RF cavities, overview, part I

RF CAS, Berlin

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Overview

- Cavity classifications
- The first accelerating cavities
- Basic RF theory
- Cavity parameters
- The pillbox cavity
- TM, TE, and TEM mode cavities
- Lumped circuit description
- Getting power into a cavity
- New ideas

How do we classify cavities?

- Fixed/variable frequency
- Accelerating/non-accelerating
- By electromagnetic mode type
- Travelling wave/standing wave
- Normal conducting/superconducting

Accelerating cavities by fixed/variable frequency

	Acceleration with changing particle velocity	Acceleration with constant particle velocity
variable RF frequency (~revolution frequency)	low-β synchrotrons (protons, ions) low-β FFAGs (protons, ions)	
fixed RF frequency	cyclotrons low-β proton/ion linacs	electron linacs high-β synchrotrons (electrons, protons, ions) high-β FFAGs (electrons, protons, ions)

Accelerating cavities by fixed/variable frequency

	Acceleration with changing particle velocity	Acceleration with constant particle velocity
variable RF frequency (~revolution frequency)	• Materials with adjustable permeability in the cavity volume, e.g. ferrites or Finemet®: allows to tune f. • Wideband RF amplifiers • Typically low voltages, high losses See H. Klingbeil, Magnetic Alloy / Ferrite Cavities	
fixed RF frequency	• In Linacs: cell length adapts to particle velocity • In Cyclotrons: the particle path becomes longer with higher energy. • The same narrowband RF amplifiers for all cavities.	• The same narrowband RF amplifiers for all cavities. • Only one type of cavity needed. Mass production. • High cavity gradients.

Non-accelerating cavities

RF deflection

- Beam chopping at low energies.
- Beam funnelling at low energy.
- CRAB crossing of colliding beams.

See G. Burt, Transverse deflecting cavities

Longitudinal manipulation

- Forming bunches out of a continuous beam (coasting beam or ion source beam).
- Keep bunches longitudinally confined during transport or acceleration.
- Phase rotation to reduce or increase momentum spread.
- Inducing longitudinal emittance growth.
- Bunch merging (e.g. via slip-stacking) or bunch splitting.

See Longitudinal dynamics & RF manipulations

THE FIRST ACCELERATING CAVITIES

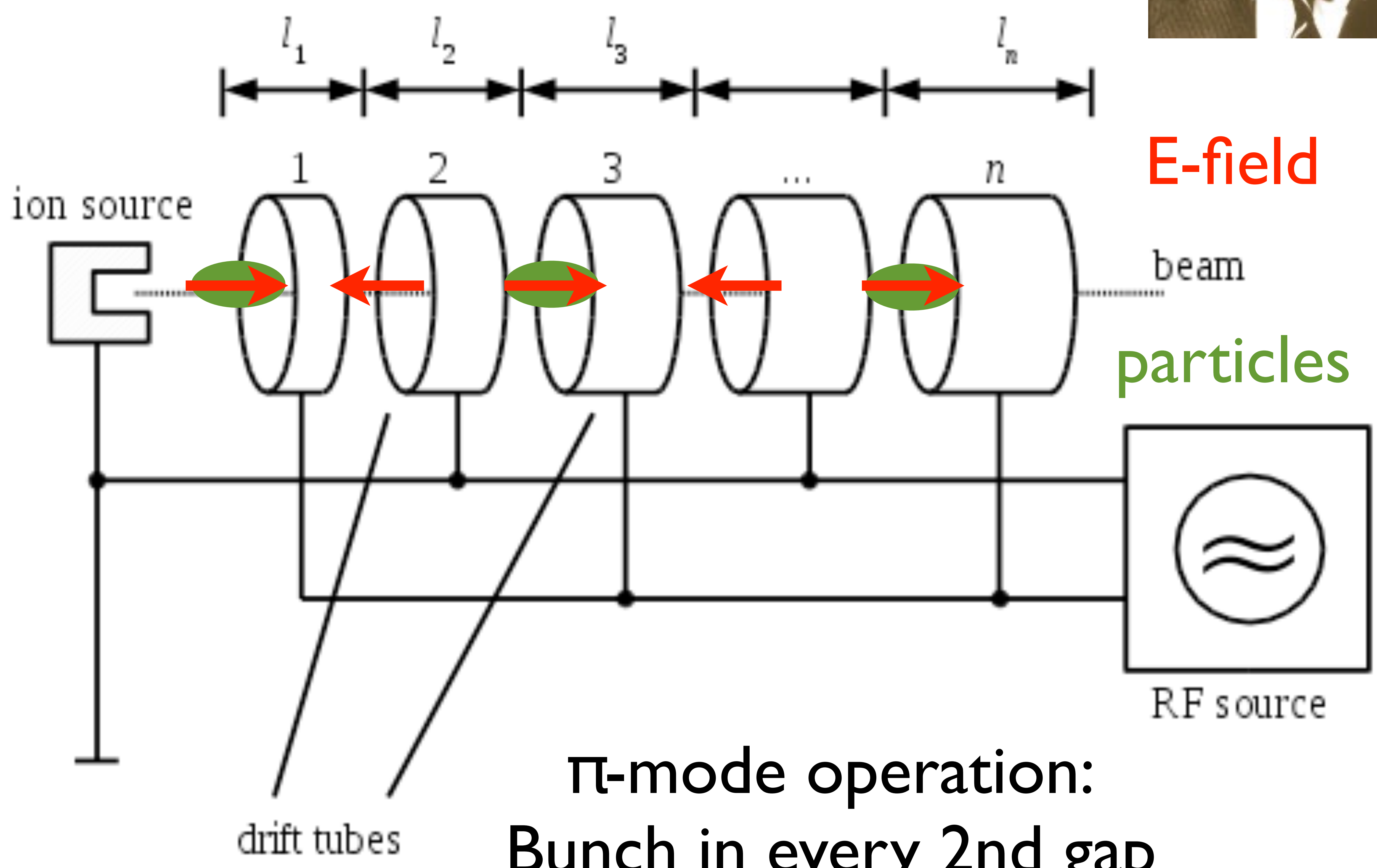
Or why we put RF fields in a box..

Not yet a cavity:
The Wideröe Linac (1927)

energy gain: $E = eN_{gap}V_{RF}$

period length increases with velocity: $l = \frac{v}{2f}$

Crucial inventions: RF power sources & synchronism between RF and particles



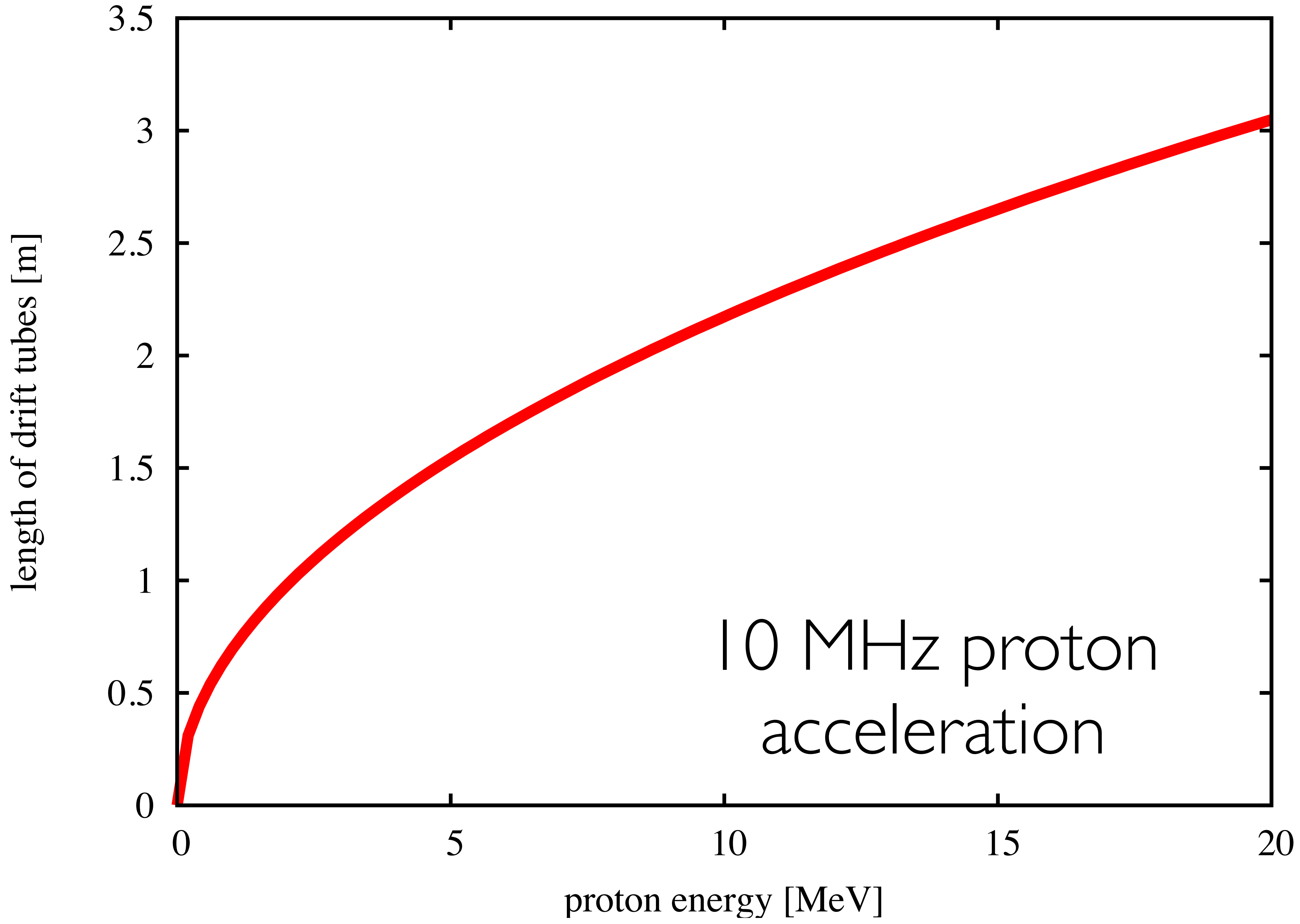
π-mode operation:
Bunch in every 2nd gap

The RF phase changes by 180°, while the particles travel from one tube to the next

Why wasn't this good enough?

- The Wideröe Linac was enclosed by a glass tube.
- At higher frequencies (> 10 MHz), the drift tubes started radiating energy (like antennas): less power was used for acceleration.
- At low frequencies and with rising particle energy, the length of the drift tubes becomes quickly unpractical.

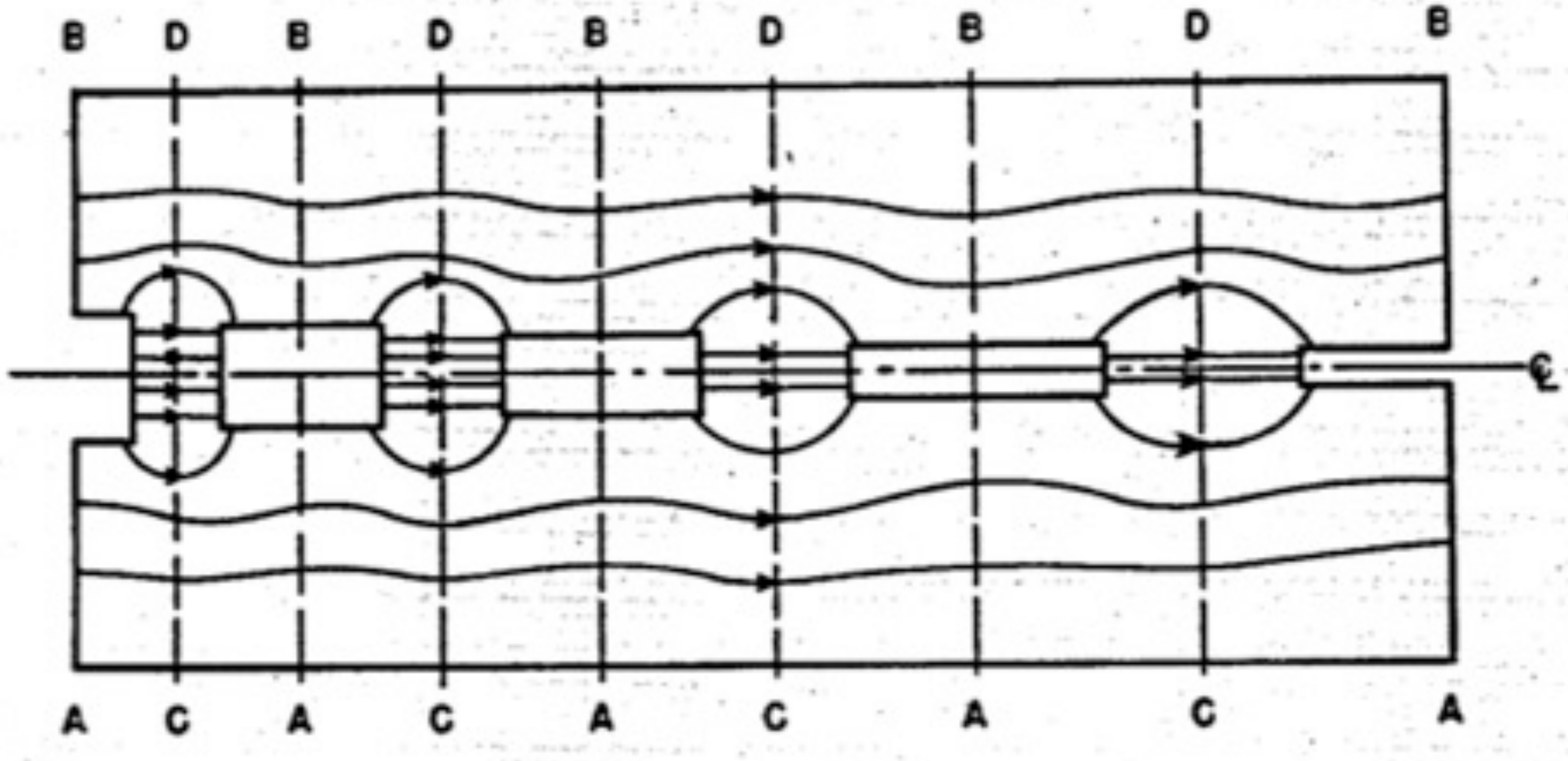
➡ The Wideröe Linac was only usable for very low-velocity particles.



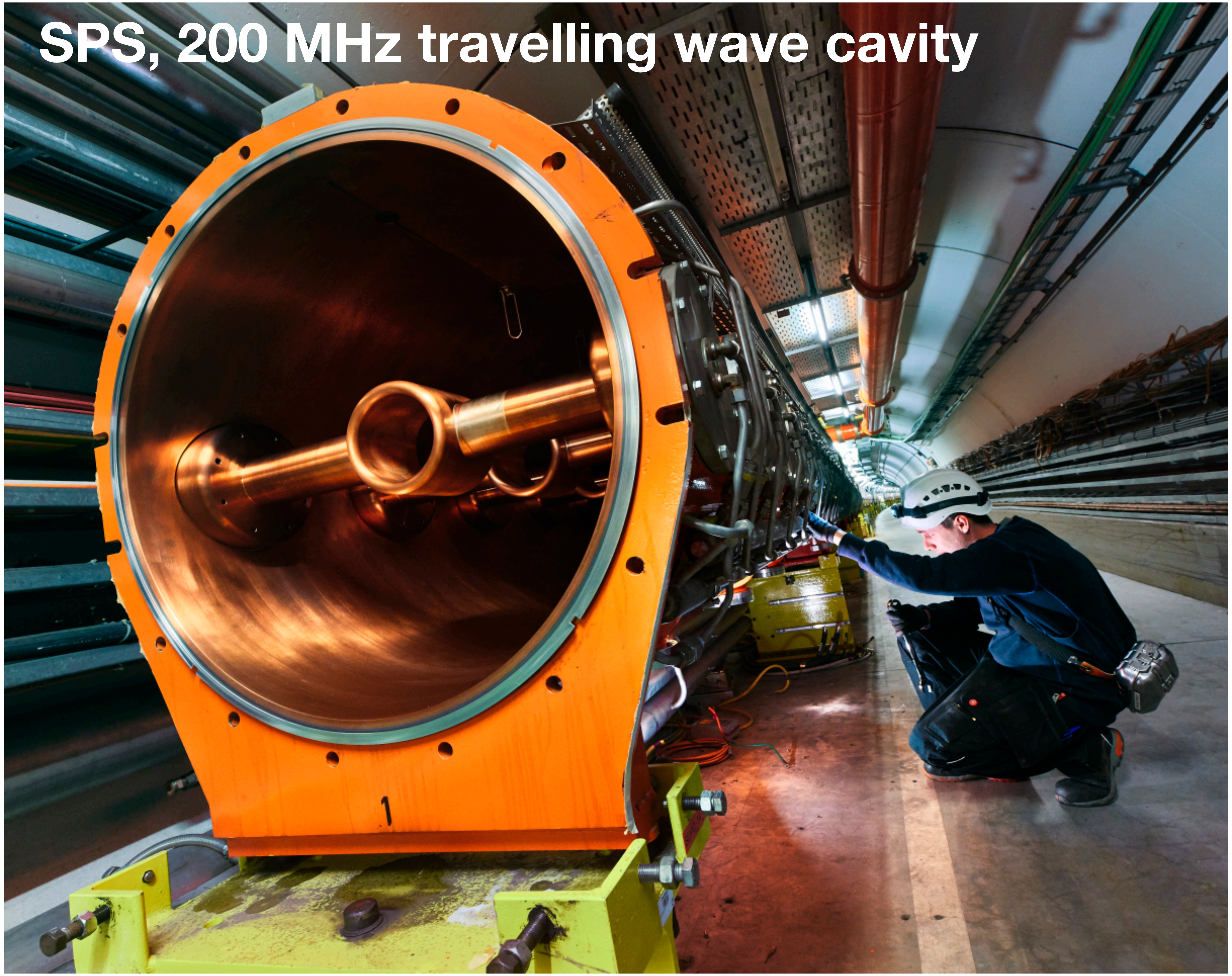
1946: Alvarez put a cavity around the Wideröe Linac and created a resonant structure

- While the resonator fields point in the "wrong" direction, the particles are shielded by the drift tubes.
- WW2 brought the development of high-power, high-frequency RF tubes for radar technology.

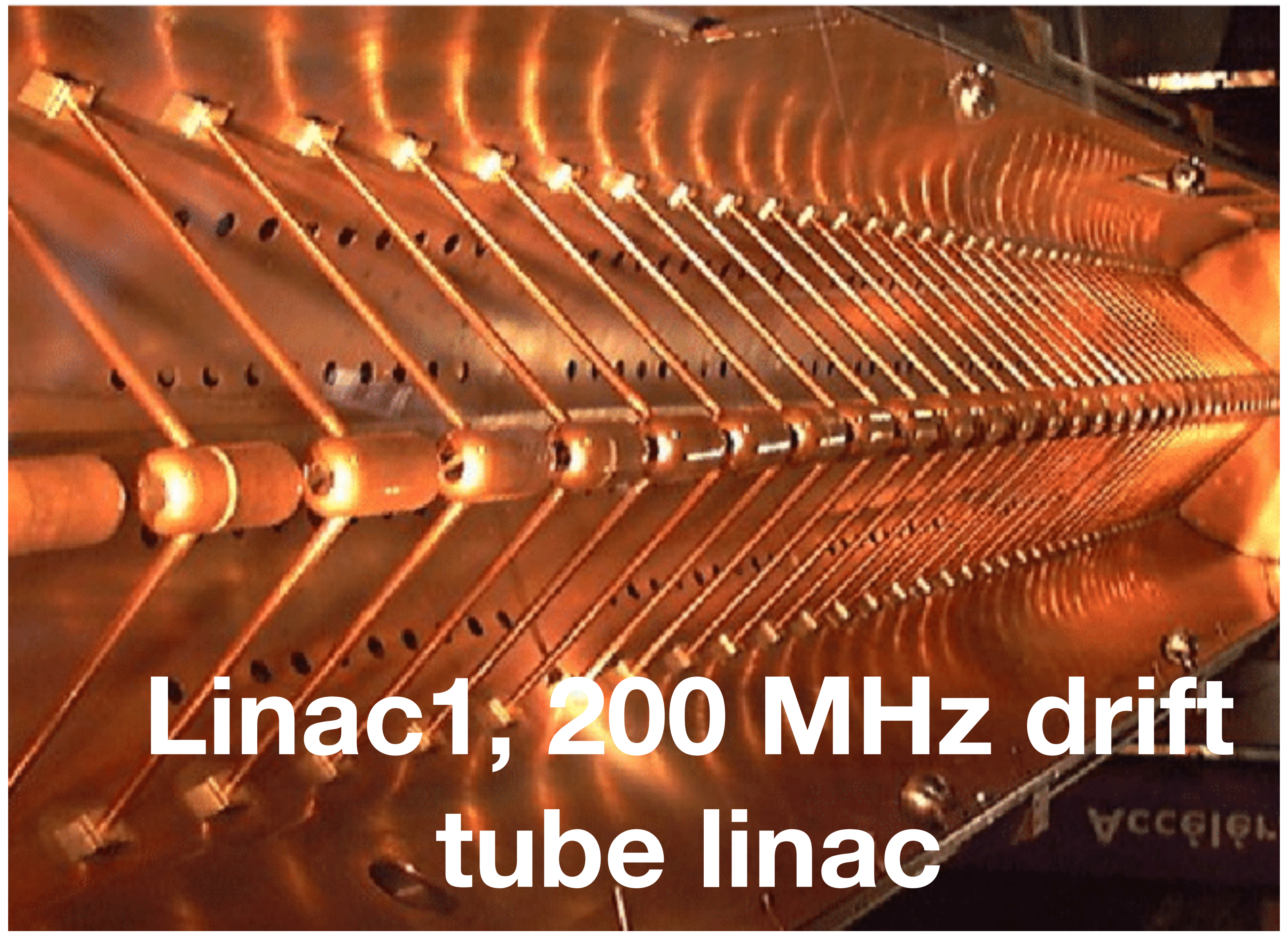
➡Therefore most of the early accelerators operated at this frequency (including Linac1, Linac2, and the SPS at CERN)



0-mode operation: bunch in every gap


SPS, 200 MHz travelling wave cavity


Linac4, 350 MHz drift tube linac


Linac1, 200 MHz drift tube linac

Basic RF theory

Going from a waveguide to a RF cavity

Complex notation for time-harmonic fields

In Radio Frequency we are usually dealing with sine-waves, which are sometimes modulated in phase or in amplitude. This means we will concentrate on time-harmonic solutions of Maxwells Equations. For this purpose we introduce the time-harmonic notation, which can be used for all linear processes. (Electric and magnetic fields can be linearly superimposed.)

Let us assume a time-harmonic electric field with amplitude E_0 and phase ϕ: $E(t) = E_0 \cos(\omega t + \varphi)$

this corresponds to the Real part of: $E(t) = \Re\left\{E_0 e^{i\varphi} e^{i\omega t}\right\} = \Re\left\{\cos(\omega t + \varphi) + i \sin(\omega t + \varphi)\right\}$

by defining a complex amplitude (or **phasor**): $\tilde{E} = E_0 e^{i\varphi}$

we can write: $E(t) = \Re\left\{\tilde{E} e^{i\omega t}\right\}$

from now on we will only use complex amplitudes and write them without tilde:

$$E_0 \cos(\omega t + \varphi) \longrightarrow \tilde{E} e^{i\omega t} \longrightarrow E$$

Why should we do that?

- Simplified, shorter expressions
- Time derivations become very simple

$$\frac{d}{dt}E(t) \longrightarrow \frac{d}{dt}\tilde{E}e^{i\omega t} = i\omega\tilde{E}e^{i\omega t}$$

$$\longrightarrow \frac{d}{dt}E = i\omega E$$

Only valid for harmonic time dependence

Complex notation of Maxwells Equations

The use of phasors yields the following form:

$$\nabla \times \mathbf{H} = i\omega\varepsilon\left(1 - i\frac{\kappa}{\omega\varepsilon}\right)\mathbf{E} \quad (I) \qquad \nabla \cdot \mathbf{E} = \frac{\rho_V}{\varepsilon} \quad (III)$$

$$\nabla \times \mathbf{E} = -i\omega\mu\mathbf{H} \quad (II) \qquad \nabla \cdot \mathbf{H} = 0 \quad (IV)$$

The general wave equations become:

$$\nabla^2\mathbf{E} - \nabla\left(\nabla \cdot \mathbf{E}\right) = -k^2\mathbf{E}$$

$$\nabla^2\mathbf{H} = -k^2\mathbf{H}$$

with the wavenumber

$$k^2 = \omega^2\mu\underline{\varepsilon} = \omega^2\mu\varepsilon\left(1 - i\frac{\kappa}{\omega\varepsilon}\right)$$

$\underline{\varepsilon} \rightarrow \varepsilon' - i\varepsilon''$

Remark: in conducting media k becomes complex (with a complex dielectric constant).
In non-conducting charge-free media the wave equations simplify to:

$$\nabla^2\mathbf{E} = -k^2\mathbf{E}$$

$$\nabla^2\mathbf{H} = -k^2\mathbf{H}$$

with the wave number

$$k^2 = \omega^2\mu\varepsilon = \frac{\omega^2}{c^2}$$

Solution of the wave equation

To find the electric and magnetic fields in free space, in wave-guides or in cavities, one needs to solve the wave equation in the appropriate coordinate system (cartesian, cylindric, spherical).

A common approach to **solve the wave equation for wave guides** is to define a vector potential for TE and TM waves, so that electric and magnetic fields can be calculated from:

$$\mathbf{E}^{TE} = \nabla \times \mathbf{A}^{TE} \quad \text{and} \quad \mathbf{H}^{TM} = \nabla \times \mathbf{A}^{TM}$$
$$\mathbf{H}^{TE} = \nabla \times (\nabla \times \mathbf{A}^{TE}) \quad \text{and} \quad \mathbf{E}^{TM} = \nabla \times (\nabla \times \mathbf{A}^{TM})$$

In both cases the vector potential fulfills the wave equation,

$$\nabla^2 \mathbf{A} = -k^2 \mathbf{A} \quad \text{with} \quad k^2 = \omega^2 \mu \varepsilon$$

which can then be solved for different coordinate systems for TE and TM waves and which has usually just one vector component:

$$\mathbf{A} = A_z \mathbf{e}_z$$

Nomenclature of modes in cavities (3 indices) and waveguides (2 indices)

TM_{mnp}-mode = E_{mnp}-mode → E-field parallel to axis, B_z =0, only transverse magn. (TM) components

TE_{mnp}-mode = H_{mnp}-mode → B-field parallel to axis, E_z =0, only transverse el. (TE) components

in a circular cavity this means:

- number of full-period variations of the field components in the azimuthal-direction

$$\mathbf{E} \text{ or } \mathbf{B} \propto \cos(m\phi) \text{ or } \sin(m\phi)$$

- number of zeros of the axial field component in radial direction.

$$E_z \text{ or } B_z \propto J_m(x_{mn} r/R_c)$$

- number of half-period variations of the field components in the longitudinal-direction

$$\mathbf{E} \text{ or } \mathbf{B} \propto \cos(p\pi z/l) \text{ or } \sin(p\pi z/l)$$

Solution of the wave equation (circular wave guides)

For circular wave guides we obtain for the vector potential:

$$A_z^{TM/TE} = C\mathrm{J}_m(k_c r)\cos(m\varphi)e^{-ik_z z} \quad \text{with} \quad k_z = \sqrt{k^2 - k_c^2}$$

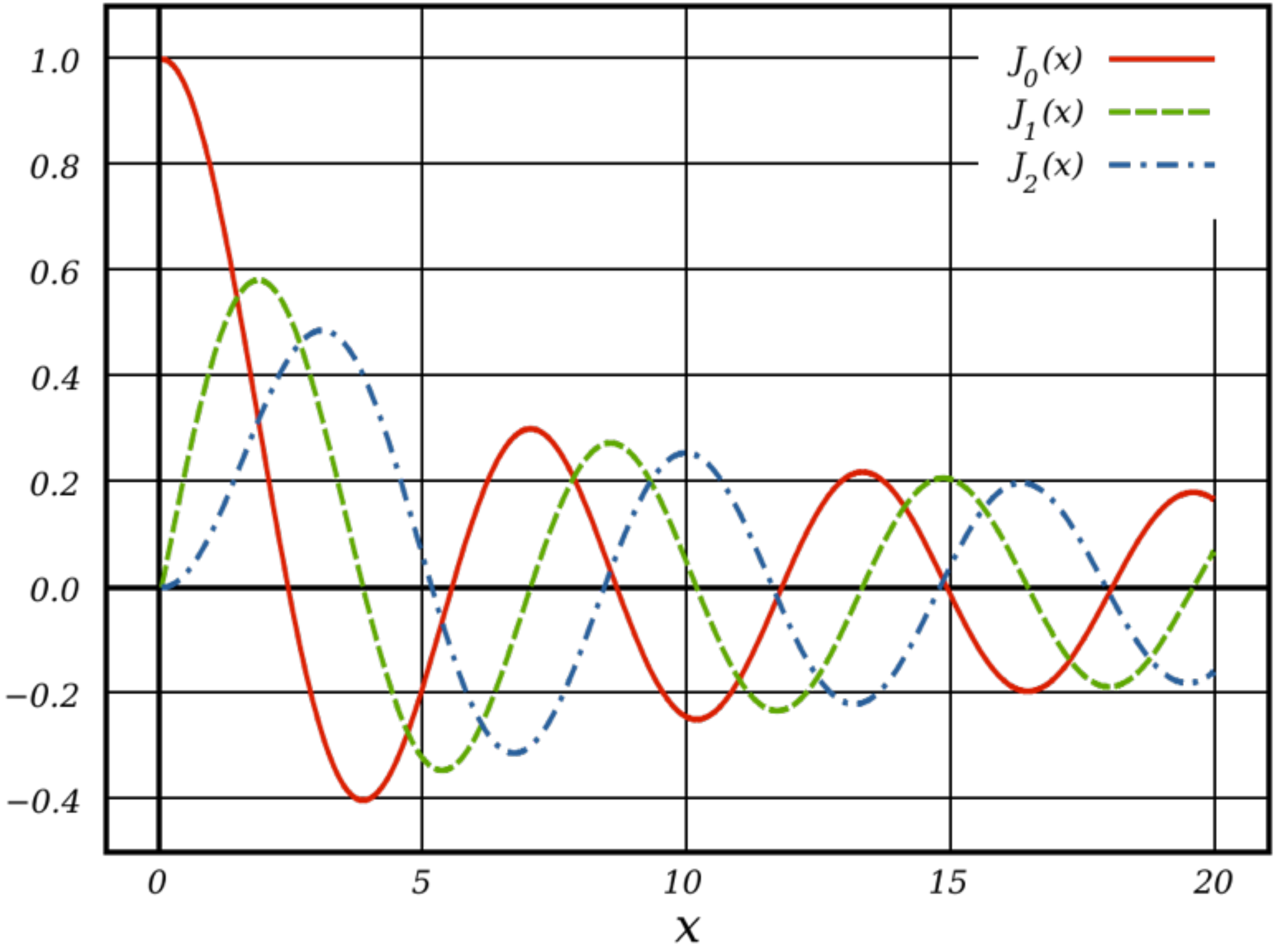
using $$\mathbf{H}^{TM} = \nabla \times \mathbf{A} \quad \text{and} \quad \mathbf{E}^{TM} = \nabla \times (\nabla \times \mathbf{A})$$

results in the following **field components for TM waves**:

$$\left.\begin{aligned}
E_r &= \frac{i}{\omega\varepsilon}\frac{\partial H_\varphi}{\partial z} &&= -C\frac{k_z k_c}{\omega\varepsilon}\mathrm{J'}_m(k_c r)\cos(m\varphi) \\
E_\varphi &= -\frac{i}{\omega\varepsilon}\frac{\partial H_r}{\partial z} &&= C\frac{m k_z}{\omega\varepsilon r}\mathrm{J}_m(k_c r)\sin(m\varphi) \\
E_z &= \frac{ik_c^2}{\omega\varepsilon}A_z &&= C\frac{ik_c^2}{\omega\varepsilon}\mathrm{J}_m(k_c r)\cos(m\varphi) \\
H_r &= \frac{1}{r}\frac{\partial A_z}{\partial\varphi} &&= -C\frac{m}{r}\mathrm{J}_m(k_c r)\sin(m\varphi) \\
H_\varphi &= -\frac{\partial A_z}{\partial r} &&= -Ck_c\mathrm{J'}_m(k_c r)\cos(m\varphi)
\end{aligned}\right\} e^{-ik_z z}$$

J_m are Bessel functions of the first kind and of m'th order

Bessel functions of the first kind

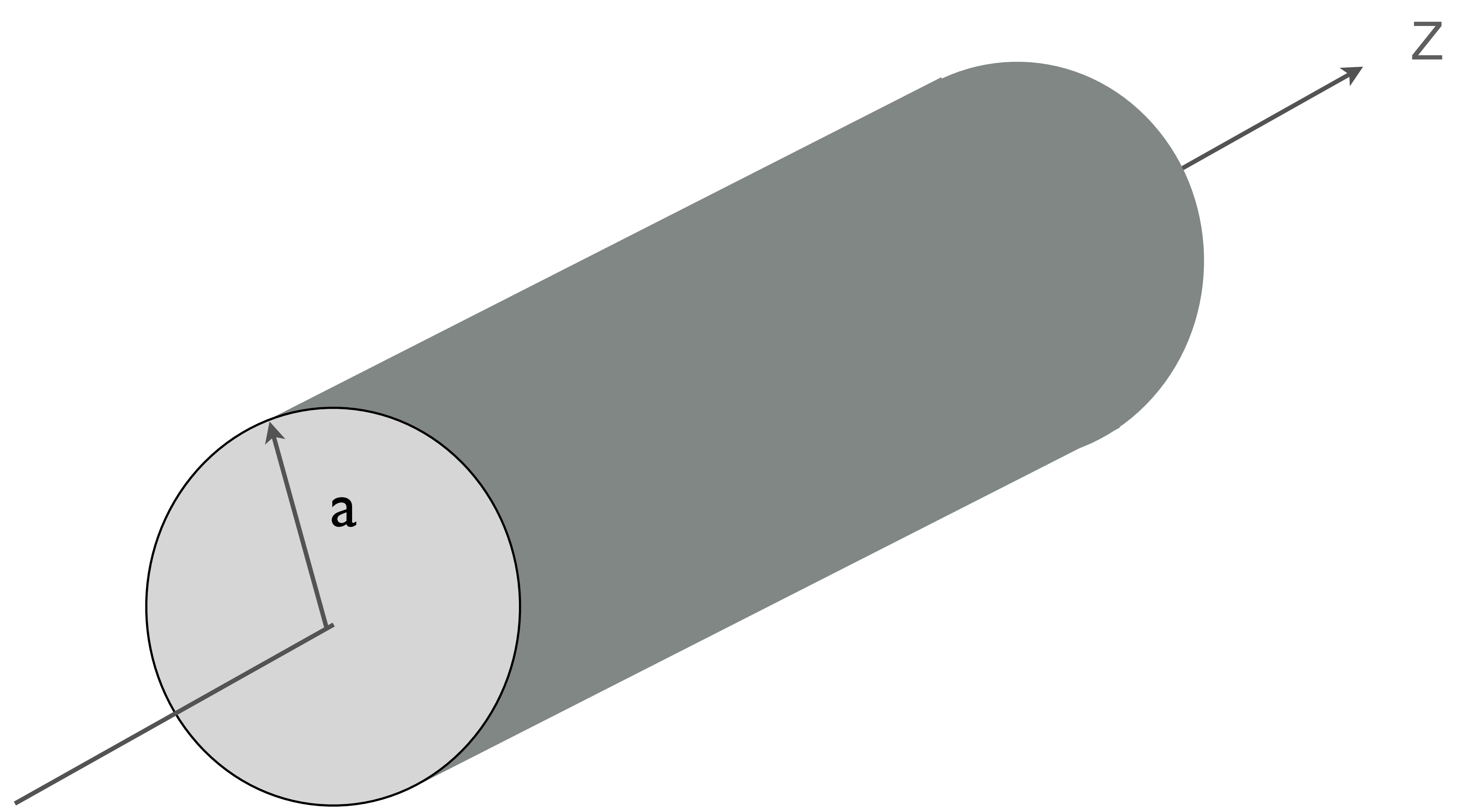


Wave propagation in a cylindrical pipe (conducting walls)

let us consider the simplest accelerating mode (electric field in z-direction): m=0, n=1, TM_{01}

using $$J_0'(r) = -J_1(r)$$

$$\left.\begin{aligned} E_r &= C\tfrac{k_z k_c}{\omega\varepsilon}\mathrm{J}_1(k_c r) \\ E_z &= -C\tfrac{ik_c^2}{\omega\varepsilon}\mathrm{J}_0(k_c r) \\ H_\varphi &= Ck_c\mathrm{J}_1(k_c r) \end{aligned}\right\} e^{-ik_z z}$$



propagation constant: $k_z^2 = k^2 - k_c^2$

wave number: $k = \dfrac{2\pi}{\lambda} = \dfrac{\omega}{c}$

k_c is determined by the boundary conditions of the wave-guide

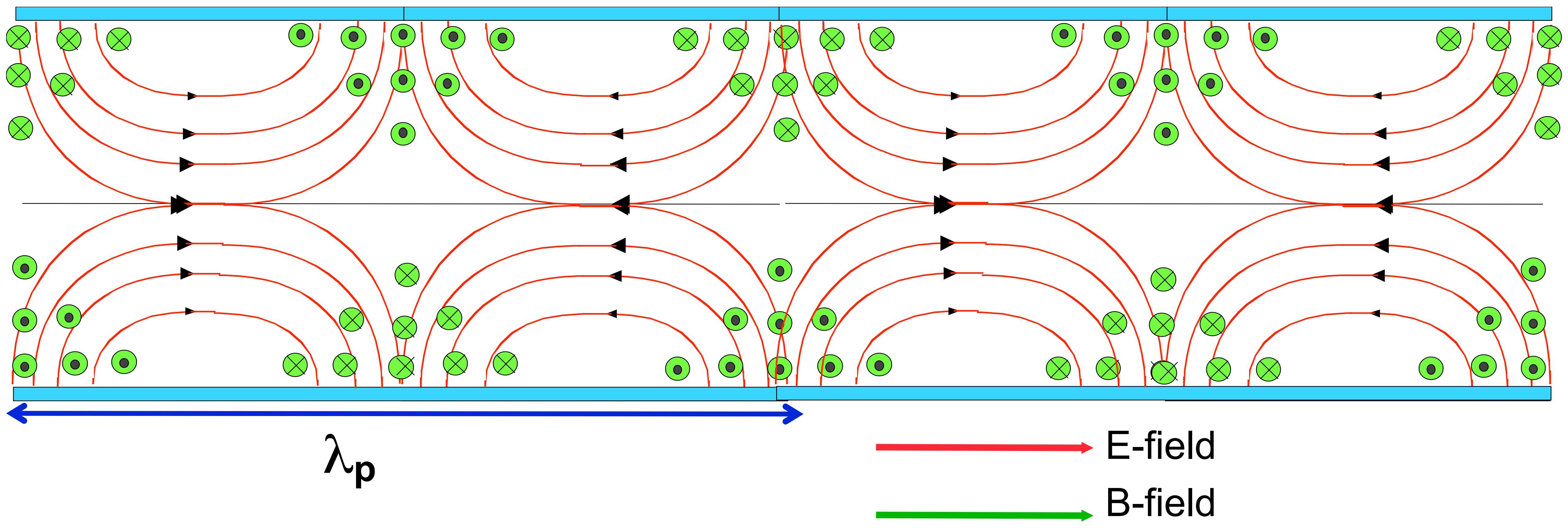
$$\mathbf{E}_{||} = 0 \Rightarrow E_z(r=a) = 0 \Rightarrow J_0(k_c a) = 0 \Rightarrow k_c a = 2.405$$

Wave propagation in a cylindrical pipe (conducting walls)

and from $k_c = \frac{2\pi}{\lambda_c} = \frac{\omega_c}{c}$ we can calculate the **cut-off frequency** for the TM_{01} mode in a cylindrical pipe

$$\omega_c = \frac{2.405c}{a}$$

TM01 field configuration

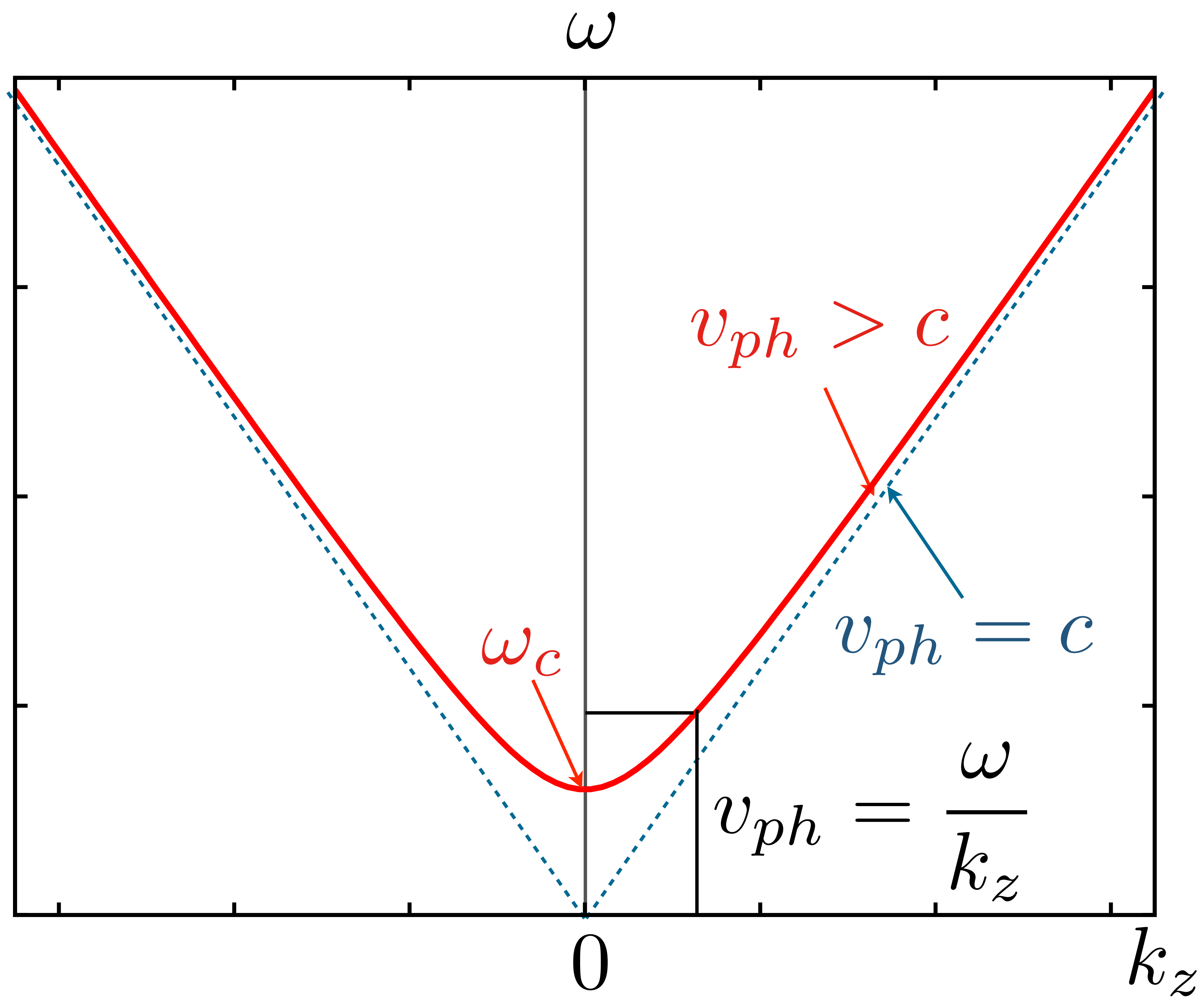


from $k_z^2 = k^2 - k_c^2$ we also get the **dispersion relation**

$$k_z^2 = \frac{\omega^2 - \omega_c^2}{c^2} = \frac{\omega^2}{v_{ph}^2}$$

- TM_{01} waves propagate for: $\omega > \omega_c$
- and are exponentially damped for: $\omega < \omega_c$
- the phase velocity is: $v_{ph} = \frac{\omega}{k_z}$

Dispersion relation (Brillouin diagram)



group velocity:

$$v_{gr} = \frac{d\omega}{dk_z}$$

phase velocity:

$$v_{ph} = \frac{\omega}{k_z}$$

- Each frequency corresponds to a certain phase velocity,
- **The phase velocity is always larger than c!** (at $\omega=\omega_c$: $k_z=0$ and $v_{ph}=\infty$),

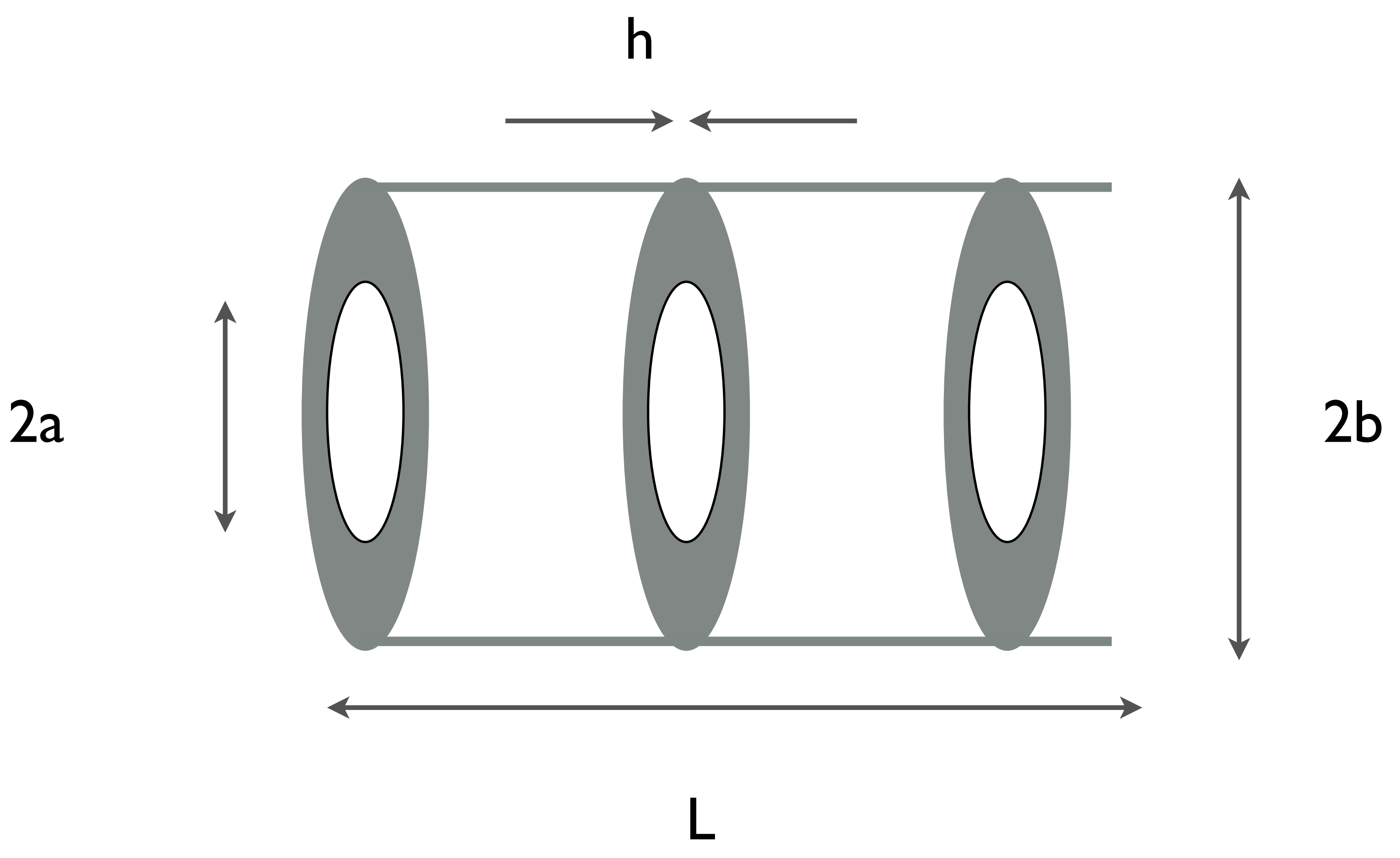
$$v_{ph}^2 = c^2 \frac{\omega^2}{\omega^2 - \omega_c^2}$$

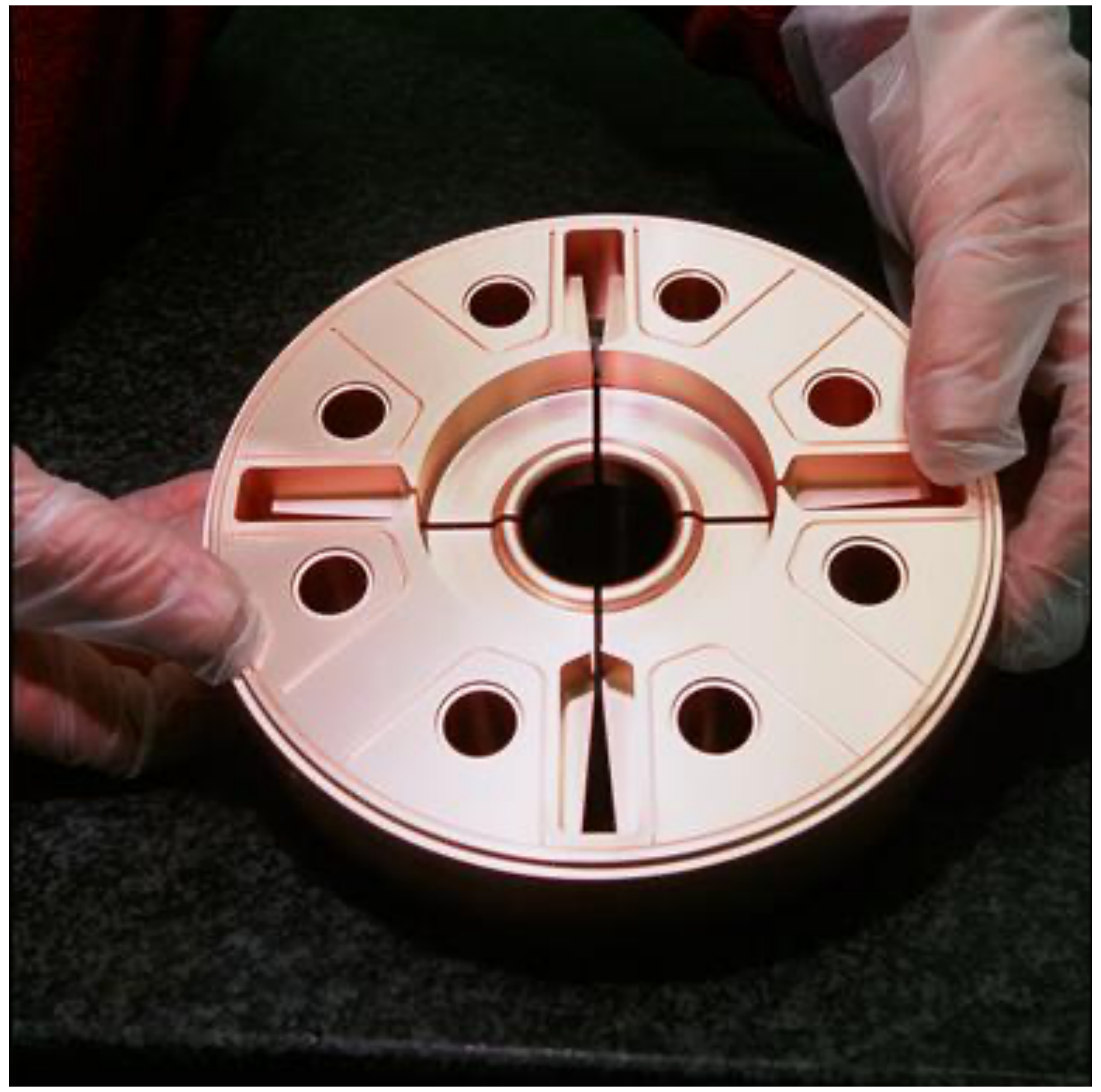
- **Synchronism with RF** (necessary for acceleration) **is impossible** because a particle would have to travel at $v=v_{ph}>c$!
- Energy (and therefore information) travels at the group velocity $v_{gr}<c$,

How can we slow down phase velocity?

How can we slow down the phase velocity?

put some obstacles into the wave-guide: e.g: discs





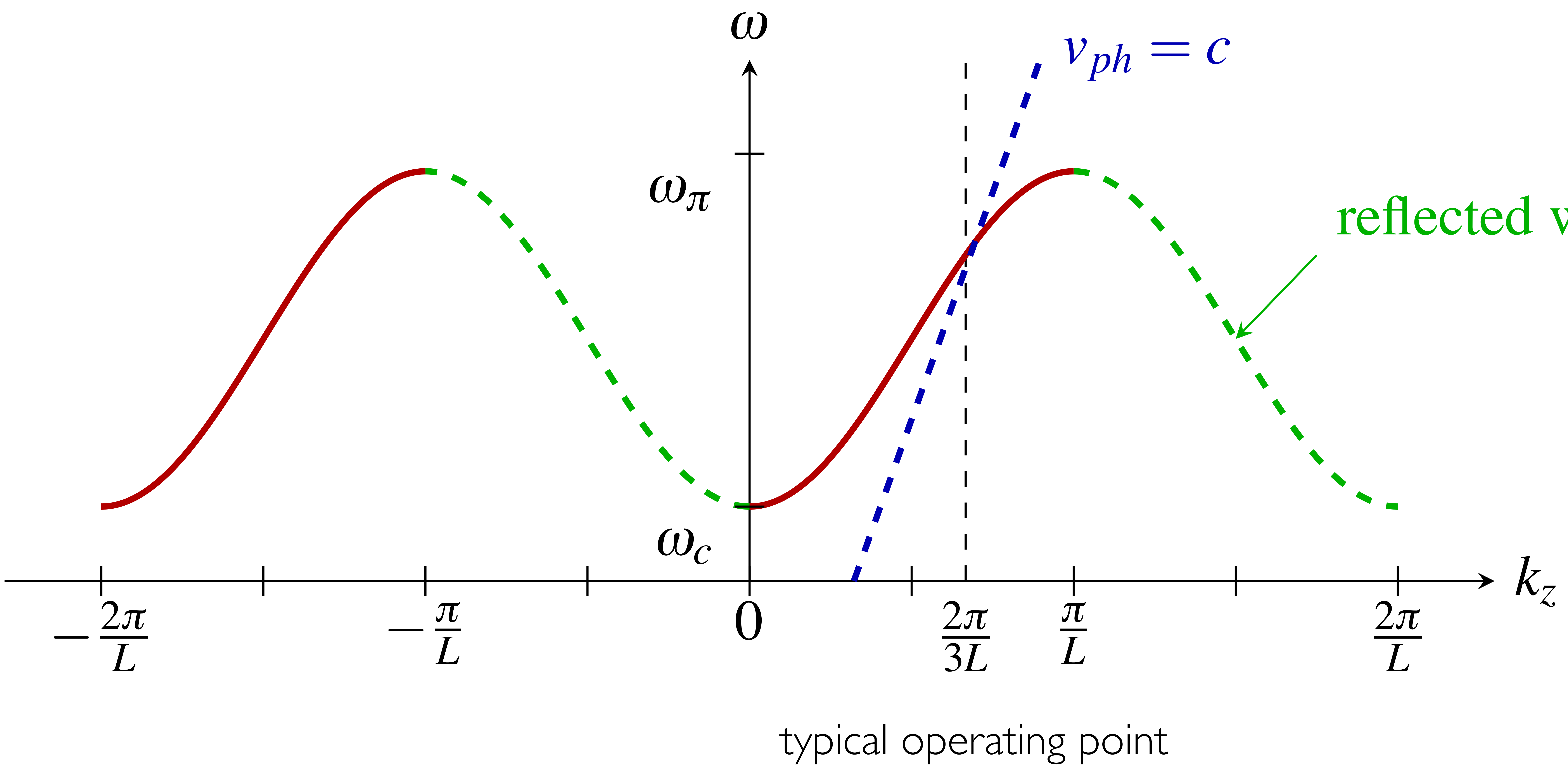
Only then can we achieve synchronism between the particles and the phase velocity of the RF wave.

Dispersion relation for disc-loaded circular wave-guides

$$\omega = \frac{2.405c}{b}\sqrt{1 + \kappa(1 - \cos(k_z L)e^{-\alpha h})}$$

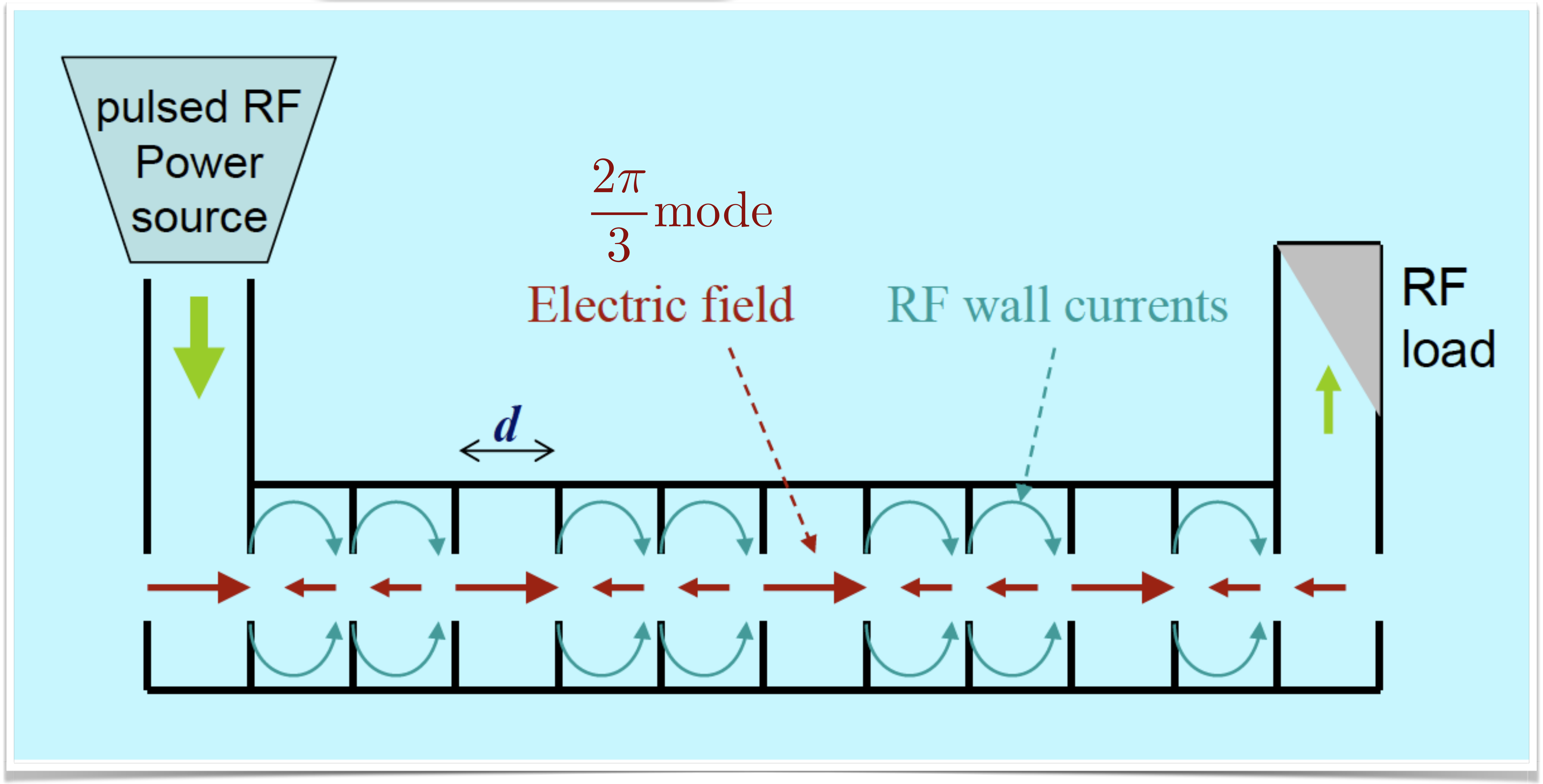
$$\kappa = \frac{4a^3}{3\pi J_1^2(2.405)b^2 L} \ll 1$$

damping: $\alpha \approx \dfrac{2.405}{a}$



Example of a 2/3 π travelling wave structure

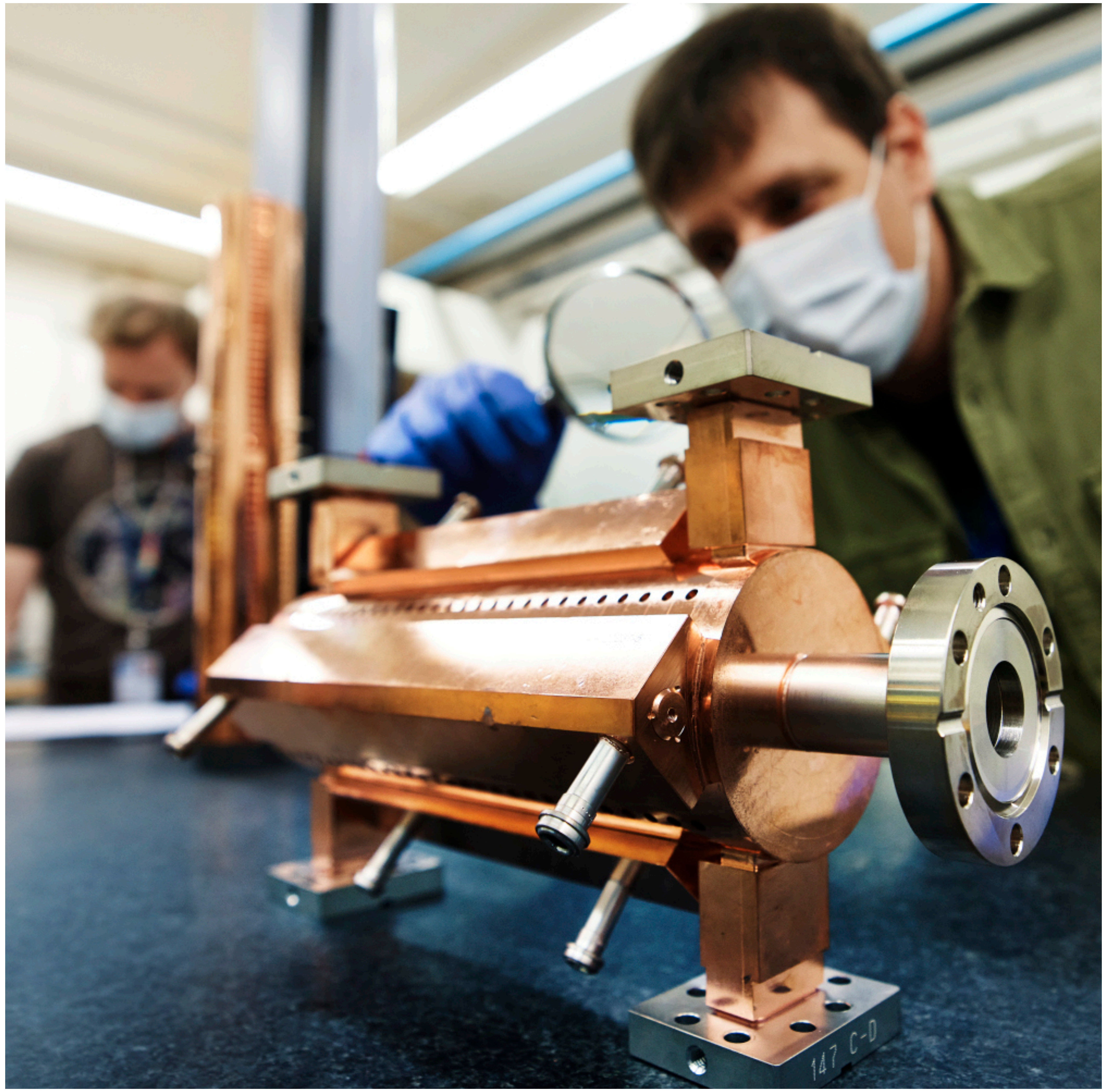
synchronism condition: $d = \frac{(\beta)\lambda}{3}$ with $\beta \approx 1$



Travelling wave structures

- Since the particles gain energy the EM-wave is damped along the structure (“**constant impedance structure**”). But by changing the bore diameter one can decrease the group velocity from cell to cell and obtain a **“constant-gradient”** structure. Here one can operate in all cells near the break-down limit and thus achieve a higher average energy gain.
- High-gradient, high-frequency traveling wave structures are mostly used for very short (us) pulses, and can reach high efficiencies, and high accelerating gradients (up to 100 MV/m, CLIC).
- Generally used for electrons at $\beta \approx 1$,

See also W. Wuensch: high-beta cavities



CLIC prototype structure

CERN-PHOTO-202202-013-8

Basic cavity parameters

Energy gain in a cavity

We drill two holes at the cavity axis to let the beam pass, and assume the field distribution does not get disturbed:

$$E_z(r=0,z,t) = E(0,z)\cos(\omega t+\varphi)$$

$$\Delta W = q\int_{-L/2}^{L/2} E(0,z)\cos(\omega t+\varphi) = qV_0 T\cos\varphi$$

with: $$V_0 = \int_{-L/2}^{L/2} E(0,z)dz = E_0 L$$

and Transit Time Factor T:

$$T = \frac{\int_{-L/2}^{L/2} E(0,z)\cos(\omega t(z))dz}{\int_{-L/2}^{L/2} E(0,z)dz} - \underbrace{\tan\phi\frac{\int_{-L/2}^{L/2} E(0,z)\sin(\omega t(z))dz}{\int_{-L/2}^{L/2} E(0,z)dz}}_{=0 \text{ if } E(0,z) \text{ is symmetric to z=0}}$$

Transit Time Factor

The Panofsky equation

$$\Delta W = qE_0 TL\cos\varphi$$

The Transit Time Factor gives the ratio between the energy gained in an RF field and a DC field and is therefore < 1. It takes into account that the electric field changes its phase during the passage of the beam.

If we assume that the velocity change is small, we can say that

$$\omega t \approx \omega\frac{z}{v} = \frac{2\pi z}{\beta\lambda}$$

which simplifies the Transit Time Factor to:

$$T = \frac{\int\limits_{-L/2}^{L/2} E(0,z)\cos\left(\frac{2\pi z}{\beta\lambda}\right)dz}{\int\limits_{-L/2}^{L/2} E(0,z)dz}$$

Shunt impedance

$$R_s = \frac{V_0^2}{P_d} \quad \text{shunt impedance}$$

$$R = \frac{(V_0 T)^2}{P_d} \quad \text{effective shunt impedance}$$

$$Z = \frac{R_s}{L} = \frac{E_0^2}{P_d/L} \quad \text{shunt impedance per unit length}$$

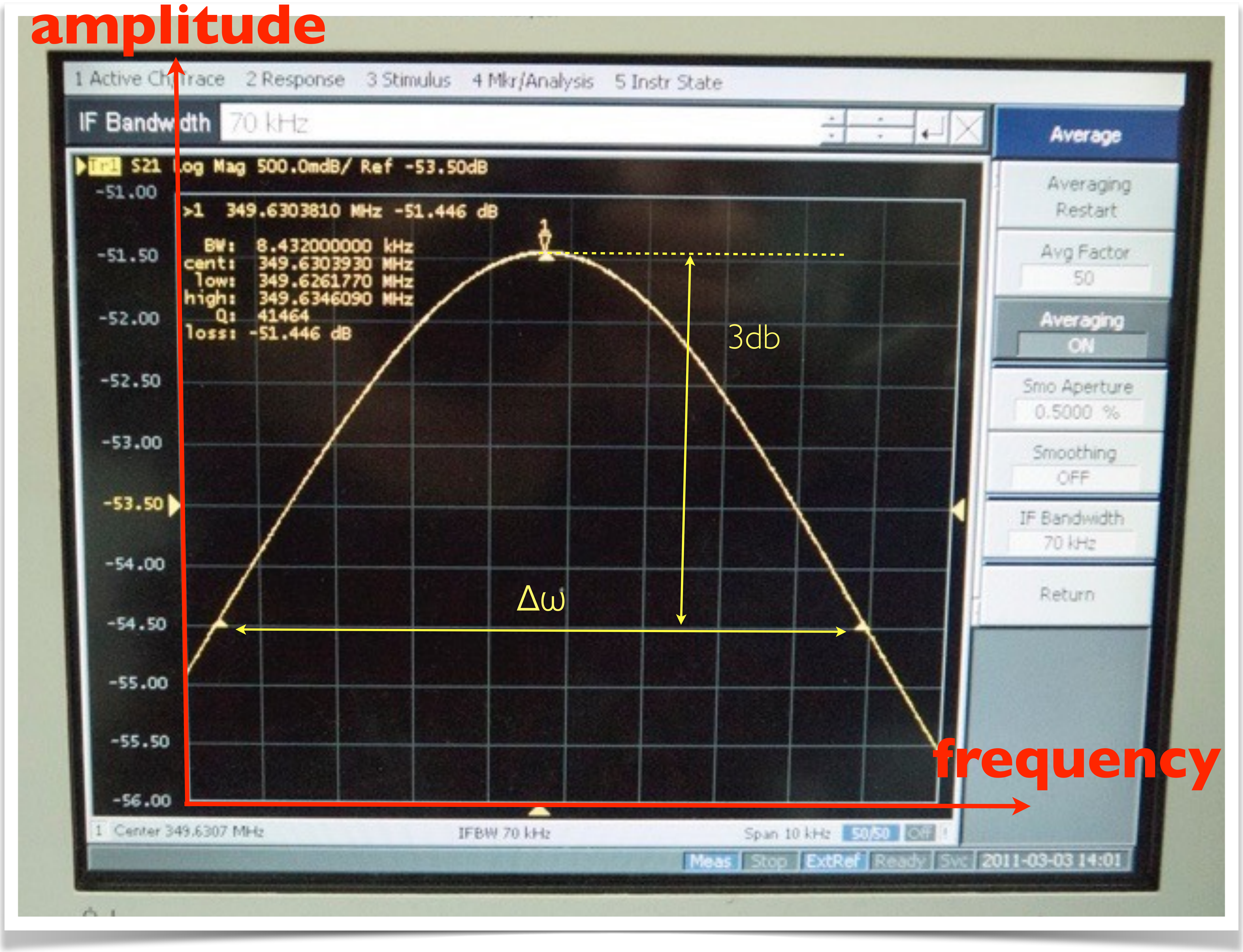
$$ZT^2 = \frac{R}{L} = \frac{(E_0 T)^2}{P_d/L} \quad \text{eff. shunt impedance per unit length}$$

The shunt impedance gives a measure of how much accelerating voltage V_0 one can get with a given power P_d, which is dissipated in the cavity walls.

Above we have used the **linac definition** of the shunt impedance, but sometimes also the **circuit definition** is used (more on that later).

$$R_S^c = \frac{V_0^2}{2P_d}$$

3 db bandwidth



Q and (R/Q)

The Quality factor Q describes the bandwidth of a resonator and is defined as the ratio of reactive power (stored energy) to real power lost in the cavity walls:

$$Q = \frac{\omega}{\Delta\omega} = \frac{\omega W}{P_d}$$

Together with the shunt impedance we can define another figure of merit, which is used to maximize the energy gain in a given length for a certain power loss.

$$\left(\frac{R}{Q}\right) = \frac{(V_0 T)^2}{\omega W}$$

This quantity is independent from the surface losses and qualifies only the geometry of the cavity!

Filling time of a cavity

The dissipated power in the cavity walls must be equal to the rate of change of the stored energy:

$$P_d = -\frac{dW}{dt} = \frac{\omega_0 W}{Q_0}$$

$$Q_0 = \frac{\omega_0 W}{P_c}$$

As a solution we find an exponential decay for the energy:

$$W(t) = W_0 e^{-\frac{2t}{\tau}} \quad \text{with} \quad \tau = \frac{2Q_0}{\omega_0}$$

For a loaded cavity (e.g. equipped with a power coupler) the filling time constant changes to:
(Q_l will be derived later)

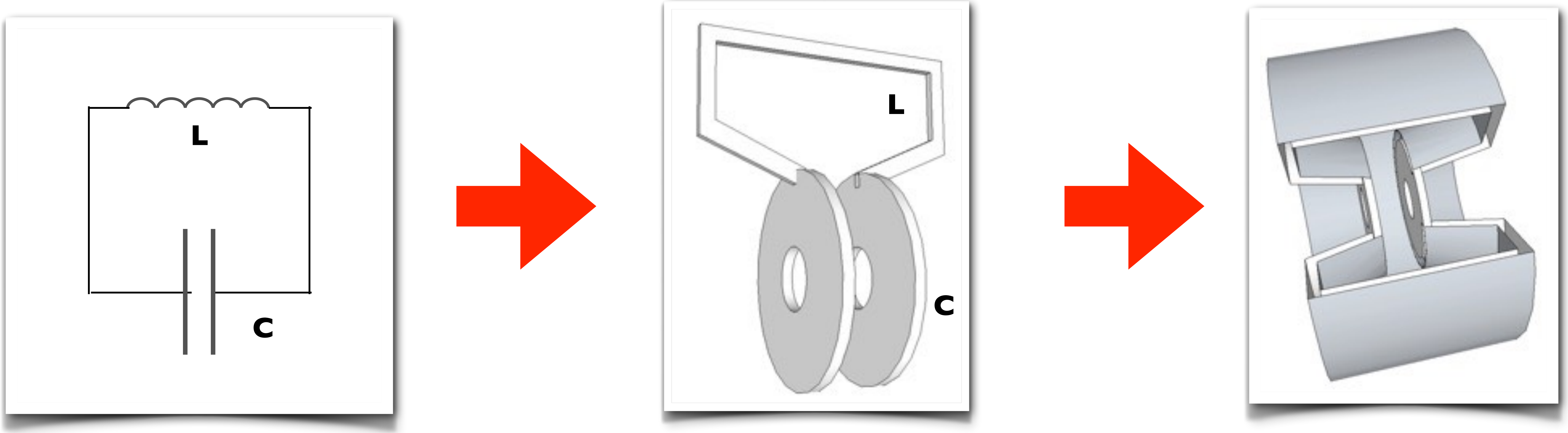
$$\tau_l = \frac{2Q_l}{\omega_0}$$

(Comment: one can also define tau as $\tau = \dfrac{Q_0}{\omega_0}$), then $W(t) \propto e^{-\frac{t}{\tau}}$

The Pillbox cavity

- a typical TM mode cavity -

The pillbox cavity



$$\omega_{res} = 2\pi f_{res} = 1/\sqrt{LC}$$

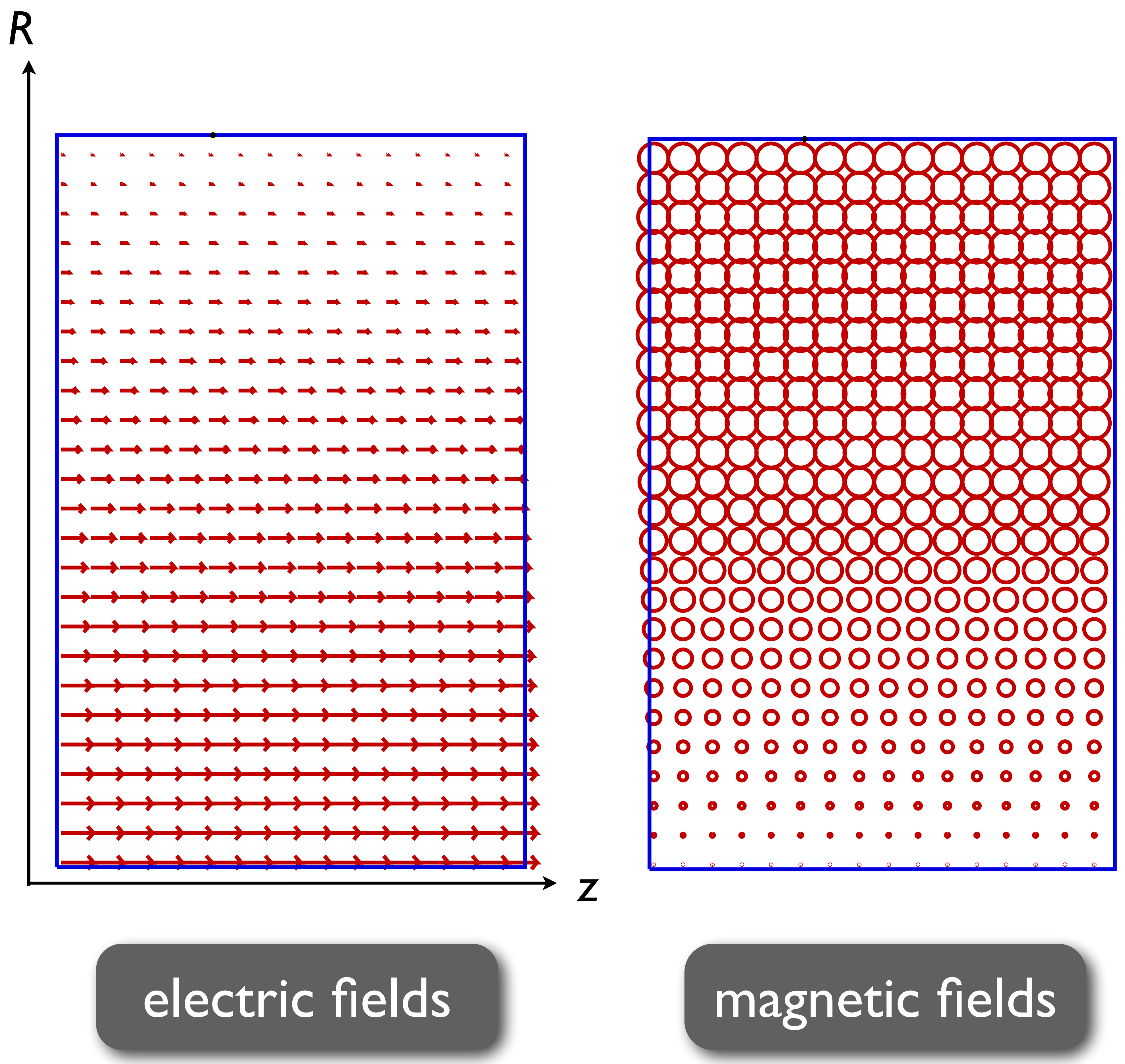
A lumped element resonator transformed into a pillbox cavity

The pillbox cavity

...an empty cylinder with conducting walls:

- with longitudinal electric field and transverse magnetic fields: TM_{010} mode (ϕ,r,z),
- no field dependence on ***z and φ***, frequency is determined by radius ***r=a***:

$$f = \frac{2.405c}{2\pi a}$$



$$E_z \propto \mathrm{J}_0(k_r r)$$

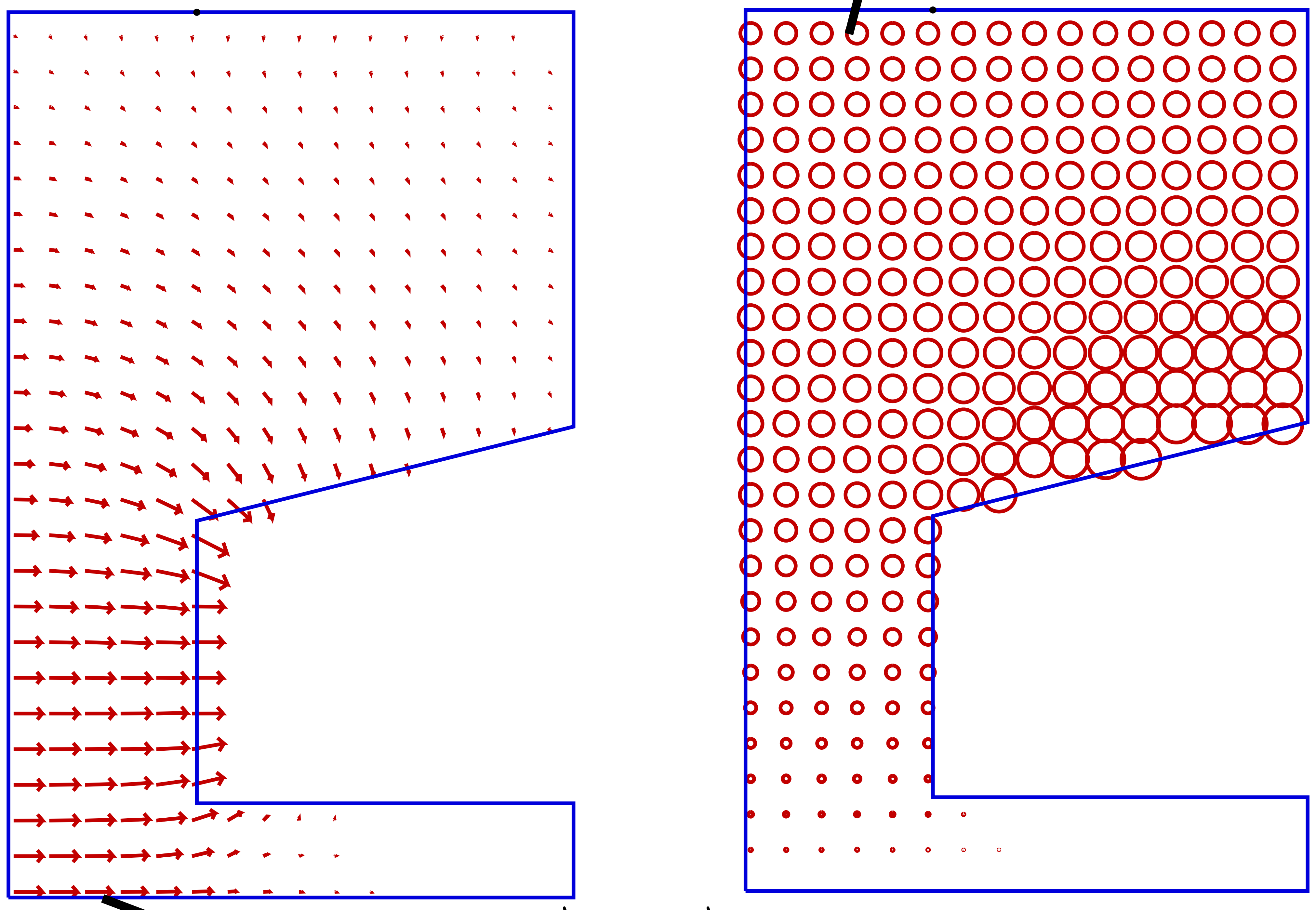
$$H_\varphi \propto \mathrm{J}_1(k_r r)$$

The typical TM mode cavity

- normal conducting, standing wave -

- usually C is increased to concentrate the electric field lines along the axis,
- diameter of the cavities is in the order of λ/2, which makes them suitable for frequencies >100 MHz - GHz range,
- exist as single/multi-cell, normal/ superconducting,
- usually fixed frequency,

$$L = \frac{\phi}{I} = \frac{\oint_s \vec{B} \cdot d\vec{S}}{\oint_l \vec{H} \cdot d\vec{l}}$$



$$C = \frac{Q}{V} = \frac{\varepsilon \oint \vec{E} \cdot d\vec{S}}{\int \vec{E} \cdot d\vec{s}}$$

$$\omega_{res} = 2\pi f_{res} = 1/\sqrt{LC}$$

Field distribution in a pillbox cavity

Using again the vector potential for circular waves and superimposing 2 waves: one in positive and one in negative z-direction

$$A_z^{TM/TE} = C\mathrm{J}_m(k_r r)\cos(m\varphi)\underbrace{\left(e^{-ik_z z} + e^{ik_z z}\right)}_{2\cos(k_z z)}$$

We can derive all TM field components using:

$$\mathbf{H}^{TM} = \nabla \times \mathbf{A}^{TM} \quad \text{and} \quad \mathbf{E}^{TM} = \nabla \times (\nabla \times \mathbf{A}^{TM})$$

$$E_r = \frac{i}{\omega\varepsilon}\frac{\partial H_\varphi}{\partial z} = i2C\frac{k_z k_r}{\omega\varepsilon}\mathrm{J'}_m(k_r r)\cos(m\varphi)\sin(k_z z)$$

$$E_\varphi = -\frac{i}{\omega\varepsilon}\frac{\partial H_r}{\partial z} = -i2C\frac{m k_z}{\omega\varepsilon r}\mathrm{J}_m(k_r r)\sin(m\varphi)\sin(k_z z)$$

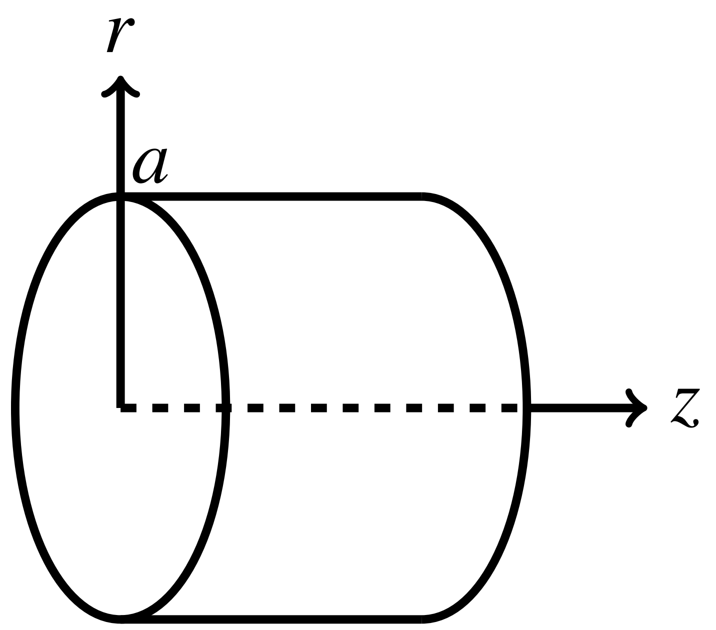
$$E_z = \frac{ik_r^2}{\omega\varepsilon}A_z = i2C\frac{k_r^2}{\omega\varepsilon}\mathrm{J}_m(k_r r)\cos(m\varphi)\cos(k_z z)$$

$$H_r = \frac{1}{r}\frac{\partial A_z}{\partial\varphi} = -2C\frac{m}{r}\mathrm{J}_m(k_r r)\sin(m\varphi)\cos(k_z z)$$

$$H_\varphi = -\frac{\partial A_z}{\partial r} = -2Ck_r\mathrm{J'}_m(k_r r)\cos(m\varphi)\cos(k_z z)$$

Field distribution in a pillbox cavity

applying the boundary conditions we can define the wave numbers:



$$E_r(z = 0/L), E_\varphi(z = 0/L) = 0 \qquad \Rightarrow k_z = \frac{n\pi}{L}$$

$$E_\varphi(r = a), E_z(r = a), H_r(r = a) = 0 \qquad \Rightarrow k_r = \frac{j_m}{a}$$

which gives us a **discrete set of frequencies**:

$$k^2 = \frac{\omega^2}{c^2} = k_z^2 + k_r^2 \qquad \Rightarrow f_{nm} = \frac{c}{2\pi}\sqrt{\left(\frac{n\pi}{L}\right)^2 + \left(\frac{j_m}{a}\right)^2}$$

dispersion relation

The mode with lowest frequency is the **TM_{010} mode:**

$$f = \frac{2.405c}{2\pi a}$$

with

$$E_z = -i2C\frac{j_{01}^2}{a^2\omega\varepsilon}\mathrm{J}_0\left(\frac{j_{01}}{a}r\right) = E_0\mathrm{J}_0\left(\frac{j_{01}}{a}r\right)$$

$$H_\varphi = 2C\frac{j_{01}}{a}\mathrm{J}_1\left(\frac{j_{01}}{a}r\right) = \frac{E_0}{Z_0}\mathrm{J}_1\left(\frac{j_{01}}{a}r\right)$$

Transit time factor (pillbox cavity)

In a pillbox cavity (TM_{010} mode) the accelerating field has no dependence on z, which simplifies our expression for T to:

$$T = \frac{\int\limits_{-L/2}^{L/2} E(0,z)\cos\left(\frac{2\pi z}{\beta\lambda}\right)dz}{\int\limits_{-L/2}^{L/2} E(0,z)dz} = \frac{\sin\left(\frac{\pi L}{\beta\lambda}\right)}{\frac{\pi L}{\beta\lambda}}$$

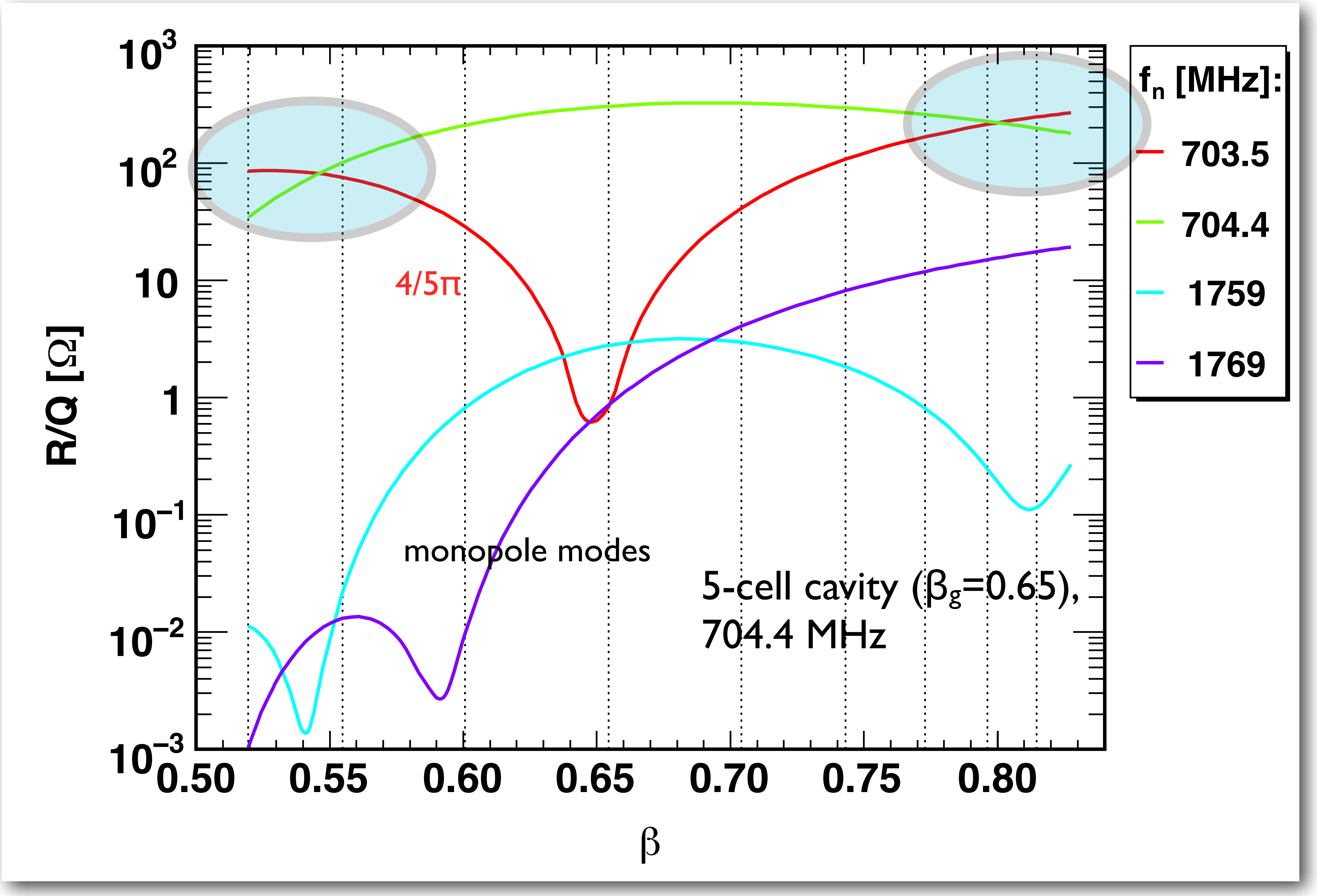
Let us assume relativistic particles (β≈1) and a cavity length of L=λ/2 (which can be cascaded to π-mode multi-cell cavities):

$$T = \frac{2}{\pi} = 0.64$$

Accelerating voltage (pillbox)

accelerating voltage: $$V_{acc} = V_0 T = E_0 L T = E_0 L \frac{\sin(\frac{\pi L}{\beta\lambda})}{\frac{\pi L}{\beta\lambda}}$$

- The accelerating voltage is a strong function of the transit time factor.
- It therefore depends on the gap length (L), and the speed of the particle (β).
- Especially in multi-cell cavities, which are used over a wide velocity range (e.g. SC multi-cell cavities for protons), this effect must be taken into account carefully.
- Also HOMs depend on the depend on the particle speed!



Q_0 (pillbox)

quality factor

$$Q_0 = \frac{\omega W}{P_d} = \frac{Z_0^2 \omega}{2R_{surf}} \frac{La}{L+a} = \frac{1}{\delta_s} \frac{La}{L+a} \propto \sqrt{\omega}$$

With the skin depth
(not derived here)

$$\delta_s = \sqrt{\frac{2}{\omega \mu \kappa}}$$

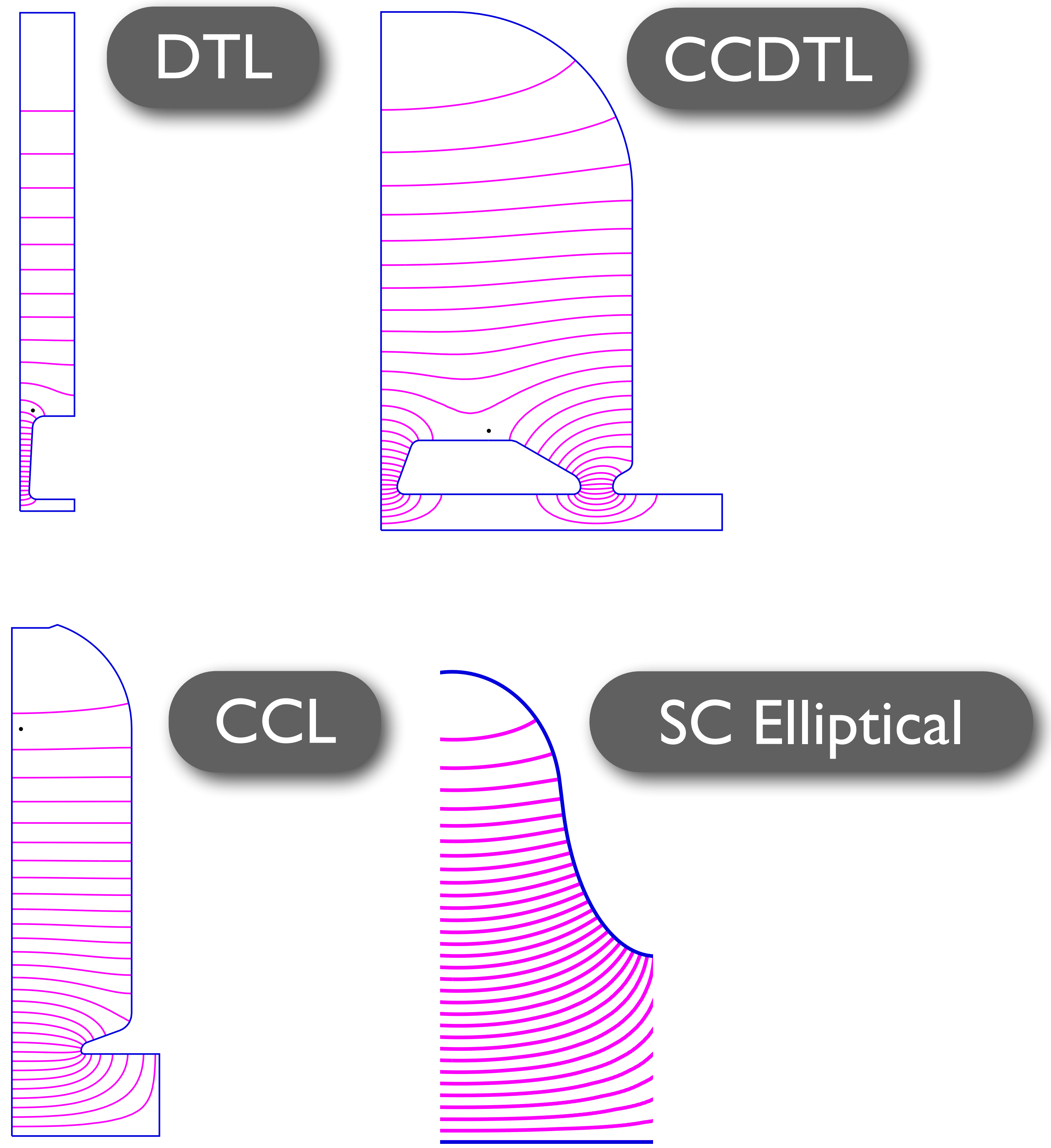
- The quality factor is a function of the material constants (el. conductance, permeability), the frequency, and the geometry of the cavity.
- Since the material is usually fixed (Cu), one can optimize the quality factor by optimizing the geometry of the cavity.
- Higher frequencies yield higher quality factors (only true for normal conducting cavities).

Shunt impedance (pillbox)

effective shunt impedance

$$R = \frac{(V_0 T)^2}{P_d} = \frac{Z_0}{\pi R_{surf} J_1^2(j_{01})} \frac{\sin\left(\frac{\pi L}{\beta\lambda}\right)}{\frac{\pi L}{\beta\lambda}} \frac{L^2}{a(a+L)}$$

- Depends on material parameters, the transit time factor and the geometry.
- This is why most normal conducting cavities have noses.
- Noses increase T and focus the electric field between them.
- Why do SC cavities not have noses?



Frequency and (R/Q) in pillboxes

frequency

$$f = \frac{2.405c}{2\pi a}$$

(R/Q)

$$\left(\frac{R}{Q}\right) = \frac{2c}{\omega\pi J_1^2(j_{01})} \frac{\sin\left(\frac{\pi L}{\beta\lambda}\right)}{\frac{\pi L}{\beta\lambda}} \frac{L}{a^2}$$

- In all TM mode cavities, the frequency is strongly influenced by the cavity diameter.

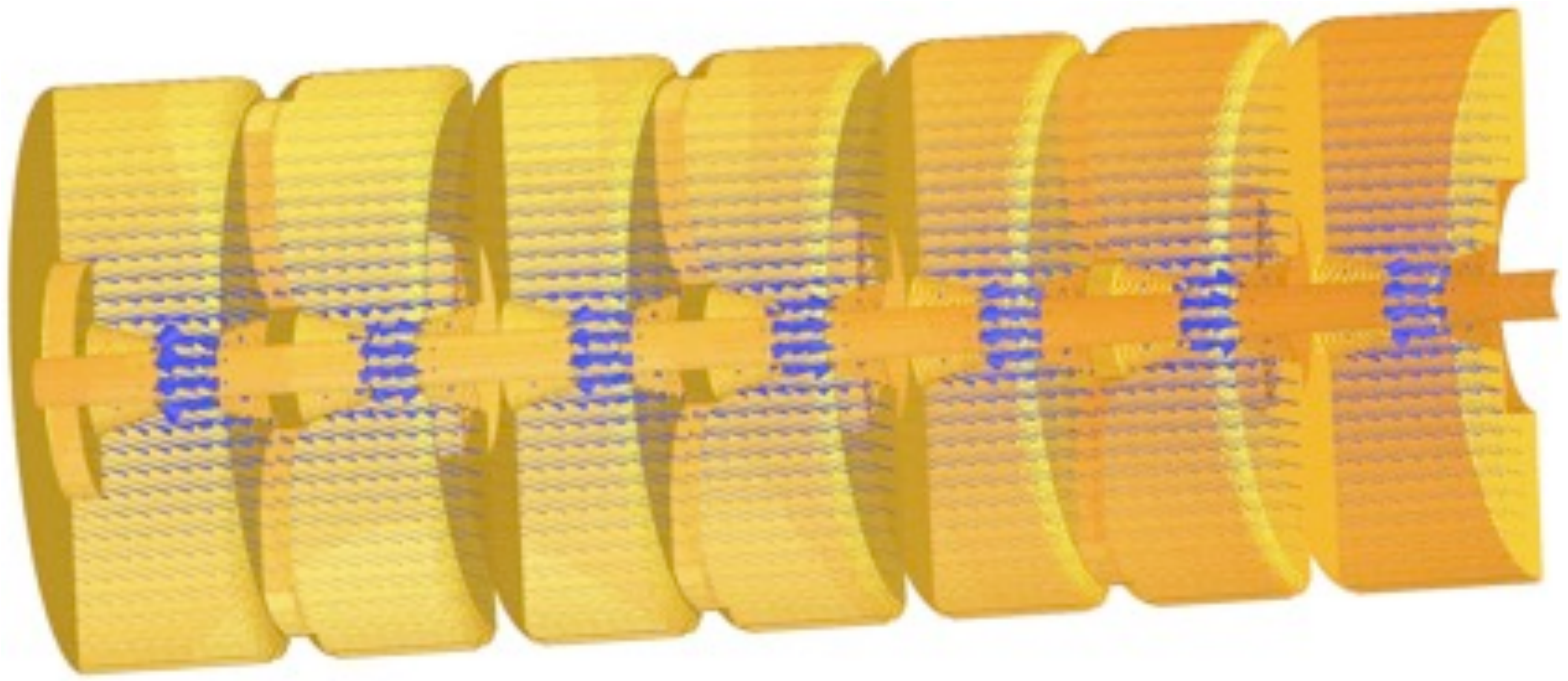
- (R/Q) does not depend on any material parameters, but is influenced by the transit time factor and the geometry and is inversely proportional to the frequency.

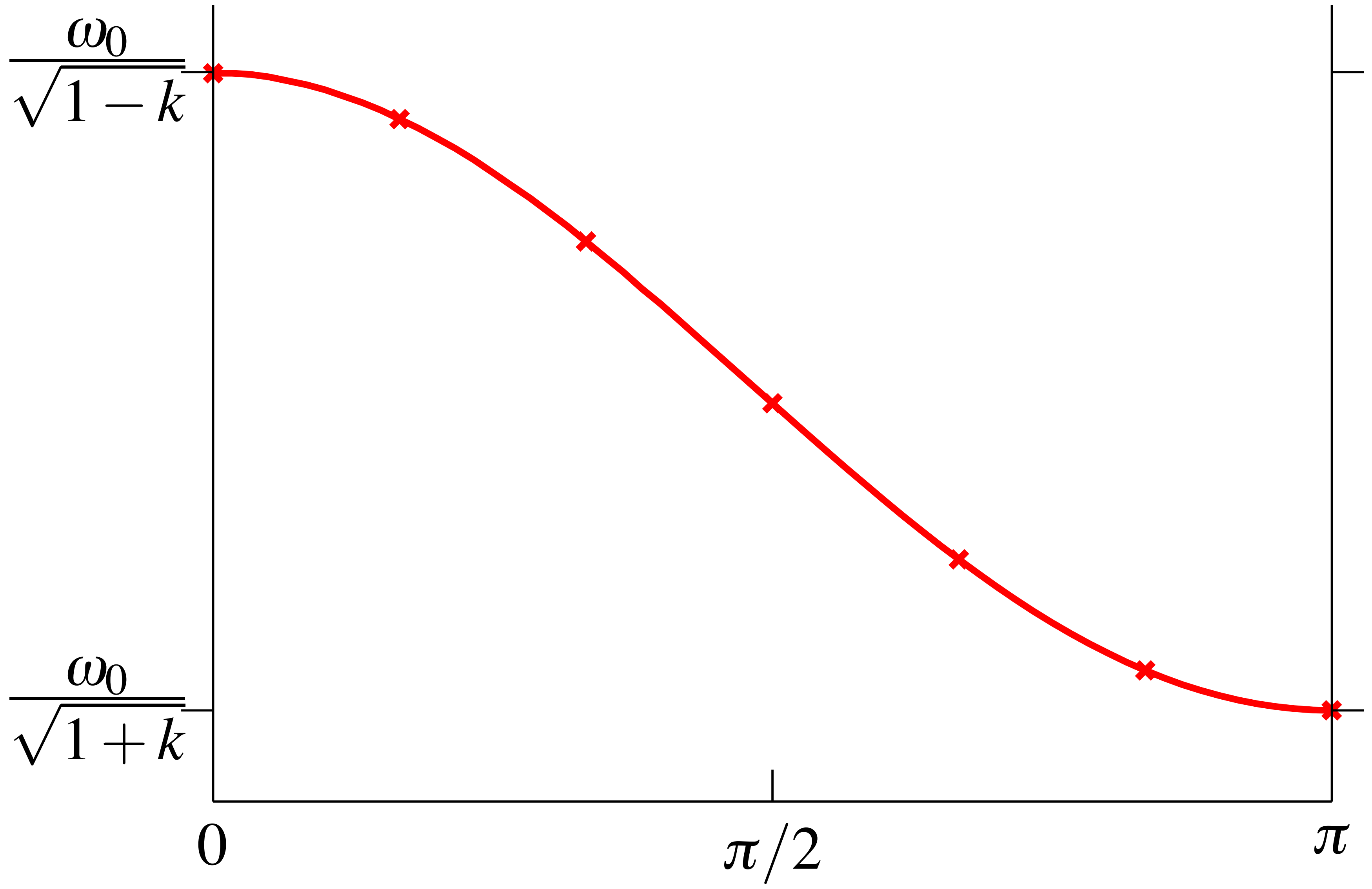
Multi-cell TM-mode cavities

- For coupled multi-cell structures one power source can be used for many cells.
- Here we assume a TM010 mode in each cell.
- A model of equivalent LC circuits is used to introduce the coupling between cells, and can be used to determine the resulting single cell frequencies.
- The mode names (0, ..,π/2, .., π) correspond to the phase difference between the gaps.

$$\omega_n = \frac{\omega_0}{\sqrt{1 + k\cos(n\pi/N)}}$$

dispersion relation





To be continued