MATCHING AND MERGING

COMPARING FIXED ORDER AND PARTON SHOWER

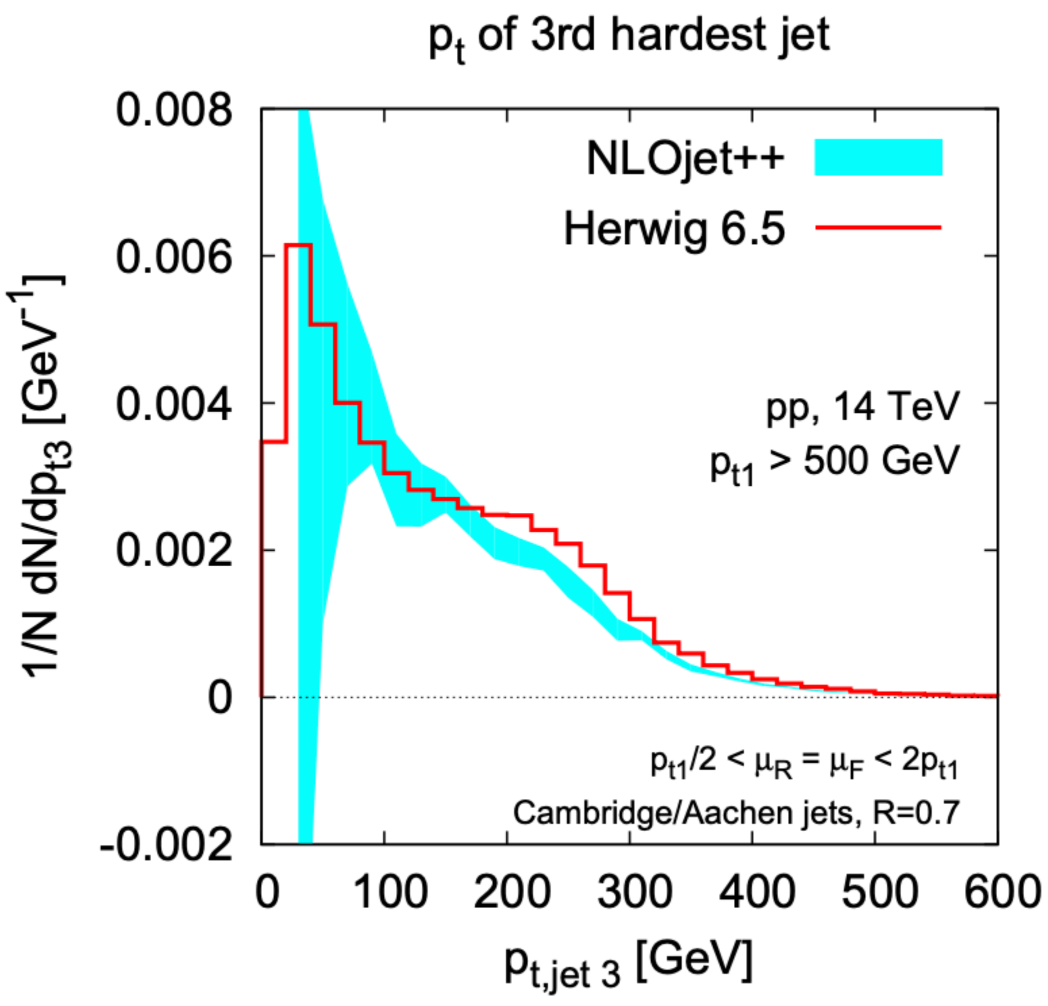
Parton shower	Fixed order
Correct only for soft/collinear radiation	Hard radiation correctly described
High multiplicity final states possible	At most ~10 particles in final state
Realistic, hadronic final states	Only partonic final states
Hard to improve accuracy	Known how to systematically improve accuracy

BRIEF ASIDE ON JETS

- Fixed order calculations describe single partons, but they are not what is observed in a detector
- Soft-collinear radiation causes a 'spray' of particles from each initiating parton
- The spray forms a cone-like shape of radiation, called a jet
- Many different ways to define jets, which must be IR-safe - definitions use a jet radius R which quantifies jet size
- Jet algorithms take partons and cluster them into jets

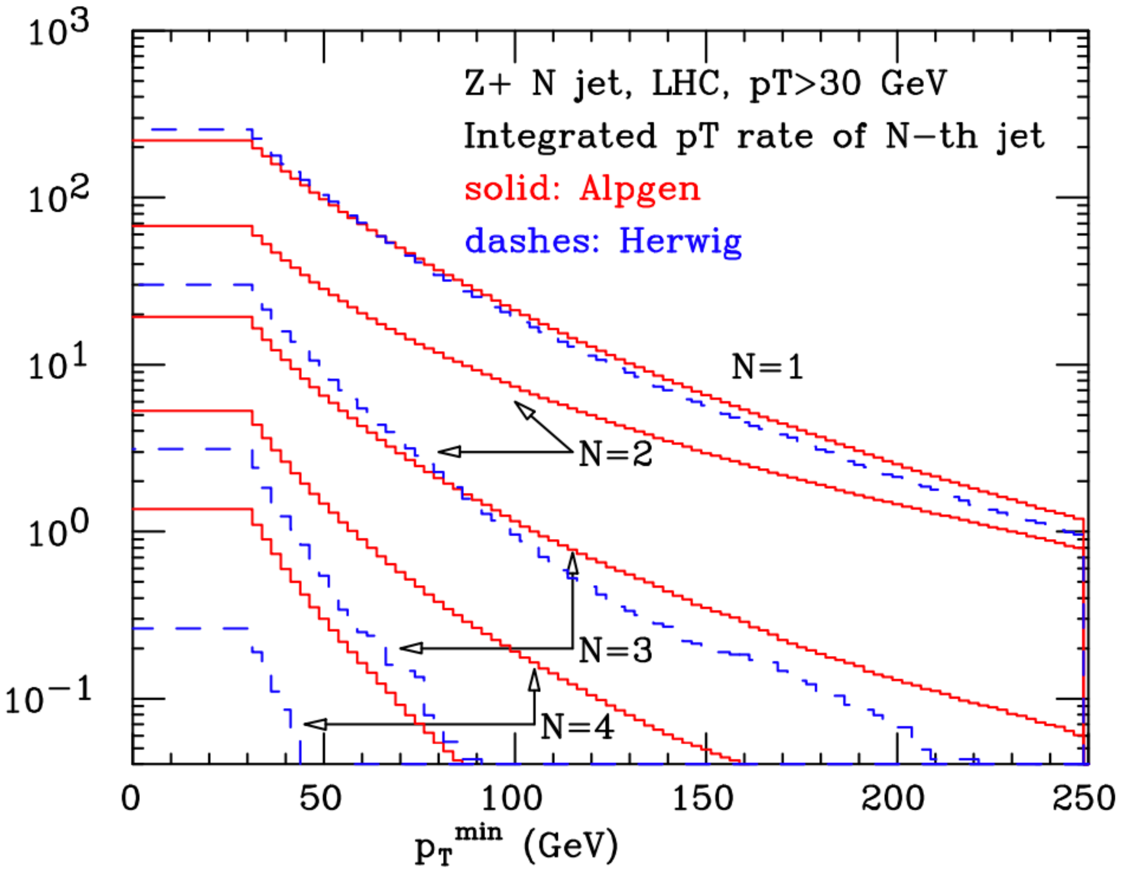
JET PRODUCTION

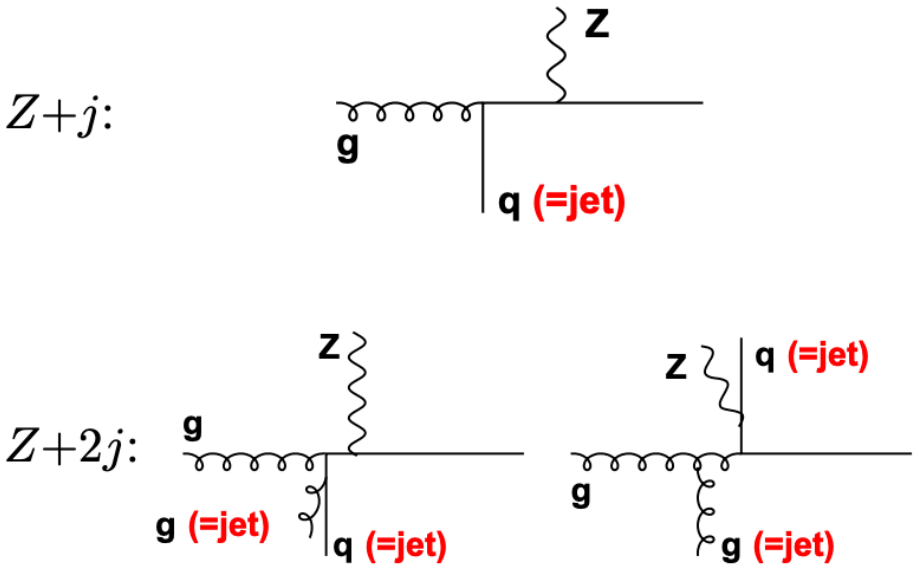
- Tree level processes are $qq \rightarrow qq$, $qg \rightarrow qg$ etc.
- Parton shower starts with these and adds extra radiation
- Only correct in soft/collinear limit, but sometimes adds hard extra emissions
- Pretty good from PS - large uncertainties on FO at small values as large logs blow up



VECTOR BOSON + JET PRODUCTION

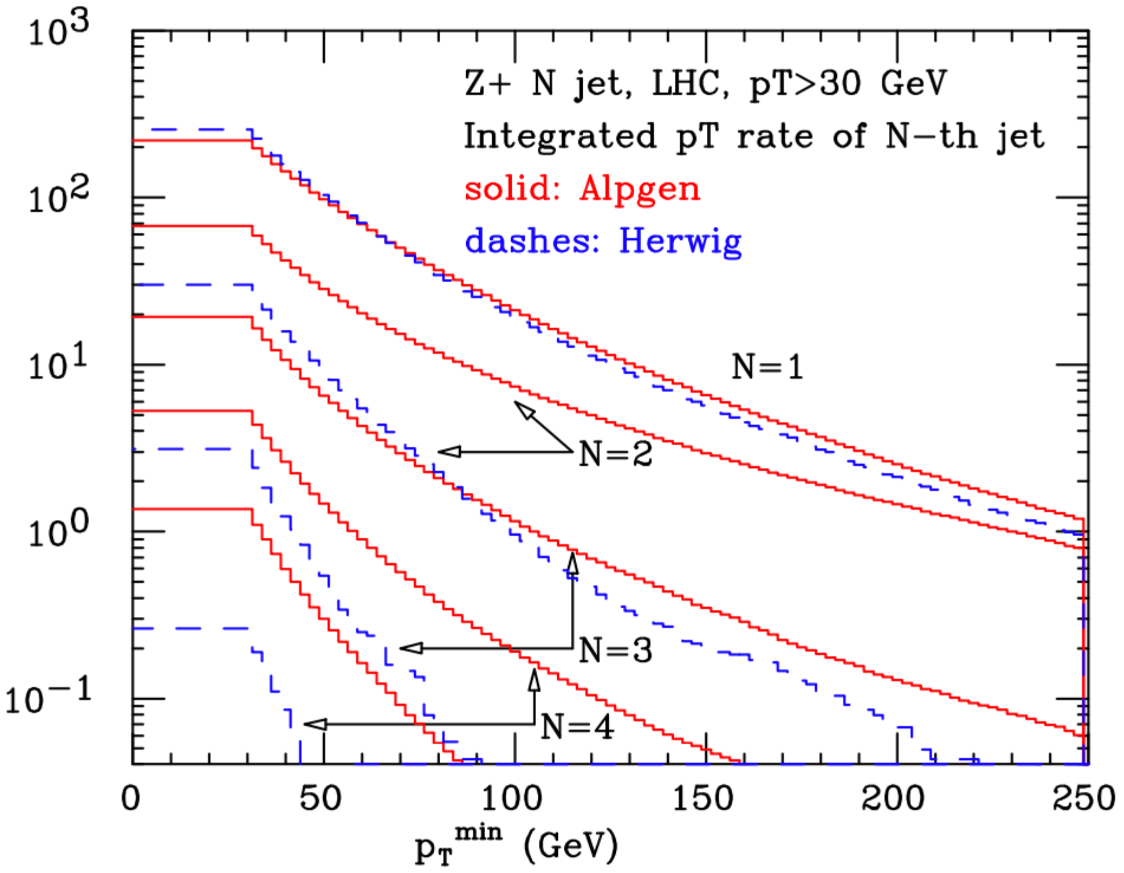
- Fig. shows cross section for $N^{\rm th}$ jet to have transverse energy above E_T
- PS and FO in agreement for 1st jet, but terrible for >2

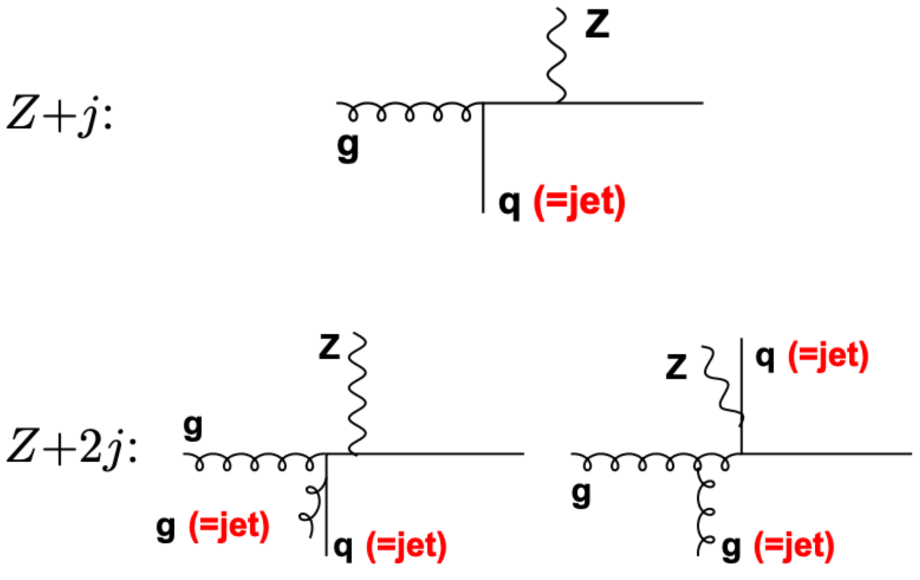




VECTOR BOSON + JET PRODUCTION

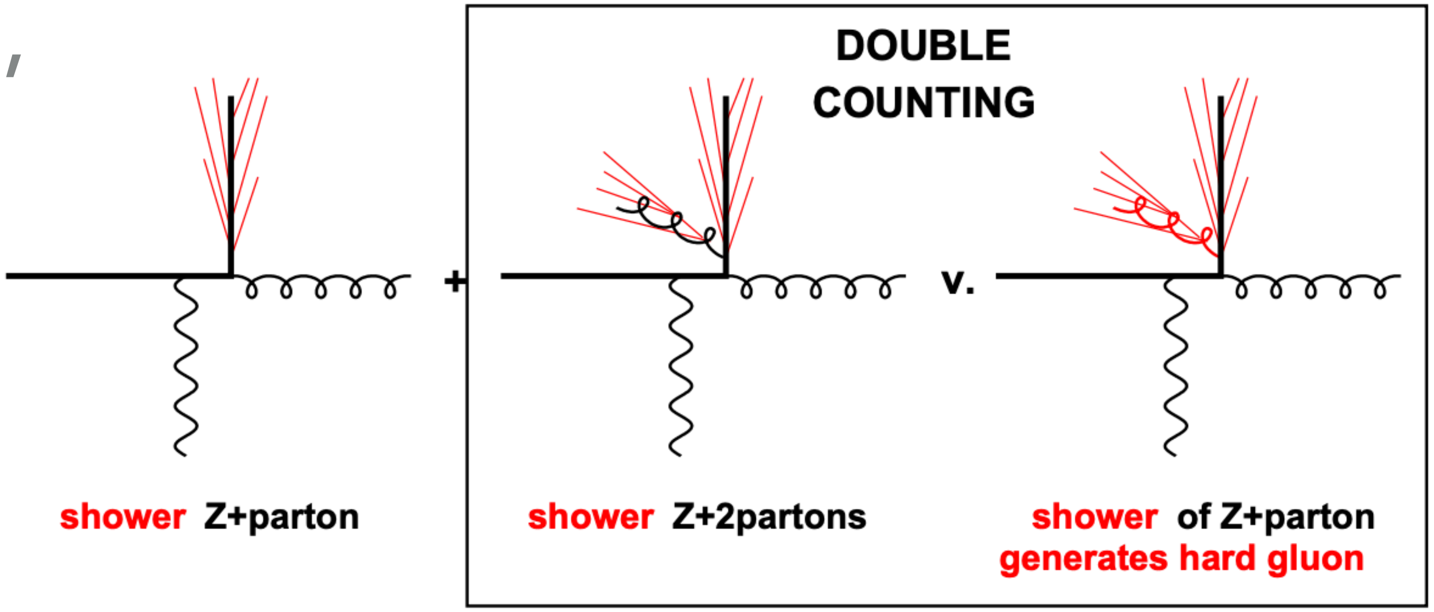
- Explanation: HERWIG generates hard $Z+j$ configs
- But: also soft/coll. enhanced events where Z is radiated off a dijet config, not captured by QCD shower alone





MATRIX ELEMENTS WITH PARTON SHOWERS (MEPS)

- How do we combine PS and FO?
- Consider $Z + j$ production as the underlying hard process. PS does a good job for $Z + 1$ parton, but terribly for more
- Naïve solution: generate $Z + 2$ with correct LO ME, then shower
- Problem: double counting!



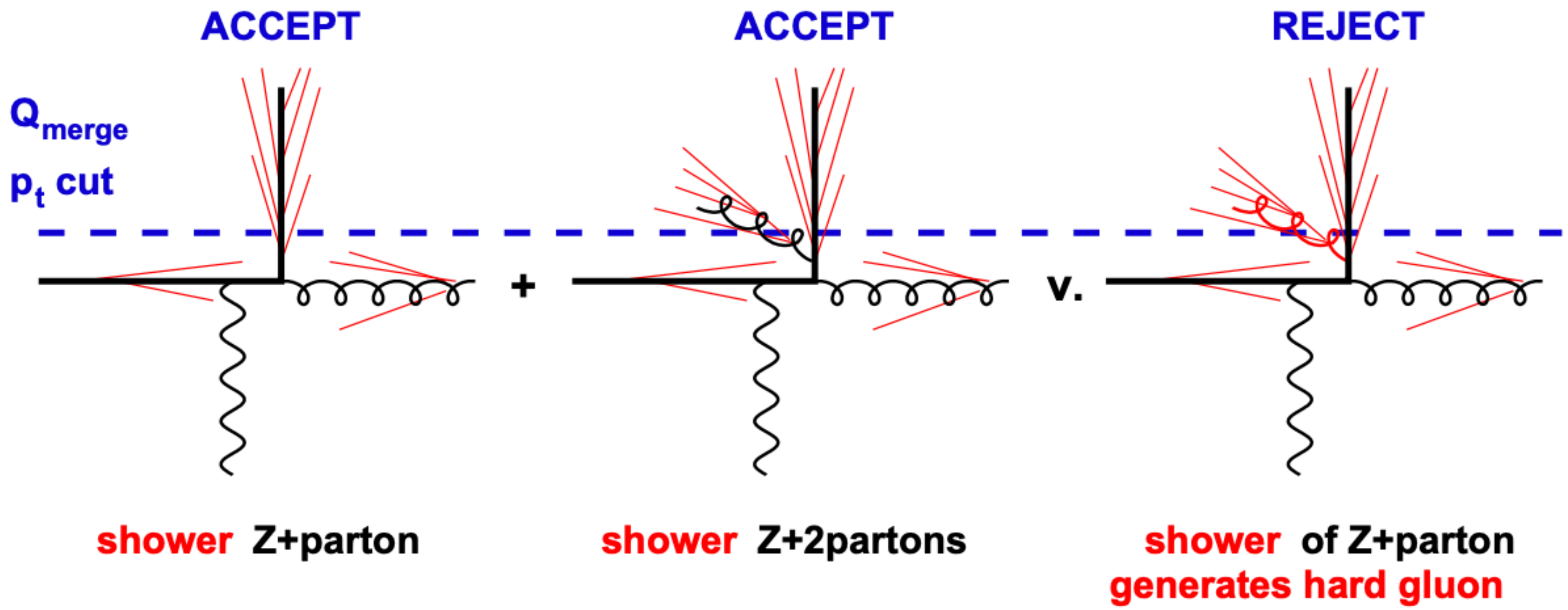
Credit: G. Salam

MLM MERGING

- Introduce transverse momentum cutoff Q_{ME} and angular cutoff R_{ME}
- Generate tree-level events for up to $Z+N$ partons, where all partons have $p_T > Q_{\mathrm{ME}}$ and separation $\theta > R_{\mathrm{ME}}$
- Shower all events
- Apply a jet algorithm to the showered event with $R > R_{\mathrm{ME}}$, and identify all jets with $p_T > Q_{\mathrm{merge}} \gtrsim Q_{\mathrm{ME}}$
- If each jet corresponds to one of the partons and there are no extra jets above Q_{merge} then accept, otherwise reject

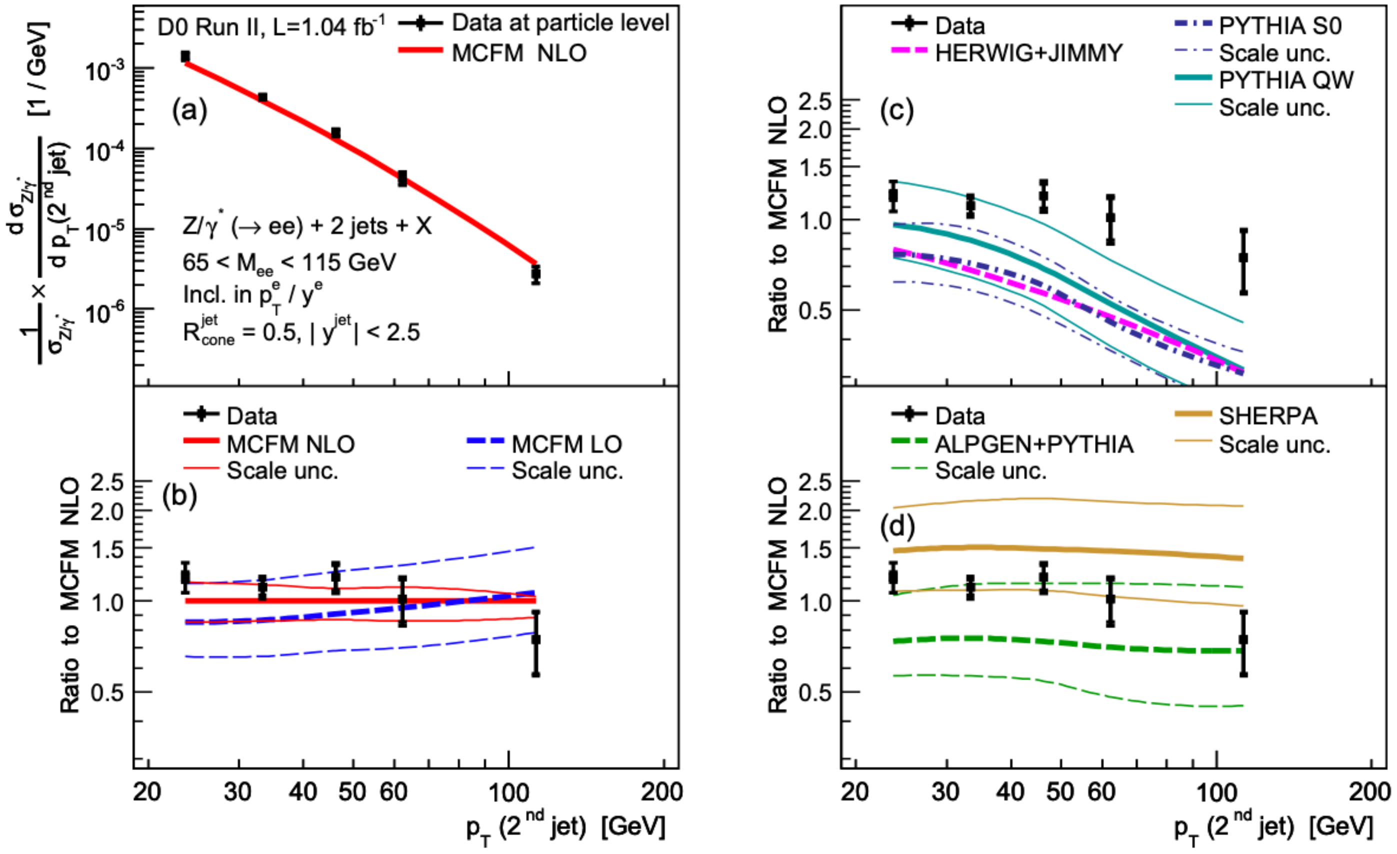
MLM MERGING

- Double-counting removed by rejection of hard radiation
- Hard jets come only from the matrix element



MLM VS FIXED ORDER AND PARTON SHOWER

- MLM (green) gets shape right
- Large scale uncertainty and normalisation wrong, much worse than NLO (red)



- Ideally, need a way to combine NLO calculations with the parton shower (or even NNLO)

MATCHING NLO TO PARTON SHOWER

- Criteria for a successful combination of NLO+PS:
  - Total cross section inherited from NLO
  - Radiation pattern (first order) follows NLO real emission
  - Logarithmic accuracy of PS is maintained
- Recall NLO structure:

$$\sigma_N^{\mathrm{NLO}} = \int \mathrm{d}\Phi_{\mathscr{B}} \left[\mathscr{B}_N(\Phi_{\mathscr{B}}) + \mathscr{V}_N(\Phi_{\mathscr{B}}) + \mathscr{I}_N^{\mathscr{S}}(\Phi_{\mathscr{B}})\right] + \int \mathrm{d}\Phi_{\mathscr{R}} \left[\mathscr{R}_N(\Phi_{\mathscr{R}}) - \mathscr{S}_N(\Phi_{\mathscr{R}})\right]$$

IMPROVING THE PARTON SHOWER - MATRIX ELEMENT CORRECTIONS

- Parton shower good for soft/collinear, bad for hard emissions
- Can we correct it to get the hardest emission right?
- In many processes, parton shower is an overestimate of exact ME:

$$\mathscr{R}_N(\Phi_{\mathscr{B}} \otimes \Phi_1) \leq \mathscr{B}_N(\Phi_{\mathscr{B}}) \otimes \mathscr{K}_N(\Phi_1)$$

- $\mathscr{K}$ is PS soft and collinear splitting kernel (we discussed P_{ij}, the collinear case only)

MATRIX ELEMENT CORRECTIONS

- First emission pattern looks like:

$$d\sigma_N = d\Phi_{\mathcal{B}}\mathcal{B}_N(\Phi_{\mathcal{B}})\left\{\boxed{\Delta_N(\mu_Q^2, t_c)} + \int_{t_c}^{\mu_Q^2} d\Phi_1 \boxed{\left[\mathcal{K}_N(\Phi_1)\Delta_N(\mu_Q^2, t(\Phi_1))\right]}\right\}$$

No emission probability

Single emission probability at a given time t

- Terms in curly brackets integrate to 1 (shower is unitary)
- Let's modify the splitting kernel to make it look more like the real matrix element, at least for the first emission:

$$\tilde{\mathcal{K}}_N(\Phi_1) = \mathcal{R}_N(\Phi_{\mathcal{B}} \otimes \Phi_1)/\mathcal{B}_N(\Phi_{\mathcal{B}})$$

MATRIX ELEMENT CORRECTIONS

- First emission pattern modified to:

$$\mathrm{d}\sigma_N = \mathrm{d}\Phi_{\mathscr{B}}\mathscr{B}_N(\Phi_{\mathscr{B}})\left\{\tilde{\Delta}_N(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1 \left[\frac{\mathscr{R}_N(\Phi_{\mathscr{B}} \otimes \Phi_1)}{\mathscr{B}_N(\Phi_{\mathscr{B}})}\tilde{\Delta}_N(\mu_Q^2, t(\Phi_1))\right]\right\}$$

- Now first emission follows real matrix element!
- Practically, use normal shower kernels and simply accept/reject points with a probability

$$\mathscr{P}_{\mathrm{MEC}} = \frac{\mathscr{R}_N(\Phi_{\mathscr{B}} \otimes \Phi_1)}{\mathscr{B}(\Phi_{\mathscr{B}}) \otimes \mathscr{K}_N(\Phi_1)}$$

NLO MATCHING - THE POWHEG METHOD

- Define Born-like configurations which give NLO-accurate cross section:

$$\overline{\mathscr{B}}_N(\Phi_{\mathscr{B}}) = \mathscr{B}_N(\Phi_{\mathscr{B}}) + \overline{\mathscr{V}}_N(\Phi_{\mathscr{B}}) + \int \mathrm{d}\Phi_1 \left[\mathscr{R}_N(\Phi_\mathrm{B} \otimes \Phi_1) - \mathscr{S}_N(\Phi_\mathrm{B} \otimes \Phi_1)\right]$$

- IR-subtracted, UV-renormalised virtual piece is

$$\overline{\mathscr{V}}_N(\Phi_{\mathscr{B}}) = \mathscr{V}_N(\Phi_{\mathscr{B}}) + \mathscr{I}_N^{\mathscr{S}}(\Phi_{\mathscr{B}})$$

- Works if $\Phi_{\mathscr{R}} = \Phi_{\mathscr{B}} \otimes \Phi_1$. $\overline{\mathscr{B}}$ terms are fully differential cross sections of Born configurations with NLO weight.

NLO MATCHING - THE POWHEG METHOD

- Unitary PS cannot spoil NLO cross section
- Still need pattern of first emission to be correct up to $\mathcal{O}(\alpha_s)$
- Get this by applying matrix element corrections!
- POWHEG formula given by

$$\mathrm{d}\sigma_N = \mathrm{d}\Phi_{\mathcal{B}}\overline{\mathcal{B}}_N(\Phi_{\mathcal{B}})\left\{\tilde{\Delta}_N(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1 \left[\frac{\mathcal{R}_N(\Phi_{\mathcal{B}}\otimes\Phi_1)}{\mathcal{B}_N(\Phi_{\mathcal{B}})}\tilde{\Delta}_N(\mu_Q^2, t(\Phi_1))\right]\right\}$$

NLO MATCHING - THE POWHEG METHOD

- POWHEG formula given by

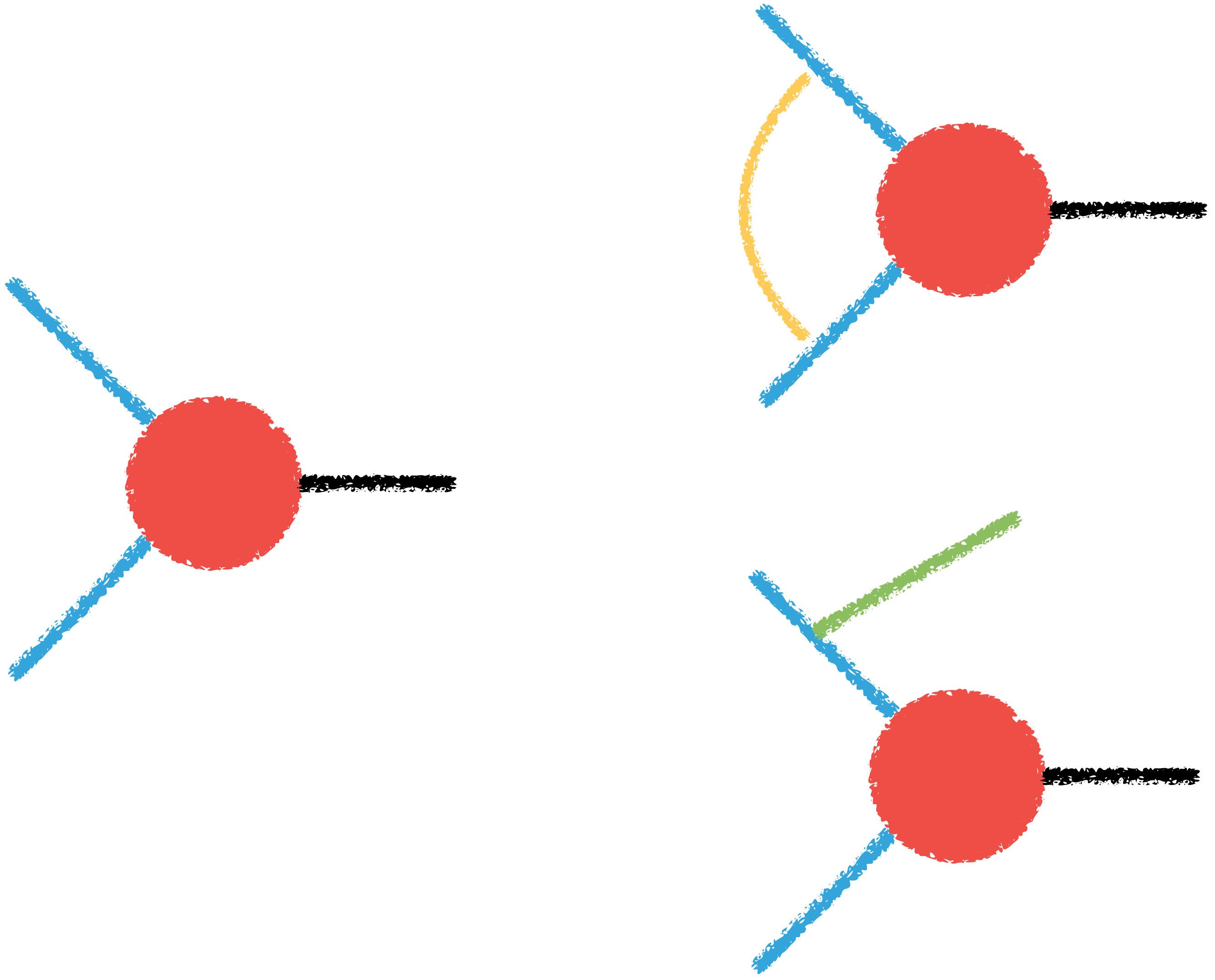
$$d\sigma_N = d\Phi_{\mathcal{B}}\overline{\mathcal{B}}_N(\Phi_{\mathcal{B}})\left\{\tilde{\Delta}_N(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} d\Phi_1 \left[\frac{\mathcal{R}_N(\Phi_{\mathcal{B}} \otimes \Phi_1)}{\mathcal{B}_N(\Phi_{\mathcal{B}})}\tilde{\Delta}_N(\mu_Q^2, t(\Phi_1))\right]\right\}$$

- Gets NLO cross section right (term in curly braces integrates to unity)
- Gets real radiation right at $\mathcal{O}(\alpha_s)$ - NLO terms in $\overline{\mathcal{B}}$ hitting $\mathcal{R}_N/\mathcal{B}_N$ are $\mathcal{O}(\alpha_s^2)$
- Subtleties in scale choices, starting scale of PS

MATCHING TO NNLO

- LO gives order of magnitude estimate, NLO is reliable, but need NNLO for precision.
- NNLO calculations are much harder than NLO.
- Many overlapping divergences - up to 2 extra emissions, can be soft and/or collinear in different combinations
- Cancellation of divergences between real and virtual diagrams is still guaranteed by the KLN theorem
- Let’s take a step back - how do we define an 'event' which is IR-finite?

DEFINING IR-FINITE EVENTS



VIRTUAL

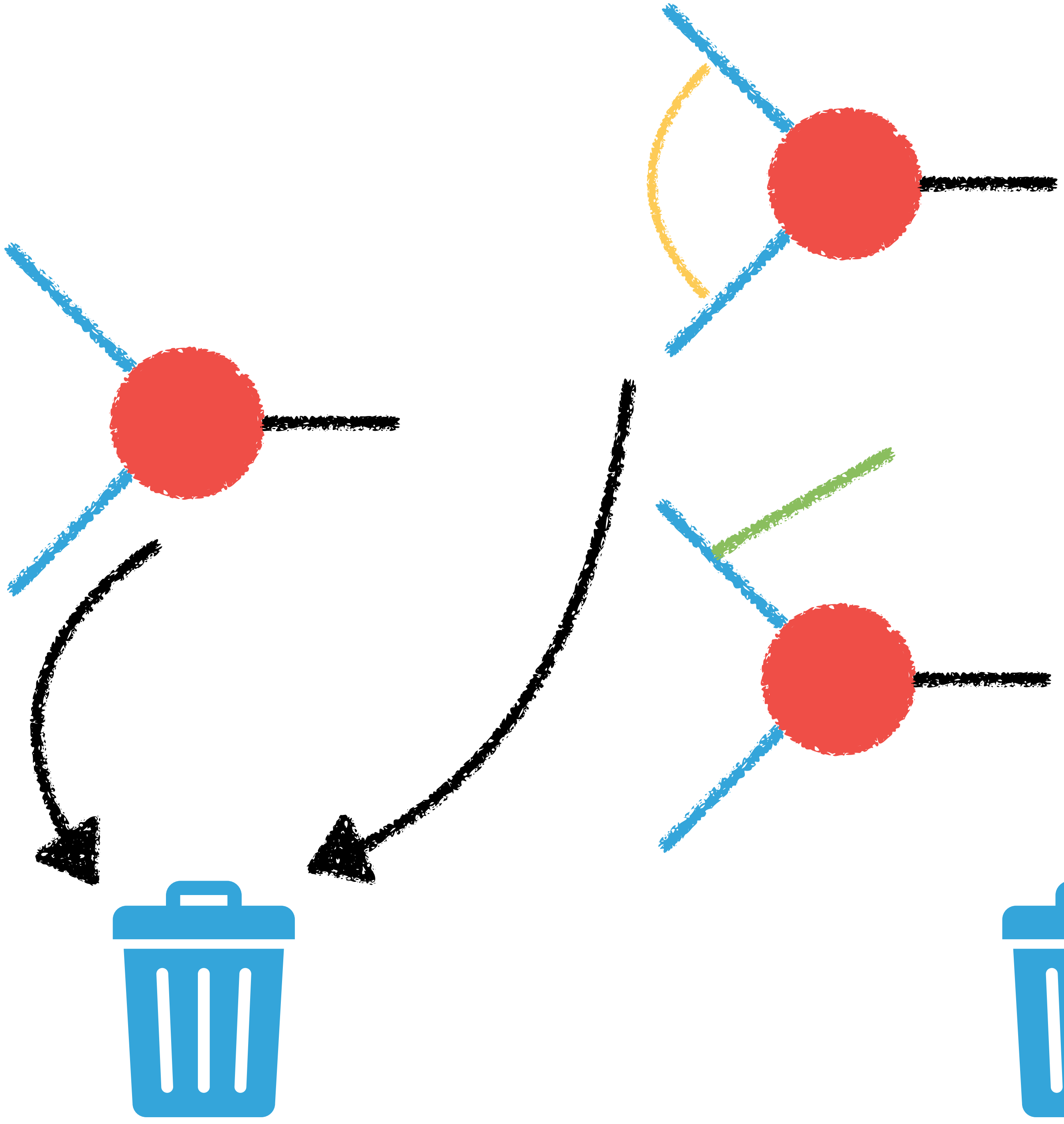
BORN

REAL

0-JET

1-JET

DEFINING IR-FINITE EVENTS



VIRTUAL

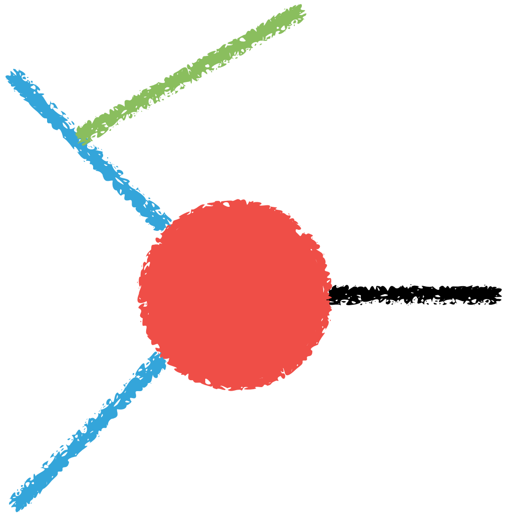
BORN

REAL

0-JET

1-JET

DEFINING IR-FINITE EVENTS



REAL

🗑

0-JET

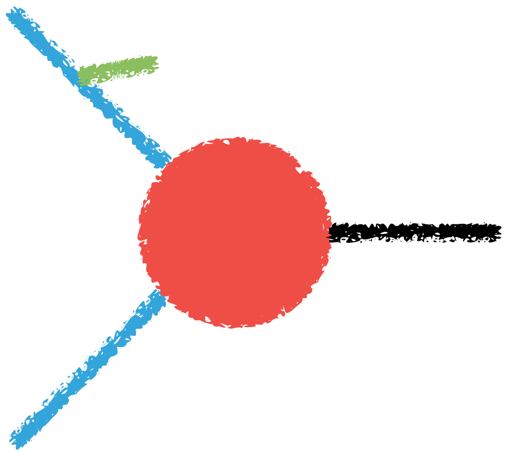
🗑

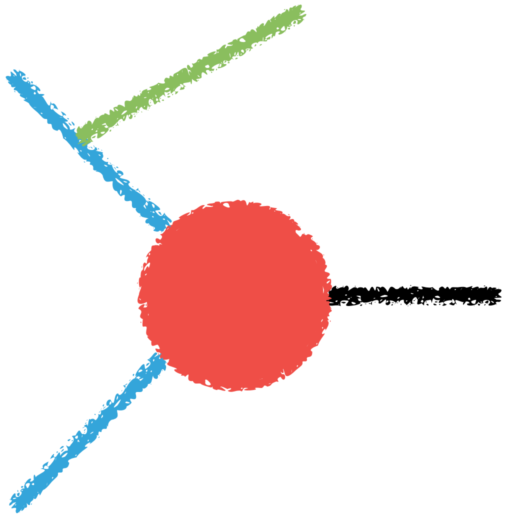
1-JET

DEFINING IR-FINITE EVENTS

SOFT/COLL. REAL

$r_0 < r_0^{\mathrm{cut}}$





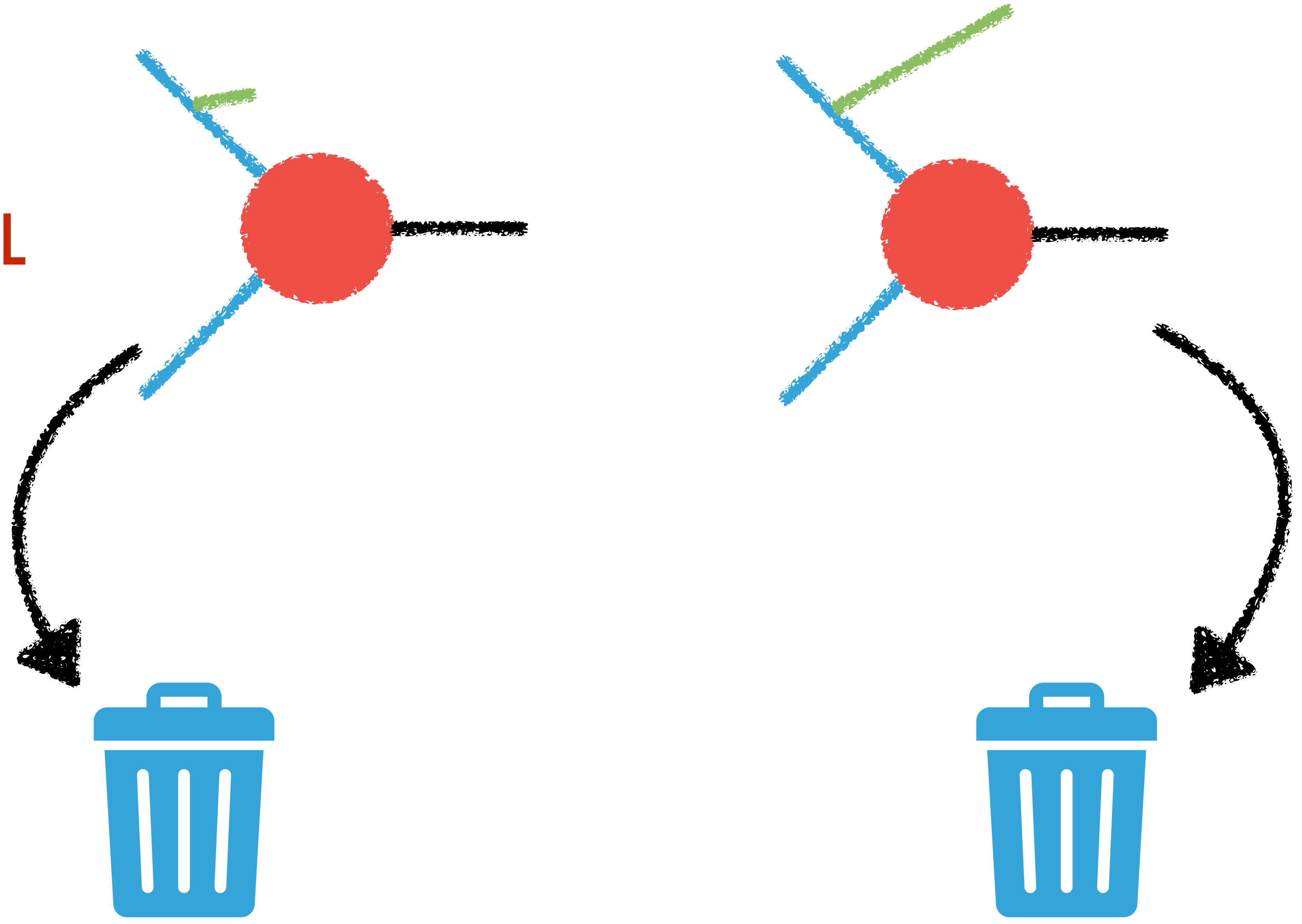
HARD REAL

$r_0 > r_0^{\mathrm{cut}}$

0-JET

1-JET

DEFINING IR-FINITE EVENTS



SOFT/COLL. REAL

$r_0 < r_0^{\mathrm{cut}}$

0-JET

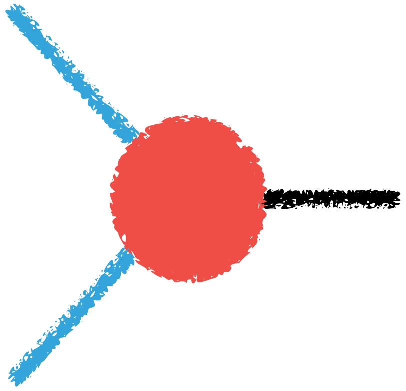
HARD REAL

$r_0 > r_0^{\mathrm{cut}}$

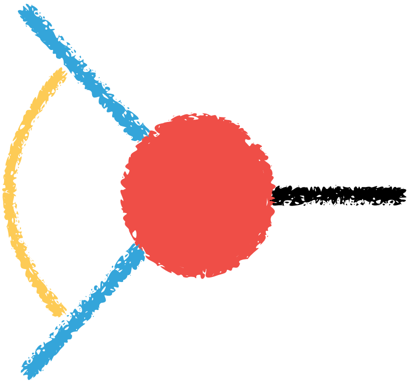
1-JET

DEFINING IR-FINITE EVENTS

BORN



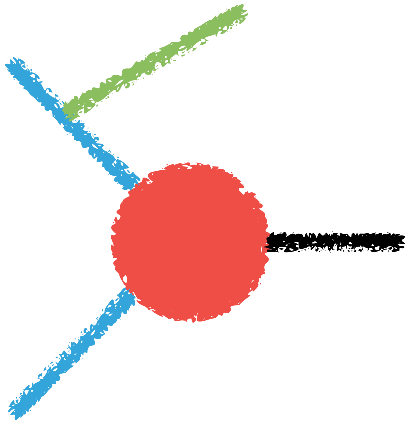
VIRTUAL



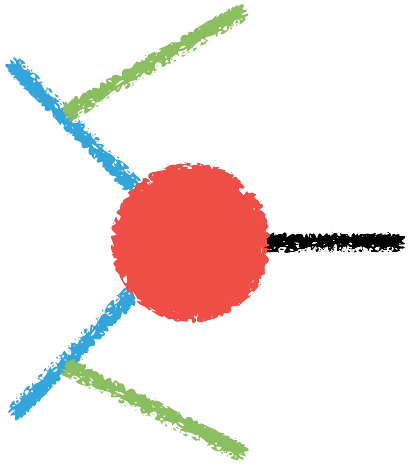
DOUBLE-VIRTUAL

0-JET

REAL



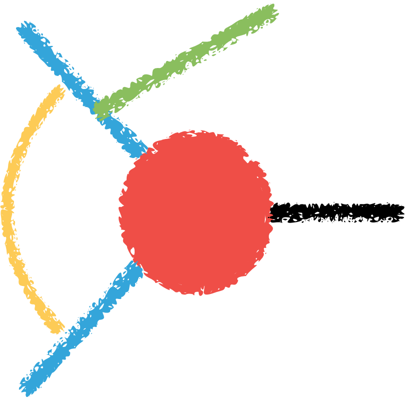
DOUBLE-REAL



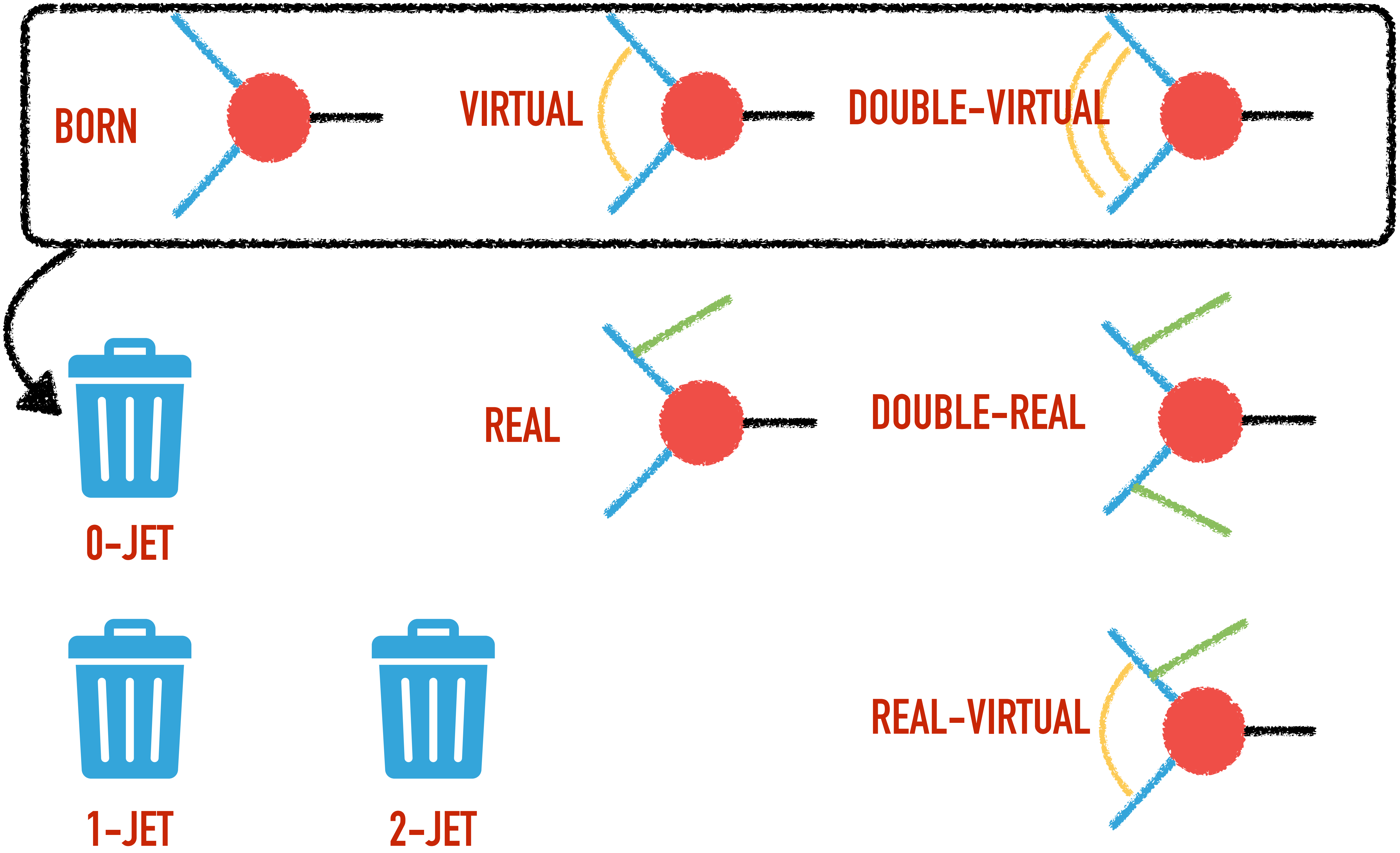
1-JET

2-JET

REAL-VIRTUAL



DEFINING IR-FINITE EVENTS



BORN

VIRTUAL

DOUBLE-VIRTUAL

REAL

DOUBLE-REAL

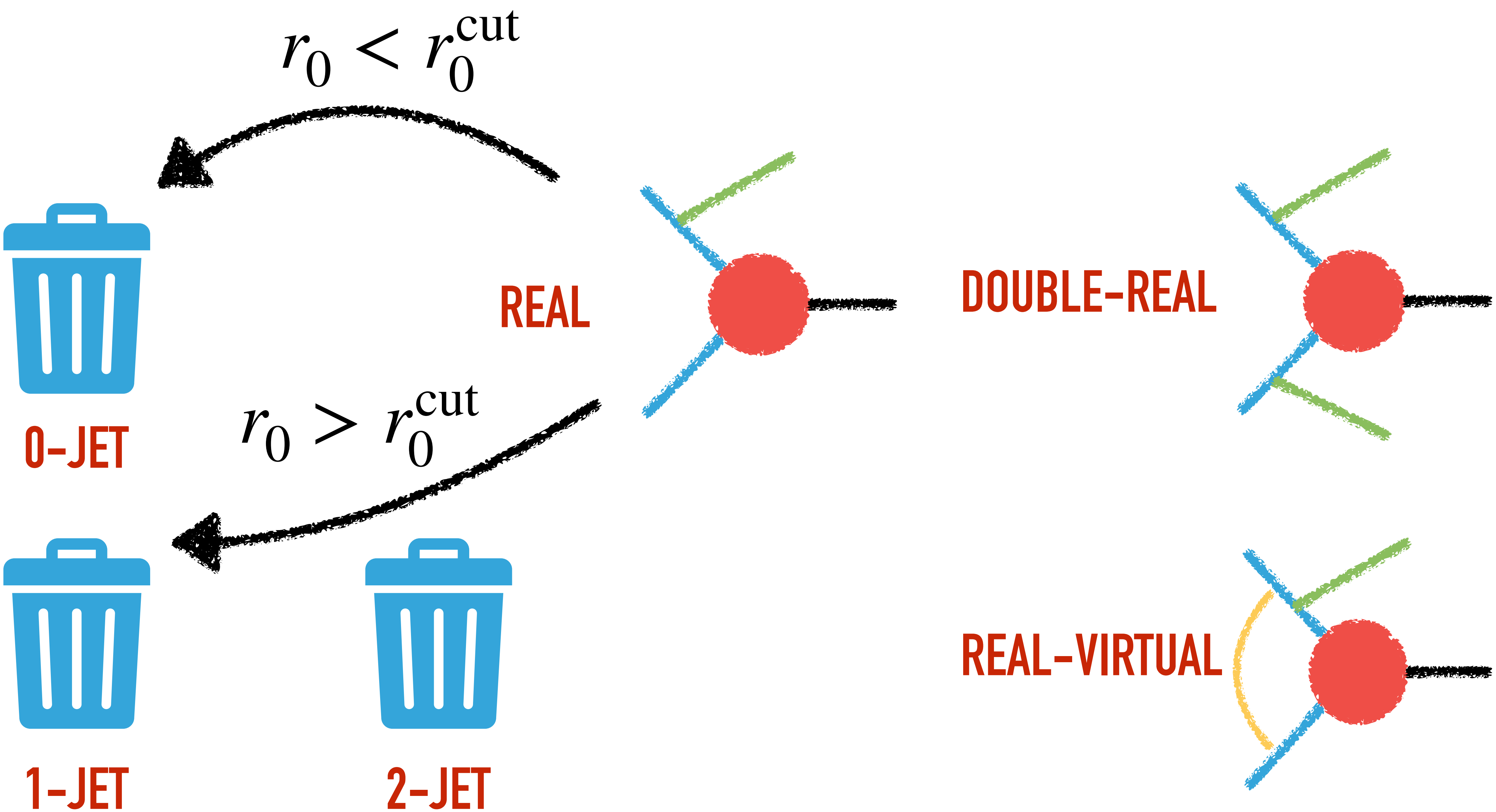
REAL-VIRTUAL

0-JET

1-JET

2-JET

DEFINING IR-FINITE EVENTS



$r_0 < r_0^{\text{cut}}$

$r_0 > r_0^{\text{cut}}$

REAL

DOUBLE-REAL

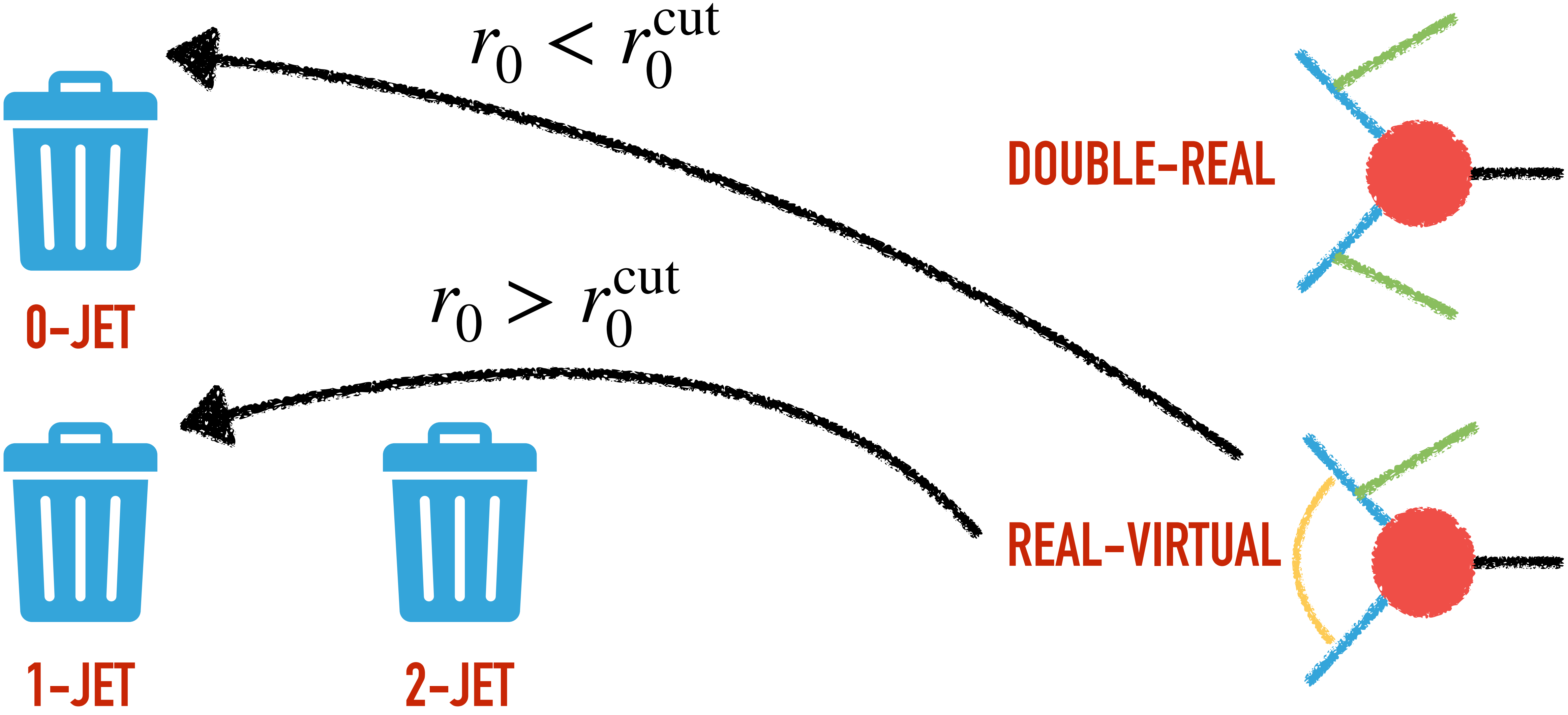
REAL-VIRTUAL

0-JET

1-JET

2-JET

DEFINING IR-FINITE EVENTS



$r_0 < r_0^{\text{cut}}$

DOUBLE-REAL

0-JET

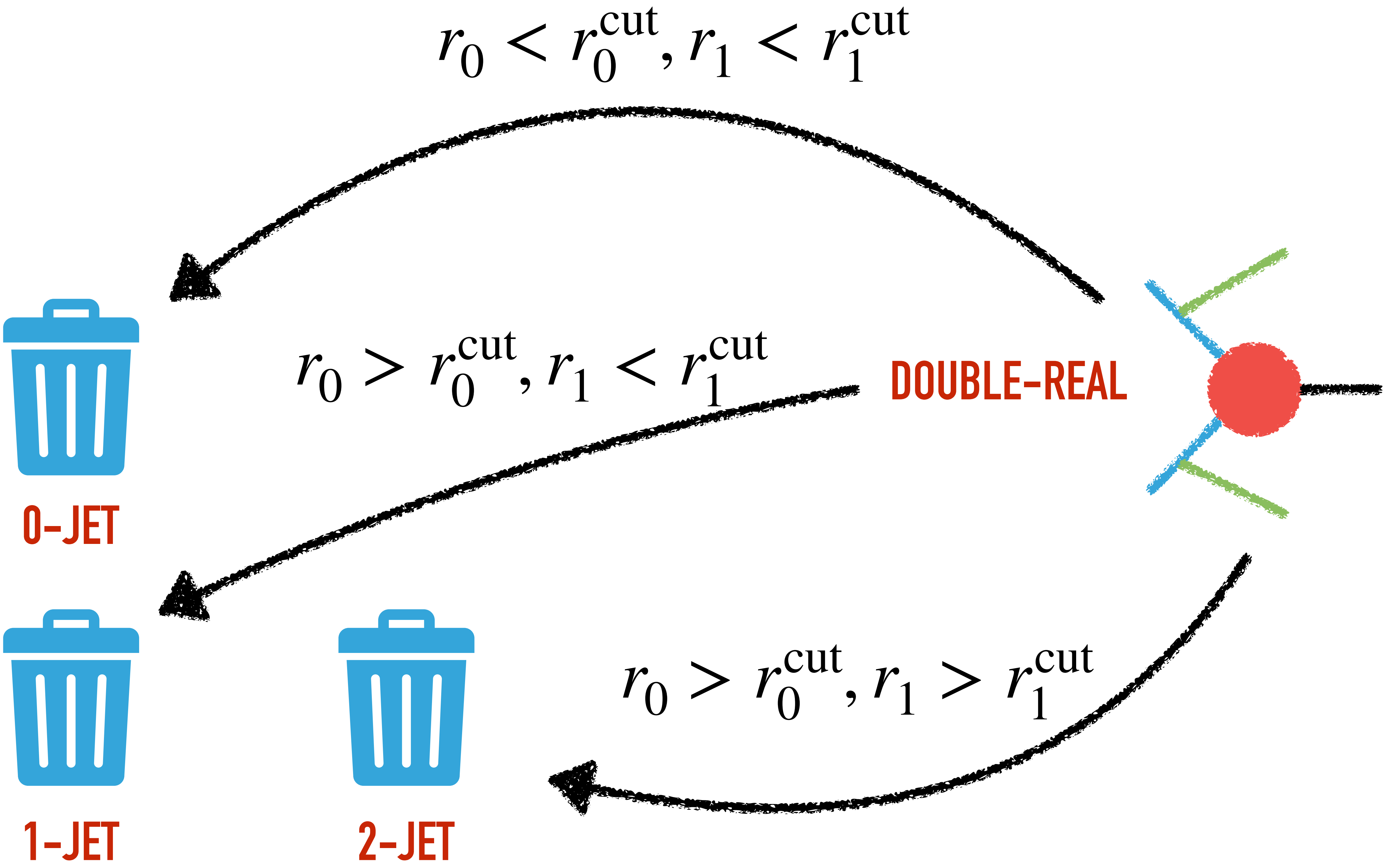
$r_0 > r_0^{\text{cut}}$

REAL-VIRTUAL

1-JET

2-JET

DEFINING IR-FINITE EVENTS



$r_0 < r_0^{\text{cut}}, r_1 < r_1^{\text{cut}}$

$r_0 > r_0^{\text{cut}}, r_1 < r_1^{\text{cut}}$

DOUBLE-REAL

$r_0 > r_0^{\text{cut}}, r_1 > r_1^{\text{cut}}$

0-JET

1-JET

2-JET

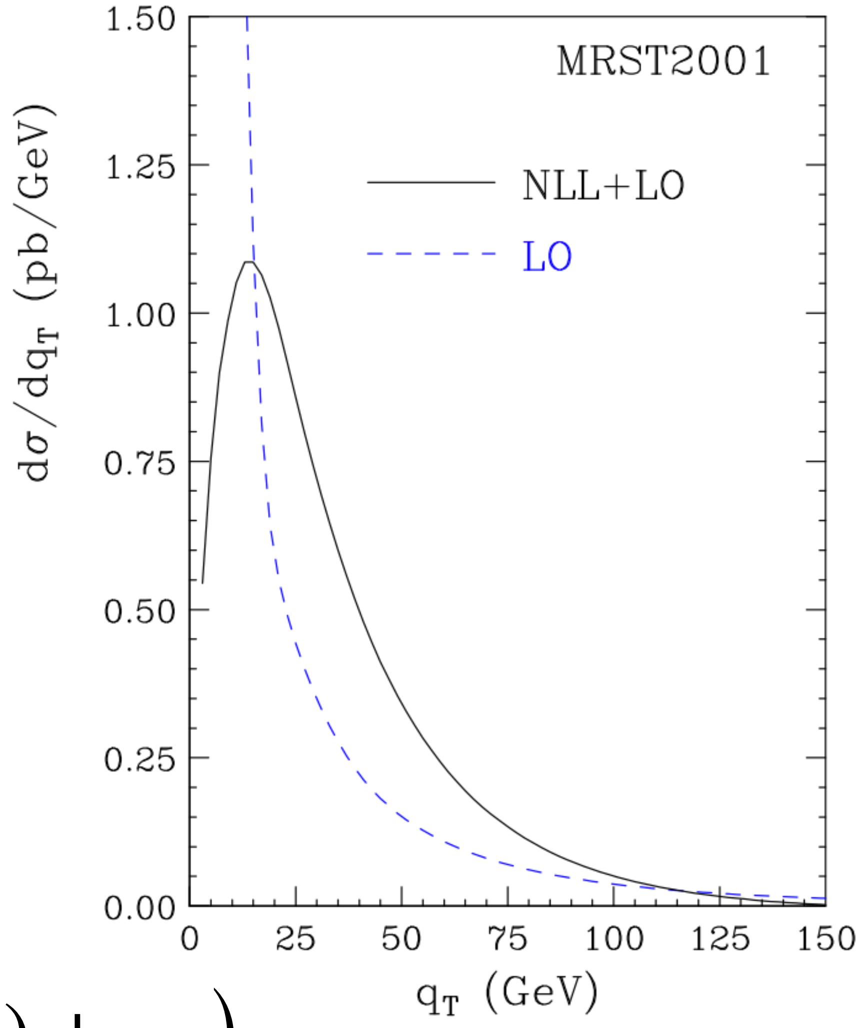
DEFINING IR-FINITE EVENTS

- Defining events this way introduced a projection from a higher multiplicity to a lower multiplicity phase space
- Results are only (N)NLO accurate up to power corrections in r_0^{cut} - as $r_0^{\text{cut}} \to 0$, exact fixed order result is recovered
- Causes large logarithms to appear which spoil perturbative convergence!

$$L = \log(Q/r_0^{\text{cut}}) \text{ becomes large...}$$

RESUMMATION – THE CURE FOR LARGE LOGS

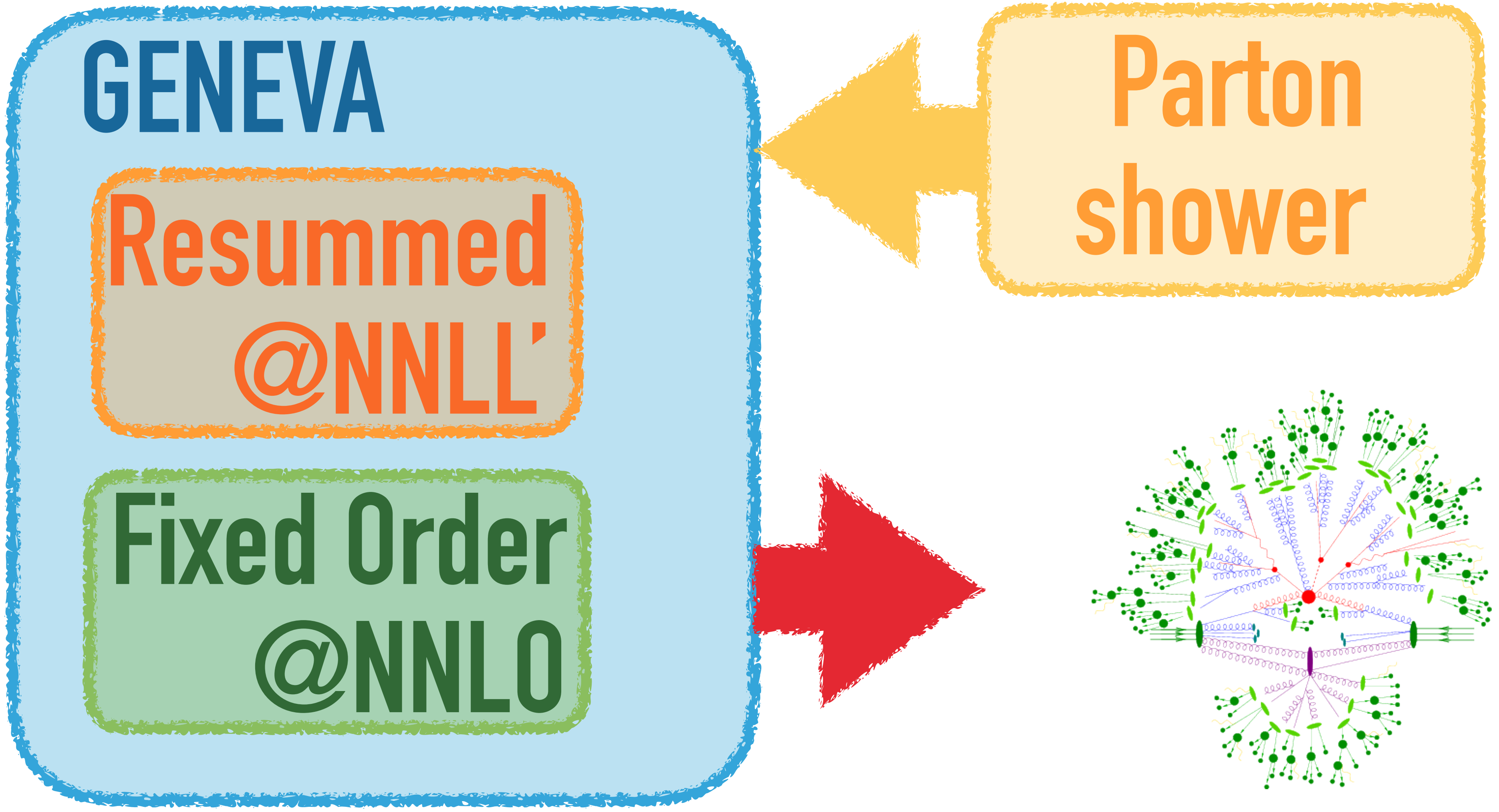
- Large logs signal the breakdown of the perturbative series in the coupling, leading term $\alpha L^2 \sim 1 \Rightarrow \alpha L \ll 1$
- Reordering the series to expand in a genuinely small parameter cures behaviour



$$d\sigma = C(\alpha_s)\,\exp\left(Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right)$$

- Different formalisms available to achieve this

COMBINING RESUMMED AND FIXED ORDER CALCULATIONS IN GENEVA



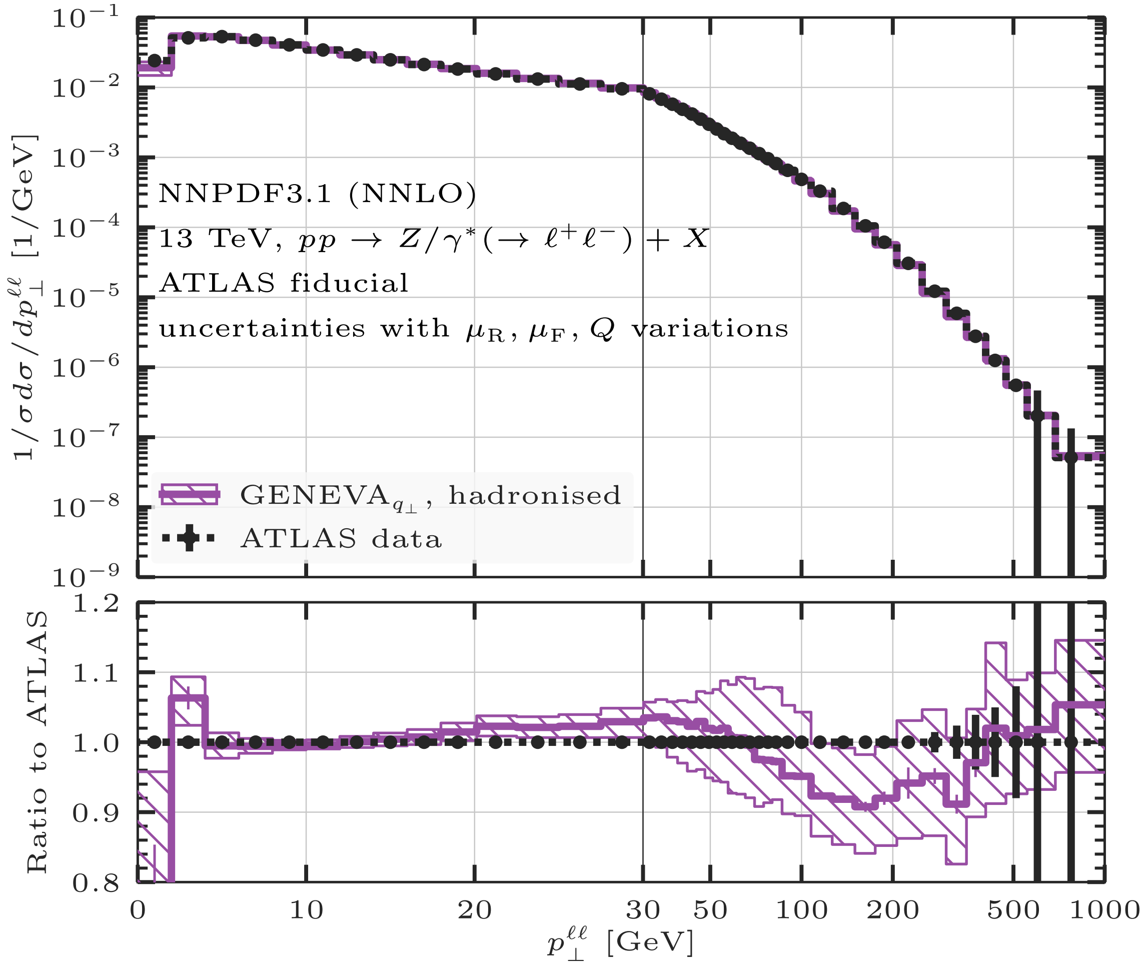
GENEVA

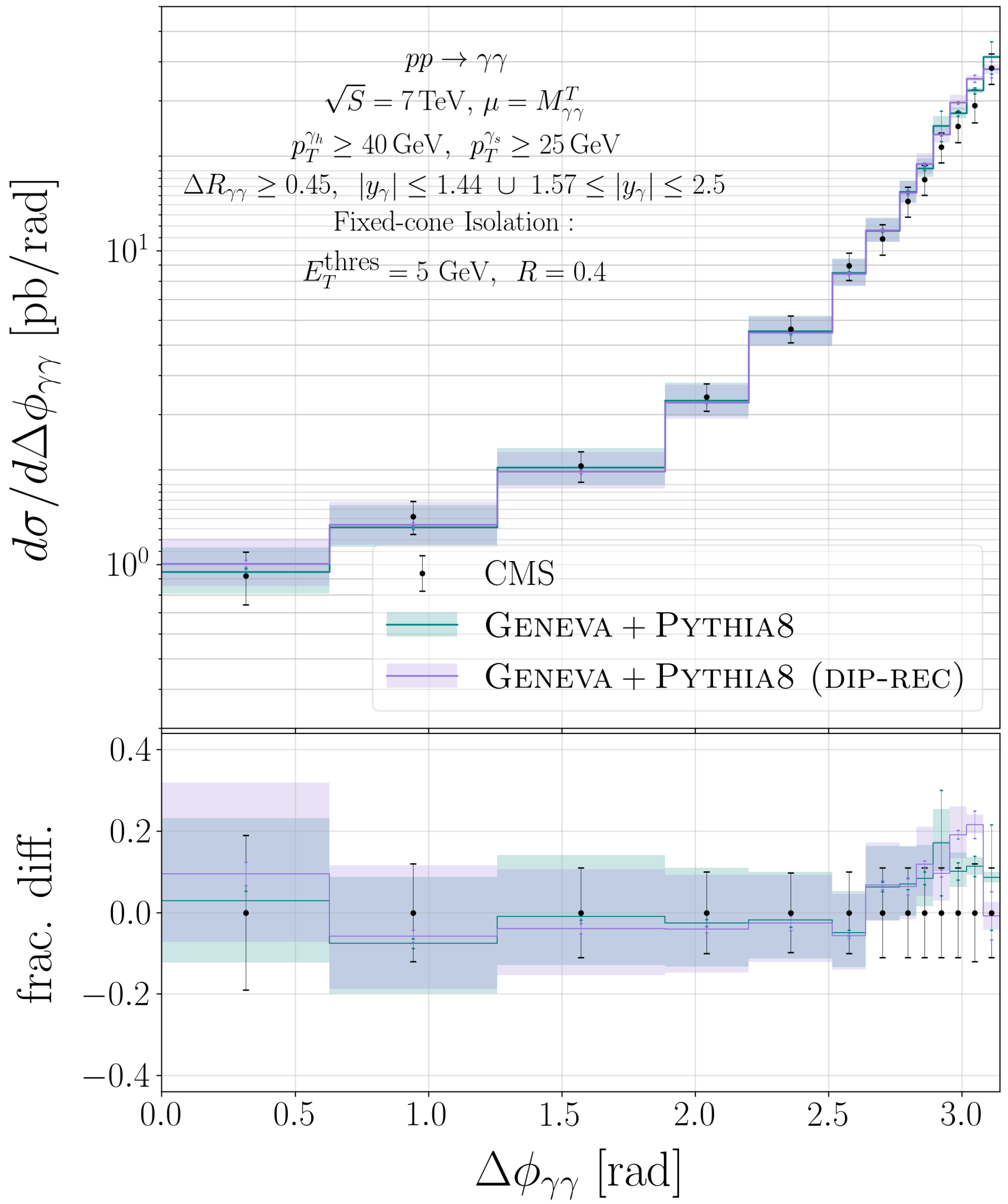
Resummed @NNLL'

Fixed Order @NNLO

Parton shower

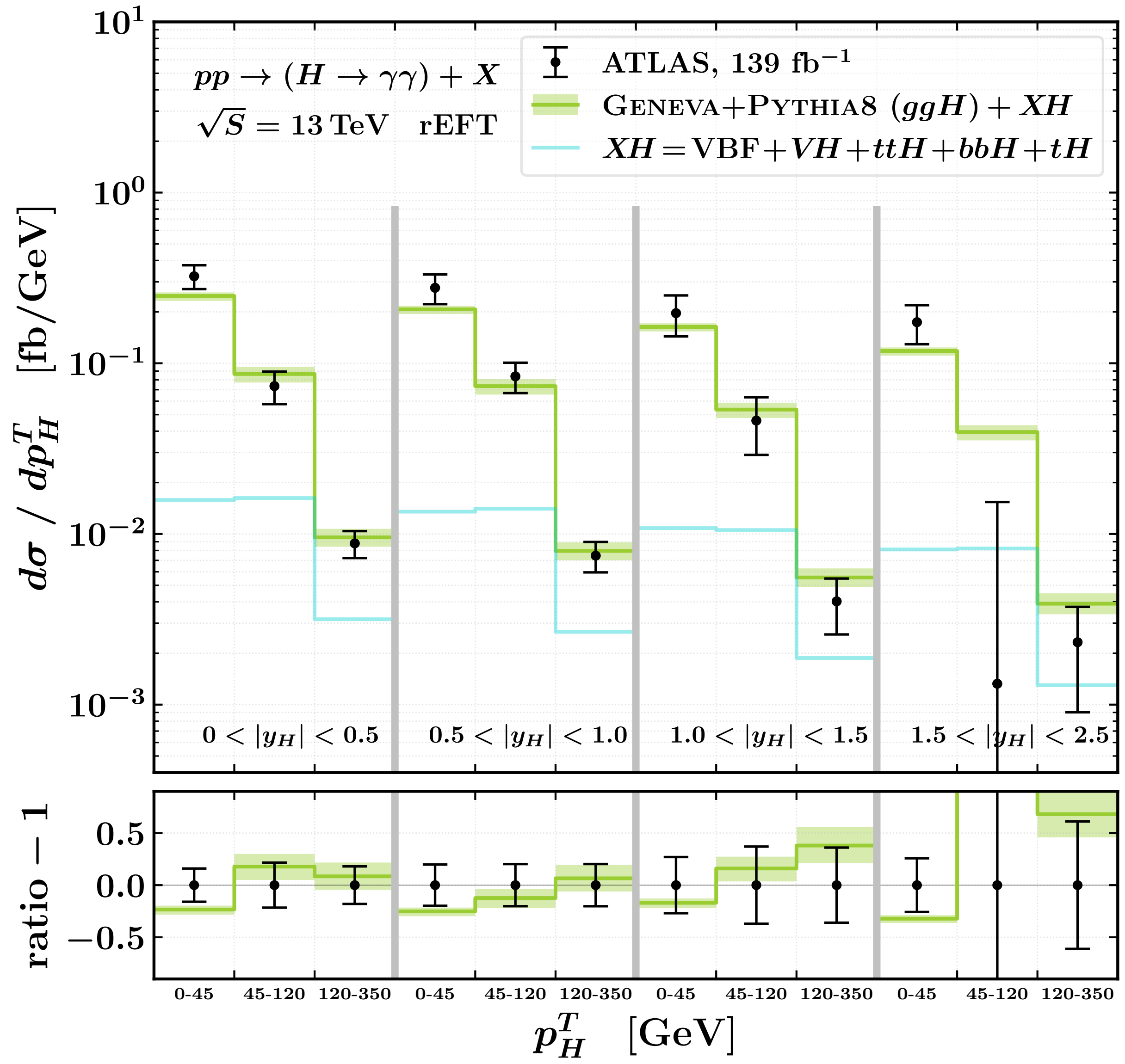
GENEVA AS AN NNLO+PS EVENT GENERATOR

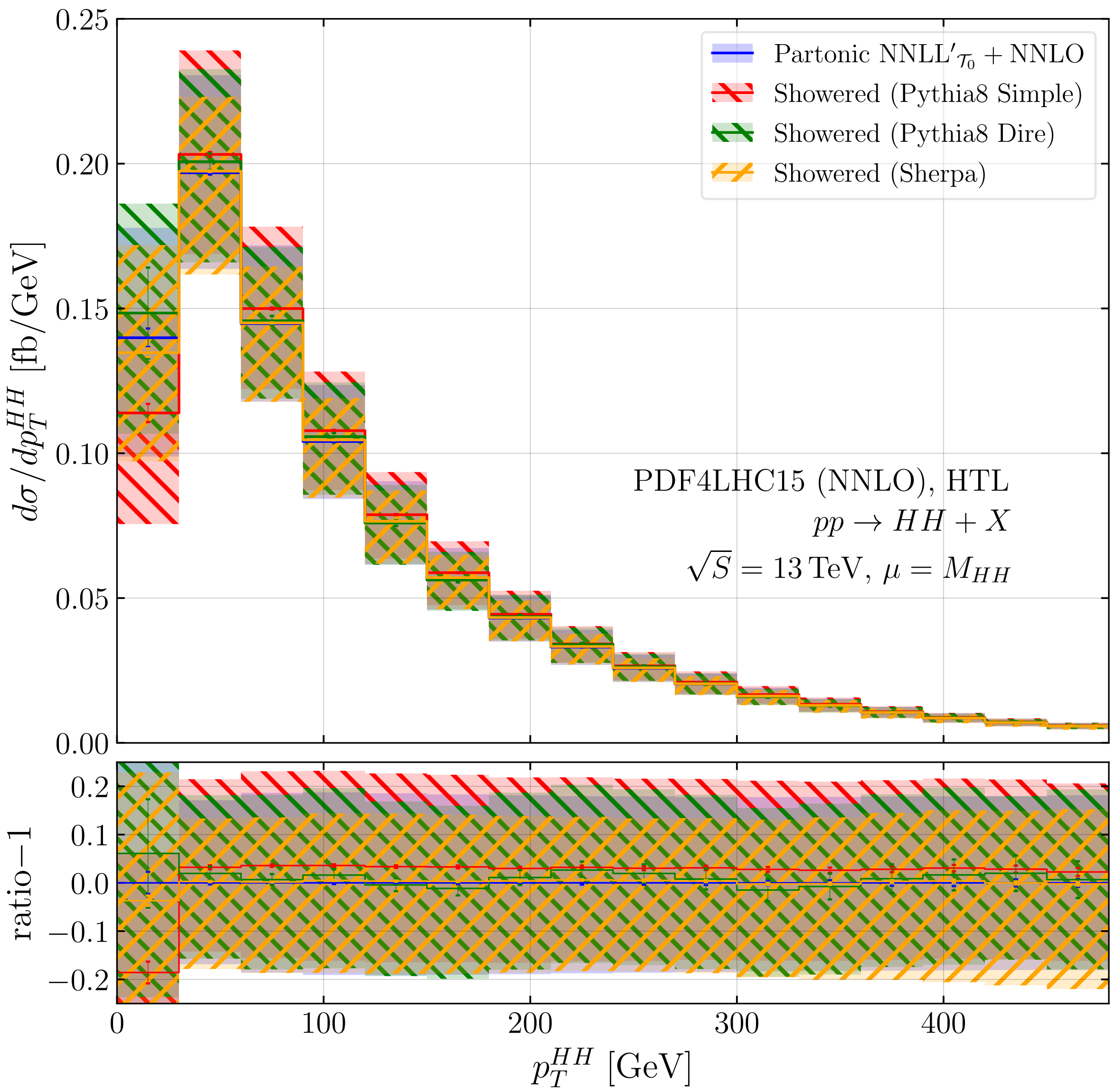




2010.10498, 2102.08390, S. Alioli, C. Bauer, A. Broggio, A. Gavardi, S. Kallweit, MAL, R. Nagar, D. Napoletano, L. Rottoli

GENEVA AS AN NNLO+PS EVENT GENERATOR





2212.10489, 2301.11875, S. Alioli, G. Billis, A. Broggio, A. Gavardi, S. Kallweit, MAL, G. Marinelli, R. Nagar, D. Napoletano

SUMMARY

- Fixed order and parton shower calculations have different advantages - important to be able to combine them to achieve best theoretical description
- Merging combines samples with different multiplicities at FO and showers them without double counting
- Matching corrects first emissions of parton shower to be (N)NLO accurate and gives events with (N)NLO weight

OVERVIEW

- I have not been exhaustive by a long shot - many topics uncovered and details omitted, see initial references for more info
- Aim has not been to bring you up-to-speed with cutting edge developments or list all available tools, but to peek inside the black-box
- Hopefully now you appreciate the power and limitations of event generators, and can debug more successfully!