Special Topic RF Course – Numerical Analysis of RF Problems

N. Baboi, H. Glock, A. Neumann, R. Singh, S. Udongwo, C. Vollinger, M. Wendt

Content

- Parametric modelling
- Eigenmode simulation
- Frequency domain simulation
- Time domain simulation
- Wakefield simulation
- Particle in Cell (PIC) simulation
- Equivalent Circuit Analysis with QucsStudio

Conventions

1.2 Conventions Used

- Bold texts represent clickable software buttons, e.g. **Start**
- Italicised texts represent texts written on the software GUI or used to describe sections of the software, e.g. *Navigation Tree*, *Messages/Progress panel*.
- Instruction box

< *title* >::< *subtitle* >::< *subsubtitle* >

These boxes display concise software steps, each with a title and body. The instructions are split into several boxes for complex models or analyses with explanatory text in between. To indicate continuation, subsequent instruction boxes are marked with *contd.* in the box title.

Icons

1.3 Icons Used

- Displays extra useful information.
- Displays useful software tips.
- Displays a warning or limitation of the CST Studio software or practices to avoid.
- **Displays questions that the students should think about and answer.**
- Keyboard keys are displayed as buttons, e.g. Enter , Esc , Ctrl , Shift , Z , etc.
- ① - Step marker.
- 3 Shift - Hold shift and click.

Numerical simulation

Maxwell Eigenvalue Problem (MEVP)

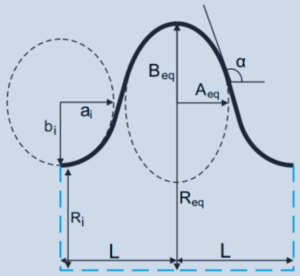
$$\nabla \times (\mu^{-1} \nabla \times \mathbf{E}(\mathbf{x})) - \lambda(\mathbf{x}) \epsilon \mathbf{E}(\mathbf{x}) = 0, \qquad \lambda = \frac{\omega^2}{c^2},$$

$$\nabla \cdot \mathbf{E} = 0 \qquad \mathbf{E} \in \Omega,$$

$$\mathbf{n} \times \mathbf{E} = 0 \qquad \mathbf{E} \in \partial\Omega_{\text{wall}},$$

$$\mathbf{n} \times (\mu^{-1} \nabla \times \mathbf{E}) = 0 \qquad \mathbf{E} \in \partial\Omega,$$

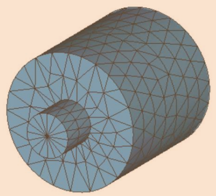
Modelling



$\partial\Omega, \Omega$

Meshing

- Tetrahedral (FEM)
- Hexahedral (FIT, FDTD)



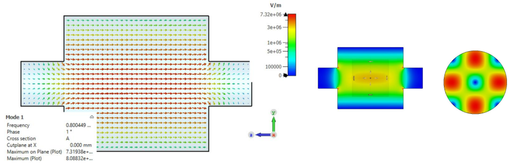
Boundary Conditions

- PEC
- PMC
- Open
- Periodic
- Waveguide

Solver Setup

- Direct method or Iterative method
- Analysis bounds
- Accuracy

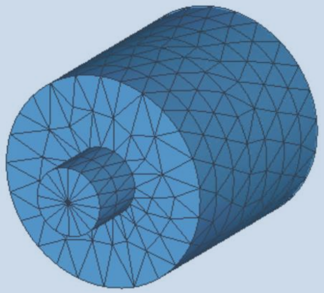
Results Post-Processing



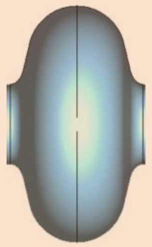
[1] T. Flisgen, Electromagnetic Simulations I&II, CAS course on “RF for Accelerators“, Berlin, Germany

Parametric Modelling

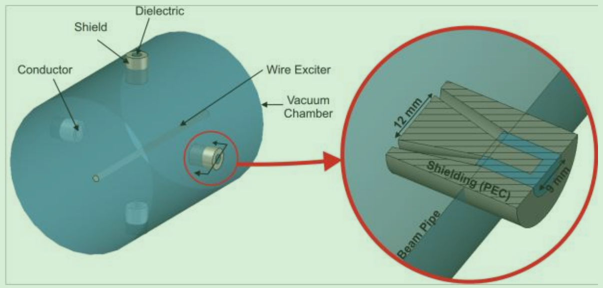
- Parametric modelling is a feature in CAD software that allows designers to create and modify 2D or 3D models flexibly and efficiently.



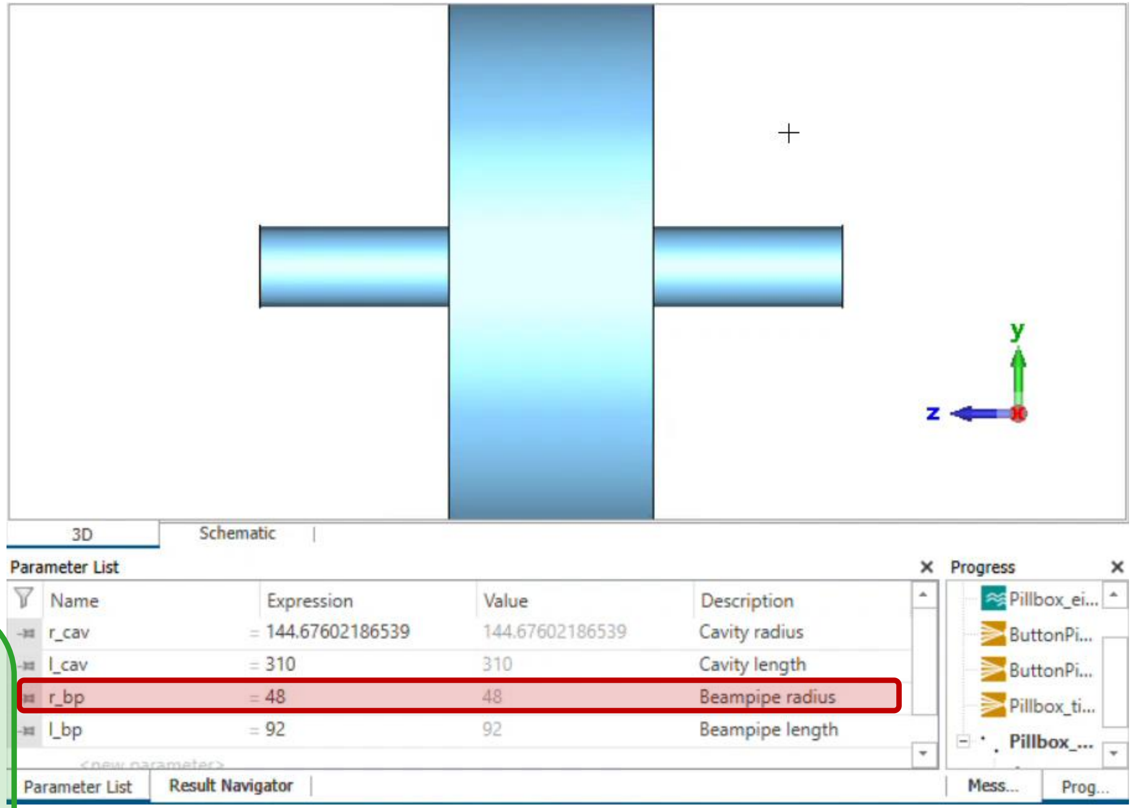
Pillbox Cavity



Elliptical cavity geometry



Button Beam Position Monitor (BPM)



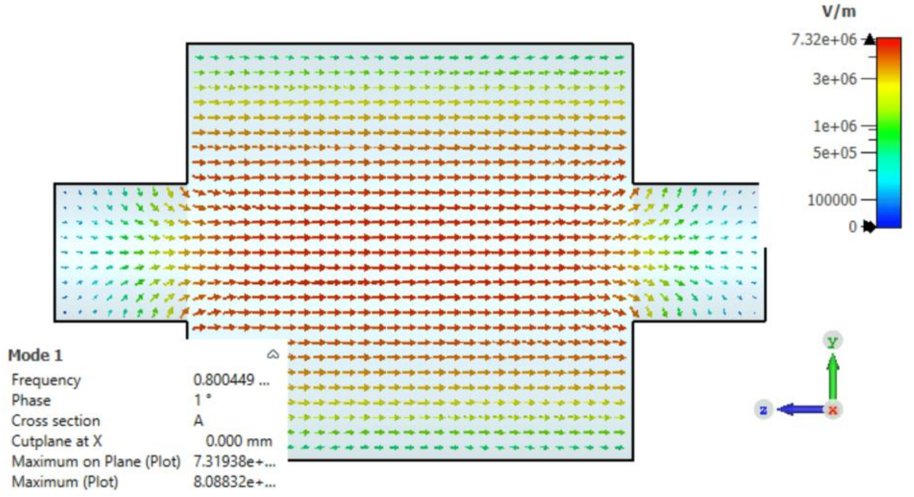
Parametric Modelling

Parametric Modelling : More material

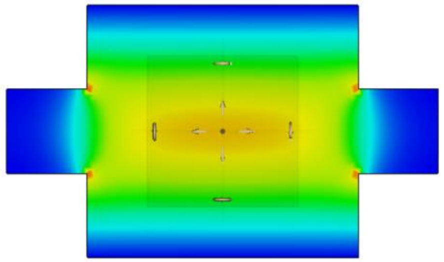
- For those interested in some more advanced modelling techniques and illustrations, check out the following links.
  - Double Quarterwave HOM Coupler Modelling: https://youtu.be/FvePLJSld2w
  - TESLA-type HOM Coupler Modelling: https://youtu.be/StuOmIS82UI

Eigenmode Simulation

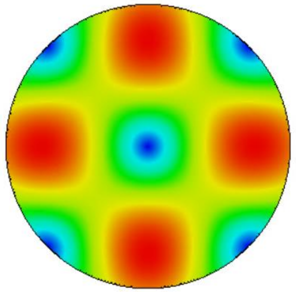
- Eigenmode analysis is a method for analysing the resonant modes of an electromagnetic system.



TM010 mode electric field simulation



TM010 mode electric field contour



HOM electric field contour

The CERN Accelerator School

Eigenmode simulation

Maxwell Eigenvalue Problem (MEVP)

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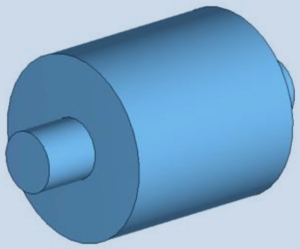
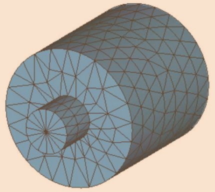
Modelling



$\partial\Omega, \Omega$

Meshing

- **Tetrahedral (FEM)**
- Hexahedral (FIT, FDTD)



Boundary Conditions

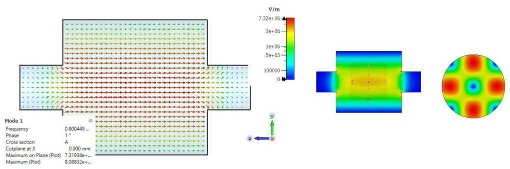
- **PEC**
- PMC
- Open
- Periodic
- Waveguide

Solver Setup

- Direct method or **Iterative method**
- **Analysis bounds:**
  - $0 < f < 1\text{GHz}$
- **Accuracy: 1e-6**

Results

Post-Processing

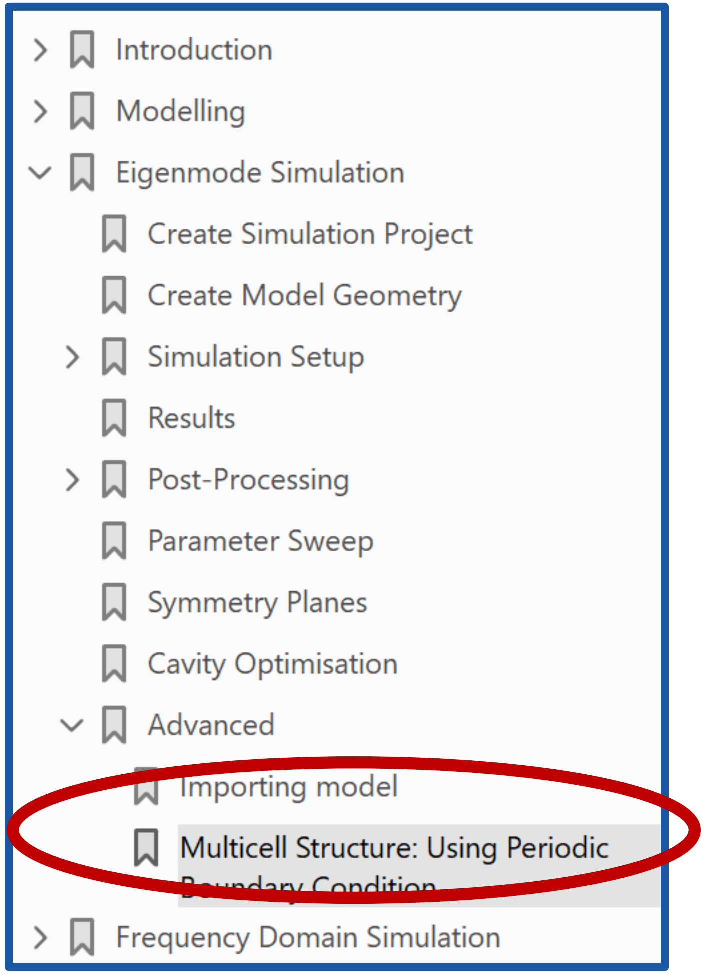


[1] T. Flisgen, Theory of EM Fields I&II, CAS course on “RF for Accelerators“, Berlin, Germany.
[2] T. Flisgen, Electromagnetic Simulations I&II, CAS course on “RF for Accelerators“, Berlin, Germany.

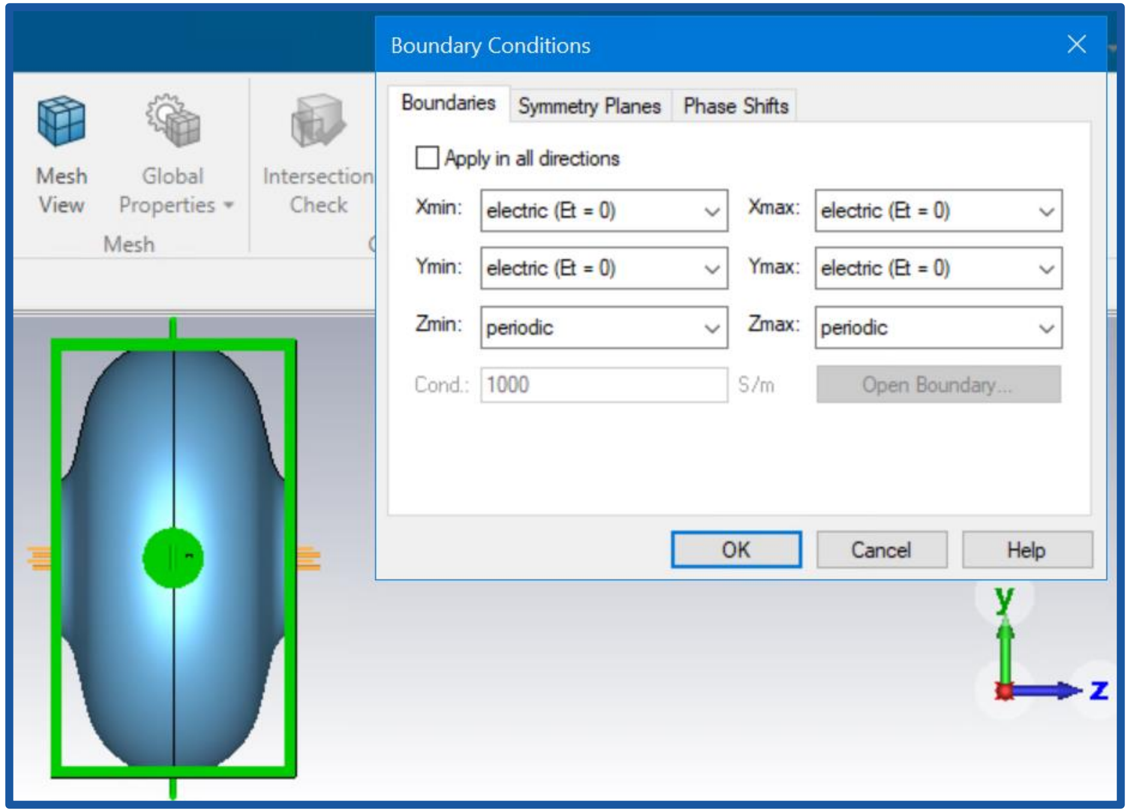
Eigenmode Simulation: More material

- For SRF elliptical cavity-specific figures of merit calculations, check
  - https://youtu.be/rDuoBRcQons

Eigenmode Simulation: Periodic Boundary

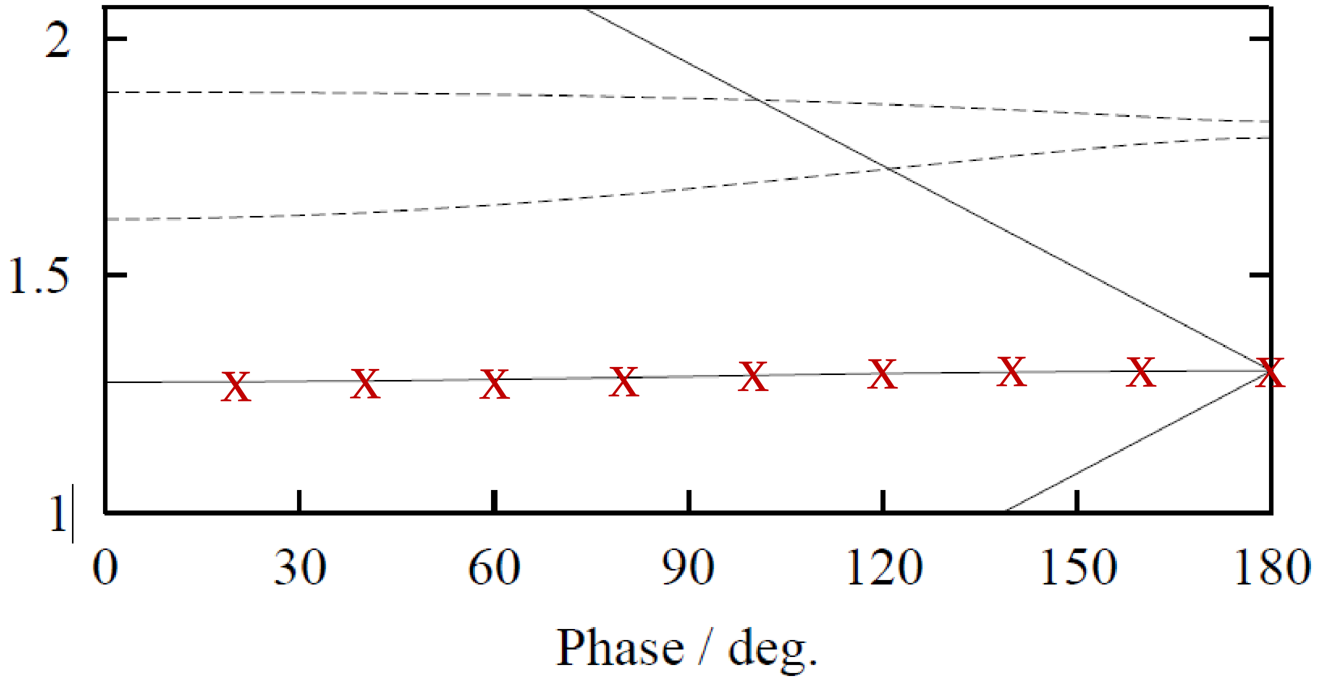


- Simulate an infinite periodic structure
  - dispersion diagram

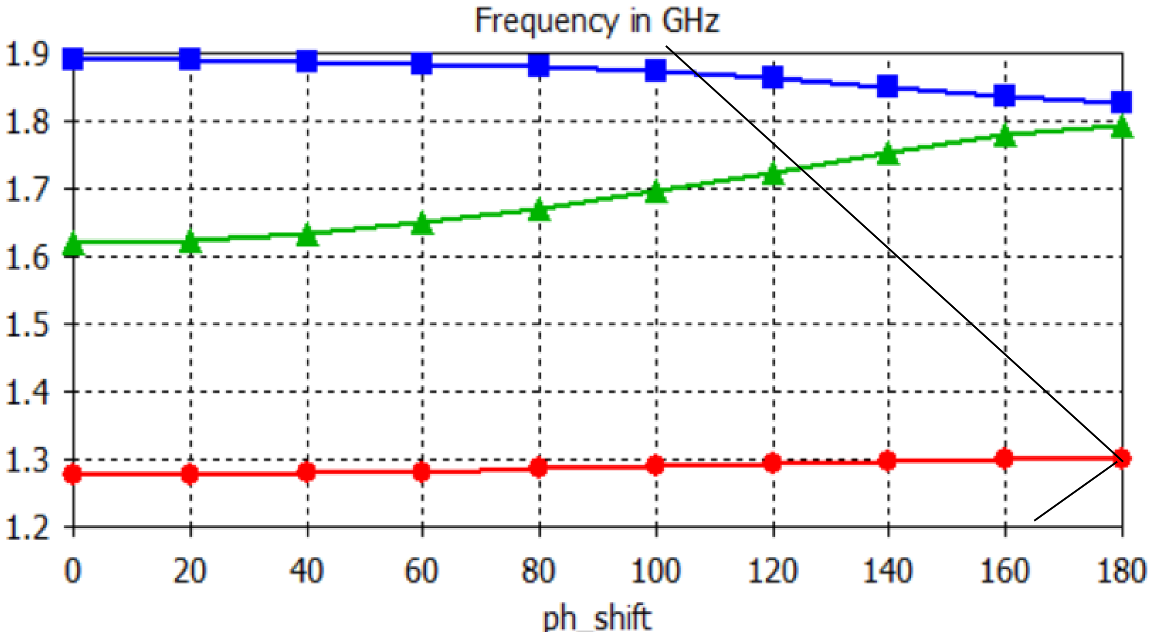


Dispersion diagram

- Infinite periodic structure



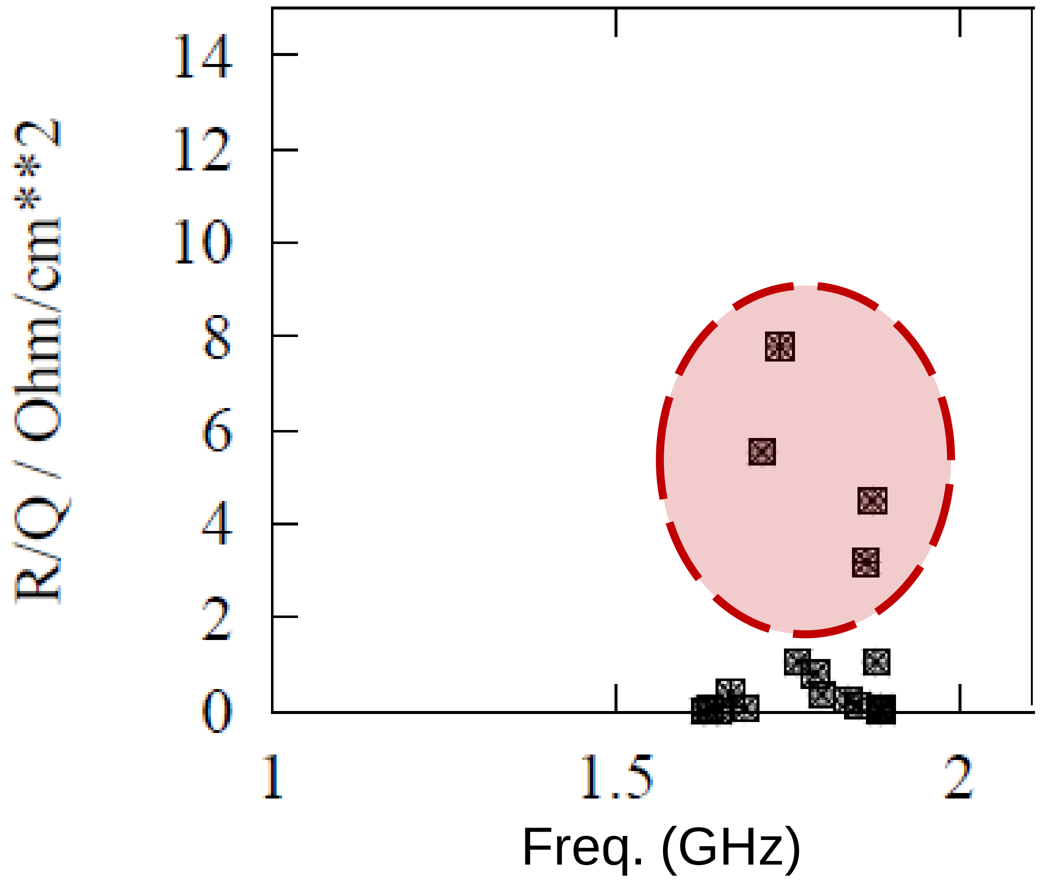
- 9-cell cavity – CST Studio result

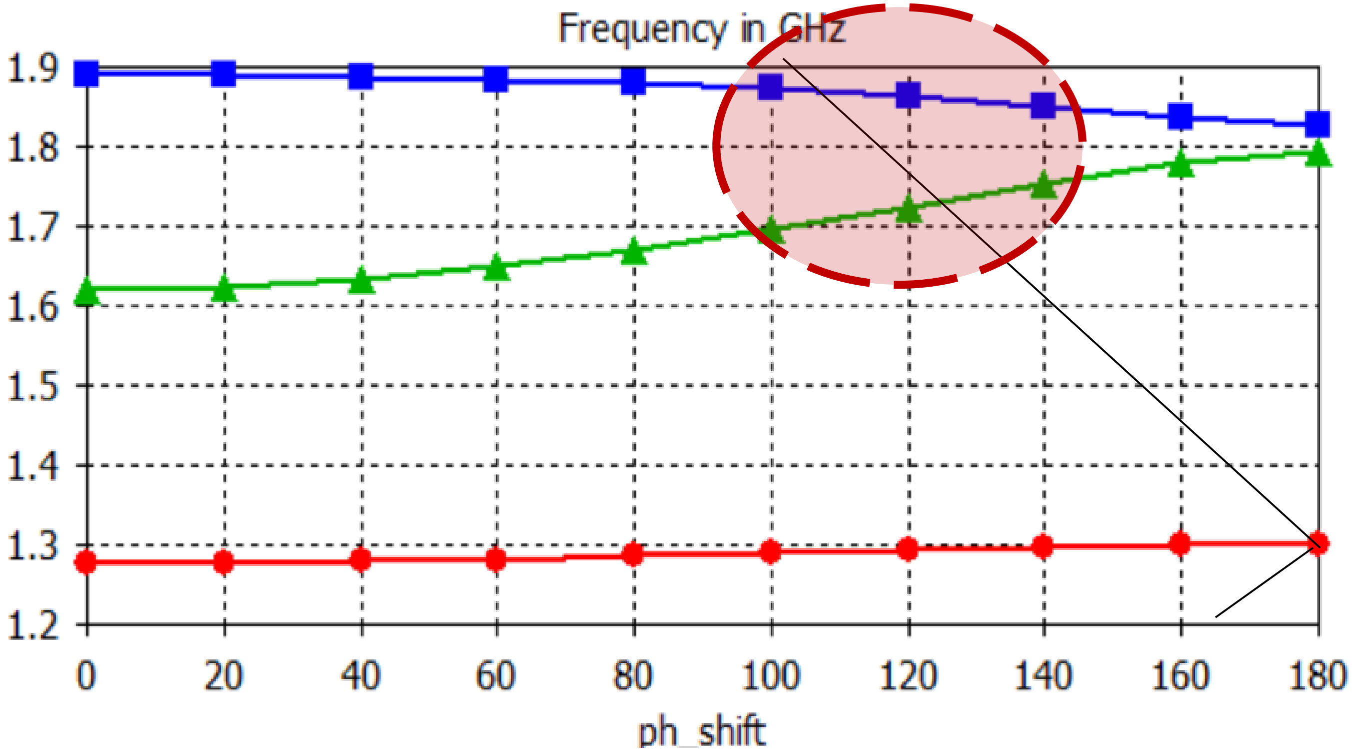


R. Wanzenberg, TESLA 2001-33

R/Q

- High R/Q value for modes close to the light line





Frequency domain simulation

Maxwell Eigenvalue Problem (MEVP)

$$\nabla \cdot \mathbf{E} = \frac{\rho}{e} \qquad \nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

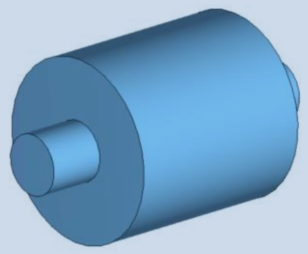
$$\nabla \cdot \mathbf{H} = 0 \qquad \nabla \times \mathbf{H} = \mathbf{J} + j\omega\epsilon\mu\mathbf{E}$$

$$\nabla \cdot \mathbf{E} = 0 \qquad \mathbf{E} \in \Omega,$$

$$\mathbf{n} \times \mathbf{E} = 0 \qquad \mathbf{E} \in \partial\Omega_{\text{wall}},$$

$$\mathbf{n} \times (\mu^{-1} \nabla \times \mathbf{E}) = 0 \qquad \mathbf{E} \in \partial\Omega,$$

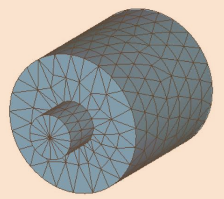
Modelling



$\partial\Omega, \Omega$

Meshing

- **Tetrahedral (FEM)**
- **Hexahedral (FIT**, FDTD)



Boundary Conditions

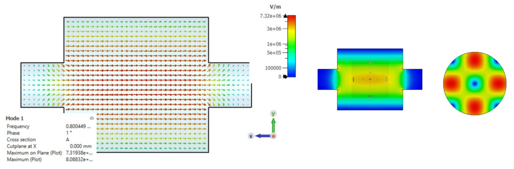
- **PEC**
- PMC
- Open
- Periodic
- **Waveguide port**

Solver Setup

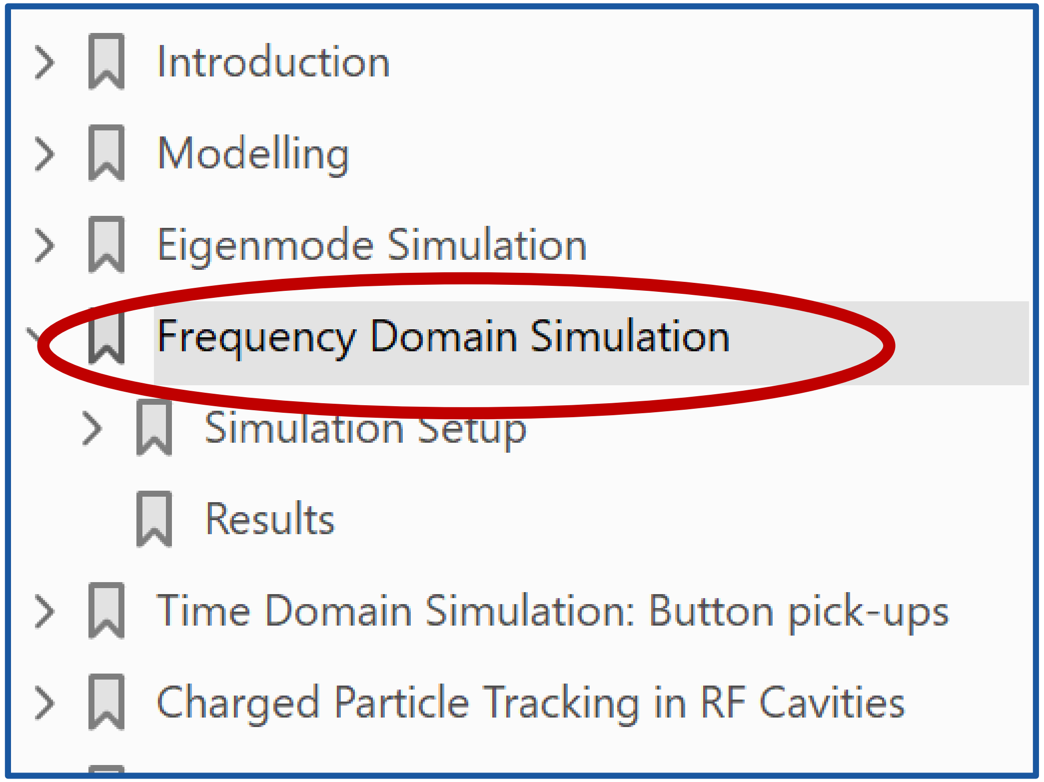
- Direct method or **Iterative method**
- **Analysis bounds:**
  - $0 < f < 1\text{GHz}$
- **Accuracy: 1e-4**

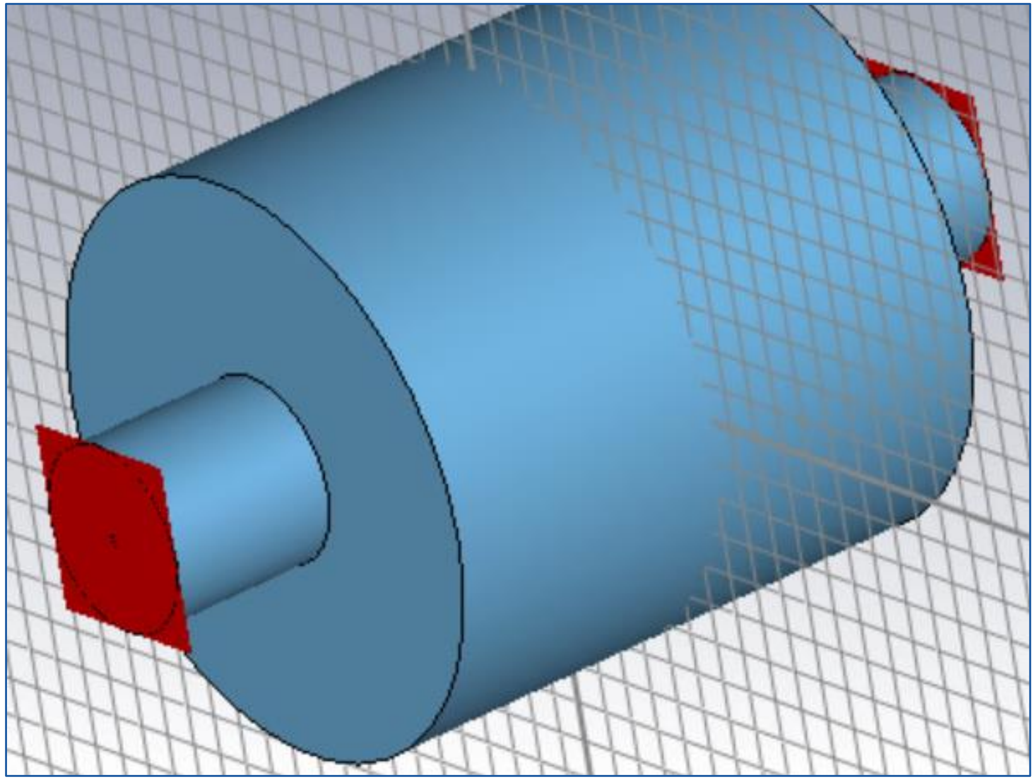
Results

Post-Processing



Frequency Domain Simulation





Day 02: General Information

- Important to install the provided CST version and not too old or busy laptop. You may encounter an error in adaptive port meshing.
  - **Good news: The error can be fixed**

- We will use the online PDF manual provided. The paper version is old and conflicting conventions between the two for today's exercises

- Only two hours left for CST: Might not get through all simulations for the button pickup

- After the coffee break, one hour with circuit simulation

- Talk with each other, ask as many questions as you want

- In the folder CAS2023->CST Simulation->modelling, make two more copies of "ButtonPickUp.cst"

Time Domain Solver

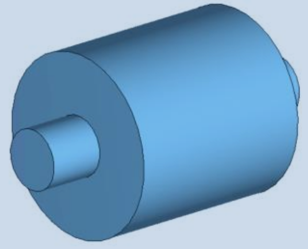
Maxwell Equations (Integral form)

$$\oiint_{\partial\Omega} \mathbf{E}\cdot\mathrm{d}\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_{\Omega}\rho\,\mathrm{d}V \qquad \oint_{\partial\Sigma}\mathbf{E}\cdot\mathrm{d}\boldsymbol{\ell} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma}\mathbf{B}\cdot\mathrm{d}\mathbf{S}$$

$$\oiint_{\partial\Omega} \mathbf{B}\cdot\mathrm{d}\mathbf{S} = 0 \qquad \oint_{\partial\Sigma}\mathbf{B}\cdot\mathrm{d}\boldsymbol{\ell} = \mu_0\left(\iint_{\Sigma}\mathbf{J}\cdot\mathrm{d}\mathbf{S} + \varepsilon_0\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma}\mathbf{E}\cdot\mathrm{d}\mathbf{S}\right)$$

$$\nabla\cdot\mathbf{E} = 0 \qquad \mathbf{E}\in\Omega$$
$$\mathbf{n}\times\mathbf{E} = 0 \qquad \mathbf{E}\in\partial\Omega_{\mathrm{wall}}$$
$$\mathbf{n}\times(\mu^{-1}\nabla\times\mathbf{E}) = 0 \qquad \mathbf{E}\in\partial\Omega$$

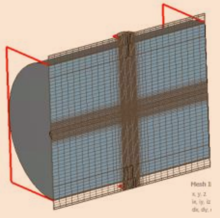
Modelling



$\partial\Omega, \Omega$

Meshing

- Tetrahedral (FEM)
- **Hexahedral (FIT,** FDTD)



Boundary Conditions

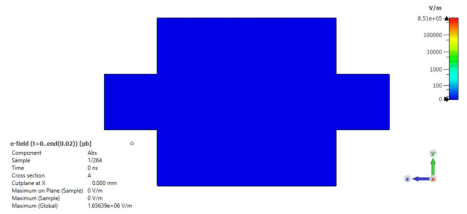
- **PEC**
- PMC
- Open
- Periodic
- **Waveguide Ports**

Solver Setup

- Direct method or Iterative method
- **Analysis bounds:**
  - $0 < f < 1\mathrm{GHz}$
- **Accuracy: 1e-6**

Results

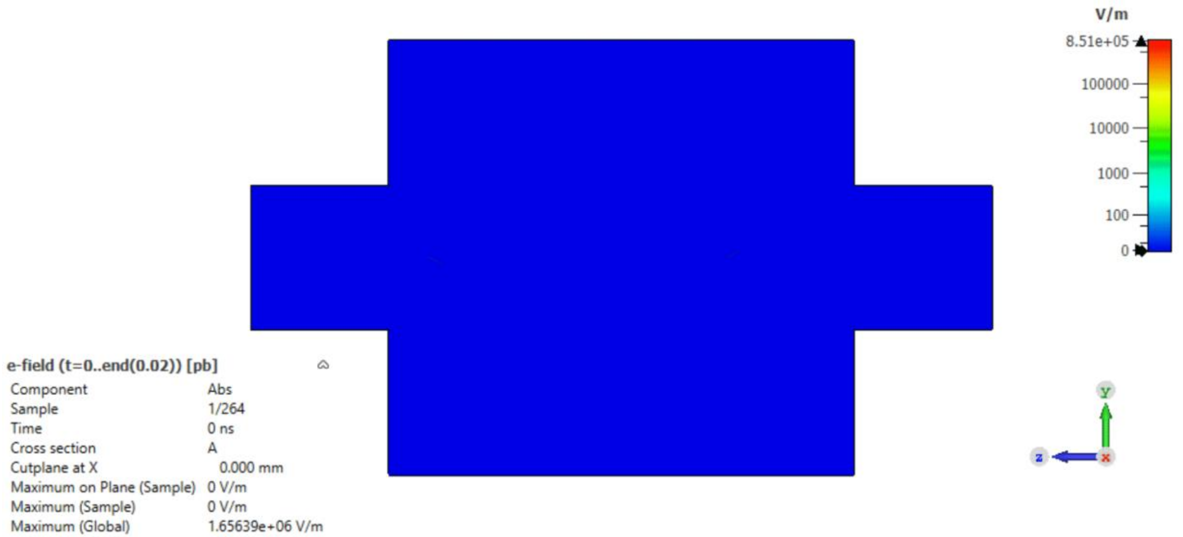
Post-Processing



[1] T. Flisgen, Electromagnetic Simulations I&II, CAS course on "RF for Accelerators", Berlin, Germany.

Wakefield Simulations

- Wakefield solver is essentially an extension of a time-domain solver to simulate the interaction of charged particle beams with the environment.



Wakefield Simulation of a pillbox cavity

Wakefield Solver

Maxwell Equations (Integral form)

$$\oiint_{\partial\Omega} \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_{\Omega} \rho\, dV \qquad \oint_{\partial\Sigma} \mathbf{E}\cdot d\boldsymbol{\ell} = -\frac{d}{dt}\iint_{\Sigma} \mathbf{B}\cdot d\mathbf{S}$$

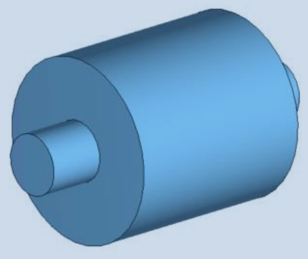
$$\oiint_{\partial\Omega} \mathbf{B}\cdot d\mathbf{S} = 0 \qquad \oint_{\partial\Sigma} \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0\left(\iint_{\Sigma} \mathbf{J}\cdot d\mathbf{S} + \varepsilon_0\frac{d}{dt}\iint_{\Sigma} \mathbf{E}\cdot d\mathbf{S}\right)$$

$$w_{\parallel}(\boldsymbol{\rho}, s) = \frac{c}{q}\int_{-\infty}^{\infty} E_z|_{z=ct-s}\, dz,$$

$$\mathbf{w}_{\perp}(\boldsymbol{\rho}, s) = \frac{c}{q}\int_{-\infty}^{\infty} (\mathbf{E}_{\perp} + c\hat{z}\times\mathbf{B})|_{z=ct-s}\, dz,$$

$$\nabla\cdot\mathbf{E} = 0 \qquad \mathbf{E}\in\Omega$$
$$\mathbf{n}\times\mathbf{E} = 0 \qquad \mathbf{E}\in\partial\Omega_{\text{wall}}$$
$$\mathbf{n}\times(\mu^{-1}\nabla\times\mathbf{E}) = 0 \qquad \mathbf{E}\in\partial\Omega$$

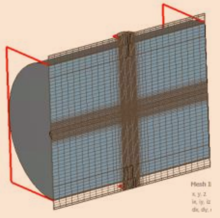
Modelling



$\partial\Omega, \Omega$

Meshing

- Tetrahedral (FEM)
- **Hexahedral (FIT,** FDTD)



Boundary Conditions

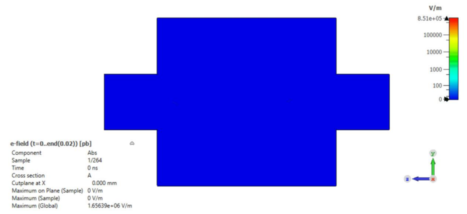
- **PEC**
- PMC
- Open
- Periodic
- **Waveguide ports**

Solver Setup

- Direct method or Iterative method
- **Analysis bounds:**
  - $\mathbf{0 < f < 1GHz}$
- **Accuracy: 1e-6**

Results

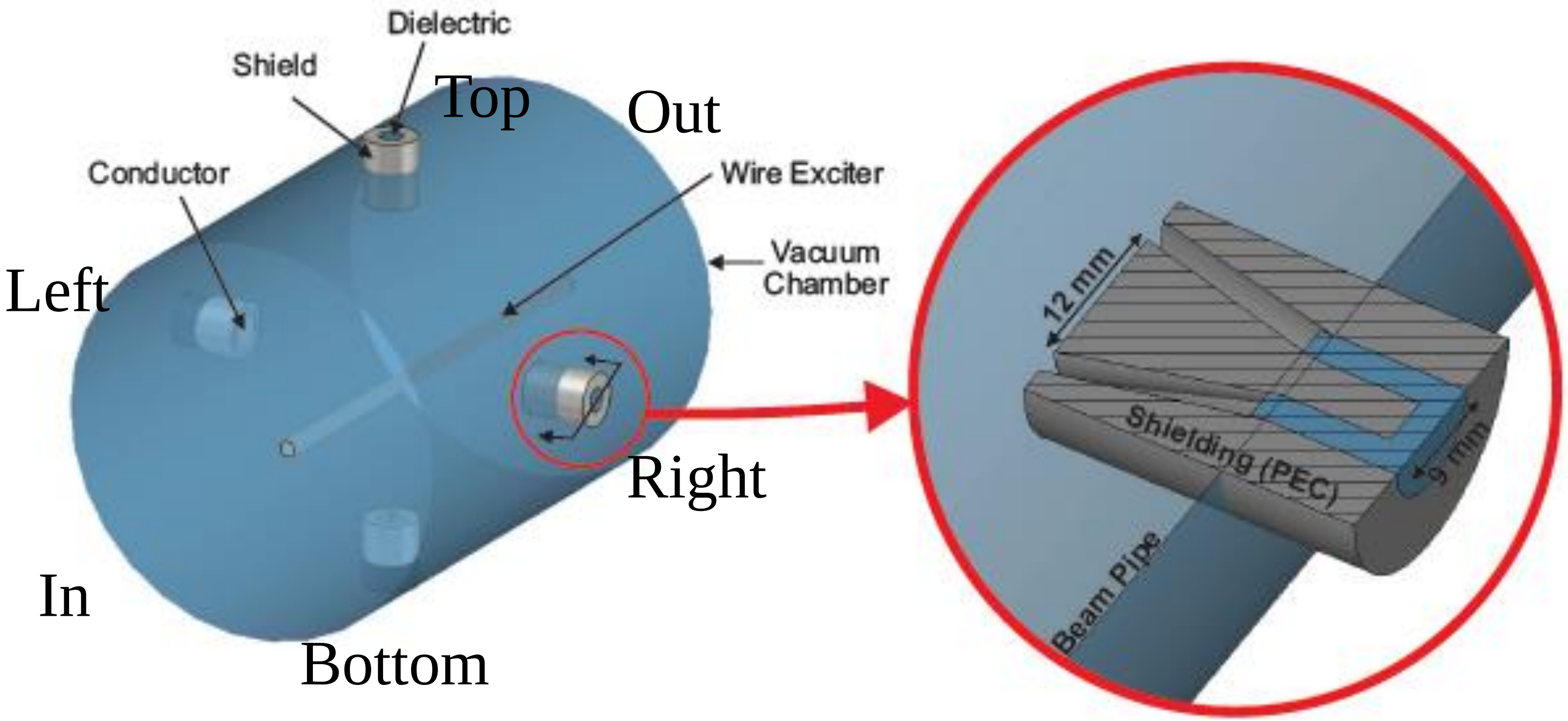
Post-Processing



[1] A. Mostacci, Impedance and Wakefield, CAS 2023, June 2023, Berlin.
[2] T. Flisgen, Electromagnetic Simulations I&II, CAS course on "RF for Accelerators", Berlin, Germany.

Pick-up/Phase Probe/Electrostatic Pick-Up/Beam Position Monitor (BPM)

- Non-interceptive diagnostic devices used in most accelerators for transverse beam position, charge distribution, time of arrival and absolute charge measurement → **Also in LLRF control loops**



Pick-up/Phase Probe/Electrostatic Pick-Up/Beam Position Monitor (BPM)

- The beam is mimicked using a wire excitation for testing in the lab before installation → TD solver

Some Figures of Merit

Transverse Impedance ($Z_{transfer}$): The ratio of voltage induced by pick-up to the current in the wire/ beam current.

$$V_{top} = Z_{transfer,top} \times I_{wire}$$

where

$$Z_{transfer,top} = \sqrt{Z_{input} Z_{top}} S_{top,in} S_{out,in}$$

Position Sensitivity (K): Difference in signal induced on opposite plates normalized to sum of signals per unit movement of the wire/beam. Used to calculate position of beam

$$Ver.Position = \left(\frac{1}{K}\right) \times \frac{V_{top} - V_{bottom}}{V_{top} + V_{bottom}}$$

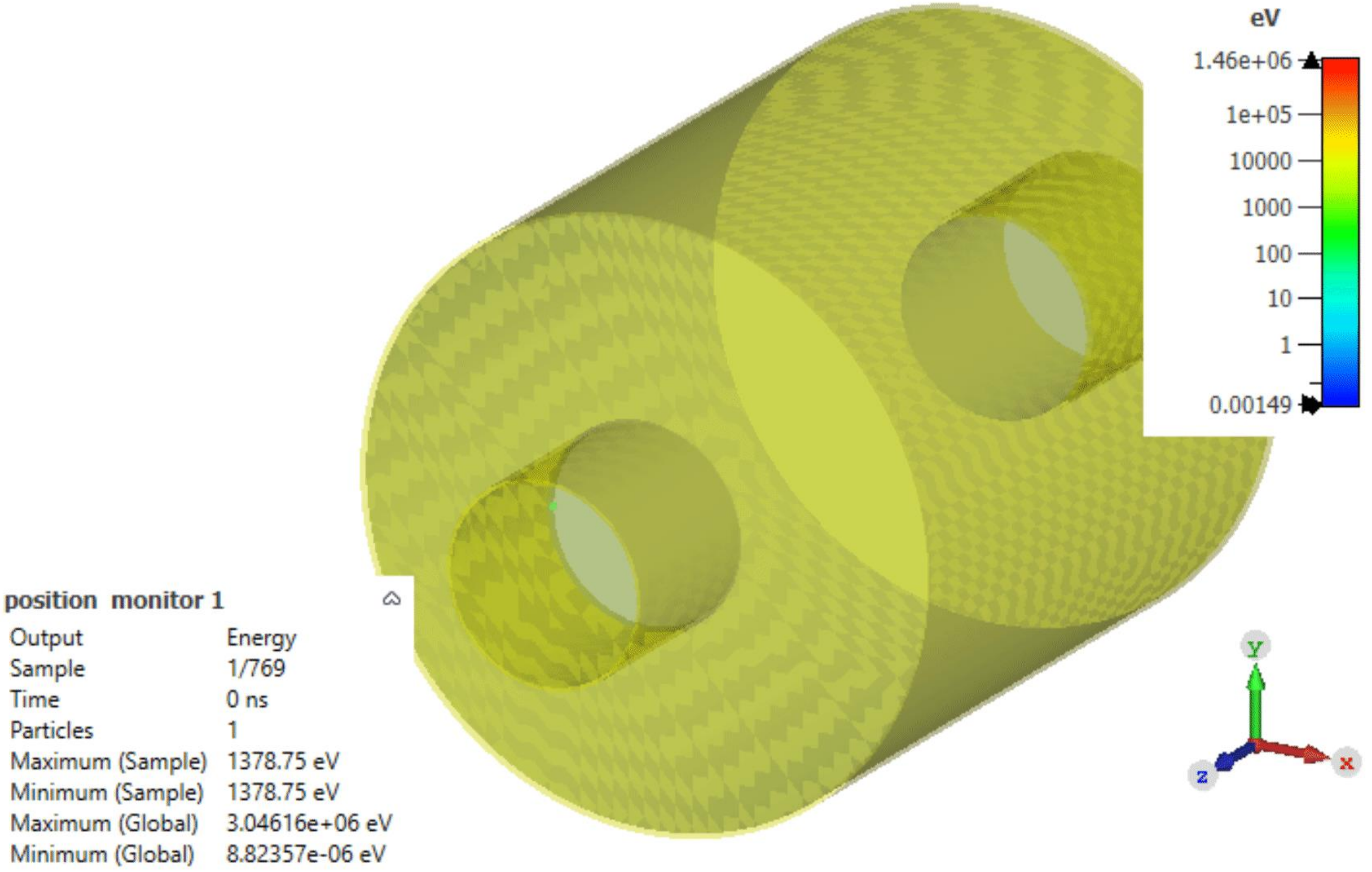
Wakefield Simulation : More material

- For transversal wakefield simulation, check
  - https://youtu.be/Zm0xEvK0e1A

Particle in Cell (PIC) Simulation

- PIC simulation is a tool for simulating the behaviour of charged particle beams in complex electromagnetic environments

- In the tutorial, multipacting is simulated in a pillbox cavity.



Multipacting in pillbox cavity

Particle-in-Cell (PIC) Simulation

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$$\nabla\cdot\mathbf{E} = 0$$
$$\mathbf{n}\times\mathbf{E} = 0$$
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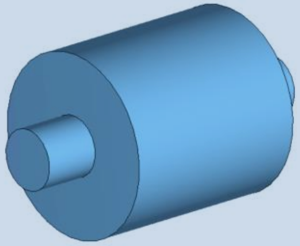
(Non)**relativistic**
Lorentz force

$$\frac{\mathrm{d}}{\mathrm{d}t}((\gamma m_0)\mathbf{v}) = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \qquad \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{r} = \mathbf{v}$$

Modelling



$\partial\Omega, \Omega$

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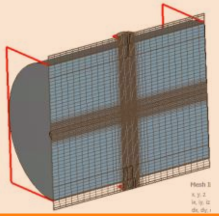

Initial/Boundary Conditions

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- Waveguide

Solver Setup

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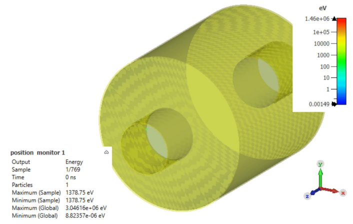
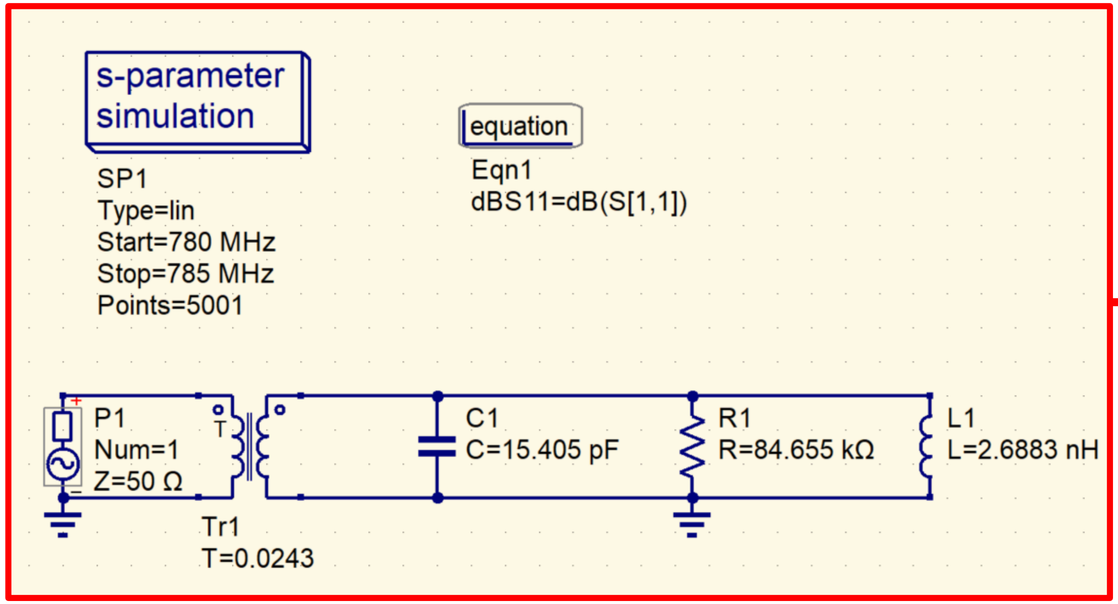
Results

Post-Processing

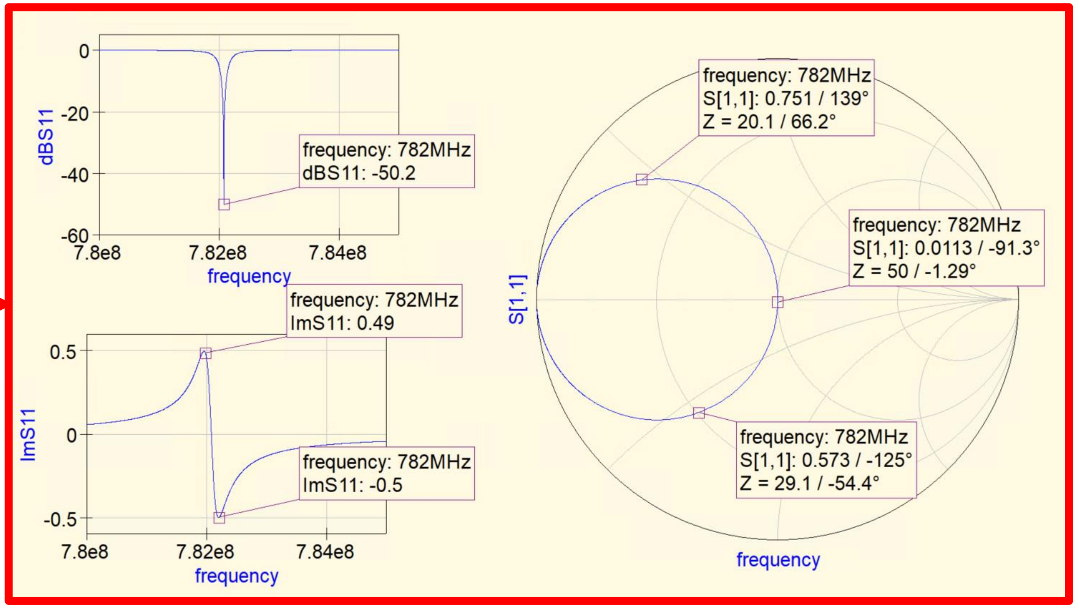


Equivalent Circuit Analysis - Qucs Studio

- Equivalent circuit simulations provide a simplified representation of complex systems, enabling efficient analysis and prediction of system behaviour.



RLC equivalent circuit of a resonant mode.



Transmission curves and Smith's chart of the equivalent circuit