



# Thermodynamics tutorials

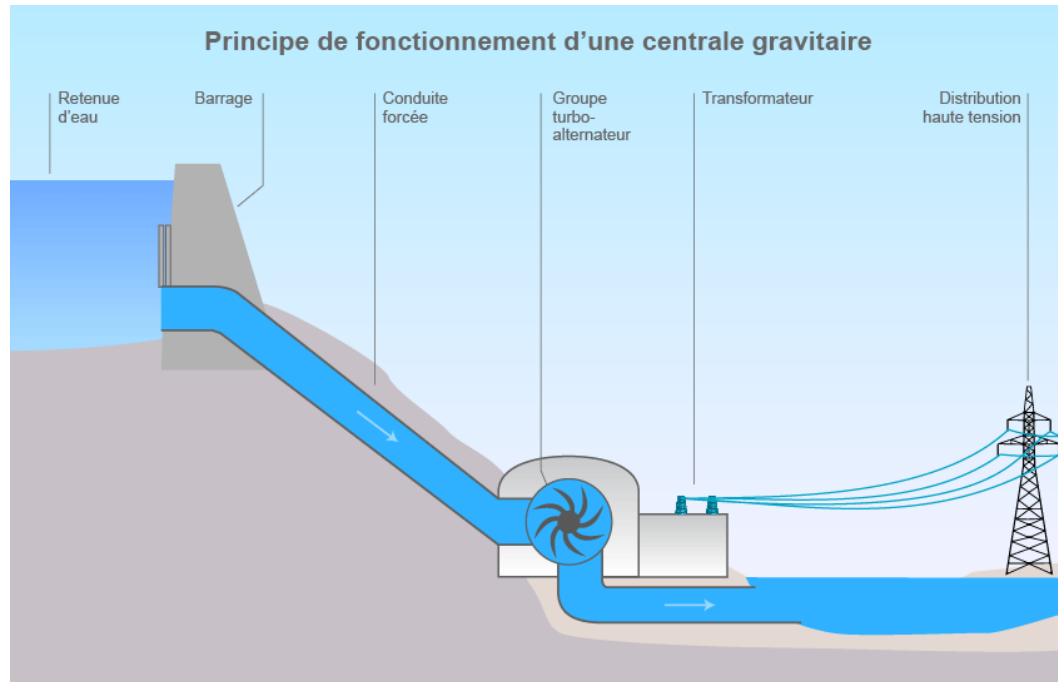
Philippe Lebrun

*European Scientific Institute, Archamps*

Cryostat Engineering for Helium Superconducting Devices  
CERN, 7-9 November 2022



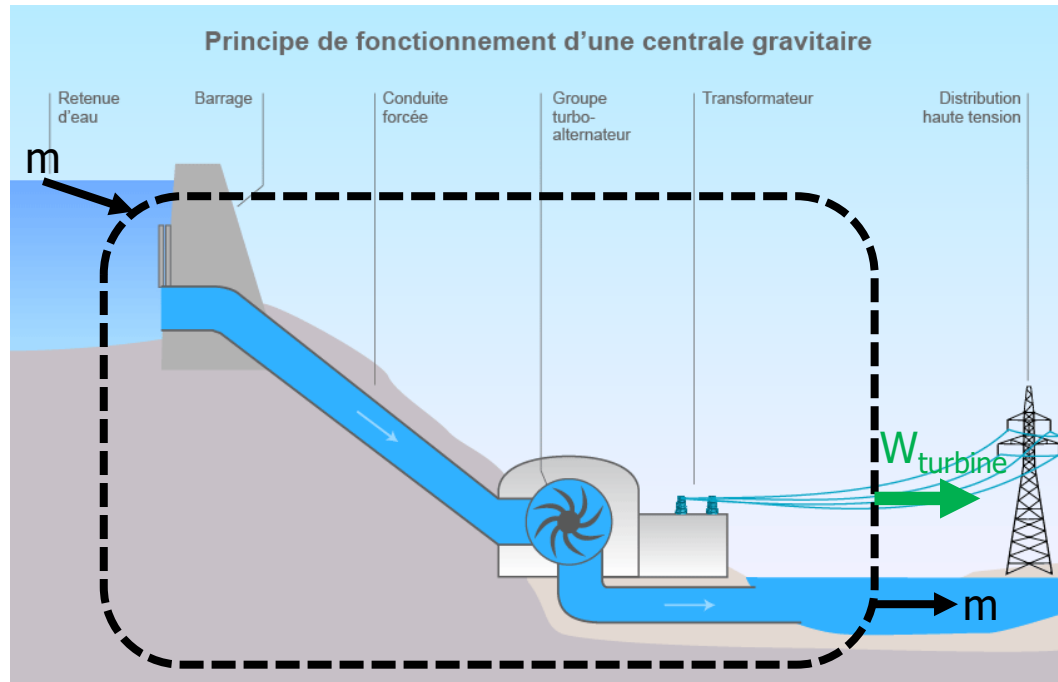
# Thermodynamics tutorial 1: text



- First principle of thermodynamics applied to hydroelectric plant
  1. Define boundary of thermodynamic system (*Hint: make sure you will not have to cope with variations of kinetic energy*)
  2. Write energy conservation of the fluid (water) traversing the system (*Hint: water is assumed non-compressible*)
  3. Doesn't this suggest to introduce a new definition of enthalpy, called  $I$ , suited to gravity flow? Rewrite energy conservation using  $I$



# Thermodynamics tutorial 1 : solution [1/2]



- Open system in steady state: system boundary
  - Water in at reservoir lake surface, no velocity
  - Water out at outlet lake, velocity negligible
  - Hence, no variation of kinetic energy



## Thermodynamics tutorial 1 : solution [2/2]

- Energy conservation
  - $\Delta U = \Delta Q + \Delta W$  with  $\Delta W$  algebraic sum of work received and exerted by the system
  - Short residence time of water in penstock and turbine makes flow adiabatic  $\Delta Q \approx 0$
  - $\Delta U = \Delta W = P_{in}V_{in} - P_{out}V_{out} + g(z_{in} - z_{out}) - W_{turbine} = -g \Delta z - W_{turbine}$
  - Work of surface forces on the fluid  $P_{in}V_{in} - P_{out}V_{out} \equiv 0$
  - Work of volume forces on the fluid  $g(z_{in} - z_{out})$
  - Hence  $\Delta U = -g \Delta z - W_{turbine}$
- Redefining enthalpy  $I$ 
  - A suitable definition of enthalpy for non-compressible flows subject to gravity is
$$I = U + g z$$
  - Then  $\Delta I = -W_{turbine}$  the work produced equals the drop in enthalpy of the fluid



## Thermodynamics tutorial 2 : text

- Specific heat of solids
  1. Why are  $C_P$  and  $C_V$  not differentiated for solids?
  2. Estimate numerically their difference for a metal, e.g. aluminium



## Thermodynamics tutorial 2 : solution

- From the definitions of  $C_P$  and  $C_V$ 
  - $H = U + P V$  hence  $C_P = C_V + d(PV)/dT = C_V + V dP/dT + P dV/dT$
  - At atmospheric pressure  $C_P = C_V + P dV/dT \approx C_V$  since  $dV/dT$  small
- Aluminium
  - Specific heat  $C = 900 \text{ J/kg}^\circ\text{C}$
  - Density  $\rho = 2.7 \text{ kg/dm}^3$
  - Coefficient of linear expansion  $\lambda = dL/L dT = 23.6 \text{ E} - 6 /^\circ\text{C}$
  - Hence, the coefficient of volume expansion is  $dV/VdT = 3\lambda = 7.1 \text{ E} - 5 /^\circ\text{C}$
  - Consider 1 kg of aluminium, having a volume of  $370 \text{ cm}^3$
  - Volume expansion when temperature increases by 1 degree  $0.026 \text{ cm}^3$
  - Resulting work done against the atmosphere ( $10^5 \text{ Pa}$ ) is 2.6 mJ, to be compared with 900 J of «specific heat»



## Thermodynamics tutorial 3 : text

- Helium dewar boiloff and pressurization
  1. Consider a 100-liter He dewar vessel initially filled at 90% with a 1 W heat inleak, vented to atmosphere (1 bar). How long will it take for the liquid level to reach 80%?
  2. At 80% level, the vessel is sealed: what happens? Which function of state is to be used to describe the evolution of the system?
  3. Assuming equilibrium conditions, calculate the evolution of pressure and temperature in the vessel with time



## Thermodynamics tutorial 3 : solution [1/3]

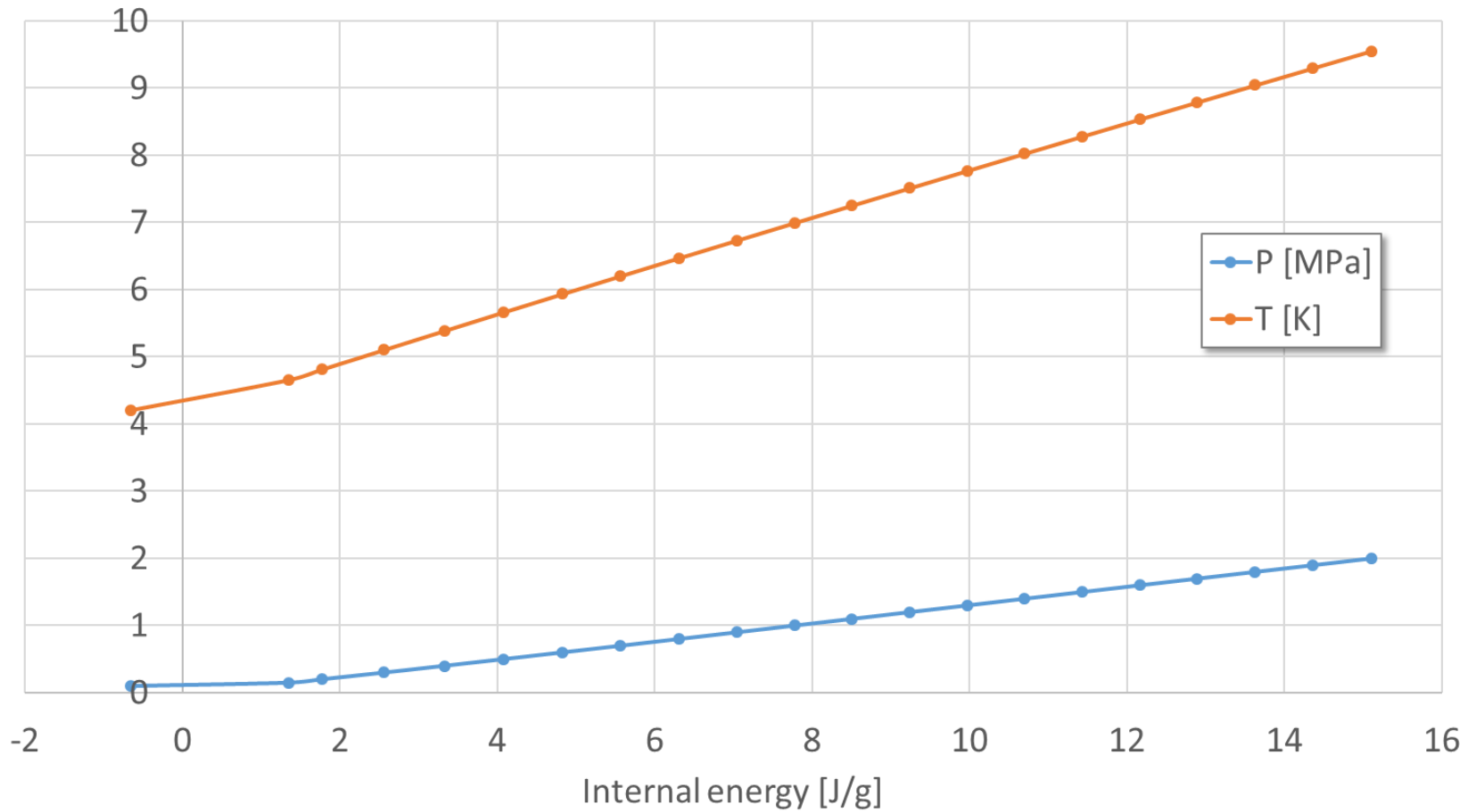
- Boiloff at 1 bar
  - Saturated helium properties
    - Liquid density 125 g/l
    - Vapour density 16.64 g/l
    - Internal energy of liquid -0.87 J/g
    - Internal energy of vapour 14.64 J/g
    - Latent heat of vaporization (difference of vapour and liquid enthalpies) 20.7 J/g
  - Boiloff rate  $\dot{Q} = L_V \dot{m} = L_V \rho \dot{V}$  hence  $\dot{V} = \frac{\dot{Q}}{\rho L_V} \cong 3.86E - 4 \text{ l/s}$
  - To vaporize 10 l (from 90% to 80%),  $t = \frac{10}{3.86E-4} \cong 25875 \text{ s} \cong 7.2 \text{ h}$
- Evolution at constant volume
  - The function of state to be used is internal energy  $\frac{dU}{dt} = \dot{Q} = 1W$
  - The total mass is constant  $M = (80 \times 125) + (20 \times 16.64) \cong 11460 \text{ g}$
  - The average density is constant  $\rho = \frac{(80 \times 125) + (20 \times 16.64)}{100} \cong 114.6 \text{ g/l}$
  - Calculate  $U(P)$  and  $U(T)$  at constant density (REFPROP)





## Thermodynamics tutorial 3 : solution [2/3]

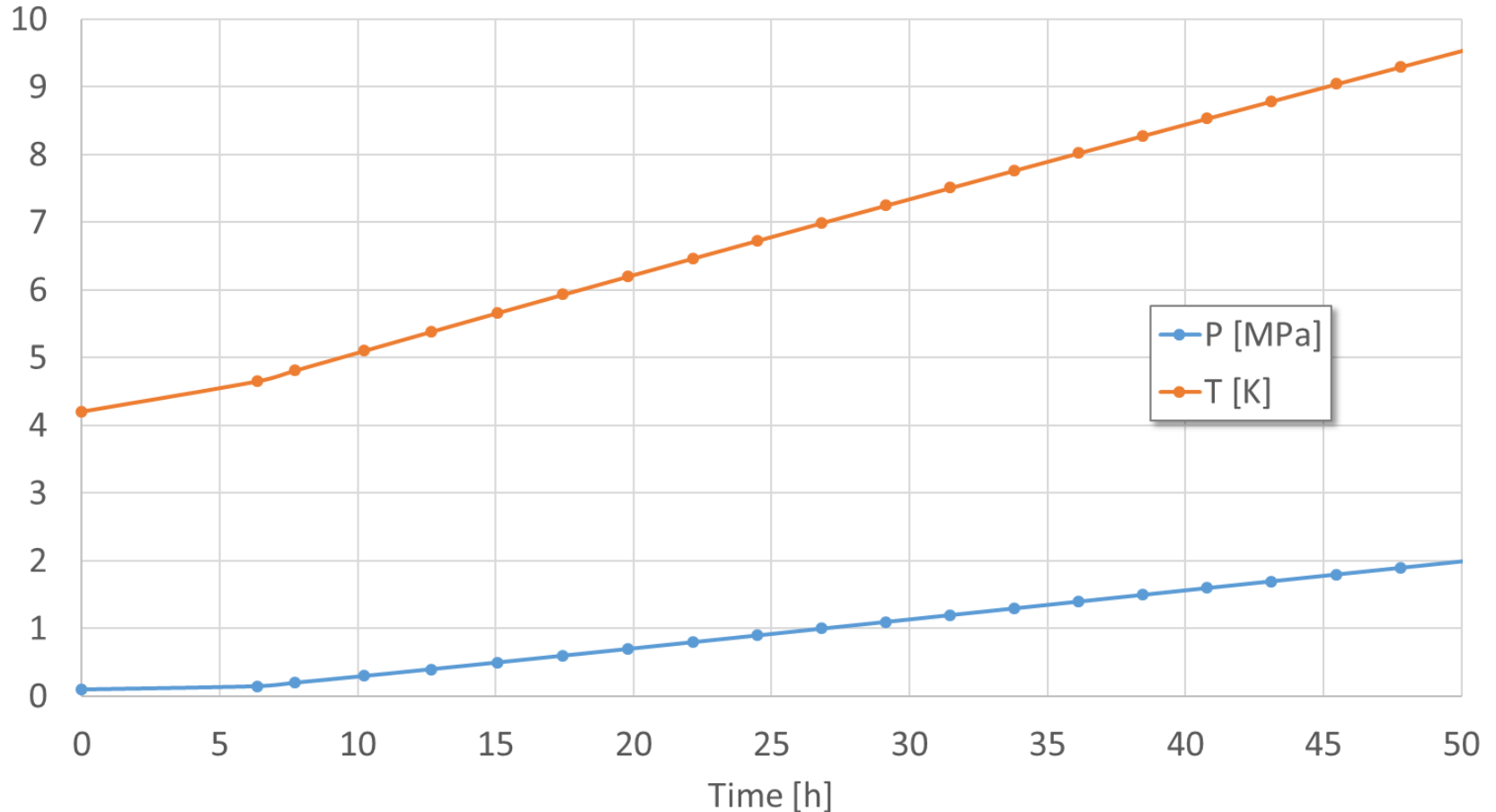
Evolution of P and T in closed 100 l LHe vessel  
Initial: 80 l liquid, 20 l vapour





## Thermodynamics tutorial 3 : solution [3/3]

Evolution of P and T in closed 100 l LHe vessel  
Initial: 80 l liquid, 20 l vapour





## Thermodynamics tutorial 4 : text

- Helium refrigeration and liquefaction work
  1. Calculate the minimum work to produce 1 W isothermal refrigeration at liquid helium temperature (4.2 K), assuming a room temperature of 290 K
  2. The most efficient helium refrigerators have a COP of 220 W/W: what is their efficiency relative to the Carnot cycle?
  3. A liquefier is a non-isothermal refrigerator to which the vaporized helium is returned at room temperature (290 K, 1 bar) instead of cold (4.2 K, 1 bar). Calculate the minimum liquefaction work for 1 g/s liquefaction. How much isothermal refrigeration at 4.2 K can that work produce?

*(Hint 1: Latent heat of vaporization of helium at 4.2 K = 20.8 J/g)*

*(Hint 2: Assume specific heat of helium at 1 bar is constant = 5.2 J/g.K)*



## Thermodynamics tutorial 4 : solution

- Minimum refrigeration work at 4.2 K
  - $W_{min}^{ref} = Q_i \left( \frac{T_0}{T_i} - 1 \right) = 1 \left( \frac{290}{4.2} - 1 \right) \cong 68 \text{ W/W}$
- Real refrigerators
  - $\eta = \frac{W_{min}^{ref}}{W_{real}^{ref}} \cong \frac{68}{220} \cong 0.31$
- Minimum liquefaction work at 4.2 K
  - Consider 1 g/s helium
  - $w_{min}^{liq} = w_{min}(\text{cooling from 290 K to 4.2 K}) + w_{min}(\text{condensation at 4.2 K})$
  - $dw_{min}(\text{cooling from 290 K to 4.2 K}) = dq \left( \frac{T_0}{T} - 1 \right) = c_P dT \left( \frac{T_0}{T} - 1 \right)$   
 $w_{min}(\text{cooling from 290 K to 4.2 K}) = c_P \left( T_0 \ln \frac{T_0}{T} - (T_0 - T) \right) \cong 4900 \text{ J/g}$
  - $w_{min}(\text{condensation at 4.2 K}) = 20.8 \left( \frac{290}{4.2} - 1 \right) \cong 1415 \text{ J/g}$
  - $w_{min}^{liq} \cong 4900 + 1415 = 6315 \text{ J/g}$
  - This amount of work could produce  $\frac{6315}{68} \cong 93 \text{ W}$  isothermal refrigeration at 4.2 K
  - *Rule of thumb: 1 g/s helium liquefaction ~ 100 W isothermal refrigeration, applicable only for small changes in the vicinity of the design point of a cryoplant*



# Measured refrigeration/liquefaction equivalence 12 kW @ 4.5 K helium refrigerators for LEP 2

