutrinos are relativistic,
$E \approx |\vec{p}| >> m_1, m_2 \Rightarrow$

$$\boxed{\Delta\phi_{12} = \left(\frac{m_1^2 - m_2^2}{2E} L\right)}$$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m_{12}^2 L}{4E}\right)$$

$$P(\nu_e \to \nu_e) = 1 - P(\nu_e \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m_{12}^2 L}{4E}\right)$$

# Neutrino Oscillations

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2_{12} L}{4E}\right)$$

$$P(\nu_e \to \nu_e) = 1 - P(\nu_e \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2_{12} L}{4E}\right)$$

$P(\nu_e \to \nu_\mu)$ is the probability $\nu_e$ oscillates to $\nu_\mu$
$P(\nu_e \to \nu_e)$ is the survival probability [consequence of unitarity of quantum mechanics]

The wavelength of an oscillation is given by

$$\boxed{\lambda_{osc}[\mathrm{km}] = \frac{\pi \, E[\mathrm{GeV}]}{1.27 \, \Delta m^2 [\mathrm{eV}^2]}}$$

# Neutrino Oscillations

<u>Three neutrino mixing</u>

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}}_{U_{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\underline{U^\dagger U = U U^\dagger = \mathbb{1}_{3\times 3}}$$

The exact same derivation applies but now:

$$|\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle$$

# Neutrino Oscillations

In general we can write an arbitrary unitary 3x3 matrix as:

$$U = e^{i\psi} \,\mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)\, CKM(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$$
$$\mathrm{diag}(e^{i\rho_e}, e^{i\rho_\mu}, 1)$$

$$CKM = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow$ 9 real parameters: $\phi_1, \phi_2, \theta_{12}, \theta_{13}, \theta_{23}, \delta$
$\rho_e, \rho_\mu, \psi$

# Neutrino Oscillations

The only "place" where $U$ appears is in CC interactions (in neutral current interactions $U^{+}$ & $U$ appear one for $\nu$ & $\bar{\nu}$ and by unitarity give the identity matrix)

$$\mathcal{L}_{CC} = \frac{-g}{2\sqrt{2}} \left[ \bar{\nu}_\alpha \gamma^\mu (1-\gamma^5) \ell_\alpha W^{+}_{\mu} + h.c \right] \qquad \alpha = e, \mu, \tau$$

$$\mathcal{L}_{CC} = \frac{-g}{2\sqrt{2}} \left[ \bar{\nu}_i U^{*}_{\alpha i} \gamma^\mu (1-\gamma^5) \ell_\alpha W_\mu^{+} + h.c \right]$$

$$\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

We can "rephase" ie just rotate by a global each generation of the charged leptons

$$e \to e^{-i(\phi_e+\phi)} e \qquad \mu \to e^{-i(\phi_\mu+\phi)} \mu \qquad \tau \to \tau e^{-i\phi}$$

# Neutrino Oscillations

This remove 3 of the 9 paramets and a further 2 phases can be removed by the Dirac neutrino mass term $\bar{\Psi} m \Psi$

- If $\nu$ Majorana particle cannot remove any phases & $\varphi_1$, $\varphi_2$ are physical phases. ie they have an effect on physical observables.