PARTON SHOWERS

LEARNING OBJECTIVES

By the end of this lecture, you will be able to:

- Define a parton shower
- Describe why parton showers are needed to simulate realistic final states
- Interpret parton showering as a probabilistic process
- Discuss the need for a hadronisation model and describe the most common approaches

LIMITATIONS OF FIXED ORDER CALCULATIONS

- Calculations at fixed order the most accurate description of high energy (hard) processes
- Accuracy is systematically improvable (though difficult)
- However: they break down when physics is low energy (soft and/or collinear)
- Describe only partonic final states
- Can only deal with a small number of particles (max ~10 at NLO, max 3 at NNLO)

PARTON SHOWER ALGORITHMS

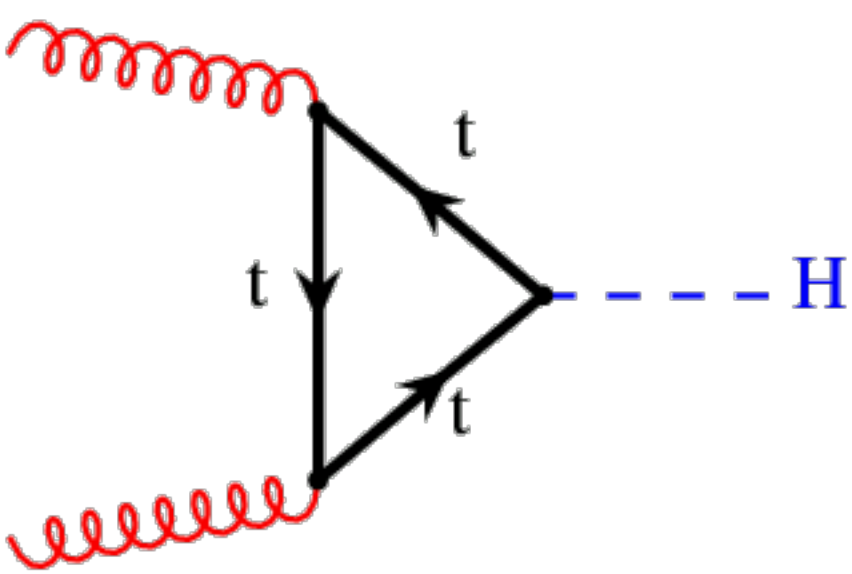
- Parton showers bridge the gap between hard (TeV) and soft (GeV) scales Q by resolving multiple gluon emissions
- They evolve down to hadronic scales Q_0 (long distances)
- Presence of multiple scales - rate is determined by large logarithms,

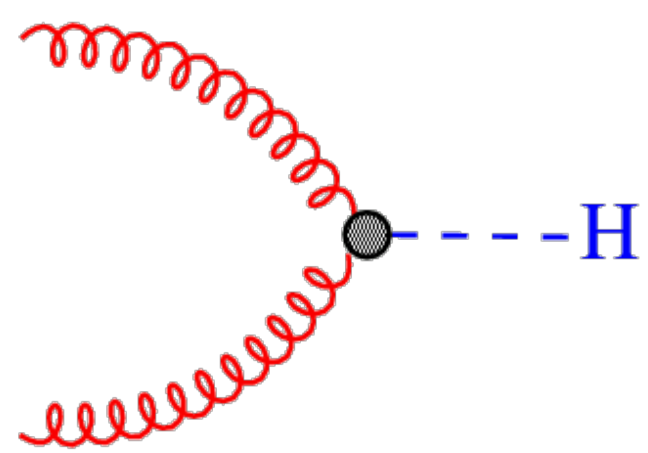
$$\alpha_s^n \log^{2n} \frac{Q}{Q_0} \sim 1$$

- Generated by emissions ordered in Q

EXAMPLE – HIGGS PRODUCTION IN GLUON FUSION

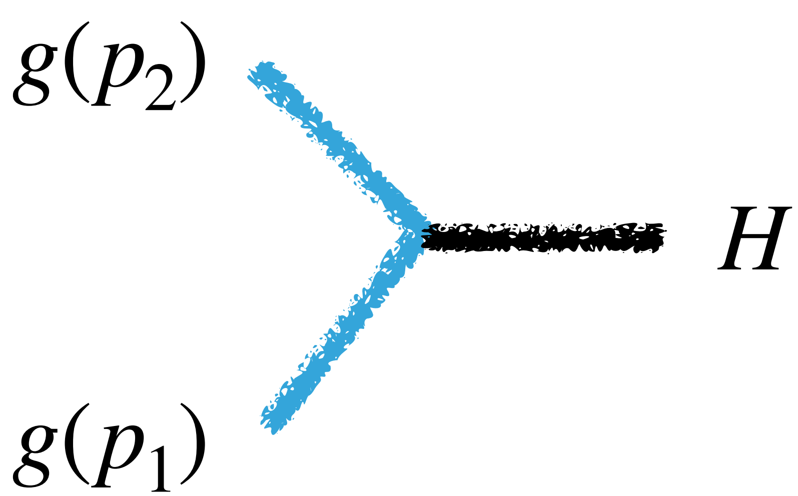
- At the LHC, Higgs bosons are mainly produced via gluon fusion. Higgs does not couple directly to gluons, but through a top-quark loop.
- Can be 'integrated out' to give an effective theory coupling Higgs and gluons - good approximation, simpler calculations



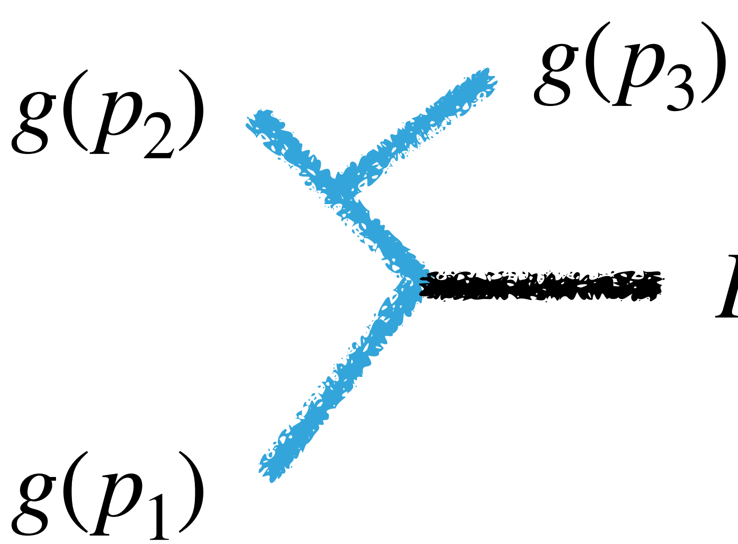


HIGGS PRODUCTION IN GLUON FUSION

- Calculate squared MEs ($C = \alpha_s / 6\pi v$):



$$|\mathscr{M}_{Hgg}|^2 = 2(N_c^2 - 1) m_H^4 C^2$$



$$|\mathscr{M}_{Hggg}|^2 = 4N_c(N_c^2 - 1)C^2 g_s^2$$
$$\times \left(\frac{m_H^8 + (2p_1 \cdot p_2)^4 + (2p_1 \cdot p_2)^4 + (2p_1 \cdot p_2)^4}{8\,(p_1 \cdot p_2)\,(p_1 \cdot p_3)\,(p_2 \cdot p_3)} \right)$$

HIGGS PRODUCTION IN GLUON FUSION - COLLINEAR LIMIT

- Examine in the limit that gluons 2 and 3 are collinear:
  $p_2 = zP, p_3 = (1-z)P$:

$$2p_1 \cdot p_2 \to z m_H^2,\ 2p_1 \cdot p_3 \to (1-z) m_H^2,\ 2p_2 \cdot p_3 \to 0$$

- The three gluon ME reduces to

$$|\mathscr{M}_{Hggg}|^2 \to 4N_c(N_c^2-1)C^2 g_s^2 m_H^4 \left(\frac{1+z^4+(1-z)^4}{2z(1-z)p_2 \cdot p_3} \right)$$

- We recognise the two gluon ME inside!

HIGGS PRODUCTION IN GLUON FUSION - COLLINEAR LIMIT

- Rewriting,

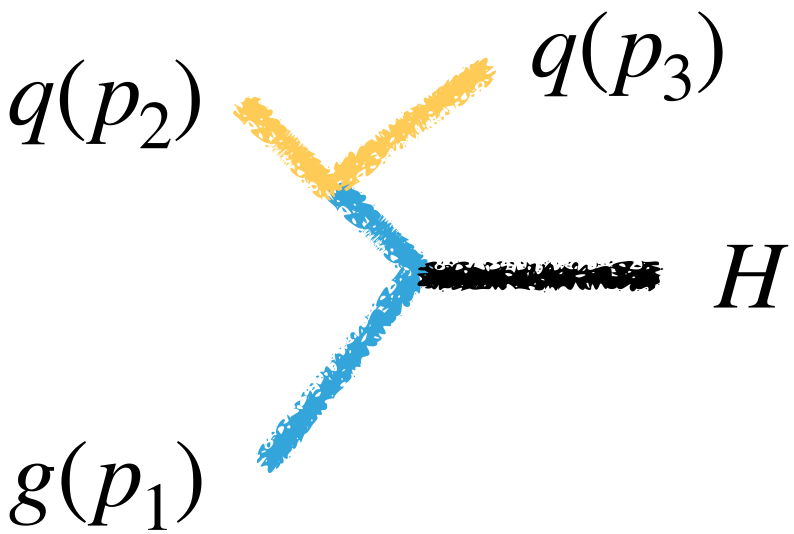
$$|\mathscr{M}_{Hggg}|^2 \to \frac{2g_s^2}{2p_2 \cdot p_3} |\mathscr{M}_{Hgg}|^2 P_{gg}(z)$$

- The collinear splitting function P_{gg} is given by

$$P_{gg}(z) = 2N_c \left(\frac{z^2 + (1-z)^2 + z^2(1-z)^2}{z(1-z)} \right)$$

HIGGS PRODUCTION IN GLUON FUSION – COLLINEAR LIMIT

- Repeat for quarks:



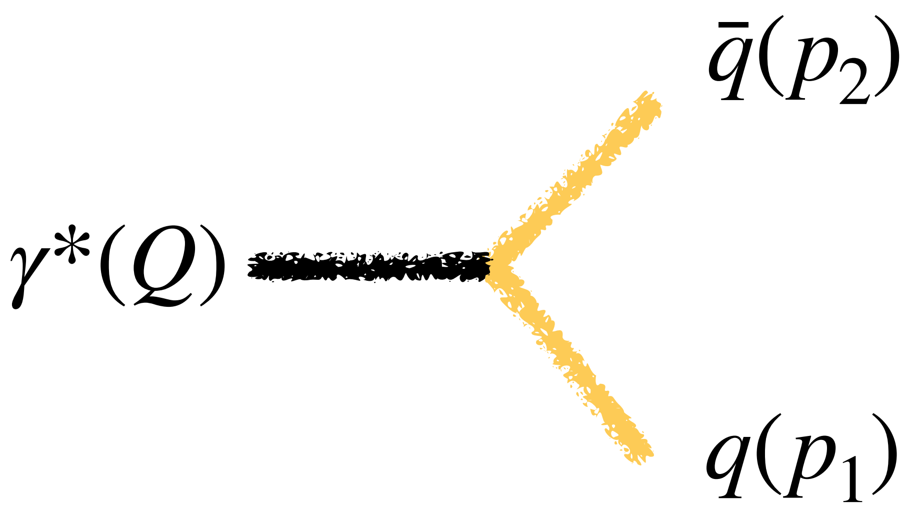
$$|\mathcal{M}_{Hg\bar{q}q}|^2 = 4T_R(N_c^2 - 1)C^2 g_s^2 \times \left(\frac{(2p_1 \cdot p_2)^2 + (2p_1 \cdot p_3)^2}{2\,(p_2 \cdot p_3)} \right)$$

$$|\mathcal{M}_{Hg\bar{q}q}|^2 \to \frac{2g_s^2}{2p_2 \cdot p_3} |\mathcal{M}_{Hgg}|^2 P_{qg}(z)$$

$$P_{qg}(z) = T_R \left[z^2 + (1-z)^2 \right]$$

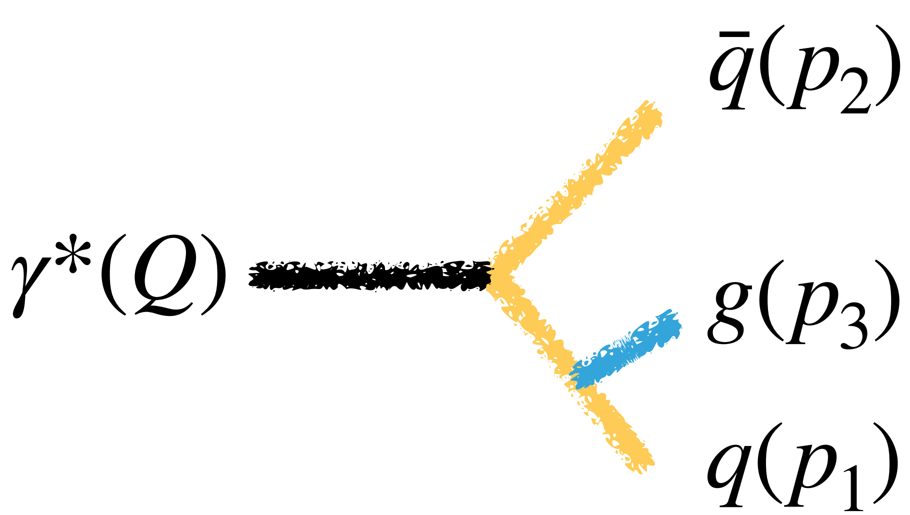
QUARK-GLUON COLLINEAR LIMIT

- Remaining possibility not present in Higgs process, but can look at e.g. $\gamma^* \to q\bar{q}$:



$$|\mathcal{M}_{\gamma^*\bar{q}q}|^2 = 4N_c e_q^2 Q^2$$

$$|\mathcal{M}_{\gamma^*\bar{q}qg}|^2 = 8N_c C_F e_q^2 g_s^2 Q^2 \left(\frac{(2p_1\cdot p_3)^2 + (2p_2\cdot p_3)^2 + 2Q^2(2p_1\cdot p_2)}{4(p_1\cdot p_3)\,(p_2\cdot p_3)} \right)$$



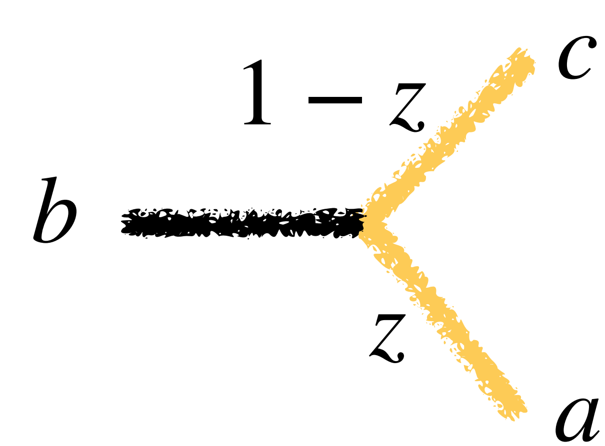
$$P_{qq}(z) = C_F\left(\frac{1+z^2}{1-z}\right)$$

SPLITTING FUNCTIONS AND FACTORISATION

- The results for the splitting functions are actually universal - they are the same for every QCD process in the collinear limit

$$|\mathcal{M}_{ac\ldots}|^2 \rightarrow \frac{2g_s^2}{2p_a \cdot p_c} |\mathcal{M}_{b\ldots}|^2 P_{ab}(z)$$

Collinear singularity



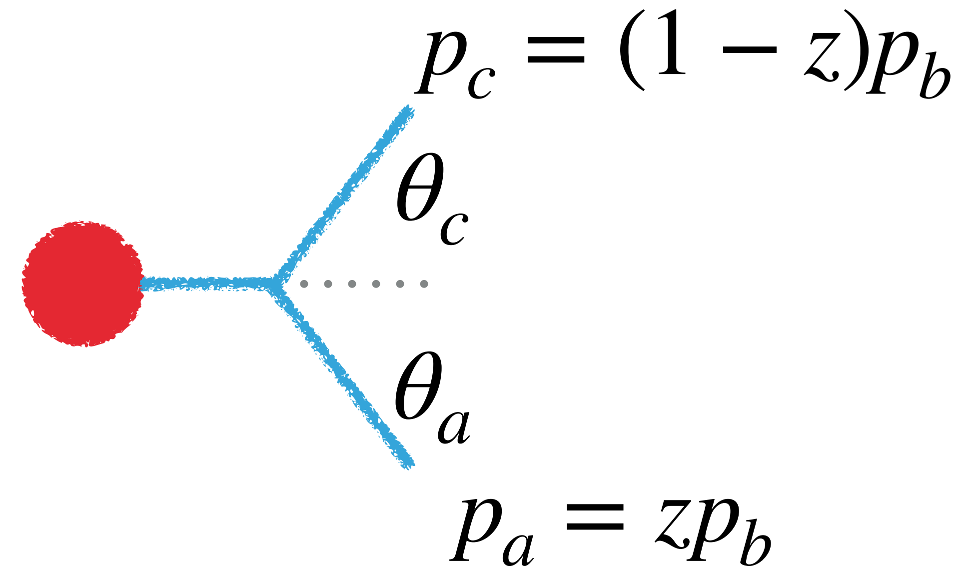
- Soft singularities also present in P_{qq}, P_{gg} as $z \rightarrow 1$.
- Singularities imply soft/collinear radiation is favoured.

PHASE SPACE FACTORISATION

- Same considerations apply to phase space:

$$\mathrm{d}\Phi_{\ldots b} = (\ldots)\frac{\mathrm{d}^3\overrightarrow{p_b}}{(2\pi)^3 2E_b}$$

$$\mathrm{d}\Phi_{\ldots ac} = (\ldots)\frac{\mathrm{d}^3\overrightarrow{p_a}}{(2\pi)^3 2E_a}\frac{\mathrm{d}^3\overrightarrow{p_c}}{(2\pi)^3 2E_c}$$



$$\mathrm{d}\Phi_{\ldots ac} = \mathrm{d}\Phi_{\ldots b}\frac{\mathrm{d}^3\overrightarrow{p_a}}{(2\pi)^3 2E_a}\frac{E_b}{E_c} \approx \mathrm{d}\Phi_{\ldots b}\frac{1}{(2\pi)^2}\frac{E_aE_b}{2E_c}\mathrm{d}E_a\theta_a\mathrm{d}\theta_a$$

Exercise - prove for small θ_a

PHASE SPACE FACTORISATION

- Small-angle kinematics, collinear limit: momentum conservation gives

$$E_a = zE_b, E_c = (1-z)E_b$$
$$z\theta_a - (1-z)\theta_c = 0$$

- Relate opening angle $\theta = \theta_a + \theta_c$ to Mandelstam:

$$t = \left(p_a + p_c\right)^2 = 2E_aE_c(1-\cos^2\theta) = \frac{zE_b^2\theta_a^2}{1-z}$$

- Change variables in phase space $(E_a, \theta_a) \to (z, t)$

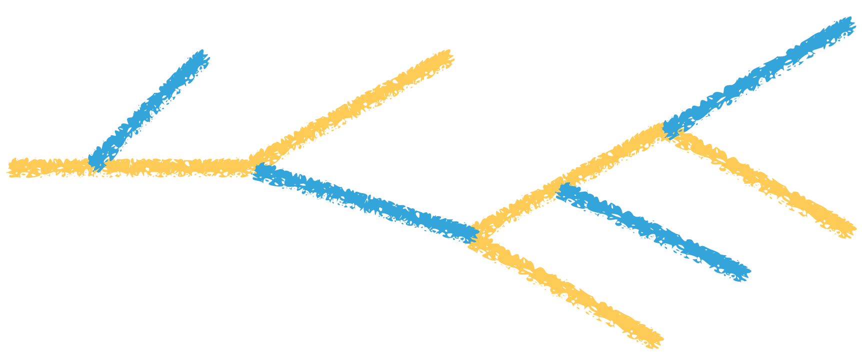
$$\mathrm{d}\Phi_{\ldots ac} = \mathrm{d}\Phi_{\ldots b}\frac{\mathrm{d}z\,\mathrm{d}t}{16\pi^2}$$

PARTON SHOWER

- Combining phase space and matrix element factorisation, we arrive at

$$\mathrm{d}\sigma_{n+1} = \mathrm{d}\sigma_n \left(\frac{\alpha_s}{2\pi}\right) \frac{\mathrm{d}t}{t} P_{ab}(z)\,\mathrm{d}z$$

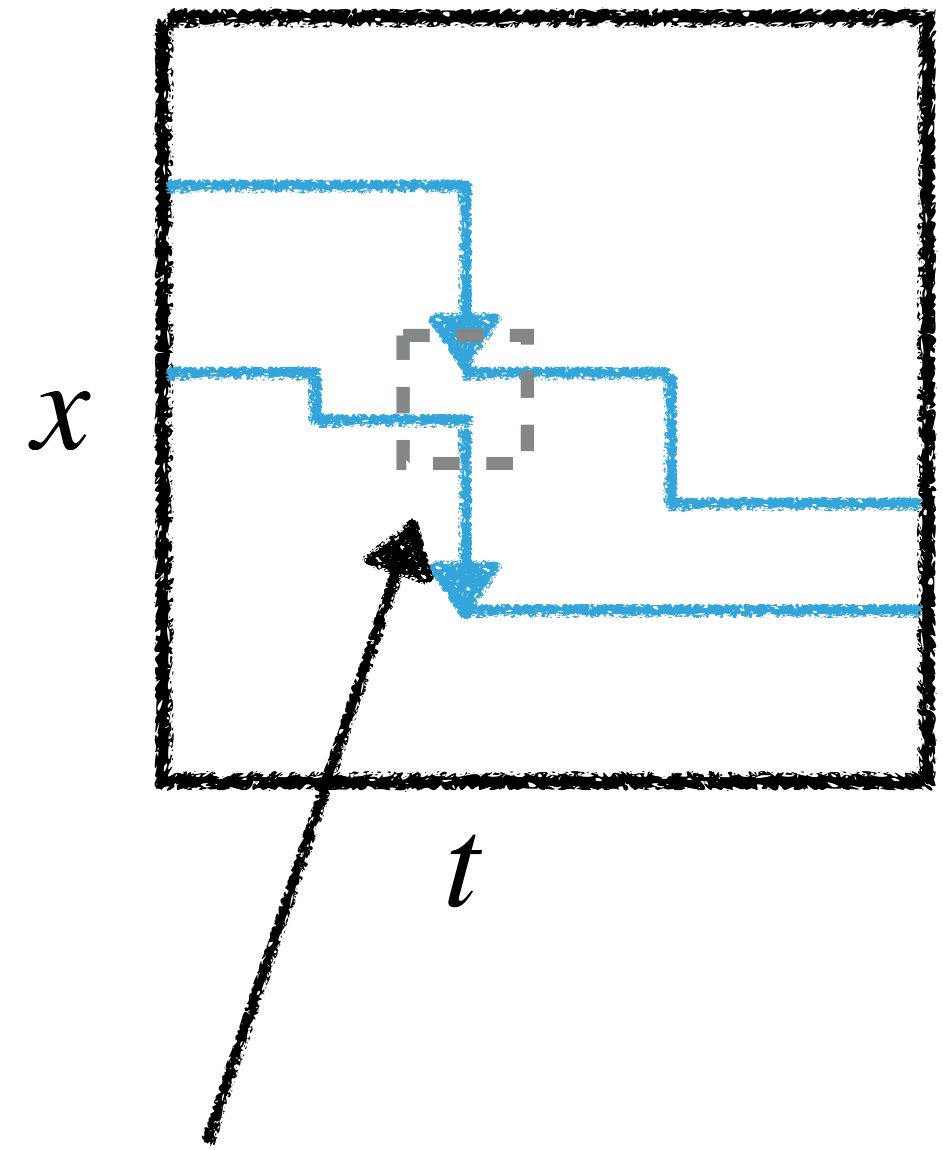
- Iterate to generate soft and collinear radiation!



- Evolution in invariant mass t reduces momentum fraction z
- Treatment so far for final state radiation ($t > 0$) - for IS, must account for PDFs

PARTON SHOWER

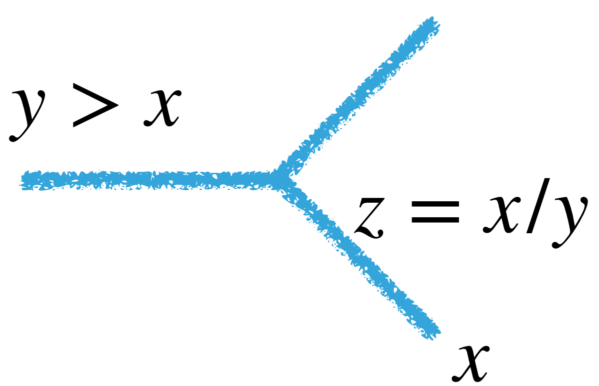
- Master equation for parton shower can be interpreted as the probability $f(x, t)$ of a given parton branching at values (x, t) - how does $f(x, t)$ change in time step $t + \delta t$?
- Represent multiple emissions as paths in (x, t) space. Branchings are vertical steps, where x changes at fixed t.



Change in $f(x, t)$ = (paths in - paths out) / δx

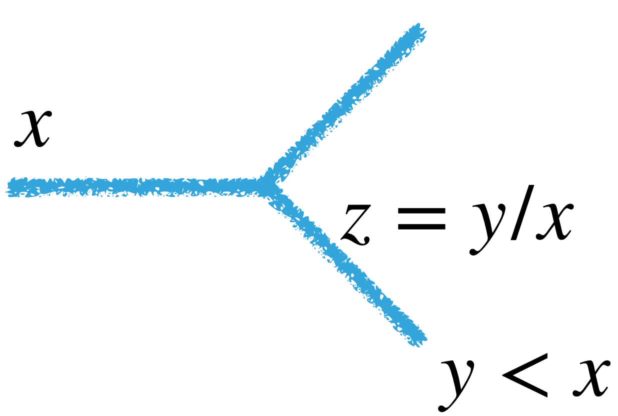
PARTON SHOWER

- Consider gluon-only case - integrate branching probabilities to find numbers in and out:



$$\delta f_{\mathrm{in}}(x,t) = \frac{\delta t}{t}\int_x^1 \mathrm{d}y\,\mathrm{d}z\left(\frac{\alpha_s}{2\pi}\right)P_{gg}(z)\,f(y,t)\,\delta(x-zy)$$

$$= \frac{\delta t}{t}\int_x^1 \frac{\mathrm{d}z}{z}\left(\frac{\alpha_s}{2\pi}\right)P_{gg}(z)\,f(x/z,t)$$



$$\delta f_{\mathrm{out}}(x,t) = \frac{\delta t}{t}f(x,t)\int_0^x \mathrm{d}y\,\mathrm{d}z\left(\frac{\alpha_s}{2\pi}\right)P_{gg}(z)\,\delta(y-xz)$$

$$= \frac{\delta t}{t}f(x,t)\int_x^1 \mathrm{d}z\left(\frac{\alpha_s}{2\pi}\right)P_{gg}(z)$$

DGLAP EVOLUTION

- Taking the difference, arrive at the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation

$$t\frac{\partial f(x,t)}{\partial t} = \int_0^1 \mathrm{d}z \left(\frac{\alpha_s}{2\pi}\right) P_{ab}(z) \left(\frac{1}{z} f(x/z,t) - f(x,t)\right)$$

- Introducing the Sudakov form factor,

$$\Delta(t) = \exp\left[-\int_{t_0}^{t} \frac{\mathrm{d}t'}{t'} \int \mathrm{d}z \left(\frac{\alpha_s}{2\pi}\right) P_{ab}(z)\right]$$

write as

$$t\frac{\partial}{\partial t}\left(\frac{f(x,t)}{\Delta(t)}\right) = \frac{1}{\Delta(t)} \int \frac{\mathrm{d}z}{z} \left(\frac{\alpha_s}{2\pi}\right) P_{ab}(z) f(x/z,t)$$

THE SUDAKOV FORM FACTOR

- Integrate previous equation to find solution

$$f(x,t) = \Delta(t) f(x,t_0) + \int_{t_0}^{t} \frac{\mathrm{d}t'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{\mathrm{d}z}{z} \left(\frac{\alpha_s}{2\pi}\right) P_{ab}(z) f(x/z, t)$$

No branching between t_0 and t

For each value of t', no branching between t' and t

- Sudakov gives no emission probability!
- Back to MC: generate $r \in [0,1]$, determine t_2 from $\Delta(t_2)/\Delta(t_1) = r$, generate z to get integral right

EXPONENTIATION AND RESUMMATION

- Have to cut-off z integration to avoid singularities - defines what is a resolvable emission for the shower

- No resolvable emissions for quark in the simplest case:

$$\Delta_q(t) = \exp\left[-\int_{t_0}^{t} \frac{\mathrm{d}t'}{t'} \int_{t_0/t'}^{1-t_0/t'} \mathrm{d}z \left(\frac{\alpha_s}{2\pi}\right) P_{qq}(z)\right]$$

$$\sim \exp\left[-C_F\left(\frac{\alpha_s}{2\pi}\right)\log^2\left(\frac{t}{t_0}\right)\right]$$

- Exponentiation sums terms with a double log - in resummation terminology, this is a leading log evolution

EVOLUTION VARIABLES

- At the moment have considered the virtuality t as the evolution variable, but other choices possible:

$$t = z(1-z)E_b^2\theta^2$$
$$p_T^2 = \theta_a^2 E_a^2 = z^2(1-z)^2 E_b^2 \theta^2$$

- For constant z, these imply

$$\frac{\mathrm{d}t}{t} = \frac{\mathrm{d}\theta^2}{\theta^2} = \frac{\mathrm{d}p_T^2}{p_T^2}$$

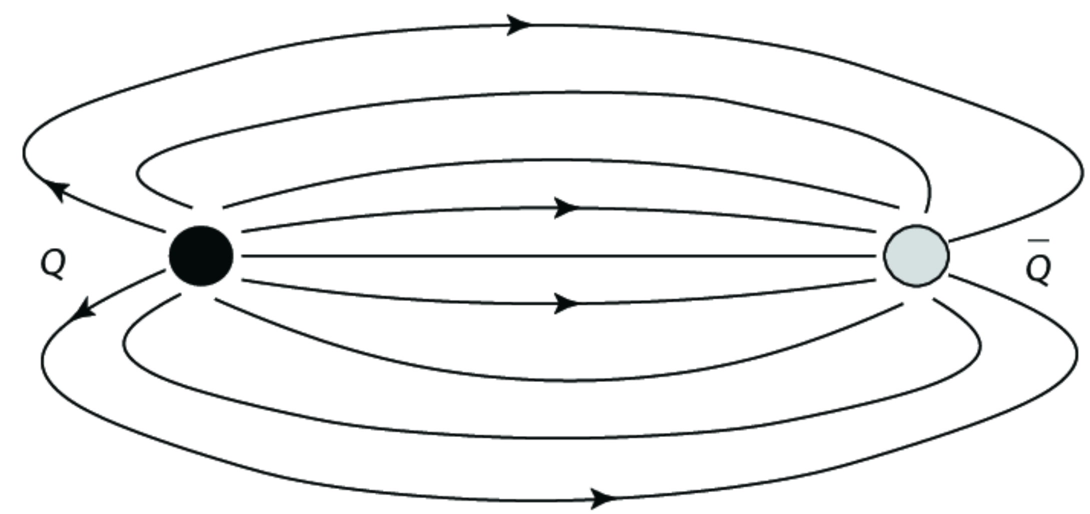
- Equivalent in collinear limit but different elsewhere. Of the Big Three, PYTHIA and SHERPA use p_T, HERWIG uses θ

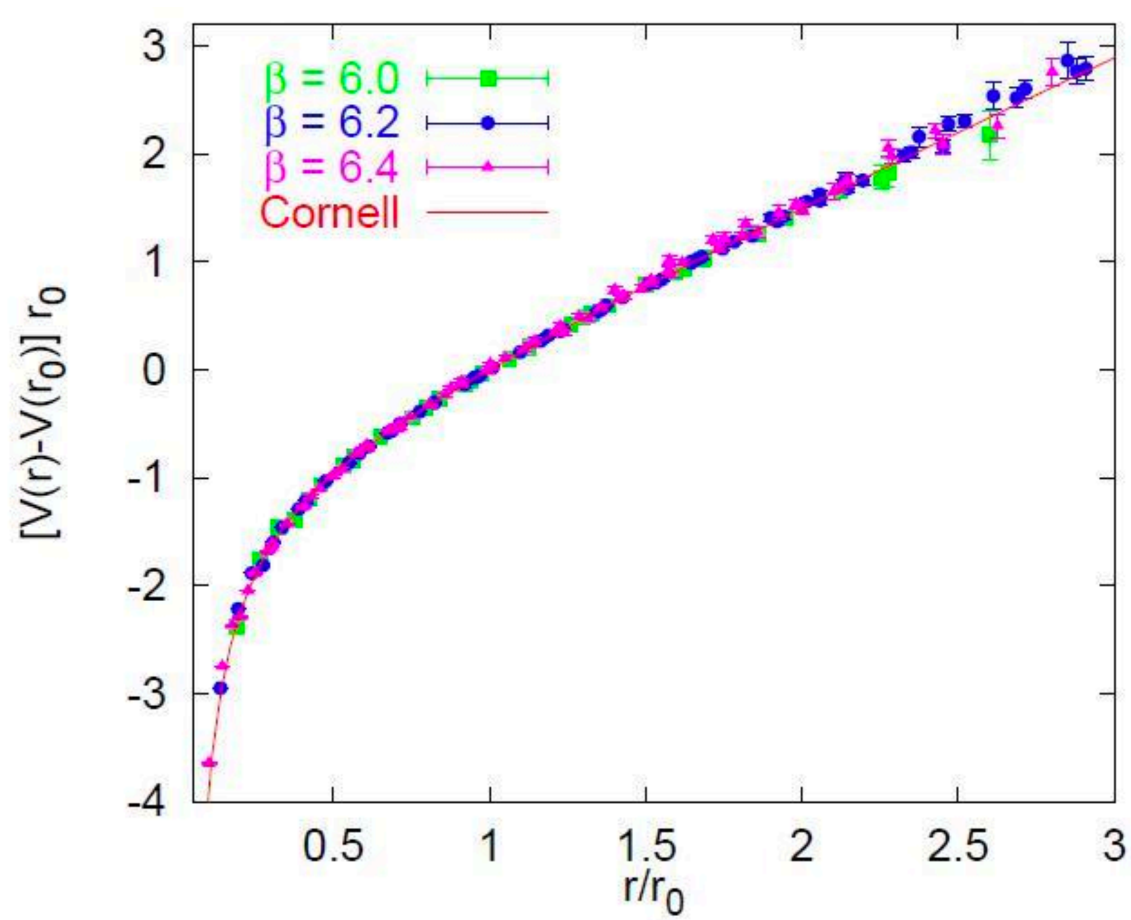
THIS TOO SHALL PASS

- Nothing gold can stay. Eventually, shower evolution brings us to ~1 GeV scales where perturbation theory breaks down. No more branching!
- Once all partons meet this fate, we are left with quarks and gluons. Detector, however, sees hadrons!
- Need a hadronisation model to describe transition from partons to hadrons. Nonperturbative physics ⇒not calculable from first principles, use empirical models

THE BROWN MUCK

- What do we actually know?
- Gluons self-couple - field lines are attractive (to each other!)
- Inter-quark potential is quasi-linear with separation (known from lattice data, hadron spectroscopy)



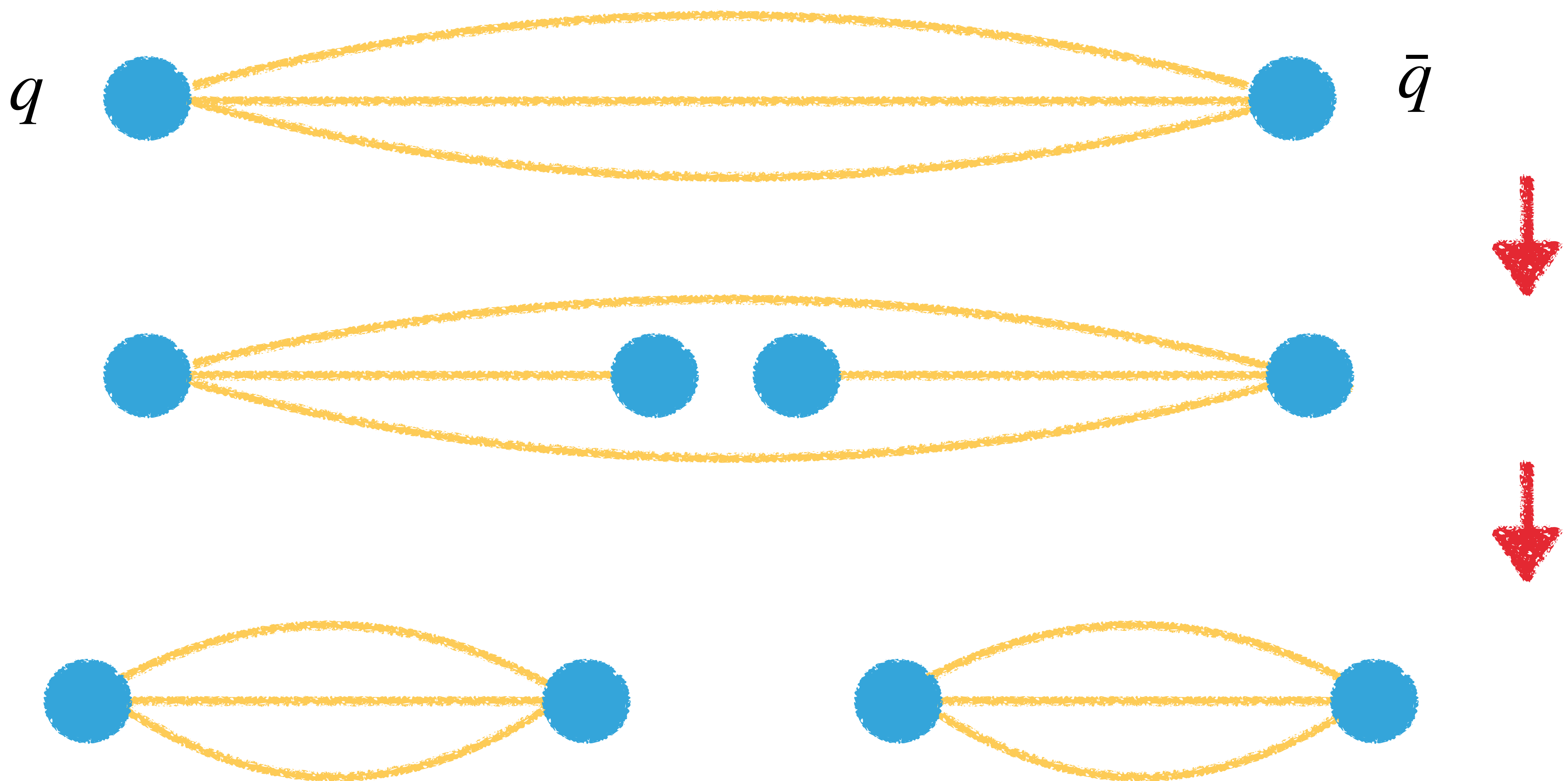


FEYNMAN-FIELD FRAGMENTATION ('78)

- $q\bar{q}$ pairs created from vacuum to dress bare quarks.
- Fragmentation functions $f_{q\to h}(z)$ give density of momentum fraction z carried away by hadron h from quark q
- Gaussian p_T distribution, recursively split $q \to q' + h$
- Flaws: frame dependent, no connection with perturbative physics, not IR safe, not a confinement model, wrong energy dependence

LUND STRING MODEL

- Model linear potential as a stretched spring (string)
- Mesons are oscillating strings



LUND STRING MODEL

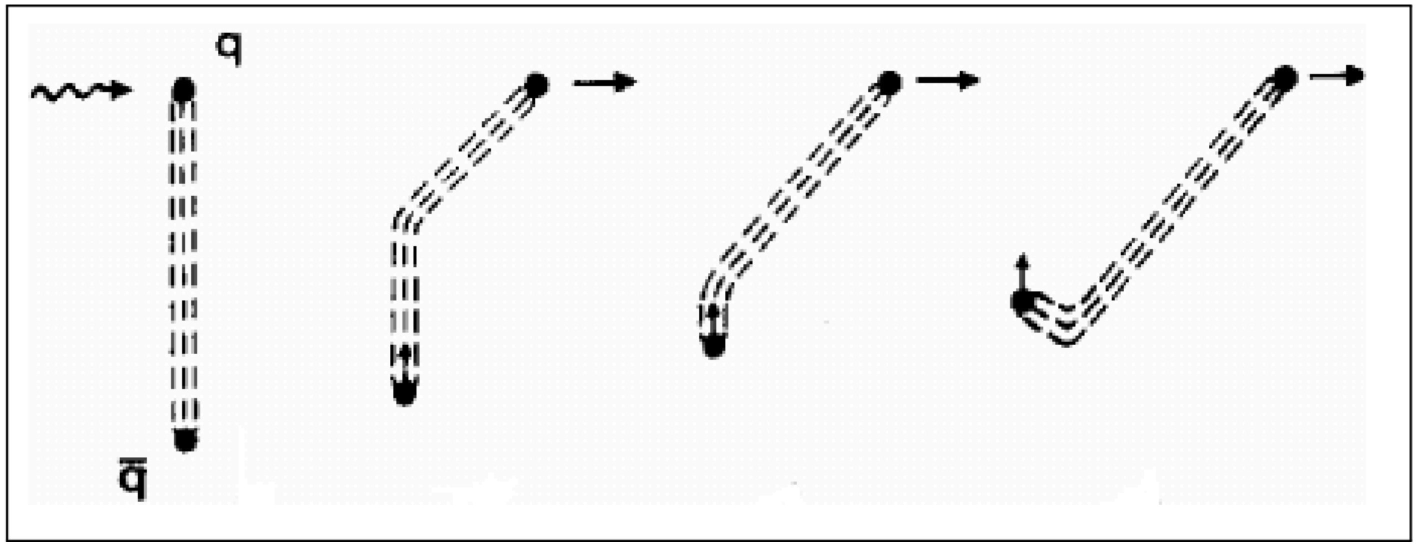
- Force between quarks eventually drops as it becomes energetically favourable for string to break
- Tunnelling probability for breaking of string with tension κ given by

$$P \propto \exp\left(\frac{\pi m_{\perp q}^{2}}{\kappa}\right)$$

- Suppressed by quark masses (so charm almost negligible)
- Suppressed by high transverse momentum - breaking gives back-to-back particles in CM frame

LUND STRING MODEL

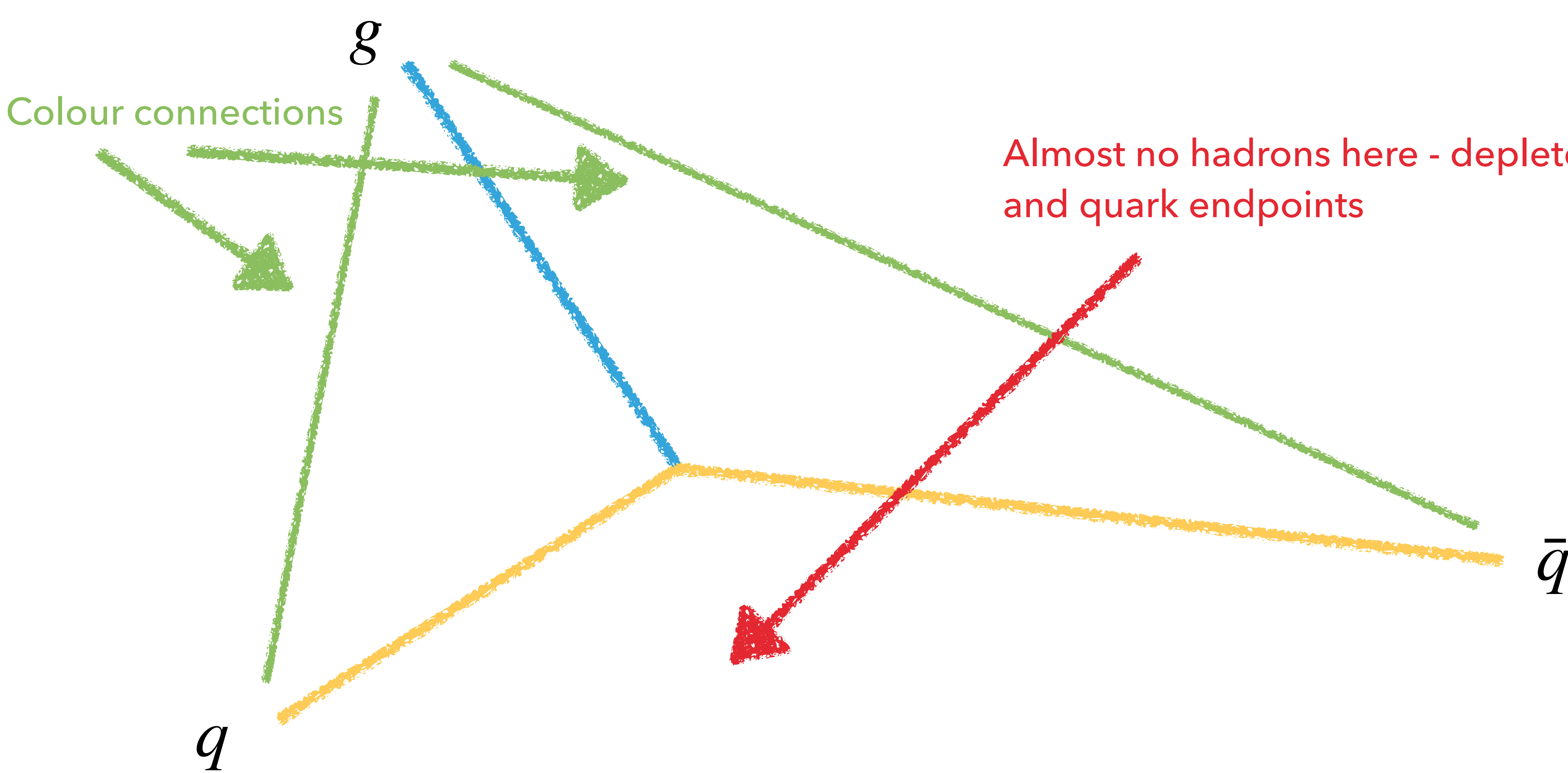
- Need to account for gluons too - model as kinks in string
- Large, instantaneous momentum transfer at initial time which stretches string



- Connected to two string segments, so loses energy twice as fast as the endpoints ($C_A/C_F \sim 2$)

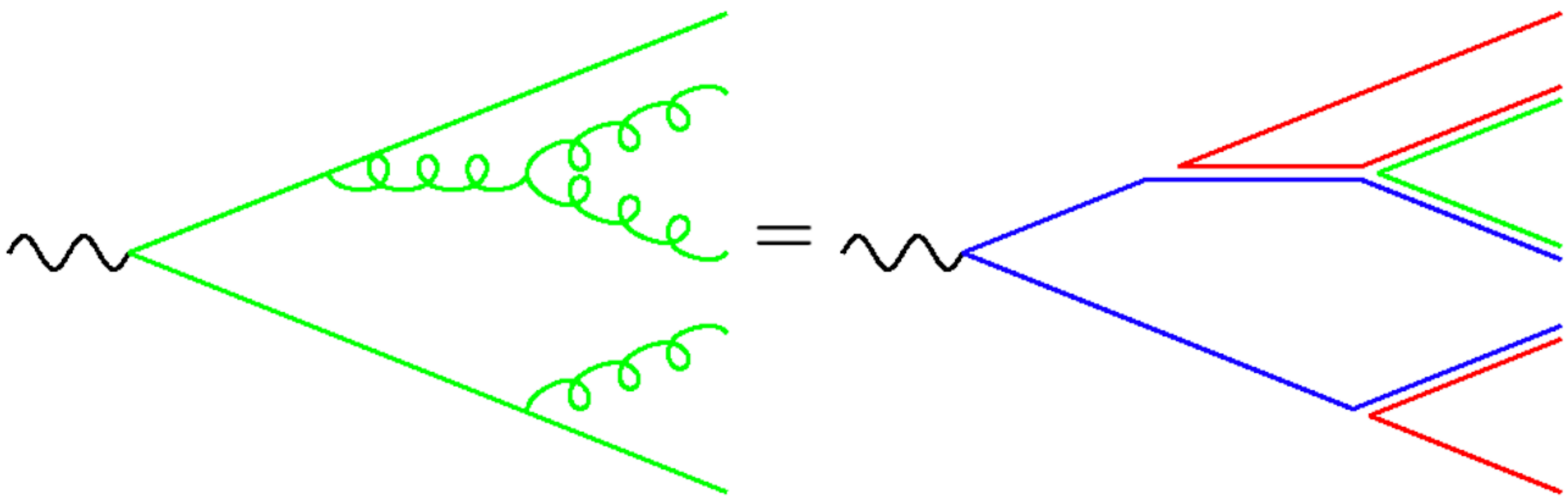
LUND STRING MODEL

- Consequence: string effect results in depleted regions



PRECONFINEMENT

- In large N_c limit, planar graphs dominate:



- Colour partners (forming a colour singlet) are produced 'close' to each other by parton shower.
- Can be paired up to form 'clusters'

CLUSTER MODEL

Steps:

- Trace colour through shower
- Convert gluons to quark pairs with heuristic model
- Collect quark pairs into colour singlet clusters
- Cluster masses peaked at low scales: decay heavy clusters into lighter ones
- Light clusters decay to hadrons

HADRONISATION MODELS

- String model based on nonperturbative physics, improved by perturbative QCD (parton shower)
- Cluster model based on perturbative physics, improved by string-like cluster fission
- Both depend on a large number of free parameters - extracted by tuning to data, usually from LEP
- Overall good agreement, ~5% over many observables and energy scales

SUMMARY

- Parton shower algorithms use a soft/collinear approximation to the matrix element to iteratively generate many emissions
- Shower evolves from a high to low scale
- Sudakov factor gives shower no-emission probability between two scales
- End of showering is described by nonperturbative hadronisation models