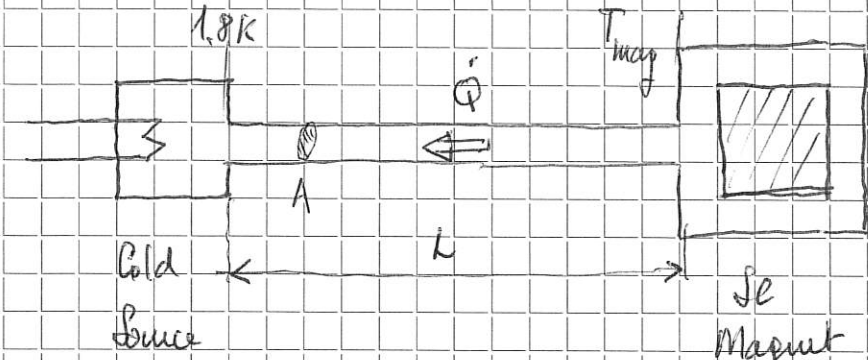
Problem 1 : Steady-state conduction in He II



1/ given $\begin{cases} T_{mag} = 1.9\,K \\ \dot{Q} = 20\,W \end{cases}$ $L = 8\,m = 800\,cm$

Equivalent conduction law $\dot{q}^{3.4} L = X(1.8K) - X(1.9K)$

From the $X(T)$ plot, in appropriate units $\begin{cases} X(1.8K) = 360 \\ X(1.9K) = 200 \end{cases}$

Hence $\dot{q}^{3.4} = \dfrac{\Delta X}{L} = \dfrac{160}{800} = 0,2$

$\dot{q} = 0,2^{1/3,4} = 0,623\ W/cm^2$

Since $\dot{q} = \dfrac{\dot{Q}}{A} \Rightarrow A = \dfrac{\dot{Q}}{\dot{q}} = \dfrac{20}{0,623}\ cm^2 = 32,1\ cm^2$

And $D^2 = \dfrac{4A}{\pi} = 40\ cm^2 \Rightarrow D = 64\ mm$

2/ Same $\begin{cases} T_{mag} = 1.9\,K \\ \dot{Q} = 20\,W \end{cases}$ but new $L = 16\,m$

Hence $\dot{q}^{3.4} = \dfrac{\Delta X}{L} = \dfrac{160}{1600} = 0,1$

$\dot{q} = 0.1^{1/3.4} = 0,508\ W/cm^2$

$$A = \frac{\dot{Q}}{\dot{q}} = \frac{20}{0,508} = 39,4 \text{ cm}^2$$

$$D^2 = \frac{4A}{\pi} = 50.1 \text{ cm}^2 \Rightarrow D = 71 \text{ mm}$$

$\Rightarrow$ Doubling the conduction length does not half the heat flux between fixed temperatures, as it would have done for "normal" conduction

3/ given $L = 8$ m and $D = 64$ mm ($A = 32.1$ cm^2)

The heat flow from the magnet is now increased to 25 W.

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{25}{32.1} = 0,779 \text{ W/cm}^2$$

$$\dot{q}^{3.4} = 0,427 \Rightarrow \dot{q}^{3.4} L = 342$$

From the equivalent conduction law, in appropriate units

$$\dot{q}^{3.4} L = 342 = X(1.8K) - X(T_{mag})$$

Hence $X(T_{mag}) = 360 - 342 = 18$

$$T_{mag} = 2.07 \text{ K}$$

$\Rightarrow$ The magnet temperature is now just below the lambda point.

4/ given $L = 8\,m$ and $D = 64\,mm$

$(A = 32{,}1\ cm^2)$

The maximum heat flux is obtained when the magnet temperature reaches the lambda point

$$T_{mag} = T_\lambda \iff X(T_{mag}) = X(T_\lambda) \equiv 0$$

The maximum heat flux is given by

$$\dot{q}_{max}^{3,4}\, L = X(1{,}8) - X(T_\lambda) = 360$$

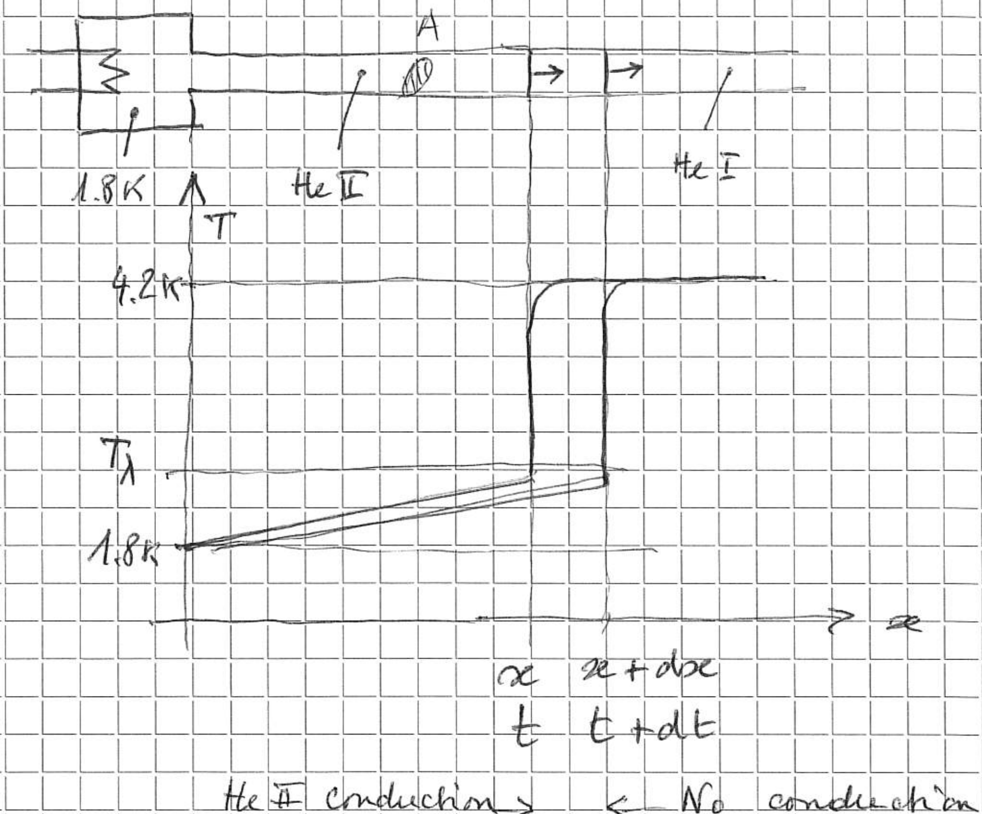
$$\dot{q}_{max}^{3,4} = \frac{360}{800} = 0{,}45$$

$$\dot{q}_{max} = 0{,}791\ W/cm^2$$

$$\dot{Q}_{max} = 0{,}791 \times 32{,}1 = 25{,}4\ W$$

WEKA

Problem 2 = Cooldown of liquid helium duct to superfluid



1/ The interface between normal and superfluid helium will be a front moving along the duct at T_λ.
The heat conducted to the cold source in He II will be used to cool the helium from its initial temperature of 4.2K to the lambda point T_λ

2/ • He II conduction $\left(\frac{\dot{Q}}{A}\right)^{3.4} x = X(1.8K) - X(T\lambda)$
$= X(1.8K)$

Hence $\dot{Q} = \left(\frac{X(1.8K)}{x}\right)^{1/3.4} A.$

Over time dt, the front progress of dx

$$d\dot{Q} = \dot{Q}\, dt = \left(\frac{X(1.8K)}{x}\right)^{1/3.4} A\, dt$$

- Cooling of slice dx $\quad d\dot{Q} = \Delta H\, dm = \rho A\, dx\, \Delta H$

enthalpy difference of helium $(4.2 - T_\lambda)$

Energy conservation :

$$\left(\frac{X(1.8K)}{x}\right)^{1/3.4} \cancel{A}\, dt = \rho \cancel{A}\, \Delta H\, dx$$

$$\Rightarrow \frac{dx}{dt} = \frac{X(1.8K)^{1/3.4}}{\rho\, \Delta H_{T_\lambda}^{4.2K}} \times \frac{1}{x^{1/3.4}}$$

$1/3.4$: 0,294

$\Rightarrow$ the velocity of the front does not depend on A

$\Rightarrow$ it slows down as distance x increases.

A.N. $\quad X(1,8K) = 360 \qquad \rho = 147\ kg/m^3 = 147\ g/dm^3 = 0,147\ g/cm^3$

$\Delta H = 9902 - 3634 = 6268\ J/kg = 6,27\ J/g$.

$$\frac{dx}{dt} = \frac{360^{0,294}}{0,147 \times 6,27} \times \frac{1}{x^{0,294}}\ cm/s$$

$$\frac{dx}{dt} \simeq 6,12 \times \frac{1}{x^{0,294}}\ cm/s \quad (x \text{ in cm})$$

For $x = 1m = 100\ cm$, $\dfrac{dx}{dt} = 1.6\ cm/s$

$x = 10\ m = 1000\ cm$, $\dfrac{dx}{dt} = 0,8\ cm/s$

3/ $t = \int_0^T dt$

$$dt = \frac{\rho \, \Delta H^{4.2}_{1.8}}{X (1.8k)^{0.294}} \, x^{0,294} \, dx$$

$$t = \frac{\rho \, \Delta H^{4,2}_{1.8}}{X (1.8k)^{0,294}} \, L^{1,294}$$

$$t \simeq \frac{0,147 \times 6,27}{360^{0,294}} \, L^{1,294}$$

$$t \simeq 0,163 \, L^{1,294}$$

t: sec, L: cm

For $\begin{cases} L = 1\,m = 100\,cm, & t = 387\,s = 6\,min\ 27\,s \\ L = 10\,m = 1000\,cm, & t = 7621\,s = 127\,min \\ & \quad = 2\,h\ 7\,min. \end{cases}$

Problem ~~4~~ 3 = Adiabatic compression of the vapor

1/ $\dot{Q} = 120 \text{ W at } 1.8\text{K}$

$\dot{Q} = \dot{m}\,\Delta H_L^V(1.8\text{K})$

$\Delta H_L^V(1.8\text{K}) = 24201 - 842 \simeq 23360 \text{ J/kg}$

$\Delta H_L^V(1.8\text{K}) \simeq 23{,}36 \text{ J/g}$

Hence $\dot{m} = \dfrac{\dot{Q}}{\Delta H_L^V} = \dfrac{120}{23{,}36} \simeq 5{,}14 \text{ g/s} = 5{,}14\times10^{-3} \text{ kg/s}$

2/ Volumetric flow $\dot{V}$ is given by $\dot{V} = \dfrac{\dot{m}}{\rho}$

$\begin{cases} \rho(16.4 \text{ mb}, 4\text{K}) = 0{,}198 \text{ kg/m}^3 \\ \rho(16.4 \text{ mb}, 290\text{K}) = 0{,}00272 \text{ kg/m}^3 \end{cases}$

Hence $\begin{cases} \dot{V}_{4K} = 0{,}026 \text{ m}^3/\text{s} = 93{,}5 \text{ m}^3/\text{h} \\ \dot{V}_{290K} = 1{,}89 \text{ m}^3/\text{s} = 6803 \text{ m}^3/\text{h} \end{cases}$

3/ For an ideal gas

$$W_{adiab} = \frac{\gamma}{\gamma-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

For He, $\gamma = 5/3 = 1{,}67 \Rightarrow \begin{cases} \dfrac{\gamma-1}{\gamma} = 0{,}4 \\ \dfrac{\gamma}{\gamma-1} = 2{,}5 \end{cases}$

$\dot{W}_{adiab\ 4K} = 2{,}5 \times 1640 \times 0{,}026 \times (61^{0{,}4} - 1)$ 4,18

$\simeq \cancel{267}$ 446 W $\simeq \cancel{0{,}27}$ 0,45 kW

$\dot{W}_{adiab\ 290K} = 2{,}5 \times 1640 \times 1{,}89 \times (61^{0{,}4} - 1)$

$\simeq \cancel{19434}$ 32390 W $= \cancel{19{,}4}$ 32,4 kW

4/ Real helium

Adiabatic reversible $\rightarrow \Delta Q = 0 \rightarrow \Delta S = 0$ isentropic

$\Delta H + \cancel{\Delta K} = \cancel{\Delta Q} + \Delta W$

The compression work is equal to the enthalpy difference, at constant entropy.

$\left\{ \begin{array}{ll} h(16.4\,mb, 4\,K) = 26.00\ kJ/kg & s(16.4\,mb, 4\,K) = 14.13\ kJ/kg\,K \\ h(16.4\,mb, 290\,K) = 1511\ kJ/kg & s(16.4\,mb, 290\,K) = 36.39\ \text{" "} \end{array} \right.$

Hold s constant $\Rightarrow$. $h(1\,bar, s = 14.13) = 112.46\ kJ/kg$

Corresponding temperature 20.8 K

. $h(1\,bar, s = 36.39) = 7804\ kJ/kg$

Corresponding temperature 1502 K

Hence, with inlet @ 4K $\quad \Delta h = 112.46 - 26.00 = 86.46\ kJ/kg$

and $\dot{W} = 5.14 \times 86.46 = 444\ W$

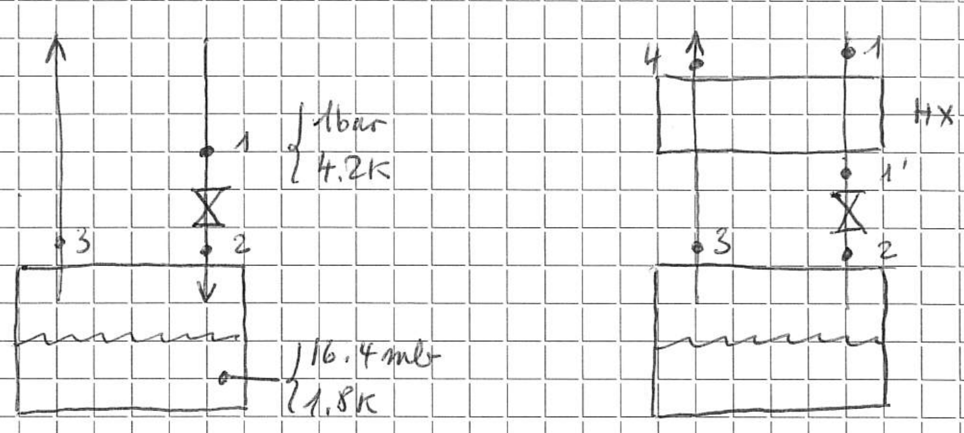
With inlet @ 290K, $\quad \Delta h = 7804 - 1511 = 6293\ kJ/kg$

and $\dot{W} = \dot{m}\,\Delta h = 5.14 \times 6293 = 32\,346\ W$

$\Rightarrow$ the ideal gas approximation is fully justified.

4

Problem ~~3~~ = Saturated superfluid by J-T expansion



1/ Saturation pressure of He at 1.8K = 1638.41 Pa

$\simeq$ 16.4 mbar

2/ Adiabatic Expansion without production of motor work ≡ isenthalpic

$$\Delta U + \underbrace{\cancel{\Delta K}}_{\text{negligible}} = \underbrace{\Delta Q}_{=0} + \Delta W \qquad \Delta W = -\Delta(PV)$$

$$\Rightarrow \Delta(U + PV) = 0$$

3/
$$\begin{cases} h\,(1\text{ bar},\ 4.2K) = 9902 \ \ J/kg \\ h_L\,(\text{sat},\ 1.8K) = 842 \ \ J/kg \\ h_V\,(\text{sat},\ 1.8K) = 24201 \ \ J/kg \end{cases}$$

Conservation of enthalpy can be written

$$h\,(1\text{ bar},\ 4.2K) = x\, h_V\,(\text{sat},\ 1.8K) + (1-x)\, h_L\,(\text{sat},\ 1.8K)$$

with x = vapour fraction produced in the expansion.

Hence $x = 0,39$

$$\begin{cases} 39\% \text{ vaporized} \\ 61\% \text{ remaining liquid} \end{cases}$$

4/ - In the high-pressure stream of the HX, the liquid can be subcooled not lower than the lambda point: the part of this stream which would contain superfluid would be a thermal short circuit.

Hence, $T(1') = 2.18\ K$

- The temperature of the returning saturated vapor is by definition 1,8K

Hence, $T(3) = 1,8K$.

- $\begin{cases} h(1\ bar, 4,2K) - h(1\ bar, 2,18\ K) = \Delta h_{HP} \\ h(16\ mb, T4) - h(16\ mb, 1.8\ K) = \Delta h_{LP} \end{cases}$

$\Delta h_{HP} = 9902 - 3634 = 6268\ J/kg$

$\Delta h_{LP} = \Delta h_{HP}$

$h(16\ mb, T4) = 6268 + 24201 = 30469\ J/kg$

Hence $T4 \simeq 3,0\ K$.

- Fraction of vapor produced in expansion x

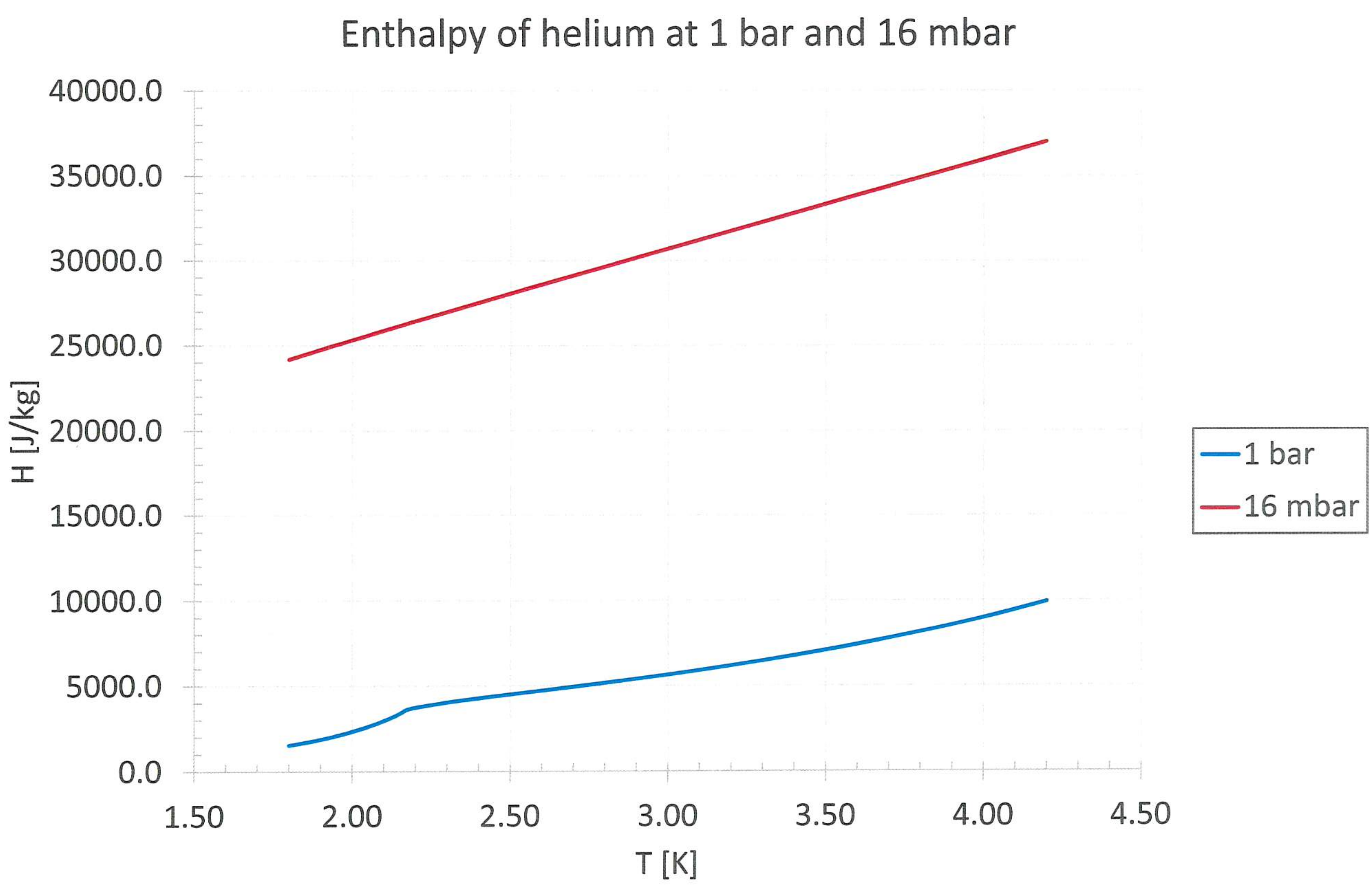
$h(1\ bar, 2,18\ K) = x\, h_V(16.4\ mb, 1,8K) + (1-x)\, h_L(16.4\ mb, 1.8K)$

$3634 = x \times 24201 + (1-x) \times 842$

Hence $x = 0,12$ $\begin{cases} 12\% \text{ vaporized} \\ 88\% \text{ remaining liquid} \end{cases}$

5/ Challenges of the HX

- High efficiency (at small T differences between
- HP and LP streams)
- Very low pressure drop on the LP stream, typically < 1 mbar.


Enthalpy of helium at 1 bar and 16 mbar
40000.0
35000.0
30000.0
25000.0
20000.0
15000.0
10000.0
5000.0
0.0
H [J/kg]
1.50
2.00
2.50
3.00
3.50
4.00
4.50
T [K]
1 bar
16 mbar