Neutrino Physics

Neutrino masses and phenomenology

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Neutrino masses

Weyl fermions any field in an irreducible representation of the Lorentz group will be a Weyl fermion

$$\chi_+ = (1/2, 0) \qquad \chi_- = (0, 1/2)$$

- two component spinors which obey:

$$i\sigma^\mu \partial_\mu \chi_+ = 0 \qquad \sigma^\mu = (1, \sigma)$$

$$i\bar{\sigma}^\mu \partial_\mu \chi_- = 0 \qquad \bar{\sigma}^\mu = (, -\sigma)$$

- χ_+ and χ_- have different chiralities:

$\chi_+ \Rightarrow$ left-handed

$\chi_- \Rightarrow$ right-handed

Neutrino masses

Dirac fermions

$$(i\not{\partial} - m)\psi = 0 \qquad \psi \equiv \text{four-component spinor}$$

$$\psi = \psi_L + \psi_R = P_L\psi + P_R\psi \qquad P_L = \frac{1-\gamma^5}{2}$$

$$P_R = \frac{1+\gamma^5}{2}$$

In chiral representation, γ_5 diagonal & ψ has the following form:

$$\psi = \begin{pmatrix} \chi_+ \\ \chi_- \end{pmatrix} \qquad \psi = \psi_L + \psi_R = \begin{pmatrix} \chi_+ \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \chi_- \end{pmatrix}$$

Ex in massless limit $(m=0)$ show that two independent Weyl equations are obeyed

Neutrino masses

Majorana fermions Majorana fermions obey the Majorana condition

$$\boxed{\psi = \psi^c = C\bar{\psi}^T}$$

- C is the charge conjugation matrix

$$C^{-1}\gamma^\mu C = -\gamma^{\mu T}$$
$$C^{-1} = C^\dagger = -C = C^T$$

quick check

$$\gamma^0 = \gamma^{0*}$$
$$\gamma^2 = -\gamma^{2*}$$

$$C = i\gamma^0\gamma^2 \Rightarrow C^\dagger = -i\gamma^{2*}\gamma^0$$
$$= i\gamma^2\gamma^0$$
$$= -i\gamma^0\gamma^2$$
$$= -C$$

- C can be defined in any basis but it has the rep $C = i\gamma^0\gamma^2$
- Majorana condition is equivalent to saying a particle is its own anti-particle.

Neutrino masses

$$\psi = \psi^c = C\bar{\psi}^T \qquad (\psi_L + \psi_R) = C(\overline{\psi_L + \psi_R})^T = C(\bar{\psi}_R^T + \bar{\psi}_L^T)$$

$$\psi = \begin{pmatrix} \chi_+ \\ \chi_- \end{pmatrix} = C\overline{\begin{pmatrix} \chi_+ \\ \chi_- \end{pmatrix}}^T = C\left[(\chi_+ \; \chi_-)^* \gamma^0\right]^T$$

$$= C\gamma^0 \begin{pmatrix} \chi_+^* \\ \chi_-^* \end{pmatrix} = \frac{i\gamma^0\gamma^2\gamma^0}{C} \begin{pmatrix} \chi_+^* \\ \chi_-^* \end{pmatrix}$$

$$= -i\gamma^2 \begin{pmatrix} \chi_+^* \\ \chi_-^* \end{pmatrix}$$ use chiral representation $\gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}$

$$= -i \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix} \begin{pmatrix} \chi_+^* \\ \chi_-^* \end{pmatrix} = -i \begin{pmatrix} \sigma^2 \chi_-^* \\ -\sigma^2 \chi_+^* \end{pmatrix} = \begin{pmatrix} -i\sigma^2 \chi_-^* \\ i\sigma^2 \chi_+^* \end{pmatrix}$$

$\therefore \; \psi = \begin{pmatrix} -i\sigma_2 \chi_-^* \\ \chi_- \end{pmatrix}$ a Majorana fermion only requires a single Weyl spinor to parametrise it.

Neutrino masses

$\Rightarrow$ Majorana fermion only has 2 degrees of freedom
$\Rightarrow$ single Weyl spinor. You can see this from
computing :

$$\psi = \psi^c \Rightarrow (\psi_L + \psi_R) = C(\bar{\psi}_R + \bar{\psi}_L)^T = C\bar{\psi}_R^T + C\bar{\psi}_L^T$$

$$\psi_L = C\bar{\psi}_R^T \qquad \psi_R = C\bar{\psi}_L^T$$

$$\psi = \psi_L + (\psi_L)^c$$

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3\, 2\omega_p} \sum_{s=1} \left[\overline{C\, u_s(p)}^T (a_p^s)^\dagger e^{ip\cdot x} + u_s(p)\, a_p^s\, e^{-ip\cdot x} \right]$$

n.b Dirac field would require a 'v' spinor as well as the 'u' spinor

Neutrino masses

Dirac mass

$$m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

$$m_{\alpha\beta}\bar{\psi}_\alpha\psi_\beta \qquad \alpha,\beta \text{ flavour index.}$$

Majorana mass

$$m\bar{\psi}\psi = m\bar{\psi}_L\psi_R + m\bar{\psi}_R\psi_L$$

$$= m\,\overline{\psi^c_L}\,\psi_L + h.c$$

$$= \frac{1}{2}m_{\alpha\beta}\,\overline{\psi^c_\alpha}\,\psi_\beta$$

Neutrino masses

Dirac fermion charged under a U(1) will transform:

$$\psi \to \psi e^{i\theta}$$

<u>Dirac mass</u>

$$m\bar{\psi}\psi \to m\bar{\psi}\psi$$

<u>Majorana mass</u>

$$m\bar{\psi}\psi^c \to m e^{-2i\theta}\bar{\psi}\psi^c$$

majorana mass term violates U(1) symmetry explicitly.

- any particle with a U(1) charge <u>cannot</u> have a Majorana mass.
- accidental symmetries of the SM, importantly L or B-L can be violated in a model with Majorana neutrinos.