PAUL SCHERRER INSTITUT

Milan Christmas Meeting 2022

$\mu - e$ scattering at NNLO with McMule

Marco Rocco for the McMule++ Team

Paul Scherrer Institut

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- higher-order predictions and comparison with precision experiments...
- ...but not at the LHC! → focus on low-energy QED scattering processes
- theoretical background for lepton experiments (Mu3e, MUSE, MUonE...)
- all this in

McMule

Monte Carlo for MUons and other LEptons

`https://mule-tools.gitlab.io/`

◇ fully-differential Monte Carlo integrator, not an event generator (yet)

- $4.2\,\sigma$ tension between $a_\mu^{\rm EXP}$ (BNL+Fermilab) & $a_\mu^{\rm SM}$
- prime suspect: $a_\mu^{\rm SM,\,HLO}$ $\leftarrow$ experimental input for dispersive methods
- lattice results moving towards $a_\mu^{\rm EXP}$
- why not a fourth way? $\to$ MUonE [Carloni Calame et al. 15; Abbiendi et al. 16]

◇ collide muons against electrons at rest (t channel!)

◇ measure scattering angles, θ_e and θ_μ

◇ reconstruct $\Delta\alpha^{\rm had}(x<0)$ (so smooth!)

◇ apply the space-like dispersive formula

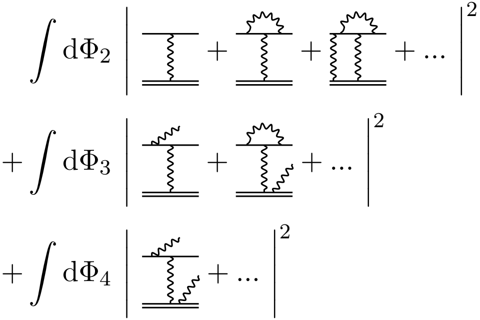
$$a_\mu^{\rm SM,\,HLO} = \frac{\alpha}{\pi}\int_0^1 {\rm d}x(1-x)\,\Delta\alpha^{\rm had}(x)$$

https://web.infn.it/MUonE/

- MUonE's signal is $\sim 10^{-3}$ $\rightarrow$ at least 10 ppm to be competitive with Fermilab
- require excellent control on background
- read: higher-order QED corrections to $e - \mu$ scattering
- probably something more: resummation, PS...

[drawings by Adrian Signer]

- steal QCD@LCH techniques
  - ◇ dim.reg. (no photon mass)
  - ◇ MIs, automation, EFT
  - ◇ subtraction method (no slicing)
  - ◇ (future) match FO w/ PS
- realise massive fermions yield simpler IR structure but harder loop amplitudes and log-enhanc.
- let the mule trot [McMule 20, 22]



- photonic & fermionic corrections
- formal leptonic charge $(q^i\, Q^j)$

1. fully-differential PS integration
   → FKS$^\ell$
2. virtual amplitudes with massive particles
   → one-loop: OpenLoops
   → two-loop: massification
3. numerical instabilities due to pseudo-singularities
   → next-to-soft stabilisation

① problem: PS integration

FKS$^\ell$ + DIMREG

1. reproduce and isolate IR behaviour from regions of the phase space where (one or more) real photons are soft:

$$\lim_{\xi\to 0} \xi^2\, \mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\, M_n^{(\ell)}$$

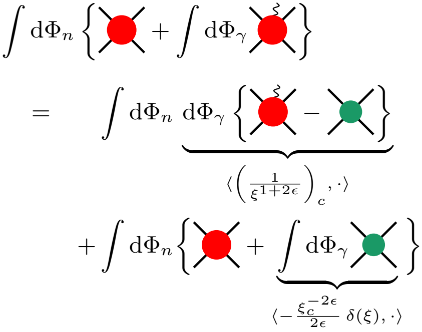
2. isolate IR-divergent behaviour from virtual amplitudes:

$$\sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = e^{-\alpha\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)\, f}$$

3. cancel analytically IR divergences and then integrate numerically in $d=4$ over the non-radiative phase space

$$\mathrm{d}\Phi_{n+r} \equiv \mathrm{d}\Phi_n \prod_{i=1}^{r} \mathrm{d}\Phi_{i,\gamma}$$

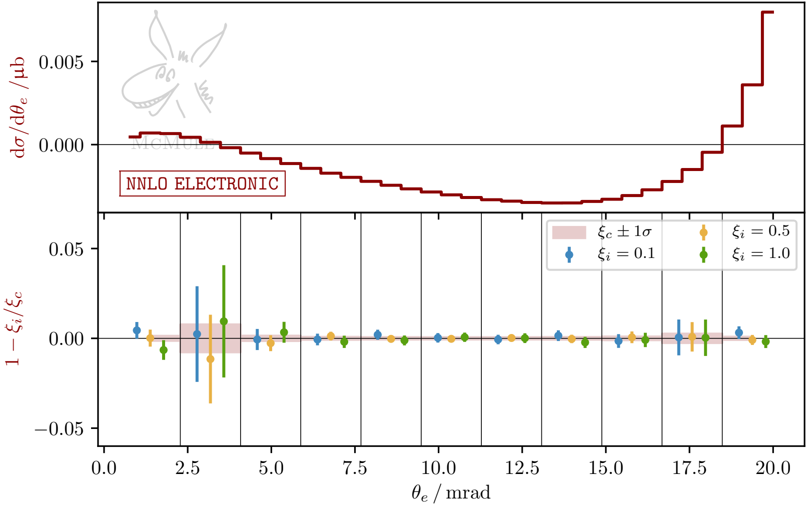
$$= \mathrm{d}\Phi_n \prod_{i=1}^{r} \mathrm{d}\Phi_{i,\gamma}^{d=4-2\epsilon}\, \xi^2\, \xi_i^{-1-2\epsilon}\, \mathrm{d}\xi_i\, \mathrm{d}\Upsilon_i^{d=4-2\epsilon}$$



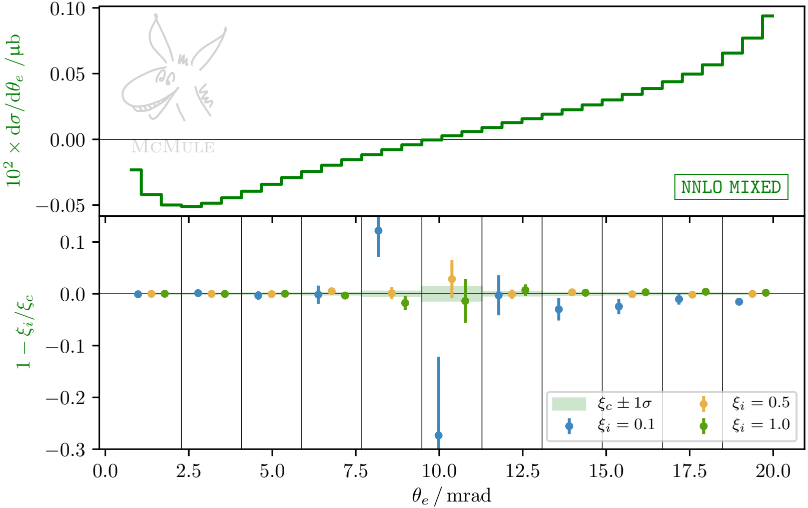
$$\begin{aligned}
\sigma^{(2)} &= \sigma^{(2)}_n(\xi_c) + \sigma^{(2)}_{n+1}(\xi_c) + \sigma^{(2)}_{n+2}(\xi_c) \\
\sigma^{(2)}_n(\xi_c) &= \int \mathrm{d}\Phi^{d=4}_n \mathcal{M}^{(2),f}_n \\
\sigma^{(2)}_{n+1}(\xi_c) &= \int \mathrm{d}\Phi^{d=4}_{n+1} \frac{1}{1!} \left(\frac{1}{\xi_1}\right)_c \xi_1\, \mathcal{M}^{(2),f}_{n+1} \\
\sigma^{(2)}_{n+2}(\xi_c) &= \int \mathrm{d}\Phi^{d=4}_{n+2} \frac{1}{2!} \left(\frac{1}{\xi_1}\right)_c \left(\frac{1}{\xi_2}\right)_c \xi_1\, \xi_2\, \mathcal{M}^{(0),f}_{n+2}
\end{aligned}$$

[Frixione, Kunszt, Signer 96; Engel, Signer, Ulrich 19]

validation: ξ_c (in)dependence for electronic corrections

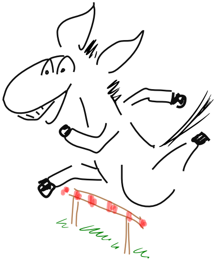


validation: ξ_c (in)dependence for mixed corrections



full muone 2-loop amplitude with $M \neq 0$, $m = 0 \rightarrow$ [Bonciani et al. 21]

full muone 2-loop amplitude with $M \neq 0$, $m \neq 0 \rightarrow$ [??]

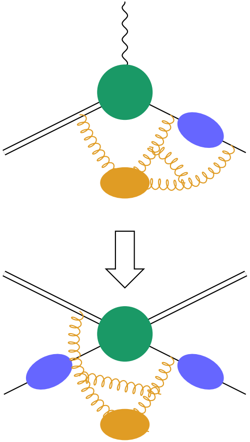


$\rightarrow$ exploit scale hierarchy $m^2 \ll M^2, Q^2$

simple process ($\mu \to e\nu\nu$ or $t \to b\ell\nu$)

- $\mathcal{A}_\mu(m) = \mathcal{S} \times Z \times \mathcal{A}_\mu(0) + \mathcal{O}(m)$
- $Z \supset \log(m)$: process indep. jet fct.
- $\mathcal{S} \supset \log(m)$: process dep. soft fct. (easy)

[Penin 06, Becher, Melnikov 07; Engel, Gnendiger, Signer, Ulrich 18]

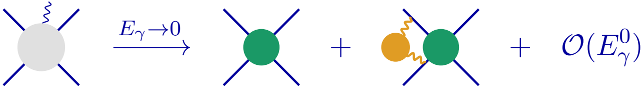


different process ($\mu e \to \mu e$)

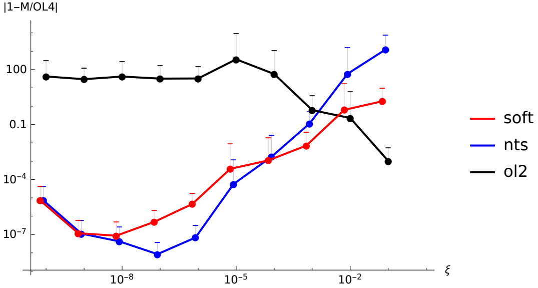
- $\mathcal{A}_{\mu e}(m) = \mathcal{S}' \times Z \times Z \times \mathcal{A}_{\mu e}(0) + \mathcal{O}(m)$

based on SCET and
method of regions as calculational tool

$\to$ massify [Bonciani et al. 21] $\to$ enhanced + constant terms real-virtual corrections 'trivial' in principle, extremely delicate numerically

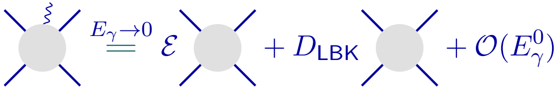


- soft limit (of collinear emission)
- OL4 $\equiv$ OpenLoops in quadruple-precision mode
- OL2 $\equiv$ OpenLoops in double/hybrid-precision mode
- OL2 vs next-to-soft limit
- stability problem solved



OpenLoops $\longrightarrow$ [Buccioni, Pozzorini, Zoller 18, Buccioni et al. 19]

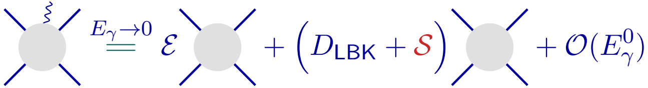
LBK theorem @ tree-level [Low 58, Burnett, Kroll 67]



LBK theorem @ one-loop [Engel, Signer, Ulrich 21]

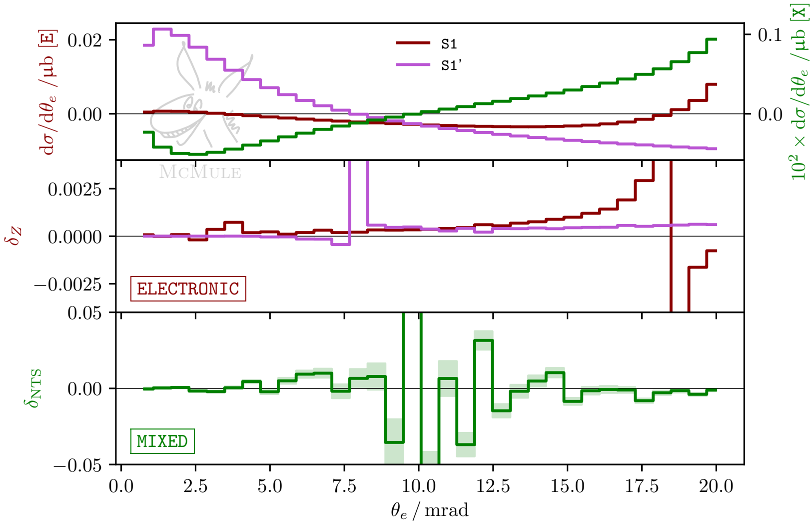
→ D_{LBK} yields hard contribution in language of MoR (HQET)

→ generic soft contribution $\mathcal{S}$

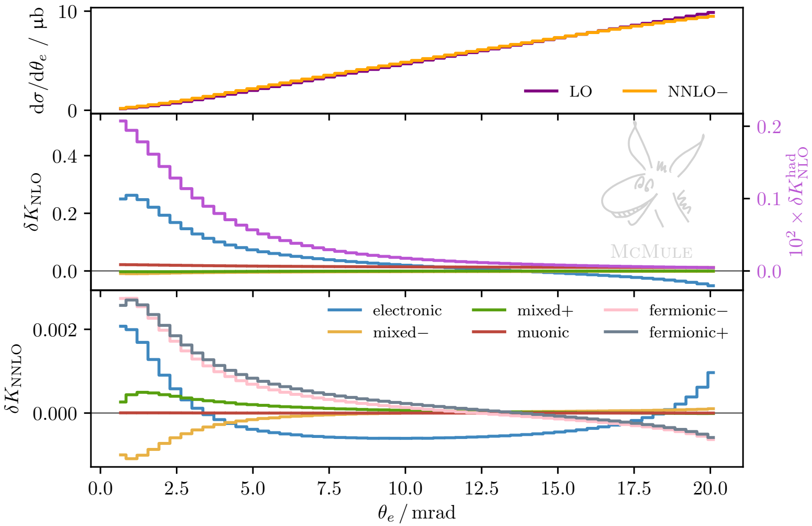


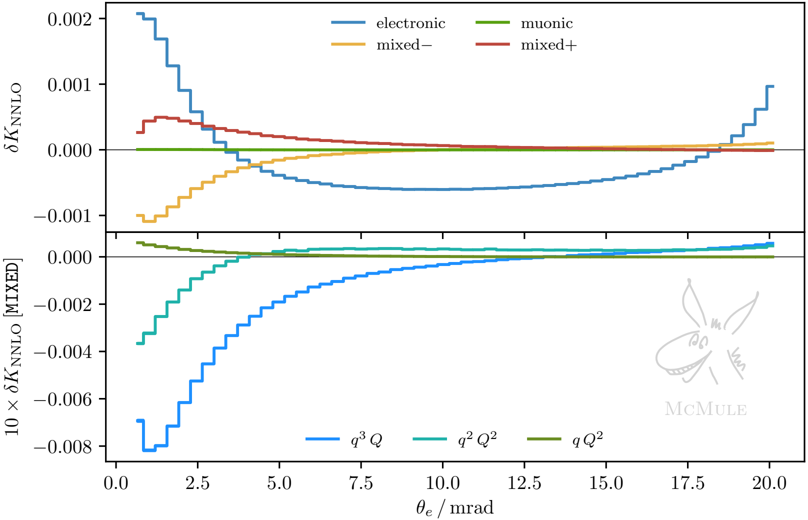
→ introduce next-to-soft stabilisation [McMule 21, 22]

validation: massification and NTS stabilisation



let the mule trot [2212.06481]




electronic
muonic
mixed−
mixed+
0.002
0.001
0.000
−0.001
δK_{NNLO}
0.000
−0.002
−0.004
−0.006
−0.008
$10 \times \delta K_{\mathrm{NNLO}}$ [MIXED]
$q^3\,Q$
$q^2\,Q^2$
$q\,Q^2$
McMule
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θ_e / mrad

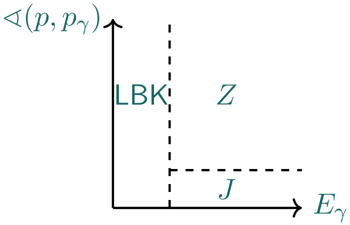
short term

- we have started thinking about $\mathrm{N^3LO}$ dominant corrections

→ subtraction scheme is ready: FKS^3

→ $\mathrm{N^3LO}$ form factor [Fael et al. 22]; higher-order massification and NTS

→



long term

- parton-shower merging to fixed-order results to cure small-angle behaviour
- STRONG 2020 Theory Workstop in Zurich – June 2023

total cross sections

	σ/μb		$\delta K^{(i)}/\%$	
	S1	S2	S1	S2
σ_0	106.44356	106.44356		
σ_1 −	106.99038(3)	102.86304(3)	0.51372(3)	−3.36377(3)
σ_1 +	107.41847(3)	103.18338(3)	0.91589(3)	−3.06283(3)
$\sigma_e^{(2)}$	0.00090	0.06595	0.00084	0.06411
$\sigma_{e\mu}^{(2)}$ −	0.00097(1)	0.01926	0.00091(1)	0.01872
$\sigma_{e\mu}^{(2)}$ +	0.00328(1)	−0.01768	0.00305(1)	−0.01713
$\sigma_\mu^{(2)}$	−0.00005	0.00002	−0.00005	0.00002
$\sigma_{\text{lep}}^{(2)}$ −	−0.01195	−0.06568	−0.01117	−0.06385
$\sigma_{\text{lep}}^{(2)}$ +	−0.00424	−0.05959	−0.00395	−0.05775
$\sigma_{\text{had}}^{(2)}$ −	−0.00045	−0.00104	−0.00042	−0.00101
$\sigma_{\text{had}}^{(2)}$ +	−0.00004	−0.00068	−0.00004	−0.00066
σ_2 −	106.97977(3)	102.88154(3)	−0.00992(4)	0.01799(4)
σ_2 +	107.41832(3)	103.19386(3)	−0.00013(4)	−0.01016(4)