Combining QED and QCD transverse-momentum resummation for electroweak boson production at hadron colliders

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Drell-Yan mechanism is a benchmark at hadron colliders and is measured with an astonishing experimental precision

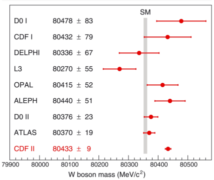
) Need of **accurate theoretical predictions**) **higher-order radiative corrections**

Distribution in q_T, in the framework of RESUMMATION FORMALISM (G. Bozzi, S. Catani, D. de Florian, M. Grazzini [hep-ph/0508068]]), are known at the high-precision level of N3LL+N3LO in QCD (S. Camarda, L. Cieri, G. Ferrera [hep-ph/2103.04974 [hep-ph]]))

EW corrections must be taken into account : $^{2}_{S}$

QED correction NLL+NLO was computed only for on-shell Z production (L. Cieri, G. Ferrera, G.F.R. Sborlini [hep-ph/1805.11948 [hep-ph]]) ! We extend the formalism to reach NLL_{QED} + NLO_{EW} accuracy for both charged and neutral current on-shell Drell-Yan processes

The last extremely precise extraction of M_W at CDFII detector shows a substantial tension with SM prediction and past M_W measurements (ATLAS, DELPHI, OPAL, ...)



CDF collaboration: High-precision measurement of the W boson mass with the CDF II detector 10.1126/science.abk1781

theoretical studies should be directed to relevant kinematic distribution (e.g. $p_T\,(W{=}Z)$, ...)

EW corrections could give O (MeV) shift of M_W value extraction

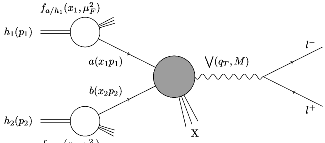
understand deeply the tension (underestimation of uncertainties, BSM effects, extension of the theory...)

Drell-Yan q_T distribution

$$\frac{d\ \ _V}{dq_T^2}(q_T; M; s) \overset{\text{factorization theorem}}{=}$$

$$= \sum_{a;b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a=h_1}(x_1; \ _F^2) f_{b=h_2}(x_2; \ _F^2)$$

$$\frac{d\hat{\ }_{V_{ab}}}{dq_T^2}\ \ q_T; M; \hat{s}; \ _S(\ _R^2); \ _R^2; \ _F^2$$



In the region $q_T \ ^> M_V$ the perturbative fixed-order expansion is reliable:

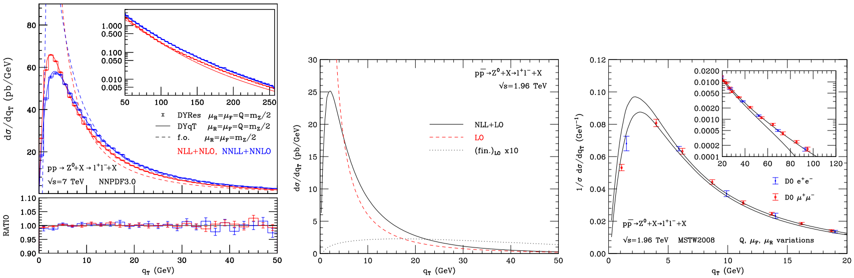
$$\frac{d\hat{\ }_{V_{ab}}}{dq_T^2} = \frac{d\hat{\ }^{(0)}_{V_{ab}}}{dq_T^2} + \frac{\ _S}{\ } \frac{d\hat{\ }^{(1)}_{V_{ab}}}{dq_T^2} + \frac{\ _S}{\ }^2 \frac{d\hat{\ }^{(2)}_{V_{ab}}}{dq_T^2} + O\ \frac{\ _S}{\ }^3$$

In the region $q_T << M_V$ (bulk of the events) large logarithmic corrections of the type $\ _S^n \ln^m(M_V^2 = q_T^2)$, due to soft and/or collinear parton radiations, spoils the convergence

+

Resummation at all perturbative orders is mandatory:

$$\frac{d\hat{\ }_{V_{ab}}}{dq_T^2} = \frac{\hat{\ }^{(0)}_V}{q_T^2} \sum_{n=1}^{+1} \sum_{m=0}^{2n\ 1} A^V_{n;m} ln^m \frac{M^2}{q_T^2}\ \ _S^n(M^2); \quad \ _S^n \ln^m(M_V^2 = q_T^2) >> 1$$



G. Bozzi, S. Catani, G. Ferrera, D. de Florian, M. Grazzini [1007.2351[hep-ph]], [1507.06937[hep-ph]]

Partonic cross section is explicitly splitted as:

$$\frac{d\hat\sigma}{dq_T^2} = \frac{d\hat\sigma^{res}}{dq_T^2} + \frac{d\hat\sigma^{fin}}{dq_T^2}; \quad \text{with} \quad \lim_{Q_T \to 0} \int_0^{Q_T} dq_T^2 \, \frac{d\hat\sigma^{fin.}}{dq_T^2} = 0$$

Resummation is performed in impact parameter ($\mathbf{b}$) space

$$\frac{d\hat\sigma^{res.}_{a_1a_2\to V}}{dq_T^2}(q_T; M, \hat s; \alpha_S(\mu_R^2), \mu_F^2, \mu_R^2) = \frac{M^2}{\hat s} \int_0^\infty db \, \frac{b}{2} J_0(bq_T) \, W^V_{a_1a_2}(b; M, \hat s; \alpha_S(\mu_R^2), \mu_F^2, \mu_R^2).$$

The resummed form factor $W^V_{a_1a_2}$ can be expressed in an exponential and factorized form in the Mellin space ! $z = M_V^2/\hat{s}$, $f_N = \int_0^1 dz\, z^{N-1} f(z)$:

$$W^V_{N;a_1a_2} = H_N(M_W;\ _S(\ ^2_R))\ \ \exp G_N(\ _S(\ ^2_R); L; M^2/\ ^2_R; M^2/Q^2);$$

$$L = \ln\left(\frac{Q^2b^2}{b_0^2}+1\right);\ b_0 = 2exp(\ _E);\ \ _E = 0.5772...$$

Hard collinear coefficient function $H^V_{N;a_1a_2}$

includes finite terms (e.g. virtual emission)

process dependent

$$H_{N;a_1a_2}\left(M;\ _S; \frac{M^2}{\ ^2_R}; \frac{M^2}{\ ^2_F}; \frac{M^2}{Q^2}\right) = \hat{\ }^V_{0;a_1a_2}(M)\left(1 + \sum_{n=1}^{\infty}\left(\frac{\ _S}{\ }\right)^n H_N^{(n)}\left(\frac{M^2}{\ ^2_R}; \frac{M^2}{\ ^2_F}; \frac{M^2}{Q^2}\right)\right)$$

Sudakov form factor G_N

contains the logarithmic enhanced contributions

universal structure

$$G^V_{N;a_1a_2}\left(\ _S(\ ^2_R); L; \frac{M^2}{\ ^2_R}; \frac{M^2}{Q^2}\right) = Lg_N^{(1)}(\ _S(\ ^2_R)L) + g_N^{(2)}\left(\ _S(\ ^2_R)L; \frac{M^2}{\ ^2_R}; \frac{M^2}{Q^2}\right) +$$

$$+ \frac{\ _S(\ ^2_R)}{\ } g_N^{(3)}\left(\ _S(\ ^2_R); \frac{M^2}{\ ^2_R}; \frac{M^2}{Q^2}\right) + \sum_{n=4}^{\infty}\left(\frac{\ _S}{\ }\right)^{n-2} g_N^{(n)}\left(\ _S(\ ^2_R)L; \frac{M^2}{\ ^2_R}; \frac{M^2}{Q^2}\right) :$$

$$+$$

Perturbative structure of the resummed component ! LL accuracy ($^n_S L^{n+1}$): $g_N^{(1)}$; NLL accuracy ($^N_S L^n$): $g_N^{(2)}$, $H_N^{(1)}$; NNLL accuracy: ($^N_S L^{n-1}$): $g_N^{(3)}$, $H_N^{(2)}$; N3LL accuracy ($^N_S L^{n-2}$): $g_N^{(4)}$, $H_N^{(3)}$

On-shell Z boson production

- QED corrections at NLL+NLO known
  - [L. Cieri, G. Ferrera, G. F. R. Sborlini 1805.11948[hep-ph]]
- We incorporated also weak corrections at one loop within the hard factor H
  - We consider one-loop renormalized form factor
  - Modifications only in the hard factor H (massive loop corrections)

On-shell W boson production (NEW)

- Charged final state ! a "naive abelianization" of QCD formulas for DY process is not suitable
- We use the formalism of tt production

Replacement: tt ! W (colour charged ! electrically charged)

Abelianization of Casimir operators: $\mathbf{T_i}$! e_i (S. Catani, S.Dittmaier, Zoltan Trocsany [9802439[hep-ph]], [0011222[hep-ph]])

We start from the formalism of $t\bar{t}$ resummation programme (**S. Catani, M. Grazzini, A. Torre 1408.4564[hep-ph]**):

$$W_N^V(b;M) = \sum_{ca_1a_2} \quad {}^{(0)}_{c\bar{c};V}(\ _S(M^2)) f_{a_1=h_1;N}(b_0^2=b^2) f_{a_2=h_2;N}(b_0^2=b^2) \quad S_c(M;b) \quad [(\mathbf{H}^V \boldsymbol{\Delta} C_1C_2)]_{c\bar{c};a_1;a_2;N}(M^2;b_0^2=b^2)]$$

$[(\mathbf{H}^V \quad C_1C_2)]$ hard factor; S_c Sudakov form factor
related to soft wide-angle radiation from final state ($\quad = 1$ for neutral final states)

$$(\mathbf{b};M; \quad ;y_{34}; \quad _3) = \mathbf{V}^y(\mathbf{b};M;\gamma;y_{34}; \quad _3)\mathbf{D}(\ _S(b_0^2=b^2);y_{34}; \quad _3)\mathbf{V}(\mathbf{b};M; \quad ;y_{34}; \quad _3)$$

V evolution operator
D color operator: resums logarithms from final-state emission
explicit expressions: **1408.4564[hep-ph]**

We can drop charge correlations involving intial and final state (abelian limit) ! **V** is diagonal ! commutation

We are only interested on **D** at the lowest order

$$\mathbf{D}(\ _S;y_{34}; \quad _3) = 1 + \frac{\ _S}{\ }\mathbf{D}^{(1)}(y_{34}; \quad _3) + O(\ _S^2)$$

$$\mathbf{D}^{(1)}(y_{34}; \quad _3) = (\mathbf{T}_3^2 + \mathbf{T}_4^2)d_a(y_{34}; \quad _3) \quad (\mathbf{T}_3^2 + \mathbf{T}_4^2)^2 d_b(y_{34}; \quad _3) + \frac{1}{2v}\mathbf{T}_3 \quad \mathbf{T}_4\, d_c(y_{34}; \quad _3)\ ; \text{for } t\bar{t}$$

Replacements and simplifications: $\mathbf{T}_3$! e_W, $\ _S$! $\quad ; y_{34} = \quad _3 = 0 = \mathbf{T}_4$, $d_b(0;0) = \frac{1}{2}$,

$\mathbf{D}$! $D = 1 + -\frac{e_W}{2} + O(\ ^2)$ (diagonal)

Sudakov form factor

The coefficient D can be absorbed in the colourless Sudakov form factor

In combined QCD-QED resummation formalism, we finally obtain the following generalization:

$$S_{cc'}(M;b) = \exp \int_{b_0^2}^{b^2} \frac{dq^2}{q^2} \; A_{c;c'}(\;_S(q^2); \;(q^2)) log \frac{M^2}{Q^2} +$$

$$B_{c;c'}(\;_S(q^2); \;(q^2)) + D_W(\;_S(q^2); \;(q^2))$$

A and B are obtainable from the QCD analogous of 0508068[hep-ph] by making an abelianization of Casimir operators, which leads to

$$2\,C_F \;\to\; (e_q^2 + e_{q'}^2)$$

D is instead an additional term, due to a charged final state, characteristic of final-state massive radiation

it generates (only) single-log terms (emission from massive leg: only-soft singularity!)

$$D^{(1)} = \frac{eW^2}{2}$$

Observation: the additional resummed contribution **implies** the replacement $B_1 \to B_1 + D_1$ in all the parts of the original formalism (, H, S)

Hard collinear coefficient function

We started from tt subtraction operator of **1408.4564[hep-ph]**, transforming it properly

The one-loop virtual renormalized form factor was included in H

- This calculation reproduces the logarithmic structure of the xed-order expansion of the Sudakov form factor
  - Cross-check of our formulas
  - Con rm the validity of the replacement tt → W, Z and abelianization procedures

Hadronic cross section: $\sigma = \sum_{ab} \int_{\tau}^{1} \frac{dz}{z} L_{ab}\left(\frac{\tau}{z}\right) \frac{1}{z} \int dq_T^2 \frac{d\hat{\sigma}_{ab}(q_T;z)}{dq_T^2}$

Partonic inclusive cross section: $\hat{\sigma}_{ab}(z) = \int_{(q_T^{cut})^2}^{(q_T^{max})^2} dq_T^2 \frac{d\hat{\sigma}_{ab}(q_T;z)}{dq_T^2}$;

Inclusive hadronic cross section, introducing $f(a) = 2\sqrt{a}(\sqrt{1+a} - \sqrt{a})$; $a \equiv \frac{q_T^{(2)}}{Q^2}$:

$$\sigma_{ud}^{(1)} = \int_{\tau}^{1-f(a)} \frac{dz}{z} L_{ud}\left(\frac{\tau}{z}\right) \frac{1}{z} \hat{\sigma}_{ud}^{(1)}(z) = \int_{\tau}^{1} \frac{dz}{z} L_{ud}\left(\frac{\tau}{z}\right) \hat{\sigma}^{(0)} \hat{G}_{ud}^{(1)}(z); \text{ with :}$$

$$\hat{G}_{ud} = \sum_{m;r} \log^m(a) a^{\frac{r}{2}} \hat{G}_{ud}^{(1;m;r)}(z);$$ power series in the cuto

Final expressionobtained:

$$\hat{G}^1_{ud} = \log(a) \quad \frac{3}{2} \; (1 \quad z) \frac{e_D^2 + e_U^2}{2} \quad \frac{1}{2} \quad P^{QED}_{\;\;dd} + e_W^2 \; (1 \quad z) \quad P^{QED}_{\;\;uu} \quad ! \; +$$

$$+ \frac{1}{2}\log^2(a) \; (1 \quad z) \frac{e_D^2 + e_U^2}{2} + {}^{p}\bar{a} \, \frac{1}{2} e_W^2 \; 2 \quad {}^{0}(1 \quad z) \quad 3 \quad (1 \quad z)$$

$$+ \text{ nite terms } + \text{ higher order terms}$$

P^{QED}_{qq} AP splitting functions in QED (D. de Florian, G. Rodrigo, G. F. R. Sborlini: 1611.04785[hep-ph]], 1512.00612[hep-ph], 1606.02887[hep-ph])

We reproduce the known A and B perturbative coe cient of the QCD resummation formalism, modulo C_F ! $\frac{e_U^2 + e_D^2}{2}$

Additional logarithmic divergence from the charged nal state $D_1 \log(a)$, $D_1 = \frac{e_W^2}{2}$

A linear power correction in the cuto (${}^{p}\bar{a}$) and proportional to the charged nal state is present

Accordingly with L. Buonocore, M. Grazzini, F. Tramontano: 1911.10166 [hep-ph] (massive leg emission ! linear power correction)

Numerical Results at hadron colliders

Code: DYqT G. Bozzi, S. Catani, G. Ferrera, D. de Florian, M. Grazzini, [1007.2351 [hep-ph]], [1007.2351 [hep-ph]], [0812.2862 [hep-ph]]

QED corrections: e ects at percent and per-thousand level

Scale variation band:

- Roughly (2:5%   4%) at LL and (1%   1:5%) at NLL+NLO
- Inclusion of NLL+NLO reduces scale variation band (roughly 1:5%   3%)
  - reduction of $^{0}_{R}$ ambiguity of

- QED corrections: e ects at percent and per-thousand level (slightly greater than the W case)
- Scale variation band:
  - Z : roughly (1% 5:5%) at LL and (2%) at NLL+NLO
  - Inclusion of NLL+NLO reduces scale variation band (roughly 1:5% 3%)
    - reduction of $^{0}_{R}$ ambiguity of
- Di erent q_T dependence at NLL+NLO (W , Z)
  - radiation from massive nal state
- Z boson: greater band-sizes

$$R(q_T) = \frac{\frac{d\;^{W}}{dq_T^2}\,\frac{1}{W}}{\frac{d\;^{Z}}{dq_T^2}\,\frac{1}{Z}};$$

LL:QED contributions and uncertainties simpli es in the ratio

- similar structure of leading logs terms
- QED impact at per thousand level
- scale variation band quite constant at $O(0{:}2\%)$

NLL+NLO : QED contributions and uncertainties do not simpli es in the ratio

- modi cation of sub-leading logs from massive charged legs
- QED impact up to percent level
- scale variation band growing with q_T and up to percent level

- Impact of EW corrections slightly smaller than Tevatron case
  - Suppression ofqq channel
  - Enhancement of gluon-induced reactions

Scale variation band:

- W : roughly (2:5%   3%) at LL and (1%) at NLL+NLO

- Impact of EW corrections slightly smaller than Tevatron case
    - Suppression ofqq channel
    - Enhancement of gluon-induced reactions

Scale variation band:

- Z: roughly (2:5% 3%) at LL and (1:5%) at NLL+NLO

- At LL, QED contributions and scale variation band (less than per thousand level) almost vanishing
- At NLL+NLO QED e ects and scale-variation band up to $O(0{:}8\%)$
- The suppression of $q\bar{q}$ channel reduce the impact of photonic radiation in comparison to the Tevatron case

- To fully exploit the potential of LHC measurements accurate theoretical predictions are required ! precise determination of SM parameters (m_W)
- We considered QED corrections to resummation formalism in QCD, to properly include photon radiation, focusing on on-shell W boson production, which has to be treat carefully due to a charged nal state
- Final state radiation is fully included by extending the resummation formalism for a coloured nal state ($t t$)
- Expansion at small q_T of the real inclusive cross section has con rmed the validity of the replacement $t t$! W
- Through the use of the numerical code DYqT we presented numerical predictions at $(\mathrm{NNLL} + \mathrm{NNLO})_{QCD} + (\mathrm{NLL})_{QED} + (\mathrm{NLO})_{EW}$, nding QED e ects from per thousand to percent level
- We considered also the ratio distribution $p_T(W)=p_T(Z)$, in the direction of m_W extraction ! a sizeable reduction of scale-variation band is observed at LL, while the predictions at NLL + NLO do not bene t from the cancellation of common uncertainties
- A natural extension of this work is the inclusion of the decay of weak boson and the radiation from leptonic nal state (QED parton shower programme, e.g. photos)

Thank you for the attention!!

BACKUP SLIDE

FCC-hh, $\sqrt{s} = 100\,\text{TeV}$

- enhacement of gluon PDF
- Larger suppression of QED effects (Tevatron, LHC at 13 TeV)
- Large uncertainties in $R(q_T)$ distribution

at FCC-hh PDF extrapolation is challenging (out of experimental accessible range)

Matching procedure

For intemerdiate q_T values, the resummed component should be properly combined with fixed order expansion ! matching procedure:

- recover fixed-order series for $q_T \ ^{<} M_V$ where $\frac{d \quad ab}{dq_T^2} \quad {}_{res:} \; ! \; 0$
- avoid double counting of logarithmic terms ! counterterm $\frac{d \quad ab}{dq_T^2} \quad {}_{asym} :$

The finite part is $\frac{d \quad ab}{dq_T^2} \quad {}_{fin:} = \frac{d \quad ab}{dq_T^2} \quad {}_{f:o:} \qquad \frac{d \quad {}^{res}_{ab}}{dq_T^2} \quad {}_{f:o:} = \frac{d \quad ab}{dq_T^2} \quad {}_{f:o:} \qquad \frac{d \quad ab}{dq_T^2} \quad {}_{asym}$

The counterterm, in impact parameter and Mellin space, is given by the expansion:

$$H^{V}_{a_1 a_2;N} \quad \exp\left(G_{a_1 a_2;N}\right) \qquad {}^{(0)}_{c\bar{c};V}(\quad {}_S; M) \quad {}_{ca_1} \quad {}_{\bar{c}a_2} \; (1 \quad z)$$

$$+ \sum_k \frac{\ \ _S}{\ } {}^{k} \quad {}^{V;(k)}_{c\bar{c}} \quad {}_{a_1 a_2}(z; L; M^2 = \ {}^2_R; M^2 = \ {}^2_F m M^2 = Q^2)$$

$$+ \sum_k \frac{\ \ _S}{\ } {}^{k} \quad H_{c\bar{c}} \quad {}_{a_1 a_2}(z; M^2 = \ {}^2_R; M^2 = \ {}^2_F; M^2 = Q^2)$$

Coloured (charged) final state S. Catani, M. Grazzini, A. Torre, [1408.4564 [hep-ph]]

Drell-Yan final state is coulourless

In case of charged final state (e.g. tt) production the hadronic resummed contribution is generalized according to:

$$W_N^V(b;M) = \sum_{c a_1 a_2} {}^{(0)}_{cc;V}(\ \ _S(M^2)) f_{a_1=h_1;N}(b_0^2=b^2) f_{a_2=h_2;N}(b_0^2=b^2)$$

$$S_c(M;b) \quad [(\mathbf{H}^V \quad C_1 C_2)]_{cc;a_1;a_2;N}(M^2; b_0^2=b^2)]$$

$[(\mathbf{H}^V \quad C_1 C_2)]$ hard factor; S_c Sudakov form factor
related to soft wide-angle radiation from final state (= 1 for colourless final states)

We extend this formalism for electrically charged emission in the proceeding of the talk