Soft Logs in Processes with Heavy Quarks

Andrea Ghira

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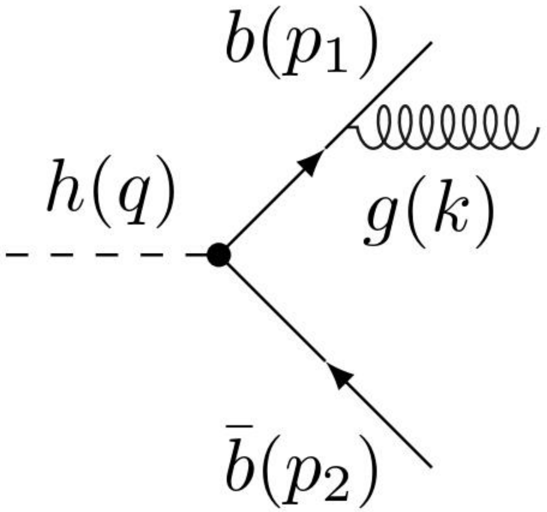
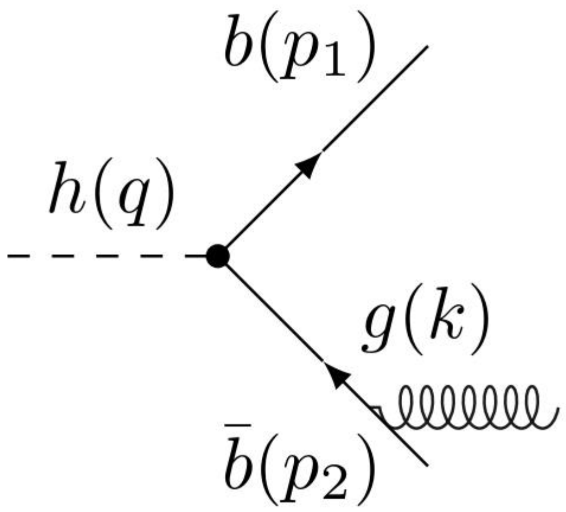
Based on arXiv:2207.13567 **in collaboration with D. Gaggero, S.Marzani and G.Ridolfi**

Introduction

We consider heavy flavour production from the decay of a Higgs boson (colour singlet)





We want to compute the differential decay rate over $x = \frac{2p_1 \cdot q}{q^2} = \frac{2E_b}{\sqrt{q^2}}$ which tends to 1 in the limit $k \to 0$

Massless vs Massive Scheme Approach

Massless Scheme:

- Quark mass used as a regulator
- Cross section computed as a convolution of a coefficient function times a fragmentation function
- Logs of $\xi = \frac{m^2}{q^2}$ resummed through DGLAP

Massive Scheme:

- All mass dependence taken into account
- Kinematics treated correctly at every order
- Large logs spoil the convergence of the series.

What is our goal?

Matching resummed scheme with fixed order calculations gives better prediction in the study of differential decay rate in various regions of ξ:

$$\tilde{\Gamma}(N,\xi) = \underbrace{\tilde{\Gamma}^{(\mathrm{F.O.})}(N,\xi)}_{\xi=\mathcal{O}(1)} + \underbrace{\tilde{\Gamma}^{(\mathrm{RES},\xi)}(N,\xi)}_{\xi\ll 1} - \mathrm{d.c}$$

- This is the FONLL scheme, where we match massive calculation with the resummed one

- We want to resum also soft logs in both schemes at NLL $\longrightarrow$ **FO(NLL)^2**

Problems with Merging

We want to merge the two different calculations of the differential decay rate resumming logs of N in the large N limit.

Massless Scheme

- Double logs of N with mass independent coefficient (arxiv:0107138, arxiv:2207.10038).

Massive Scheme

- Single logs of N with mass dependent coefficient
- If we perform the limit $\xi \to 0$ after the large N limit, we do not recover the massless case

Different logarithmic structure of the bremsstrahlung radiation in the two cases, in the soft limit

Example at Fixed order

If we compute the process with one emission at fixed order in the small mass and soft limit we will find: (see also arxiv:0311101v1)

Massless Scheme

Performing the massless limit first :

$$\lim_{x\to 1}\lim_{\xi\to 0}\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = -\frac{\alpha_\mathrm{s} C_\mathrm{F}}{\pi}\left[\frac{\log\xi}{1-x} + \frac{\log(1-x)}{1-x} + \frac{7}{4}\frac{1}{1-x} + \ldots\right]$$

Massive Scheme

Performing the soft limit first

$$\lim_{\xi\to 0}\lim_{x\to 1}\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = -\frac{2\alpha_\mathrm{s} C_\mathrm{F}}{\pi}\left[\frac{1+\log\xi}{1-x} + \ldots\right]$$

The two limits do not commute, comparing the accuracy of the soft logs can be confusing

Another Problem with Merging

The soft resummation formula in the massive scheme is the product of a coefficient function times a soft function:

$$\widetilde{\Gamma}(N,\xi) = C(\xi,\alpha_{\rm s}) e^{-2\int_0^1 {\rm d}x \frac{x^{N-1}-1}{1-x}\gamma_{\rm soft}\left(\beta,\alpha_{\rm s}\left((1-x)^2 q^2\right)\right)}$$

where $\gamma_{\rm soft}(\beta,\alpha_{\rm s})$ is the soft anomalous dimension.

If we perform the massless limit of the first order coefficient $C^{(1)}(\xi)$

we find:

$$C^{(1)}(\xi) = C_{\rm F}\left(\frac{1}{2}\log^2\xi + \log\xi + \frac{\pi^2}{2} + \mathcal{O}(\xi)\right)$$

This term is not predicted by DGLAP!

Our Strategy

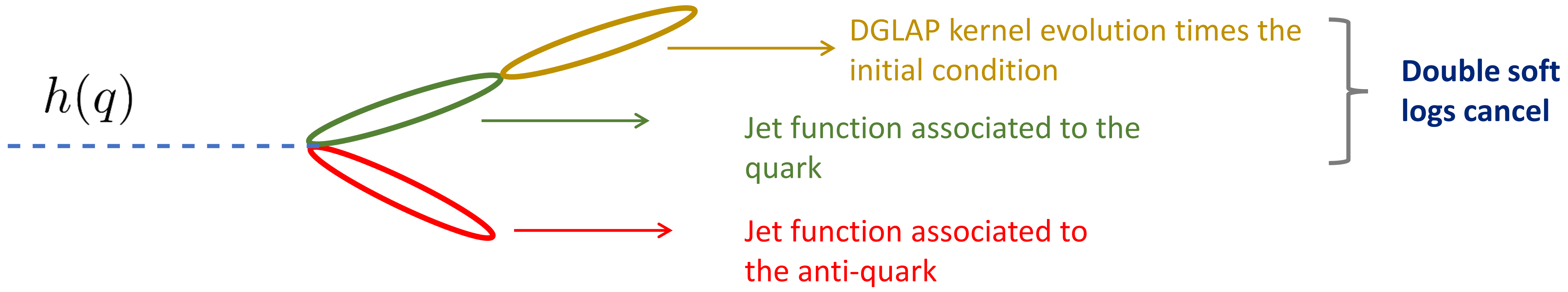
In the same spirit of FONLL, we would like to define a matching scheme:

$$\tilde{\Gamma}(N,\xi) = \underbrace{\tilde{\Gamma}^{(\mathrm{F.O.})}(N,\xi)}_{\xi=\mathcal{O}(1) \text{ and } N=\mathcal{O}(1)} + \underbrace{\tilde{\Gamma}^{(\mathrm{RES\ soft})}(N,\xi)}_{\xi=\mathcal{O}(1) \text{ and } N\gg 1} + \underbrace{\tilde{\Gamma}^{(\mathrm{RES\ soft},\xi)}(N,\xi)}_{\xi\ll 1 \text{ and } N\gg 1} - \mathrm{d.c}$$

- The problem is that we cannot identify an all-order subtraction term.
- We modify the resummed massless scheme expression in order to behave like the massive expression if $\frac{1}{N} \ll \xi \ll 1$

The origin of the Problem

At NLL in the FF calculation one can see the resummation as a product of two independent jet functions:



In the measured leg the double logarithmic structure cancel between the quark jet function and the fragmentation function.

Towards a Solution

Only the recoil jet function exhibit double soft logs, this is the function to modify:

- We can modify it function so that:
  1. When $\xi \ll (1-x) \ll 1$ we recover Cacciari-Catani formula (arxiv:0107138).
  2. When $(1-x) \ll \xi \ll 1$ we recover the massive case in the small mass limit.

This is achieved by computing the recoil jet function in the quasi-collinear limit.

Calculation in the quasi-collinear limit

In the quasi-collinear limit we keep $\xi \simeq \theta^2 \ll 1$. Setting $\rho = \frac{1}{N}$

$$J_{\bar{b}} = \int_0^1 \mathrm{d}x \int_0^1 \frac{\mathrm{d}\theta^2}{\theta^2 + \xi} P_{qq}^{(m)}(x) \frac{\alpha_s((1-x)^2\theta^2 q^2)}{\pi} \left[\Theta\Big((1-x)(\theta^2 + \xi) < \rho\Big) - 1\right]$$

Massive splitting function

Massive contribution to the jet mass

At fixed coupling we obtain:

$$\rho > \xi : \qquad J_{\bar{b}} = -\frac{1}{2}\log^2\frac{1}{\rho} + \frac{3}{4}\log\frac{1}{\rho} + \mathcal{O}(\xi^0, \rho^0),$$

$$\rho < \xi : \qquad J_{\bar{b}} = \frac{1}{2}\log^2\frac{1}{\xi} - \log\frac{1}{\rho}\log\frac{1}{\xi} + \frac{3}{4}\log\frac{1}{\xi} + \log\frac{\xi}{\rho} + \mathcal{O}(\xi^0, \rho^0)$$

Resummed formula

Our resummed formula looks like:

$$\log \tilde{\Gamma}(N,\xi) = \log\left(1+\alpha_s \delta C^{(1)}\right) + J_{\bar{b},\xi}\left(N, \frac{m^2}{q^2}, \frac{\mu_R^2}{q^2}, \alpha_s(\mu_R^2)\right)$$
$$+ J_b\left(N, \frac{\mu_F^2}{q^2}, \frac{\mu_R^2}{q^2}, \alpha_s(\mu_R^2)\right) + E\left(N, \frac{\mu_F^2}{\mu_{0F}^2}, \alpha_s(\mu_F^2)\right)$$
$$+ D_0\left(N, \frac{\mu_{0F}^2}{m^2}, \frac{\mu_{0R}^2}{m^2}, \alpha_s(\mu_{0R}^2)\right) - 2\int_{1/\bar{N}}^{1} \frac{dz}{z} \frac{\alpha_s(z^2 m^2)}{\pi} \tilde{\gamma}_{\rm soft}(\xi)$$

- In the first line there is the modified jet function of the $\bar{b}$ quark which accounts for the mass effects .
- In the following lines there is quark jet function plus the evolved fragmentation function.
- The last contribution is the $\mathcal{O}(\xi)$ contribution to the soft dimension.

Conclusions and Outlook

- The merging of the massive and massless calculation is far from trivial because of the fact that the massless and massive limit do not commute.
- We build a joint resummation in such a way that if we are in the regime in which $\frac{1}{N} \ll \xi \ll 1$ we recover the massive scheme resummation and if $\xi \ll \frac{1}{N} \ll 1$ we have the resummed expression by CC at NLL accuracy.

- What is the numerical impact of these finite mass corrections?
- Is it possible to extend this framework at NNLL accuracy?

Thanks for your attention