Interpretation of Electroweak Precision Measurements

Thirteenth NExT PhD Workshop - Queen Mary University of London

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CERN

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Aim of this Lecture

- The electroweak theory has withstood stringent experimental test for almost 60 years
- Today, we test it at the highest energies yet with the LHC
- A large number of BSM theories in the electroweak sector have been excluded (not covered here!)

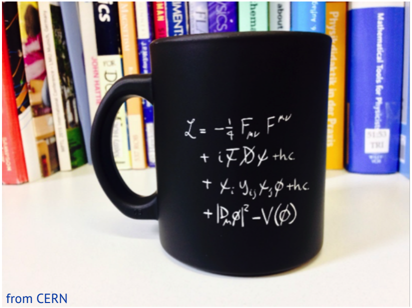
Outline

- The Electroweak Theory
- Key measurements at the LHC
  - Measurements with single W and Z bosons
  - Multiboson measurements
  - Measurements in the Higgs Sector (without the Higgs)
- ...and their *model independent* interpretations

Structure of lecture driven by experiment, resulting in a somewhat historical discussion of the theory!

The Standard Model

- The Standard Model describes the fundamental constituents of matter and their interactions
  - strong and electroweak (EW) interaction
- Rich variety of interactions derived from a rather simple set of symmetries



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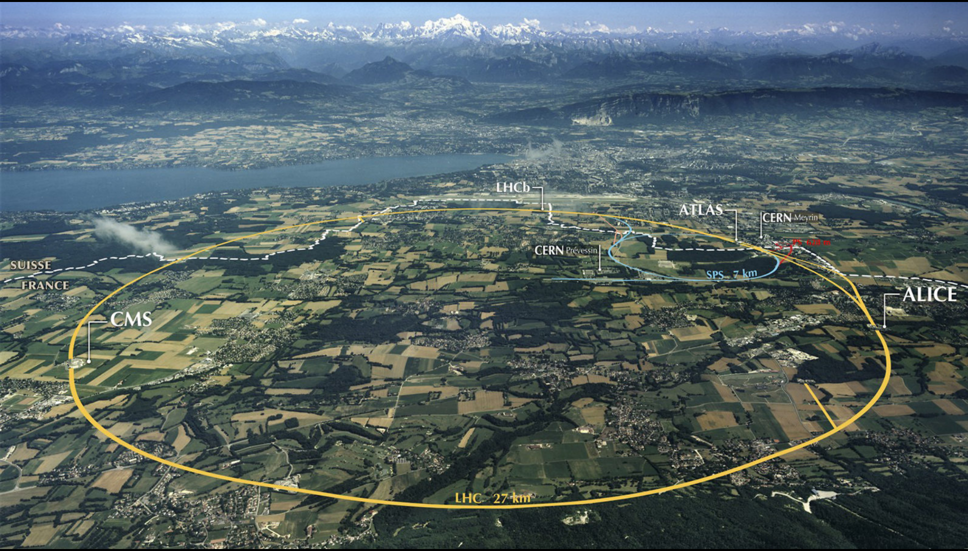
- Electroweak symmetry is broken

$\rightarrow$ Mediators of the electroweak force have mass

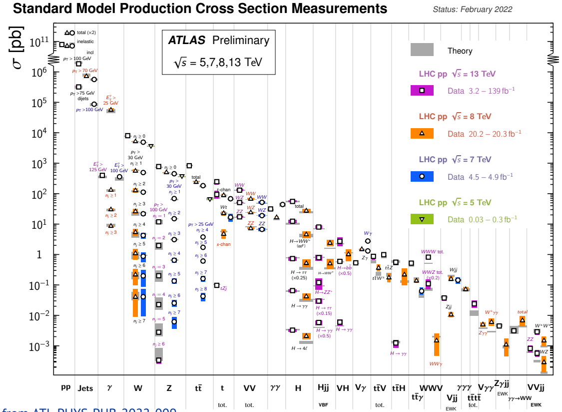
- At the LHC, we test the electroweak theory
  - precision measurements of single W/Z bosons (hoping for deviations through radiative corrections)
  - measurements of multi-bosons production (where the same physics may lead to large deviations)

$SU(3)_C$	$\times$	$SU(2)_L$	$\times$	$U(1)_Y$
colour red, green, blue		weak isospin $I_3 = 0, \pm\frac{1}{2}$		weak hypercharge Y
8 gluons		W^1, W^2, W^3		B

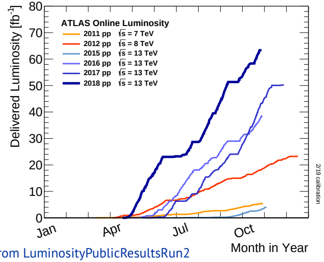
$\rightarrow W^+, W^-, Z, \gamma$


LHCb
ATLAS
CERN Meyrin
PS 628 m
CERN Prévessin
SPS 7 km
SUISSE
FRANCE
ALICE
CMS
LHC 27 km

Electroweak Physics at the LHC



from ATL-PHYS-PUB-2022-009



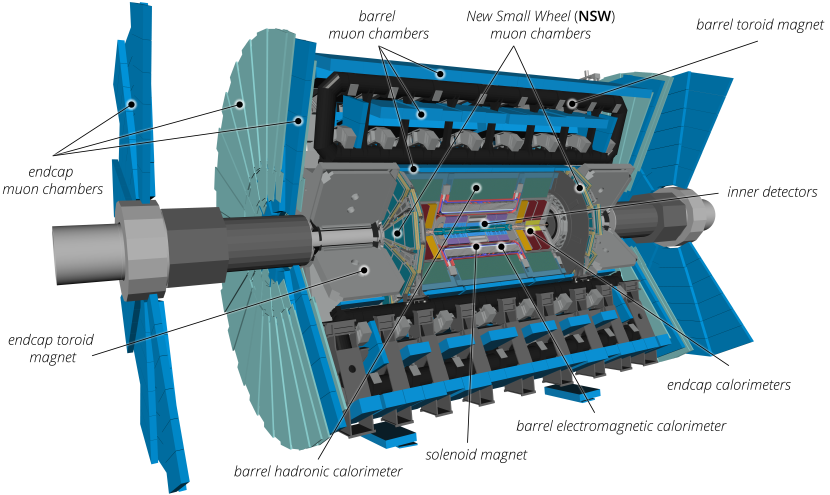
from LuminosityPublicResultsRun2

In the LHC run-2:

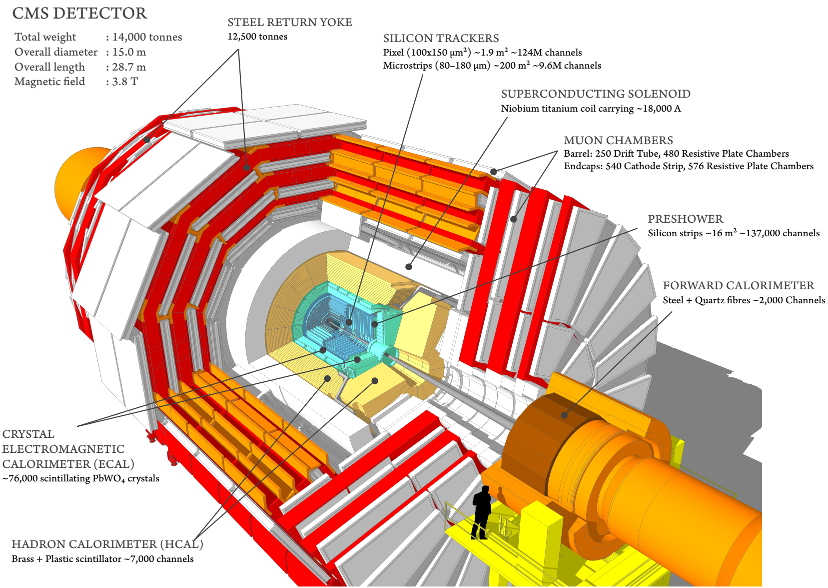
- W bosons: 25 billion
- Z bosons: 7 billion
- Higgs bosons: 8 million

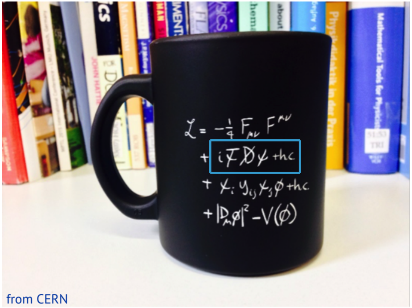
- A hadron collider is an excellent device to do electroweak physics
- Electrons and muons are a clear and unambiguous signature of an electroweak process
- Detectors designed with emphasis on electron and muon reconstruction

The ATLAS Detector



The CMS Detector





from CERN

Electroweak Interactions of single W and Z bosons

A Model of Leptons

Family			T	T_3	Q	Y
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	1/2	+1/2 −1/2	0 −1	−1 −1
e_R	μ_R	τ_R	0	0	−1	−2
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	1/2	+1/2 −1/2	+2/3 −1/3	+1/3 +1/3
u_R	c_R	t_R	0	0	+2/3	+4/3
d_R	s_R	b_R	0	0	−1/3	−2/3

- In the Standard Model, fermions are arranged into:
  - weak-isospin doublets for left-handed quarks and leptons (“L”)
  - weak-isospin singlets for right-handed quarks and leptons (“R”)
- Three families with identical quantum numbers
  - weak isospin and its third component, T and T_3
  - weak hypercharge, Y
  - electric charge, $Q = T_3 + \frac{1}{2}Y$
- Fermion current densities:

$$J^i_\mu = \bar{\chi}_L \gamma_\mu \frac{1}{2}\tau_i \chi_L \quad \text{and} \quad j^Y_\mu = \bar{\psi}\gamma_\mu Y \psi = -2(\bar{e}_R\gamma_\mu e_R) - 1(\bar{\chi}_L\gamma_\mu\chi_L)$$

The Electroweak Interaction

$$\boxed{-ig\left(\mathbf{J}^T\right)^\mu \mathbf{W}_\mu - i\frac{g'}{2}\left(j^Y\right)^\mu B_\mu}$$

- vector fields $\mathbf{W}_\mu$ coupling to weak isospin current $\mathbf{J}^T$ with strength g
- vector field B_μ coupling to weak hypercharge current j^Y with strength $\frac{g'}{2}$

- The electroweak theory is broken and the fields mix

⇒ Massive, charged $W^\pm$ bosons:

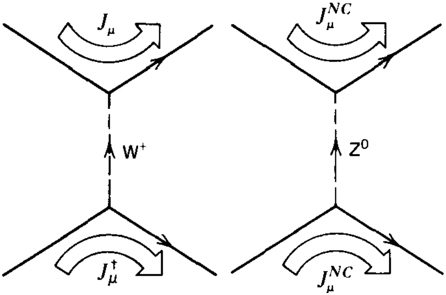
$$W_\mu^\pm = \sqrt{1/2}\left(W_\mu^1 \mp W_\mu^2\right)$$

⇒ Neutral γ (massless) and Z boson (massive):

$$\begin{aligned} A_\mu &= W_\mu^3 \sin\theta_W + B_\mu \cos\theta_W \\ Z_\mu &= W_\mu^3 \cos\theta_W - B_\mu \sin\theta_W \end{aligned}$$

where θ_W is the weak mixing angle

Charged and Neutral Currents



- Electromagnetic current:

$$j_\mu^{\text{em}} = J_\mu^3 - \frac{1}{2} j_\mu^Y$$

$$\Rightarrow g \sin\theta_W = g' \cos\theta_W = e$$

- Weak charged current:

$$J_\mu^\pm = \frac{1}{2}\left(J_\mu^1 + iJ_\mu^2\right)$$

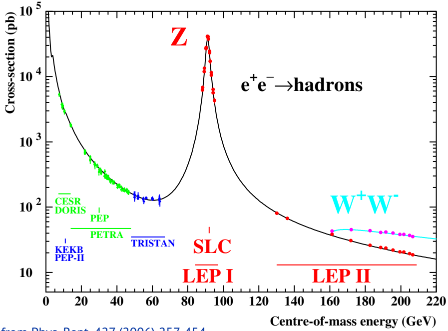
- Weak neutral current:

$$J_\mu^{\text{NC}} = J_\mu^3 - \sin^2\theta_W j_\mu^{\text{em}}$$

- Amplitudes (low energy):

$$\left.\begin{aligned} \mathfrak{M}^{\text{CC}} &\sim \left(\frac{g}{\sqrt{2}} J_\mu\right)\left(\frac{1}{m_W^2}\right)\left(\frac{g}{\sqrt{2}} J^{\mu\dagger}\right) \\ \mathfrak{M}^{\text{NC}} &\sim \left(\frac{g}{\cos\theta_W} J_\mu^{\text{NC}}\right)\left(\frac{1}{m_Z^2}\right)\left(\frac{g}{\cos\theta_W} J^{\mu\text{NC}}\right) \end{aligned}\right\} \quad \boxed{\rho = \frac{m_W^2}{m_Z^2\cos^2\theta_W}}$$

Measurement of the Z-boson mass



from Phys. Rept. 427 (2006) 257-454

Number of Events [×10³]		
Year	$Z \to qq$	$Z \to \ell\ell$
1990/91	1660	186
1992	2741	294
1993	2607	296
1994	5910	657
1995	2579	291
Total	15497	1724

- The Z-boson mass was precisely measured at LEP:

$$m_Z = 91.1876 \pm 0.0021 \text{ GeV}$$

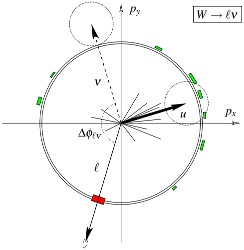
- Largest source of uncertainty in the LEP beam energy

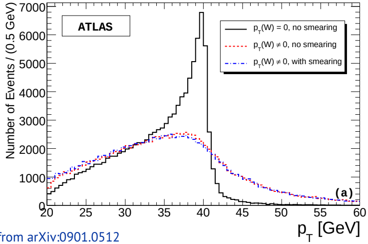
Measurement of the W-boson mass

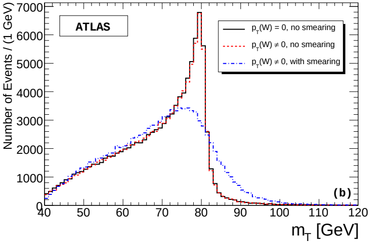
- Due to the presence of a neutrino the W boson cannot be fully reconstructed at the LHC
- Measurement from lepton p_T or transverse mass

$$m_T = \sqrt{2 p_T^\ell p_T^{\text{miss}} \left(1 - \cos \Delta\phi\right)}$$

$\Rightarrow$ Jacobian peak at $^{m_W}/_2$ and m_W, respectively





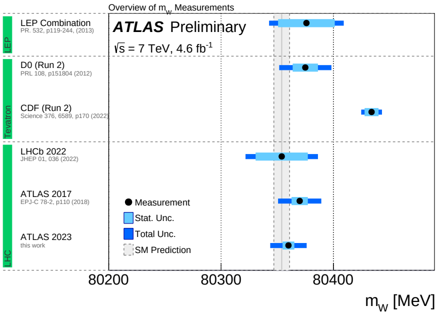


from arXiv:0901.0512

Smearing mainly due to PDF and QCD effects

Smearing mainly due to detector resolution

Results and Comparison to Theory



from ATLAS-CONF-2023-004

- Most recent result from ATLAS:

$$m_W = 80360 \pm 5\ \text{(stat.)} \pm 15\ \text{(syst.)}$$

- LHCb result with $\sigma_{m_W} = 32$ MeV partially anti-correlated to ATLAS

- The measurement of m_W is *the* most demanding concerning experimental and theoretical requirements at the LHC
- *Naive* prediction from theory:

$$m_W^2 = \frac{m_Z^2}{2}\left(1 + \sqrt{1 - 4\frac{\pi\alpha}{\sqrt{2}G_F m_Z^2}}\right) = 80939\ \text{MeV} \quad \Rightarrow 36\sigma!!$$

The ρ-parameter

- The relative strength of charged and neutral currents is given by:

$$\rho_0 = \frac{m_W^2}{m_Z^2 \cos^2\theta_W}$$

  with $\rho_0 = 1$ predicted in the Standard Model
- In practise, ρ_0 is not what is measured experimentally

$$\Rightarrow \quad \rho = 1 + \Delta\rho$$

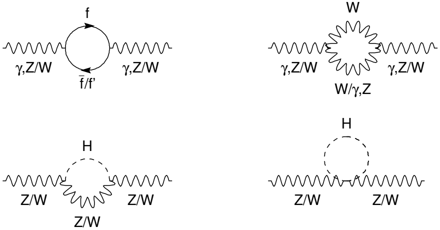
- Relation often expressed removing direct dependence on m_W:

$$m_W^2 = \frac{m_Z^2}{2}\left(1 + \sqrt{1 - 4\frac{\pi\alpha}{\sqrt{2}G_F m_Z^2}\frac{1}{1-\Delta r}}\right) \text{ with } \Delta r = \Delta\alpha - \frac{\cos^2\theta_W}{\sin^2\theta_W}\Delta\rho$$

- ...using the most precisely measured quantities as parameters

α :	from electron anomalous magnetic moment	$\Delta\alpha/\alpha = 2\cdot 10^{-10}$
G_F :	from muon lifetime	$\Delta m_{G_F}/G_F = 2\cdot 10^{-7}$
m_Z:	from LEP (shown before)	$\Delta m_Z/m_Z = 2\cdot 10^{-5}$

Radiative Corrections



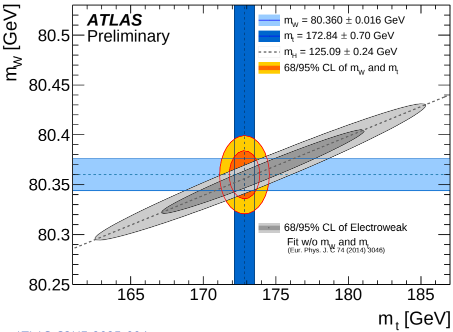
from Phys. Rept. 427 (2006) 257-454

- Contributions to Δr are radiative corrections
- Most importantly, corrections to the propagator self energies:

$$\Delta\rho_{\mathrm{se}} = \frac{3G_{\mathrm{F}}m_W^2}{8\sqrt{2}\pi^2}\left[\frac{m_t^2}{m_W^2} - \frac{\sin^2\theta_W}{\cos^2\theta_W}\left(\ln\frac{m_H^2}{m_W^2} - \frac{5}{6}\right) + \ldots\right]$$

- Loops induce quadratic dependence on the top quark mass and a weaker, logarithmic dependence on the Higgs boson mass

Comparison of Measurement and Theory



from ATLAS-CONF-2023-004

- The W boson mass is measured in agreement with the Standard Model
- Precision of the prediction from theory is more precise than the direct measurement
- Ultimately, LHC experiments are targetting $\sigma_{m_W} = 10$ MeV

Struture of Neutral Currents

- The neutral current involves both vector and axial vector couplings:

$$-i\frac{g}{\cos\theta_W}\left(J_\mu^3 - \sin^2\theta_W j_\mu^{\text{em}}\right)Z^\mu = -i\frac{g}{\cos\theta_W}\bar{\psi}\gamma^\mu\frac{1}{2}\Big(\underbrace{(1-\gamma^5)T^3 - 2\sin^2\theta_W Q}_{c_V - c_A\gamma^5}\Big)\psi Z_\mu$$

with the vector and axial vector couplings

$$c_V^f = T_f^3 - 2\sin^2\theta_W Q_f \quad \text{and} \quad c_A^f = T_f^3$$

- The V–A structure of neutral current introduces asymmetries in $e^+e^-\to f\bar{f}$ cross section:

$$\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} &= \frac{\pi\alpha^2(s)}{2s}\left[F_1\left(1+\cos^2\theta\right) + 2F_2\cos\theta\right] \\
F_1 &= Q_e^2 Q_f^2 - 2\chi Q_e Q_f c_V^e c_V^f \cos\delta_R + \chi^2\left(c_V^{e2} + c_A^{e2}\right)\left(c_V^{f2} + c_A^{f2}\right) \\
F_2 &= -2\chi Q_e Q_f c_A^e c_A^f \cos\delta_R + 4\chi^2 c_V^e c_A^e c_V^f c_A^f
\end{aligned}$$

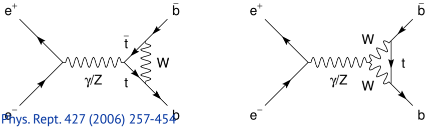
with

$$\chi = \frac{G_F}{2\sqrt{2}\pi\alpha(s)}\frac{s m_Z^2}{\sqrt{\left(m_Z^2 - s\right)^2 + m_Z^2\Gamma_Z^2}}$$

Radiative corrections to $\sin^2\theta_W$

- Also production and decay vertices receive radiative corrections

$$\sin^2\theta_W \rightarrow \kappa_f \sin^2\theta_W = \sin^2\theta^f_{\text{eff}}$$
$$c^f_V \rightarrow \sqrt{\rho_f}(T^3_f - 2Q_f \sin^2\theta^f_{\text{eff}})$$
$$c^f_A \rightarrow \sqrt{\rho_f}T^3_f$$



from Phys. Rept. 427 (2006) 257-454

- The corrections are flavour specific, and largest for $b\bar{b}$ production

$$\Delta\kappa_b = \frac{G_F m_t^2}{4\sqrt{2}\pi^2} + \ldots \quad \text{and} \quad \Delta\rho_b = -2\Delta\kappa_b + \ldots$$

- Experimentally, $\sin^2\theta^f_{\text{eff}}$ can be measured from asymmetries in Z/γ^* production

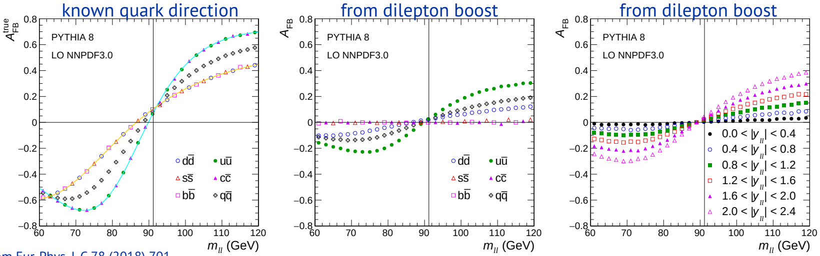
$$A_{\text{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

with $\sigma_F = \int_0^1 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\mathrm{d}\cos\theta, \sigma_B = \int_{-1}^0 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\mathrm{d}\cos\theta$

Forward-Backward Asymmetry and $\sin^2\theta^{\ell}_{\text{eff}}$

- At the LHC, the initial state (*pp*) is symmetric
- Only contributions from valence quarks generate a forward-backwards asymmetry

⇒ achieved by measuring the asymmetry in bins of the boost of the dilepton system

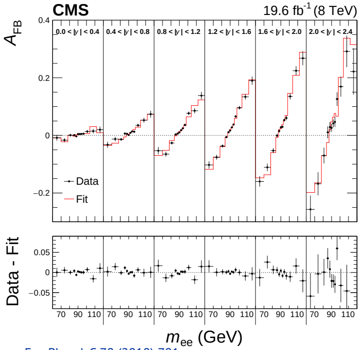


from Eur. Phys. J. C 78 (2018) 701

- Contributions from sea quarks dilute the measurement

⇒ PDF uncertainties are primary source of uncertainty for this measurement at the LHC

Measurement at the LHC



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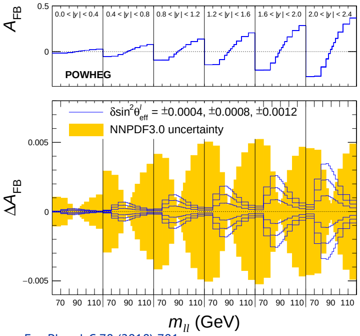


from Eur. Phys. J. C 78 (2018) 701

- The measured value of $\sin^2\theta^\ell_{\rm eff}$ is:

$$\sin^2\theta^\ell_{\rm eff} = 0.23101 \pm 0.00036\,(\text{stat}) \;\pm 0.00018\,(\text{syst}) \;\pm 0.00016\,(\text{theo}) \;\pm 0.00031\,(\text{PDF})$$

- In agreement with measurements at the Tevatron, LEP and SLD, but (still) less precise

Testing the Electroweak Theory

- Test the electroweak theory in a fit of precision measurements

→ Use more than minimal set to constrain theory

Free parameters:

- G_F, m_Z (mentioned before)
- evolution of fine structure constant:

$$\Delta\alpha(s) = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha^{(5)}_{\text{had}}(s) + \Delta\alpha_{\text{top}}(s)$$

relative precision: $1{\cdot}10^{-6}$ $2{\cdot}10^{-4}$ $1{\cdot}10^{-7}$

- Fermion masses (m_c, m_b, m_t)
- α_S
- Higgs mass

Several groups routinely perform these fits, e.g. HEPFit, GFitter, ZFitter

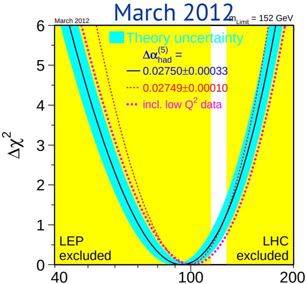
link to GFitter webpage

Parameter	Input value	Source
M_H [GeV]	125.1 ± 0.2	LHC
M_W [GeV]	80.379 ± 0.013	LHC, Tevatron, LEP
Γ_W [GeV]	2.085 ± 0.042	Tevatron, LEP
M_Z [GeV]	91.1875 ± 0.0021	LEP
Γ_Z [GeV]	2.4952 ± 0.0023	LEP
σ^0_{had} [nb]	41.540 ± 0.037	LEP
R^0_ℓ	20.767 ± 0.025	LEP
$A^{0,\ell}_{\text{FB}}$	0.0171 ± 0.0010	LEP
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	SLD, LEP
$\sin^2\theta^\ell_{\text{eff}}(Q_{\text{FB}})$	0.2324 ± 0.0012	LEP
$\sin^2\theta^\ell_{\text{eff}}(\text{TEV})$	0.2318 ± 0.0003	Tevatron
A_c	0.670 ± 0.027	SLD
A_b	0.923 ± 0.020	SLD
$A^{0,c}_{\text{FB}}$	0.0707 ± 0.0035	LEP
$A^{0,b}_{\text{FB}}$	0.0992 ± 0.0016	LEP
R^0_c	0.1721 ± 0.0030	SLD, LEP
R^0_b	0.21629 ± 0.00066	SLD, LEP
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	
m_t [GeV]$^{(\triangledown)}$	173.06 ± 0.94	LHC, Tevatron
$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ $^{(\dagger\triangle)}$	2758 ± 10	
$\alpha_s(M_Z^2)$	–	

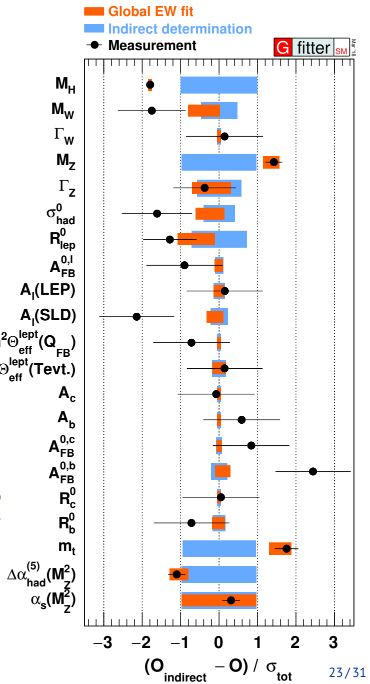
Electroweak Fit Results

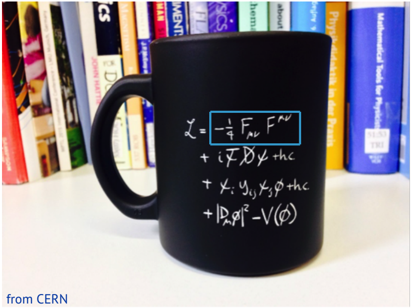
- ● Experimental measurement
- ■ Full fit (orange)
- ■ Fit excluding specific measurement (blue)
- No individual measurement differes by more than 3σ from fit
- Largest deviations in b-sector
- Small pulls for m_H and m_Z
- Historically, such a fit was used to constrain the Higgs boson mass

LEP/TEV EW WG



link to GFitter webpage





from CERN

Gauge-boson Self-interactions

Gauge Symmetries

U(1) Local Gauge Symmetry (QED) | $SU(2)_L \times U(1)_Y$ Local Gauge Symmetry

- Lagrangian density of freely propagating ...

fermions:

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi$$

isospin doublets and singlets:

$$\mathcal{L} = \bar{\chi}_L\left(i\gamma^{\mu}\partial_{\mu} - m\right)\chi_L + \bar{\psi}_R\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi_R$$

- Require $\mathcal{L}$ to be invariant under local gauge transformation:

$$\psi \rightarrow e^{i\alpha(x)}\psi$$

$$\chi_L \rightarrow e^{i\beta(x)Y + i\alpha_a(x)\tau_a}\chi_L$$
$$\psi_R \rightarrow e^{i\beta(x)Y}\psi_R$$

- Achieved by introducing vector fields in the derivative:

$$\partial_{\mu} \rightarrow \partial_{\mu} - ieA_{\mu}$$

$$\partial_{\mu} \rightarrow \partial_{\mu} - ig\frac{\tau_a}{2}W_{\mu}^a + \frac{g'}{2}B_{\mu}$$

- With transformation properties:

$$A_{\mu} \rightarrow A_{\mu} + \frac{1}{e}\partial_{\mu}\alpha$$

$$B_{\mu} \rightarrow B_{\mu} + \frac{1}{g'}\partial_{\mu}\beta$$
$$W_{\mu}^a \rightarrow W_{\mu}^a + \frac{1}{g}\partial_{\mu}\alpha_a - \epsilon_{abc}\alpha_b W_{\mu}^c$$

Lagrangian Densitiy

- Substitution yields the Lagrangian densities:

$$\mathcal{L}_{\text{QED}} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi + e\bar{\psi}\gamma^\mu A_\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\mathcal{L}_{\text{EWK}} = \bar{\chi}_L\gamma^\mu\left(i\partial_\mu - g\frac{\tau_a}{2}W^a_\mu + \frac{g'}{2}B_\mu\right)\chi_L + \bar{\psi}_R\gamma^\mu\left(i\partial_\mu + g'B_\mu\right)\psi_R - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

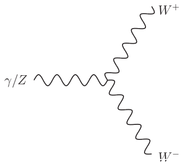
- The kinetic terms of the gauge fields contain the field strength tensors

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial\nu B_\mu$$
$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon_{abc}W^b_\mu W^c_\nu$$

- As previously, the physical fields are $W^\pm_\mu$, Z_μ and A_μ

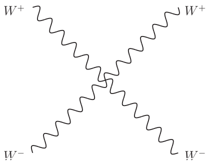
Self-Interactions

- The more complicated structure of the field strength tensor leads to self interactions between the three vector fields $\mathbf{W}_\mu$
- It generates cubic and quartic couplings



$$\mathcal{L}_3 = ie_{V=\gamma,Z}\left[W^+_{\mu\nu}W^{-\mu}V^\nu - W^-_{\mu\nu}W^{+\mu}V^\nu + W^+_\mu W^-_\nu V^{\mu\nu}\right]$$

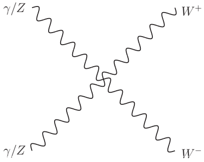
$$\mathcal{L}_4 = e_W^2 \left[W^-_\mu W^{+\mu}W^-_\nu W^{+\nu} - W^-_\mu W^{-\mu}W^+_\nu W^{+\nu}\right]$$
$$+ e^2_{V=\gamma,Z}\left[W^-_\mu W^{+\mu}V_\nu V^\nu - W^-_\mu V^\mu W^+_\nu Z^\nu\right]$$
$$+ e_\gamma e_Z\left[2W^-_\mu W^{+\mu}Z_\nu A^\nu - W^-_\mu Z^\mu W^+_\nu A^\nu - W^-_\mu A^\mu W^+_\nu Z^\nu\right]$$

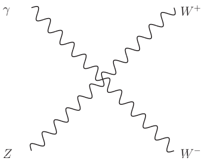


- With predictions of the coupling strength:

$$e = g\sin\theta_W, e_W = \frac{e}{2\sqrt{2}\sin\theta_W} \text{ and } e_Z = e\cot\theta_W$$

- They *always* involve a pair of W bosons, there are no neutral vertices





Vector-Boson Pair Production

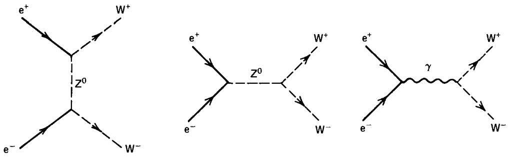
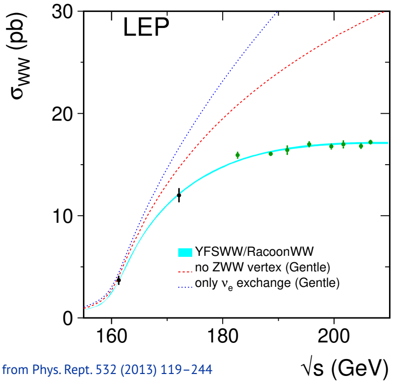


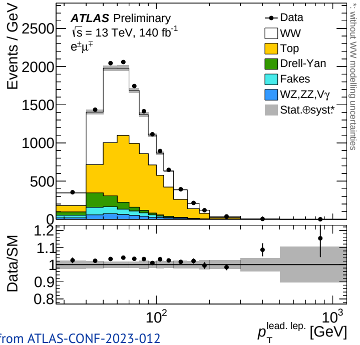
Fig. 15.3 Three contributions to the process $e^-e^+ \rightarrow W^-W^+$.



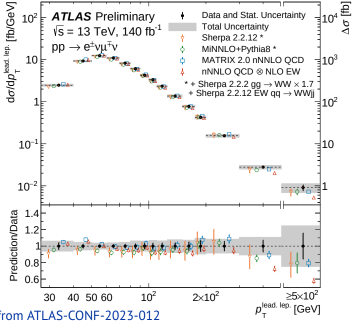
from Phys. Rept. 532 (2013) 119–244

- Self interactions can be probed in the pair production of W and Z bosons
- Individual diagrams diverge with energy
- The gauge cancellations through self interactions remove the divergencies
- There are no divergencies up to $\mathcal{O}(\alpha^4)$ (counting V production and decay)

W-Boson Pair Production



from ATLAS-CONF-2023-012



from ATLAS-CONF-2023-012

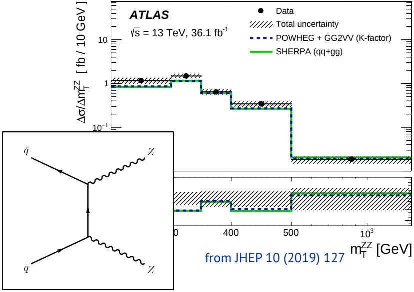
- The measurement of W-Boson pair production confirms the mechanism of gauge cancellations up to the TeV scale
- Contributions from beyond-SM physics could destroy the exact cancellations, leading to potentially large effects

In the LHC run-2:

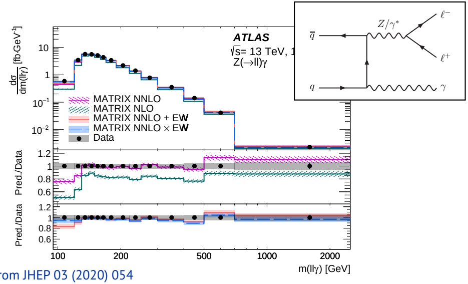
- 17 million WW events
- 7 million WZ events
- 2.5 million ZZ events

Neutral gauge Couplings

- The pair production of $\gamma\gamma$, $Z\gamma$ and ZZ do *not* exhibit gauge cancellations



from JHEP 10 (2019) 127



from JHEP 03 (2020) 054

- Also in processes sensitive to non-SM neutral couplings, no deviations are observed tomorrow's lecture

How to use these measurements to quantify our understanding of the electroweak theory will be discussed in tomorrow's lecture

Literature

- *Quarks and Leptons: An Introductory Course in Modern Particle Physics*; Francis Halzen, Alan Douglas Martin;Wiley, 1984
- *Precision electroweak measurements on the Z resonance*; ALEPH, DELPHI, L3, OPAL, SLD; Phys. Rept. **427** (2006), 257-454 [arXiv:hep-ex/0509008 [hep-ex]].
- *Review of Particle Physics*; Particle Data Group; PTEP **2022** (2022), 083C01; `https://pdg.lbl.gov/`

Backup