2023 CAS course on “RF for Accelerators”

# RF Measurements

## – *An Introduction* –

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- ~~Part III: Scattering parameters~~
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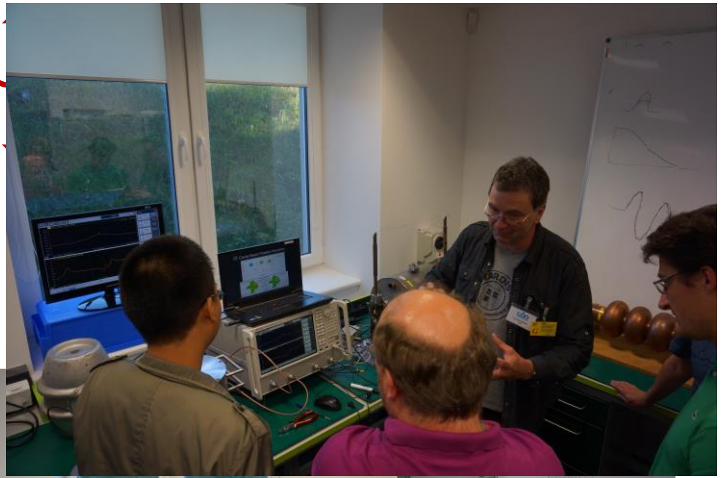
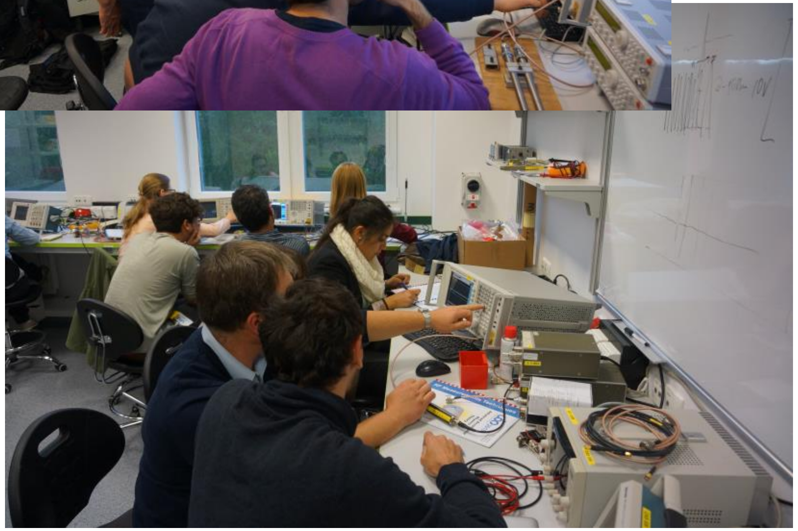
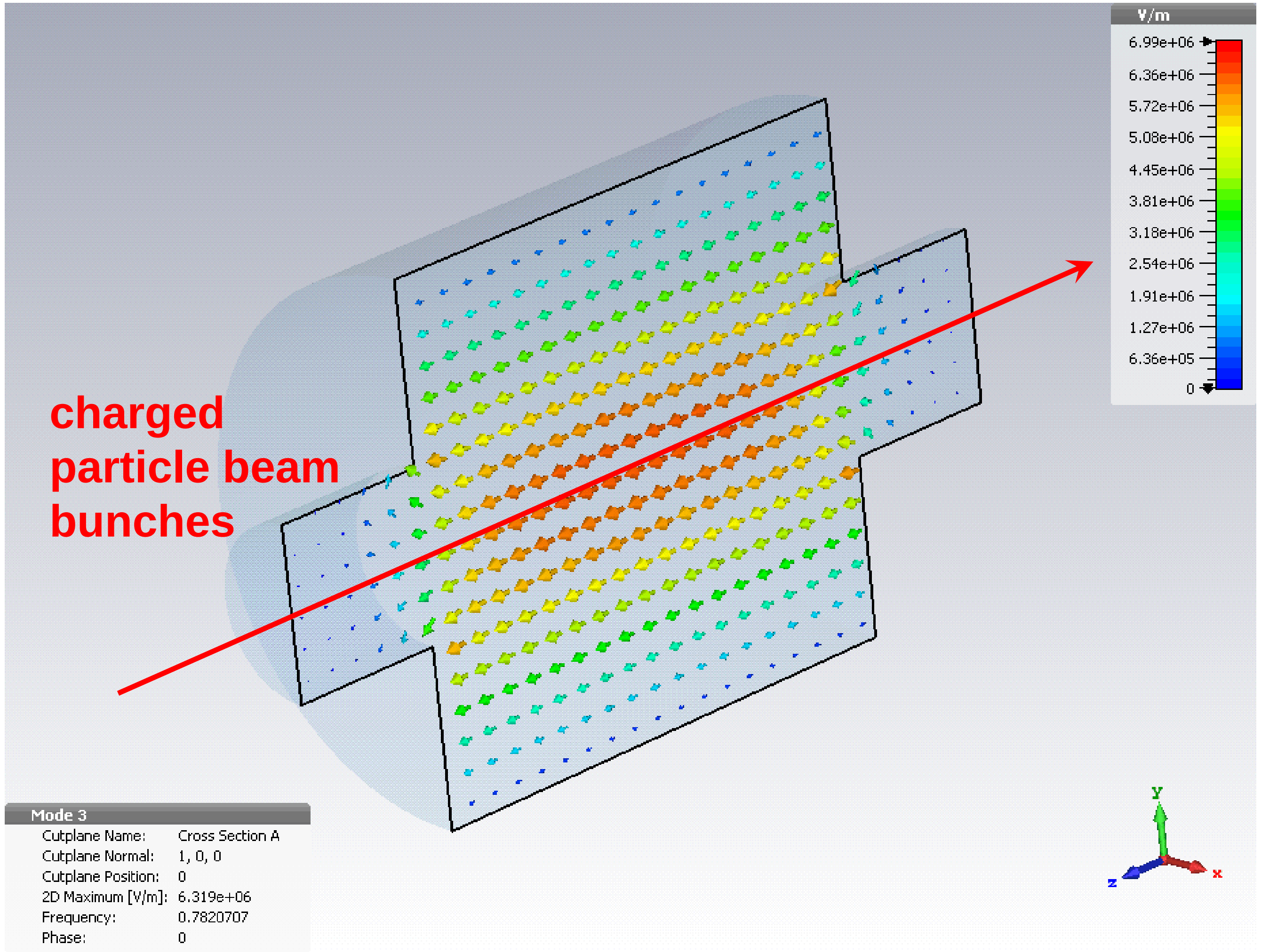
  If you want to know more…



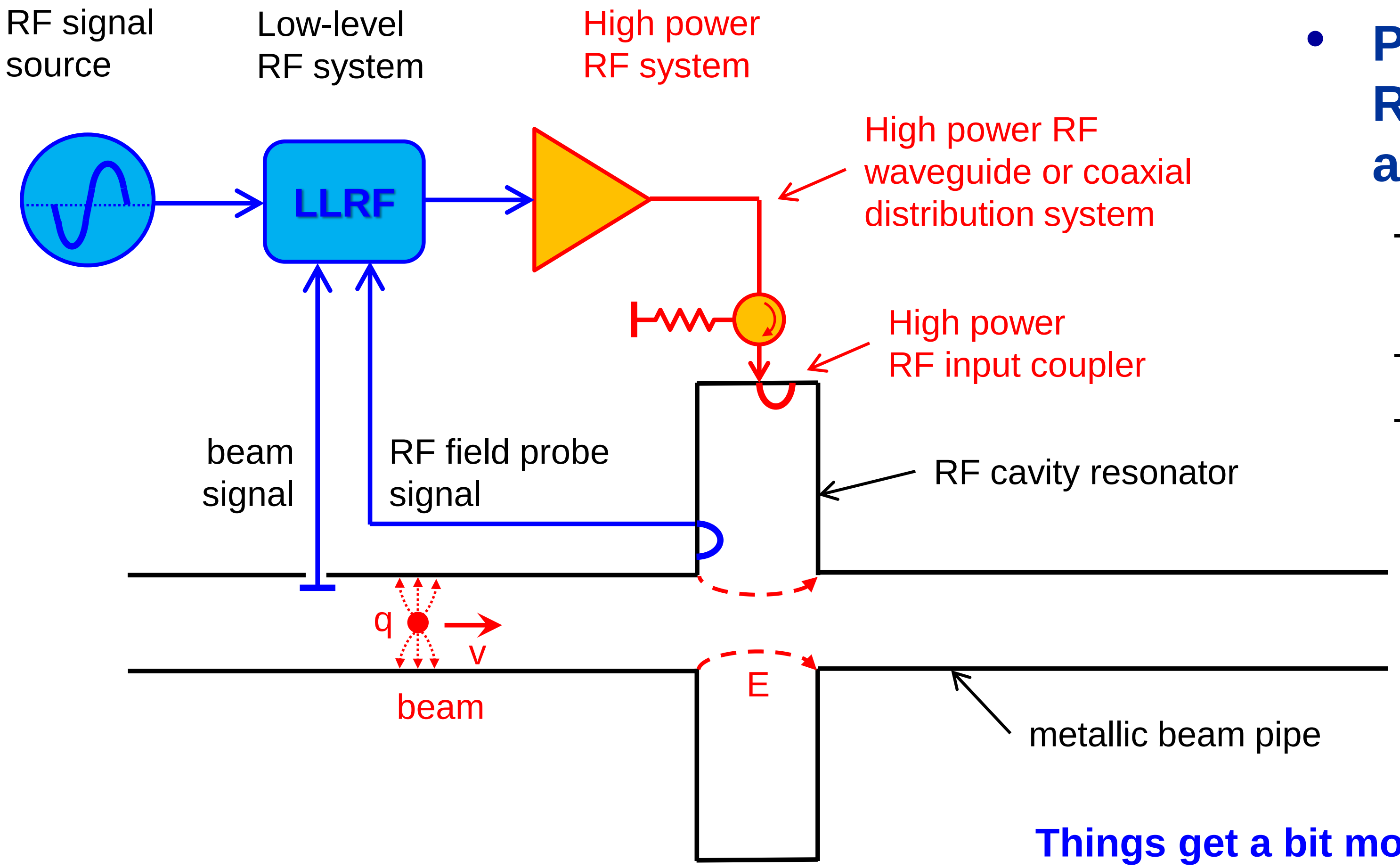


# Introduction – A resonant cavity




- **EM-fields, RF technology, material science, etc.**
  - **High acceleration gradient, up to 100MV/m**
- **Beam dynamics**
  - **Particle interaction with EM-fields**
  - **HOMs, wakefields**
  - $f_{res}$, **harmonic number** $h$
  - $V_{RF}$ **defines a stable RF bucket (potential well)**
    - …and therefore, $f_s$, *etc.*
  - **Transit time**

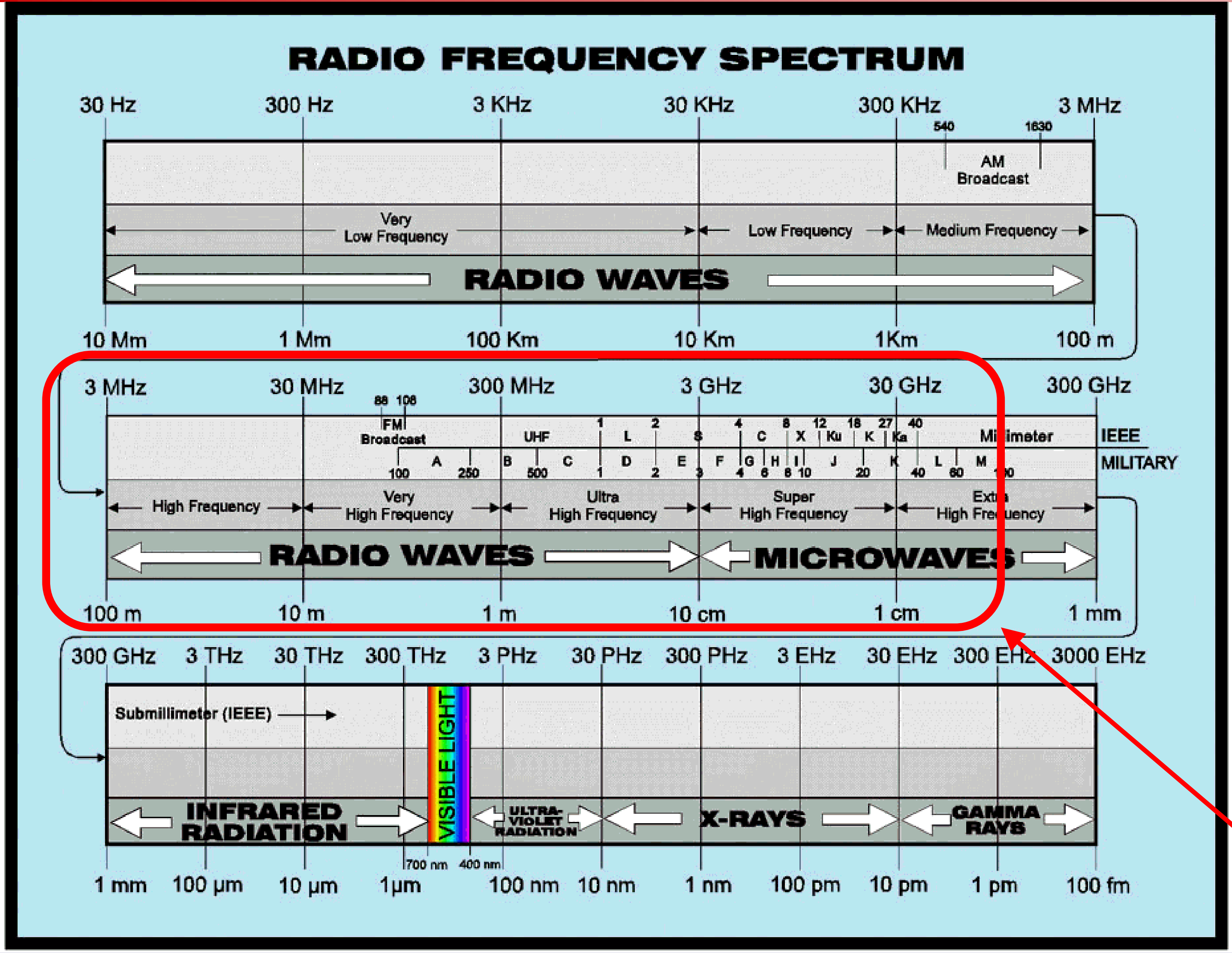
# Introduction – A simple RF system




- **Please note: RF (measurement) techniques are not limited to accelerator RF**
  - **Beam / bunch instrumentation and diagnostics**
  - **Injection / extraction elements**
  - **Pulsed accelerator systems, etc.**

**Things get a bit more complicated in the real world: pulsed power RF, multi-cell resonators or traveling wave structures, non-relativistic beams, HOM's, SRF, etc.**

# Introduction – A simple RF system




**Free space wavelength:**

$$\lambda = \frac{c}{f}$$

**We care about RF concepts if the physical dimensions of an apparatus is $> \lambda/10$**

RF frequencies typically utilized in accelerator applications

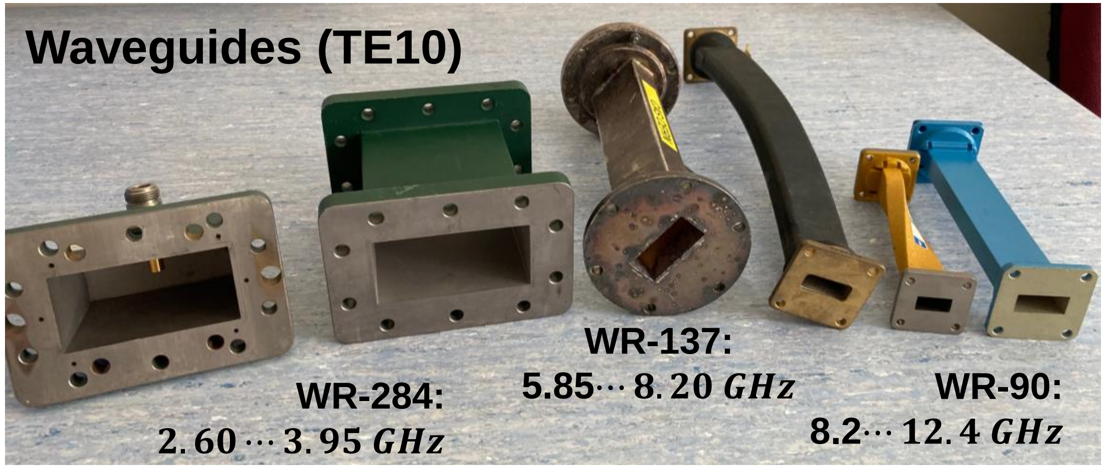
# Digital (mathematical) RF: The RF-SoC

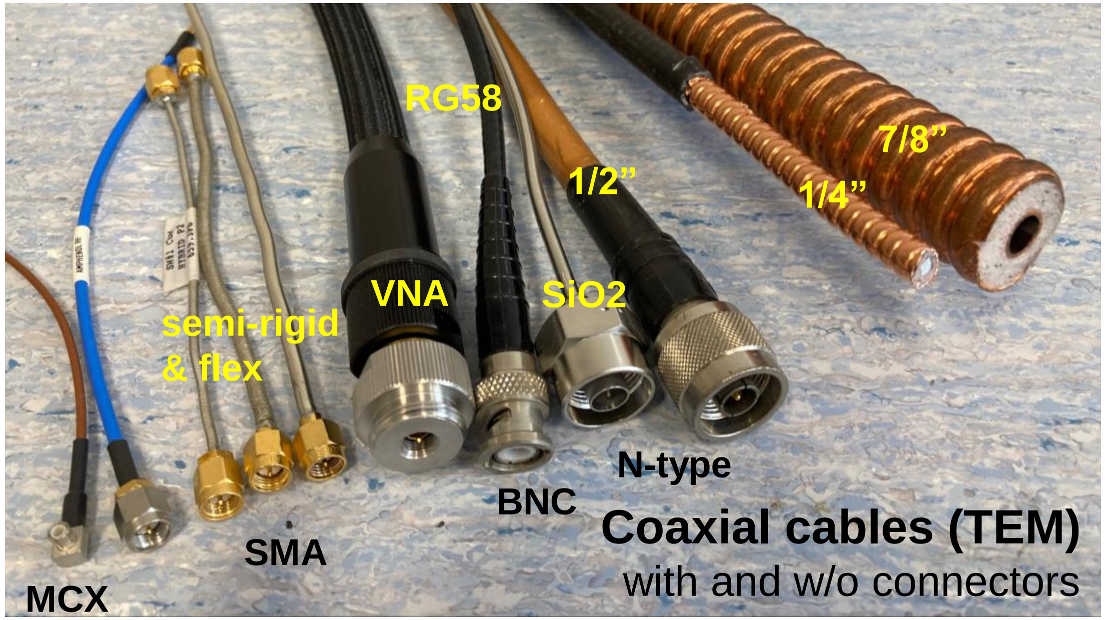

- **Low-power RF signals can be processed digitally**
  - **RF System-on-Chip: processors, FPGA, ADCs and DACs**
    - 8x ADC (5 GBPS, 12-bit, 70 dB dynamic range), 8x DAC (9.8 GBPS), incl. up-down converters
    - Multi-core processors, on-chip memory, many I/O options
  - **For frequencies <2 GHz, operation in the 1$^{st}$ *Nyquist* passband**
- **Not part of this lecture…**

# Part I: Transmission-lines

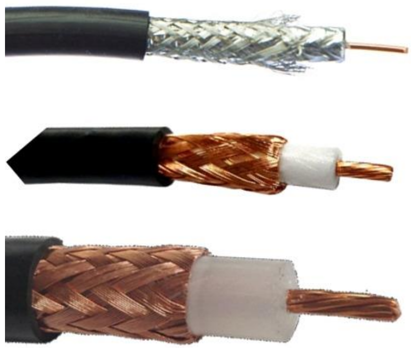
- **Outline and Learning objectives**
  - Introduction and some minimum theoretical background
  - Reflections effects of pulse signals on transmission-lines due to characteristic-termination impedance mismatch
  - Standing waves on transmission-lines for continuous wave (CW) sinusoidal signals, definition of the reflection coefficient
  - Relation between reflection coefficient, standing wave ration and return loss

# Transmission-lines (1)




Semi-rigid coaxial cables 0.141", SMA, $DC \cdots 18\ GHz$

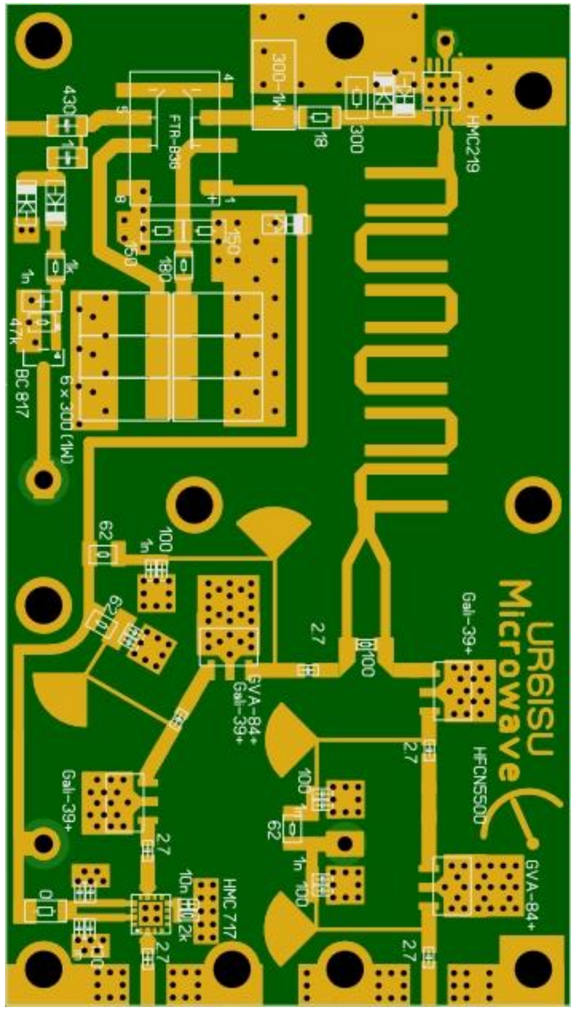


RG-type coaxial cables

corrugated coaxial cables, foamed PE & air

Low-loss, high-power air coaxial transmission-line

PCB microstrip-line (TEM)



# Transmission-lines (2)

- **Transmission-lines transport RF energy (waves) between components and sub-systems and exist in a large variety. Most popular are**
  - **Coaxial cables, always in TEM operation below the TE11 cut-off frequency**
    - Popular connectors are BNC (*Bayonet Neill–Concelman*), N-type and SMA (Sub-Miniature version A)
    - There exist a large variety of coaxial cables and connectors for HOM-free operation up to $110\ GHz$
  - **PCB (printed circuit board)-based planar quasi-TEM transmission-lines**
    - Popular are microstrip, stripline and coplanar-waveguide structures
  - **Rectangular waveguides, usually in TE10-mode operation**
    - For low-loss (high-power), high-frequency RF signal transmission
    - Clumsy and expensive, however, for some applications the only solution
- **Strictly speaking: Transmission-lines are 2-dimensional objects with infinite length**
  - **The cross-section geometry defines the characteristic impedance $Z_0$**
- **TEM transmission-lines (coaxial cables, PCB planar lines) operate from DC (direct current: $0\ Hz$) to a frequency given by the 1$^{st}$ higher-order mode**
  - **Various frequency depended losses contribute, therefore, high insertion losses at high frequencies**
- **Rectangular waveguides do not operate at DC or low frequencies!**
  - **Instead utilize the TE10 mode for the signal transmission**

# Transmission-lines (3)

- **TEM: coaxial cable, PCB stripline, micro-stripline, co-planar waveguide, etc.**
  - **but also, TE / TM: waveguides (rectangular, circular, elliptical)**
- **Transport RF energy (EM waves) from a RF source to a load.**
- **Physical length $\ell$ becomes relevant for**

$$\ell \gtrsim \frac{\lambda_g}{10}, \qquad \text{with guide wavelength: } \lambda_g = \frac{v_p}{f}$$

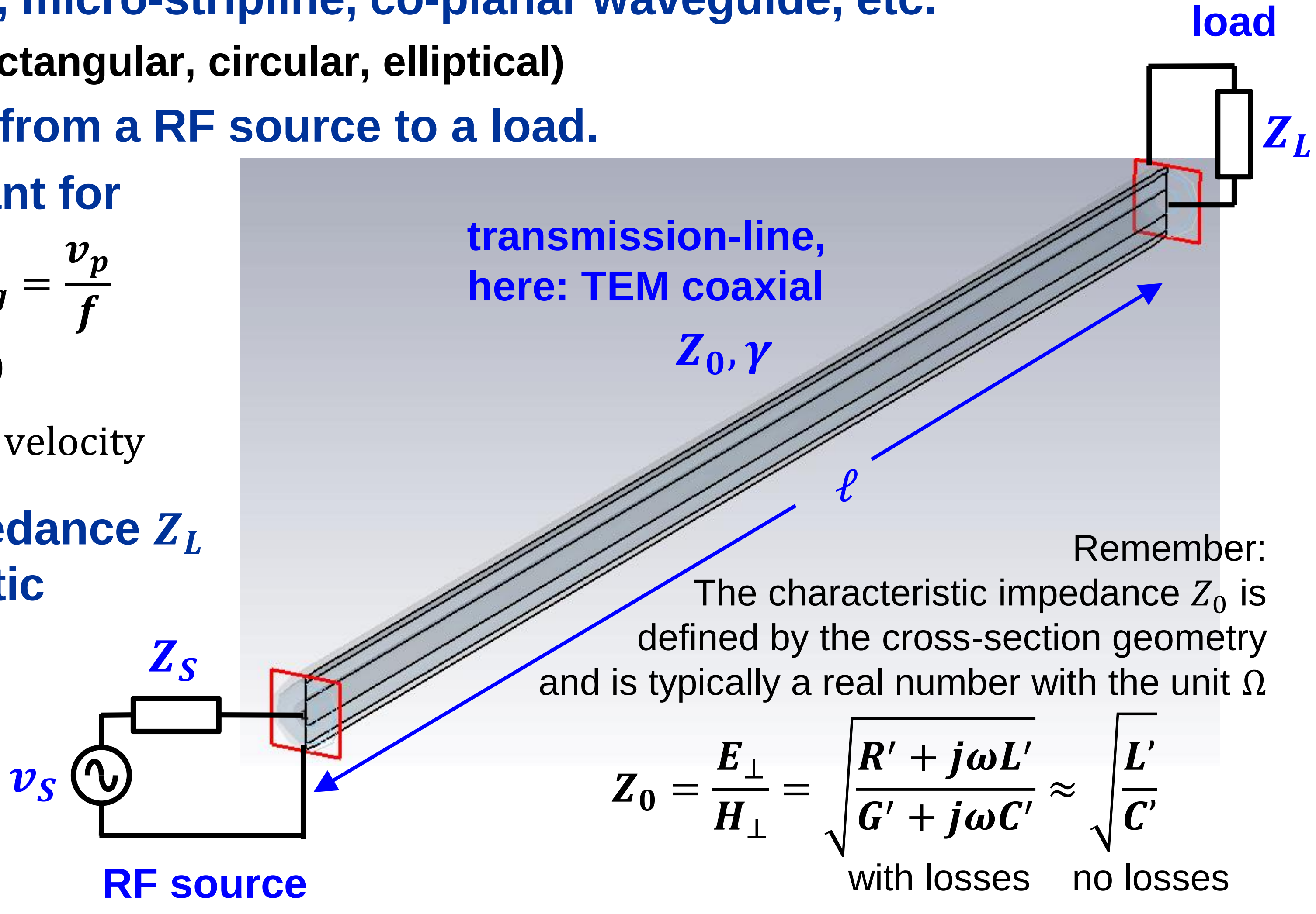
$f$: operating frequency (max.)

$$v_p = \frac{1}{\sqrt{L'C'}} = \frac{c}{\sqrt{\epsilon_r}} \text{ propagation velocity}$$

- **Reflections occur if the load impedance $Z_L$ is not matched to the characteristic impedance $Z_0$ of the TL**

$$Z_L = Z_0 \Rightarrow \Gamma = 0\text{: no reflections} ☺$$

$$Z_L \neq Z_0 \Rightarrow \Gamma \neq 0\text{: reflections!} ☹$$




Remember:
The characteristic impedance $Z_0$ is defined by the cross-section geometry and is typically a real number with the unit Ω

$$Z_0 = \frac{E_\perp}{H_\perp} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \approx \sqrt{\frac{L'}{C'}}$$

with losses — no losses

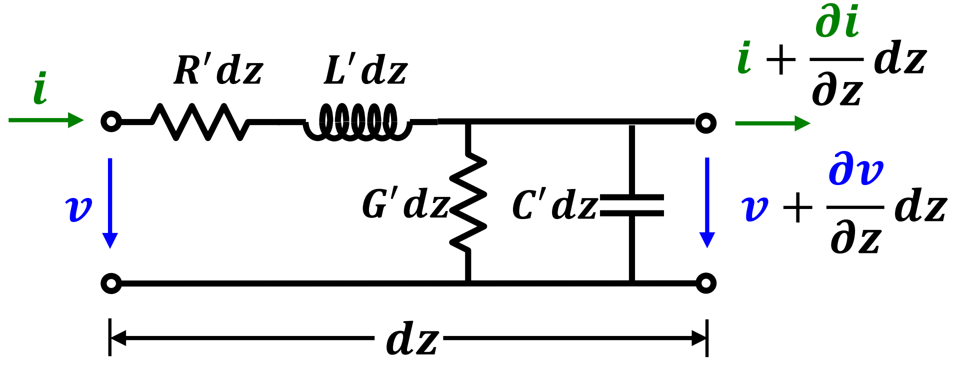
# Telegrapher's Equation for TEM transmission-lines



## A more general approach:

$$\frac{\partial v(z,t)}{\partial z} = -\left(R' + L'\frac{\partial}{\partial t}\right) i(z,t)$$

$$\frac{\partial i(z,t)}{\partial z} = -\left(G' + C'\frac{\partial}{\partial t}\right) v(z,t)$$




**phase velocity**

$$v_p = \frac{\lambda}{T} = \frac{\omega}{\beta}$$

**in steady state:**

$$\frac{dV}{dz} = -(R' + j\omega L')I$$

$$\frac{dI}{dz} = -(G' + j\omega C')V$$

$$\frac{d^2V}{dz^2} = \gamma^2 V$$

**voltage and current along a transmission-line:**

$$V(z) = V_0 \cosh \gamma z - Z_0 I_0 \sinh \gamma z$$

$$I(z) = I_0 \cosh \gamma z - {V_0}/{Z_0} \sinh \gamma z$$

$V_0, I_0$: voltage and current at the beginning of the line ($z = 0$)

**wave number**

$$k = \frac{2\pi}{\lambda} = \beta$$

**propagation constant** — attenuation constant — phase constant

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta$$

**characteristic impedance**

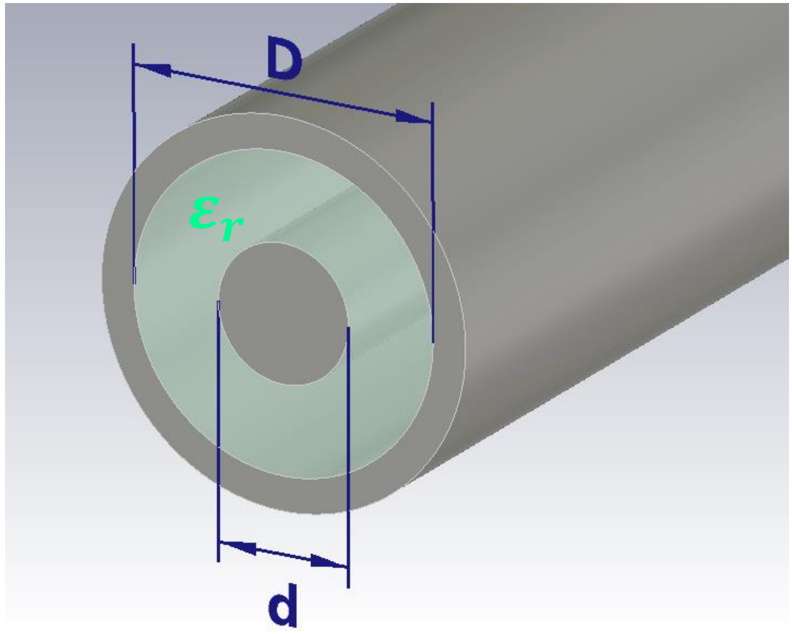
$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

# Transmission-lines – Coaxial Cables

- **The characteristic impedance is defined by the cross-section geometry:**

$$Z_0 = \frac{\eta_0}{2\pi\sqrt{\varepsilon_r}} \ln\left(\frac{D}{d}\right) \approx \frac{60\ \Omega}{\sqrt{\varepsilon_r}} \ln\left(\frac{D}{d}\right) \quad [\Omega]$$



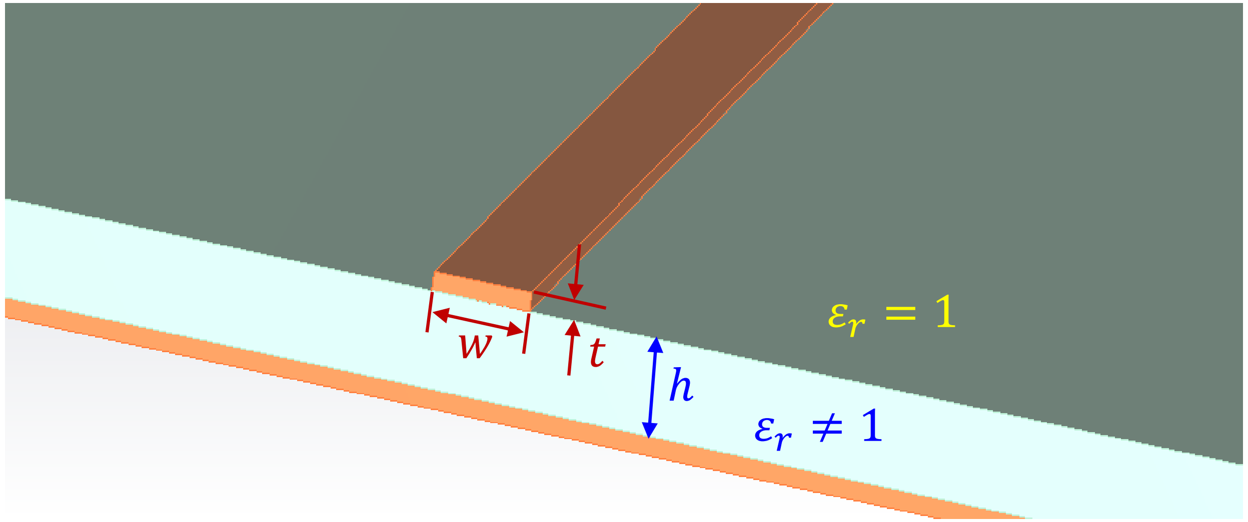

- **The attenuation losses in the propagation constant $\gamma$ are dominated by the material properties:**
  - **Propagation constant:** $\gamma = \alpha + j\beta$
  - **Attenuation constant:** $\alpha = \alpha_c + \alpha_d + \alpha_r + \alpha_l$
    - ➢ The attenuation losses are dominated by conductor and dielectric losses

For inner and outer conductor in copper:
$\mu_{rD} = \mu_{rd} = \mu_{rCu} = 1; \quad \rho_D = \rho_d = \rho_{Cu} = 1.72 \cdot 10^{-8}\ \Omega m$

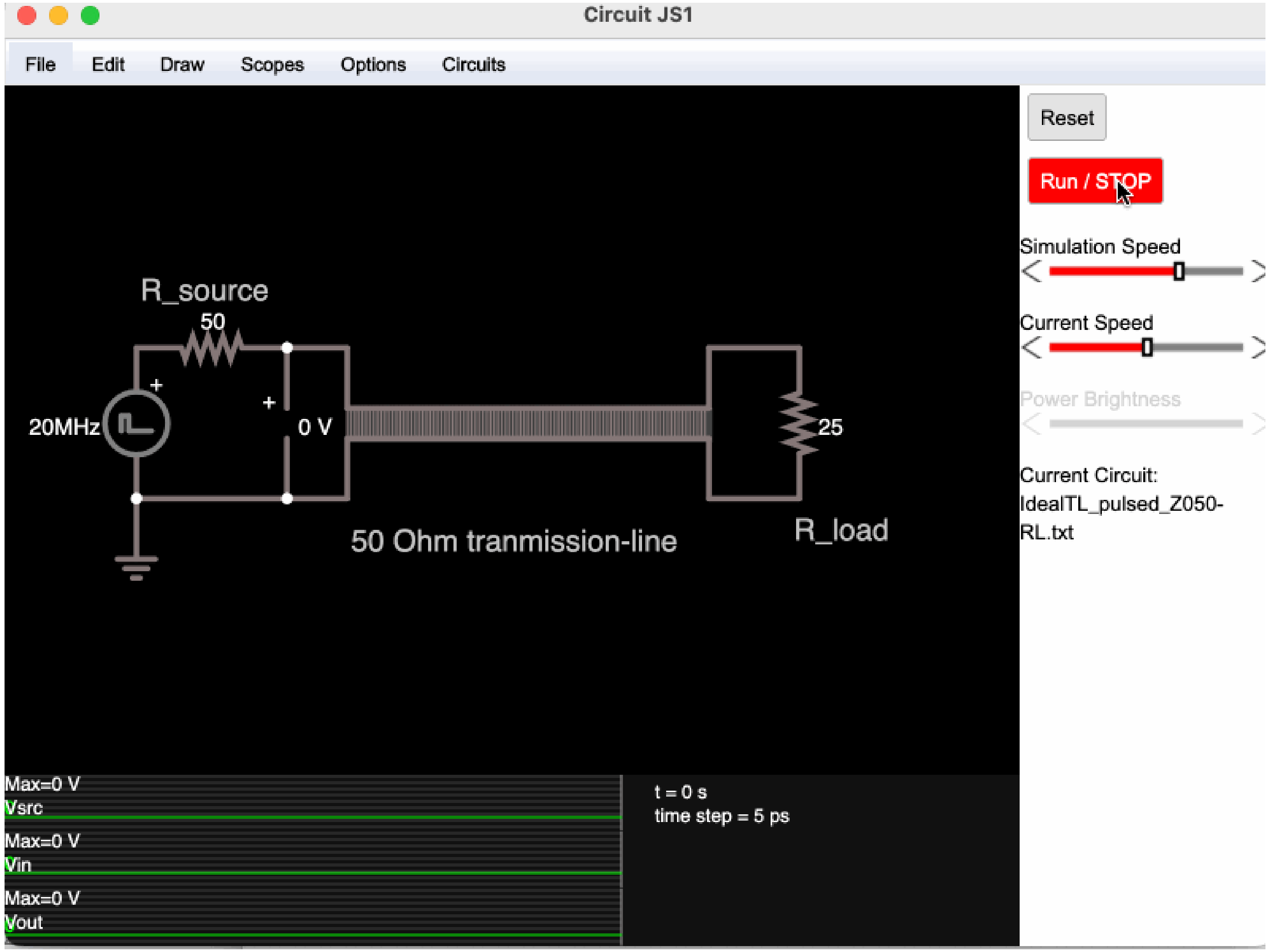
  - **Conductor losses:** $\alpha_c = \frac{\sqrt{f\mu_0/\pi}}{2Z_0}\left(\frac{\sqrt{\mu_{rD}\rho_D}}{D} + \frac{\sqrt{\mu_{rd}\rho_d}}{d}\right) \ [Np/m]$ $\qquad \alpha_c = 6 \cdot 10^{-9}\sqrt{f\varepsilon_r}\frac{D+d}{dD\ln(D/d)} \ [dB/m]$
  - **Dielectric losses:** $\alpha_d = \pi f\sqrt{\mu_0\varepsilon_0\varepsilon_r}\tan\delta_\varepsilon \ [Np/m]$ $\qquad \alpha_d = 91 \cdot 10^{-9} f\sqrt{\varepsilon_r}\tan\delta \ [dB/m]$
    - ➢ with: resistivity: $\rho = \frac{1}{\sigma}\ [\Omega m]$; loss tangent: $\tan\delta_\varepsilon = \frac{\varepsilon''}{\varepsilon'}$; permittivity: $\varepsilon = \varepsilon' - j\varepsilon''\ [F/m]$; permeability: $\mu = \mu' - j\mu''\ [H/m]$

# Transmission-lines – Microstrip-line

- **Microstrip-line:**
  - **Metallic strip conductor of width $w$ and thickness $t$ on a dielectric substrate of height $h$ over a conductive ground plane.**
    - Typically, the dielectric layer is a low-loss printed circuit board substrate
    - Quasi-TEM field as the EM-field propagates in two medias of different $\varepsilon_r$
    - Complicated analytical approximations to calculate the properties ($Z_0$, $\varepsilon_{eff}$, losses, high-order modes, etc.)
- **Other popular planar structures are coplanar waveguides and striplines**

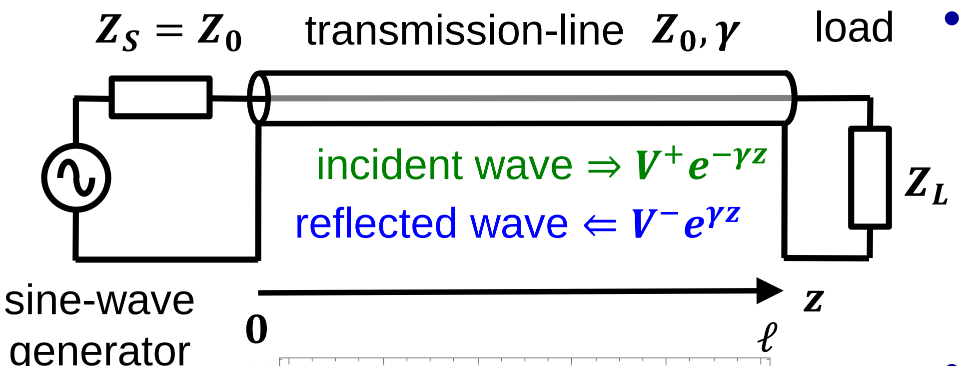


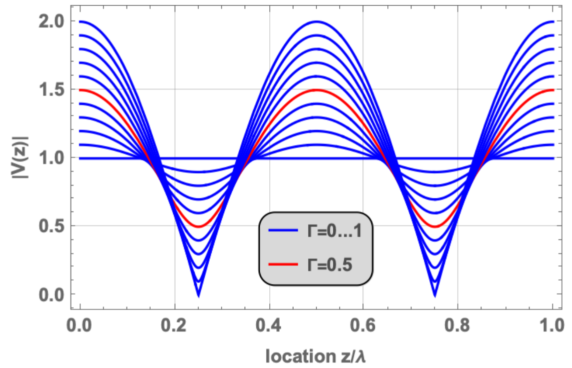
# TL: Signal visualization in time-domain



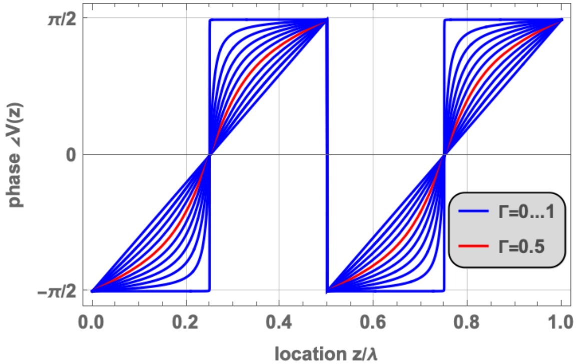

- **Circuit simulator applet: [https://www.falstad.com/circuit/](https://www.falstad.com/circuit/)**
  - **Load file: *IdealTL_DCswitched_Z050-RL.txt***
    - ➢ Change the load resistor value: *RL = 50, 100, 25 Ω*
    - ➢ Operate the switch and observe the signals at the beginning, and at the end of the transmission-line.
  - **Load file: *IdealTL_pulsed_Z050-RL.txt***
    - ➢ Change the load resistor value: *RL = 50, 100, 25 Ω*
    - ➢ Observe the signal waveforms! Can you predict the values?!
      - (Press *Run/STOP* and hover with the mouse over the waveform)

# TL: Operating with sinusoidal signals (FD)




here: $V(z)$ for a lossless TL ($\gamma = j\beta$)

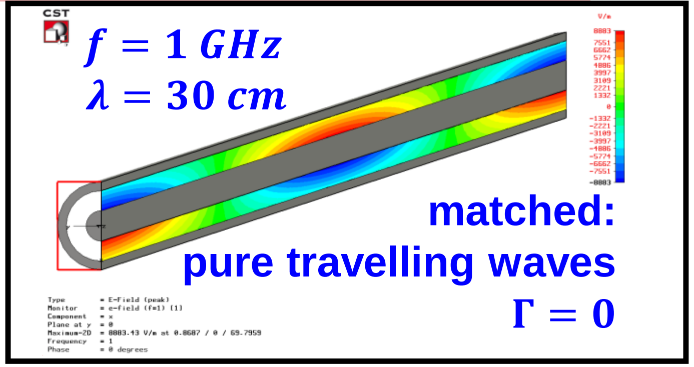


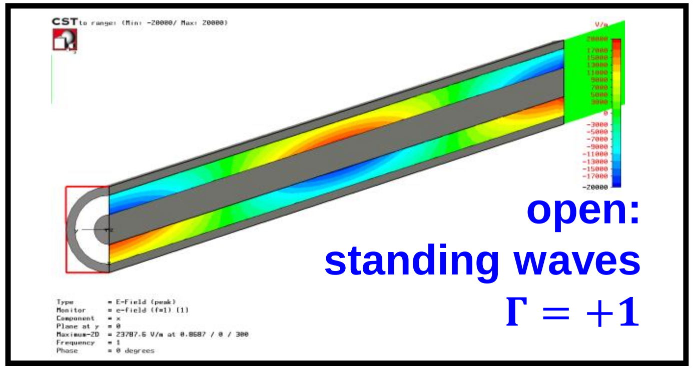

- **For a single frequency, continues wave (CW) signal on a transmission-line**
  - **Superposition of forward $a, E^{inc}, V^+$ and backward b, $E^{refl}, V^-$ traveling waves**
  - **$\Rightarrow$ standing waves**
- **Reflection coefficient $\Gamma$**

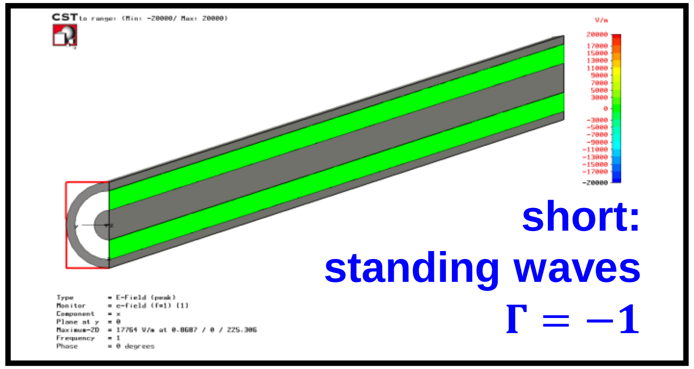
$$\Gamma = \frac{b}{a} = \frac{E^{refl}}{E^{inc}} = \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

waves E-fields voltages impedances

  - **$\Gamma$ is related to the impedance ratio: $Z_L/Z_0$**
    - The load impedance $Z_L$ can be complex
  - **$\Gamma$ is a complex number**
- **Reflections also originate due to discontinuities of the TL along $z$**
  - **Same $Z_0$ does not prevent reflections! EM-field effect!**



# TL: Voltage Standing Wave Ratio (VSWR)

- **The voltage standing wave ratio (VSWR) expresses the ratio between the maximum and minimum voltage of a standing wave along a transmission-line**

$$VSWR = \frac{|V_{max}|}{|V_{min}|} = \frac{|a|+|b|}{|a|-|b|} = \frac{1+|\Gamma|}{1-|\Gamma|} = \left|\frac{Z_L}{Z_0}\right|$$

with: $\begin{cases} |V_{max}| = |V^+| + |V^-| \\ |V_{min}| = |V^+| - |V^-| \end{cases}$

**symbols:** incident (forward) wave: $a, X^+$
reflected (backward) wave: $b, X^-$

  - **The VSWR is a function of the frequency.**
  - **The generalized standing wave ratio (SWR), e.g., expressed as power ratio is less popular**

$$SWR = \frac{1+\sqrt{P^-/P^+}}{1-\sqrt{P^-/P^+}}$$

- **The return loss (RL) is another way to express reflection effects**

$$RL\,[dB] = 10\log_{10}\frac{P^+}{P^-} = -20\log_{10}|\Gamma|$$

| $\Gamma$ | $VSWR = Z_L/Z_0$ | Return Loss [dB] | Refl. Power $\|\Gamma\|^2$ | Inc. Power $1-\|\Gamma\|^2$ |
|---|---|---|---|---|
| 0.0 | 1.00 | ∞ | 0.00 | 1.00 |
| 0.1 | 1.22 | 20.0 | 0.01 | 0.99 |
| 0.2 | 1.50 | 14.0 | 0.04 | 0.96 |
| 0.3 | 1.87 | 10.5 | 0.09 | 0.91 |
| 0.4 | 2.33 | 8.0 | 0.16 | 0.84 |
| 0.5 | 3.00 | 6.0 | 0.25 | 0.75 |
| 0.6 | 4.00 | 4.4 | 0.36 | 0.64 |
| 0.7 | 5.67 | 3.1 | 0.49 | 0.51 |
| 0.8 | 9.00 | 1.9 | 0.64 | 0.36 |
| 0.9 | 19.00 | 0.9 | 0.81 | 0.19 |
| 1.0 | ∞ | 0 | 1.00 | 0.00 |

# "dB" [*dee-bee*], or not to be…

- **dezi-Bel: 1 dB = 0.1 B (Bel)**
  - **Logarithmic scaling to compare large, e.g., power ratios:** $P_{dB} = 10\log_{10}\left(\frac{P_1}{P_2}\right)$
  - **or large ratios of other quantities, e.g.:** $V_{dB} = 20\log_{10}\left(\frac{V_1}{V_2}\right)$

$$I_{dB} = 20\log_{10}\left(\frac{I_1}{I_2}\right)$$

| dB ratio | $P_1/P_2$ | $V_1/V_2$ |
|---|---|---|
| n x 10 dB | $10^n$ | $10^{n/2}$ |
| 40 dB | 10000 | 100 |
| 20 dB | 100 | 10 |
| 10 dB | 10 | ~3.16 |
| 6 dB | ~4 | ~2 |
| 3 dB | ~2 | ~1.41 |
| **0 dB** | **1** | **1** |
| -3 dB | ~0.5 | ~0.71 |
| -20 dB | 0.01 | 0.1 |

$$\frac{P_1}{P_2} = 10^{\left(\frac{P_{dB}}{10}\right)} \qquad \frac{V_1}{V_2} = 10^{\left(\frac{V_{dB}}{20}\right)}$$

**The 3 dB ratio (half power) is a common specification for the bandwidth**

# "dB" is not "dBm"

- *dBm* is defined as a logarithmic power unit
  - based on dB and $P_{ref} = 1\,mW$

$$P_{dBm} = 10\log_{10}\left(\frac{P}{P_{ref}}\right)$$

- *dBm* can also be used as logarithmic voltage unit
  - e.g., for $Z_0 = 50\ \Omega$: $V_{ref} = 0.2236\,V$

$$V_{dBm} = 20\log_{10}\left(\frac{V}{V_{ref}}\right)$$

| dBm | P | V (RMS) |
|---|---|---|
| 30 dBm | 1 W | 7.07 V |
| 20 dBm | 100 mW | 2.24 V |
| 10 dBm | 10 mW | 707 mV |
| 6 dBm | 4.0 mW | 446 mV |
| **0 dBm** | **1.0 mW** | **224 mV** |
| -20 dBm | 10 µW | 22.4 mV |
| -60 dBm | 1.0 nW | 224 µV |
| -120 dBm | 1.0 fW | 224 nV |
| - 174 dBm | 4.0e-21 W | 0.446 nV |

← noise power in a bandwidth *BW = 1 Hz* at room temperature

$$P = P_{ref}10^{\left(\frac{P_{dBm}}{10}\right)}$$

$$V = V_{ref}10^{\left(\frac{V_{dBm}}{20}\right)}$$

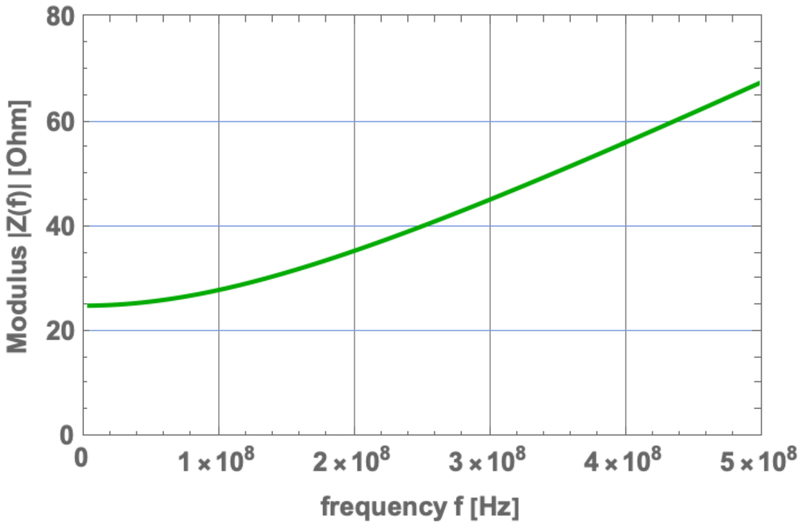
# Part II: *Smith*-Chart

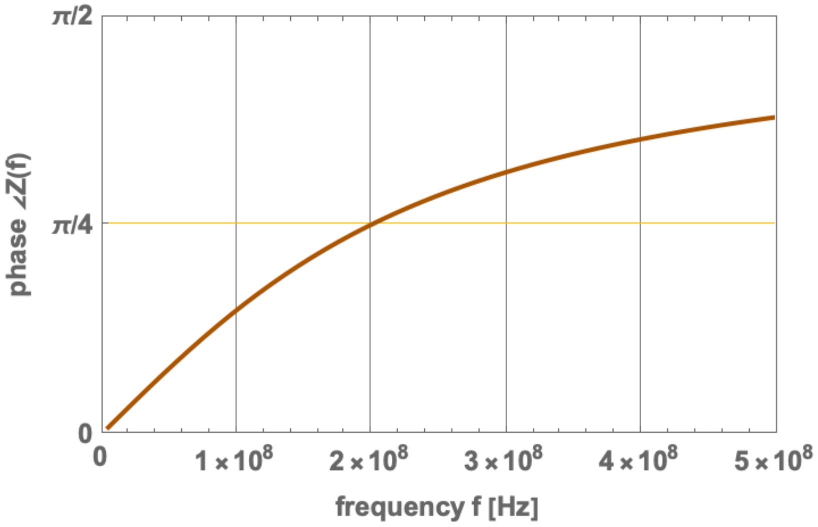
- **Outline and Learning objectives**
  - **Refresher: Visualization of a complex impedance in the frequency domain**
  - **Definition of the *Smith* chart, mapping the complex impedance / admittance plane with the complex reflection coefficient**
  - **Basic facts and important points on the *Smith* chart**
  - **Simple example**

    More examples and information in the backup section:

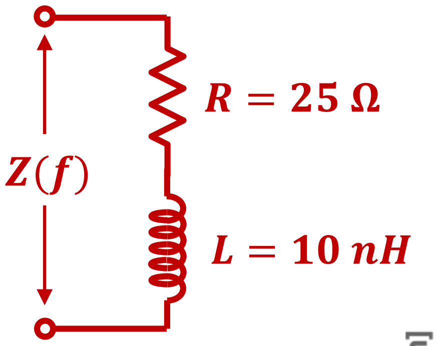
    - Examples for a RL and RC series circuit,
      and for a transmission-line terminated with a RL series circuit.
    - Operation of a $\lambda/4$ transformer based on a transmission-line
      as a (normalized) impedance inverter

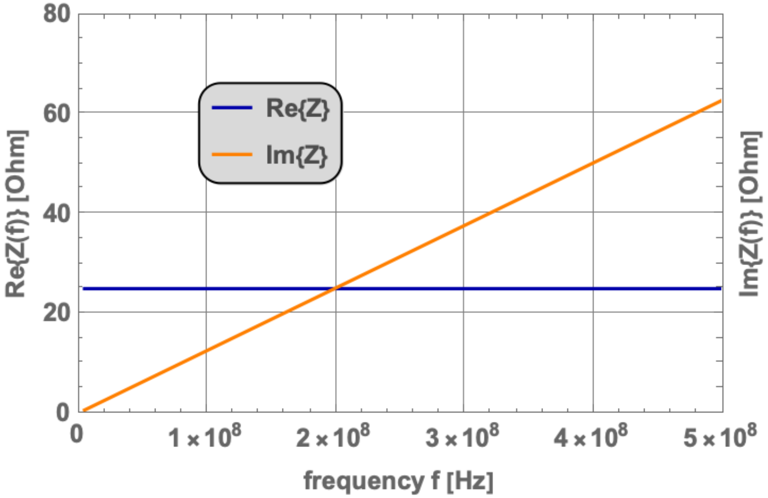
# Reminder: The complex $Z$-Plane

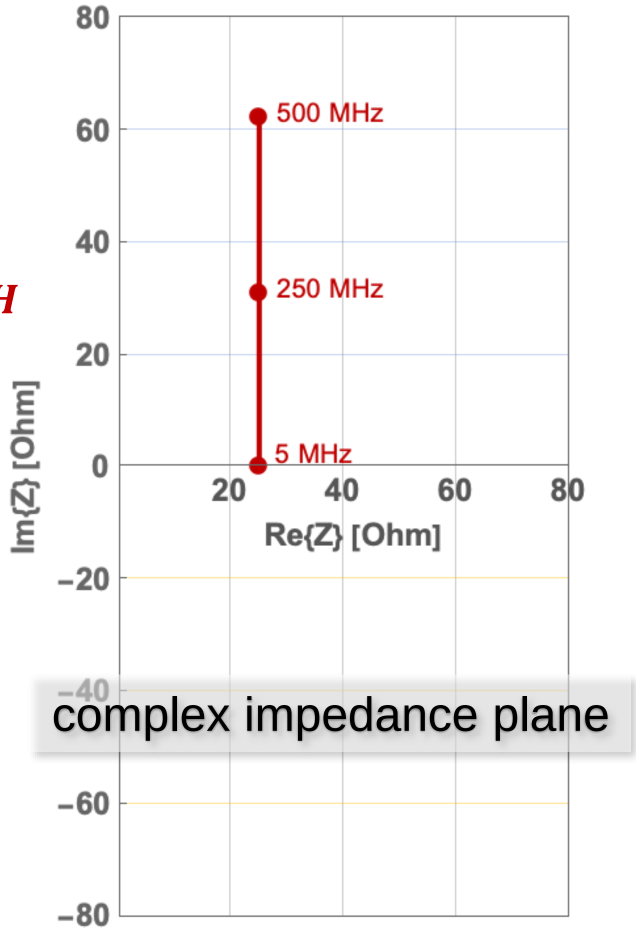



- **Different ways to visualize $Z(f)$**
  - magnitude / phase
  - real / imaginary
  - complex $Z$-plane
    - Plot in the complex plane with $f$ as parameter



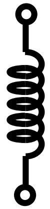



# Reminder: Circuit Vocabulary

conductor, with:
**conductance** $G\ [S]$

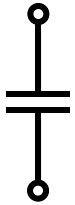
resistor, with:
**resistance** $R\ [\Omega]$



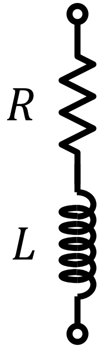
inductor, with:
**inductance** $L\ [H]$
**reactance** $X_L = \omega L\ [\Omega]$

**susceptance** $B_L = 1/\omega L\ [S]$

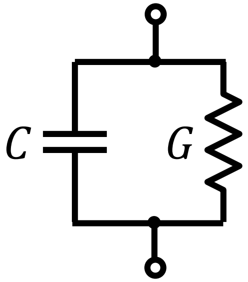
capacitor, with:
**capacitance** $C\ [F]$
**susceptance** $B_C = \omega C\ [S]$



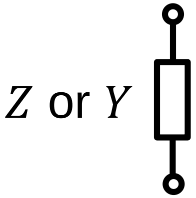
**reactance** $X_C = -1/\omega C\ [\Omega]$




**complex impedance example:**
$Z = R + j\omega L\ [\Omega]$




**complex admittance example:**
$Y = G + j\omega C\ [S]$

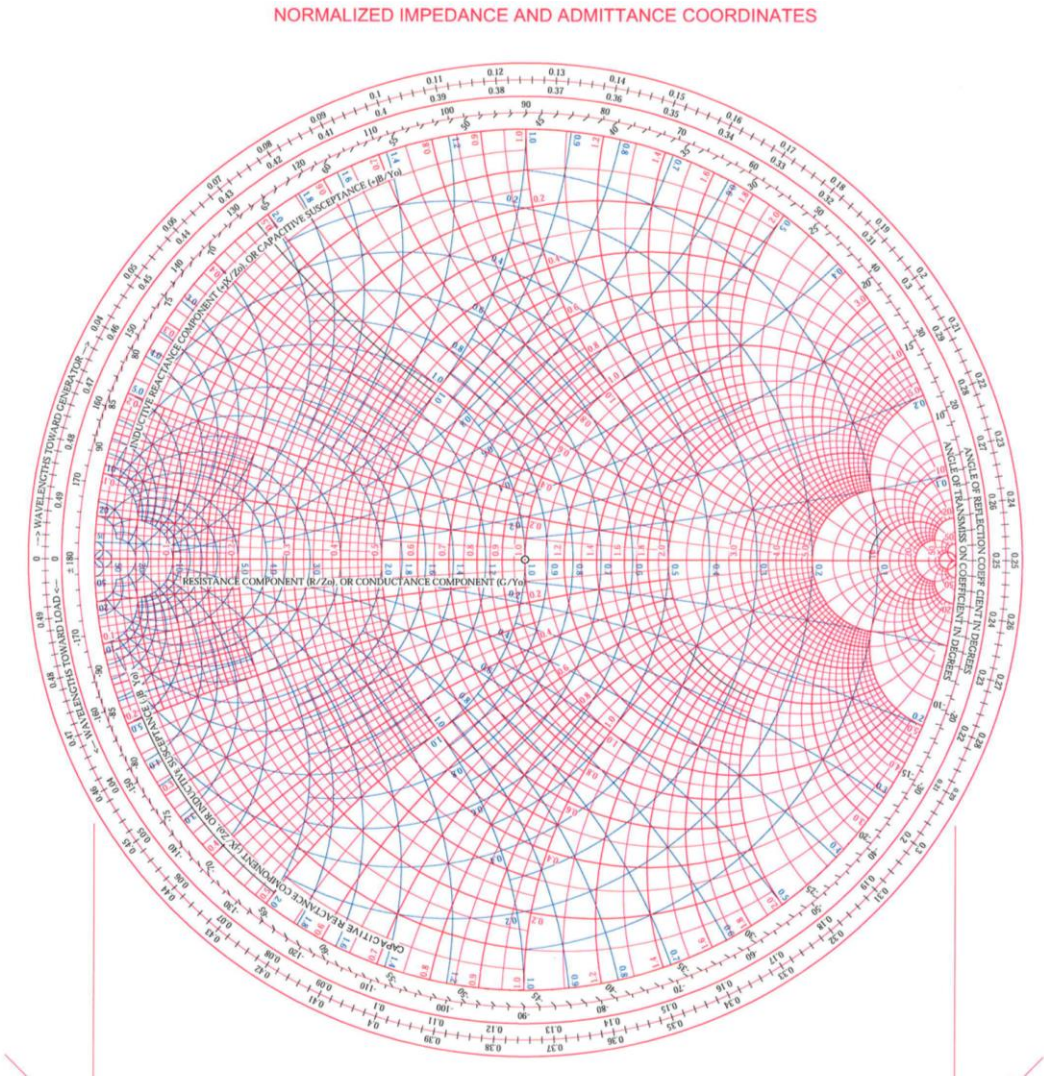



complex impedance $Z$
or
complex admittance $Y$

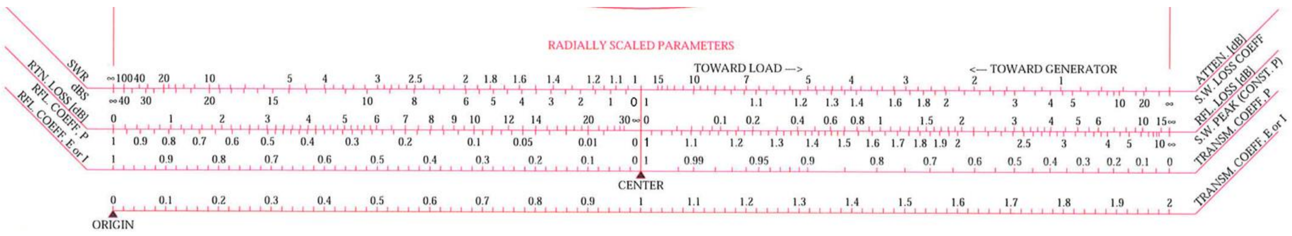
- **Resistance, impedance, reactance are inverse proportional to conductance, susceptance, admittance**

$$R = \frac{1}{G} \qquad Z = \frac{1}{Y} \qquad X = \frac{-1}{B}$$

# The *Smith* Chart (1)



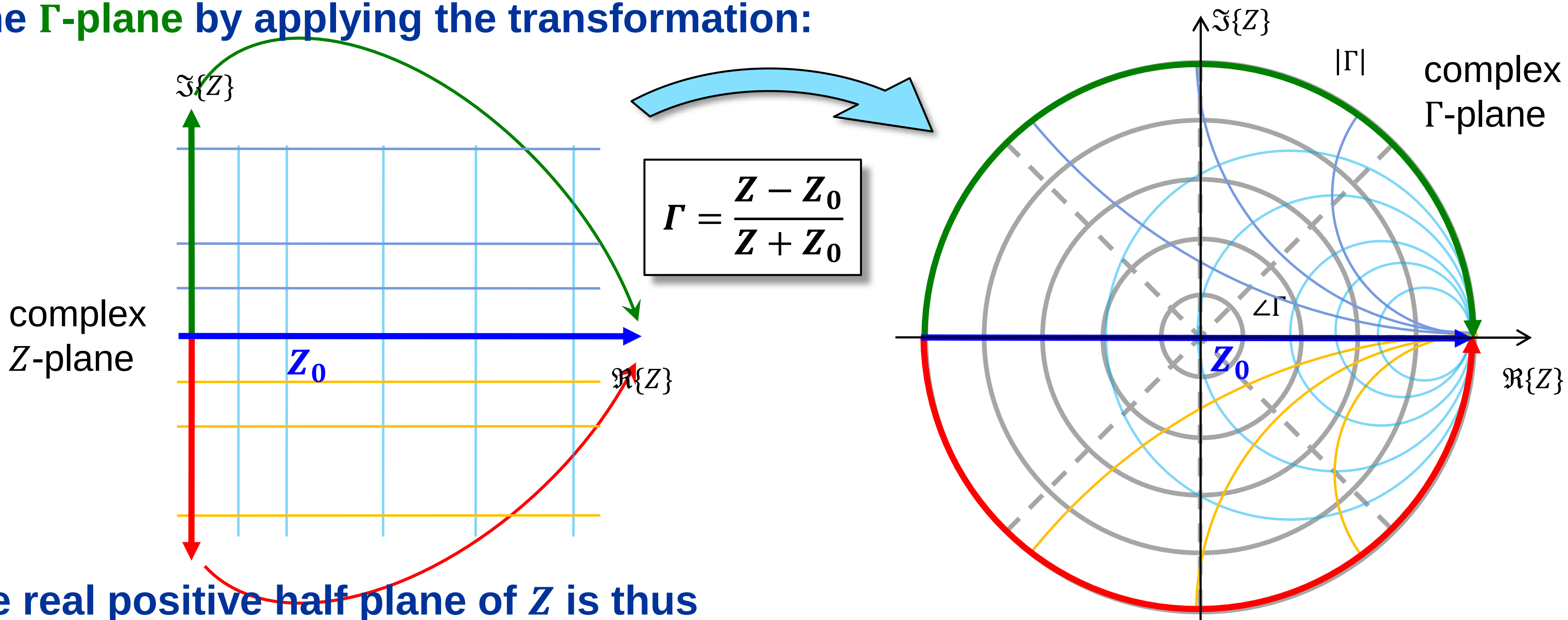

- **The *Smith* chart is a calculation tool, divided in 2 parts:**
  - **A transformation of the complex, normalized impedance $z$ and admittance $y$-planes on a circle.**
  - **A set of "rulers" below, for additional computations**
    - VSWR, return and reflection loss, etc.




- **At a 1st look the Smith chart is quite overwhelming**
  - **In this introduction the focus is on the complex z-plane**

# The *Smith* Chart (2)

- The *Smith* chart (in impedance / admittance coordinates) represents the complex Γ-plane (in polar coordinates) within the unit circle.
- It is a conformal mapping of the complex $Z$-plane on the Γ-plane by applying the transformation:




- ⇒ the real positive half plane of $Z$ is thus transformed (*Möbius*) into the interior of the unit circle!

# The *Smith* Chart (3)

- In the classical paper *Smith* chart the impedance $Z$ is **normalized**:

$$z = \frac{Z}{Z_0}$$

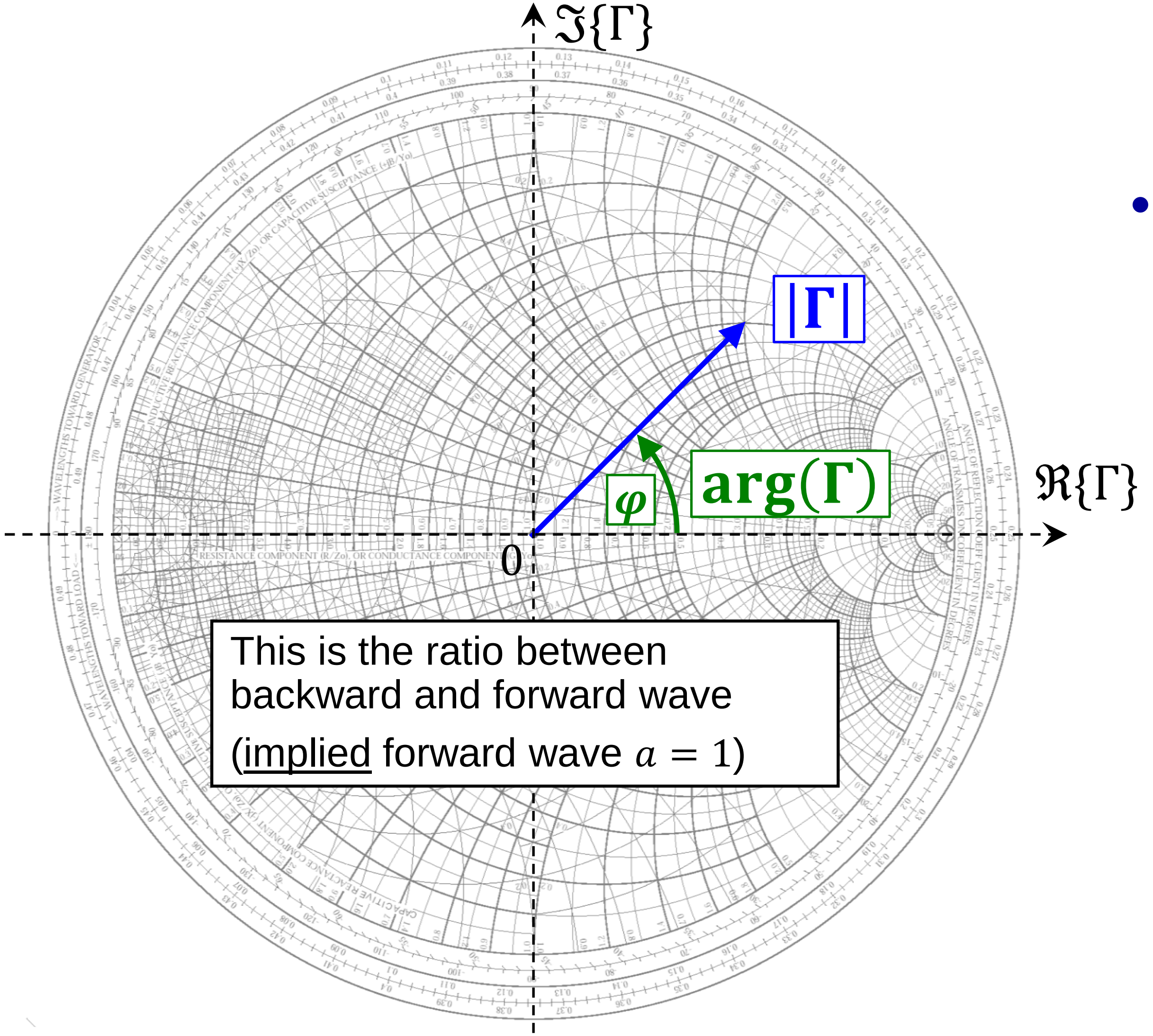
  to a **reference impedance** $Z_0$,
  typically, to the characteristic impedance of the coaxial cable transmission-lines used in RF / microwave engineering: $Z_0 = 50\ \Omega$.
- The normalized form of the transformation follows then as:

$$\Gamma = \frac{z-1}{z+1} \Rightarrow \frac{Z}{Z_0} = z = \frac{1+\Gamma}{1-\Gamma}$$

- The Smith chart is a **parametric graph**
  - with the **frequency $f$ as parameter**
  - and the **normalized, complex impedance $z$** and **complex reflection coefficient $\Gamma$** as variables
    - also, the normalized, complex admittance $y = 1/z$ is mapped and can be used as variable.

- In the past
  - The *Smith* chart was used as a calculation tool for impedance matching, e.g., antennas to transmitters or receivers, amplifier input / output stages, couplers of accelerating cavities, etc.
- At presence, the *Smith* chart is still popular
  - for visualization purposes, e.g., vector network analyzer (VNA) measurement of input / output impedances (display of the Snn scattering parameters)
  - for the optimization of the coupling between RF source and a cavity resonator.
  - …

# The *Smith* Chart (4)




- **In the *Smith* chart, the complex reflection factor**

$$\Gamma = |\Gamma| e^{j\varphi} = \frac{b}{a}$$

**is expressed in linear polar coordinates, representing the ratio of backward $b$ vs. forward $a$ traveling waves.**

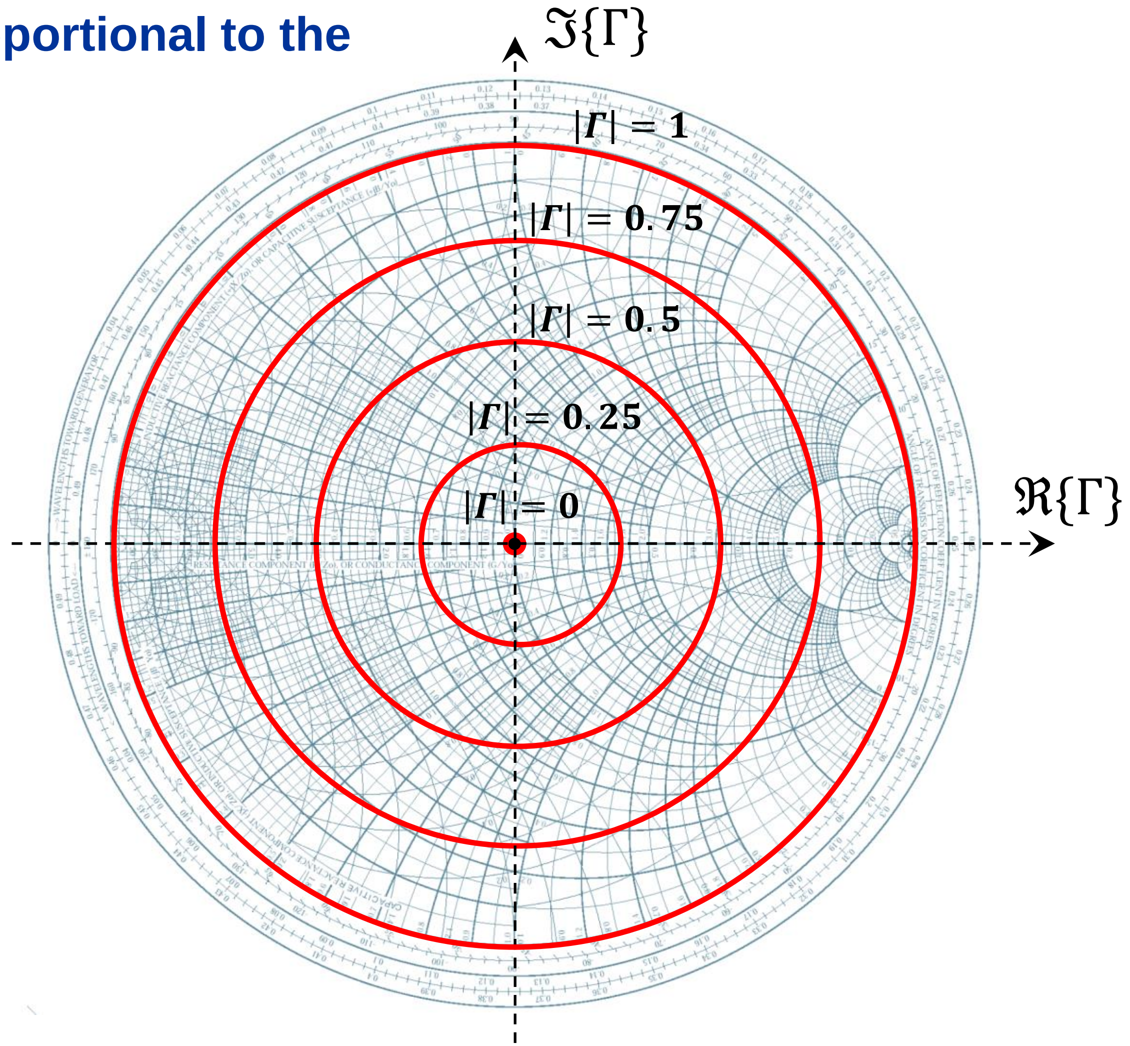
# The *Smith* Chart (5)

- The distance from the center of the directly proportional to the magnitude of the reflection factor $|\Gamma|$ and permits an easy visualization of the matching performance.
  - In particular, the perimeter of the diagram represents total reflection: $|\Gamma| = 1$.
  - (power dissipated in the load) = (forward power) – (reflected power)

$$\mathbf{P} = |a|^2 - |b|^2$$
$$= |a|^2 \left(1 - |\Gamma|^2\right)$$

available source power

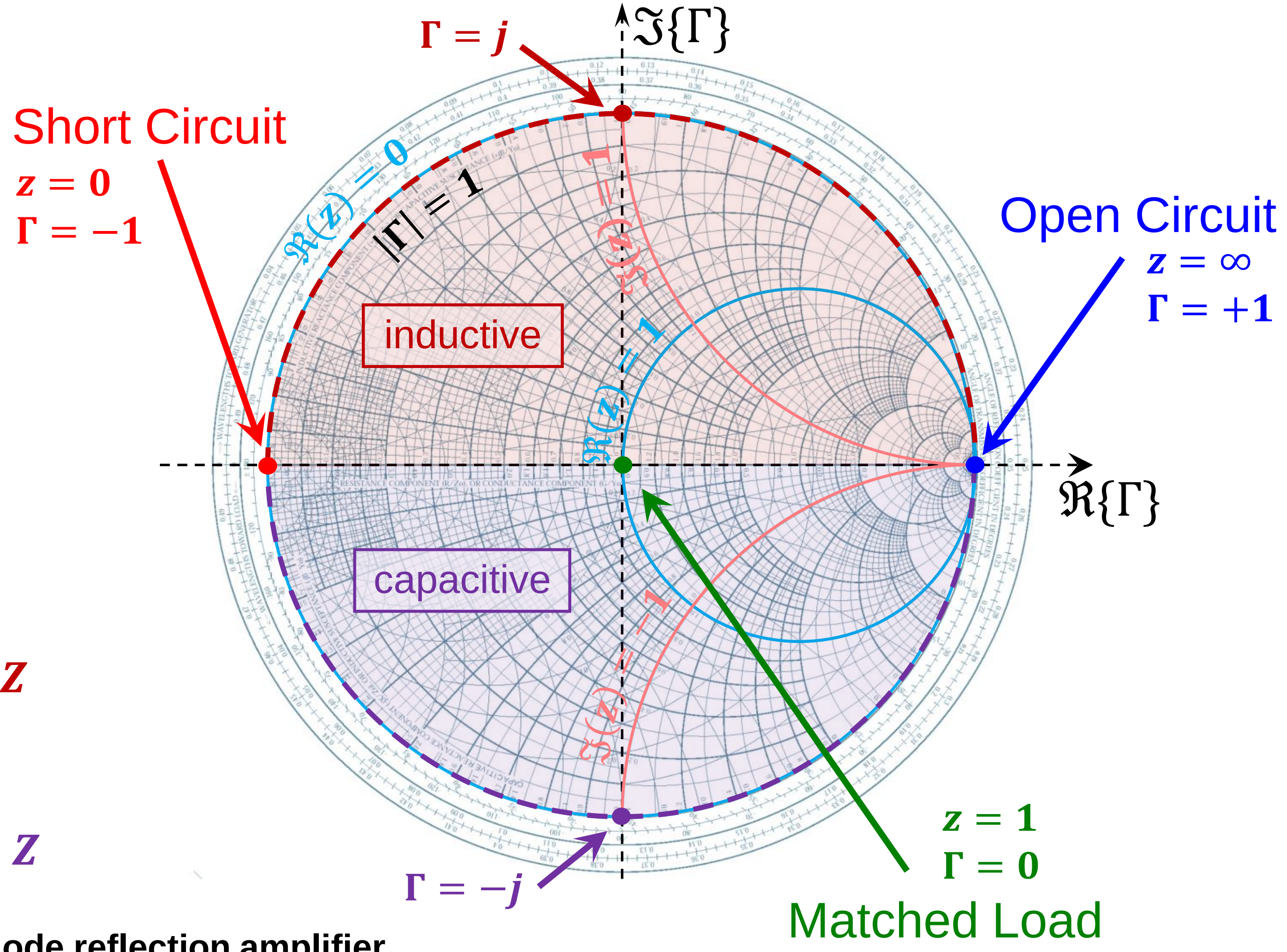
mismatch losses



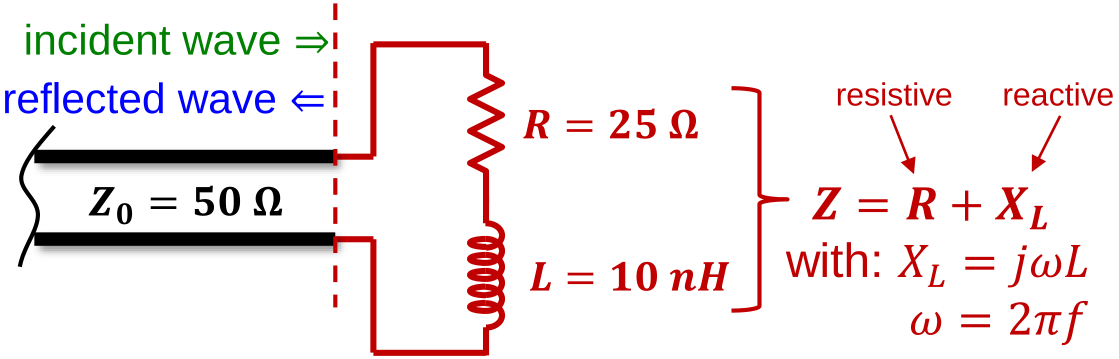
# The *Smith* Chart – "Important Points"

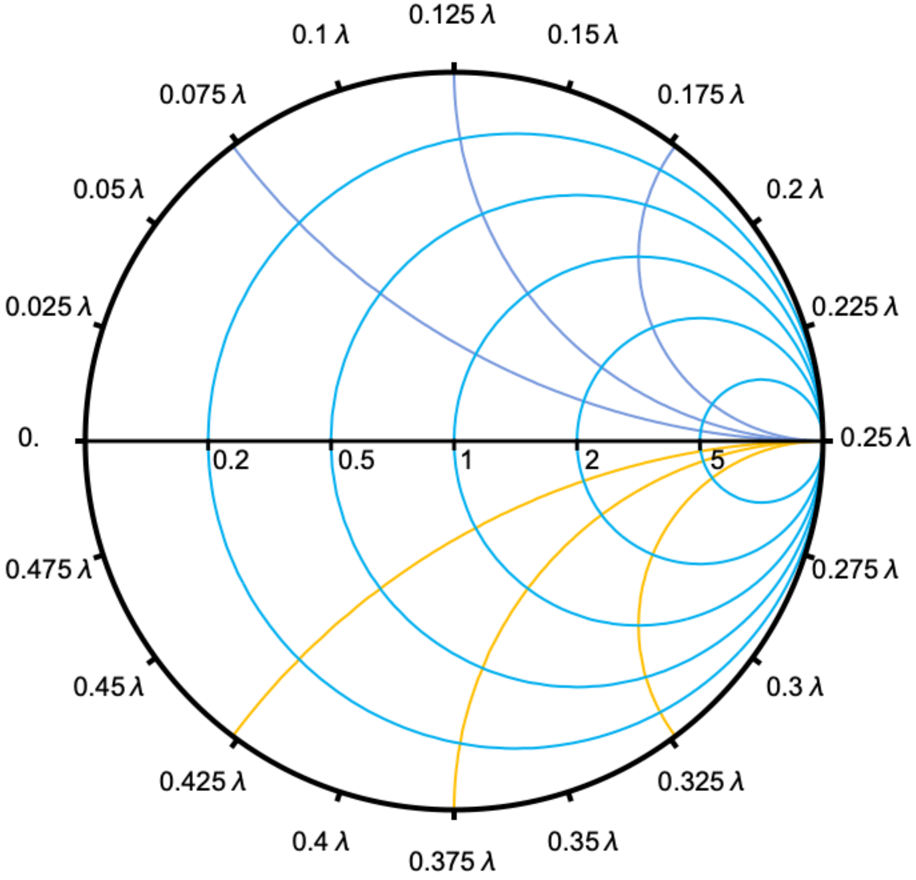
**Important Points:**

- **Short Circuit**
  $\Gamma = -1, z = 0$
- **Open Circuit**
  $\Gamma = +1, z \rightarrow \infty$
- **Matched Load**
  $\Gamma = 0, z = 1$
- **On the circle $|\Gamma| = 1$:**
  **lossless element**
- **Upper half:**
  **"inductive" =**
  **positive imaginary part of $Z$**
- **Lower half:**
  **"capacitive" =**
  **negative imaginary part of $Z$**
  - **Outside the circle, $\Gamma > 1$:**
    **active element, e.g., tunnel diode reflection amplifier**



# The *Smith* Chart – Basic Example




**Complex impedance based on lumped element components**

- **Calculate $Z$ for a given frequency, e.g., $f = 50\ MHz$: $Z = (25 + j6.28)\ \Omega$**
- **Calculate the normalized impedance $z = Z/Z_0 = 0.5 + j0.126$**
  - **Locate $z$ in the Smith chart**
  - **Retrieve $\Gamma = 0.34\angle 161° = 0.34e^{j2.81}$**
- **Repeat for other frequencies…**

# Remarks on transmission-lines and the *Smith* chart

- **The *Smith* chart is a special type of a parametric plot for the complex impedance or admittance, mapped to the plane of the complex reflection coefficient.**
- **The use of the *Smith* chart can be viewed in two ways:**
  - **As a calculation and impedance matching tool as originally envisioned.**
    - However, today this will happen rather rarely!
    - Please notice: All the examples presented (in the backup slides) are based on the paper-style *Smith* chart, which is always based on an unitless, normalized impedance $z = Z/Z_0$
  - **As a visualization tool for the complex impedance, along with the reflection coefficient**
    - Still very popular and useful for displaying and analyzing $S_{ii}$ on a vector network analyzer, also used in data-sheets, and RF simulation and education software.
    - Here the *Smith* chart utilizes the actual complex impedance $Z$ in units of $\Omega$! Markers on the parametric trace give all relevant information, including the element values of a selected equivalent circuit.
- **More information and basics examples are found in the backup slides**
- **Old, but excellent information on transmission-lines and standing waves:**
  - https://www.youtube.com/watch?v=I9m2w4DgeVk
  - https://www.youtube.com/watch?v=DovunOxlY1k&t=38s
- ***Smith* chart education software (only for MS-Windows):**
  - https://www.fritz.dellsperger.net/smith.html

# Part III: Scattering Parameters

- **A brief recap on electrical networks**
  - **A simplified way to describe electrical, electronics and RF circuits**
  - **Electrical network composed out of lumped and distributed elements**
  - **Two-port RC-filter example using admittance (Y) parameters**

- **Introduction to scattering (S) parameters**
  - **General concept of incident / reflected waves scattered at the ports of an RF network**
  - **Reference impedance $Z_0$**
  - **$1$-port and $2$-port S-parameters**
  - **Properties of the S-matrix: reciprocity, symmetry, losses**
    - ➢ with more examples in the backup slides
  - **A few S-matrix examples for RF networks with $1$-, $2$-, $3$-, and $4$-ports**
  - **S-parameters in practice: the SnP Touchstone file format**
  - **General $n$-ports**

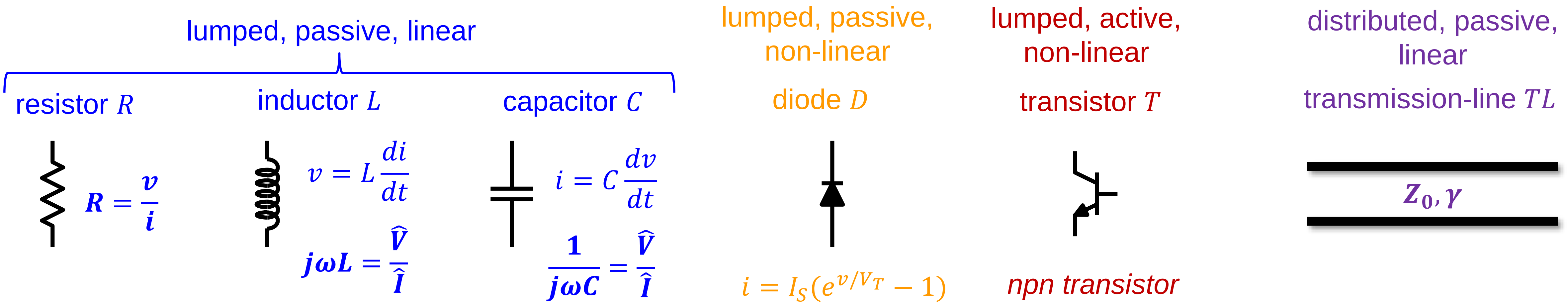
# Electrical Networks (1)

- **The electromagnetic behavior or RF circuits and systems, like any other electrical / electronics circuit or system can be described by Maxwell's equations**

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon}, \qquad \nabla \times \boldsymbol{B} - \frac{\partial \boldsymbol{E}}{c^2 \partial t} = \mu \boldsymbol{J}, \qquad \nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0, \qquad \nabla \cdot \boldsymbol{B} = 0$$

  - **These equations need to be solved, taking all the boundaries and materials into account**

- **However, this is far too complicated and inconvenient for most practical situations!**
  - **simplified electrical network description based on approximative lumped or distributed elements**
    - ➢ With given characteristics and values of each circuit element represented by a symbol in an electrical network, following the laws of *Ohm* and *Kirchhoff*. Here some examples:



# Electrical Networks (2)

- Complex electrical, electronics and RF systems are divided into functional blocks, i.e., networks, with $n$-ports, each port has two terminals
  - The two-port network is most popular, typical examples for two-port networks are filters, attenuators, amplifiers, etc.


**$Y$- parameters**

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

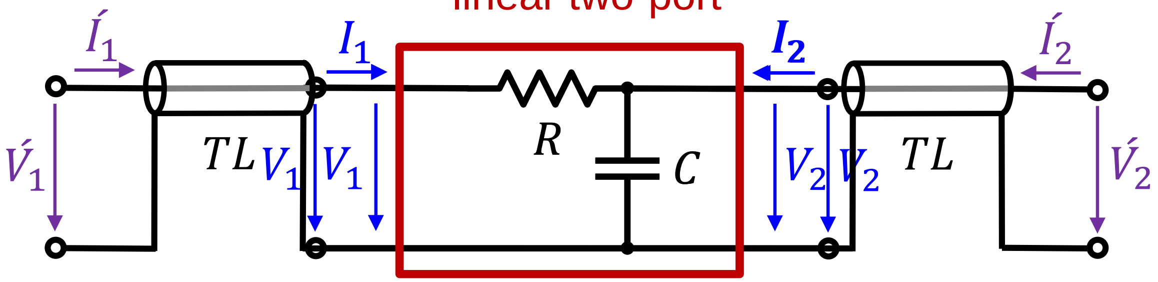
$I_1$, $I_2$: dependent
$V_1$, $V_2$: independent

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$
$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

- The characteristic behavior of the $n$-port network is defined by a set of $2n$ parameters, linked to their ports.
  - A two-port network has four parameters as port voltages ($V_1$, $V_2$) and port currents ($I_1$, $I_2$), two are independent, the other two are dependent parameters.
  - Various combinations of dependent and independent port voltages and currents exist, accordingly various $n \times n$ matrix definitions for linear $n$-port networks exist, known as $Y$-, $Z$-, $h$- and $g$- parameters.

# Electrical Networks (3)

- It is more convenient to express the parameters for linear networks in the frequency domain
  - Avoids solving differential equations! Example: Simple RC two-port network
    - ➢ Only for time-invariant and linear networks!




$Y$- parameters:

$$Y = \frac{1}{R}\begin{bmatrix} 1 & -1 \\ -1 & 1 + j\omega RC \end{bmatrix}$$

$Z$- parameters:

$$Z = \frac{1}{j\omega C}\begin{bmatrix} 1 + j\omega RC & 1 \\ 1 & 1 \end{bmatrix}$$

- Voltage/current-based network parameters fail at high frequencies!
  - Now voltages and currents are a function of frequency (or time) AND space: $V(\omega, z)$, $I(\omega, z)$
    - ➢ Originating from time/space varying EM-fields
  - Example: RC two-port network embedded between transmission-lines
    - ➢ While it is still possible to solve the network problem, it becomes complicated and cumbersome based on $V$ and $I$
  - The circuit may become unstable or might be damaged, when operating on a short or open end for characterizing the network parameters
  - Due to parasitic effects, a reliable measurement of $V$ and $I$ becomes almost impossible.
- Resolution: RF scattering parameters based on power-waves for linear networks which include distributed elements
  - The magnitude of a traveling wave is independent of the location $z$ in a lossless transmission-line

# Principle of Scattering (S)-Parameters

- **Analogy to optical waves**
  - **Light falls on a car window**
    - Some parts of the incident light is reflected (you see the mirror image)
    - Other parts of the light is transmitted through the window (you can still see objects inside the car)
  - **Optical reflection and transmission coefficients of the window glass define the ratio between reflected and transmitted light.**

- **Similar in RF networks:**
  **The scattering (S)-parameters of an $n$-port RF network (DUT) is characterized by incident and reflected / transmitted (power) waves.**

# Generalized S-Parameters

- **for an arbitrary $n$-port microwave or RF network are defined by a set of normalized complex voltage waves:**

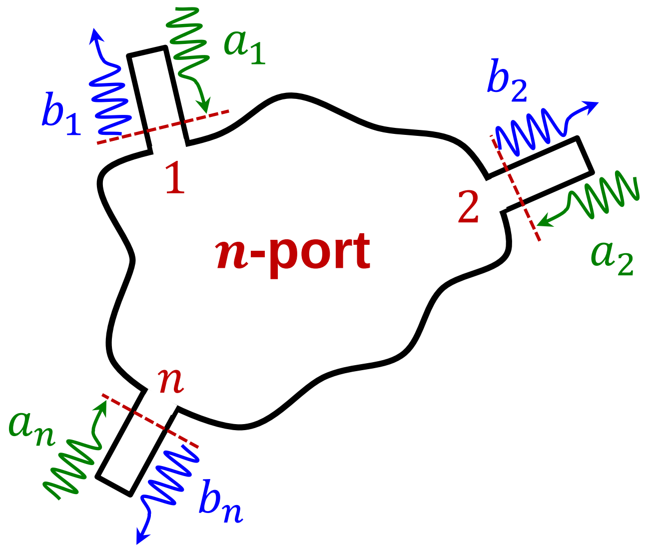
incident wave at port $i$:

$$a_i = \frac{V_i + Z_i I_i}{2\sqrt{\Re\{Z_i\}}} = \frac{V_i^{inc}}{\sqrt{\Re\{Z_i\}}}$$

reflected wave at port $i$:

$$b_i = \frac{V_i - Z_i^* I_i}{2\sqrt{\Re\{Z_i\}}} = \frac{V_i^{refl}}{\sqrt{\Re\{Z_i\}}}$$

$Z_i^*$: conjugate complex

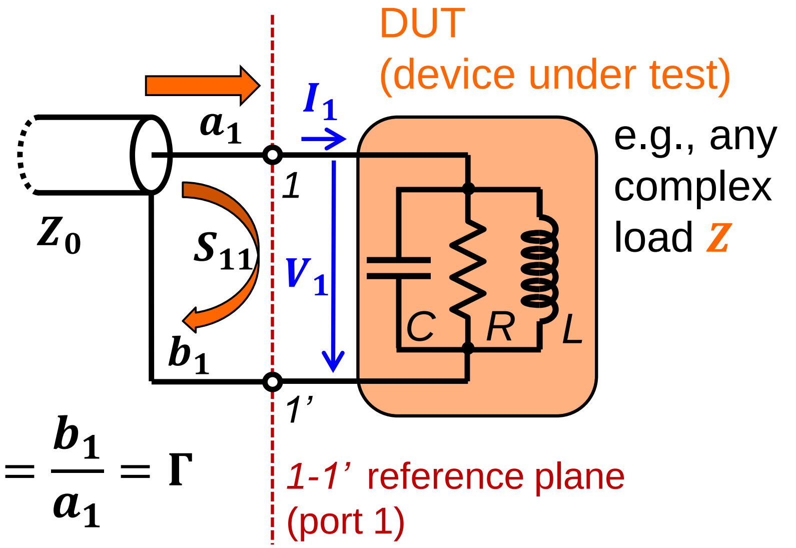



  - **as incident $a_i$ and reflected / transmitted $b_i$ power waves at the $i^{th}$ port of the network, defined by the terminal voltage $V_i$ and current $I_i$, and an arbitrary reference impedance $Z_i$**
    - Please note the complex notation implies linear, time-invariant networks describe in the frequency-domain

- **Today, for most practical cases the RF network, also called "device under test" (DUT) is characterized by a vector network analyzer (VNA), connected with coaxial cables (transmission-lines) with a characteristic impedance of $Z_0 = 50\ \Omega$ to the ports.**
  - **Usually the S-parameters are defined for a port reference impedance: $Z_i = Z_0 = 50\ \Omega$**
  - **Some VNAs with a physical reference impedance of $Z_0 = 50\ \Omega$ allow a mathematical port impedance conversion to adapt to a port reference impedance $Z_i \neq 50\ \Omega$**

# S-Parameters – 1-port

- **Electrical / electronics networks**
  - $1 \cdots n$-port electronics circuits
  - Defined by voltages $V_i(\omega)$ or $v_i(t)$ and currents $I_i(\omega)$ or $i_i(t)$ at the port terminals
  - Characterized by circuit matrices, e.g., $Z$, $Y$, $h$, etc.
- **RF / microwave networks**
  - $1 \cdots n$-port RF DUT circuit or subsystem, e.g., filter, amplifier, transmission-line, hybrid, circulator, resonator, etc., which may include distributed elements
  - Defined by incident $a_i(\omega)$ and reflected / transmitted waves $b_i(\omega)$ at a reference plane $s$ (physical position) at the ports.
  - Characterized by a scattering parameter (S-parameter) matrix of the reflected and transmitted power waves, typically as a function of the frequency $f = 2\pi/\omega$
  - Normalized to a reference impedance of typically $Z_0 = 50\ \Omega$

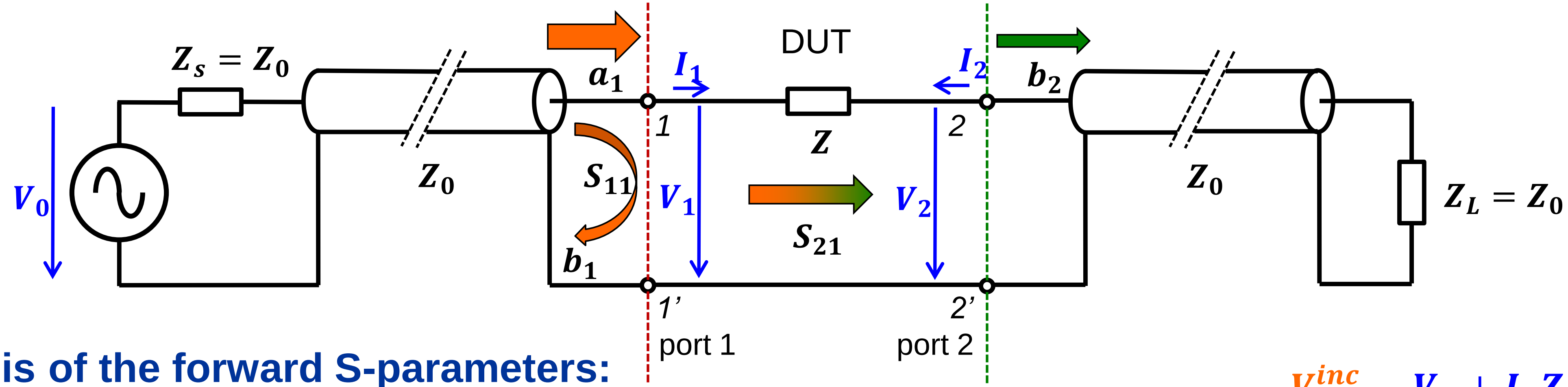



$$S_{11} = \frac{b_1}{a_1} = \Gamma$$

**1-port RF network (DUT) example**

- S-Parameters allow to characterize the DUT with the measurement equipment located at some physical distance
- All high frequency effects of distributed elements are included with respect to the reference plane

# S-Parameters – 2-port (1)




- **Analysis of the forward S-parameters:**

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \equiv \text{input reflection coefficient}$$

$$(Z_L = Z_0 \Rightarrow a_2 = 0)$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \equiv \text{forward transmission gain}$$

  – **3 +-port networks still can be fully characterized with a 2-port VNA, but always remember:**

  – **Terminated unused ports in their characteristic impedance!**

– **Independent parameters:**

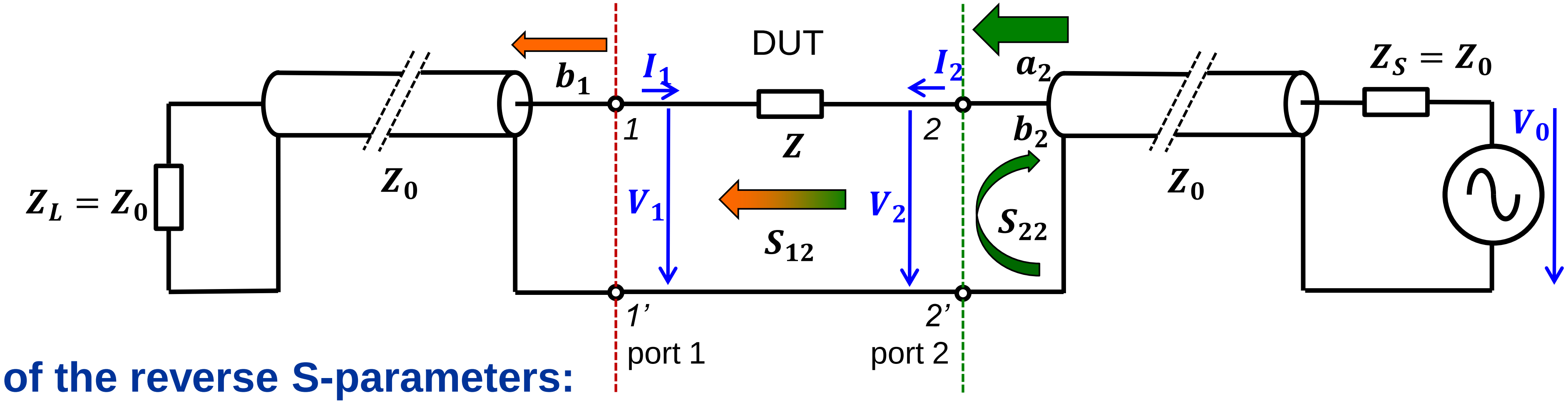
$$a_1 = \frac{V_1^{inc}}{\sqrt{Z_0}} = \frac{V_1 + I_1 Z_0}{2\sqrt{Z_0}}$$

– **Dependent parameters:**

$$b_1 = \frac{V_1^{refl}}{\sqrt{Z_0}} = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}}$$

$$b_2 = \frac{V_2^{refl}}{\sqrt{Z_0}} = \frac{V_2 - I_2 Z_0}{2\sqrt{Z_0}}$$

# S-Parameters – 2-port (2)




- **Analysis of the reverse S-parameters:**

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \equiv \text{output reflection coefficient}$$

$$(Z_L = Z_0 \Rightarrow a_1 = 0)$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \equiv \text{reverse transmission gain}$$

  - **Examples of 2-ports DUT: filters, amplifiers, attenuators, transmission-lines (cables), etc.**
  - **ALL ports ALWAYS need to be terminated in their characteristic impedance!**

- **Independent parameters:**

$$a_2 = \frac{V_2^{inc}}{\sqrt{Z_0}} = \frac{V_2 + I_2 Z_0}{2\sqrt{Z_0}}$$

- **Dependent parameters:**

$$b_1 = \frac{V_1^{refl}}{\sqrt{Z_0}} = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}}$$

$$b_2 = \frac{V_2^{refl}}{\sqrt{Z_0}} = \frac{V_2 - I_2 Z_0}{2\sqrt{Z_0}}$$

# S-Parameters – 2-port (3)

- **Linear equations for a 2-port network (DUT):**

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

  - **with:**

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \equiv \text{input reflection coefficient}$$
$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \equiv \text{output reflection coefficient}$$

} impedance measurements

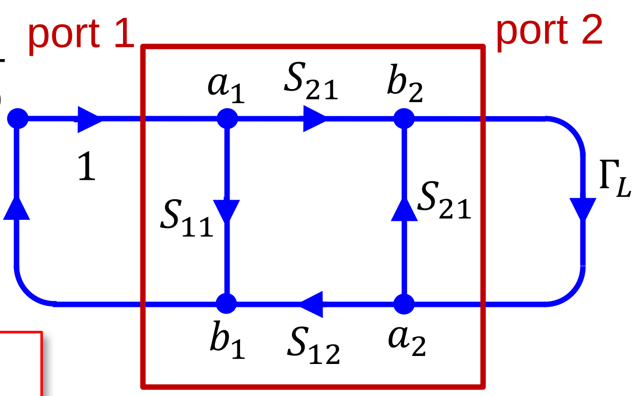
$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \equiv \text{forward transmission (insertion) gain}$$
$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \equiv \text{reverse transmission (insertion) gain}$$

} transmission (insertion) measurements

$$b_S = \frac{V_S\sqrt{Z_0}}{Z_S + Z_0}$$

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$




signal flow graph (SFG) of a 2-port network

- port1: $V_S$ with $Z_s \neq Z_0$
- port 2: $Z_L \neq Z_0$

# S-Parameters – $n$-port

- **Reflection coefficient and impedance at the $i^{th}$-port of a RF network (DUT):**

$$S_{ii} = \frac{b_i}{a_i} = \frac{\frac{V_i}{I_i} - Z_0}{\frac{V_i}{I_i} + Z_0} = \frac{Z_i - Z_0}{Z_i + Z_0} = \Gamma_i$$

$$Z_i = Z_0 \frac{1 + S_{ii}}{1 - S_{ii}} \text{ with } Z_i = \frac{V_i}{I_i} \text{ being the input impedance at the } i^{th} \text{port}$$

- **Power reflection and transmission for a $n$-port network (DUT):**

$$|S_{ii}|^2 = \frac{\text{power reflected from port } i}{\text{power incident on port } i}$$

$$|S_{ij}|^2 = \text{transmitted power between ports } i \text{ and } j$$

with all ports terminated in their characteristic impedance $Z_0$ and $Z_s = Z_0$

Here the US notion is used, where power $= |a_i|^2$.
European notation (often): power $= |a_1|^2/2$
These conventions have no impact on the S-parameters, they are only relevant for absolute power calculations

# The Scattering Matrix (1)

- **Waves traveling towards the *n*-port:** $(\boldsymbol{a}) = (\boldsymbol{a_1}, \boldsymbol{a_2}, \boldsymbol{a_2}, \ldots \boldsymbol{a_n})$
- **Waves traveling away from the *n*-port:** $(\boldsymbol{b}) = (\boldsymbol{b_1}, \boldsymbol{b_2}, \boldsymbol{b_2}, \ldots \boldsymbol{b_n})$

- **The relation between $\boldsymbol{a_i}$ and $\boldsymbol{b_i}$ ($\boldsymbol{i = 1 \ldots n}$) can be written as a system of $\boldsymbol{n}$ linear equations**
  ($a_i$ = the independent variable, $b_i$ = the dependent variable)

| | |
|---|---|
| one-port | $b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + S_{14}a_4 + \cdots$ |
| two-port | $b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + S_{24}a_4 + \cdots$ |
| three-port | $b_3 = S_{31}a_1 + S_{32}a_2 + S_{33}a_3 + S_{34}a_4 + \cdots$ |
| four-port | $b_4 = S_{41}a_1 + S_{42}a_2 + S_{43}a_3 + S_{44}a_4 + \cdots$ |

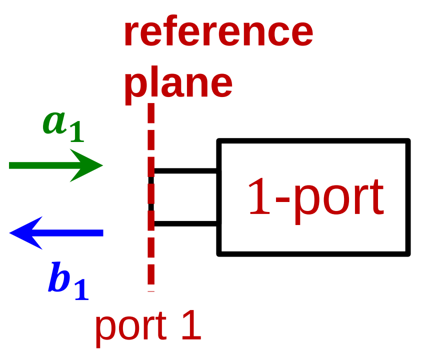
  – **in compact matrix form follows**

$$(\boldsymbol{b}) = (\boldsymbol{S})(\boldsymbol{a})$$

# The Scattering Matrix (2)

- **Its simplest form is for a passive 1-port network:**

$$(\mathbf{S}) = S_{11} \quad \Rightarrow \quad b_1 = S_{11} a_1$$

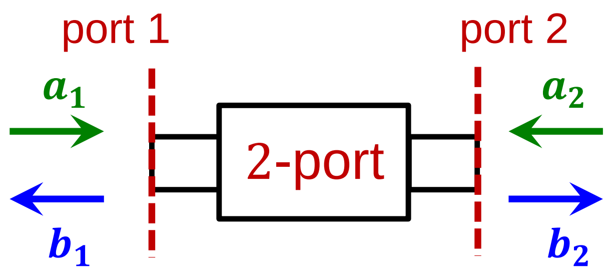



  - with the **reflection coefficient:**

$$\Gamma = S_{11} = \frac{b_1}{a_1}$$

- **Most popular is the 2-port network:**

$$(\mathbf{S}) = \begin{pmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{pmatrix} \quad \Rightarrow \quad \begin{aligned} b_1 &= S_{11} a_1 + S_{12} a_2 \\ b_2 &= S_{21} a_1 + S_{22} a_2 \end{aligned}$$




  - An unmatched load, present at port 2 with a **reflection coefficient** $\Gamma_{load}$ transfers to the input port as:

$$\Gamma_{in} = S_{11} + \frac{S_{21} \Gamma_{load} S_{12}}{1 - S_{22} \Gamma_{load}}$$

# S-Matrix Properties

- **A port of the network is matched if:** $\boldsymbol{S_{ii}=0}$
  - **i.e., no reflections!**
- **A $\boldsymbol{n}$-port is reciprocal if:** $(\boldsymbol{S})^{\boldsymbol{T}}=(\boldsymbol{S}) \;\Rightarrow\; \boldsymbol{S_{ij}=S_{ji}} \;\forall\, \boldsymbol{i,j}$ $\quad (S)^T$: transpose

  Matrix symmetry
  - **Most passive components are reciprocal, e.g., resistor, capacitor, inductor, transformer, etc.**
    - But not components with inhomogeneous material properties, e.g., magnetized ferrites, plasma, etc.
  - **Active components, like amplifiers are non-reciprocal**
- **A $\boldsymbol{n}$-port is symmetric if:** $\boldsymbol{S_{ij}=S_{ji}} \;\wedge\; \boldsymbol{S_{ii}=S_{jj}}$

  Matrix symmetry and electrical symmetry
  - **It needs to be reciprocal, and input and output reflection coefficient need to be equal.**
- **A $\boldsymbol{n}$-port is passive and lossless if the matrix $(\boldsymbol{S})$ is unitary:** $(\boldsymbol{S})^{\dagger}(\boldsymbol{S})=(\boldsymbol{S})^{\boldsymbol{T}}(\boldsymbol{S})^{*}=(\boldsymbol{I})$

  $(S)^{\dagger}=(S^{*})^{T}$: conjugate transpose
  $(I)$: identity matrix
  - **Example: passive, lossless 2-port:**

$$(S^{*})^{T}(S)=\begin{pmatrix}S_{11}^{*} & S_{21}^{*}\\ S_{12}^{*} & S_{22}^{*}\end{pmatrix}\begin{pmatrix}S_{11} & S_{12}\\ S_{21} & S_{22}\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \;\Rightarrow\; \begin{gathered}\angle S_{11}-\angle S_{12}=\angle S_{21}-\angle S_{22}-\pi\\ |S_{11}|=|S_{22}|,\qquad |S_{12}|=|S_{21}|\\ |S_{11}|=\sqrt{1-|S_{12}|^{2}}\end{gathered}$$

# S-Matrix Examples – 1-port

- Examples for 1-port S-matrices are any simple, passive (complex) impedances $Z$
  - Any R, L, C, RL, RC, LC and RLC circuit or any combinations of those elements leading to a single port network, which of course also my include distributed (transmission-line) elements
  - "Special" cases are:
    - $Z = Z_0 \Rightarrow S_{11} = 0$ (matched, ideal termination)
    - $Z = 0 \Rightarrow S_{11} = -1$ (ideal short)
    - $Z = \infty \Rightarrow S_{11} = +1$ (ideal open)
  - If $|S_{11}| > 1$ an active element is involved, e.g., a reflection amplifier

$$(\boldsymbol{S}) = \boldsymbol{S}_{11} = \boldsymbol{Z}$$

- Strictly speaking, a simple RF resonator, e.g., a "pill-box" cavity, is a 3-port
  - One coaxial or waveguide port as RF power coupler, plus two beam (waveguide) ports.
- However, for many practical cases it can be treated as 1-port
  - The mode of interest,<br>e.g., TM010, is trapped with no or negligible fields contribution near the beam-ports
  - We consider only a single coupler to characterize,<br>e.g., the TM010 mode in terms of a 1-port S-parameter measurement
    - Typically applying an RLC-parallel equivalent circuit

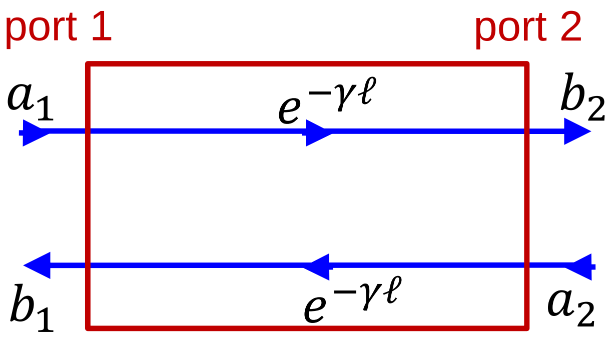
# S-Matrix Examples – 2-port

- **Ideal (matched: $Z = Z_0$) transmission-line of length $\ell$**

$$(S) = \begin{pmatrix} 0 & e^{-\gamma\ell} \\ e^{-\gamma\ell} & 0 \end{pmatrix}$$

$\gamma = \alpha + j\beta$: propagation constant
$\alpha$: attenuation constant in [*Np/m*]
$\beta = 2\pi/\lambda$: phase constant [*rad/m*]

signal flow graph (SFG):




  – **For a lossless transmission-line:** $\alpha = 0 \;\Rightarrow\; |S_{21}| = |S_{12}| = 1$

  – **For a lossless line of length $\ell = \lambda/4$:** $(S) = \begin{pmatrix} 0 & -j \\ -j & 0 \end{pmatrix}$

- **Ideal attenuator**

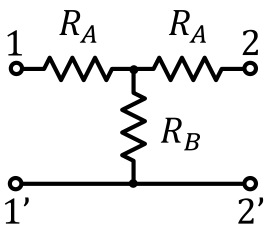
$$(S) = \begin{pmatrix} 0 & k \\ k & 0 \end{pmatrix}$$

$\mathrm{k} = V_2/V_1 = 10^{-(\Delta dB/20)}$: attenuation $k < 1, k \in \mathbb{R}$
$\Delta dB = 20\log_{10} V_1/V_2$: attenuation in *dB*
$\alpha = -\ln k$: attenuation in *neper*

SFG example: *3 dB* attenuator

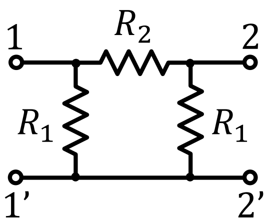
$T$-attenuator:

$$R_A = \frac{1-k}{1+k} Z_0$$
$$R_B = \frac{2k}{1-k^2} Z_0$$

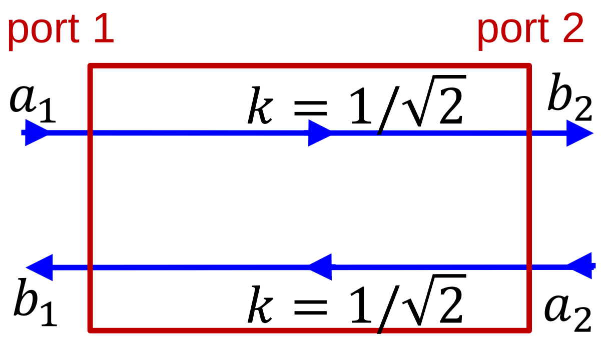



$\pi$-attenuator:

$$R_1 = \frac{1-k}{1+k} Z_0$$
$$R_2 = \frac{1-k^2}{2k} Z_0$$




$$S_{21} = \frac{b_2}{a_1} = \frac{1}{\sqrt{2}}$$
$$S_{12} = \frac{a_2}{b_1} = \frac{1}{\sqrt{2}}$$

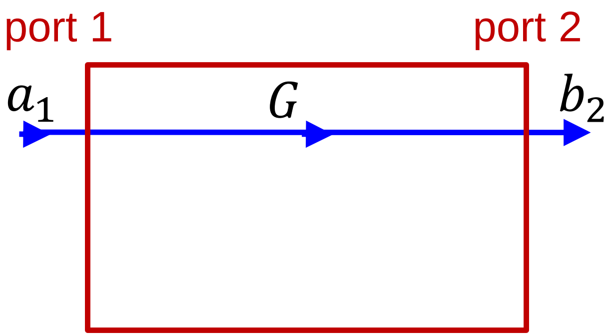


# S-Matrix Examples – 2-port

- **Ideal amplifier (gain stage)**

$$(\boldsymbol{S}) = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \boldsymbol{G} & \mathbf{0} \end{pmatrix}$$

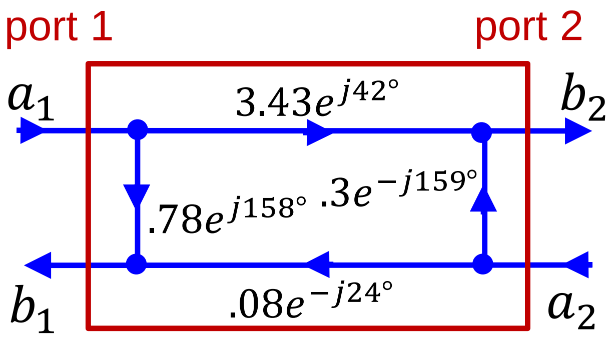
$\mathrm{G} = V_{out}/V_{in} = 10^{g/20}$: voltage gain $\mathrm{G} > 1$
$\mathrm{g} = 20\log_{10} V_{out}/V_{in}$: voltage gain in *dB*




- **Low-noise RF transistor**

$$(\boldsymbol{S}) = \begin{pmatrix} \mathbf{0.78}e^{j158°} & \mathbf{0.08}e^{-j24°} \\ \mathbf{3.43}e^{j42°} & \mathbf{0.3}e^{-j159°} \end{pmatrix}$$

Datasheet *Avago* VMMK-1218:
$f = 10\ GHz, Z_0 = 50\Omega, T_A = 25°C,$
$V_{ds} = 2V, I_{ds} = 20mA$



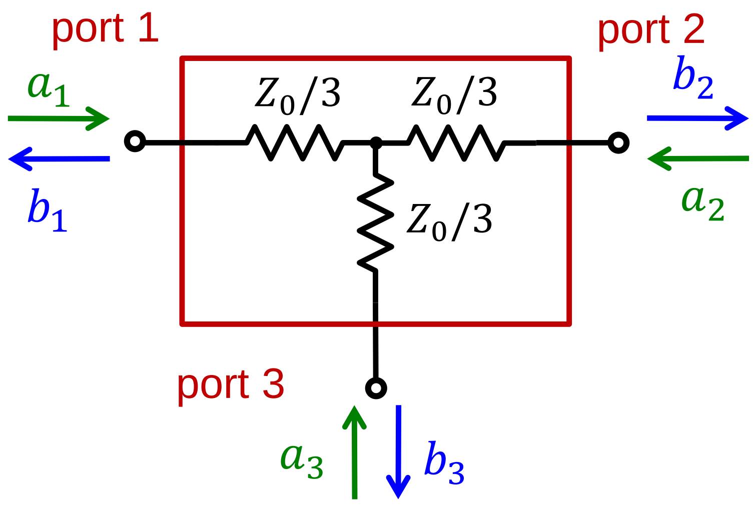

- ***Avago* VMMK-1218**
- **E-pHEMT GaAs FET**
  - The S-parameters are different at other frequencies and operational conditions
  - The transistor requires impedance matching networks at in- and output

# S-Matrix Examples – 3-port

- **3-port resistive power divider**

$$(S) = \frac{1}{2}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$b_1 = \frac{1}{2}(a_2 + a_3)$$
$$b_2 = \frac{1}{2}(a_1 + a_3)$$
$$b_3 = \frac{1}{2}(a_1 + a_2)$$

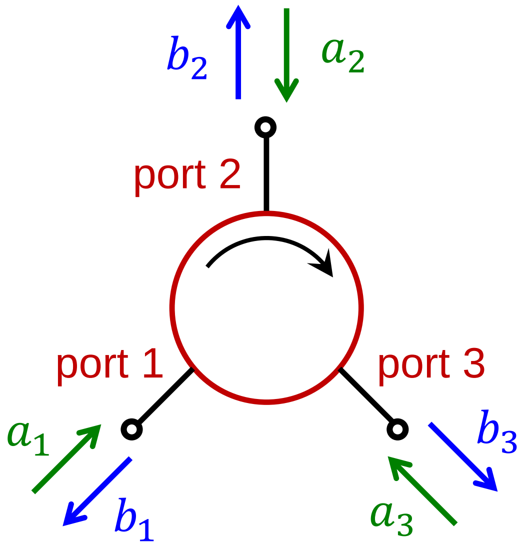



  - **The transfer-loss between *ij*-ports is 6 dB.**

- **Ideal circulator**

$$(S) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

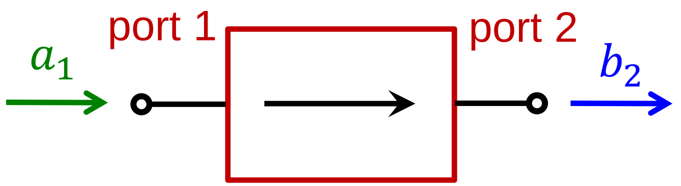
$$b_1 = a_3$$
$$b_2 = a_1$$
$$b_3 = a_2$$




  - **Matched, but not reciprocal**

- **Isolator, based on the circulator**
  - **Terminating, e.g., port 3 internally results in a 2-port, called isolator**

$$(S) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad b_2 = a_1$$

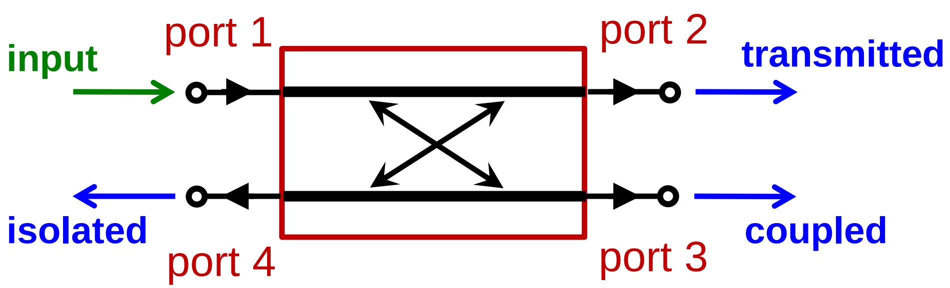


# S-Matrix Example – 4-port

- **Ideal directional coupler**

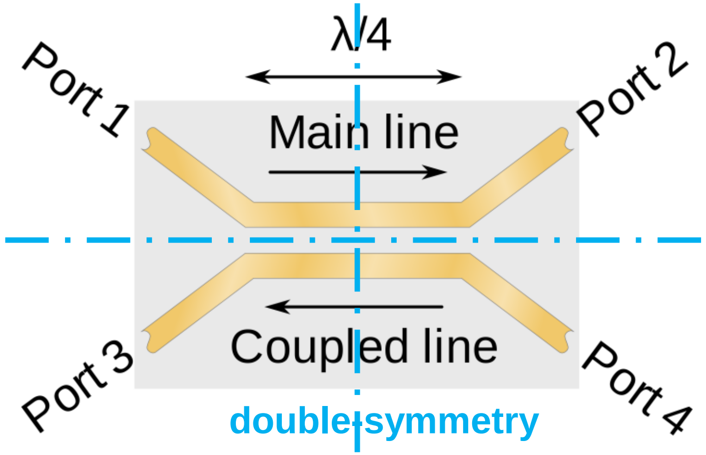
coupling coefficient: $k = \left|\frac{b_2}{a_1}\right|$; $\kappa dB = 20\log_{10} k$

$$(S) = \begin{pmatrix} 0 & -j\sqrt{1-k^2} & k & 0 \\ -j\sqrt{1-k^2} & 0 & 0 & k \\ k & 0 & 0 & -j\sqrt{1-k^2} \\ 0 & k & -j\sqrt{1-k^2} & 0 \end{pmatrix}$$




- **Operating at the center frequency**
- **Figures of merit (ideal, lossless):**
  - Coupling factor $C_{3,1} = \kappa dB = 10\log_{10}(P_3/P_1)\ [dB]$
  - Insertion loss $L_{i2,1} = -10\log_{10}(P_2/P_1)\ [dB]$
  - Coupling loss $L_{c2,1} = -10\log_{10}[1-(P_3/P_1)]\ [dB]$

  no losses: $L_{i2,1} = L_{c2,1}$
- **Coupler with losses, imperfections, etc.**
  - Isolation $I_{4,1} = -10\log_{10}(P_4/P_1)\ [dB]; I_{3,2} = -10\log_{10}(P_3/P_2)\ [dB]$
  - Directivity $D_{3,4} = -10\log_{10}(P_3/P_4) = -10\log_{10}(P_4/P_1) + 10\log_{10}(P_3/P_1)\ [dB]$




https://en.wikipedia.org/wiki/Power_dividers_and_directional_couplers

# S-Parameters in Practice

- In practice, S-parameters are a function of the frequency: $S(f)$
  - Some instruments or applications can also provide time-domain S-parameters

- In most real-world practical situations, S-parameters are acquired by a measurement, e.g., characterization of a RF component or sub-system by a VNA.
  - By characterizing the DUT over a range of frequencies, $f_{min} < f < f_{max}$ in steps of $\Delta f$

- Also, numerical RF analysis tools (Qucs, ADS, Microwave Office, etc.) generate S-parameters through linear RF circuit / systems simulations.
  - Numerical EM software tools (CST, HFSS, etc.) and PCB tools (Cadence Allegro) can also generate S-parameters

- Both application types, VNA measurements and RF/EM simulation software exchange S-parameters on a file basis
  - The *SnP Touchstone* ASCII file format is de-facto the industry standard for S-parameters
  - Example *Touchstone s2p* file:

# *SnP Touchstone* S-Parameter Files

BRSTM_refline_2-4GHz_20001.s2p

```
!Keysight Technologies,P5024A,MY58100247,A.15.20.07
!Date: Wednesday, October 06, 2021 16:19:17
!Correction: S11(C  2-Port  )
!S21(C  2-Port  )
!S12(C  2-Port  )
!S22(C  2-Port  )
!S2P File: Measurements: S11, S21, S12, S22:
# Hz S  dB   R 50
2000000000 -2.5430779 -88.497566 -18.274168 38.763039 -18.26178 38.742687 -2.4251425 -85.792152
2000100000 -2.5365531 -88.473915 -18.266272 38.606499 -18.269154 38.716461 -2.4215624 -85.861908
2000200000 -2.5314419 -88.471634 -18.280306 38.559021 -18.258684 38.624985 -2.4253747 -85.806236
2000300000 -2.5216722 -88.617905 -18.269596 38.352692 -18.266785 38.342678 -2.4210978 -85.8368
2000400000 -2.5178108 -88.521271 -18.257862 38.298622 -18.275055 38.266792 -2.4244151 -85.914787
2000500000 -2.5327342 -88.484985 -18.263821 38.31945 -18.27046 38.20409 -2.4263382 -85.85775
2000600000 -2.5191193 -88.462044 -18.262426 38.195797 -18.246525 38.250874 -2.3989511 -85.885635
2000700000 -2.5219827 -88.44445 -18.253748 38.216946 -18.245417 38.138355 -2.4155877 -85.859711
2000800000 -2.5198817 -88.588783 -18.255999 37.902744 -18.257757 38.035915 -2.415241 -85.887199
2000900000 -2.5370498 -88.496101 -18.256392 38.004234 -18.258446 37.872906 -2.4170656 -85.854294
2001000000 -2.5363033 -88.544846 -18.252661 37.830475 -18.259445 37.925297 -2.4102871 -85.901245
2001100000 -2.5417418 -88.47406 -18.270754 37.731613 -18.252403 37.74762 -2.4158244 -85.822517
```

! header

\# format

2-port VNA file

*Touchstone v1.1* example file

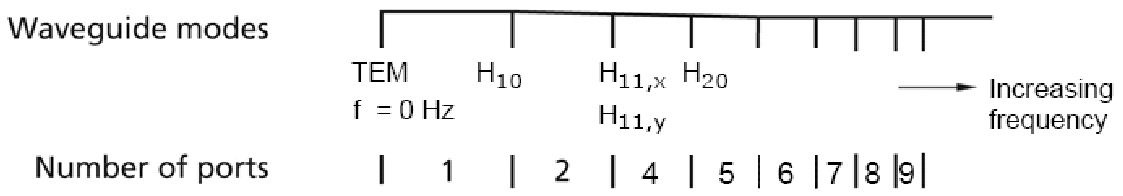
- v2.0 is different, file ext. **.ts*

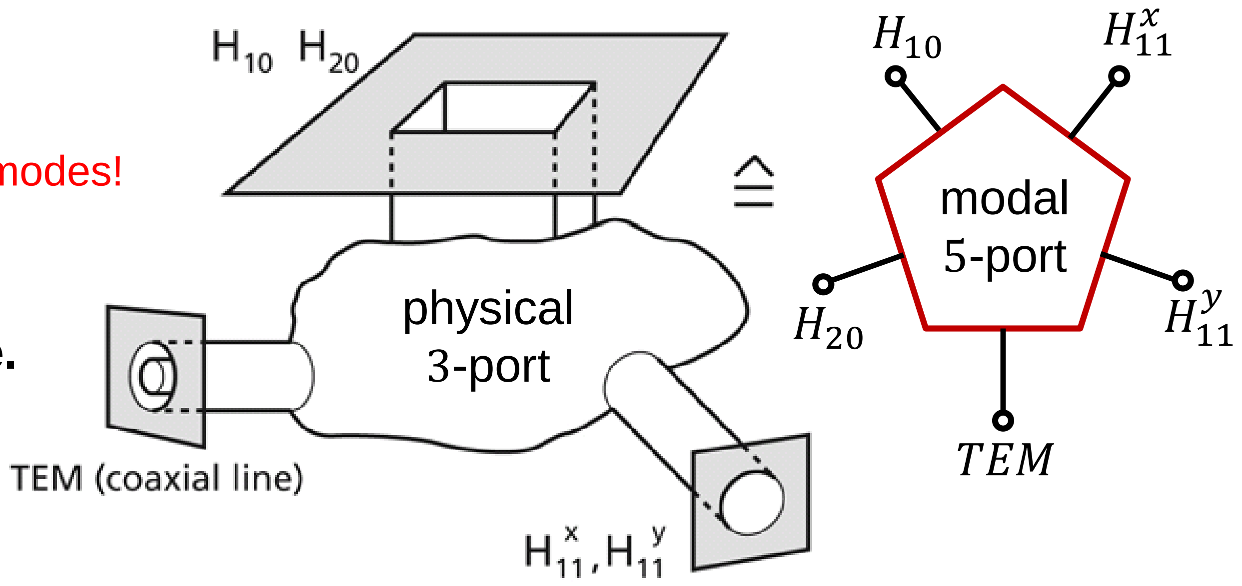
frequency $f$ $|S_{11}|$[dB] $\angle S_{11}$[deg]

- **The file name extension specifies the number $n$ of ports**
  - **Attention: NOT equal to the number of columns! The carriage return (CR) is different between s1p, s2p and s3p, s4p files!**
- **The comment header (!) includes general information, e.g., type of instrument, measurement time, etc.**
- **The format line (#) defines the format (mag[dB],angle[deg], mag/angle, real/imag), stimulus units and the reference impedance**
- **The column delimiter varies, e.g., space, comma, semicolon, etc.**
  - **Column order in the example file: *f S11dB S11a S21dB S21a S12dB S12a S22dB S22a***

# General $n$-Ports

- **A general $n$-port may include ports of different technologies, i.e., waveguides, as well as TEM transmission-lines, such as coaxial lines, microstrip lines etc.**
  - **In the frequency range of interest different modes may propagate at each physical port, e.g., several waveguide modes in a rectangular waveguide and/or higher order modes in a coaxial line..**
  - **Each EM-mode must then be represented by a distinct modal port.**
    - ➢ This is very important in EM-simulation to ensure the absorption of the energy for all modes!
  - **The number of modal ports needed generally, increases with frequency, as more waveguide modes can propagate.**




$H_{11}^x, H_{11}^y$: $x, y$-polarization of the $E_{11}$ circular mode

# S-Parameter Summary

- The scattering (S) parameters are based on incident and reflected normalized complex voltage waves (power waves), defined at the ports of a RF network.
- S-Parameters are used to characterize a linear, time-invariant RF component, circuit or sub-system as function of frequency under realistic operational conditions
  - The S-parameters are given in a matrix notation, and have complex values
- The characteristic of the S-matrix may provide additional details about the network, such as reciprocity, symmetry, losses.
- Typically, the S-parameters matrix of a RF network is acquired by measurement characterization with a vector network analyzer (VNA), or by a numerical analysis, e.g., circuit analysis or electromagnetic simulation software
- The S-parameter matrices of a set of networks can be converted to transfer (T) parameter matrices to enable a simple cascading of those networks
- The number of logical, modal ports might be higher than the number of physical ports for a general RF network utilizing various transmission-line technologies.

# Part IV: RF Measurement Methods

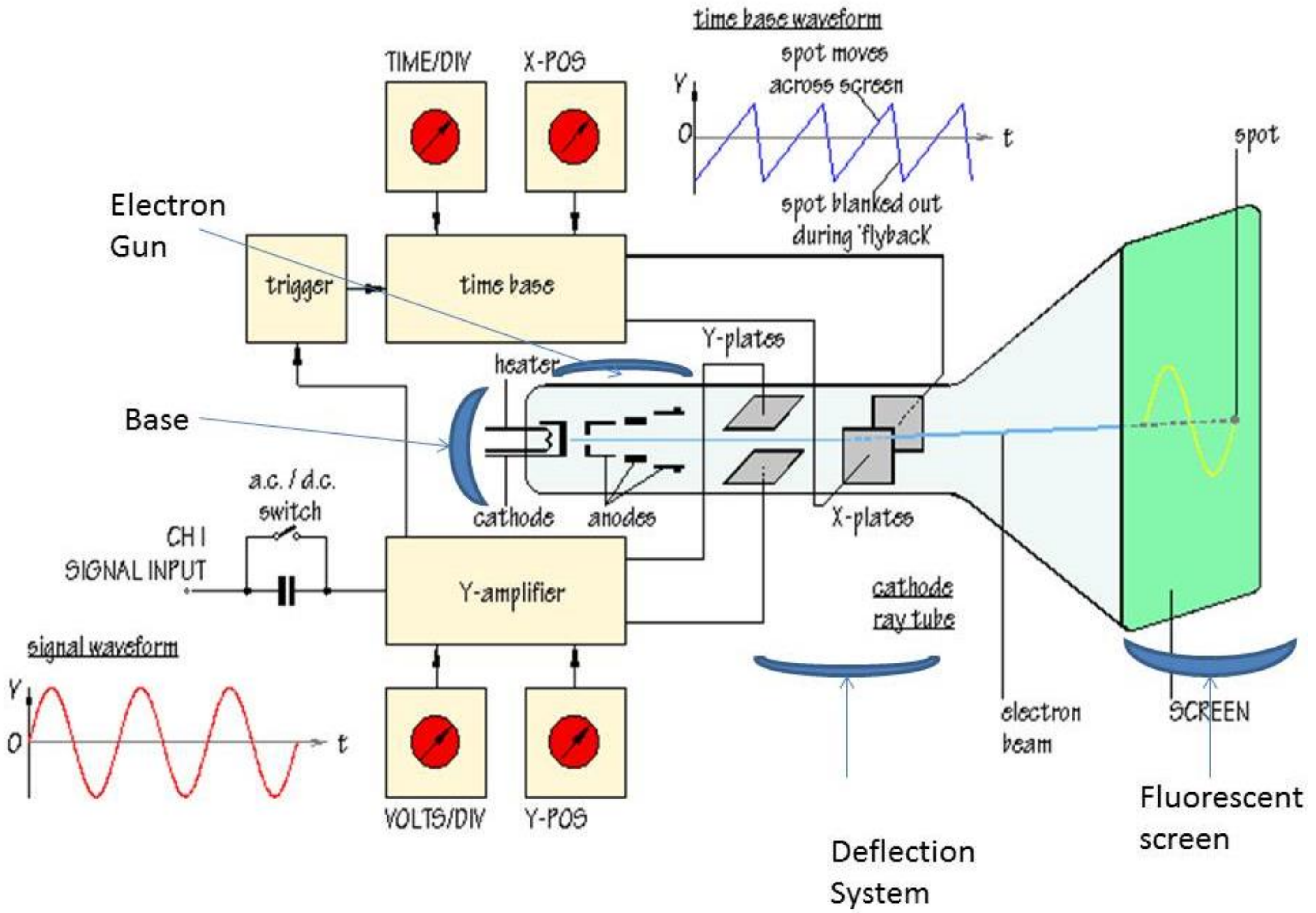
- **Overview of RF measurement instruments**
  - Oscilloscope, spectrum analyzer (SA), signal (FFT) analyzer, slotted measurement line, vector network analyzer (VNA)
- **The super-heterodyne receiver principle**
  - Modulation, down-conversion, mixer, spectrum analyzer block schematics
- **Reflection measurement with the slotted coaxial air-line**
- **S-parameter measurements**
  - Simple measurement setup, VNA block schematics
  - VNA calibration
  - Features of modern RF measurement equipment
  - Synthetic pulse measurements with the VNA
  - Measurement example: pillbox resonator characterization
    - ➢ Equivalent circuit parameters, $Q$-factor measurement in the *Smith*-chart, $R/Q$ measurement
  - Measurement of the beam-coupling impedance with a stretched-wire

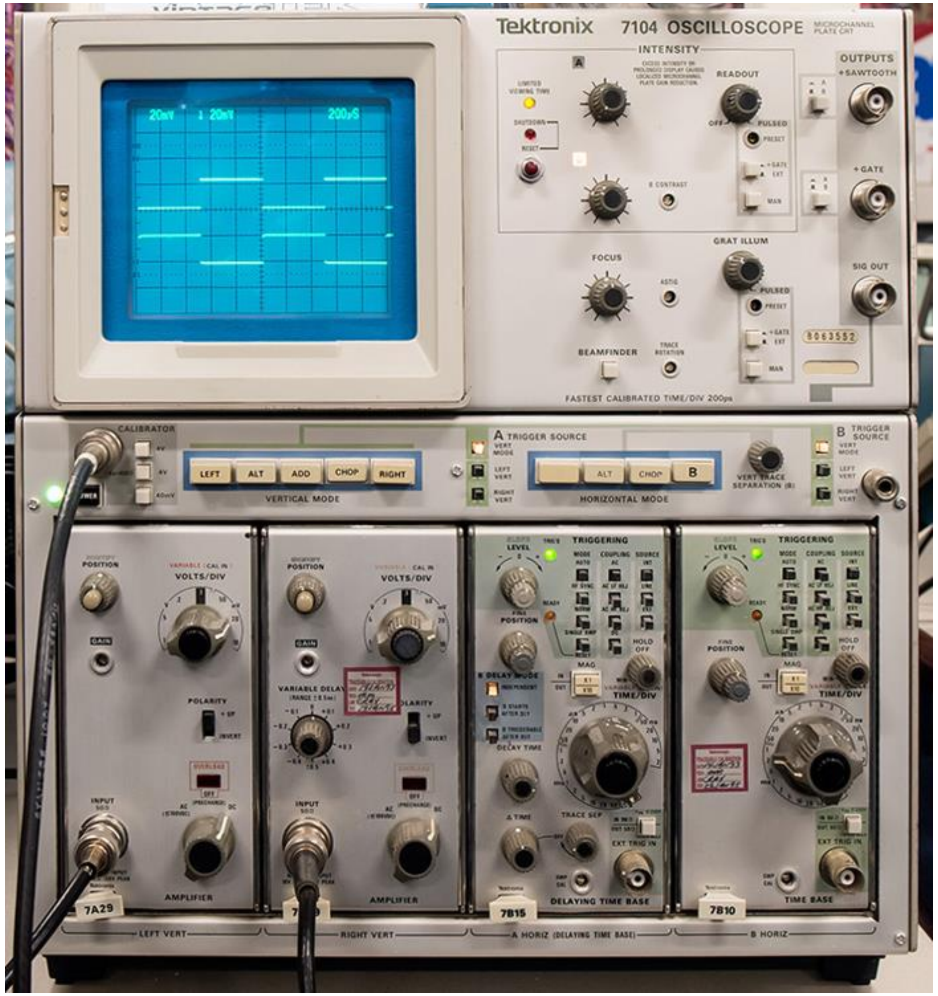
# RF Measurement Methods (1)

**There are different options to observe RF signals**
**Here some typical measurement tools:**

- **Oscilloscope: to observe signals in time-domain**
  - **periodic signals**
  - **burst and transient signals with arbitrary waveforms**
  - **application: direct observation of signals from a beam pick-up, from a test generator, or from other sources**
  - **visualizes the shape of a waveform, etc.**
  - **limited performance for the evaluation of non-linear effects.**

# Cathode Ray Tube (CRT) Oscilloscope

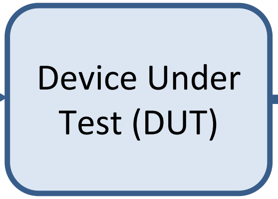


# Today: Digital Storage Oscilloscope (DSO)

…and digital signal generator
(AWG: arbitrary waveform generator)

50 GS/s, 10-bit AWG (*Tektronix*)




100 GHz bandwidth
240 GS/s oscilloscope
(*LeCroy*)

- **Signal processing based on fast ADCs and DACs**
  - **Similar "look and feel" as analog oscilloscopes, but better performance**
    - 8…12-bit multi-GS/s ADCs, still, be aware of aliasing effects!
    - Fast sampling oscilloscope require sufficient memory resources.
- **AWG or pulse generator & digital oscilloscope: Time-domain (TD) test setup**
  - **Device under test (DUT) characterization and trouble shooting**
    - Impulse, step, or arbitrary waveform (e.g., beam signal) as stimulus signal
    - High impedance probe for measurements on the printed circuit board (PCB)

# RF Measurements Methods (2)

- **Spectrum analyzer: to observe signals in a "frequency-domain like" fashion**
  - **sweeps in equidistant steps through a given frequency range**
  - **application: observation of spectrum from the beam, or from a signal generator or RF source, or the spectrum emitted from an antenna to locate EMI issues in the accelerator tunnel, etc.**
    - Also, DUT characterization in the laboratory, e.g., noise figure measurement on amplifiers (requires a noise source), intermodulation measurements on amplifiers (requires two RF generators).
  - **Requires periodic signals**
  - **Assumes time-invariance of the measurement object (DUT) throughout the frequency sweep**
  - **Large dynamic range!**

- **RF detection (Schottky) diode (RF power meter)**
  - **Supplies a rectified (video) output signal proportional to the RF signal level**
  - **Delivers no frequency or phase information but operates over a very broad frequency range few MHz to many GHz, and up to 90 dB dynamic range.**

# RF Measurements Methods (3)

- **Vector signal analyzer (VSA), sometimes called FFT analyzer**
  - **Acquires the RF signal, after down-conversion to an intermediate (IF) signal, in time-domain by fast sampling**
  - **Further numerical treatment in digital signal processors (DSPs)**
  - **Spectrum calculated using Fast Fourier Transform (FFT)**
  - **Combines features of an oscilloscope and a spectrum analyzer: Signals can be observed directly in time-domain, or in a frequency-domain like fashion**
  - **Contrary to the SA, also the spectrum of non-periodic signals and transients can be measured**
  - **Application: Observation of tune sidebands, transient behavior of a phase locked loop, single pass beam signal spectrum, etc.**
  - **Digital oscilloscopes and FFT analyzers share similar technologies, i.e., fast sampling and digital signal processing, and therefore can provide similar measurement options**
    - The digital oscilloscope directly digitizes the RF signal → limited dynamic range, large instantaneous bandwidth
    - The FFT analyzer digitizes the down-converted IF signal → large dynamic range, but (still) limited instantaneous bandwidth

# RF Measurements Methods (4)

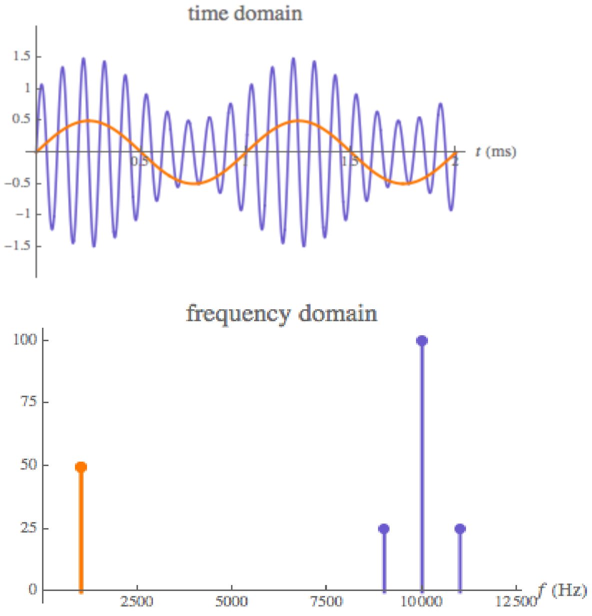
**Tools to characterize RF components and sub-systems:**

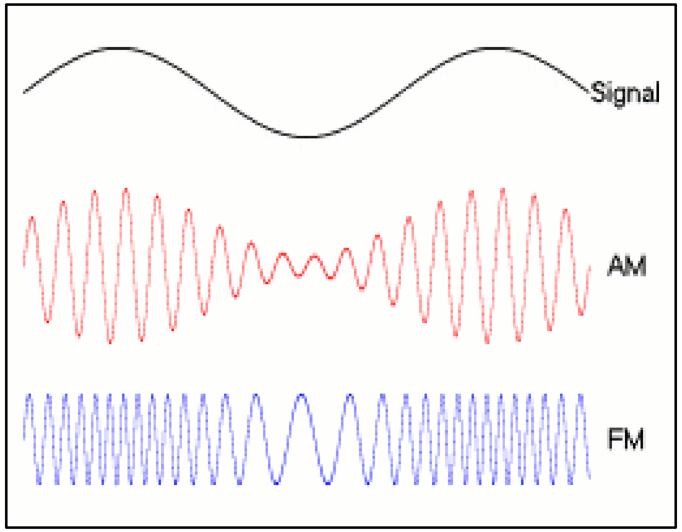
- **Slotted coaxial (or waveguide) measurement transmission-line**
  - **For study and illustration purposes only –<br>not anymore used in today's RF laboratory environment.**
- **Vector Network Analyzer (VNA)**
  - **Combines the functions of a vector spectrum analyzer (FFT analyzer),<br>a RF sweep generator, and a S-parameter test set (directional coupler)**
  - **Excites a *Device Under Test* (DUT, e.g., circuit, antenna, amplifier, etc.) network at a given sinusoidal *continuous wave* (CW) frequency, and measures the response in magnitude and phase => determines the S-parameters**
  - **Covers a selectable frequency range by measuring step-by-step at subsequent frequency points (like a spectrum analyzer, again requires the DUT to be time-invariant!)**
  - **Applications: characterization of passive and active RF components,<br>*Time Domain Reflectometry* (TDR) by Fourier transformation of the reflection response, etc.**
    - Also, power sweep measurements (1 $dB$ compression point),<br>4-port VNAs enable virtual ports: e.g., single-ended / differential port DUT characterization.
  - **The VNA is the most versatile and comprehensive tool in the RF laboratory!**

# RF Signals & Modulation, without Math!

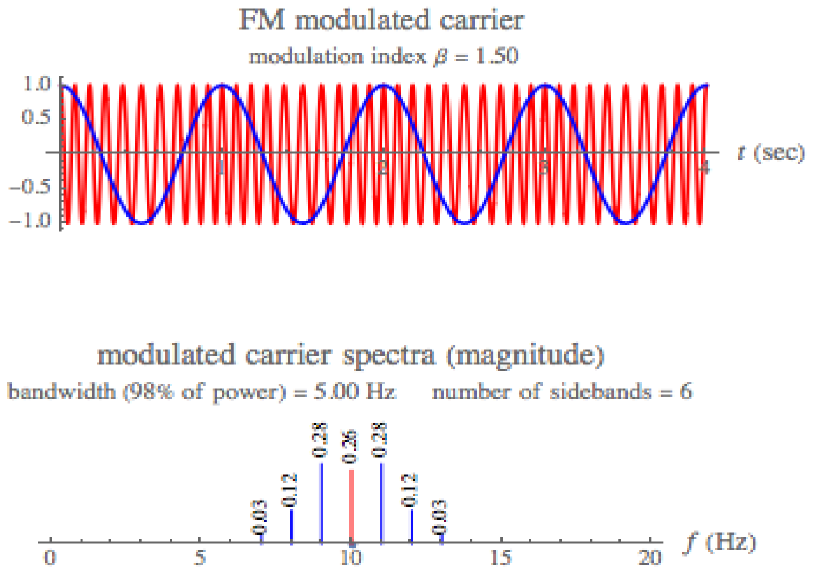
- **RF signals are continuous wave (CW),sinusoidal signals**
  - **Often, a high frequency carrier is modulated with low frequency information**
  - **Modulation appears "naturally" in ring accelerators as:**
    - Modulation is also provided through the LLRF system to the accelerating structures

AM: Betatron oscillations

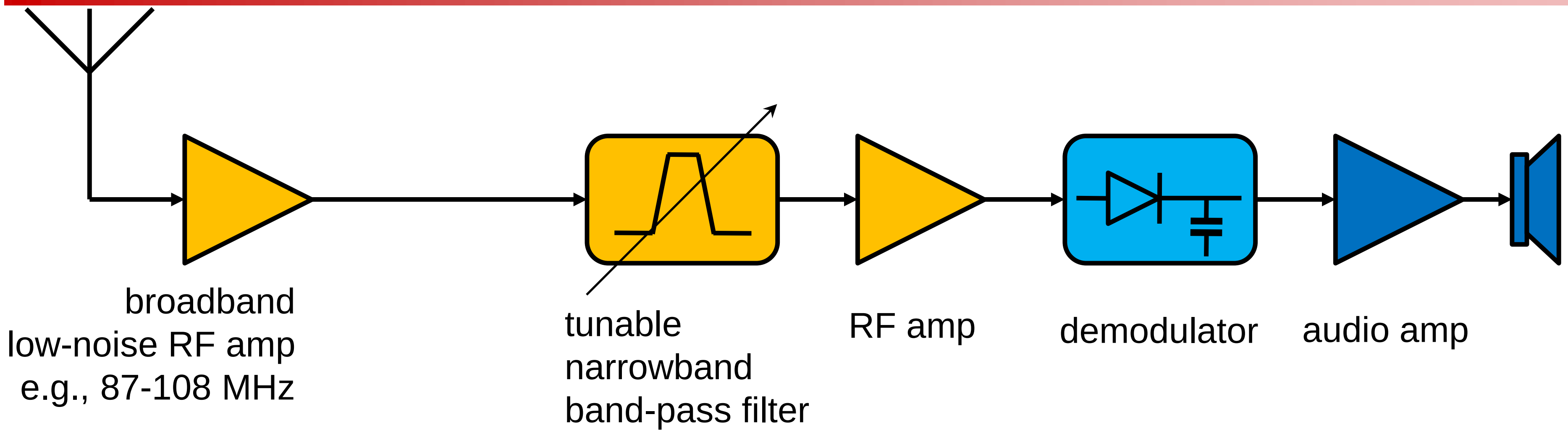



FM: Synchrotron oscillations



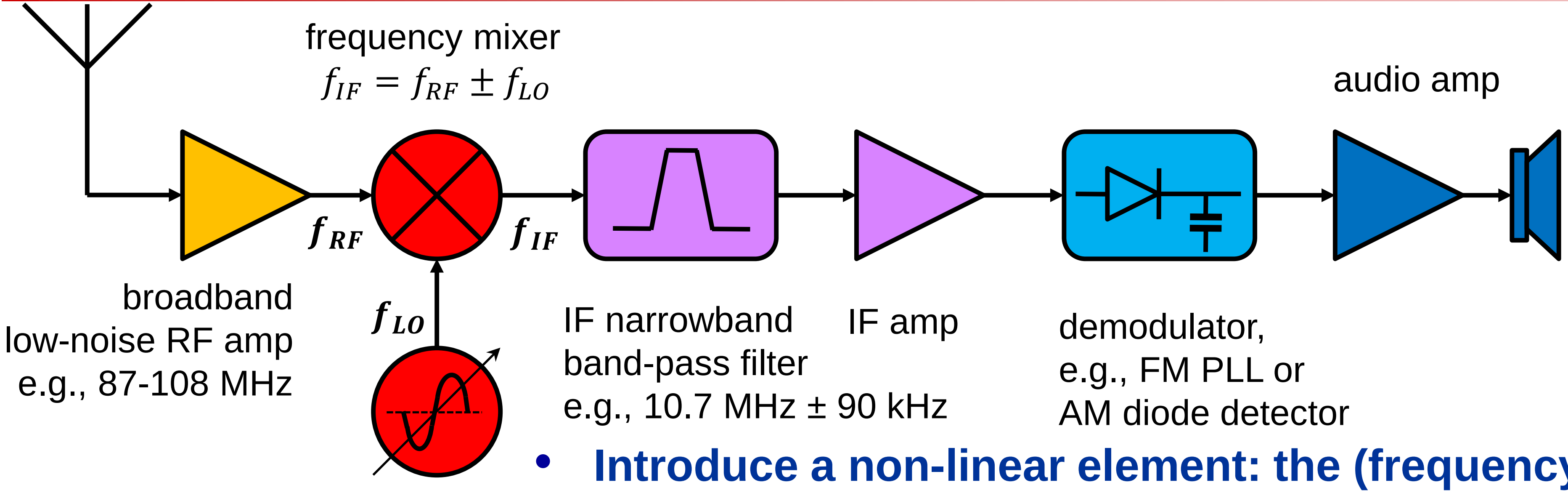
# A (too) simple Radio Receiver




- **…or: How does a ”traditional” analog radio works?**
  - **It was, and still is, difficult to make precisely tunable narrowband, band-pass filters for high frequencies (~100 MHz)!!**
  - **high frequency low-noise amplifiers are expensive!**
  - **high frequency demodulators are not trivial.**

  - **direct detection of radio and RF signals is challenging!**

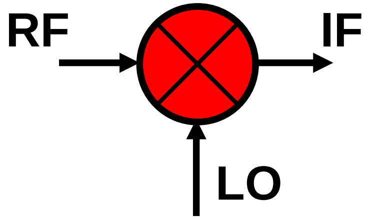
# The Super-Heterodyne Receiver




- **Introduce a non-linear element: the (frequency) mixer!**
  - **”down-convert” the RF band to a fixed ”intermediate” frequency (IF):** $f_{IF} = f_{RF} \pm f_{LO}$
  - **requires a tunable local oscillator (LO)**
  - **well manageable IF section:**
    - narrowband band-pass filter(s) (BPF) and amplifier(s)
  - **RF telecommunication standard**
  - **Often multiple mixing stages are used in modern RF instruments, e.g., spectrum and network analyzers**

# The RF Mixer as Down-Converter

$y_{RF}(t) = A_{RF}\sin(\omega_{RF}t + \varphi_{RF})$

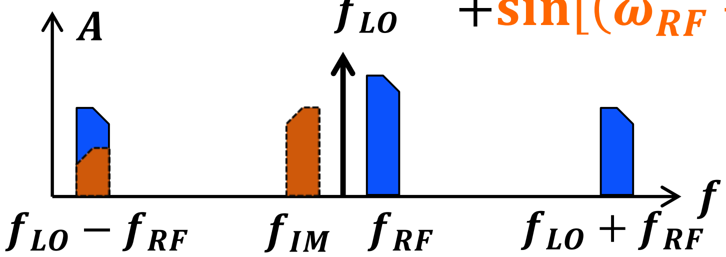



$y_{IF}(t) = y_{RF}(t)y_{LO}(t)$

$y_{LO}(t) = A_{LO}\sin(\omega_{LO}t + \varphi_{LO})$

- **Ideal mixer:** $f_{IF} = f_{RF} \pm f_{LO}$

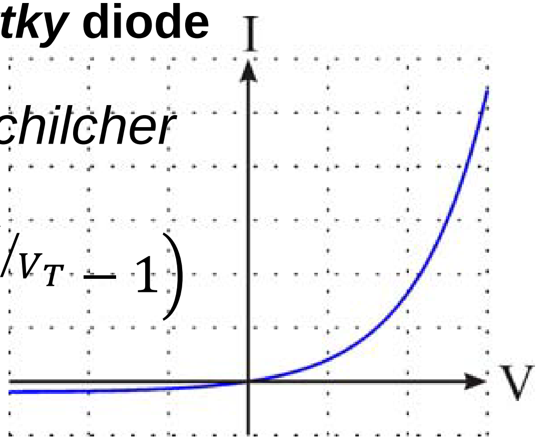
$$y_{IF}(t) = \frac{1}{2}A_{LO}A_{RF}\{\sin[(\omega_{RF} - \omega_{LO})t + (\varphi_{RF} - \varphi_{LO})] \quad \text{upper sideband}$$
$$+\sin[(\omega_{RF} + \omega_{LO})t + (\varphi_{RF} + \varphi_{LO})]\} \quad \text{lower sideband}$$




$I = f(V)$ of a *Schottky* diode

courtesy *T. Schilcher*

$$I = I_0\left(e^{V/V_T} - 1\right)$$

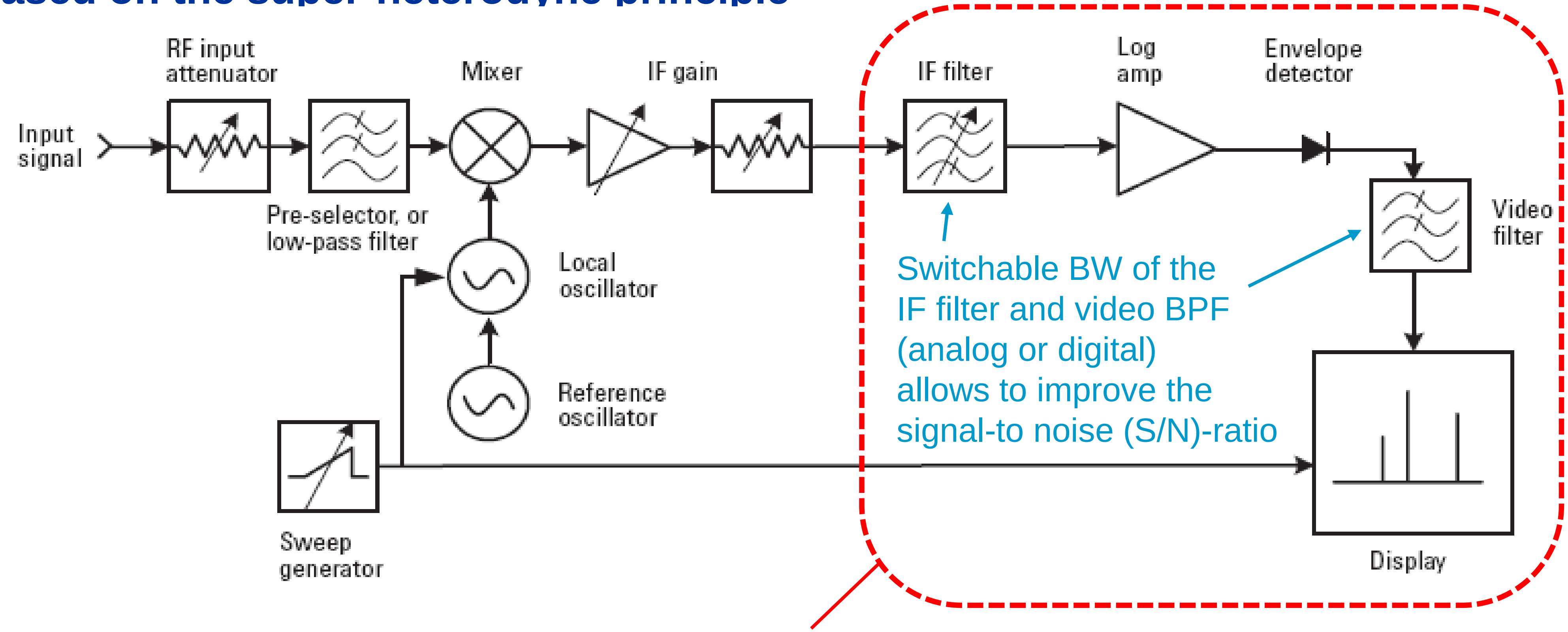



- **Frequency conversion**
  - $f_{RF} \neq f_{LO}$: heterodyne receiver
  - $f_{RF} = f_{LO}$: homodyne, demodulator
- **Real-world mixer:** $f_{IF} = mf_{RF} \pm nf_{LO}$
  - Image frequency: $f_{IM} = f_{LO} - f_{IF}$

$$\Delta I = I_0 e^{V/V_T}\left[\frac{\Delta V}{V_T} + \frac{1}{2}\left(\frac{\Delta V}{V_T}\right)^2 + \frac{1}{6}\left(\frac{\Delta V}{V_T}\right)^3 + \cdots\right]$$

# Simplified Spectrum Analyzer

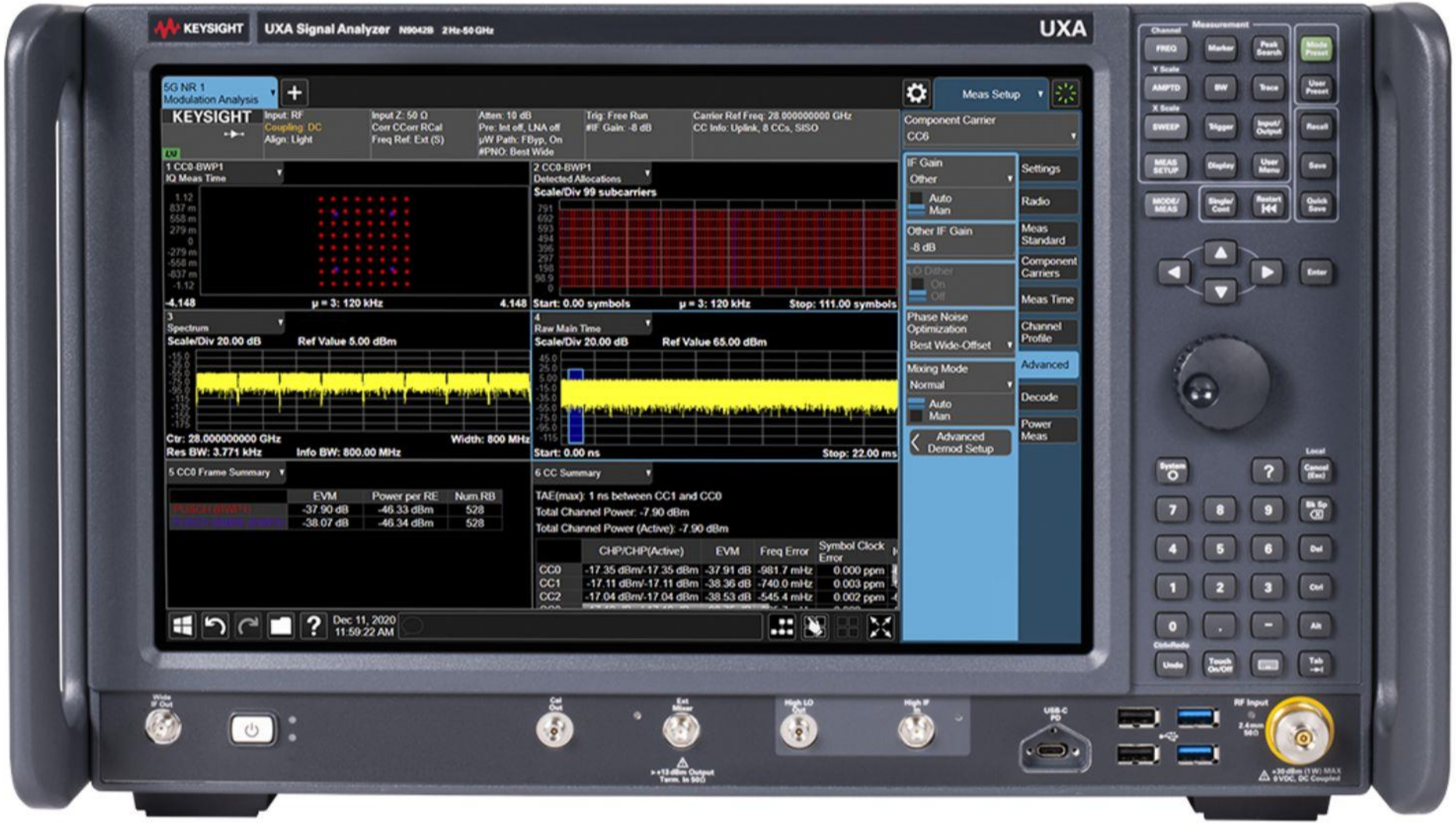
- **based on the super-heterodyne principle**



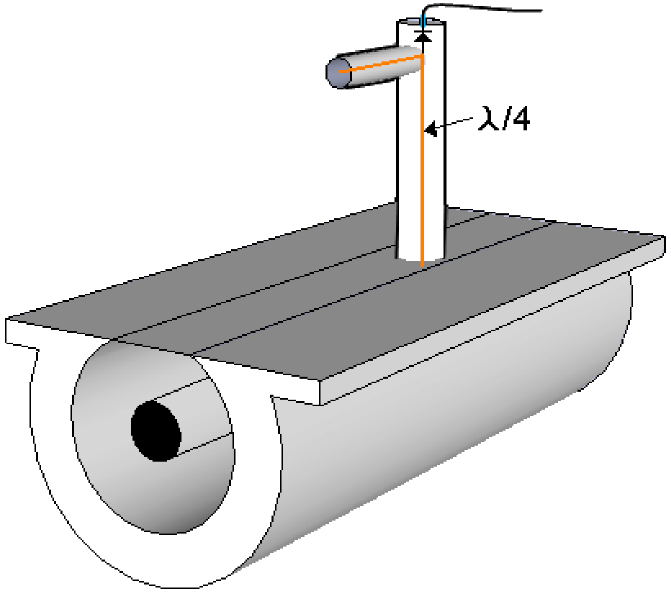

**Today, the IF, demodulation, video and display sections of a spectrum analyzer are realized digitally**

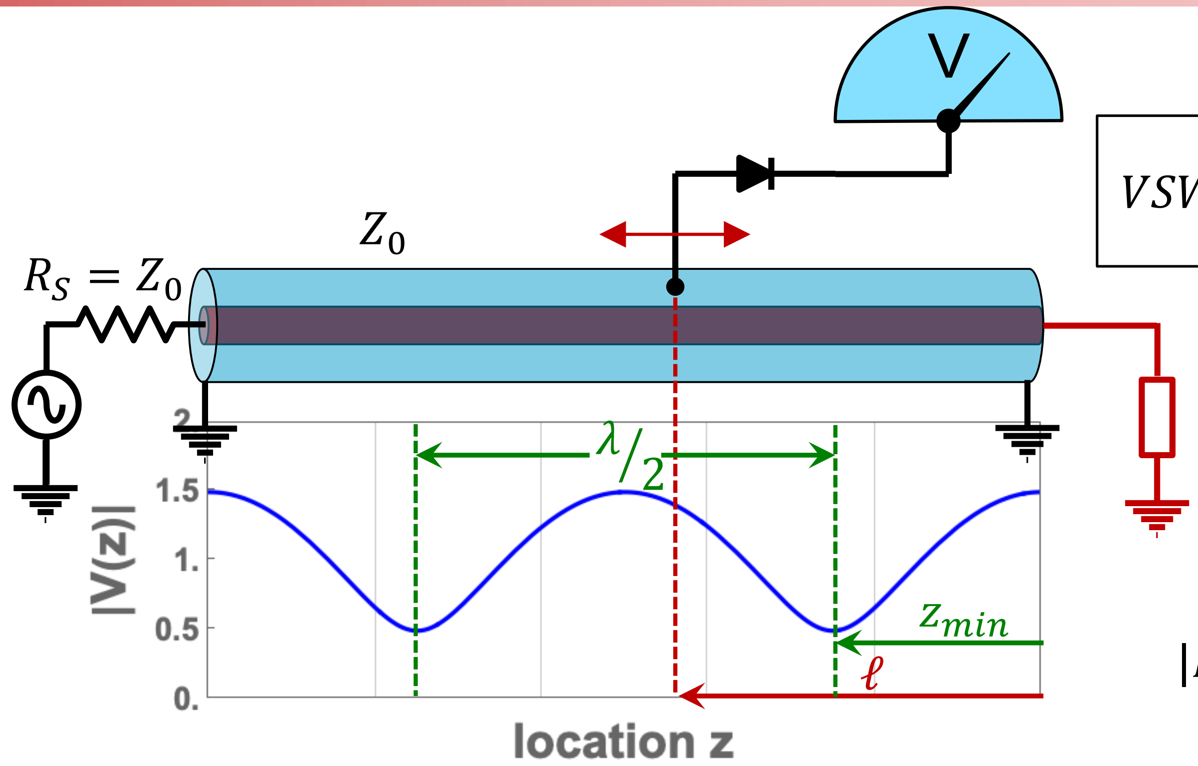
– **Requires an analog-digital converter (ADC) with sufficient dynamic range**

# Modern Spectrum (RF Signal) Analyzer



# Reflection (VSWR) Measurement




$$VSWR = \frac{|V_{max}|}{|V_{min}|}$$

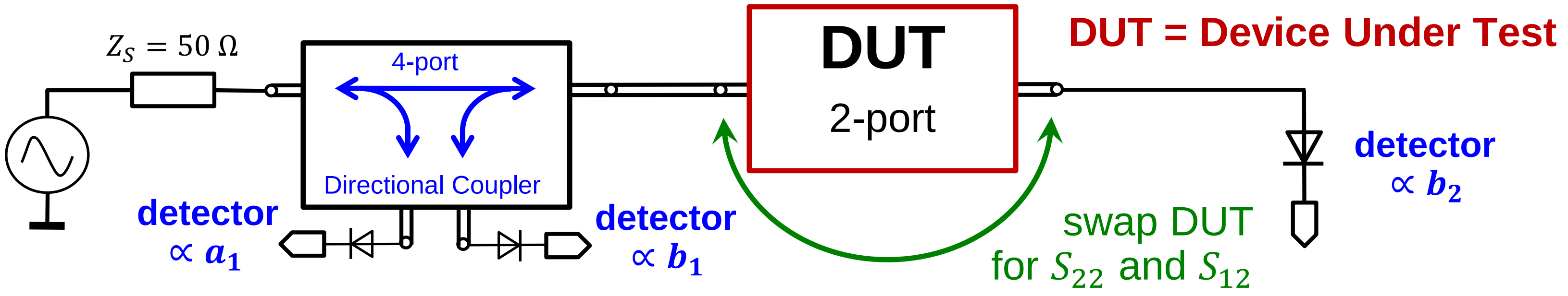
$$|\Gamma| = \frac{VSWR - 1}{VSWR + 1}$$

$$\angle\Gamma = \frac{4\pi}{\lambda} z_{min} - \pi$$

$$Z_x = Z_0 \frac{1 - \Gamma}{1 + \Gamma}$$

- **Slotted coaxial air-line is used as standing wave detector**
  - **Probes the radial electric field along the slotted line.**
  - **Measurement of E-field minima's $E_{min}$ and maxima's $E_{max}$ with a diode detector, thus detect $|V_{min}|$ and $|V_{max}|$ along the line.**
  - **Evaluate the reflection coefficient Γ of a DUT of unknown $Z_X$ at the end of the line**

# How to measure S-Parameters?

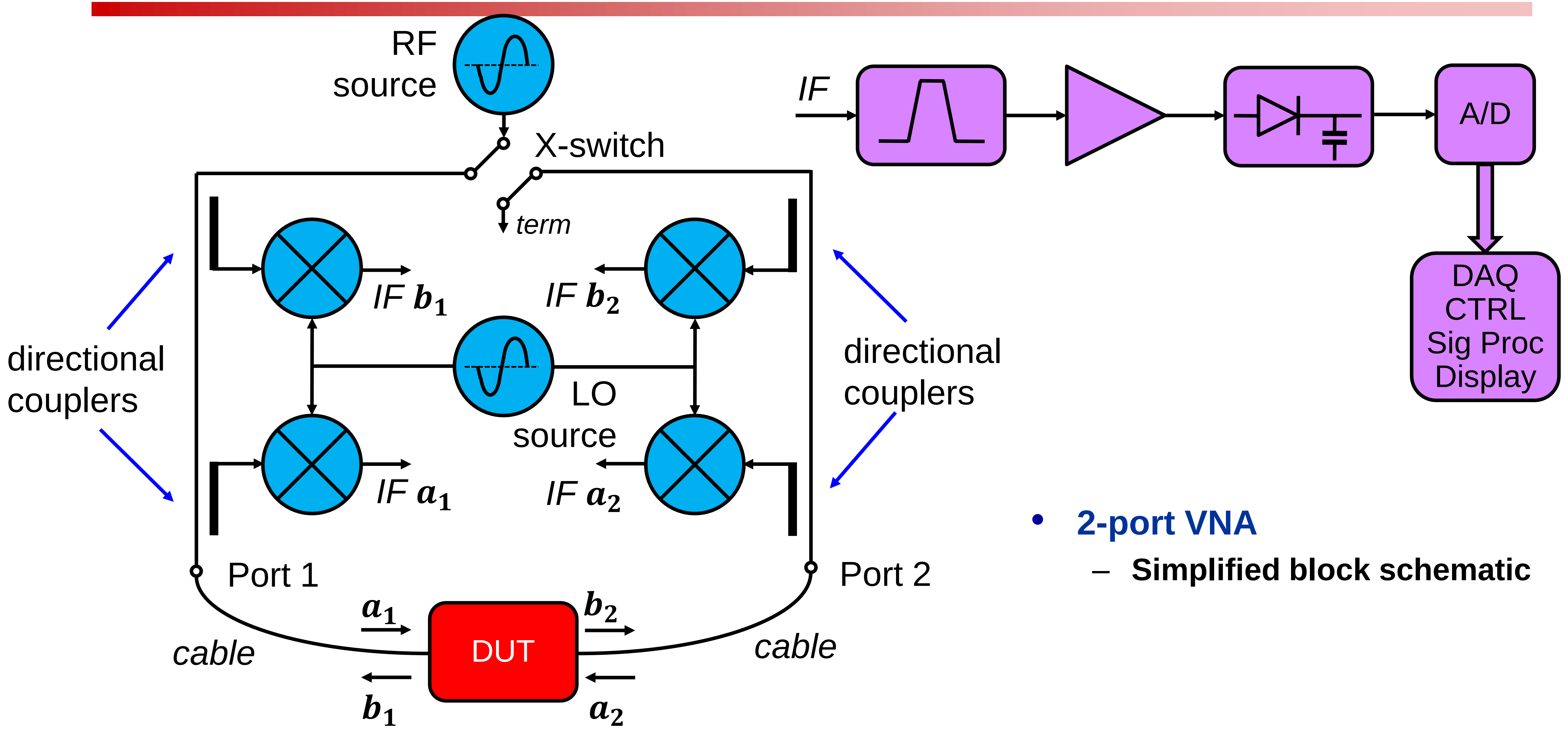



- **Performed in the "frequency domain"**
  - **Single or swept frequency generator, stand-alone or as part of a VNA or SA**
  - **Requires a directional coupler and RF detector(s) or receiver(s)**
- **Evaluate $S_{11}$ and $S_{21}$ of a 2-port DUT**
  - **Ensure $a_2 = 0$, i.e., the detector at port 2 offers a well-matched impedance**
  - **Measure incident wave $a_1$ and reflected wave $b_1$ at the directional coupler ports and compute for each frequency**
  - **Measure transmitted wave $b_2$ at DUT port 2 and compute**
- **Evaluate $S_{22}$ and $S_{12}$ of the 2-port DUT**
  - **Perform the same methodology as above by exchanging the measurement equipment on the DUT ports**

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$

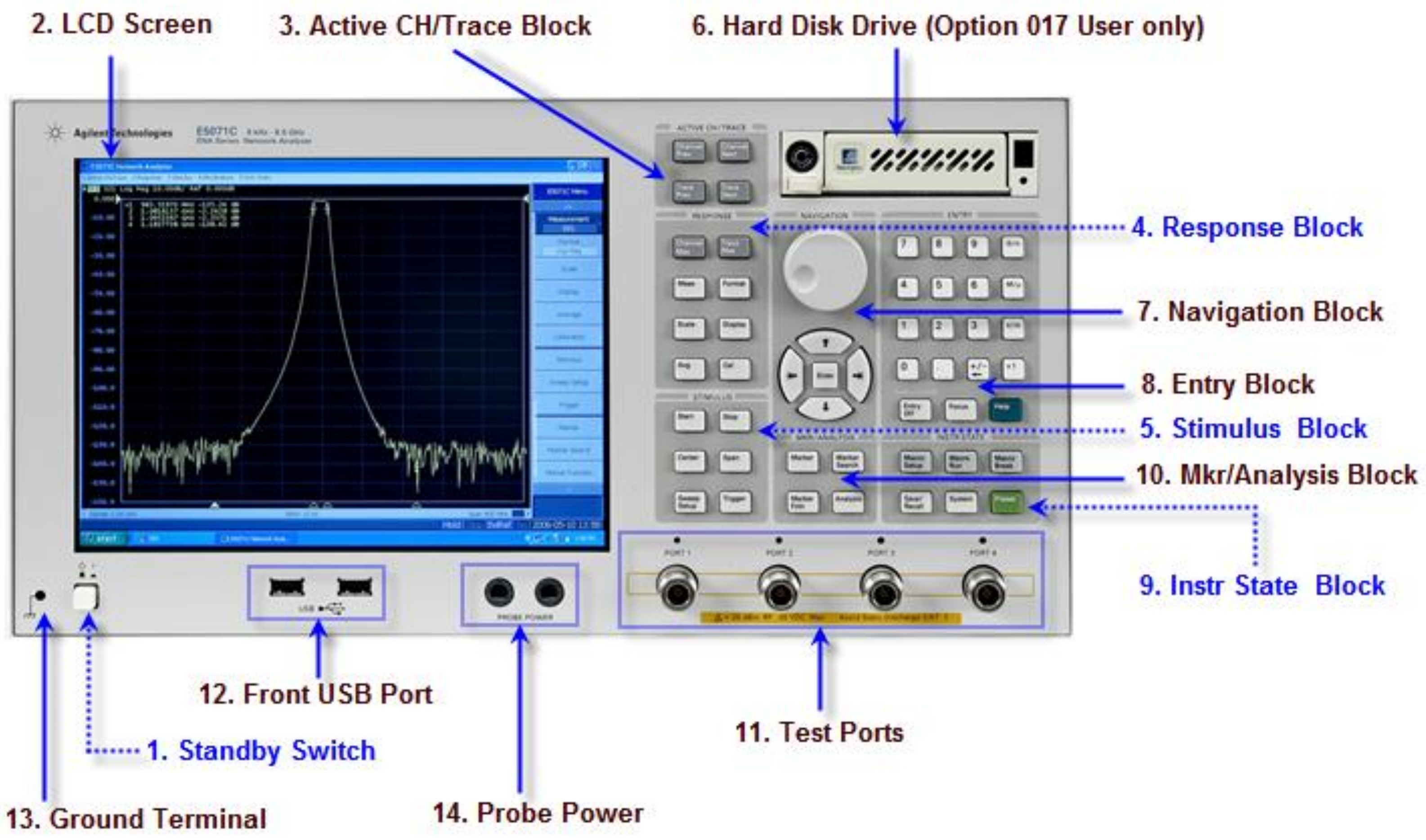
$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

# The Vector Network Analyzer (VNA)




- **2-port VNA**
  - **Simplified block schematic**

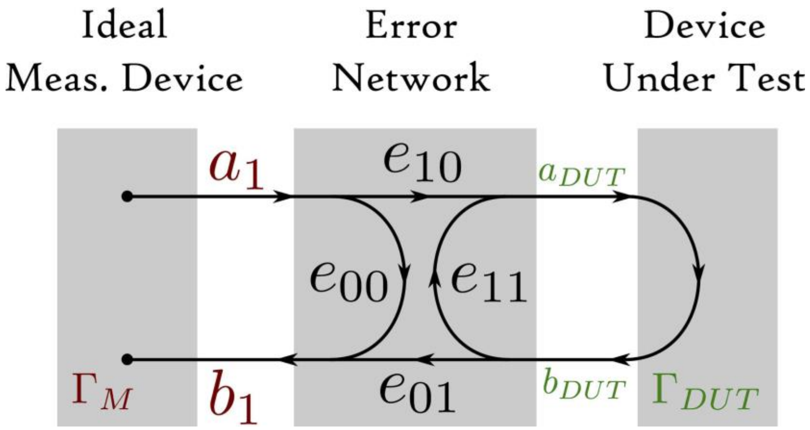
# Fun with the VNA!



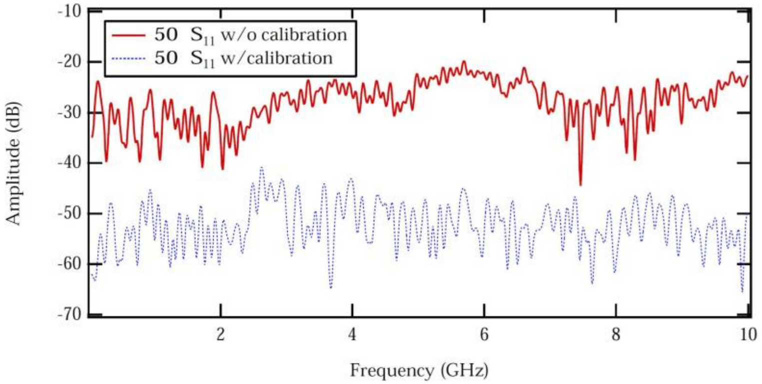

- **The "*look and feel*" between VNAs vary between manufacturers and models**
  - **Concepts and operation is still very similar**

# VNA Calibration (1)

- **Calibration is not necessary for pure frequency or phase measurements**
- **Before calibrating the VNA measurement setup, perform a brief measurement and chose appropriate VNA settings:**
  - **Frequency range (center, span or start, stop)**
  - **Number of frequency points**
    - Can be sometimes increased  by rearranging the VNA memory (# of channels)
  - **IF filter bandwidth**
  - **Output power level**
- **Calibrate the setup, preferable with an electronic calibration system if more than 2 ports are used!**
  - **Each port and combination needs to be calibrated, with the cables attached**
  - **Choose the appropriate connector type and sex**
  - **The instrument establishes a correction matrix and displays the "CAL" status.**



# VNA Calibration (2)



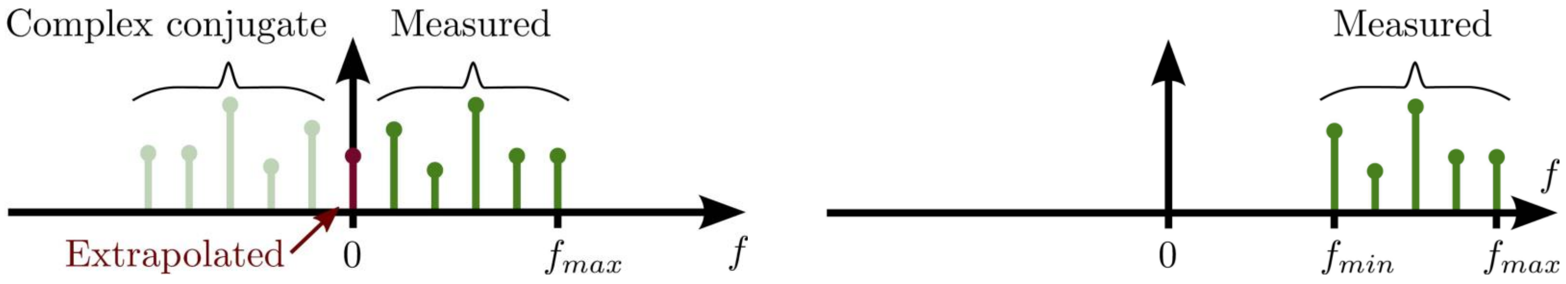

- **Calibration improves the measurement performance**
  - **Return loss improvement by typically 20 dB. Enables mdB accuracy measurements!**
  - **Full 2-port or 4-port calibration with manual calibration kits is prone to errors, better use electronic calibration systems.**
  - **Change VNA settings will cause the instrument to inter- and extrapolate, and the calibration status becomes uncertain.**
- **Cables are included in the calibration**
  - **However, changing coaxial connector types not.**
  - **Special VNA cables allows the adaption of different connector types and sex, without requiring a re-calibration of the setup!**

# RF Measurement Instrument Features

- **Modern VNAs (SAs, oscilloscopes, etc. as well) have many "features"**
- **Hardware features, e.g.**
  - **Automatic calibration system, down to DC**
  - **4 and more ports**
  - **Additional 2$^{nd}$ source, for downconverter / mixer measurements**
  - **Integrated spectrum analyzer function**
- **Software, control and data post processing options, e.g.**
  - **Far too many to list all**
  - **Sweep options, e.g., lin., log., segmented, in frequency or power**
  - **iDFT (or iFFT), gating**
  - **TDR, TDT for BP or LP step or impulse, segmented (advanced) TDR**
    - Only for linear, time-invariant systems!
  - **Port extension, virtual ports (4-port VNA), $Z_{0e}, Z_{0o}$ characterization, virtual baluns, etc.**
  - **Data transformations, e.g., $\Gamma \Rightarrow Z$**
  - **Noise figure measurements**
  - **Measurements following telecommunication standards**

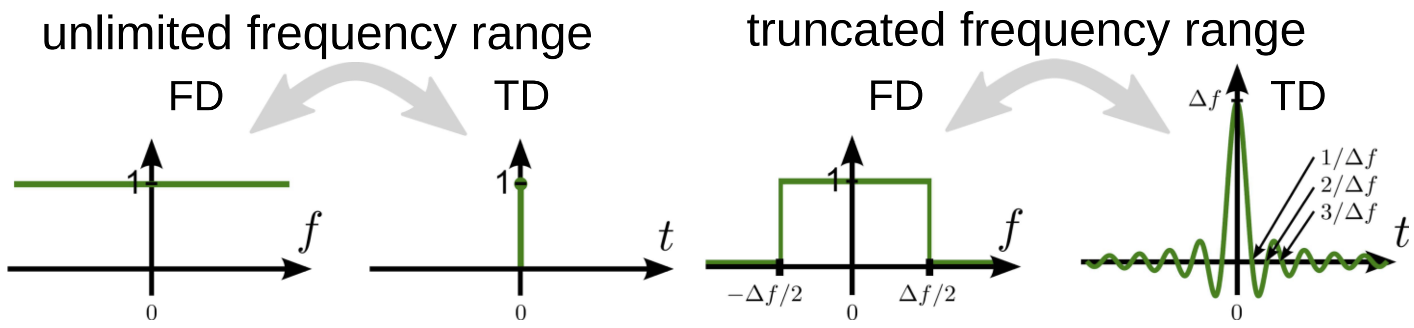
# Synthetic Pulse TD Measurements (1)

- **Based on an inverse discrete *Fourier* transformation (iDFT) option in the VNA**

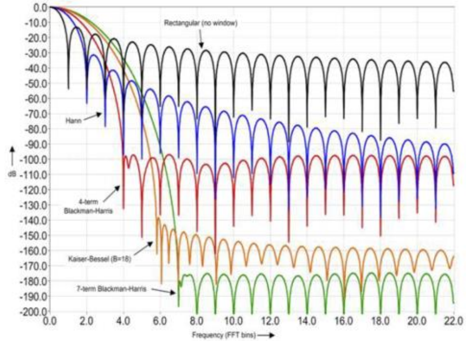



  – **Low-pass mode: Impulse or step response, relying on equidistant samples over the extrapolated (to DC) frequency range.**
    - The VNA does not measure at DC!
    - Manually match frequency range and # of points for DC extrapolation, e.g., 1…1000 MHz -> 1001 points, to enable extrapolation exactly to DC, or let the instrument chose the extrapolation settings automatically
  – **Enables time-domain reflectometry (TDR)**
    - Very useful on portable VNAs, troubleshooting RF cable problems
  – **Band-pass response (no DC extrapolation)**
  – **Allows time-domain gating and de-embedding of non-resonant sub-systems, e.g., measurements on a PCB**
  – **Limited to linear systems**
  – **Select the "real" format for $S_{11}$ or $S_{21}$ for time-domain transformations (*Keysight* instruments)!**
    - or dB magnitude to detect small reflections in TDR analysis

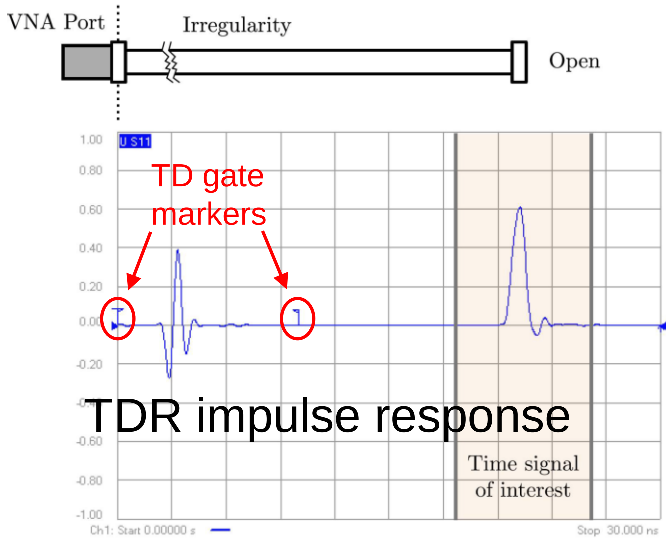
# Synthetic Pulse TD Measurements (2)

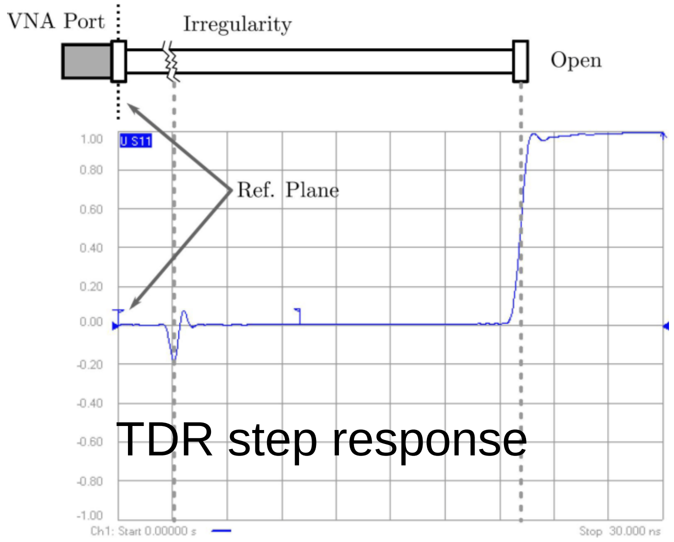



$$\mathrm{rect}\left(\frac{f}{\Delta f}\right) \qquad \frac{\sin(\Delta f \pi t)}{\pi t}$$

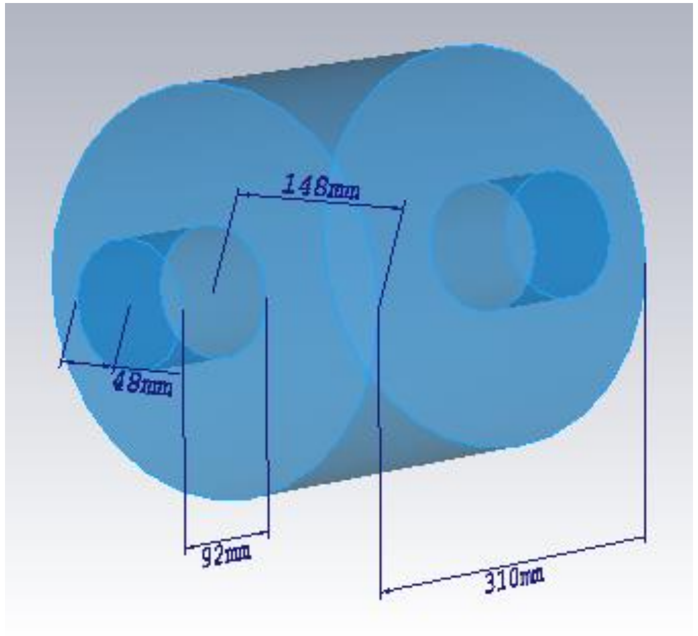


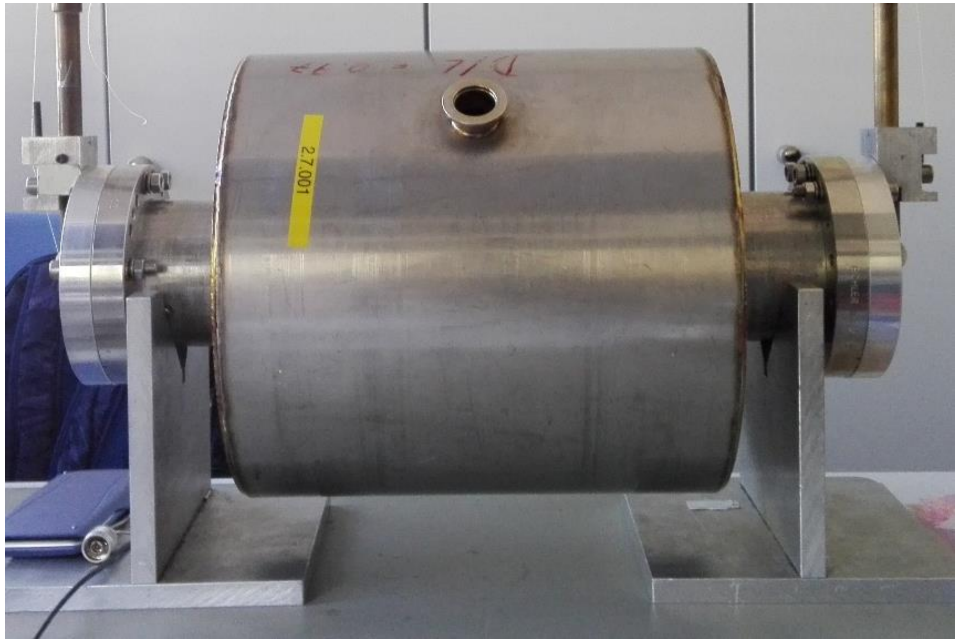
smoothing window functions



# An Example – Pillbox* $TM_{010}$ Eigenmode



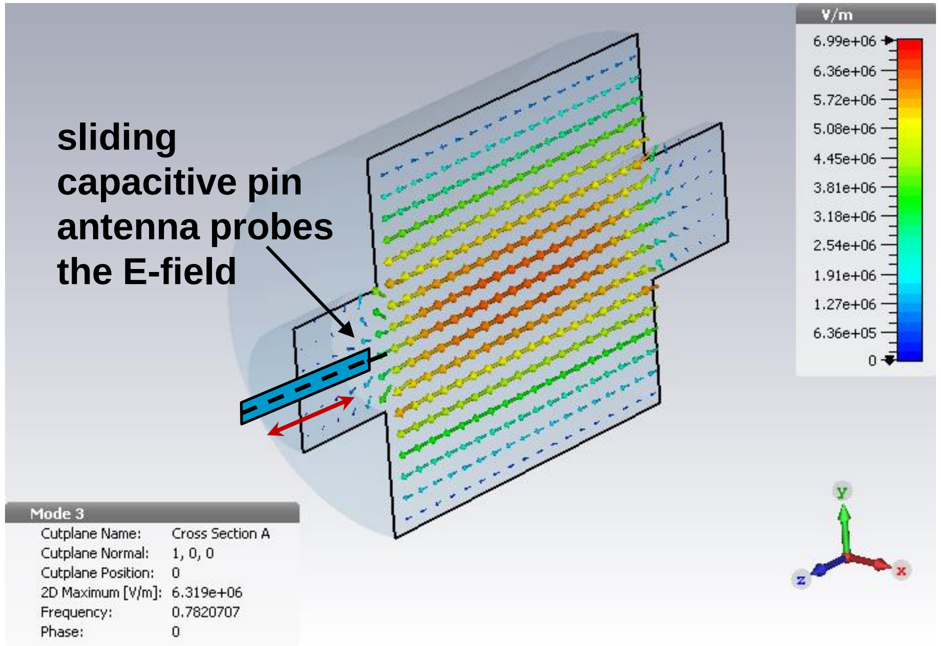



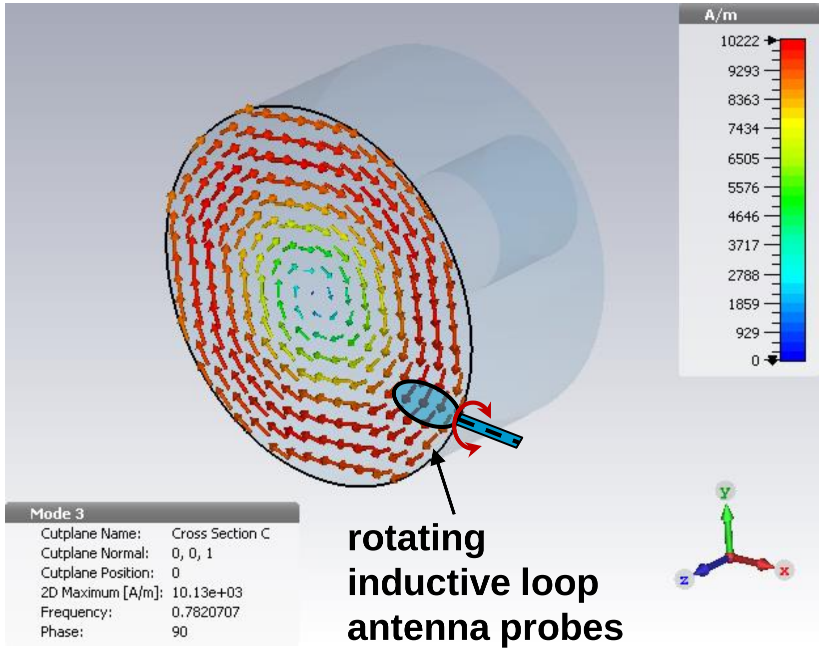
* Characterize the accelerating $TM_{010}$ mode of a cylindrical cavity with beam ports
  – The $TM_{010}$ does not have to be the lowest frequency mode
* Compare the measured values of $f_{res}$, $Q_0$ and $R/Q$
  – with an analytical analysis of a perfect cylinder (no beam ports)
  – with a numerical analysis

* normal conducting!

# Excite the Modes while Measuring $S_{11}$





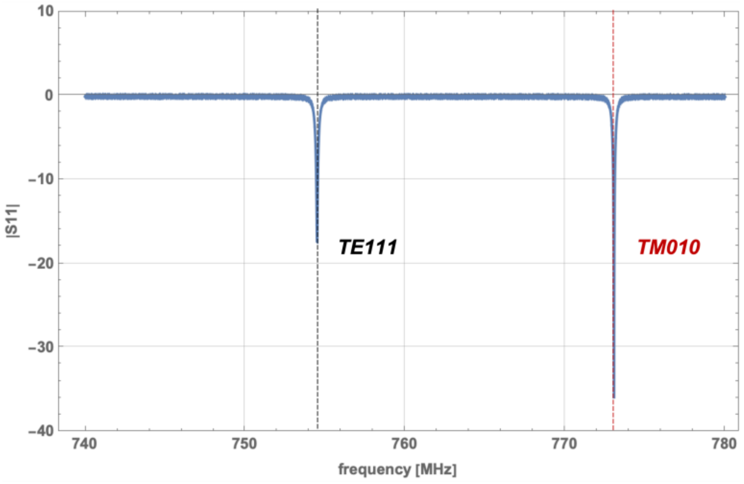



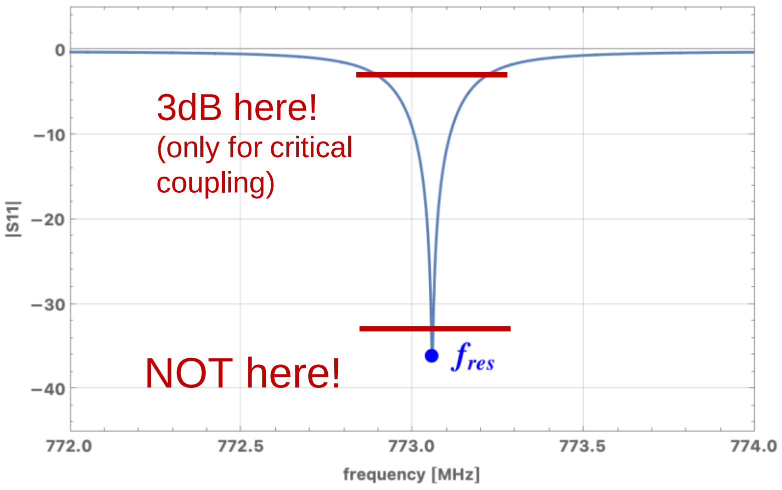

rotating inductive loop antenna probes the H-field

- $S_{11}$ **measurement with tunable coupling antenna**
  - **E-field on z-axis using a capacitive coupling pin**
    - Center pin, e.g., of semi-rigid coaxial cable
  - **H-field on the cavity rim using an inductive coupling loop**
    - Bend the center conductor to a closed loop connected to ground

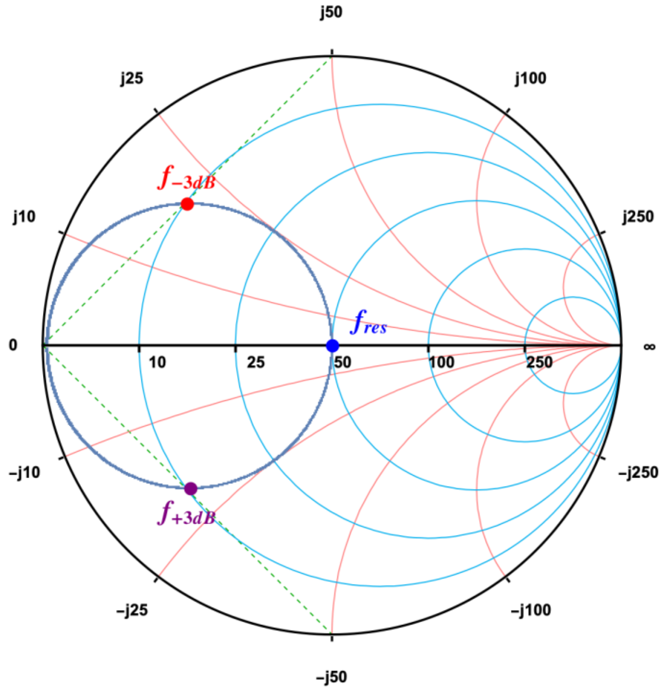
# Measurement of Frequency and Q-value





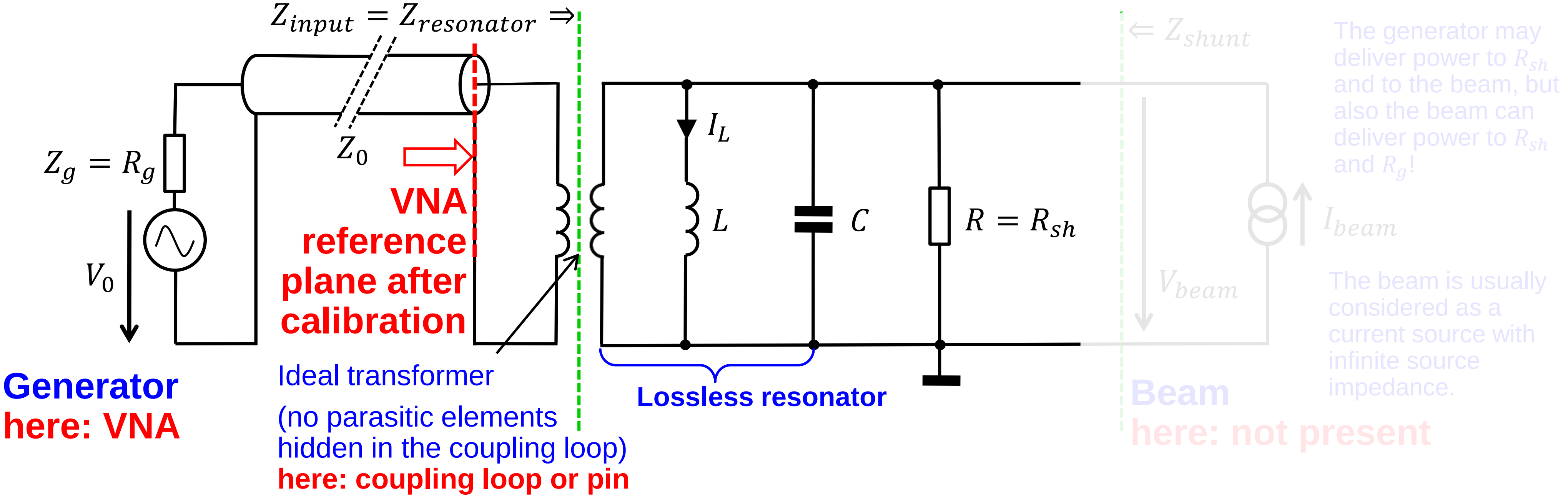




classical mistake!




- **Identify the correct ($TM_{010}$) mode frequency**
  - **Introduce a small perturbation, e.g., metallic rod or wire on the z-axis, and observe the shift of the mode frequencies**
- **Calibrate the VNA and measure $S_{11}$**
  - **Tune the coupling loop for critical coupling**
  - **Display the resonant circle in the *Smith* chart using enough points!**

# The Equivalent Circuit of a Resonant Mode




$R = R_{sh}$: shunt resistor, representing the losses of the resonator

**We have resonance condition, when** $\omega L = \frac{1}{\omega C}$

**→ Resonance frequency:** $\omega_{res} = 2\pi f_{res} = \frac{1}{LC} \Rightarrow$ $f_{res} = \frac{1}{2\pi\sqrt{LC}}$

# Useful Formulas of the Equivalent Circuit

- **Characteristic impedance "R over Q"**

$$X = \frac{R}{Q} = \omega_{res} L = \frac{1}{\omega_{res} C} = \sqrt{L/C}$$

- **Stored energy at resonance**

$$U = U_e + U_m = \frac{1}{4}|V_C|^2 C + \frac{1}{4}|I_L|^2 L$$

$V_C$ ... Voltage at the capacitor
$I_L$ … Current in the inductor

- **Dissipated power**

$$P = \frac{V^2}{2\,R}$$

- **Q-factor**

$$Q = \frac{R}{X} = \frac{\omega_{res}\, U}{P}$$

$U$ ... stored energy
$P$ ... dissipated power over 1 period

- **Shunt impedance (circuit definition)**

$$R = \frac{V^2}{2\,P}$$

$$\frac{R}{Q} = \frac{V^2}{2\omega_{res}\, U}$$

- **Tuning sensitivity**

$$\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta C}{C} = -\frac{1}{2}\frac{\Delta L}{L}$$

- **Coupling parameter (shunt impedance over generator or feeder impedance)**

$$k^2 = \frac{R}{R_{input}}$$

tune for critical coupling

# The Quality Factor ($Q$-Factor)

- The quality ($Q$) factor of a resonant circuit is defined as ratio of the stored energy $U$ over the energy dissipated $P$ in one oscillation cycle:

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy dissipated in 1 cycle}} = \frac{\omega_{res} U}{P}$$

- The $Q$-factor of an impedance loaded resonator:
  - $Q_0$: unloaded Q-value of the unperturbed system
  - $Q_L$: loaded Q-value, e.g., measured with the impedance of the connected generator
  - $Q_{ext}$: external Q-factor, representing the effects of the external circuit (generator and coupling circuit)
- Q-factor and bandwidth
  - This is how we actually "measure" the Q-factor!

$$Q = \frac{f_{res}}{f_{BW}}$$

with: $f_{BW} = f_{+3dB} - f_{-3dB}$

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

tune $k$ for critical coupling:

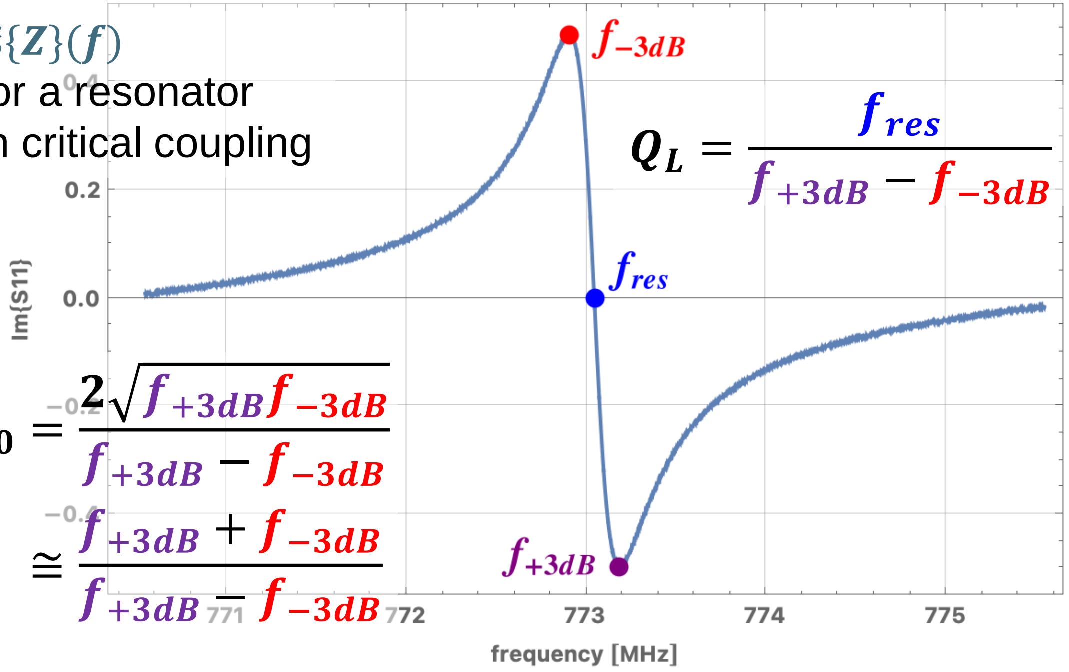
$Q_0 = Q_{ext}$

$\Rightarrow Q_0 = 2\,Q_L$

With $Q_L$ being our measured Q-value

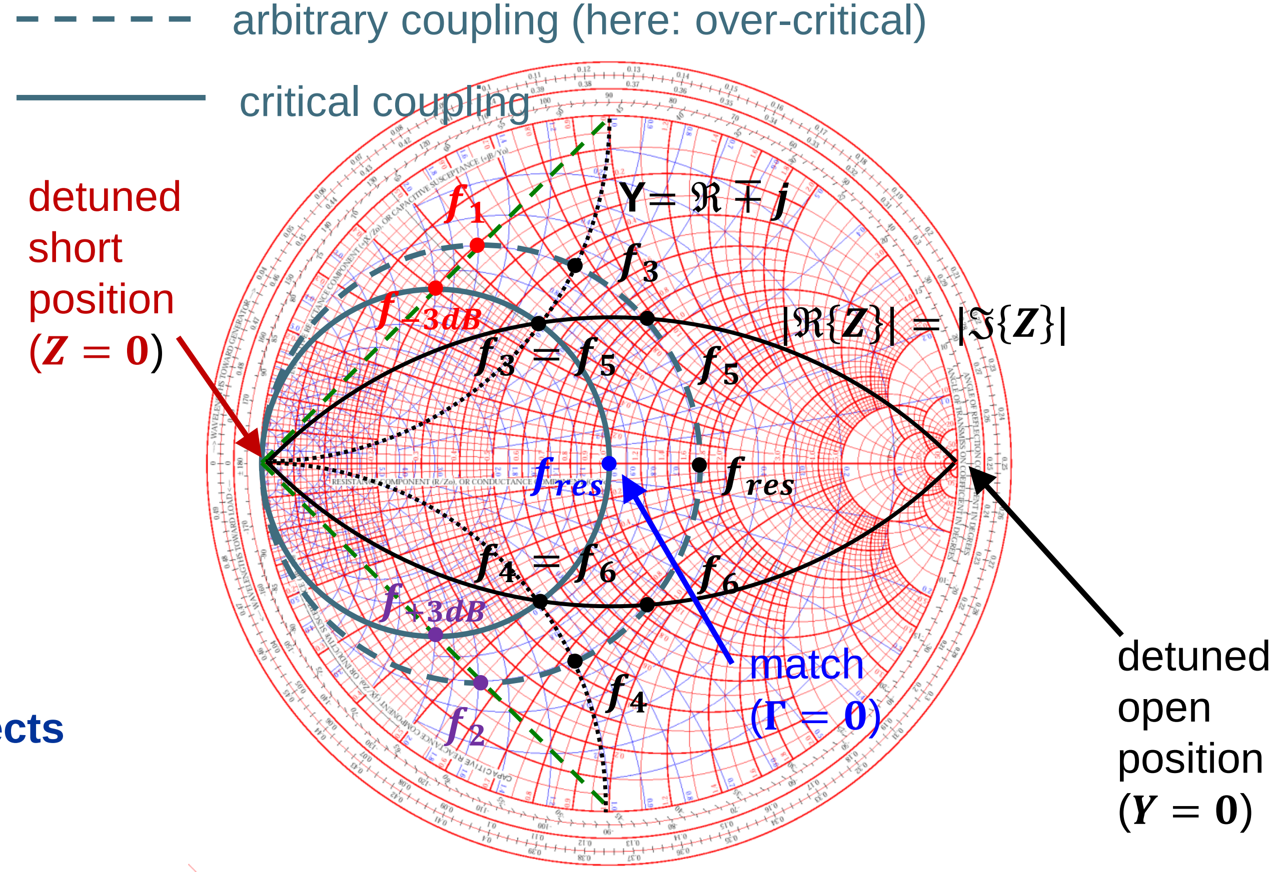
# $Q$-factor from $S_{11}$ Measurement

$\Im\{Z\}(f)$ for a resonator in critical coupling




$$Q_L = \frac{f_{res}}{f_{+3dB} - f_{-3dB}}$$

$$Q_0 = \frac{2\sqrt{f_{+3dB} f_{-3dB}}}{f_{+3dB} - f_{-3dB}} \cong \frac{f_{+3dB} + f_{-3dB}}{f_{+3dB} - f_{-3dB}}$$




Frequency marker points in the *Smith* chart:

$f_{1,2}$ ($f_{-3dB}$, $f_{+3dB}$): $|\Im\{S_{11}\}| = max.$ to calculate $Q_L$

$f_{3,4}$: $Y = \Re \mp j$ to calculate $Q_{ext}$

$f_{5,6}$: $|\Re\{Z\}| = |\Im\{Z\}|$ to calculate $Q_0$

- **Correct for the uncompensated transmission-line effects between calibration reference and the coupling loop**
  - **Electrical length adjustment: "straight" $\Im\{Z\}(f)$**
- **Adjust the locus circle to the detuned short location**
  - **Phase offset**
- **Verify no evanescent fields penetrating outside the beam ports**
  - **i.e., no frequency shifts if the boundaries at the beam ports are altered**

# $R/Q$ Measurement – *Slater*'s Theorem

- **Remember from the equivalent circuit:**

$$\frac{R}{Q} = \frac{V_{acc}^2 / 2P_d}{\omega_{res} U / P_d} = \frac{V_{acc}^2}{2\omega_{res} U} \qquad \text{with:} \qquad V_{acc} = \left| \int E_z(z) \cos\left(\frac{\omega z}{\beta c_0}\right) dz \right|$$

transit time related

  - $V_{acc}$ is based on the integrated longitudinal E-field component $E_z$ along the z-axis ($x = y = 0$)

- **Based on Slater's perturbation theorem:**

$$\frac{\Delta f}{f_{res}} = \frac{1}{U}\left[\mu_0 \left(k_{\parallel}^H |H_{\parallel}|^2 + k_{\perp}^H |H_{\perp}|^2\right) - \varepsilon_0 \left(k_{\parallel}^E |E_{\parallel}|^2 + k_{\perp}^E |E_{\perp}|^2\right)\right]$$

  - Resonance frequency shift due to a small perturbation object, expressed in longitudinal and transverse E and H field components
  - $k$: coefficients proportional to the electric or magnetic polarizability of the perturbation object (here: only $k_{\parallel}^E$ for a longitudinal metallic object)

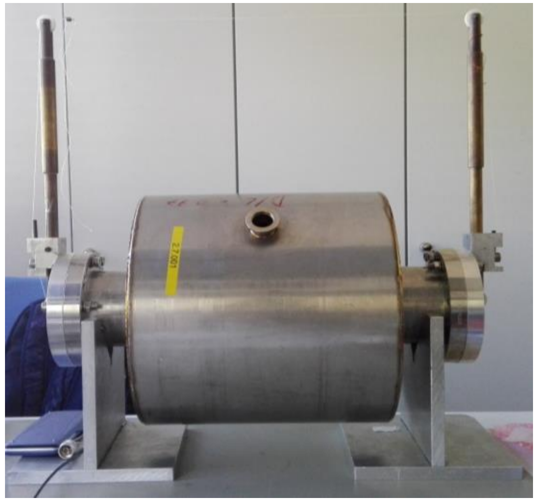
- **E-field characterization along the z-axis**

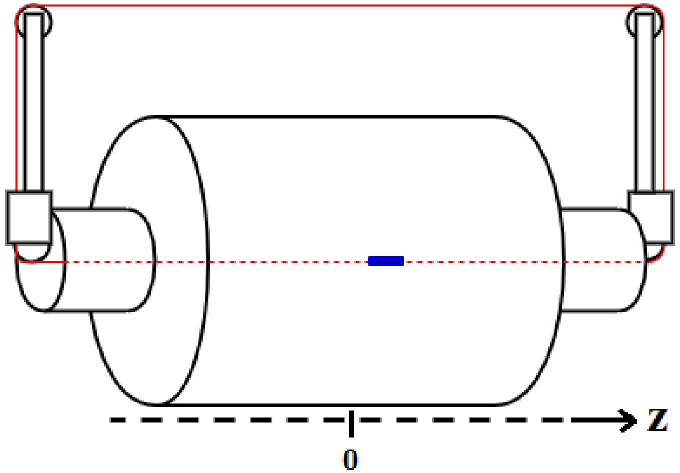
$$E(z) = E_{\parallel}(z) = \sqrt{U \frac{\Delta f(z)}{f_{res}} \cdot \frac{-1}{k_{\parallel}^E \varepsilon_0}}$$

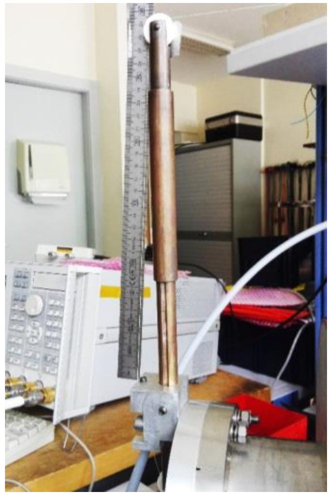
with: $k_{\parallel}^E = \frac{\pi}{3} l^3 \left[\sinh^{-1}\left(\frac{2}{3\pi}\frac{l}{a}\right)\right]^{-1}$

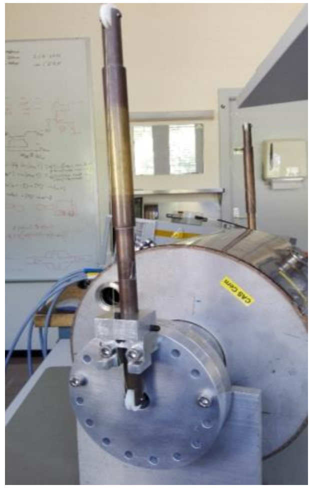
(metallic ellipsoid, e.g., syringe needle of half length $l$ and radius $a$)

# $R/Q$ Measurement - Bead Pull Method







- **E-field characterization by evaluating**
  - **The frequency shift $\Delta f$ ($S_{11}$ reflection measurement with a single probe) or**
  - **The phase shift $\phi$ at $f_{res}$ ($S_{21}$ transmission measurement with 2 probes)**

$$\frac{\Delta f}{f_{res}} = \frac{1}{2\,Q_0} \tan \phi$$

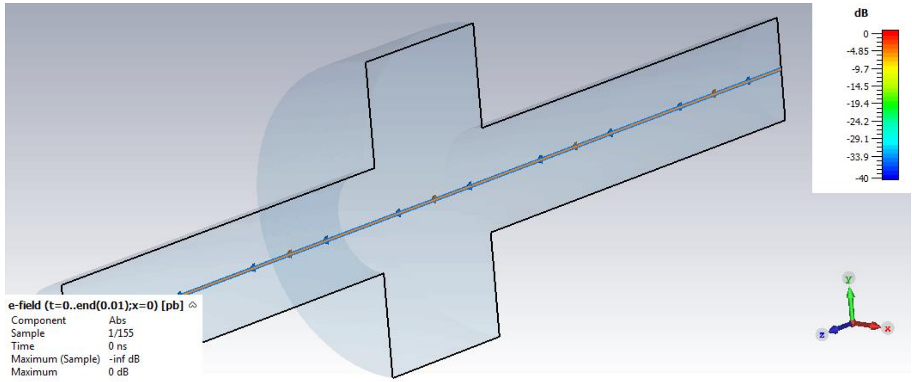
- **Exercise with a manual bead-pull through a known cavity**
  - **requires: fishing wire, syringe needle, ruler and VNA**
  - **Compare the measured $E_z$ at the maximum $f$ or $\phi$ shift (in the center of the cavity) with the theoretical estimation (e.g., numerical computed value)**

# Beam Coupling Impedance

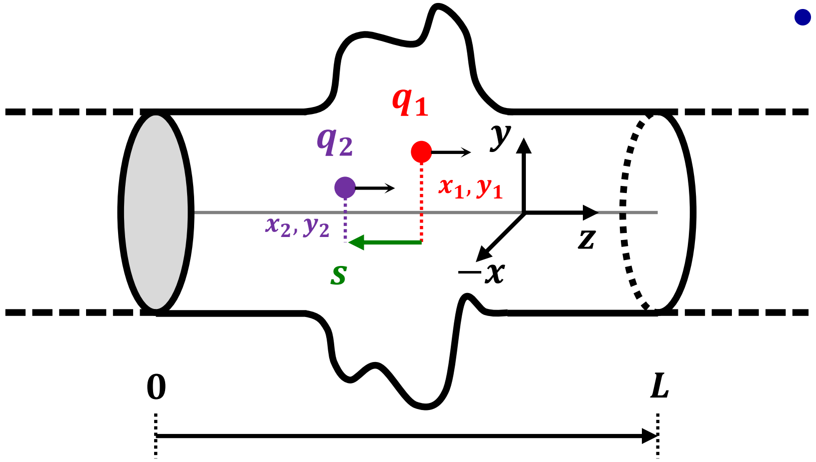
- **The wake potential**
  - Lorenz force on $q_2$ by the wake field of $q_1$:

$$\vec{F} = \frac{d\vec{p}}{dt} = q_2\left(\vec{E} + c_0\vec{e}_z \times \vec{B}\right)$$

  - Wake potential of a structure, e.g., a discontinuity driven by $q_1$



$$\vec{w}(x_1, y_1, x_2, y_2, s) = \frac{1}{q_1} \int_{-\infty\ (\text{or } 0)}^{+\infty\ (\text{or} L)} dz \left[\vec{E}(x_2, y_2, z, t) + c_0\vec{e}_z \times \vec{B}(x_2, y_2, z, t)\right]_{t=(s+z)/c}$$

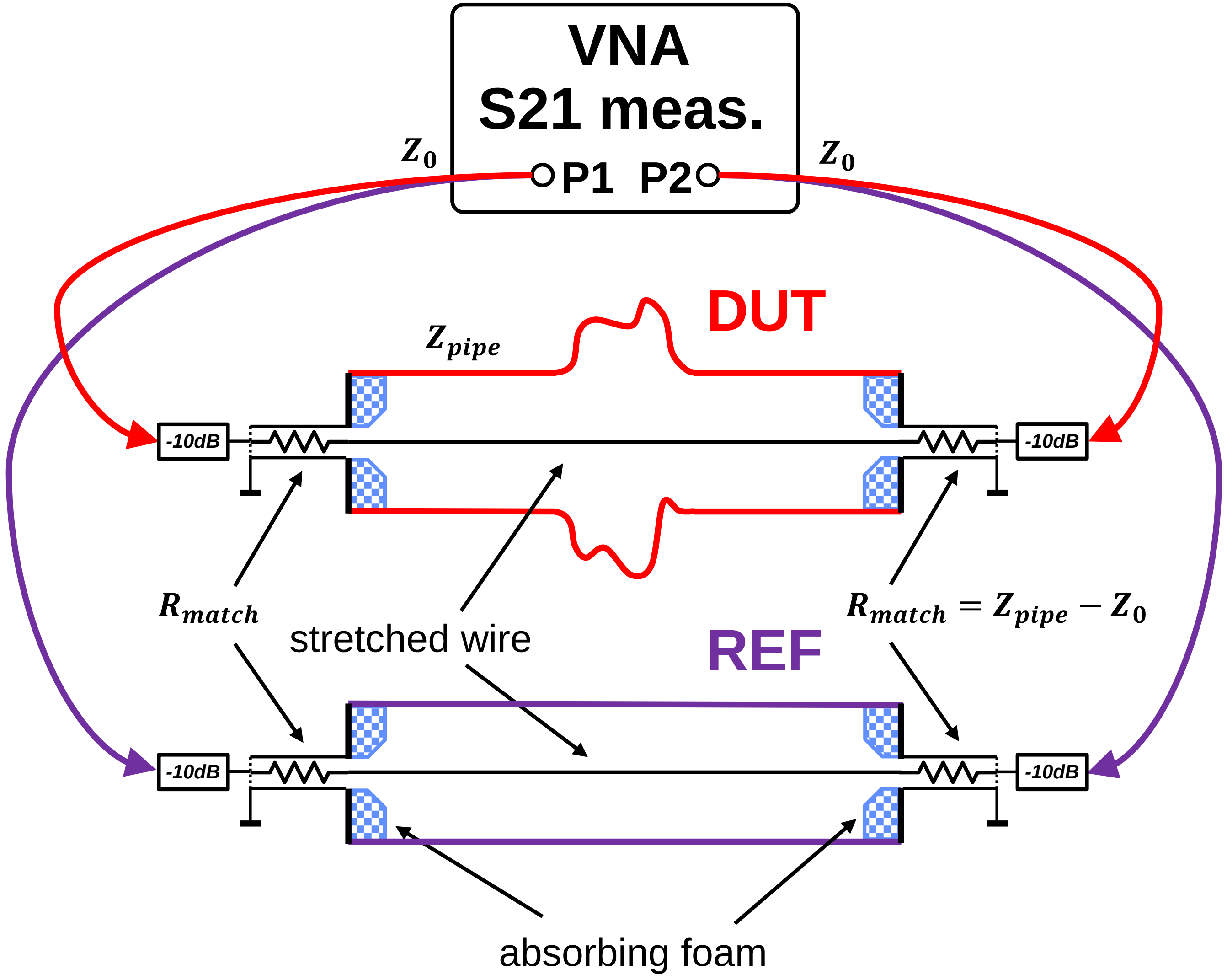



- **Beam coupling impedance**
  - Frequency domain representation of the wake potential

$$Z(x_1, y_1, x_2, y_2, \omega) = -\frac{1}{c_0} \int_{-\infty}^{+\infty} ds\, \vec{w}(x_1, y_1, x_2, y_2, s) e^{-j\omega s/c_0}$$

  - Can be decomposed in longitudinal $Z_{\parallel}$ and transverse $Z_{\perp}$ components (*Panofsky-Wenzel* theorem)
  - Resonant structures, $i^{th}$ mode: $R_{sh,i} = Z_{\parallel,i} = \frac{2k_{loss,i}Q_i}{\omega_i}$

# Stretched-Wire $Z_{\parallel}$ Measurement




- **Formulas:**
  - **Normalized electrical length:** $\Theta = 2\pi \frac{L}{\lambda}$
  - **Lumped impedance formula**

$$Z_{\parallel} = 2Z_{pipe} \frac{1 - S_{21}}{S_{21}} \quad \Theta \leq 1, \; L < D_{pipe}$$

  - **Log formula**

$$Z_{\parallel} = -2Z_{pipe} \ln S_{21}$$

  - **Improved log formula**

$$Z_{\parallel} = -2Z_{pipe} \ln S_{21} \left(1 + j \frac{\ln S_{21}}{2\Theta}\right)$$

  - **Transmission coefficient**

$$S_{21} = \frac{S_{21,DUT}}{S_{21,REF}}$$

  - **Circular beam pipe impedance**

$$Z_{pipe} = \frac{\eta_0}{2\pi\sqrt{\varepsilon_r}} \ln \frac{D}{d} \cong 60\ \Omega \ln \frac{D_{pipe}}{d_{wire}}$$

# Summary

- **No summary, just thank you for listening!**
- **Also, a big THANK YOU to all the help from the hands-on instructors!**
- **More THANKS to *Dassault / CST*, *SIMUSERV (Frank)* and *Computer Controls / Keysight***
- **THANK YOU to my colleagues at CERN, in particular *Joel D.*!**

**Thank you!**

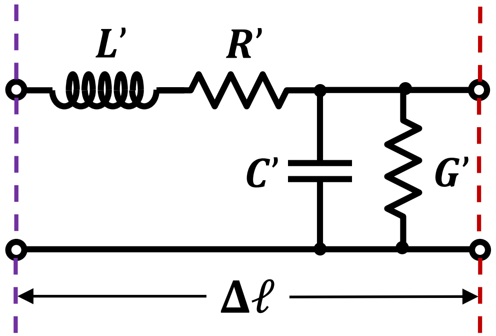
# Backup Slides

# Refresher: Some TL Equations (1)

- **Characteristic impedance**
  - **for a TEM transmission-line**
    - with losses

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \quad [\Omega]$$

**Equivalent circuit TEM TL segment**




- lossless, non-magnetic media ($\mu_r = 1$)

$$\boldsymbol{Z_0 \cong \sqrt{\frac{L'}{C'}}} \quad [\Omega]$$

$$Z_0 \cong \frac{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}}{C'} = \frac{\sqrt{\varepsilon_r}}{cC'} = \frac{1}{v_p C'} \quad [\Omega]$$

The characteristic impedance can be calculated from 2D electrostatic equations

$$Z_0 \cong \frac{L'}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}} = \frac{cL'}{\sqrt{\varepsilon_r}} = v_p L' \quad [\Omega]$$

- **Propagation constant**
  - **for a TEM transmission-line**

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

- attenuation constant

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} - 1\right)} \quad [\frac{Np}{m}]$$

- phase constant

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} + 1\right)} \quad [\frac{rad}{m}]$$

$$\boldsymbol{\beta = \frac{2\pi}{\lambda_g}} \quad [\frac{rad}{m}]$$

$$\beta = \omega\sqrt{L'C'}$$

# Refresher: Some TL Equations (2)

- **Wave impedance**

in media

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \quad [\Omega]$$

Characteristic impedance of free space

$$\boldsymbol{\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cong 120\pi \ [\Omega] \cong 377 \ [\Omega]}$$

- **Speed of light**

$$\boldsymbol{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \cong 2.997925 \cdot 10^8} \ \left[\frac{m}{s}\right]$$

- **Phase velocity**

$$v_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \quad \left[\frac{m}{s}\right]$$

  - non-magnetic media ($\mu_r = 1$)

$$\boldsymbol{v_p \cong \frac{c}{\sqrt{\varepsilon_r}}} \quad \left[\frac{m}{s}\right]$$

- **Wavelength**

in free space

$$\boldsymbol{\lambda_0 = \lambda = \frac{c}{f}} \quad [m]$$

guide wavelength (in media)

$$\lambda_g = \frac{c}{f\sqrt{\mu_r \varepsilon_r}} = \frac{\lambda_0}{\sqrt{\mu_r \varepsilon_r}} \quad [m]$$

  - non-magnetic media: ($\mu_r = 1$)

$$\boldsymbol{\lambda_g \cong \frac{c}{f\sqrt{\varepsilon_r}}} \quad [m]$$

- **Electrical length**
  - for a TEM line of physical length $\ell$

$$\boldsymbol{\theta = \beta\ell = 2\pi \frac{\ell}{\lambda_g}} \quad [rad]$$

$$\theta = \beta\ell = 360 \frac{\ell}{\lambda_g} \quad [deg]$$

- **Permeability:** $\boldsymbol{\mu = \mu_0 \mu_r}$ $\quad \mu_0 \cong 4\pi \cdot 10^{-7} \ [H/m]$

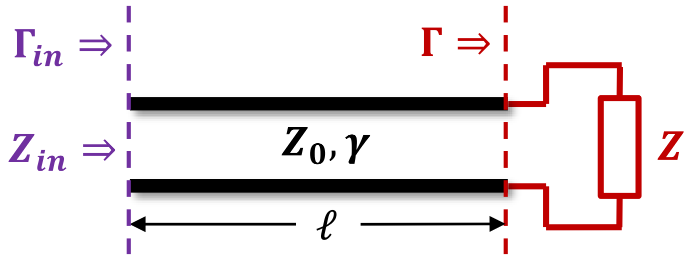
- **Permittivity:** $\boldsymbol{\varepsilon = \varepsilon_0 \varepsilon_r}$ $\quad \varepsilon_0 \cong 8.854 \cdot 10^{-12} \ [F/m]$

# Lossless Transmission-lines

- **Transmission-line terminated with an arbitrary impedance $Z$:**

$$Z_{in} = Z_0 \frac{1+\Gamma_{in}}{1-\Gamma_{in}} \quad \text{with: } \Gamma_{in} = \Gamma e^{-2\gamma\ell} \quad \text{and: } \Gamma = \frac{Z-Z_0}{Z+Z_0}$$

$$\boxed{Z_{in} = Z_0 \left( \frac{Z \cosh\gamma\ell + Z_0 \sinh\gamma\ell}{Z_0 \cosh\gamma\ell + Z \sinh\gamma\ell} \right) = Z_0 \frac{Z + Z_0 \tanh\gamma\ell}{Z_0 + Z \tanh\gamma\ell} \quad [\Omega]}$$




- **Lossless transmission-line:**

$$\alpha = 0 \quad \Rightarrow$$

$$\boxed{\boldsymbol{Z_{in} = Z_0 \left( \frac{Z \cos\beta\ell + jZ_0 \sin\beta\ell}{Z_0 \cos\beta\ell + jZ \sin\beta\ell} \right) \quad [\Omega]}}$$

– **Popular applications**
  - Quarter-wave line: $\ell = \frac{\lambda}{4} \Rightarrow \theta = \frac{\pi}{2} \Rightarrow \boxed{Z_{in} = \frac{Z_0^2}{Z}}$
  - Terminated (matched) line: $Z = Z_0 \Rightarrow \boxed{Z_{in} = Z_0}$
  - Open line: $Z \to \infty \Rightarrow \boxed{Z_{in} = -j\, Z_0 \cot\beta\ell}$
  - Shorted line: $Z = 0 \Rightarrow \boxed{Z_{in} = j\, Z_0 \tan\beta\ell}$

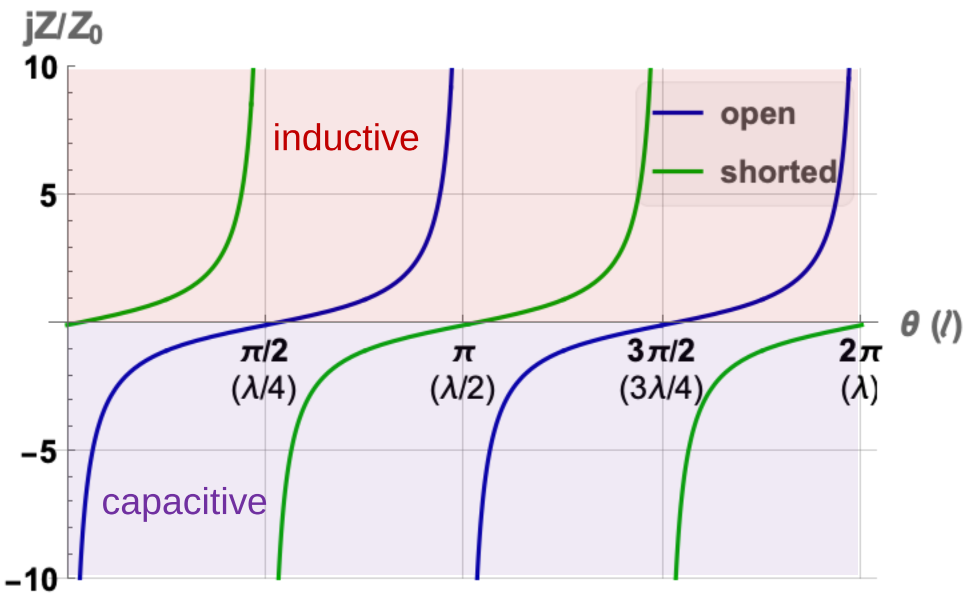
# Lossless Transmission-lines

$$Z_{in,open} = -j\,Z_0 \cot \beta\ell$$

$$Z_{in,short} = j\,Z_0 \tan \beta\ell$$

| physical length $\ell$ <br> electrical length $\theta = \beta\ell$ | $0 < \ell < \lambda_g/4$ <br> $0 < \theta < \pi/2$ | $\lambda_g/4 < \ell < \lambda_g/2$ <br> $\pi/2 < \theta < \pi$ | $\lambda_g/2 < \ell < 3\lambda_g/4$ <br> $\pi < \theta < 3\pi/2$ | $3\lambda_g/4 < \ell < \lambda_g$ <br> $3\pi/2 < \theta < 2\pi$ |
|---|---|---|---|---|
| lossless TL open | "capacitive" | "inductive" | "capacitive" | "inductive" |
| lossless TL shorted | "inductive" | "capacitive" | "inductive" | "capacitive" |

- **A lossless TL with open ($Z = 0$) or shorted ($Z \to \infty$) termination can approximate a lumped reactive element (capacitor or inductor)**
  - **A "capacitive" element has the form:** $Z_C = \dfrac{1}{j\omega C} = -j\dfrac{1}{\omega C}$
  - **An "inductive" element has the form:** $Z_L = j\omega L$
    - ➢ A more precise method follows a "$T$" or "$\pi$" $LCL$ or $CLC$ equivalent circuit of the lossless TL.
    - ➢ In case of $\theta \ll 1$, we can simplify $\tan\theta \cong \theta$, etc.
    - ➢ In practice it is useful to select a low $Z_0$ for a capacitive, or a high $Z_0$ for an inductive semi-lumped element approximation

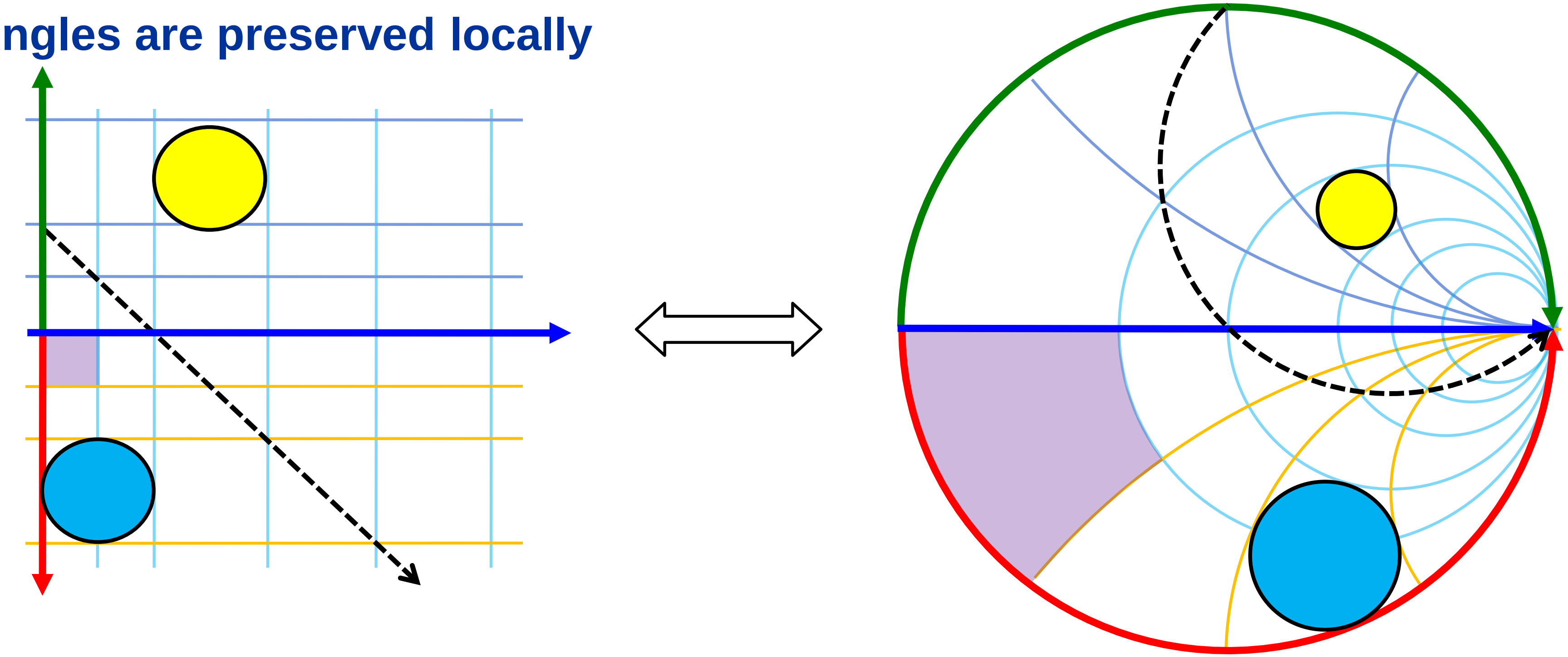


# Navigation in the *Smith* Chart (1)

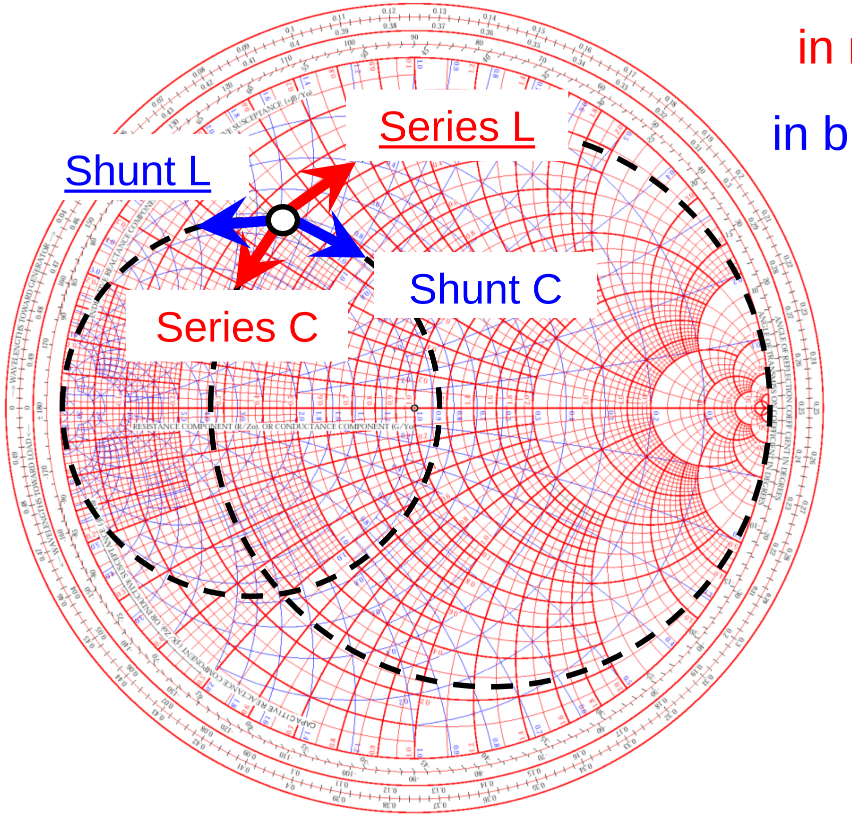
**This is a “bilinear” transformation with the following properties:**

- **Generalized circles are transformed into generalized circles**
  - **circle → circle**
  - **straight line → circle**
  - **circle → straight line**
  - **straight line → straight line**
- **Angles are preserved locally**

➢ a straight line is equivalent to a circle with infinite radius
➢ a circle is defined by 3 points
➢ a straight line is defined by 2 points



# Navigation in the *Smith* Chart (2)

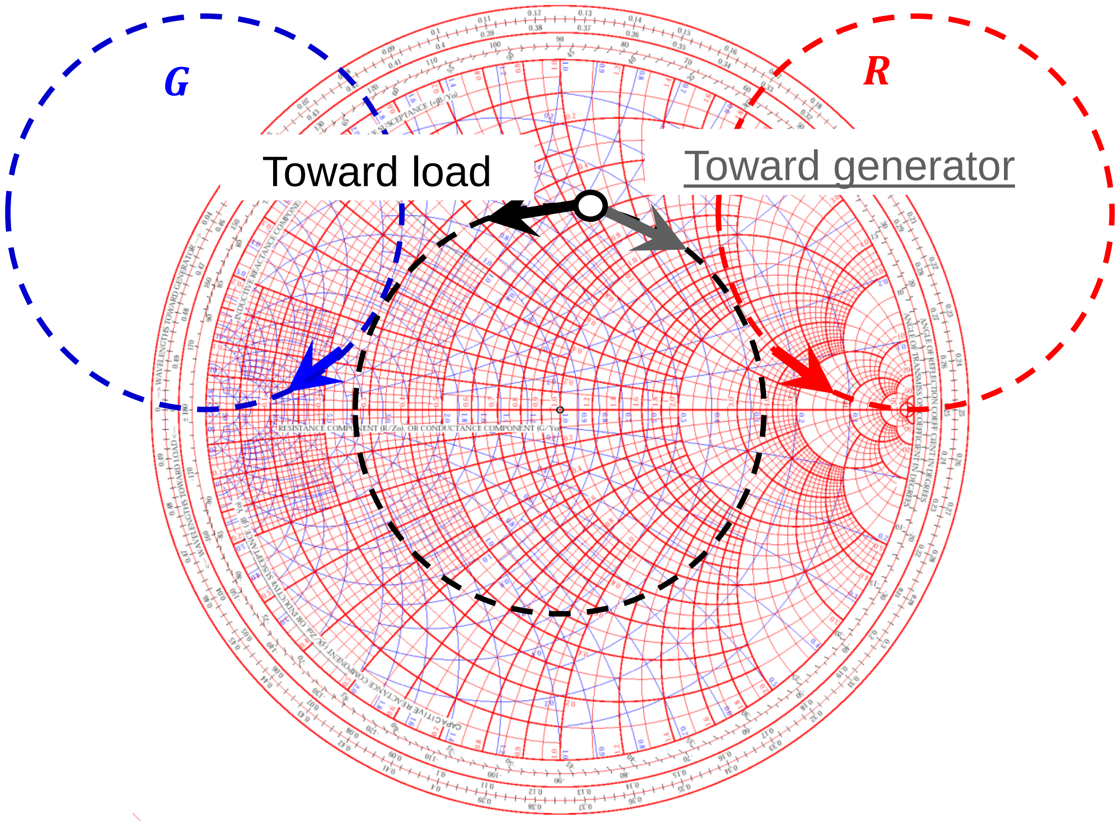



in red: impedance plane $(= \boldsymbol{z})$

in blue: admittance plane $(= \boldsymbol{y})$

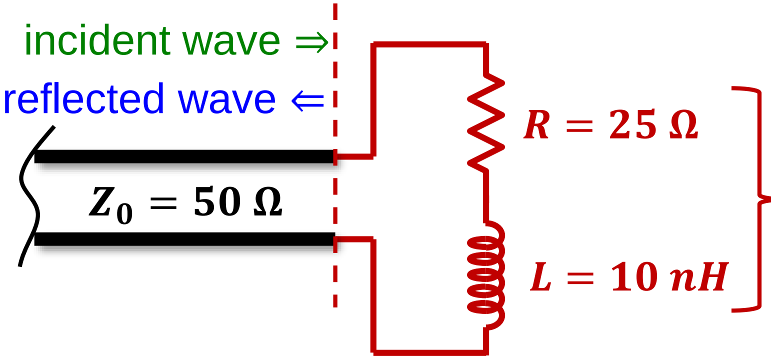
| | Up | Down |
|---|---|---|
| **Red circles** | **Series L** | **Series C** |
| **Blue circles** | **Shunt L** | **Shunt C** |

# Navigation in the *Smith* Chart (3)




| | |
|---|---|
| Red arcs | Resistance $R$ |
| Blue arcs | Conductance $G$ |
| Con-centric circle | Transmission line going<br>Toward load<br>Toward generator |

# The *Smith* Chart – Basic Example (1)

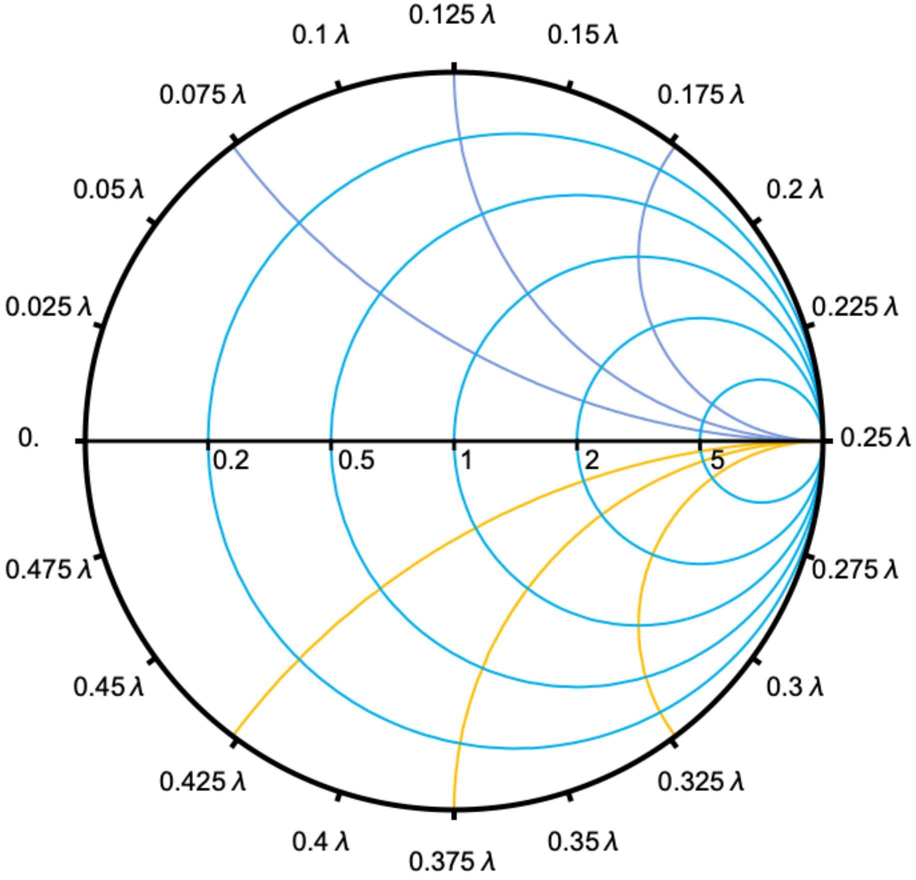



resistive reactive

$$Z = R + X_L$$

with: $X_L = j\omega L$

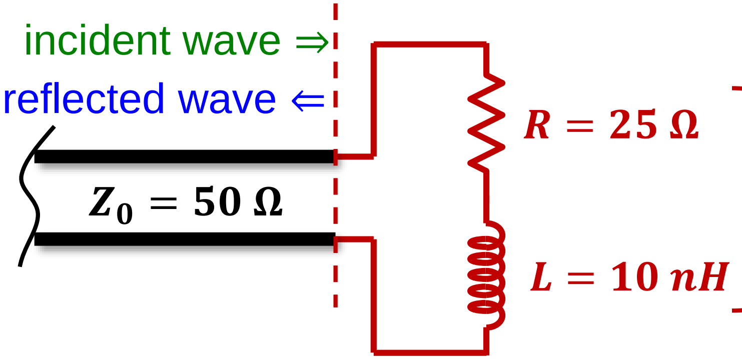
$\omega = 2\pi f$




**Complex impedance based on lumped element components**

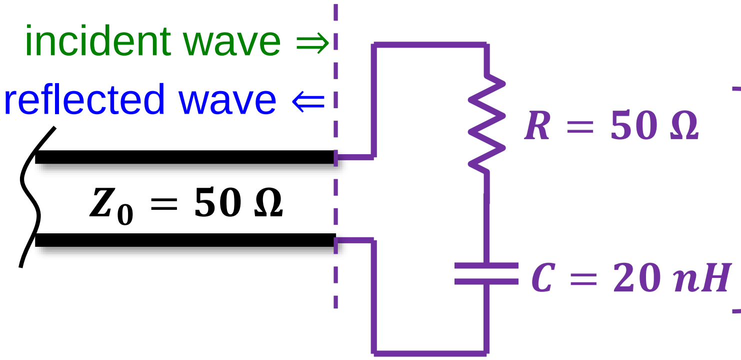
- **Calculate $Z$ for a given frequency, e.g., $f = 50\ MHz$: $Z = (25 + j6.28)\ \Omega$**
- **Calculate the normalized impedance $z = Z/Z_0 = 0.5 + j0.126$**
  - **Locate $z$ in the Smith chart**
  - **Retrieve $\Gamma = 0.34\angle 161° = 0.34e^{j2.81}$**
- **Repeat for other frequencies…**

# The *Smith* Chart – Basic Example (2)

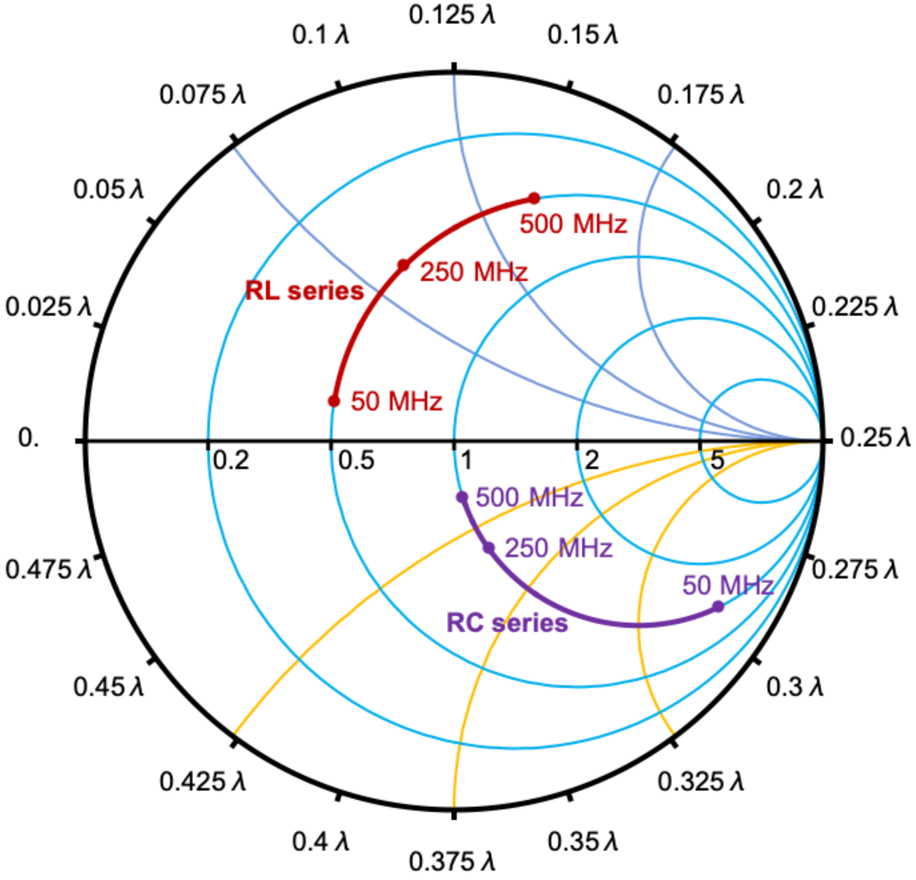



$Z = R + jX_L$
with: $X_L = \omega L$

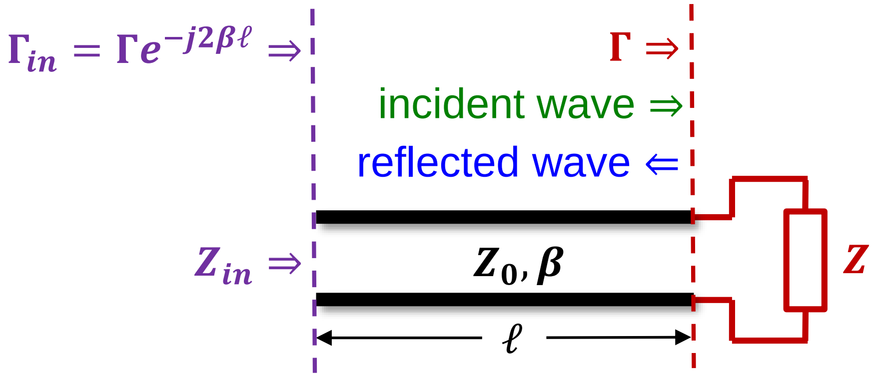
- …and for different component values and circuit combinations




$Z = R + jX_C$
with: $X_C = \frac{-1}{\omega C}$



# The *Smith* Chart – TL Transformer (1)




- **S-parameter of a lossless transmission-line:**

$$S = \begin{bmatrix} 0 & e^{-j\beta\ell} \\ e^{-j\beta\ell} & 0 \end{bmatrix}$$

backward transmission coefficient S12

forward transmission coefficient S21

  - **Phase delay (electrical length) of**

$$\theta = \beta\ell$$

**with:** $\beta = \frac{2\pi}{\lambda_g}$

- **The lossless transmission-line adds a phase delay of $2\beta\ell$, seen at its input, to the reflection coefficient at its output:**

$$\Gamma_{in} = \Gamma e^{-j2\beta\ell}$$

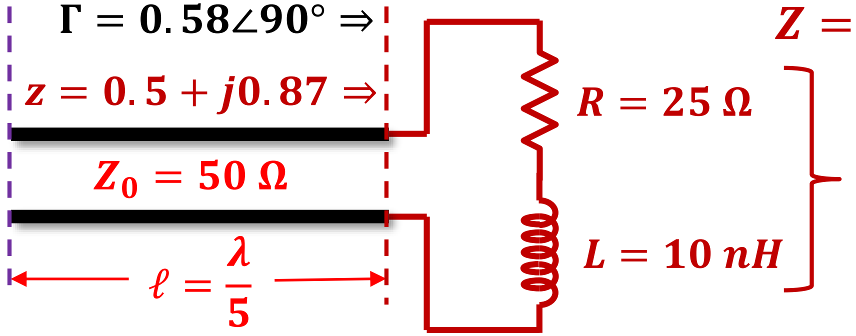
- **This results in a transformation of the impedance $Z$ at the end of the line to a different impedance $Z_{in}$ at the input of the line**
  - **The Smith chart offers an effective, simple graphical way to calculate this transmission-line based impedance transformation**

# The *Smith* Chart – TL Transformer (2)

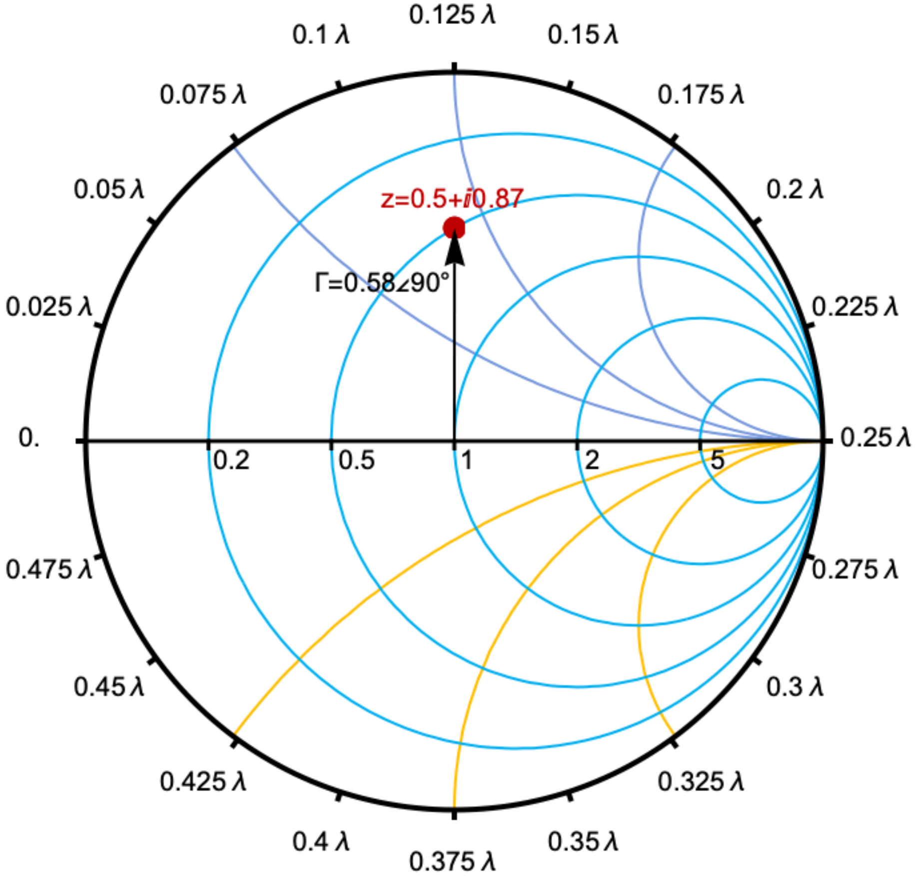
$\Gamma_{in} = 0.58\angle -54° \Rightarrow$ $\Gamma = 0.58\angle 90° \Rightarrow$

$z_{in} = 1.02 - j1.43 \Rightarrow$ $z = 0.5 + j0.87 \Rightarrow$

$Z_{in} = (50.8 - j71.3)\ \Omega \Rightarrow$



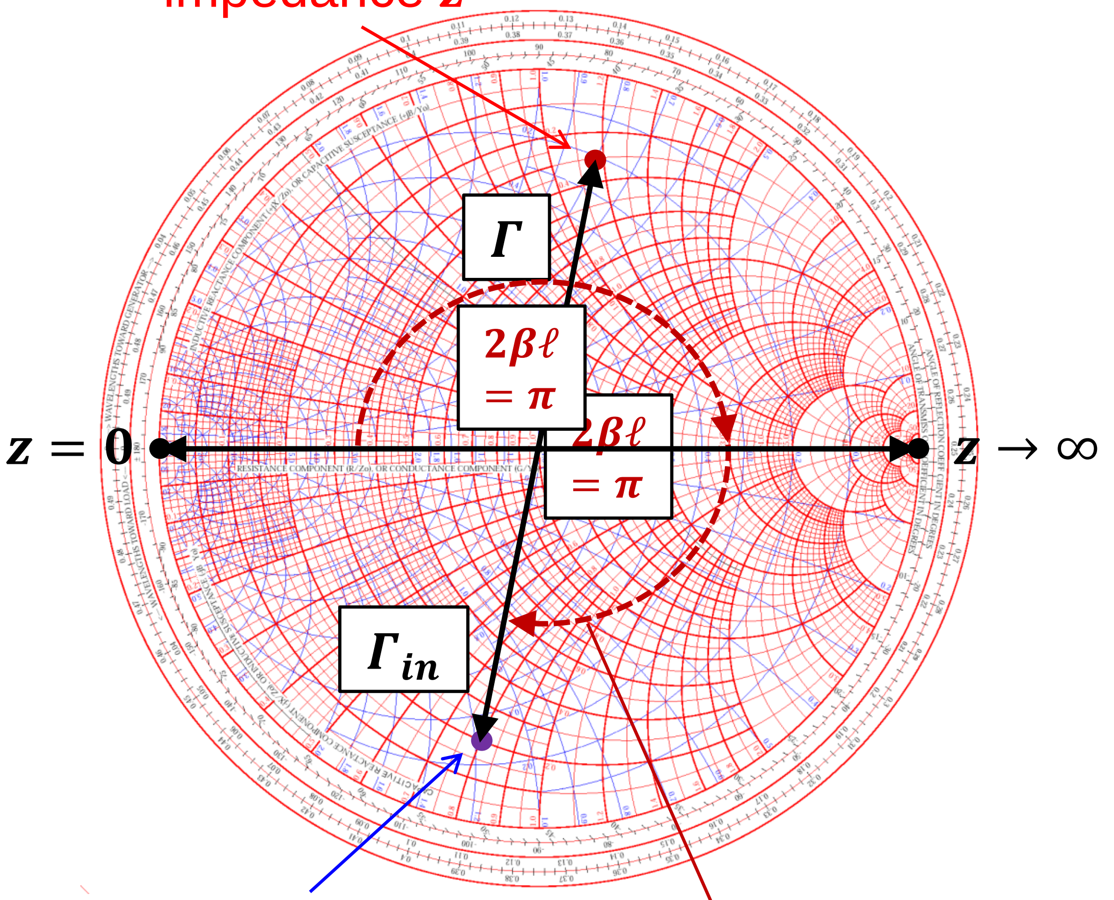

$Z = (25 + j43.4)\ \Omega$ @ $f = 345\ MHz$




**Based on the previous example**

- **Calculate the normalized impedance $z$ for $f = 345\ MHz$ and locate the point in the Smith chart.**
  - **Notice the corresponding value of $\Gamma$, and read the $\lambda$-length value on the rim**
- **Add $\ell = \lambda/5$ by rotating $\Gamma$ by $2\beta\ell = 4\pi/5 \equiv 144°$**
  - **From $0.125\ \lambda$ to $0.325\ \lambda$**
    - ➢ The phase is subtracted, therefore clockwise rotation!
- **Notice the value of $\Gamma_{in}$ and read the corresponding value of the normalized impedance $z_{in}$**
  - **Calculate the transformed impedance $Z_{in}@f = 345\ MHz$**
  - **What is the equivalent circuit, and what are the compon**
    - ➢ …and what is the physical length $\ell$ of the transmission-line?
      - assuming a coaxial cable as transmission-line with a dielectric constant of $\epsilon_r = 2.1$

# The *Smith* Chart – TL Transformer (3)




- A transmission-line of length

$$\ell = \lambda/4 \equiv \beta\ell = \frac{\pi}{2}$$

  transforms a reflection Γ at the end of the line to its input as

$$\Gamma_{in} = \Gamma\, e^{-j2\beta\ell} = \Gamma\, e^{-j\pi} = -\Gamma$$

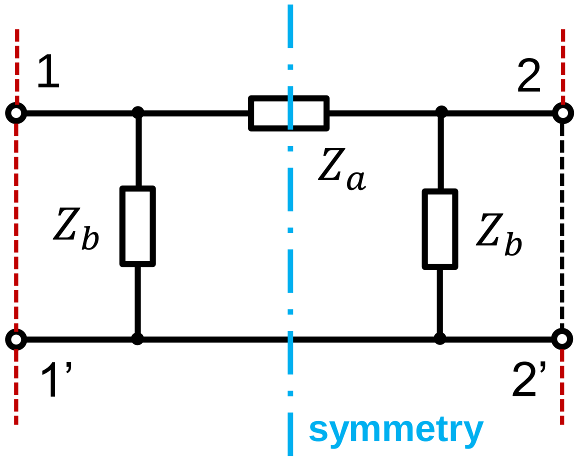
- This results the unitless, normalized impedance $z$ at the end of the line to be transformed into:

$$z_{in} = 1/z$$

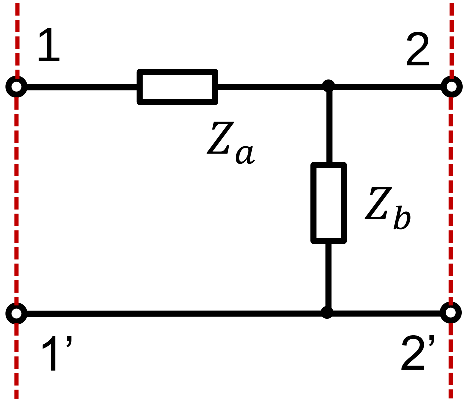
  at the beginning of the line
- A short circuit at the end of the $\lambda/4$-transformer is transformed to an open, and vice versa
  - This is the principle of the $\lambda/4$-resonator.

# Examples for Symmetry and Reciprocity

**$\pi$-network**




**divider-network**




$$(S_\pi) = \frac{1}{\Delta}\begin{pmatrix} Z_a Z_b^2 - Z_0^2(Z_a + 2Z_b) & 2Z_0 Z_b^2 \\ 2Z_0 Z_b^2 & Z_a Z_b^2 - Z_0^2(Z_a + 2Z_b) \end{pmatrix}$$

with: $\Delta = (Z_a + Z_b)[Z_a Z_b + Z_0(Z_a + 2Z_b)]$

$S_{12} = S_{21} \wedge S_{11} = S_{22} \Rightarrow$ reciprocal and symmetric

$$(S_{div}) = \frac{1}{\Delta}\begin{pmatrix} Z_a Z_b - Z_0(Z_0 - Z_a) & 2Z_0 Z_b \\ 2Z_0 Z_b & Z_a Z_b - Z_0(Z_0 + Z_a) \end{pmatrix}$$

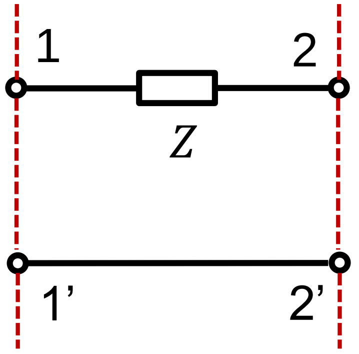
with: $\Delta = Z_0(Z_0 + Z_a) + Z_b(2Z_0 + Z_a)$

$S_{12} = S_{21} \wedge S_{11} \neq S_{22} \Rightarrow$ reciprocal, but not symmetric

- **Without prof: The S-matrix is always symmetric for reciprocal networks.**

# Numerical lossless / lossy Examples

## 2-port series-network




$$(S_{ser}) = \frac{1}{2Z_0 + Z}\begin{pmatrix} Z & 2Z_0 \\ 2Z_0 & Z \end{pmatrix}$$

$$S_{11} = S_{22} \wedge S_{12} = S_{21}$$

| $\mathbf{Z = j\omega L = j10}$ | $\mathbf{Z = R = 10}$ |
|---|---|
| $\lvert S_{11}\rvert = \sqrt{1 - \lvert S_{12}\rvert^2}$ | |
| $\frac{1}{\sqrt{101}} = \sqrt{1 - \left(\frac{10}{\sqrt{101}}\right)^2}$ | $\frac{1}{11} \neq \sqrt{1 - \left(\frac{10}{11}\right)^2}$ |
| $\angle S_{11} - \angle S_{12} = \angle S_{21} - \angle S_{22} - \pi$ | |
| $\tan^{-1}(10) + \tan^{-1}\frac{1}{10} = -\tan\frac{1}{10} - \tan(10) - \pi$ | $0 - 0 \neq 0 - 0 - \pi$ |
| ⇒ **lossless** | ⇒ **lossy** |

## 4-port ideal directional coupler

$$(S_{dc}) = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{3} & j & 0 \\ \sqrt{3} & 0 & 0 & j \\ j & 0 & 0 & \sqrt{3} \\ 0 & j & \sqrt{3} & 0 \end{pmatrix}$$

- **It is evident, this ideal 4-port coupler is symmetric and reciprocal**

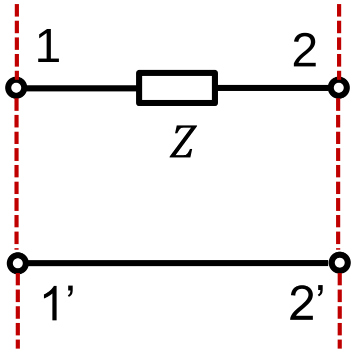
  $$S_{ij} = S_{ji} \wedge S_{ii} = S_{jj}$$

- **It also is matched:** $S_{ii} = 0$

- **But is it lossless or lossy?**

# Numerical lossless / lossy Examples

## 2-port series-network




$$(S_{ser}) = \frac{1}{2Z_0 + Z}\begin{pmatrix} Z & 2Z_0 \\ 2Z_0 & Z \end{pmatrix}$$

$$S_{11} = S_{22} \wedge S_{12} = S_{21}$$

| $\mathbf{Z = j\omega L = j10}$ | $\mathbf{Z = R = 10}$ |
|---|---|
| $\lvert S_{11}\rvert = \sqrt{1 - \lvert S_{12}\rvert^2}$ | |
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| $\angle S_{11} - \angle S_{12} = \angle S_{21} - \angle S_{22} - \pi$ | |
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| ⇒ **lossless** | ⇒ **lossy** |

## 4-port ideal directional coupler

$$(S_{dc}) = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{3} & j & 0 \\ \sqrt{3} & 0 & 0 & j \\ j & 0 & 0 & \sqrt{3} \\ 0 & j & \sqrt{3} & 0 \end{pmatrix}$$

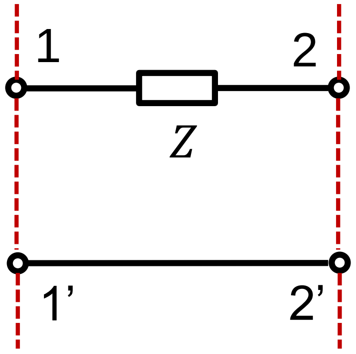
$$(S)^{\dagger}(S) = (I) \Rightarrow \sum_{k=1}^{N} S_{ki}S_{ki}^{*} = 1 \wedge \sum_{k=1}^{N} S_{ki}S_{kj}^{*} = 0 \; \forall \; i \neq j$$

- **Multiply matrix columns by itself with the conjugate complex**
  - **and test for = 1**

$$S_{11}S_{11}^{*} + S_{21}S_{21}^{*} + S_{31}S_{31}^{*} + S_{41}S_{41}^{*} = (0 \cdot 0 + \sqrt{3} \cdot \sqrt{3} + j \cdot (-j) + 0 \cdot 0)/2^2 = 1$$
$$S_{12}S_{12}^{*} + S_{22}S_{22}^{*} + S_{32}S_{32}^{*} + S_{42}S_{42}^{*} = (\sqrt{3} \cdot \sqrt{3} + 0 \cdot 0 + 0 \cdot 0 + j \cdot (-j))/2^2 = 1$$
$$S_{13}S_{13}^{*} + S_{23}S_{23}^{*} + S_{33}S_{33}^{*} + S_{43}S_{43}^{*} = (j \cdot (-j) + 0 \cdot 0 + 0 \cdot 0 + \sqrt{3} \cdot \sqrt{3})/2^2 = 1$$
$$S_{14}S_{14}^{*} + S_{24}S_{24}^{*} + S_{34}S_{34}^{*} + S_{44}S_{44}^{*} = (0 \cdot 0 + j \cdot (-j) + \sqrt{3} \cdot \sqrt{3} + 0 \cdot 0)/2^2 = 1$$

# Numerical lossless / lossy Examples

## 2-port series-network




$$(S_{ser}) = \frac{1}{2Z_0 + Z}\begin{pmatrix} Z & 2Z_0 \\ 2Z_0 & Z \end{pmatrix}$$

$$S_{11} = S_{22} \wedge S_{12} = S_{21}$$

| $\mathbf{Z = j\omega L = j10}$ | $\mathbf{Z = R = 10}$ |
|---|---|
| $\lvert S_{11}\rvert = \sqrt{1 - \lvert S_{12}\rvert^2}$ | |
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| $\angle S_{11} - \angle S_{12} = \angle S_{21} - \angle S_{22} - \pi$ | |
| $\tan^{-1}(10) + \tan^{-1}\frac{1}{10} = -\tan\frac{1}{10} - \tan(10) - \pi$ | $0 - 0 \neq 0 - 0 - \pi$ |
| $\Rightarrow$ **lossless** | $\Rightarrow$ **lossy** |

## 4-port ideal directional coupler

$$(S_{dc}) = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{3} & j & 0 \\ \sqrt{3} & 0 & 0 & j \\ j & 0 & 0 & \sqrt{3} \\ 0 & j & \sqrt{3} & 0 \end{pmatrix}$$

$$(S)^\dagger(S) = (I) \Rightarrow \sum_{k=1}^{N} S_{ki}S_{ki}^* = 1 \wedge \sum_{k=1}^{N} S_{ki}S_{kj}^* = 0 \; \forall \; i \neq j$$

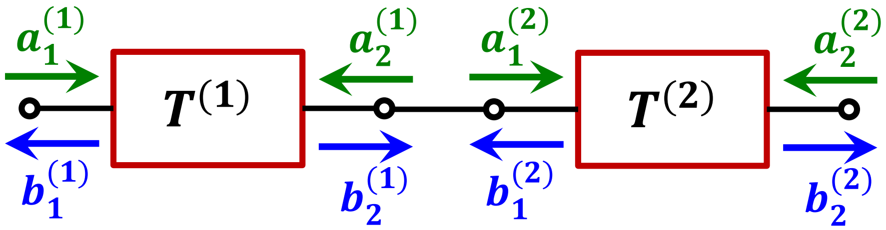
- **Multiply all different matrix columns with the conjugate complex**
  - **and test for = 0**

$$S_{11}S_{12}^* + S_{21}S_{22}^* + S_{31}S_{32}^* + S_{41}S_{42}^* = (0 \cdot \sqrt{3} + \sqrt{3} \cdot 0 + j \cdot 0 + 0 \cdot (-j))/2^2 = 0$$
$$S_{11}S_{13}^* + S_{21}S_{23}^* + S_{31}S_{33}^* + S_{41}S_{43}^* = (0 \cdot (-j) + \sqrt{3} \cdot 0 + j \cdot 0 + 0 \cdot \sqrt{3})/2^2 = 0$$
$$S_{11}S_{14}^* + S_{21}S_{24}^* + S_{31}S_{34}^* + S_{41}S_{44}^* = (0 \cdot 0 + \sqrt{3} \cdot (-j) + j \cdot \sqrt{3} + 0 \cdot 0)/2^2 = 0$$
$$S_{12}S_{13}^* + S_{22}S_{23}^* + S_{32}S_{33}^* + S_{42}S_{43}^* = (\sqrt{3} \cdot (-j) + 0 \cdot 0 + 0 \cdot 0 + j \cdot \sqrt{3})/2^2 = 0$$
$$S_{12}S_{14}^* + S_{22}S_{24}^* + S_{32}S_{34}^* + S_{42}S_{44}^* = (\sqrt{3} \cdot 0 + 0 \cdot (-j) + 0 \cdot \sqrt{3} + j \cdot 0)/2^2 = 0$$
$$S_{13}S_{14}^* + S_{23}S_{24}^* + S_{33}S_{34}^* + S_{43}S_{44}^* = (j \cdot 0 + 0 \cdot (-j) + 0 \cdot \sqrt{3} + \sqrt{3} \cdot 0)/2^2 = 0$$

# T-Parameters

- **Cascading e.g., 2-port S-parameter files is important to characterize a larger RF system.**
  - **Solution: Transfer (T) parameters, which directly relates the waves at input and output**

$$(\boldsymbol{T}) = \begin{pmatrix} \boldsymbol{T}_{11} & \boldsymbol{T}_{12} \\ \boldsymbol{T}_{12} & \boldsymbol{T}_{22} \end{pmatrix} \Rightarrow \begin{array}{l} \boldsymbol{b}_1 = \boldsymbol{T}_{11}\boldsymbol{a}_2 + \boldsymbol{T}_{12}\boldsymbol{b}_2 \\ \boldsymbol{a}_1 = \boldsymbol{T}_{21}\boldsymbol{a}_2 + \boldsymbol{T}_{22}\boldsymbol{b}_2 \end{array}$$




  - **T-parameters enable cascaded 2-port networks by simply multiplying their matrices:**

$$(T) = \left(T^{(1)}\right)\left(T^{(2)}\right)\cdots\left(T^{(N)}\right) = \prod_{i=1}^{N}\left(T^{(i)}\right)$$

  - **Relation between 2-port T-parameters and S-parameters:**

$$(\mathrm{T}) = \frac{1}{S_{21}}\begin{pmatrix} -\det(S) & S_{11} \\ -S_{22} & 1 \end{pmatrix}$$

with: $\det(S) = S_{11}S_{22} - S_{12}S_{21}$

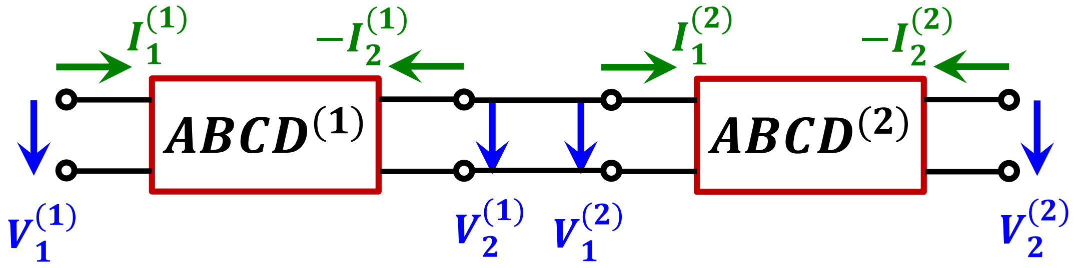
$$(\mathrm{S}) = \frac{1}{T_{22}}\begin{pmatrix} T_{12} & \det(T) \\ 1 & -T_{21} \end{pmatrix}$$

with: $\det(T) = T_{11}T_{22} - T_{12}T_{21}$

# ABCD-Parameters

- **Also called "chain" parameters, used for cascading networks based on V and I**
  - **Useful for chaining a mix of lumped elements and transmission-lines**

$$(ABCD) = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \Rightarrow \begin{matrix} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{matrix}$$




  - **Chaining 2-port ABCD-parameter networks:**

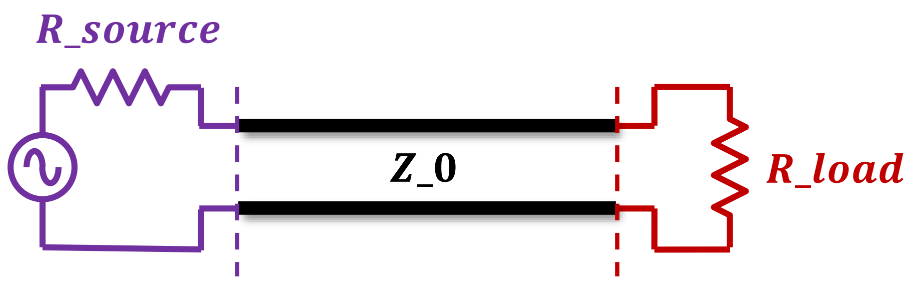
$$(ABCD) = \left(ABCD^{(1)}\right)\left(ABCD^{(2)}\right) \cdots \left(ABCD^{(N)}\right) = \prod_{i=1}^{N} \left(ABCD^{(i)}\right)$$

  - **Relation between 2-port ABCD-parameters and S-parameters:**

$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} \qquad S_{12} = \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D}$$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D} \qquad S_{11} = \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D}$$

# Quiz (1)



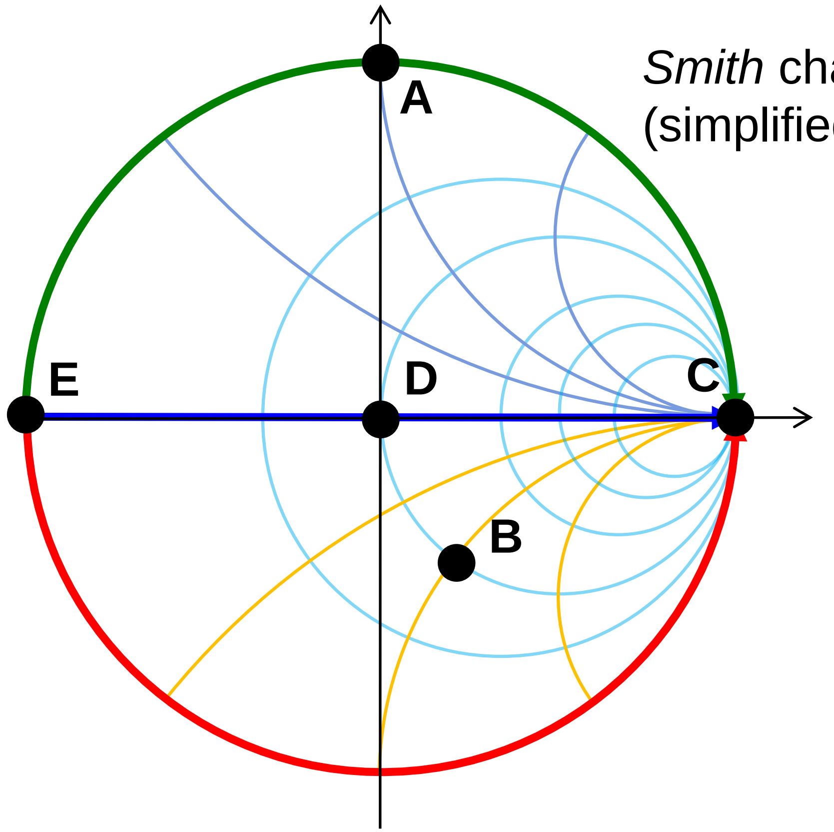

**1.** **When do no signal reflections occur at the end of a transmission-line?**

- ○ R_source = R_load
- ○ R_source = Z_0
- ✗ Z_0 = R_load
- ✗ R_source = Z_0 = R_load

**2.** **The Smith chart transforms the complex impedance plane onto the complex Gamma (reflection coefficient) plane within the unit circle.**

# Quiz (2)

| Prompts | | Possible Answers |
|---|---|---|
| A. Point A | A5 | 1. Gamma = +1, z -> infinity |
| B. Point B | B4 | 2. Gamma = -j |
| C. Point C | C1 | 3. Gamma = 0, z = 1, match |
| D. Point D | D3 | 4. Point in the capacitive half plane |
| E. Point E | E6 | 5. Gamma = +j |
| | | 6. Gamma = -1, z = 0 |
| | | 7. Point in the inductive half plane |

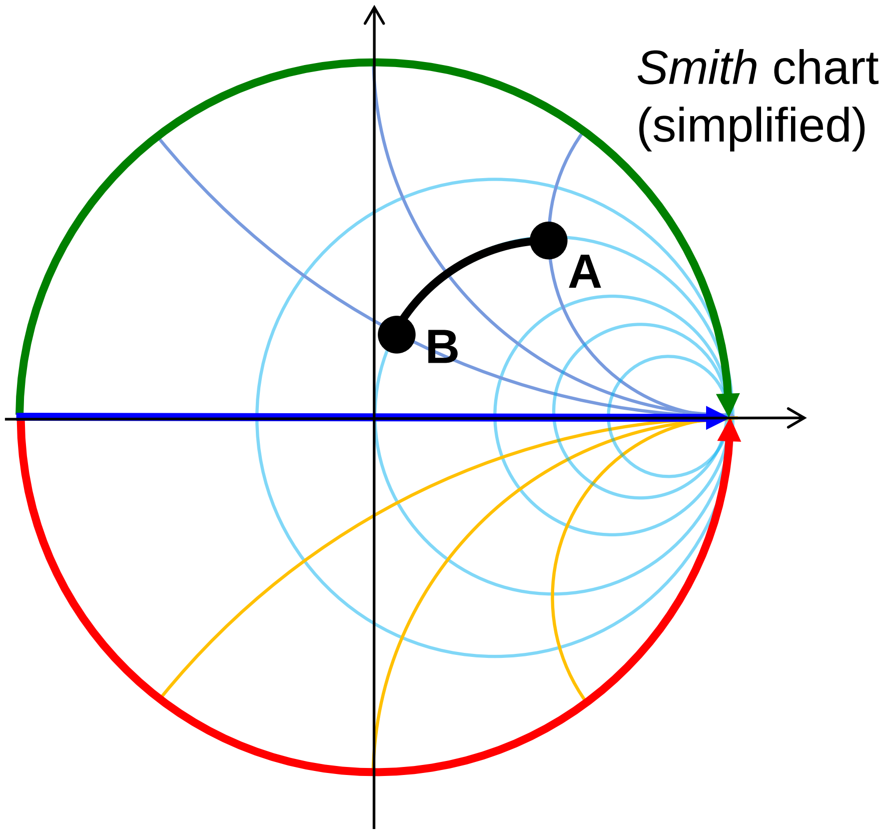



*Smith* chart (simplified)

# Quiz (3)

**4. Trace with marker points in the simplified Smith chart for an RL series impedance**

- ○ **Frequency f at point B > frequency f at point A**
- ✗ **Frequency f at point B < frequency f at point A**
- ○ **There is no frequency related to points A and B**
- ○ **Frequency f at point A = frequency f at point B**



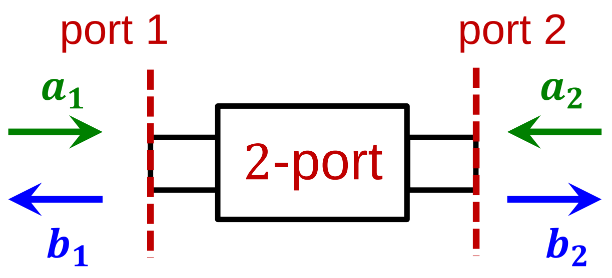
# Quiz (4)

1. **Select all correct answers**
   - ○ **Y- and Z-parameters of electrical networks require a reference impedance Z_0**
   - ✗ **Scattering parameters of RF networks are based on normalize complex voltage waves incident and reflected / transmitted at their ports**
   - ✗ **DUT stands for “Device Under Test”, as acronym for the RF network to be characterized**
   - ○ **S-parameters are only defined for a reference impedance of Z_0 = 50 Ohm.**
   - ✗ **Unused ports in a S-parameter measurement setup always need to be terminated in their characteristic port impedance**

# Quiz (5)

| Prompts | | Possible Answers |
|---|---|---|
| A. matched | A4 | 1. $S_{ii} = S_{ij}$ |
| B. symmetric | B3 | 2. $(S^*)^T = (i)$ |
| C. reciprocal | C5 | 3. $S_{ij} = S_{ji}$ and $S_{ii} = S_{jj}$ |
| D. passive and lossless | D2 | 4. $S_{ii} = 0$ |
| | | 5. Gamma = +j |
| | | 6. $S_{ij} = S_{ji}$ |

# Quiz (6)




**3.** **Mark all correct answers for the S-parameters of a 2-port RF network**

- ○ **a_1 and b_1 are independent parameters**
- ✗ **S_11 is the input reflection coefficient**
- ✗ **a_1 and a_2 are the incident waves at port 1 and port 2, respectively.**
- ○ **b_1 and b_2 are the transmitted waves between port 1 and port 2, and vice versa.**
- ✗ **S_21 and S_12 are the forward and reverse transmission gains.**
- ✗ **To characterize the S-parameters at port 2, port 1 needs to be terminated in its characteristic port impedance.**