The axion hot dark matter bound

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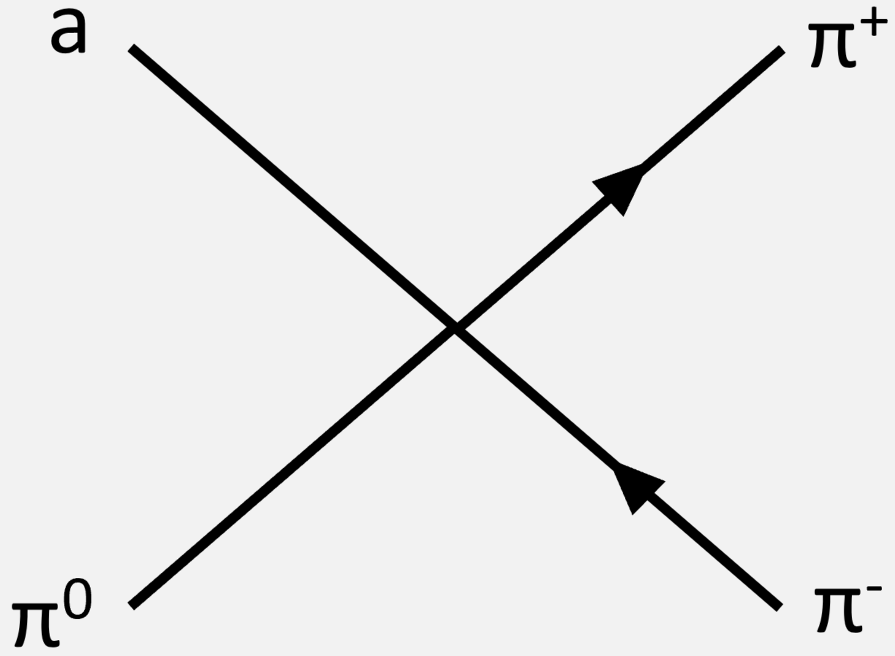


Axions as dark matter

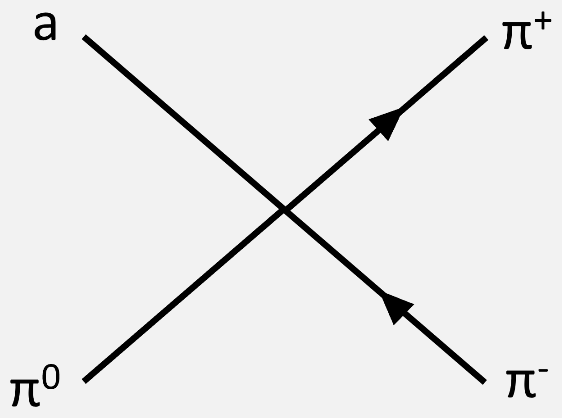
- Axions/axion-like particles (ALPs) are pseudo-scalar bosons.
- Introduced by Peccei and Quinn in their solution to the strong CP problem.
- Generally light, but still massive so are a leading candidate for dark matter.
- Axion dark matter would have a hot dark matter (HDM) component – bounded by cosmological observations.

Axion HDM

- Axion weakly couples with the SM.
- In thermal equilibrium with SM early in the universe.
- Relic HDM population left after decoupling.
- Decoupling temperature, T_D, affects the number of effective relativistic d.o.f., which is constrained by observations.
- Cosmological observations can thus bound the axion mass.

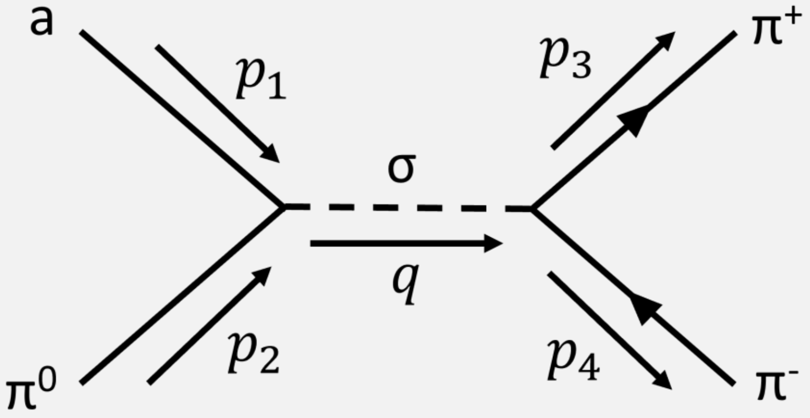
Breakdown of chiral perturbation theory

- For T_D<200 MeV the dominant axion interaction with SM is $a\pi \Leftrightarrow \pi\pi$ via a derivative axion coupling to the axial current.
- Non-linear sigma model breaks down near these temperatures (Di Luzio et al, 2021).
- Perturbative calculations no longer reliable for the axion HDM bound.



The solution: non-perturbative QFT

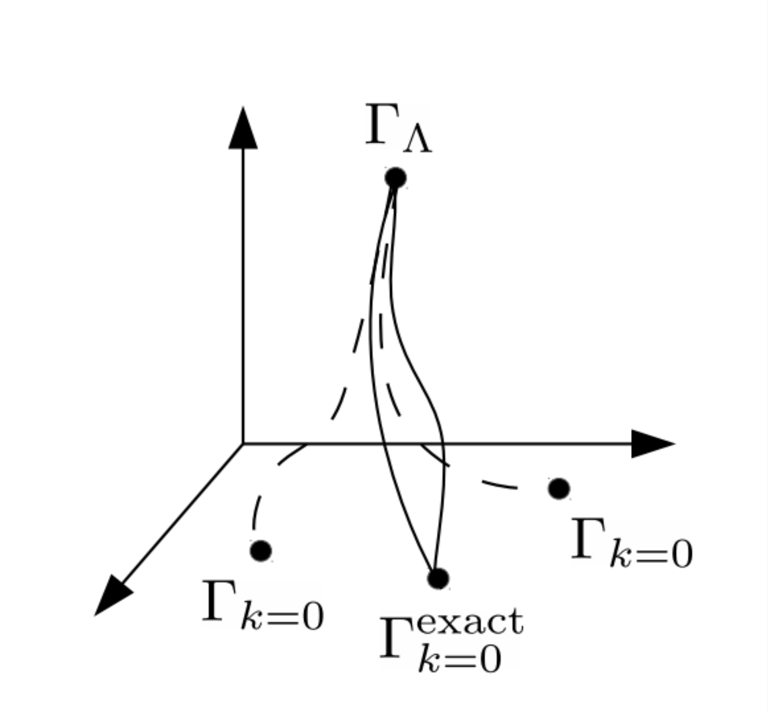
- Use the linear sigma model, so that the sigma mode is still dynamical as it should be at these temperatures (before chiral symmetry breaking).
- aπ⇔ππ now requires a sigma mediator, however in the low energy limit this produces the same amplitude.



Functional Renormalisation Group

- FRG: a non-perturbative flow equation for the effective action.
- Regulator function suppresses high energy modes above momentum scale *k*.

$$\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\left\{\partial_k R_k \left(\Gamma_k^{(2)}[\phi] + R_k\right)^{-1}\right\}$$



Effective action

- Define your effective action

$$\Gamma_k = \int d^4x \Big[Z_\pi \partial_\mu \pi^- \partial^\mu \pi^+ + \frac{Z_0}{2} \partial_\mu \pi^0 \partial^\mu \pi^0 + \frac{Z_\sigma}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{Z_a}{2} \partial_\mu a \partial^\mu a$$
$$+ \frac{m_a^2}{2} a^2 + m_\pi^2 \pi^+ \pi^- + \frac{m_0^2}{2} (\pi^0)^2 + \frac{m_\sigma^2}{2} \sigma^2$$
$$+ \frac{\lambda}{4} \left(\sigma^2 + 2\pi^+ \pi^- + (\pi^0)^2 \right)^2 + \frac{C_{a\pi}}{f_a} \partial_\mu a \left(\sigma \partial^\mu \pi^0 - \pi^0 \partial^\mu \sigma \right) \Big]$$

Projecting out the flows

- Project out the flows

$$\partial_t Z_a = \left[\frac{1}{2}\frac{\partial^2}{\partial q^2}\int d^4p \frac{\delta^2 \partial_t \Gamma_k}{\delta a(p)\delta a(q)}\right]_{a,\sigma,\pi^0,\pi^1,\pi^2,q\to 0}$$

$$\partial_t m_a^2 = \left[\int d^4p \frac{\delta^2 \partial_t \Gamma_k}{\delta a(p)\delta a(q)}\right]_{a,\sigma,\pi^0,\pi^1,\pi^2,q\to 0}$$

$$\partial_t C_{a\pi} = f_a \left[\frac{1}{2}\frac{\partial^2}{\partial q^2}\int d^4p \frac{\delta^3 \partial_t \Gamma_k}{\delta a(p)\delta\sigma(r)\delta\pi^0(q)}\right]_{a,\sigma,\pi^0,\pi^1,\pi^2,q,r\to 0}$$

$$\partial_t \lambda = \frac{1}{6}\left[\int d^4p \frac{\delta^4 \partial_t \Gamma_k}{\delta\sigma(p)\delta\sigma(r)\delta\sigma(s)\delta\sigma(q)}\right]_{a,\sigma,\pi^0,\pi^1,\pi^2,q,r,s\to 0}$$

Projecting out the flows

```
lp[k_] :=
  1/k * 1/(6720 π^2 (m02[k] + k^2 Z0[k])^3) k^6 (-1/(f^2 (ma2[k] + k^2 Za[k])^2) 21 k^4 Caπ[k]^2 l[k] (20 ma2[k] × Z0[k] + 10 (m02[k] + 3 k^2 Z0[k]) × Za[k] + 2 k (ma2[k] + k^2 Za[k]) Z0'[k] + k (m02[k] + k^2 Z0[k]) Za'[k]) + 1/(f^4 (ma2[k] + k^2 Za[k])^3) 20 k^8 Caπ[k]^4 (ma2[k] (14 Z0[k] + k Z0'[k]) + Za[k] (14 m02[k] + k^2 (28 Z0[k] + k Z0'[k])) + k (m02[k] + k^2 Z0[k]) Za'[k]) +
    1/((mpi2[k] + k^2 Zpi[k])^3 (ms2[k] + k^2 Zs[k])^3) 35 l[k]^2 (mpi2[k]^3 (216 (m02[k] + k^2 Z0[k])^3 Zs[k] + 6 k^6 Z0[k] Zs[k]^3 + k^7 Zs[k]^3 Z0'[k] + ms2[k]^3 (6 Z0[k] + k Z0'[k]) + 3 k^2 ms2[k]^2 Zs[k] (6 Z0[k] + k Z0'[k]) + 3 k^4 ms2[k] Zs[k]^2 (6 Z0[k] + k Z0'[k]) + 36 k (m02[k] + k^2 Z0[k])^3 Zs'[k]) +
      3 k^2 mpi2[k]^2 Zpi[k] (216 (m02[k] + k^2 Z0[k])^3 Zs[k] + 6 k^6 Z0[k] Zs[k]^3 + k^7 Zs[k]^3 Z0'[k] + ms2[k]^3 (6 Z0[k] + k Z0'[k]) + 3 k^2 ms2[k]^2 Zs[k] (6 Z0[k] + k Z0'[k]) + 3 k^4 ms2[k] Zs[k]^2 (6 Z0[k] + k Z0'[k]) + 36 k (m02[k] + k^2 Z0[k])^3 Zs'[k]) +
      3 k^4 mpi2[k] Zpi[k]^2 (216 (m02[k] + k^2 Z0[k])^3 Zs[k] + 6 k^6 Z0[k] Zs[k]^3 + k^7 Zs[k]^3 Z0'[k] + ms2[k]^3 (6 Z0[k] + k Z0'[k]) + 3 k^2 ms2[k]^2 Zs[k] (6 Z0[k] + k Z0'[k]) + 3 k^4 ms2[k] Zs[k]^2 (6 Z0[k] + k Z0'[k]) + 36 k (m02[k] + k^2 Z0[k])^3 Zs'[k]) +
      2 m02[k]^3 (ms2[k]^3 (6 Zpi[k] + k Zpi'[k]) + 3 k^2 ms2[k]^2 Zs[k] (6 Zpi[k] + k Zpi'[k]) + 3 k^4 ms2[k] Zs[k]^2 (6 Zpi[k] + k Zpi'[k]) + k^6 (6 Zpi[k] Zs[k]^3 + k Zs[k]^3 Zpi'[k] + 18 Zpi[k]^3 (6 Zs[k] + k Zs'[k]))) + 6 k^2 m02[k]^2 Z0[k] (ms2[k]^3 (6 Zpi[k] + k Zpi'[k]) + 3 k^2 ms2[k]^2 Zs[k] (6 Zpi[k] + k Zpi'[k]) + 3 k^4 ms2[k] Zs[k]^2 (6 Zpi[k] + k Zpi'[k]) + k^6 (6 Zpi[k] Zs[k]^3 + k Zs[k]^3 Zpi'[k] + 18 Zpi[k]^3 (6 Zs[k] + k Zs'[k]))) +
      6 k^4 m02[k] Z0[k]^2 (ms2[k]^3 (6 Zpi[k] + k Zpi'[k]) + 3 k^2 ms2[k]^2 Zs[k] (6 Zpi[k] + k Zpi'[k]) + 3 k^4 ms2[k] Zs[k]^2 (6 Zpi[k] + k Zpi'[k]) + k^6 (6 Zpi[k] Zs[k]^3 + k Zs[k]^3 Zpi'[k] + 18 Zpi[k]^3 (6 Zs[k] + k Zs'[k]))) +
      k^6 (ms2[k]^3 (6 Z0[k] Zpi[k]^3 + k Zpi[k]^3 Z0'[k] + 2 Z0[k]^3 (6 Zpi[k] + k Zpi'[k])) + 3 k^2 ms2[k]^2 Zs[k] (6 Z0[k] Zpi[k]^3 + k Zpi[k]^3 Z0'[k] + 2 Z0[k]^3 (6 Zpi[k] + k Zpi'[k])) + 3 k^4 ms2[k] Zs[k]^2 (6 Z0[k] Zpi[k]^3 + k Zpi[k]^3 Z0'[k] + 2 Z0[k]^3 (6 Zpi[k] + k Zpi'[k])) + k^6 (6 Z0[k] Zpi[k]^3 Zs[k]^3 + k Zpi[k]^3 Zs[k]^3 Z0'[k] + 2 Z0[k]^3 (6 Zpi[k] Zs[k]^3 + k Zs[k]^3 Zpi'[k] + 18 Zpi[k]^3 (6 Zs[k] + k Zs'[k]))))));

Caπp[k_] :=
  1/k *
   (k Caπ[k] (1/3 f^2 k^5 l[k] (ma2[k] + k^2 Za[k])^2 (-ms2[k] (6 Z0[k] + k Z0'[k]) + Zs[k] (6 m02[k] - k^3 Z0'[k]) + k (m02[k] + k^2 Z0[k]) Zs'[k]) -
       1/40 k^9 Caπ[k]^2 (3 k^2 ms2[k] (k Za[k] Z0'[k] + Z0[k] (20 Za[k] + k Za'[k])) + ma2[k] (3 ms2[k] (10 Z0[k] + k Z0'[k]) + Zs[k] (-50 m02[k] - 20 k^2 Z0[k] + 3 k^3 Z0'[k]) - 5 k (m02[k] + k^2 Z0[k]) Zs'[k]) + k^4 (3 k Za[k] × Zs[k] Z0'[k] + Z0[k] (10 Za[k] × Zs[k] + 3 k Zs[k] Za'[k] - 5 k Za[k] Zs'[k])) + m02[k] (3 ms2[k] (10 Za[k] + k Za'[k]) + k^2 (3 k Zs[k] Za'[k] - 5 Za[k] (4 Zs[k] + k Zs'[k])))))) /
    (4 f^2 π^2 (m02[k] + k^2 Z0[k])^2 (ma2[k] + k^2 Za[k])^2 (ms2[k] + k^2 Zs[k])^2);
Zap[k_] := 1/k * ((k^8 Caπ[k]^2 (ms2[k] (8 Z0[k] + k Z0'[k]) + Zs[k] (8 m02[k] + k^2 (16 Z0[k] + k Z0'[k])) + k (m02[k] + k^2 Z0[k]) Zs'[k])) / (48 f^2 π^2 (m02[k] + k^2 Z0[k])^2 (ms2[k] + k^2 Zs[k])^2));
ma2p[k_] := 0;
Zsp[k_] := 1/k * (-((k^8 Caπ[k]^2 (ma2[k] (8 Z0[k] + k Z0'[k]) + Za[k] (8 m02[k] + k^2 (16 Z0[k] + k Z0'[k])) + k (m02[k] + k^2 Z0[k]) Za'[k])) / (48 f^2 π^2 (m02[k] + k^2 Z0[k])^2 (ma2[k] + k^2 Za[k])^2)));
ms2p[k_] :=
  1/k * 1/(960 π^2 (m02[k] + k^2 Z0[k])^2) k^6
   (1/(f^2 (ma2[k] + k^2 Za[k])^2) 3 k^4 Caπ[k]^2 (ma2[k] (10 Z0[k] + k Z0'[k]) + Za[k] (10 m02[k] + k^2 (20 Z0[k] + k Z0'[k])) + k (m02[k] + k^2 Z0[k]) Za'[k]) -
     1/((mpi2[k] + k^2 Zpi[k])^2 (ms2[k] + k^2 Zs[k])^2) 5 l[k] (k^4 ms2[k]^2 (Zpi[k] (6 Z0[k] (2 Z0[k] + Zpi[k]) + k Zpi[k] Z0'[k]) + 2 k Z0[k]^2 Zpi'[k]) + 2 k^6 ms2[k] × Zs[k] × (Zpi[k] (6 Z0[k] (2 Z0[k] + Zpi[k]) + k Zpi[k] Z0'[k]) + 2 k Z0[k]^2 Zpi'[k]) +
       mpi2[k]^2 (ms2[k]^2 (6 Z0[k] + k Z0'[k]) + 2 k^2 ms2[k] × Zs[k] (6 Z0[k] + k Z0'[k]) + Zs[k] (36 m02[k]^2 + 72 k^2 m02[k] × Z0[k] + k^4 (6 Z0[k] (6 Z0[k] + Zs[k]) + k Zs[k] Z0'[k])) + 6 k (m02[k] + k^2 Z0[k])^2 Zs'[k]) + 2 k^2 mpi2[k] × Zpi[k] (ms2[k]^2 (6 Z0[k] + k Z0'[k]) + 2 k^2 ms2[k] × Zs[k] (6 Z0[k] + k Z0'[k]) + Zs[k] (36 m02[k]^2 + 72 k^2 m02[k] × Z0[k] + k^4 (6 Z0[k] (6 Z0[k] + Zs[k]) + k Zs[k] Z0'[k])) + 6 k (m02[k] + k^2 Z0[k])^2 Zs'[k]) +
       2 m02[k]^2 (ms2[k]^2 (6 Zpi[k] + k Zpi'[k]) + 2 k^2 ms2[k] × Zs[k] (6 Zpi[k] + k Zpi'[k]) + k^4 (Zs[k] (6 Zpi[k] (3 Zpi[k] + Zs[k]) + k Zs[k] Zpi'[k]) + 3 k Zpi[k]^2 Zs'[k])) + 4 k^2 m02[k] × Z0[k] (ms2[k]^2 (6 Zpi[k] + k Zpi'[k]) + 2 k^2 ms2[k] × Zs[k] (6 Zpi[k] + k Zpi'[k]) + k^4 (Zs[k] (6 Zpi[k] (3 Zpi[k] + Zs[k]) + k Zs[k] Zpi'[k]) + 3 k Zpi[k]^2 Zs'[k])) +
       k^8 (6 Z0[k] Zpi[k]^2 Zs[k]^2 + k Zpi[k]^2 Zs[k]^2 Z0'[k] + 2 Z0[k]^2 (Zs[k] (6 Zpi[k] (3 Zpi[k] + Zs[k]) + k Zs[k] Zpi'[k]) + 3 k Zpi[k]^2 Zs'[k]))));
Z0p[k_] := 1/k * (-((k^8 Caπ[k]^2 (ms2[k] (8 Za[k] + k Za'[k]) + Zs[k] (8 ma2[k] + k^2 (16 Za[k] + k Za'[k])) + k (ma2[k] + k^2 Za[k]) Zs'[k])) / (48 f^2 π^2 (ma2[k] + k^2 Za[k])^2 (ms2[k] + k^2 Zs[k])^2)));
m02p[k_] :=
  1/k * 1/(960 π^2 (ms2[k] + k^2 Zs[k])^2) k^6 (-(12 k^5 Caπ[k]^2 (ms2[k] + k^2 Zs[k]) Za'[k])/(f^2 (ma2[k] + k^2 Za[k])^2) + (15 k^4 Caπ[k]^2 (ms2[k] + k^2 Zs[k]) (2 Za[k] + k Za'[k]))/(f^2 (ma2[k] + k^2 Za[k])^2) - (12 k^5 Caπ[k]^2 Zs'[k])/(f^2 (ma2[k] + k^2 Za[k])) + 10 k l[k] (2 (ms2[k] + k^2 Zs[k])^2 ((3 Z0'[k])/(m02[k] + k^2 Z0[k])^2 + Zpi'[k]/(mpi2[k] + k^2 Zpi[k])^2) + Zs'[k]) + (15 k^4 Caπ[k]^2 (2 Zs[k] + k Zs'[k]))/(f^2 (ma2[k] + k^2 Za[k])) -
     1/((m02[k] + k^2 Z0[k])^2 (mpi2[k] + k^2 Zpi[k])^2) 15 l[k] (2 k^4 (Zpi[k] × (Z0[k] (2 ms2[k]^2 (Z0[k] + 3 Zpi[k]) + 4 k^2 ms2[k] × (Z0[k] + 3 Zpi[k]) × Zs[k] + k^4 Zs[k] × (Z0[k] × Zpi[k] + 2 (Z0[k] + 3 Zpi[k]) × Zs[k])) + 3 k Zpi[k] (ms2[k] + k^2 Zs[k])^2 Z0'[k]) + k Z0[k]^2 (ms2[k] + k^2 Zs[k])^2 Zpi'[k]) + k^9 Z0[k]^2 Zpi[k]^2 Zs'[k] +
       mpi2[k]^2 (2 m02[k]^2 Zs[k] + 2 Z0[k] (6 ms2[k]^2 + k^2 (2 m02[k] + 12 ms2[k] + k^2 Z0[k]) Zs[k] + 6 k^4 Zs[k]^2) + 6 k (ms2[k] + k^2 Zs[k])^2 Z0'[k] + k (m02[k] + k^2 Z0[k])^2 Zs'[k]) + 2 k^2 mpi2[k] × Zpi[k] (2 m02[k]^2 Zs[k] + 2 Z0[k] (6 ms2[k]^2 + k^2 (2 m02[k] + 12 ms2[k] + k^2 Z0[k]) Zs[k] + 6 k^4 Zs[k]^2) + 6 k (ms2[k] + k^2 Zs[k])^2 Z0'[k] + k (m02[k] + k^2 Z0[k])^2 Zs'[k]) +
       m02[k]^2 (2 ms2[k]^2 (2 Zpi[k] + k Zpi'[k]) + 4 k^2 ms2[k] × Zs[k] (2 Zpi[k] + k Zpi'[k]) + k^4 (2 Zs[k] (Zpi[k]^2 + 2 Zpi[k] × Zs[k] + k Zs[k] Zpi'[k]) + k Zpi[k]^2 Zs'[k])) + 2 k^2 m02[k] × Z0[k] (2 ms2[k]^2 (2 Zpi[k] + k Zpi'[k]) + 4 k^2 ms2[k] × Zs[k] (2 Zpi[k] + k Zpi'[k]) + k^4 (2 Zs[k] (Zpi[k]^2 + 2 Zpi[k] × Zs[k] + k Zs[k] Zpi'[k]) + k Zpi[k]^2 Zs'[k]))));

Zpip[k_] := 0;
mpi2p[k_] :=
  1/k *
   (-((k^6 l[k] (k^4 ms2[k]^2 (Zpi[k] (6 Z0[k] (7 Z0[k] + Zpi[k]) + k Zpi[k] Z0'[k]) + 7 k Z0[k]^2 Zpi'[k]) + 2 k^6 ms2[k] × Zs[k] × (Zpi[k] (6 Z0[k] (7 Z0[k] + Zpi[k]) + k Zpi[k] Z0'[k]) + 7 k Z0[k]^2 Zpi'[k]) + mpi2[k]^2 (ms2[k]^2 (6 Z0[k] + k Z0'[k]) + 2 k^2 ms2[k] × Zs[k] (6 Z0[k] + k Z0'[k]) + Zs[k] (6 ((m02[k] + k^2 Z0[k])^2 + k^4 Z0[k] × Zs[k]) + k^5 Zs[k] Z0'[k]) + k (m02[k] + k^2 Z0[k])^2 Zs'[k]) +
        2 k^2 mpi2[k] × Zpi[k] (ms2[k]^2 (6 Z0[k] + k Z0'[k]) + 2 k^2 ms2[k] × Zs[k] (6 Z0[k] + k Z0'[k]) + Zs[k] (6 ((m02[k] + k^2 Z0[k])^2 + k^4 Z0[k] × Zs[k]) + k^5 Zs[k] Z0'[k]) + k (m02[k] + k^2 Z0[k])^2 Zs'[k]) + k^8 (Zs[k] × (Zpi[k] (6 Z0[k] × (Zpi[k] × Zs[k] + Z0[k] × (Zpi[k] + 7 Zs[k])) + k Zpi[k] × Zs[k] Z0'[k]) + 7 k Z0[k]^2 Zs[k] Zpi'[k]) + k Z0[k]^2 Zpi[k]^2 Zs'[k]) +
        m02[k]^2 (7 ms2[k]^2 (6 Zpi[k] + k Zpi'[k]) + 14 k^2 ms2[k] × Zs[k] (6 Zpi[k] + k Zpi'[k]) + k^4 (Zs[k] (6 Zpi[k] × (Zpi[k] + 7 Zs[k]) + 7 k Zs[k] Zpi'[k]) + k Zpi[k]^2 Zs'[k])) + 2 k^2 m02[k] × Z0[k] (7 ms2[k]^2 (6 Zpi[k] + k Zpi'[k]) + 14 k^2 ms2[k] × Zs[k] (6 Zpi[k] + k Zpi'[k]) + k^4 (Zs[k] (6 Zpi[k] × (Zpi[k] + 7 Zs[k]) + 7 k Zs[k] Zpi'[k]) + k Zpi[k]^2 Zs'[k])))) /
     (192 π^2 (m02[k] + k^2 Z0[k])^2 (mpi2[k] + k^2 Zpi[k])^2 (ms2[k] + k^2 Zs[k])^2)));
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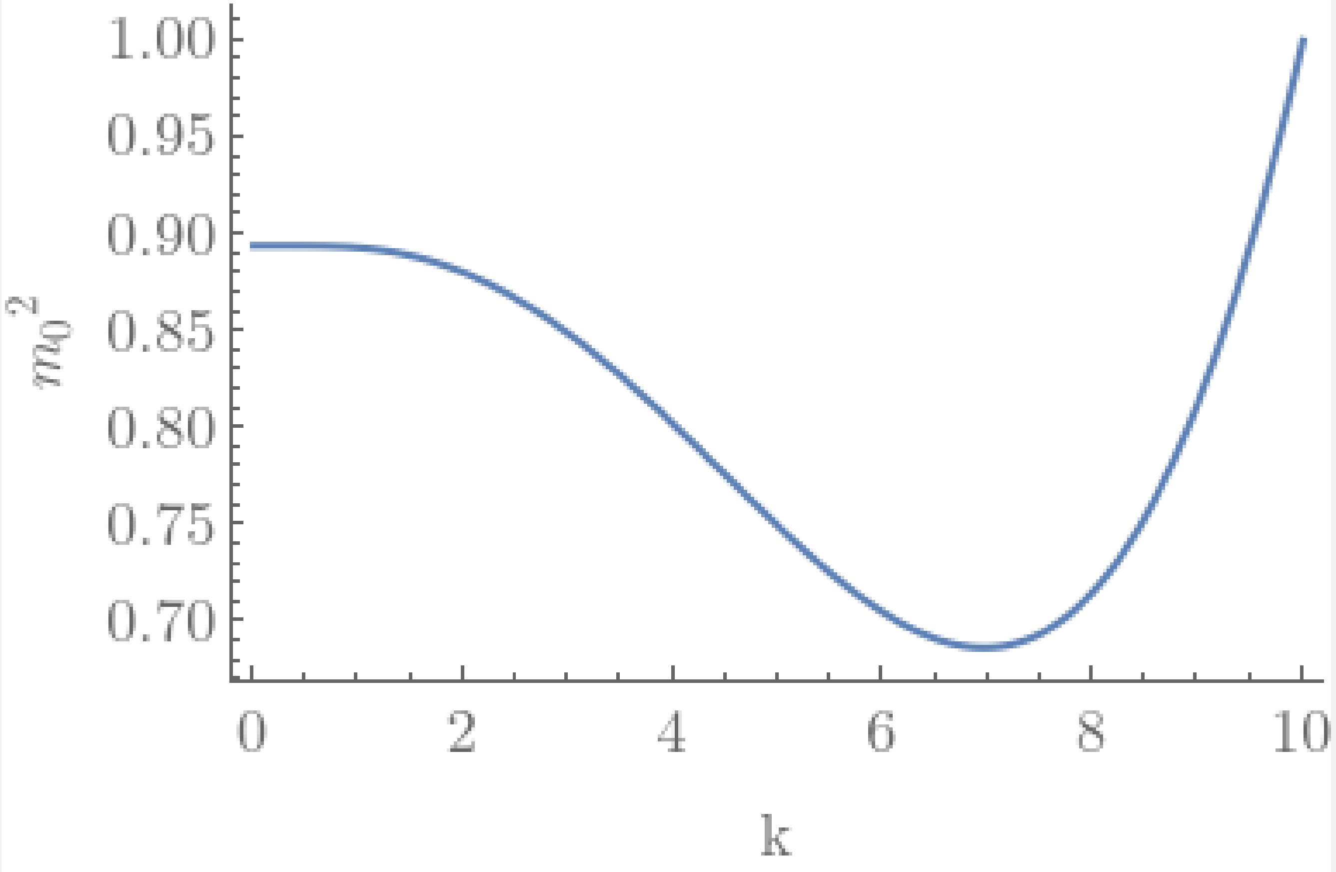
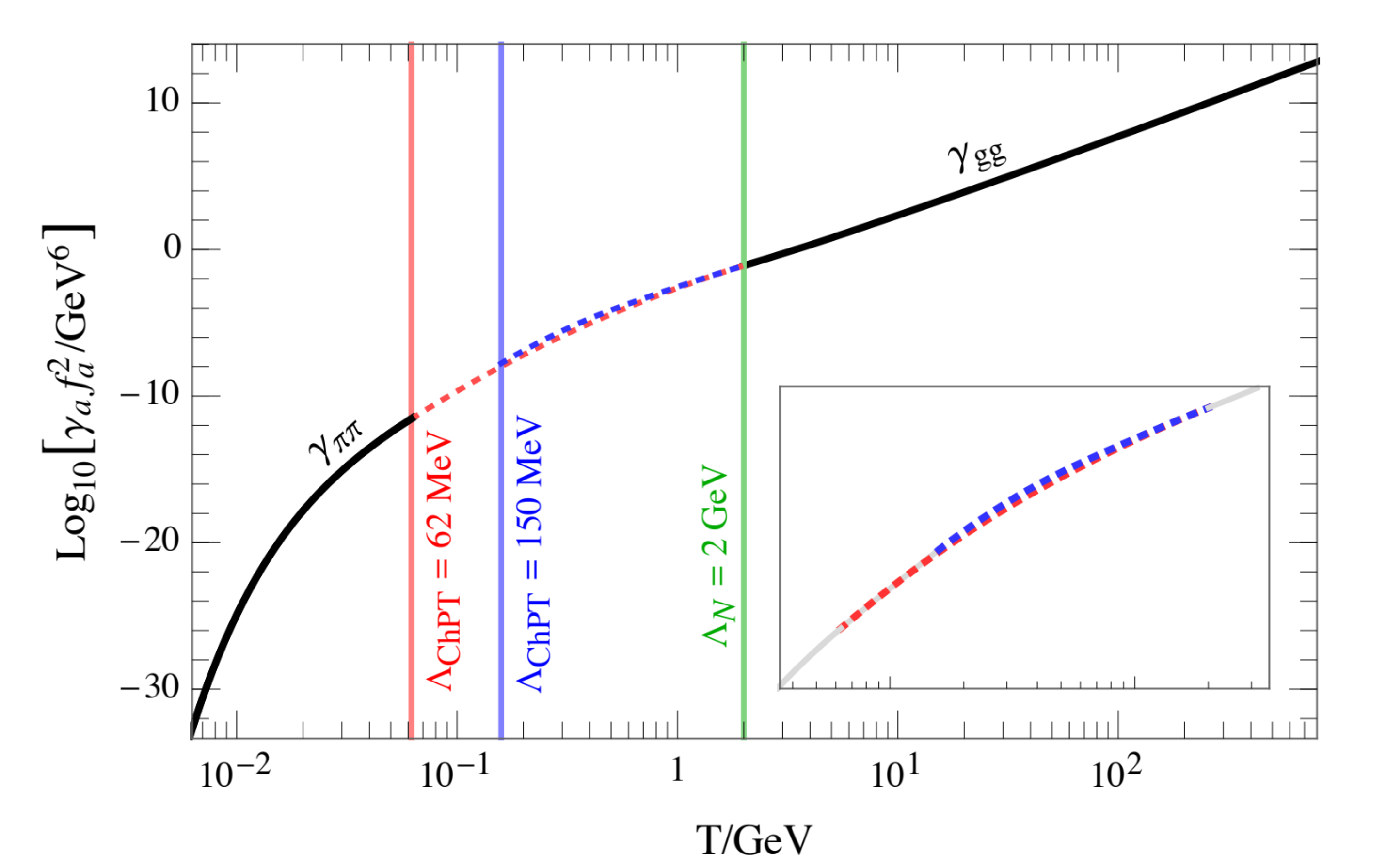
Numerically solve the coupled DEs



Next steps



10
0
−10
−20
−30
$\mathrm{Log}_{10}\left[\gamma_a f_a^2/\mathrm{GeV}^6\right]$
10^{-2}
10^{-1}
1
10^{1}
10^{2}
T/GeV
γ_{gg}
$\gamma_{\pi\pi}$
Λ_{ChPT} = 62 MeV
Λ_{ChPT} = 150 MeV
Λ_N = 2 GeV

Thanks for watching!

- Any questions?