2023 CAS course on “RF for Accelerators”

RF Measurements

– Only Part 2 –

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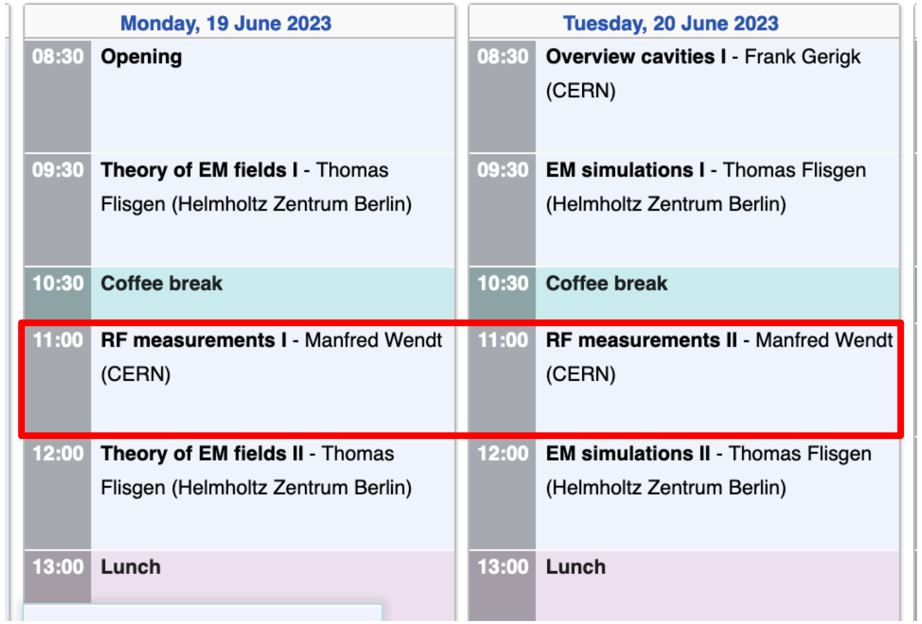
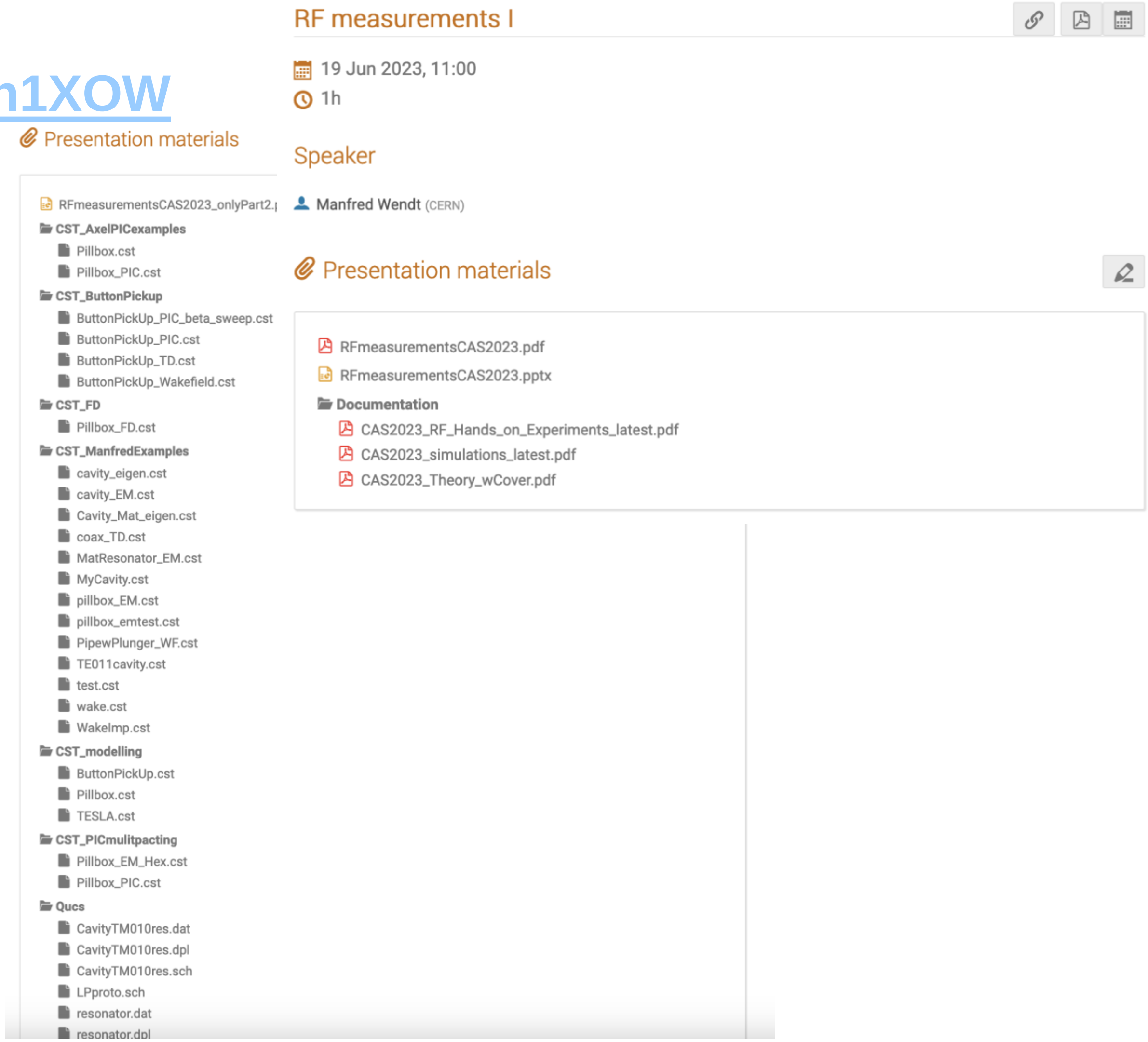
Documentation & Software Examples

- **CERNBox link:** https://cernbox.cern.ch/s/2OHukn46LDn1XOW
  - **Printed and updated documentation**
  - **Software (CST and freeware)**
  - **Software examples**





Summary Part 1

- **Transmission-lines**
  - TEM lines and (rect.) TE10 waveguides
  - Telegrapher's equation and characteristics: $Z_0, \gamma = \alpha + j\beta$
  - load impedance Z_L, forward / backward traveling waves, reflection coefficient $\Gamma = b/a$
- ***Smith* chart**
  - Mapping of the complex reflection coefficient and the complex impedance plane
  - Characteristic points and areas in the *Smith* chart
- **Scattering (S)- parameters**
  - Lumped and distributed circuit elements, linear networks described by I-V port parameters
  - n-port networks described by incident (a_i) and reflected / transmitted (b_i) normalized complex voltage waves
  - Termination of unused ports in their characteristic impedance!
  - reflection (S_{ii}) and transmission coefficients (S_{ij})
  - S-parameter matrix, properties of S matrices

S-Matrix Examples – 1-port

- Examples for 1-port S-matrices are any simple, passive (complex) impedances Z
  - Any R, L, C, RL, RC, LC and RLC circuit or any combinations of those elements leading to a single port network, which of course also my include distributed (transmission-line) elements
  - "Special" cases are:
    - $Z = Z_0 \Rightarrow S_{11} = 0$ (matched, ideal termination)
    - $Z = 0 \Rightarrow S_{11} = -1$ (ideal short)
    - $Z = \infty \Rightarrow S_{11} = +1$ (ideal open)
  - If $|S_{11}| > 1$ an active element is involved, e.g., a reflection amplifier

$$(\boldsymbol{S}) = \boldsymbol{S}_{11} = \boldsymbol{Z}$$

- Strictly speaking, a simple RF resonator, e.g., a "pill-box" cavity, is a 3-port
  - One coaxial or waveguide port as RF power coupler, plus two beam (waveguide) ports.
- However, for many practical cases it can be treated as 1-port
  - The mode of interest,
e.g., TM010, is trapped with no or negligible fields contribution near the beam-ports
  - We consider only a single coupler to characterize,
e.g., the TM010 mode in terms of a 1-port S-parameter measurement
    - Typically applying an RLC-parallel equivalent circuit

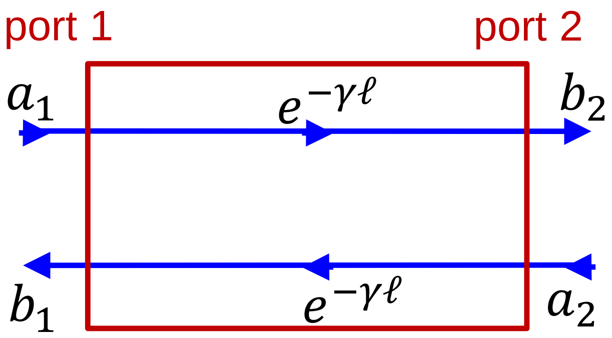
S-Matrix Examples – 2-port

- **Ideal (matched: $Z = Z_0$) transmission-line of length ℓ**

$$(S) = \begin{pmatrix} 0 & e^{-\gamma\ell} \\ e^{-\gamma\ell} & 0 \end{pmatrix}$$

$\gamma = \alpha + j\beta$: propagation constant
α: attenuation constant in [*Np/m*]
$\beta = 2\pi/\lambda$: phase constant [*rad/m*]

signal flow graph (SFG):



  – **For a lossless transmission-line:** $\alpha = 0 \;\Rightarrow\; |S_{21}| = |S_{12}| = 1$

  – **For a lossless line of length $\ell = \lambda/4$:** $(S) = \begin{pmatrix} 0 & -j \\ -j & 0 \end{pmatrix}$

- **Ideal attenuator**

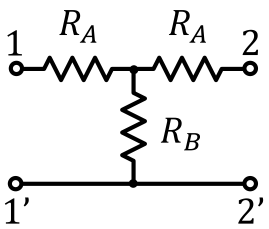
$$(S) = \begin{pmatrix} 0 & k \\ k & 0 \end{pmatrix}$$

$k = V_2/V_1 = 10^{-(\Delta dB/20)}$: attenuation $k < 1, k \in \mathbb{R}$
$\Delta dB = 20\log_{10} V_1/V_2$: attenuation in *dB*
$\alpha = -\ln k$: attenuation in *neper*

SFG example: *3 dB* attenuator

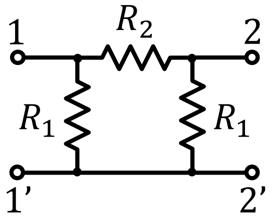
T-attenuator:

$$R_A = \frac{1-k}{1+k} Z_0$$
$$R_B = \frac{2k}{1-k^2} Z_0$$

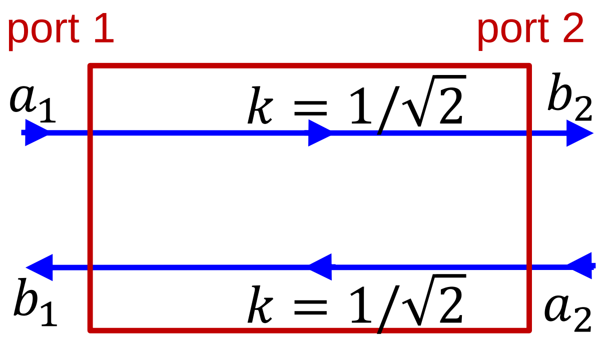


π-attenuator:

$$R_1 = \frac{1-k}{1+k} Z_0$$
$$R_2 = \frac{1-k^2}{2k} Z_0$$



$$S_{21} = \frac{b_2}{a_1} = \frac{1}{\sqrt{2}}$$
$$S_{12} = \frac{a_2}{b_1} = \frac{1}{\sqrt{2}}$$

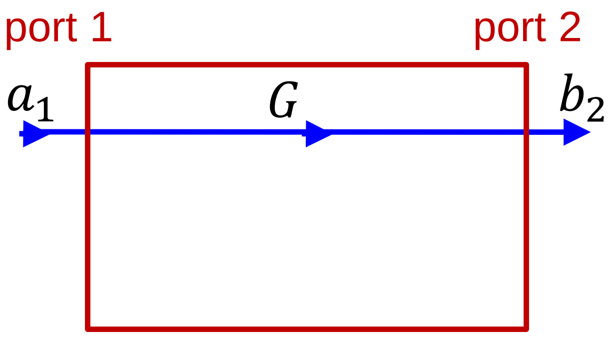


S-Matrix Examples – 2-port

- **Ideal amplifier (gain stage)**

$$(S) = \begin{pmatrix} 0 & 0 \\ G & 0 \end{pmatrix}$$

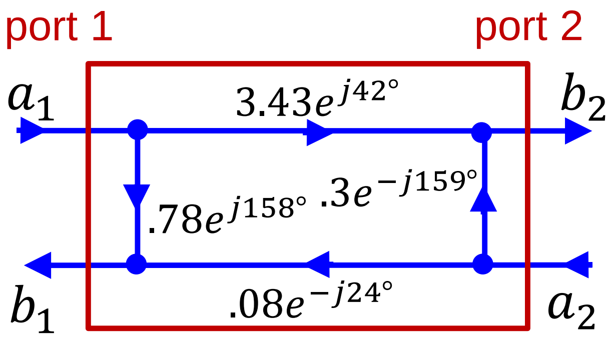
$G = V_{out}/V_{in} = 10^{g/20}$: voltage gain $G > 1$
$g = 20\log_{10} V_{out}/V_{in}$: voltage gain in *dB*



- **Low-noise RF transistor**

$$(S) = \begin{pmatrix} 0.78e^{j158°} & 0.08e^{-j24°} \\ 3.43e^{j42°} & 0.3e^{-j159°} \end{pmatrix}$$

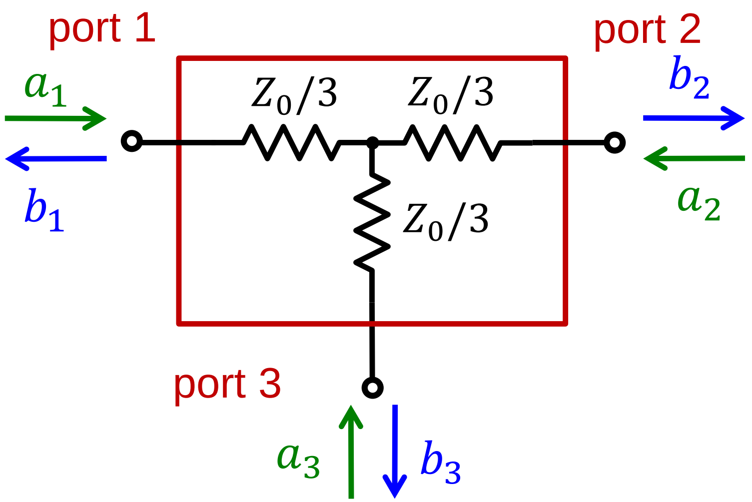
Datasheet *Avago* VMMK-1218:
$f = 10\ GHz, Z_0 = 50\Omega, T_A = 25°C,$
$V_{ds} = 2V, I_{ds} = 20mA$



- ***Avago* VMMK-1218**
- **E-pHEMT GaAs FET**
  - The S-parameters are different at other frequencies and operational conditions
  - The transistor requires impedance matching networks at in- and output

S-Matrix Examples – 3-port

- **3-port resistive power divider**



$$(\boldsymbol{S}) = \frac{1}{2}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$b_1 = \frac{1}{2}(a_2 + a_3)$$
$$b_2 = \frac{1}{2}(a_1 + a_3)$$
$$b_3 = \frac{1}{2}(a_1 + a_2)$$

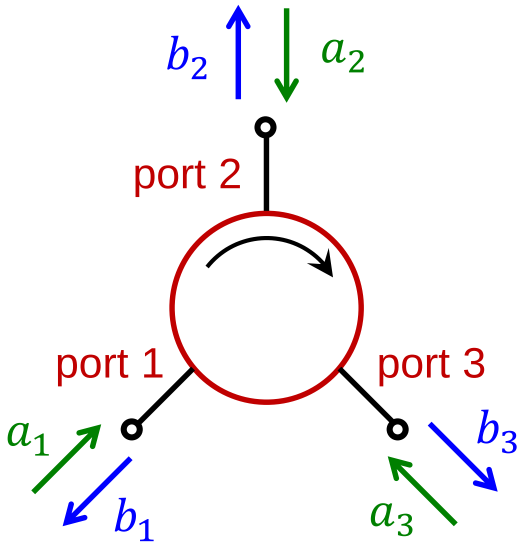
  – **The transfer-loss between *ij*-ports is 6 dB.**

- **Ideal circulator**

$$(\boldsymbol{S}) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

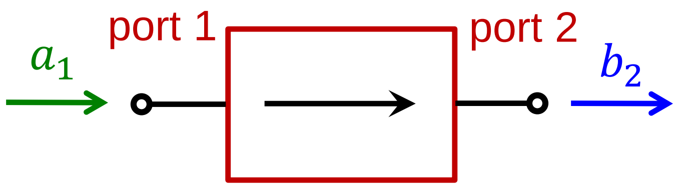
$$b_1 = a_3$$
$$b_2 = a_1$$
$$b_3 = a_2$$

  – **Matched, but not reciprocal**



- **Isolator, based on the circulator**
  – **Terminating, e.g., port 3 internally results in a 2-port, called isolator**

$$(\boldsymbol{S}) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad b_2 = a_1$$

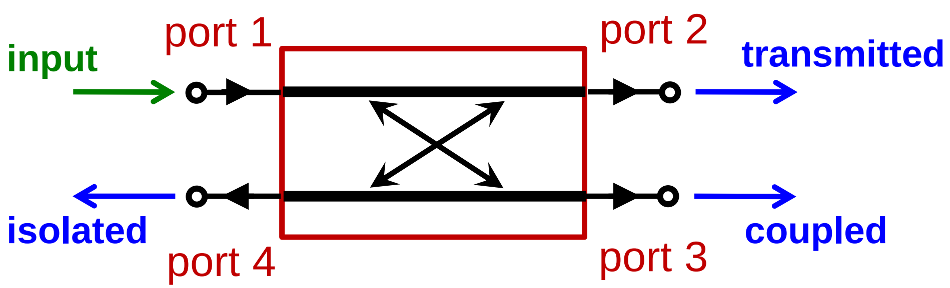


S-Matrix Example – 4-port

- **Ideal directional coupler**

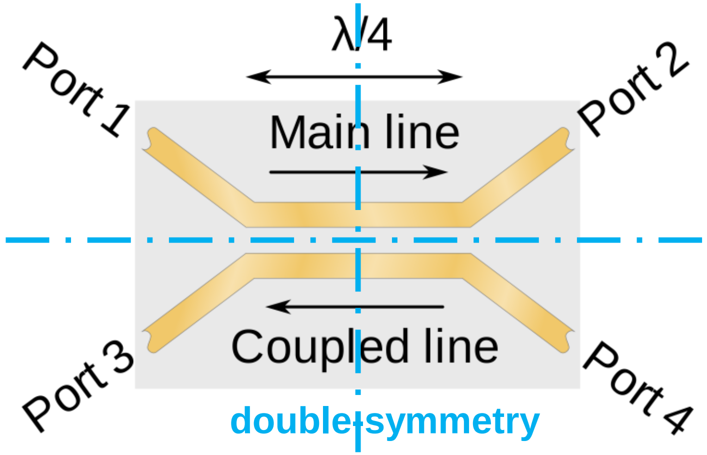
coupling coefficient: $k = \left|\frac{b_2}{a_1}\right|$; $\kappa dB = 20\log_{10} k$

$$(\boldsymbol{S}) = \begin{pmatrix} \mathbf{0} & -j\sqrt{1-k^2} & k & \mathbf{0} \\ -j\sqrt{1-k^2} & \mathbf{0} & \mathbf{0} & k \\ k & \mathbf{0} & \mathbf{0} & -j\sqrt{1-k^2} \\ \mathbf{0} & k & -j\sqrt{1-k^2} & \mathbf{0} \end{pmatrix}$$



- **Operating at the center frequency**
- **Figures of merit (ideal, lossless):**
  - Coupling factor $C_{3,1} = \kappa dB = 10\log_{10}(P_3/P_1)\ [dB]$
  - Insertion loss $L_{i2,1} = -10\log_{10}(P_2/P_1)\ [dB]$
  - Coupling loss $L_{c2,1} = -10\log_{10}[1-(P_3/P_1)]\ [dB]$

  no losses: $L_{i2,1} = L_{c2,1}$
- **Coupler with losses, imperfections, etc.**
  - Isolation $I_{4,1} = -10\log_{10}(P_4/P_1)\ [dB]; I_{3,2} = -10\log_{10}(P_3/P_2)\ [dB]$
  - Directivity $D_{3,4} = -10\log_{10}(P_3/P_4) = -10\log_{10}(P_4/P_1) + 10\log_{10}(P_3/P_1)\ [dB]$



https://en.wikipedia.org/wiki/Power_dividers_and_directional_couplers

S-Parameters in Practice

- In practice, S-parameters are a function of the frequency: $S(f)$
  - Some instruments or applications can also provide time-domain S-parameters

- In most real-world practical situations, S-parameters are acquired by a measurement, e.g., characterization of a RF component or sub-system by a VNA.
  - By characterizing the DUT over a range of frequencies, $f_{min} < f < f_{max}$ in steps of Δf

- Also, numerical RF analysis tools (Qucs, ADS, Microwave Office, etc.) generate S-parameters through linear RF circuit / systems simulations.
  - Numerical EM software tools (CST, HFSS, etc.) and PCB tools (Cadence Allegro) can also generate S-parameters

- Both application types, VNA measurements and RF/EM simulation software exchange S-parameters on a file basis
  - The *SnP Touchstone* ASCII file format is de-facto the industry standard for S-parameters
  - Example *Touchstone s2p* file:

SnP Touchstone S-Parameter Files

BRSTM_refline_2-4GHz_20001.s2p

```
!Keysight Technologies,P5024A,MY58100247,A.15.20.07
!Date: Wednesday, October 06, 2021 16:19:17
!Correction: S11(C  2-Port  )
!S21(C  2-Port  )
!S12(C  2-Port  )
!S22(C  2-Port  )
!S2P File: Measurements: S11, S21, S12, S22:
# Hz S  dB  R 50
2000000000 -2.5430779 -88.497566 -18.274168 38.763039 -18.26178 38.742687 -2.4251425 -85.792152
2000100000 -2.5365531 -88.473915 -18.266272 38.606499 -18.269154 38.716461 -2.4215624 -85.861908
2000200000 -2.5314419 -88.471634 -18.280306 38.559021 -18.258684 38.624985 -2.4253747 -85.806236
2000300000 -2.5216722 -88.617905 -18.269596 38.352692 -18.266785 38.342678 -2.4210978 -85.8368
2000400000 -2.5178108 -88.521271 -18.257862 38.298622 -18.275055 38.266792 -2.4244151 -85.914787
2000500000 -2.5327342 -88.484985 -18.263821 38.31945 -18.27046 38.20409 -2.4263382 -85.85775
2000600000 -2.5191193 -88.462044 -18.262426 38.195797 -18.246525 38.250874 -2.3989511 -85.885635
2000700000 -2.5219827 -88.44445 -18.253748 38.216946 -18.245417 38.138355 -2.4155877 -85.859711
2000800000 -2.5198817 -88.588783 -18.255999 37.902744 -18.257757 38.035915 -2.415241 -85.887199
2000900000 -2.5370498 -88.496101 -18.256392 38.004234 -18.258446 37.872906 -2.4170656 -85.854294
2001000000 -2.5363033 -88.544846 -18.252661 37.830475 -18.259445 37.925297 -2.4102871 -85.901245
2001100000 -2.5417418 -88.47406 -18.270754 37.731613 -18.252403 37.74762 -2.4158244 -85.822517
```

! header

format

2-port VNA file

Touchstone v1.1 example file

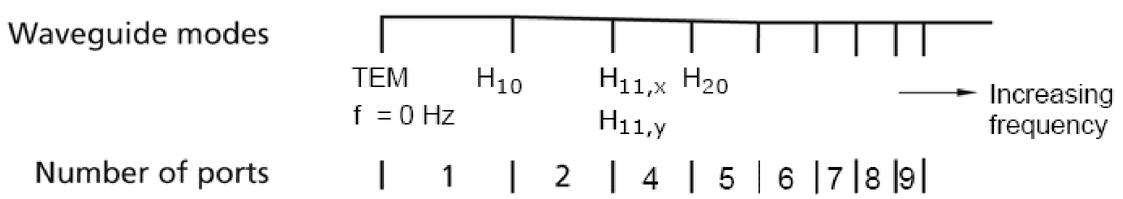
- v2.0 is different, file ext. **.ts*

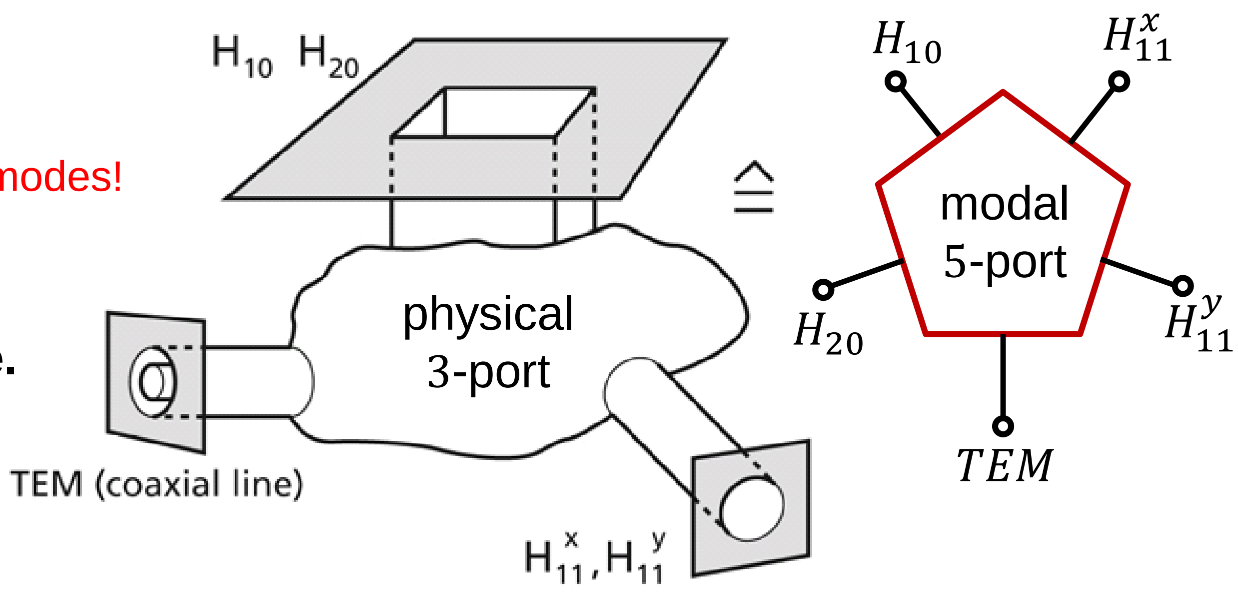
frequency f $|S_{11}|$[dB] $\angle S_{11}$[deg]

- **The file name extension specifies the number n of ports**
  - **Attention: NOT equal to the number of columns! The carriage return (CR) is different between s1p, s2p and s3p, s4p files!**
- **The comment header (!) includes general information, e.g., type of instrument, measurement time, etc.**
- **The format line (#) defines the format (mag[dB],angle[deg], mag/angle, real/imag), stimulus units and the reference impedance**
- **The column delimiter varies, e.g., space, comma, semicolon, etc.**
  - **Column order in the example file: *f S11dB S11a S21dB S21a S12dB S12a S22dB S22a***

General n-Ports

- **A general n-port may include ports of different technologies, i.e., waveguides, as well as TEM transmission-lines, such as coaxial lines, microstrip lines etc.**
  - **In the frequency range of interest different modes may propagate at each physical port, e.g., several waveguide modes in a rectangular waveguide and/or higher order modes in a coaxial line..**
  - **Each EM-mode must then be represented by a distinct modal port.**
    - This is very important in EM-simulation to ensure the absorption of the energy for all modes!
  - **The number of modal ports needed generally, increases with frequency, as more waveguide modes can propagate.**





H_{11}^{x}, H_{11}^{y}: x, y-polarization of the E_{11} circular mode

S-Parameter Summary

- The scattering (S) parameters are based on incident and reflected normalized complex voltage waves (power waves), defined at the ports of a RF network.
- S-Parameters are used to characterize a linear, time-invariant RF component, circuit or sub-system as function of frequency under realistic operational conditions
  - The S-parameters are given in a matrix notation, and have complex values
- The characteristic of the S-matrix may provide additional details about the network, such as reciprocity, symmetry, losses.
- Typically, the S-parameters matrix of a RF network is acquired by measurement characterization with a vector network analyzer (VNA), or by a numerical analysis, e.g., circuit analysis or electromagnetic simulation software
- The S-parameter matrices of a set of networks can be converted to transfer (T) parameter matrices to enable a simple cascading of those networks
- The number of logical, modal ports might be higher than the number of physical ports for a general RF network utilizing various transmission-line technologies.

Part IV: RF Measurement Methods

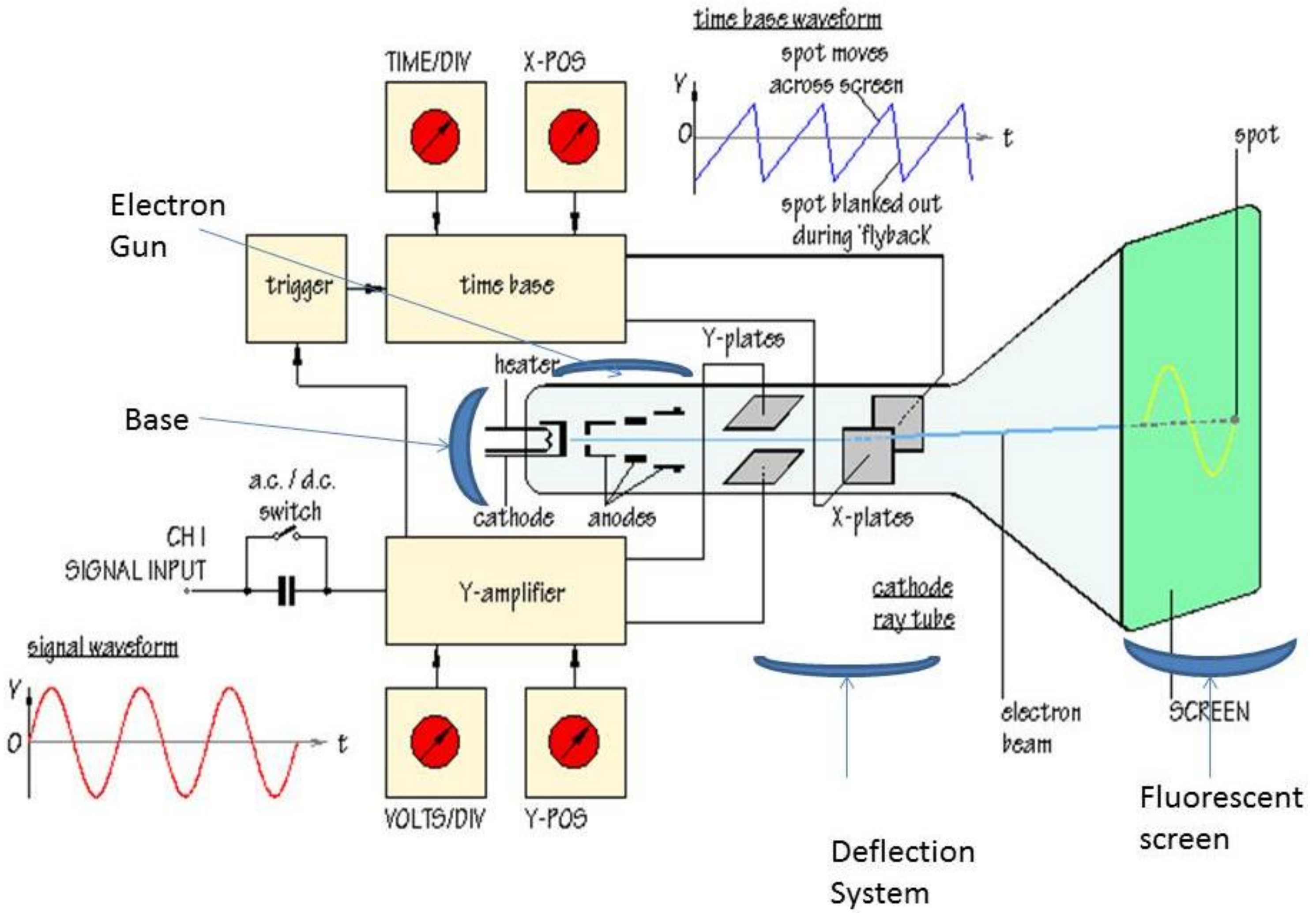
- **Overview of RF measurement instruments**
  - **Oscilloscope, spectrum analyzer (SA), signal (FFT) analyzer, slotted measurement line, vector network analyzer (VNA)**
- **The super-heterodyne receiver principle**
  - **Modulation, down-conversion, mixer, spectrum analyzer block schematics**
- **Reflection measurement with the slotted coaxial air-line**
- **S-parameter measurements**
  - **Simple measurement setup, VNA block schematics**
  - **VNA calibration**
  - **Features of modern RF measurement equipment**
  - **Synthetic pulse measurements with the VNA**
  - **Measurement example: pillbox resonator characterization**
    - Equivalent circuit parameters, Q-factor measurement in the *Smith*-chart, R/Q measurement
  - **Measurement of the beam-coupling impedance with a stretched-wire**

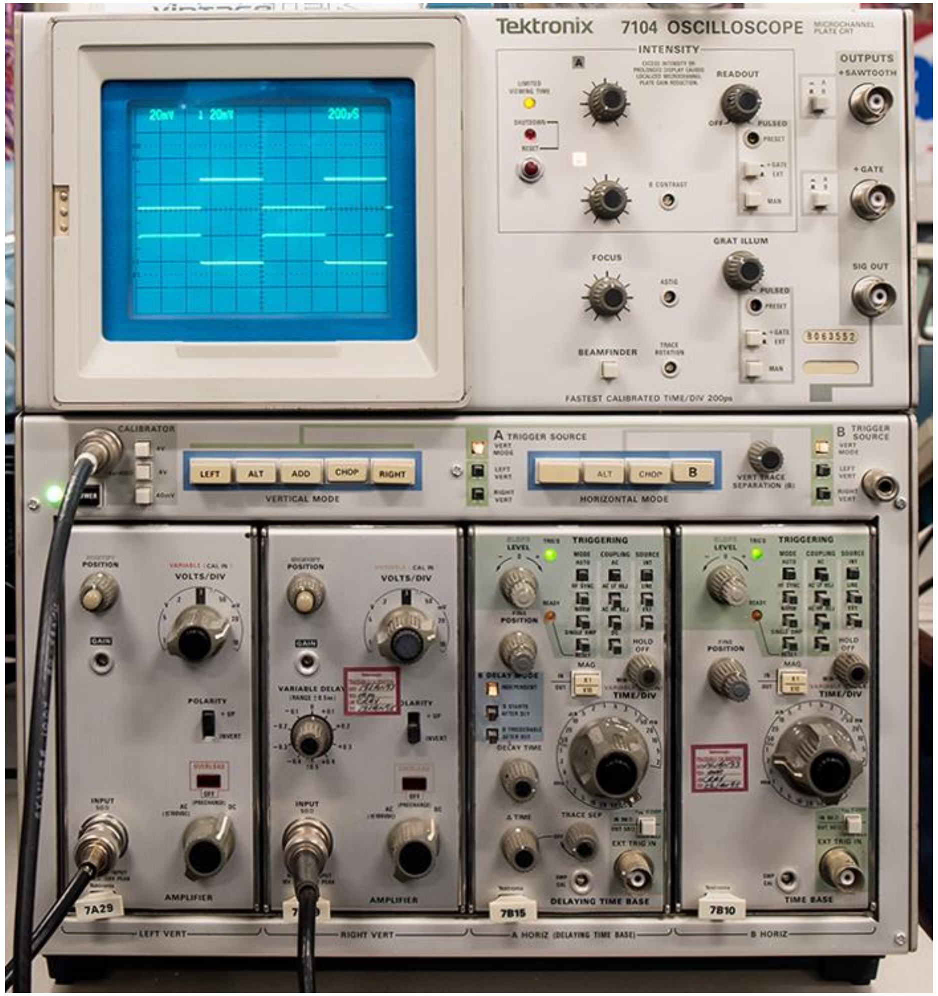
RF Measurement Methods (1)

There are different options to observe RF signals
Here some typical measurement tools:

- **Oscilloscope: to observe signals in time-domain**
  - **periodic signals**
  - **burst and transient signals with arbitrary waveforms**
  - **application: direct observation of signals from a beam pick-up, from a test generator, or from other sources**
  - **visualizes the shape of a waveform, etc.**
  - **limited performance for the evaluation of non-linear effects.**

Cathode Ray Tube (CRT) Oscilloscope

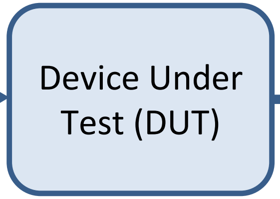




Today: Digital Storage Oscilloscope (DSO)

…and digital signal generator
(AWG: arbitrary waveform generator)

50 GS/s, 10-bit AWG (*Tektronix*)



100 GHz bandwidth
240 GS/s oscilloscope
(*LeCroy*)

- **Signal processing based on fast ADCs and DACs**
  - **Similar "look and feel" as analog oscilloscopes, but better performance**
    - ➢ 8…12-bit multi-GS/s ADCs, still, be aware of aliasing effects!
    - ➢ Fast sampling oscilloscope require sufficient memory resources.
- **AWG or pulse generator & digital oscilloscope: Time-domain (TD) test setup**
  - **Device under test (DUT) characterization and trouble shooting**
    - ➢ Impulse, step, or arbitrary waveform (e.g., beam signal) as stimulus signal
    - ➢ High impedance probe for measurements on the printed circuit board (PCB)

RF Measurements Methods (2)

- **Spectrum analyzer: to observe signals in a "frequency-domain like" fashion**
  - **sweeps in equidistant steps through a given frequency range**
  - **application: observation of spectrum from the beam, or from a signal generator or RF source, or the spectrum emitted from an antenna to locate EMI issues in the accelerator tunnel, etc.**
    - Also, DUT characterization in the laboratory, e.g., noise figure measurement on amplifiers (requires a noise source), intermodulation measurements on amplifiers (requires two RF generators).
  - **Requires periodic signals**
  - **Assumes time-invariance of the measurement object (DUT) throughout the frequency sweep**
  - **Large dynamic range!**

- **RF detection (Schottky) diode (RF power meter)**
  - **Supplies a rectified (video) output signal proportional to the RF signal level**
  - **Delivers no frequency or phase information but operates over a very broad frequency range few MHz to many GHz, and up to 90 dB dynamic range.**

RF Measurements Methods (3)

- **Vector signal analyzer (VSA), sometimes called FFT analyzer**
  - **Acquires the RF signal, after down-conversion to an intermediate (IF) signal, in time-domain by fast sampling**
  - **Further numerical treatment in digital signal processors (DSPs)**
  - **Spectrum calculated using Fast Fourier Transform (FFT)**
  - **Combines features of an oscilloscope and a spectrum analyzer: Signals can be observed directly in time-domain, or in a frequency-domain like fashion**
  - **Contrary to the SA, also the spectrum of non-periodic signals and transients can be measured**
  - **Application: Observation of tune sidebands, transient behavior of a phase locked loop, single pass beam signal spectrum, etc.**
  - **Digital oscilloscopes and FFT analyzers share similar technologies, i.e., fast sampling and digital signal processing, and therefore can provide similar measurement options**
    - The digital oscilloscope directly digitizes the RF signal → limited dynamic range, large instantaneous bandwidth
    - The FFT analyzer digitizes the down-converted IF signal → large dynamic range, but (still) limited instantaneous bandwidth

RF Measurements Methods (4)

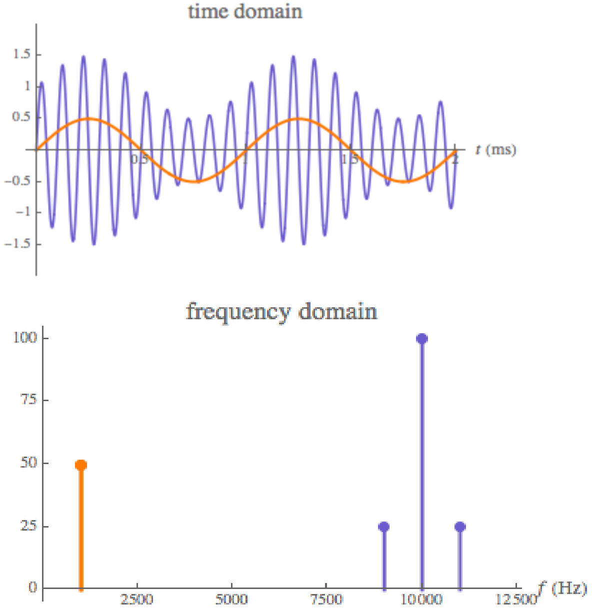
Tools to characterize RF components and sub-systems:

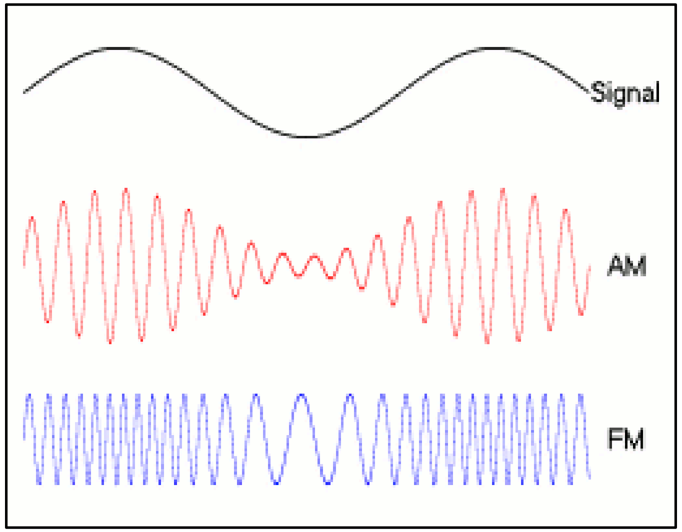
- **Slotted coaxial (or waveguide) measurement transmission-line**
  - **For study and illustration purposes only –**
    not anymore used in today's RF laboratory environment.
- **Vector Network Analyzer (VNA)**
  - **Combines the functions of a vector spectrum analyzer (FFT analyzer), a RF sweep generator, and a S-parameter test set (directional coupler)**
  - **Excites a *Device Under Test* (DUT, e.g., circuit, antenna, amplifier, etc.) network at a given sinusoidal *continuous wave* (CW) frequency, and measures the response in magnitude and phase => determines the S-parameters**
  - **Covers a selectable frequency range by measuring step-by-step at subsequent frequency points (like a spectrum analyzer, again requires the DUT to be time-invariant!)**
  - **Applications: characterization of passive and active RF components, *Time Domain Reflectometry* (TDR) by Fourier transformation of the reflection response, etc.**
    - Also, power sweep measurements (1 dB compression point),
      4-port VNAs enable virtual ports: e.g., single-ended / differential port DUT characterization.
  - **The VNA is the most versatile and comprehensive tool in the RF laboratory!**

RF Signals & Modulation, without Math!

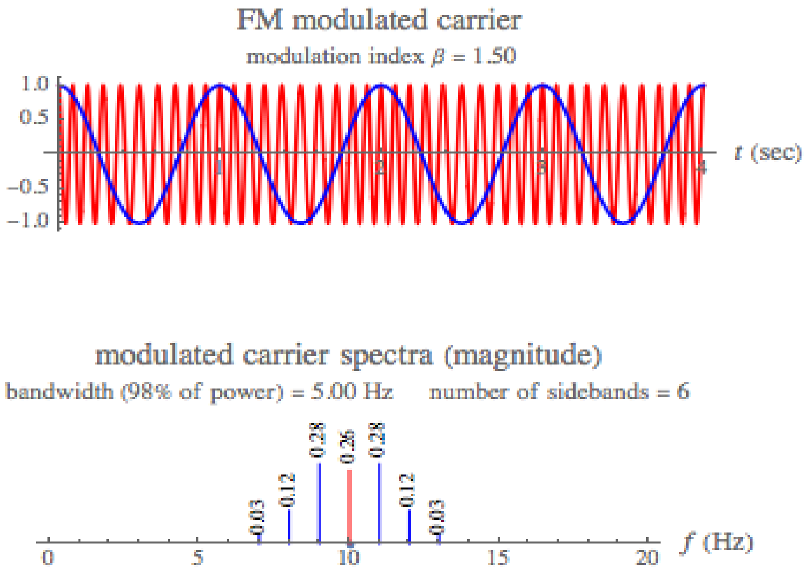
- **RF signals are continuous wave (CW),sinusoidal signals**
  - **Often, a high frequency carrier is modulated with low frequency information**
  - **Modulation appears "naturally" in ring accelerators as:**
    - Modulation is also provided through the LLRF system to the accelerating structures

AM: Betatron oscillations

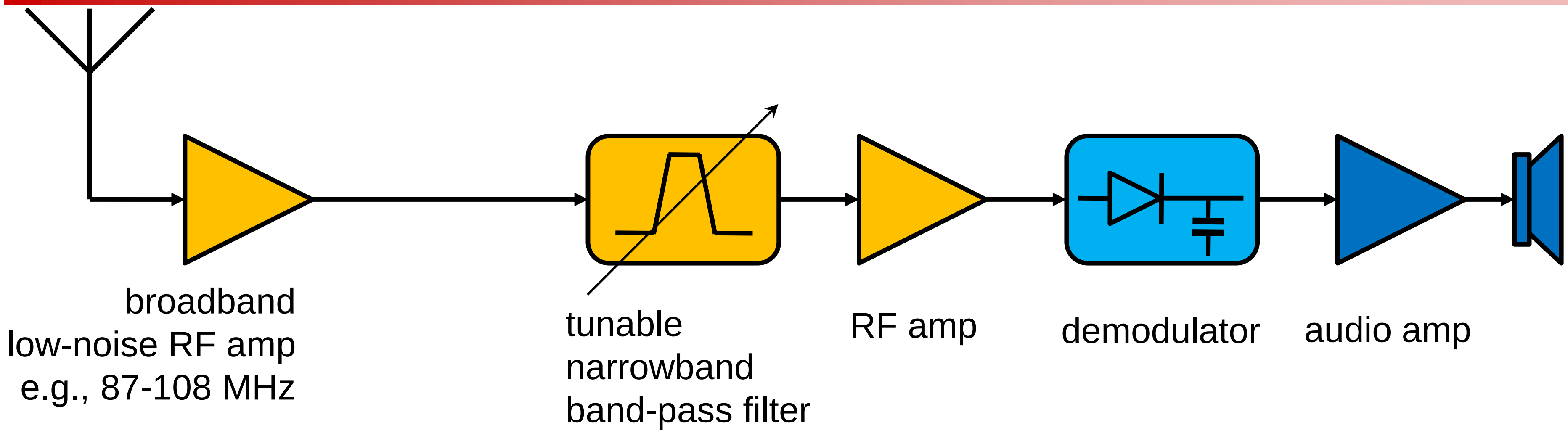




FM: Synchrotron oscillations

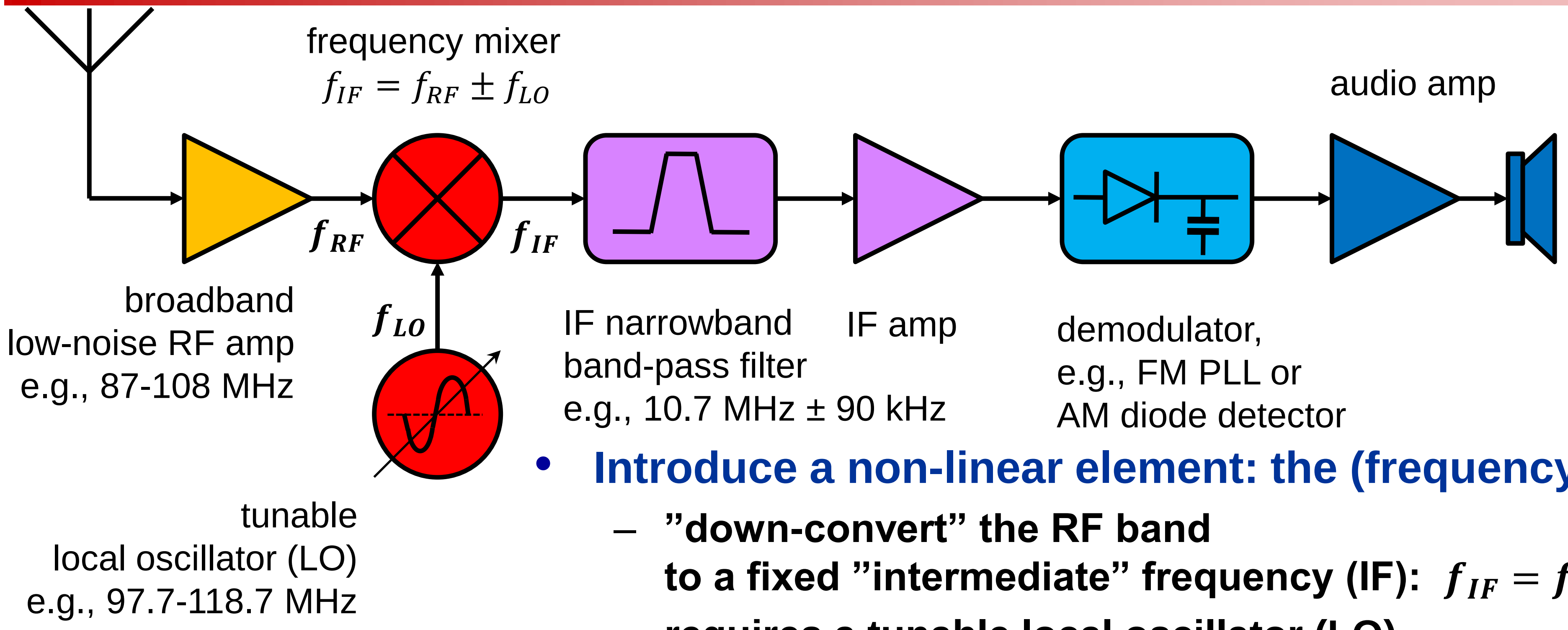


A (too) simple Radio Receiver



- **…or: How does a ”traditional” analog radio works?**
  - **It was, and still is, difficult to make precisely tunable narrowband, band-pass filters for high frequencies (~100 MHz)!!**
  - **high frequency low-noise amplifiers are expensive!**
  - **high frequency demodulators are not trivial.**
  - **direct detection of radio and RF signals is challenging!**

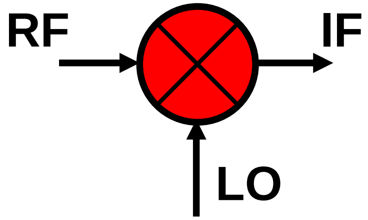
The Super-Heterodyne Receiver



- **Introduce a non-linear element: the (frequency) mixer!**
  - **"down-convert" the RF band to a fixed "intermediate" frequency (IF): $f_{IF} = f_{RF} \pm f_{LO}$**
  - **requires a tunable local oscillator (LO)**
  - **well manageable IF section:**
    - narrowband band-pass filter(s) (BPF) and amplifier(s)
  - **RF telecommunication standard**
  - **Often multiple mixing stages are used in modern RF instruments, e.g., spectrum and network analyzers**

The RF Mixer as Down-Converter

$$y_{RF}(t) = A_{RF}\sin(\omega_{RF}t + \varphi_{RF})$$

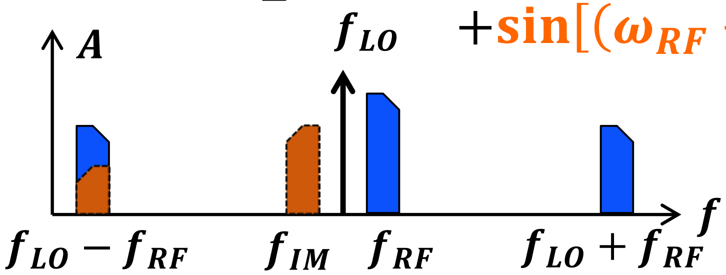


$$y_{IF}(t) = y_{RF}(t)\,y_{LO}(t)$$

$$y_{LO}(t) = A_{LO}\sin(\omega_{LO}t + \varphi_{LO})$$

- **Ideal mixer:** $f_{IF} = f_{RF} \pm f_{LO}$

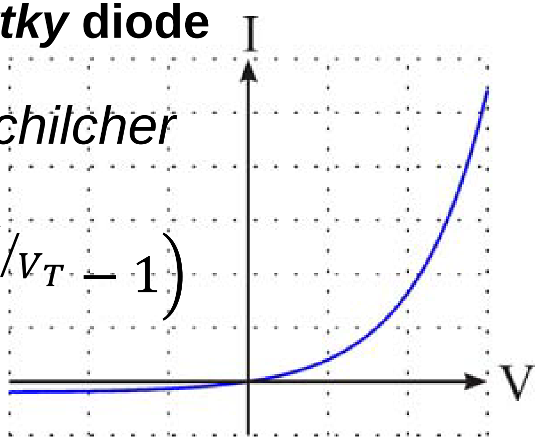
$$y_{IF}(t) = \frac{1}{2}A_{LO}A_{RF}\{\sin[(\omega_{RF} - \omega_{LO})t + (\varphi_{RF} - \varphi_{LO})] \quad \text{upper sideband}$$
$$+\sin[(\omega_{RF} + \omega_{LO})t + (\varphi_{RF} + \varphi_{LO})]\} \quad \text{lower sideband}$$



$I = f(V)$ of a *Schottky* diode

courtesy *T. Schilcher*

$$I = I_0\left(e^{V/V_T} - 1\right)$$

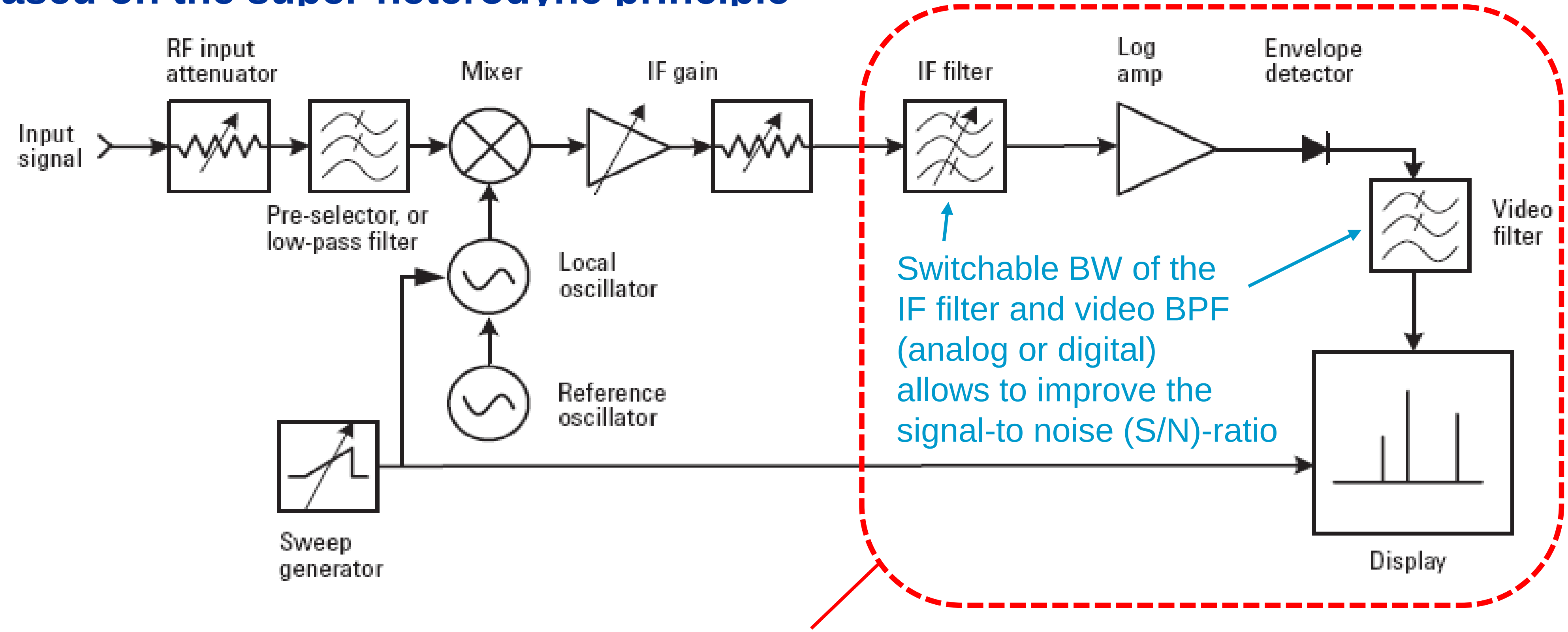


- **Frequency conversion**
  - $f_{RF} \neq f_{LO}$: heterodyne receiver
  - $f_{RF} = f_{LO}$: homodyne, demodulator
- **Real-world mixer:** $f_{IF} = mf_{RF} \pm nf_{LO}$
  - Image frequency: $f_{IM} = f_{LO} - f_{IF}$

$$\Delta I = I_0 e^{V/V_T}\left[\frac{\Delta V}{V_T} + \frac{1}{2}\left(\frac{\Delta V}{V_T}\right)^2 + \frac{1}{6}\left(\frac{\Delta V}{V_T}\right)^3 + \cdots\right]$$

Simplified Spectrum Analyzer

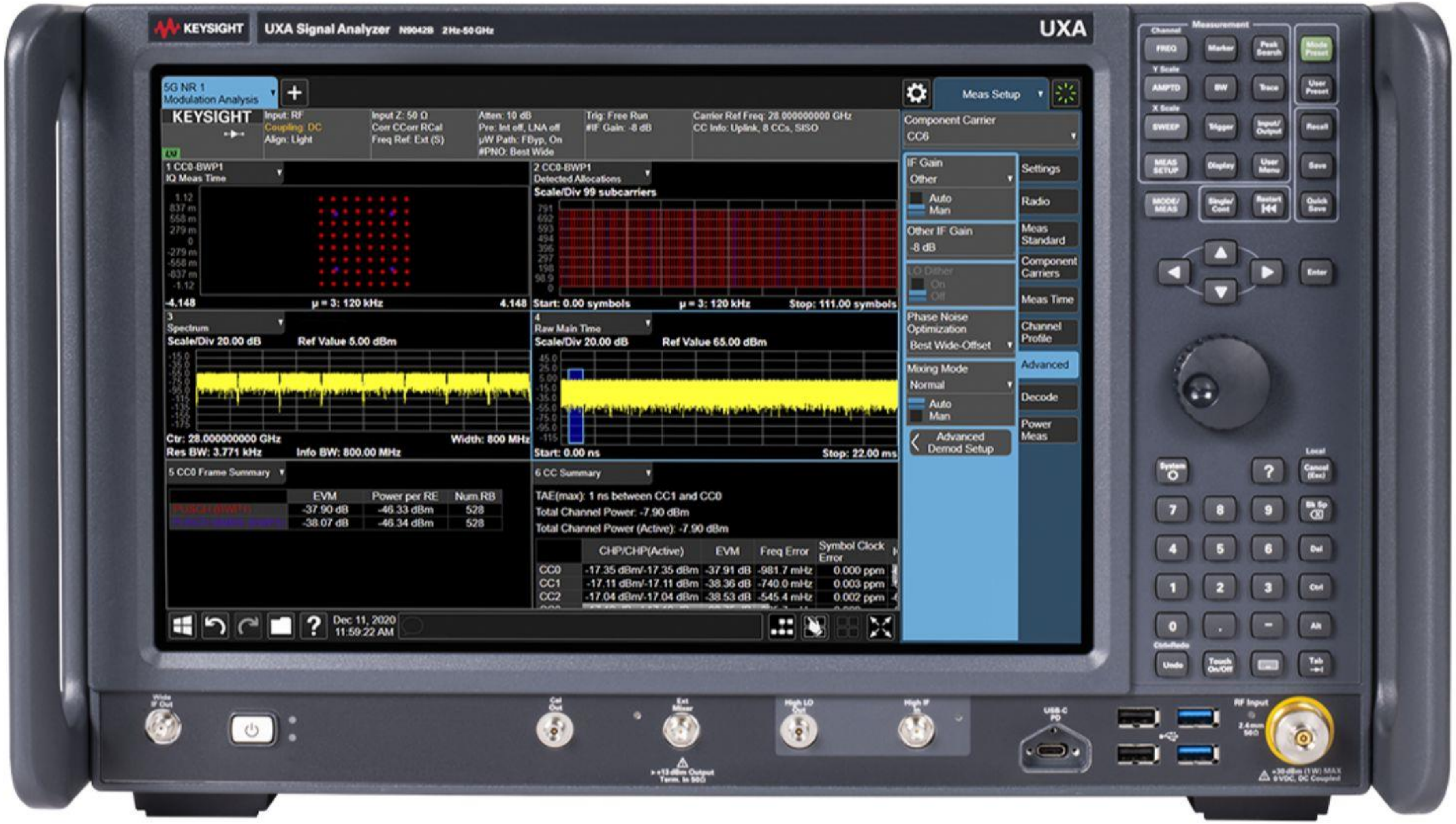
- **based on the super-heterodyne principle**



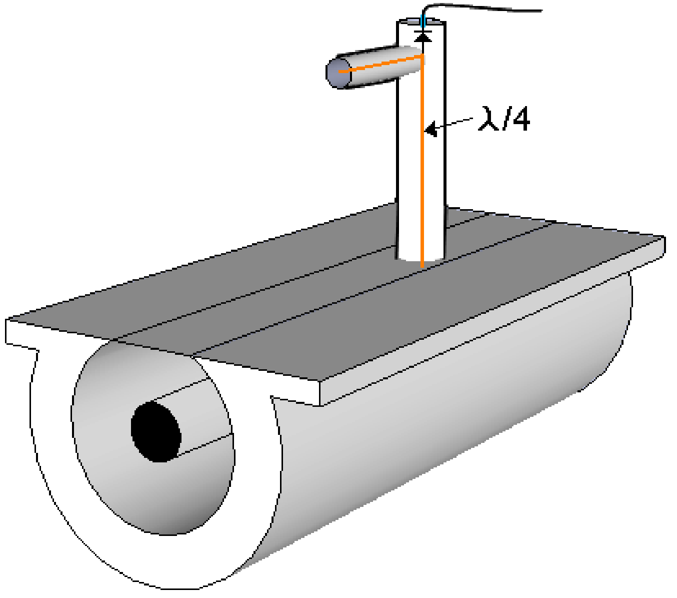
Today, the IF, demodulation, video and display sections of a spectrum analyzer are realized digitally

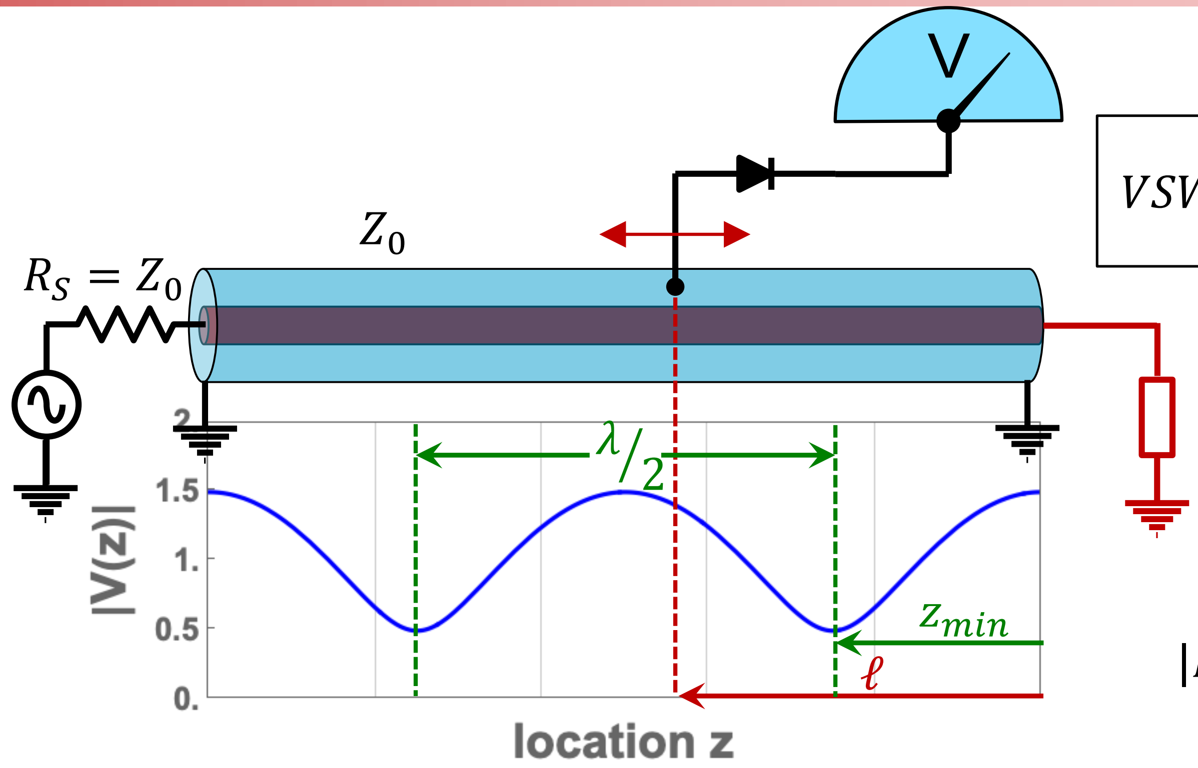
- **Requires an analog-digital converter (ADC) with sufficient dynamic range**

Modern Spectrum (RF Signal) Analyzer



Reflection (VSWR) Measurement





$$VSWR = \frac{|V_{max}|}{|V_{min}|}$$

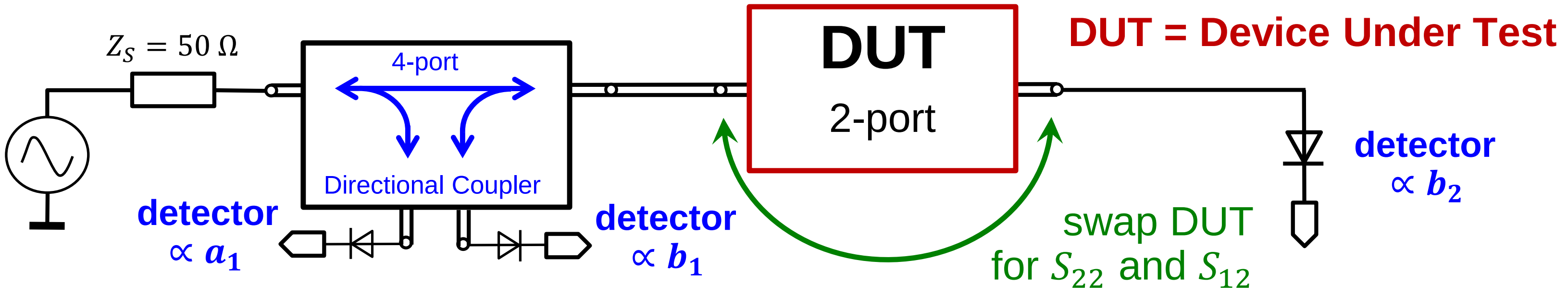
$$|\Gamma| = \frac{VSWR - 1}{VSWR + 1}$$

$$\angle\Gamma = \frac{4\pi}{\lambda} z_{min} - \pi$$

$$Z_x = Z_0 \frac{1-\Gamma}{1+\Gamma}$$

- **Slotted coaxial air-line is used as standing wave detector**
  - **Probes the radial electric field along the slotted line.**
  - **Measurement of E-field minima's E_{min} and maxima's E_{max} with a diode detector, thus detect $|V_{min}|$ and $|V_{max}|$ along the line.**
  - **Evaluate the reflection coefficient Γ of a DUT of unknown Z_X at the end of the line**

How to measure S-Parameters?

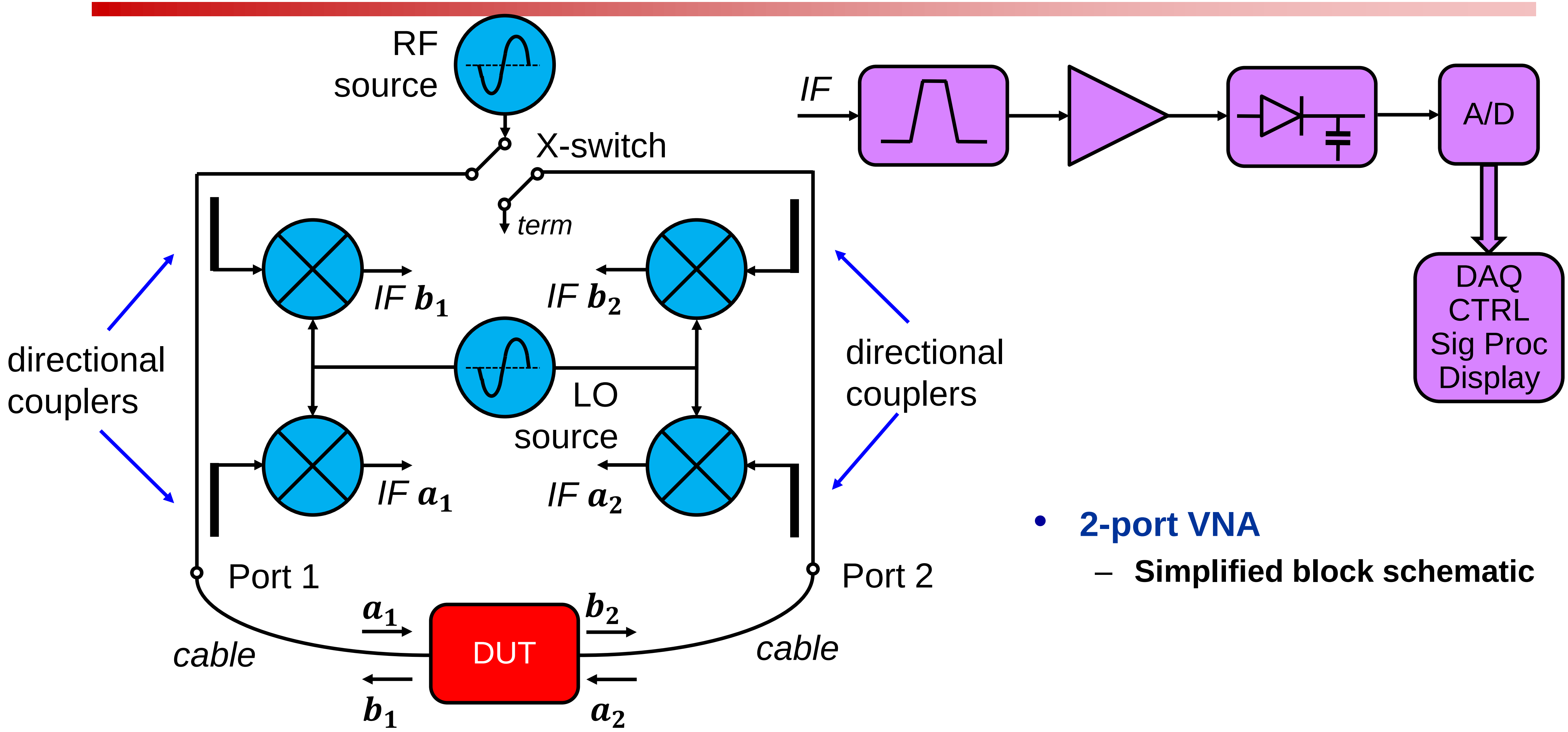


- Performed in the "frequency domain"
  - Single or swept frequency generator, stand-alone or as part of a VNA or SA
  - Requires a directional coupler and RF detector(s) or receiver(s)
- Evaluate S_{11} and S_{21} of a 2-port DUT
  - Ensure $a_2 = 0$, i.e., the detector at port 2 offers a well-matched impedance
  - Measure incident wave a_1 and reflected wave b_1 at the directional coupler ports and compute for each frequency
  - Measure transmitted wave b_2 at DUT port 2 and compute
- Evaluate S_{22} and S_{12} of the 2-port DUT
  - Perform the same methodology as above by exchanging the measurement equipment on the DUT ports

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$

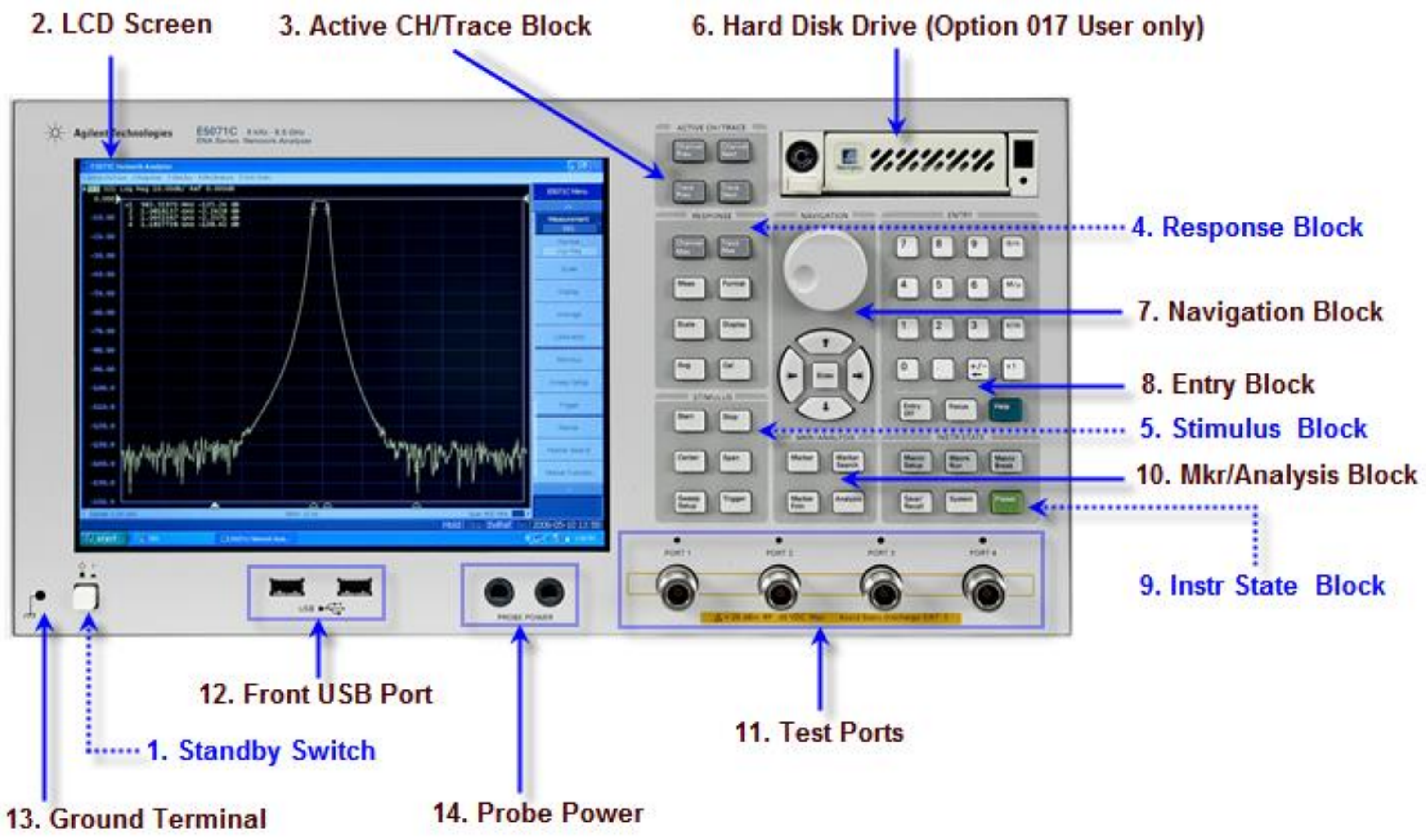
$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

The Vector Network Analyzer (VNA)



- **2-port VNA**
  - **Simplified block schematic**

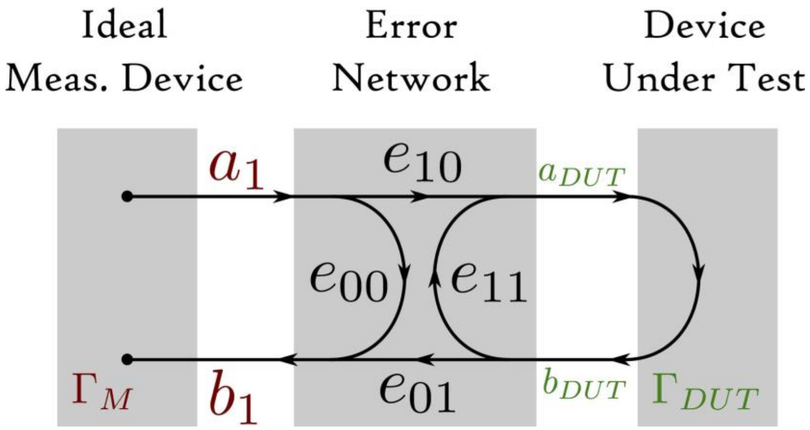
Fun with the VNA!



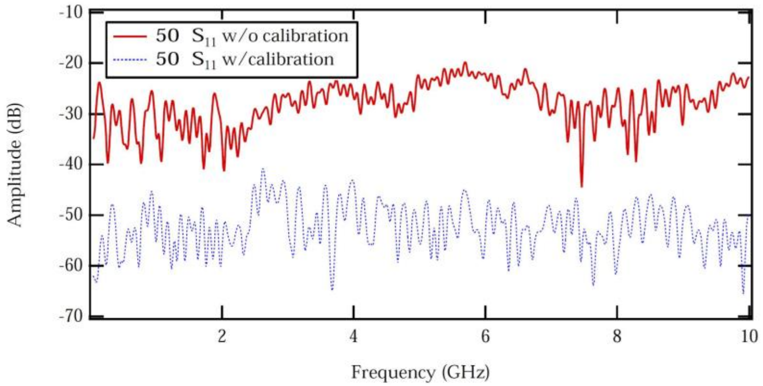
- The "*look and feel"* between VNAs vary between manufacturers and models
  - Concepts and operation is still very similar

VNA Calibration (1)

- **Calibration is not necessary for pure frequency or phase measurements**
- **Before calibrating the VNA measurement setup,
  perform a brief measurement and chose appropriate VNA settings:**
  - **Frequency range (center, span or start, stop)**
  - **Number of frequency points**
    - Can be sometimes increased by rearranging the VNA memory (# of channels)
  - **IF filter bandwidth**
  - **Output power level**
- **Calibrate the setup, preferable with an electronic calibration system if more than 2 ports are used!**
  - **Each port and combination needs to be calibrated, with the cables attached**
  - **Choose the appropriate connector type and sex**
  - **The instrument establishes a correction matrix and displays the "CAL" status.**



VNA Calibration (2)



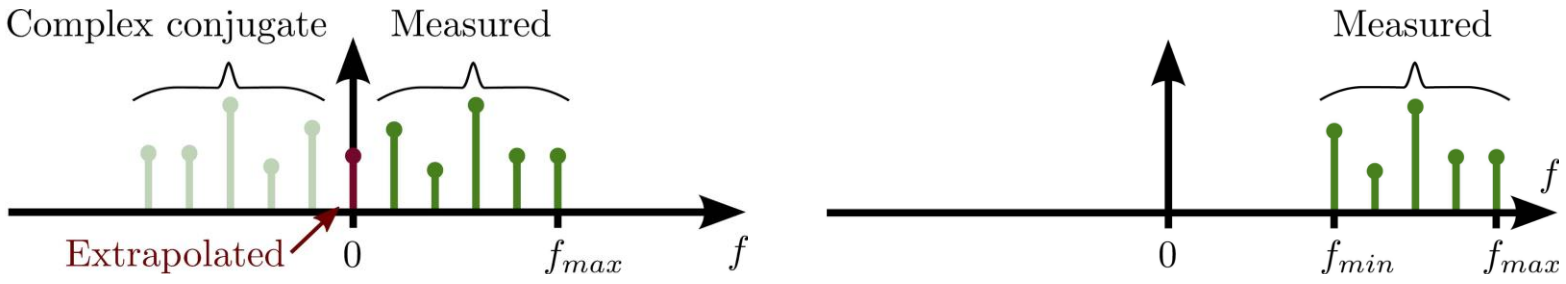
- **Calibration improves the measurement performance**
  - **Return loss improvement by typically 20 dB. Enables mdB accuracy measurements!**
  - **Full 2-port or 4-port calibration with manual calibration kits is prone to errors, better use electronic calibration systems.**
  - **Change VNA settings will cause the instrument to inter- and extrapolate, and the calibration status becomes uncertain.**
- **Cables are included in the calibration**
  - **However, changing coaxial connector types not.**
  - **Special VNA cables allows the adaption of different connector types and sex, without requiring a re-calibration of the setup!**

RF Measurement Instrument Features

- **Modern VNAs (SAs, oscilloscopes, etc. as well) have many "features"**
- **Hardware features, e.g.**
  - **Automatic calibration system, down to DC**
  - **4 and more ports**
  - **Additional 2nd source, for downconverter / mixer measurements**
  - **Integrated spectrum analyzer function**
- **Software, control and data post processing options, e.g.**
  - **Far too many to list all**
  - **Sweep options, e.g., lin., log., segmented, in frequency or power**
  - **iDFT (or iFFT), gating**
  - **TDR, TDT for BP or LP step or impulse, segmented (advanced) TDR**
    - Only for linear, time-invariant systems!
  - **Port extension, virtual ports (4-port VNA), Z_{0e}, Z_{0o} characterization, virtual baluns, etc.**
  - **Data transformations, e.g., $\Gamma \Rightarrow Z$**
  - **Noise figure measurements**
  - **Measurements following telecommunication standards**

Synthetic Pulse TD Measurements (1)

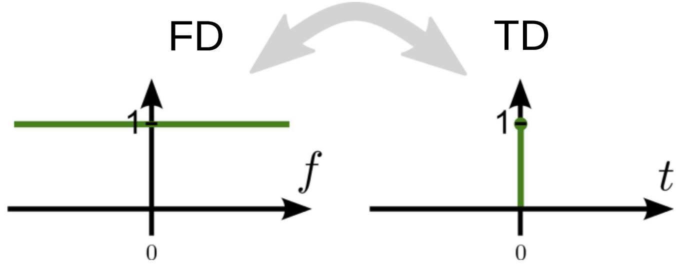
- **Based on an inverse discrete *Fourier* transformation (iDFT) option in the VNA**



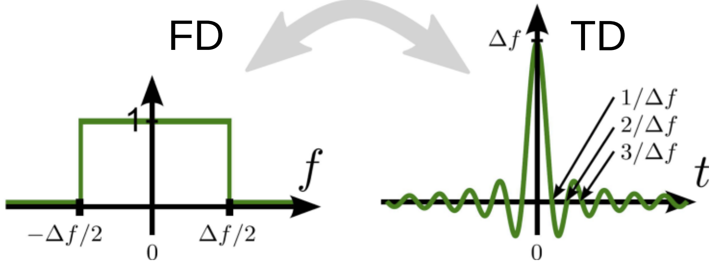
  - **Low-pass mode: Impulse or step response, relying on equidistant samples over the extrapolated (to DC) frequency range.**
    - The VNA does not measure at DC!
    - Manually match frequency range and # of points for DC extrapolation, e.g., 1…1000 MHz -> 1001 points, to enable extrapolation exactly to DC, or let the instrument chose the extrapolation settings automatically
  - **Enables time-domain reflectometry (TDR)**
    - Very useful on portable VNAs, troubleshooting RF cable problems
  - **Band-pass response (no DC extrapolation)**
  - **Allows time-domain gating and de-embedding of non-resonant sub-systems, e.g., measurements on a PCB**
  - **Limited to linear systems**
  - **Select the ”real” format for S_{11} or S_{21} for time-domain transformations (*Keysight* instruments)!**
    - or dB magnitude to detect small reflections in TDR analysis

Synthetic Pulse TD Measurements (2)

unlimited frequency range

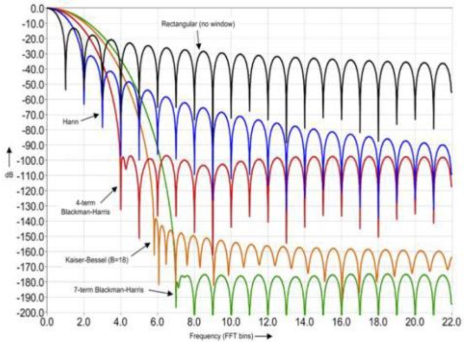


truncated frequency range

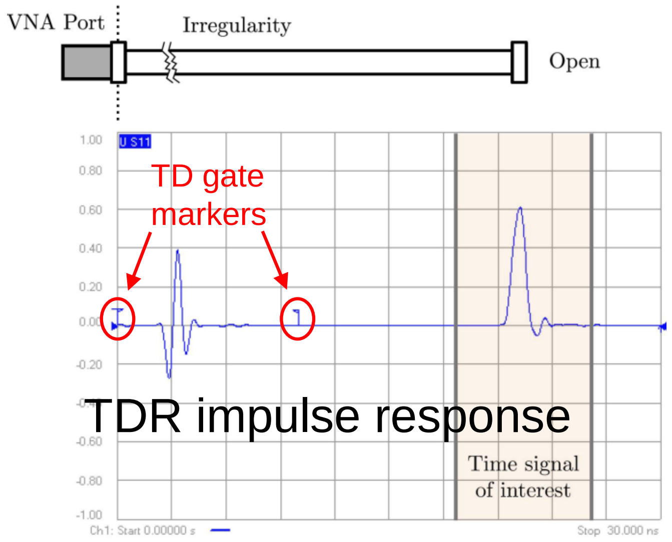


$$\mathrm{rect}\left(\frac{f}{\Delta f}\right)$$

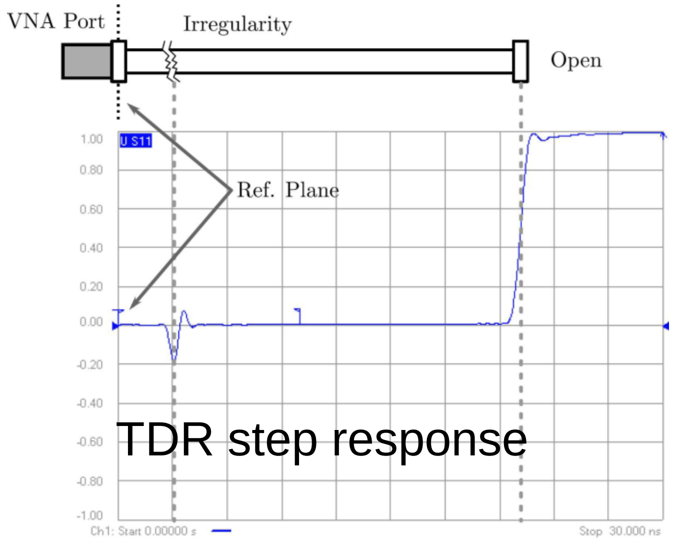
$$\frac{\sin(\Delta f \pi t)}{\pi t}$$



smoothing window functions

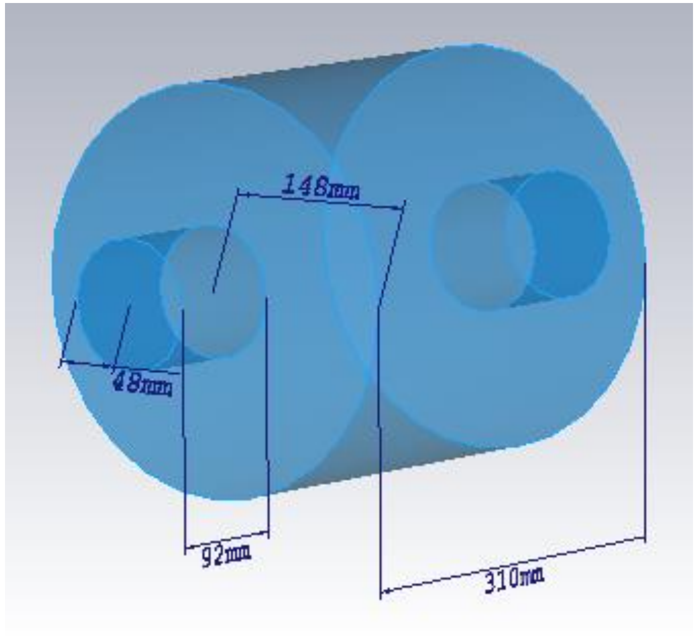


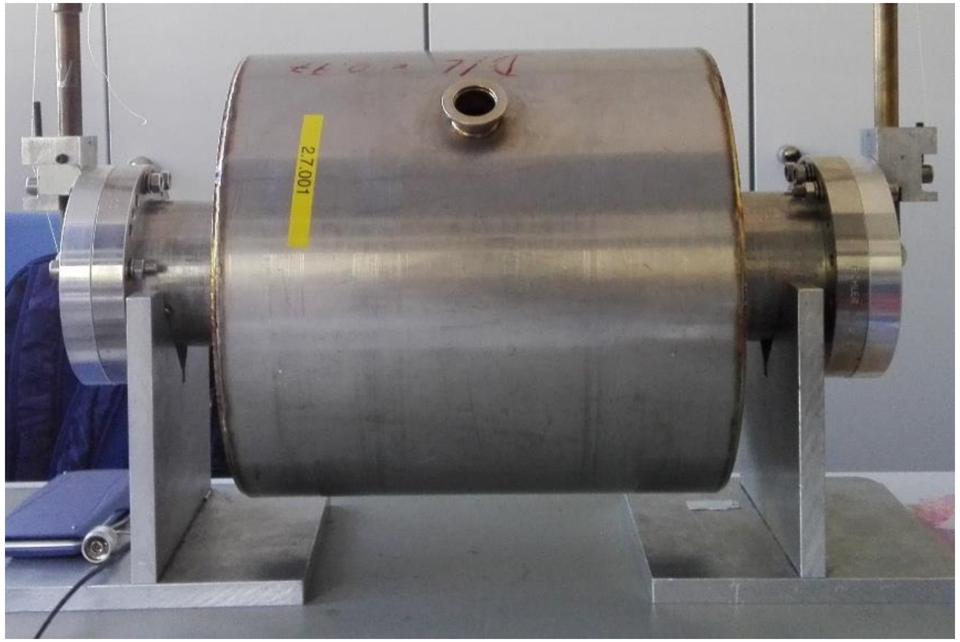
TDR impulse response



TDR step response

An Example – Pillbox* TM_{010} Eigenmode

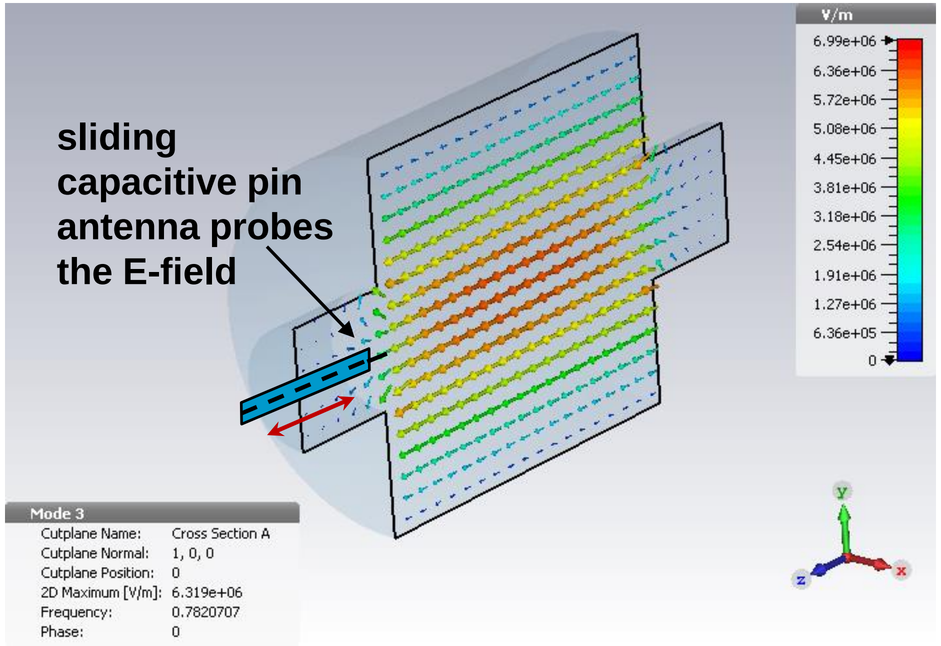


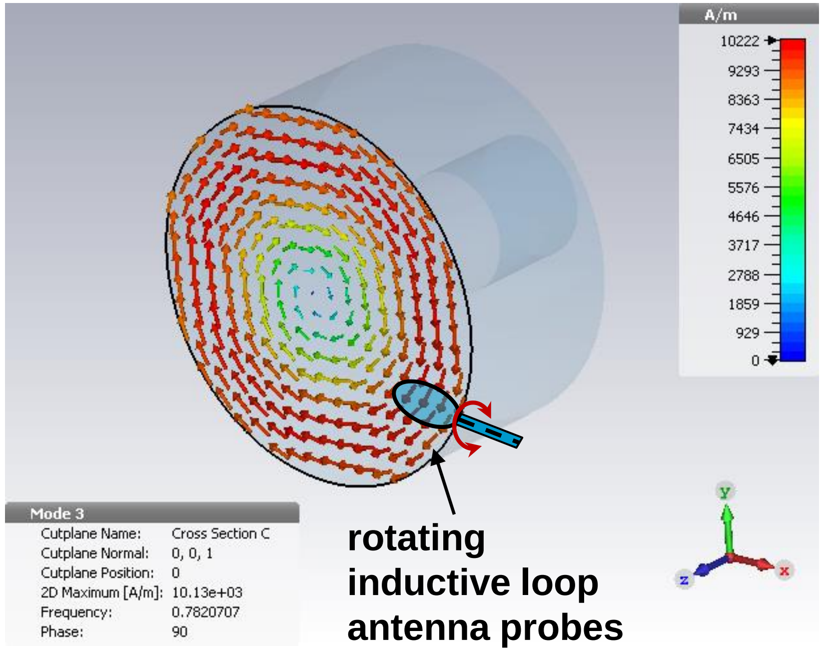


* Characterize the accelerating TM_{010} mode of a cylindrical cavity with beam ports
  – The TM_{010} does not have to be the lowest frequency mode
* Compare the measured values of f_{res}, Q_0 and R/Q
  – with an analytical analysis of a perfect cylinder (no beam ports)
  – with a numerical analysis

* normal conducting!

Excite the Modes while Measuring S_{11}

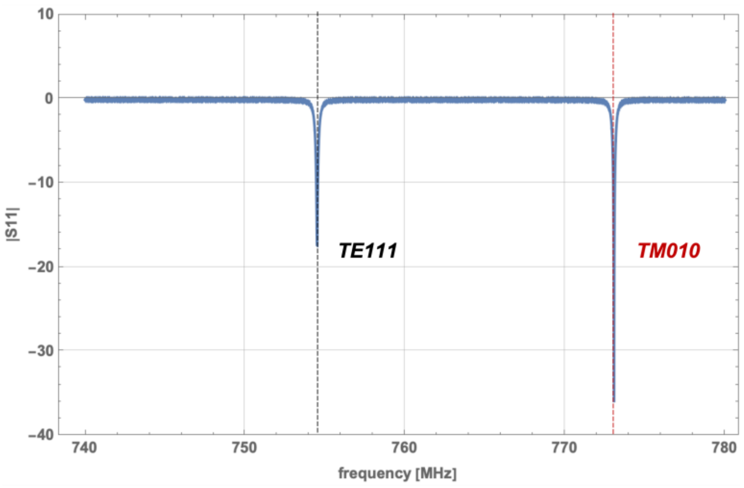


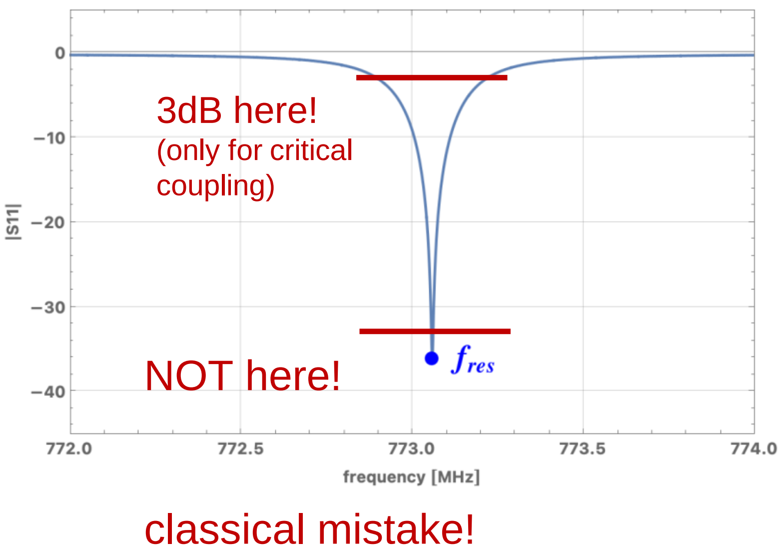


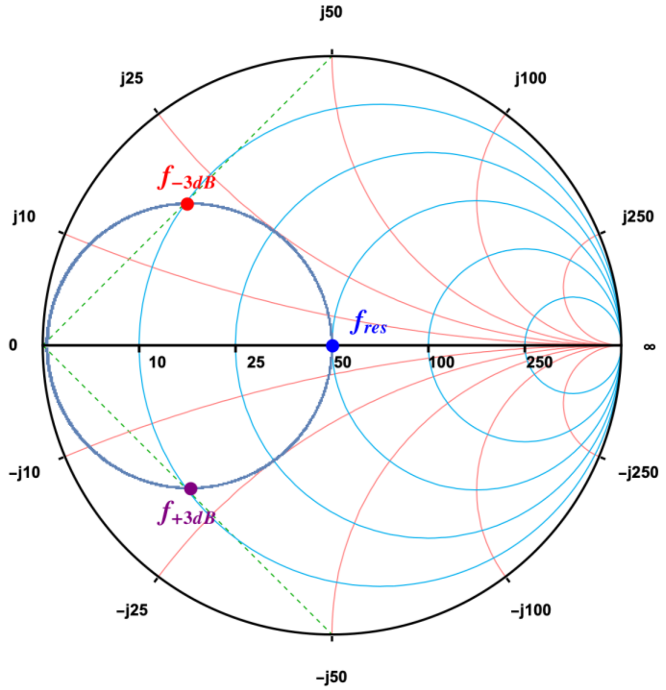
rotating
inductive loop
antenna probes
the H-field

- S_{11} **measurement with tunable coupling antenna**
  - **E-field on z-axis using a capacitive coupling pin**
    - Center pin, e.g., of semi-rigid coaxial cable
  - **H-field on the cavity rim using an inductive coupling loop**
    - Bend the center conductor to a closed loop connected to ground

Measurement of Frequency and Q-value

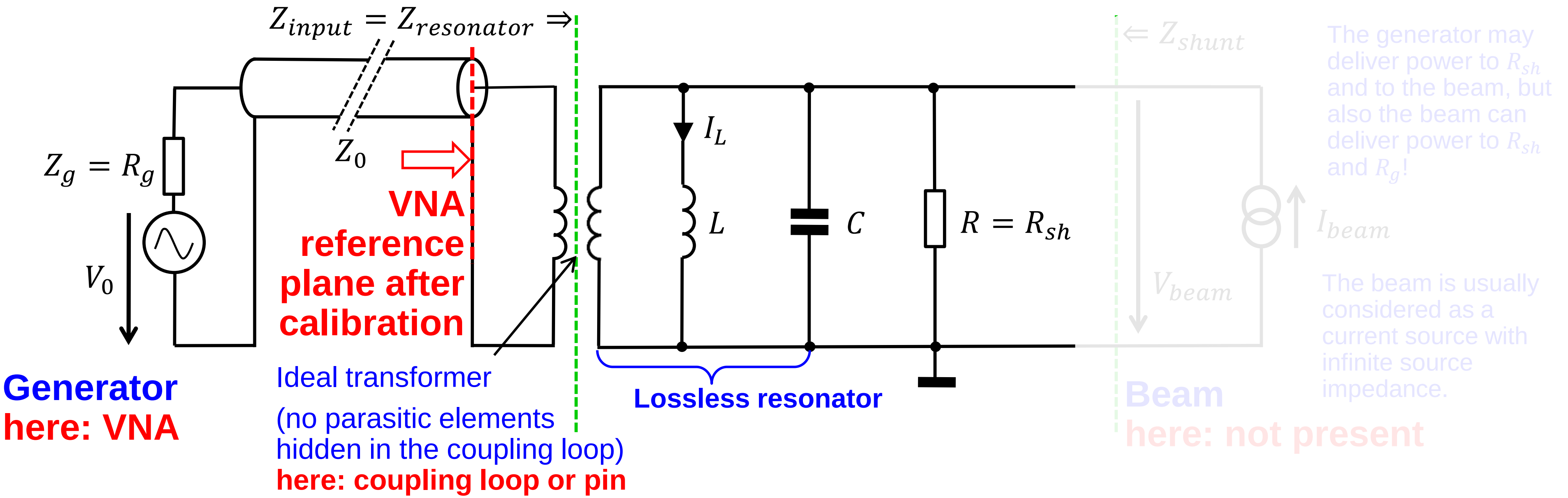






- **Identify the correct (TM_{010}) mode frequency**
  - **Introduce a small perturbation, e.g., metallic rod or wire on the z-axis, and observe the shift of the mode frequencies**
- **Calibrate the VNA and measure S_{11}**
  - **Tune the coupling loop for critical coupling**
  - **Display the resonant circle in the *Smith* chart using enough points!**

The Equivalent Circuit of a Resonant Mode



$R = R_{sh}$: shunt resistor, representing the losses of the resonator

We have resonance condition, when $\omega L = \frac{1}{\omega C}$

→ Resonance frequency: $\omega_{res} = 2\pi f_{res} = \frac{1}{LC} \Rightarrow f_{res} = \frac{1}{2\pi\sqrt{LC}}$

Useful Formulas of the Equivalent Circuit

- **Characteristic impedance "R over Q"** $X = \frac{R}{Q} = \omega_{res} L = \frac{1}{\omega_{res} C} = \sqrt{L/C}$
- **Stored energy at resonance** $U = U_e + U_m = \frac{1}{4}|V_C|^2 C + \frac{1}{4}|I_L|^2 L$
  - V_C ... Voltage at the capacitor
  - I_L … Current in the inductor
- **Dissipated power** $P = \frac{V^2}{2\,R}$
- **Q-factor** $Q = \frac{R}{X} = \frac{\omega_{res}\,U}{P}$
  - U ... stored energy
  - P ... dissipated power over 1 period
- **Shunt impedance (circuit definition)** $R = \frac{V^2}{2\,P}$
  - $\frac{R}{Q} = \frac{V^2}{2\omega_{res}\,U}$
- **Tuning sensitivity** $\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta C}{C} = -\frac{1}{2}\frac{\Delta L}{L}$
- **Coupling parameter (shunt impedance over generator or feeder impedance)** $k^2 = \frac{R}{R_{input}}$
  - tune for critical coupling

The Quality Factor (Q-Factor)

- **The quality (Q) factor of a resonant circuit is defined as ratio of the stored energy U over the energy dissipated P in one oscillation cycle:**

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy dissipated in 1 cycle}} = \frac{\omega_{res} U}{P}$$

- **The Q-factor of an impedance loaded resonator:**
  - Q_0: unloaded Q-value of the unperturbed system
  - Q_L: loaded Q-value, e.g., measured with the impedance of the connected generator
  - Q_{ext}: external Q-factor, representing the effects of the external circuit (generator and coupling circuit)
- **Q-factor and bandwidth**
  - This is how we actually "measure" the Q-factor!

$$Q = \frac{f_{res}}{f_{BW}}$$

with: $f_{BW} = f_{+3dB} - f_{-3dB}$

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

tune k for critical coupling:

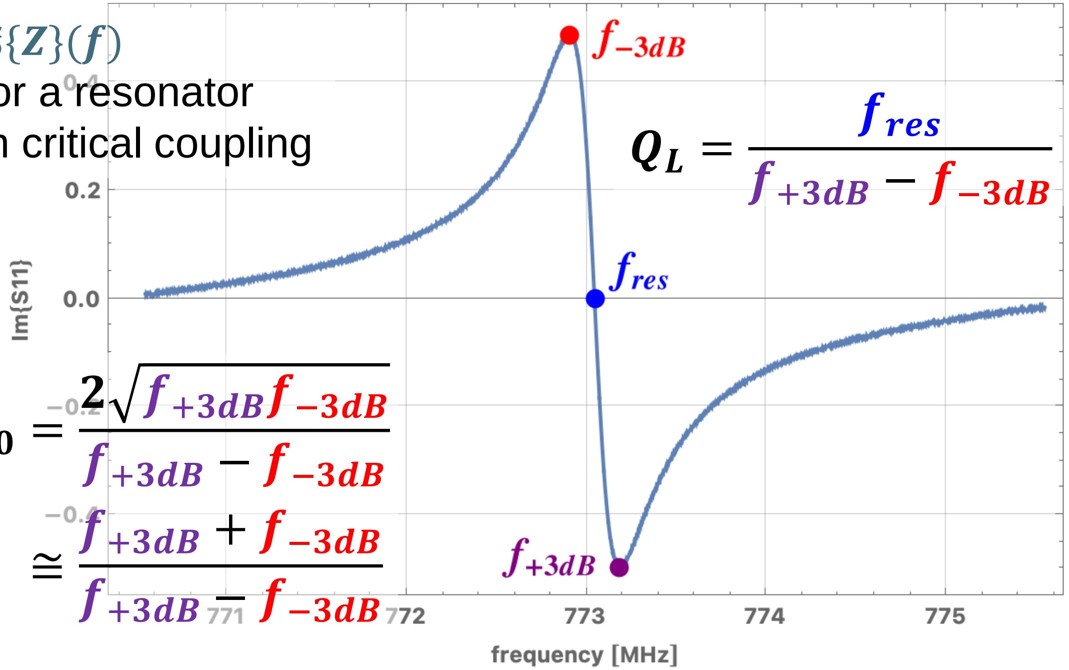
$Q_0 = Q_{ext}$

$\Rightarrow Q_0 = 2\,Q_L$

With Q_L being our measured Q-value

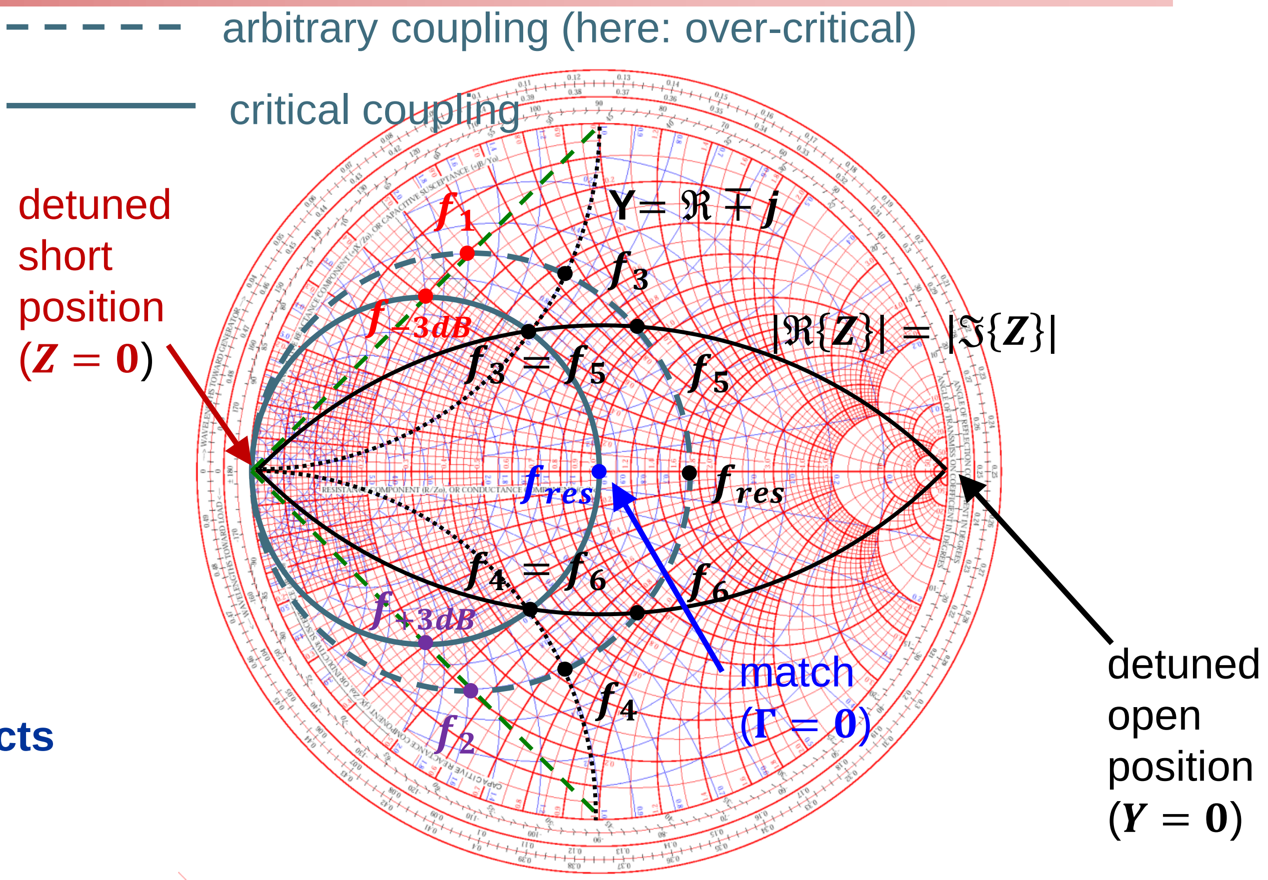
Q-factor from S_{11} Measurement

$\Im\{Z\}(f)$ for a resonator in critical coupling



$$Q_L = \frac{f_{res}}{f_{+3dB} - f_{-3dB}}$$

$$Q_0 = \frac{2\sqrt{f_{+3dB} f_{-3dB}}}{f_{+3dB} - f_{-3dB}} \cong \frac{f_{+3dB} + f_{-3dB}}{f_{+3dB} - f_{-3dB}}$$



- **Correct for the uncompensated transmission-line effects between calibration reference and the coupling loop**
  - **Electrical length adjustment: "straight" $\Im\{Z\}(f)$**
- **Adjust the locus circle to the detuned short location**
  - **Phase offset**
- **Verify no evanescent fields penetrating outside the beam ports**
  - **i.e., no frequency shifts if the boundaries at the beam ports are altered**

Frequency marker points in the *Smith* chart:

$f_{1,2}$ (f_{-3dB}, f_{+3dB}): $|\Im\{S_{11}\}| = max.$ to calculate Q_L

$f_{3,4}$: $Y = \Re \mp j$ to calculate Q_{ext}

$f_{5,6}$: $|\Re\{Z\}| = |\Im\{Z\}|$ to calculate Q_0

R/Q Measurement – *Slater*'s Theorem

- **Remember from the equivalent circuit:**

$$\frac{R}{Q}=\frac{V_{acc}^2/2P_d}{\omega_{res}U/P_d}=\frac{V_{acc}^2}{2\omega_{res}U}$$

with: $V_{acc}=\left|\int E_z(z)\cos\left(\frac{\omega z}{\beta c_0}\right)dz\right|$ (transit time related)

  - V_{acc} is based on the integrated longitudinal E-field component E_z along the z-axis ($x=y=0$)

- **Based on Slater's perturbation theorem:**

$$\frac{\Delta f}{f_{res}}=\frac{1}{U}\left[\mu_0\left(k_\parallel^H|H_\parallel|^2+k_\perp^H|H_\perp|^2\right)-\varepsilon_0\left(k_\parallel^E|E_\parallel|^2+k_\perp^E|E_\perp|^2\right)\right]$$

  - Resonance frequency shift due to a small perturbation object, expressed in longitudinal and transverse E and H field components
  - k: coefficients proportional to the electric or magnetic polarizability of the perturbation object (here: only $k_\parallel^E$ for a longitudinal metallic object)

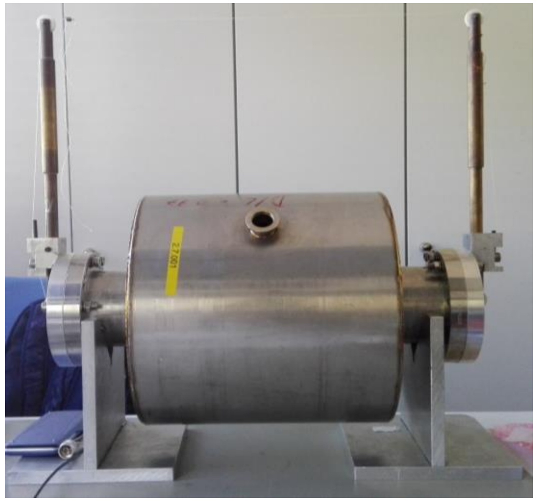
- **E-field characterization along the z-axis**

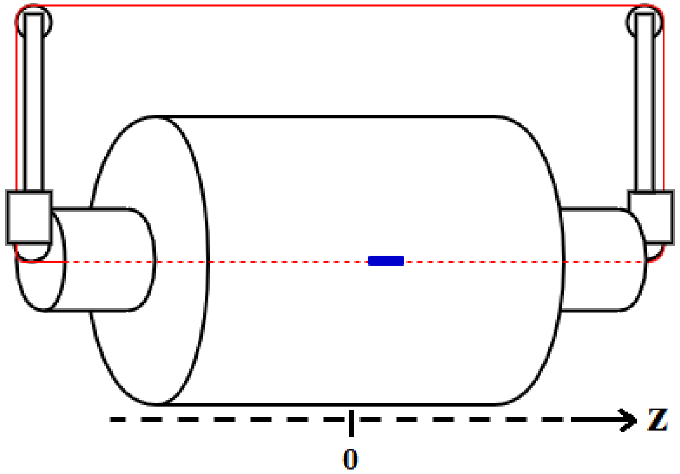
$$E(z)=E_\parallel(z)=\sqrt{U\frac{\Delta f(z)}{f_{res}}\cdot\frac{-1}{k_\parallel^E\varepsilon_0}}$$

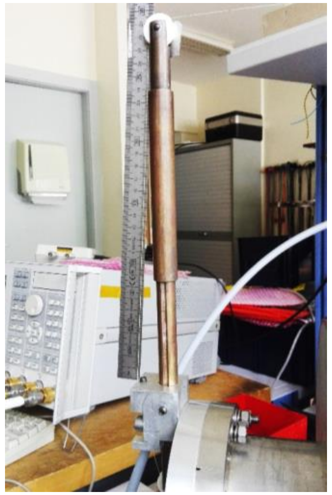
with: $k_\parallel^E=\frac{\pi}{3}l^3\left[\sinh^{-1}\left(\frac{2}{3\pi}\frac{l}{a}\right)\right]^{-1}$

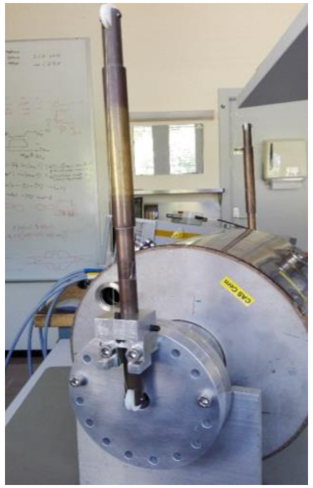
(metallic ellipsoid, e.g., syringe needle of half length l and radius a)

R/Q Measurement - Bead Pull Method









- **E-field characterization by evaluating**
  - The frequency shift Δf (S_{11} reflection measurement with a single probe) or
  - The phase shift ϕ at f_{res} (S_{21} transmission measurement with 2 probes)

$$\frac{\Delta f}{f_{res}} = \frac{1}{2\,Q_0} \tan \phi$$

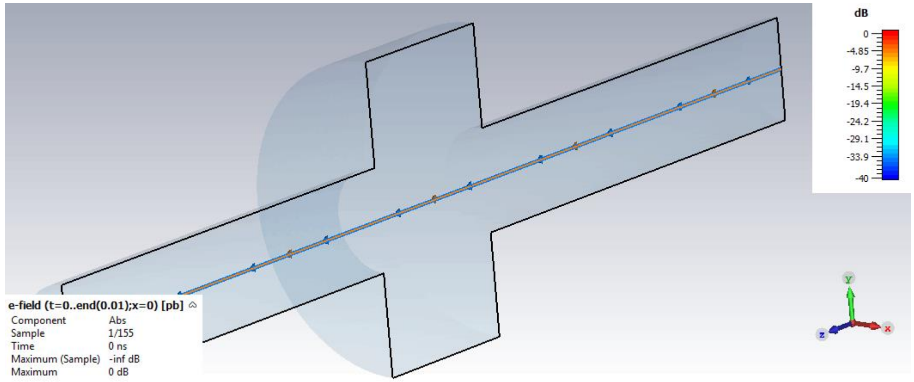
- **Exercise with a manual bead-pull through a known cavity**
  - requires: fishing wire, syringe needle, ruler and VNA
  - Compare the measured E_z at the maximum f or ϕ shift (in the center of the cavity) with the theoretical estimation (e.g., numerical computed value)

Beam Coupling Impedance

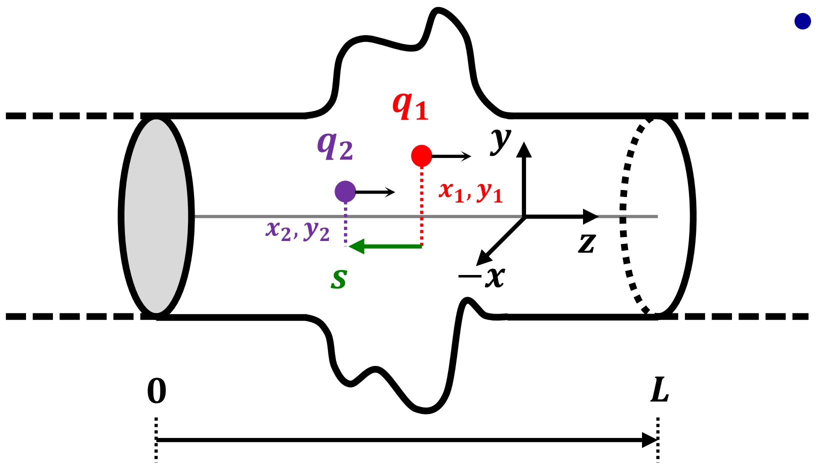
- **The wake potential**
  - **Lorenz force on q_2 by the wake field of q_1:**

$$\vec{F} = \frac{d\vec{p}}{dt} = q_2\left(\vec{E} + c_0\vec{e}_z \times \vec{B}\right)$$

  - **Wake potential of a structure, e.g., a discontinuity driven by q_1**



$$\vec{w}(x_1, y_1, x_2, y_2, s) = \frac{1}{q_1}\int_{-\infty\,(\text{or }0)}^{+\infty\,(\text{or}L)} dz\left[\vec{E}(x_2, y_2, z, t) + c_0\vec{e}_z \times \vec{B}(x_2, y_2, z, t)\right]_{t=(s+z)/c}$$

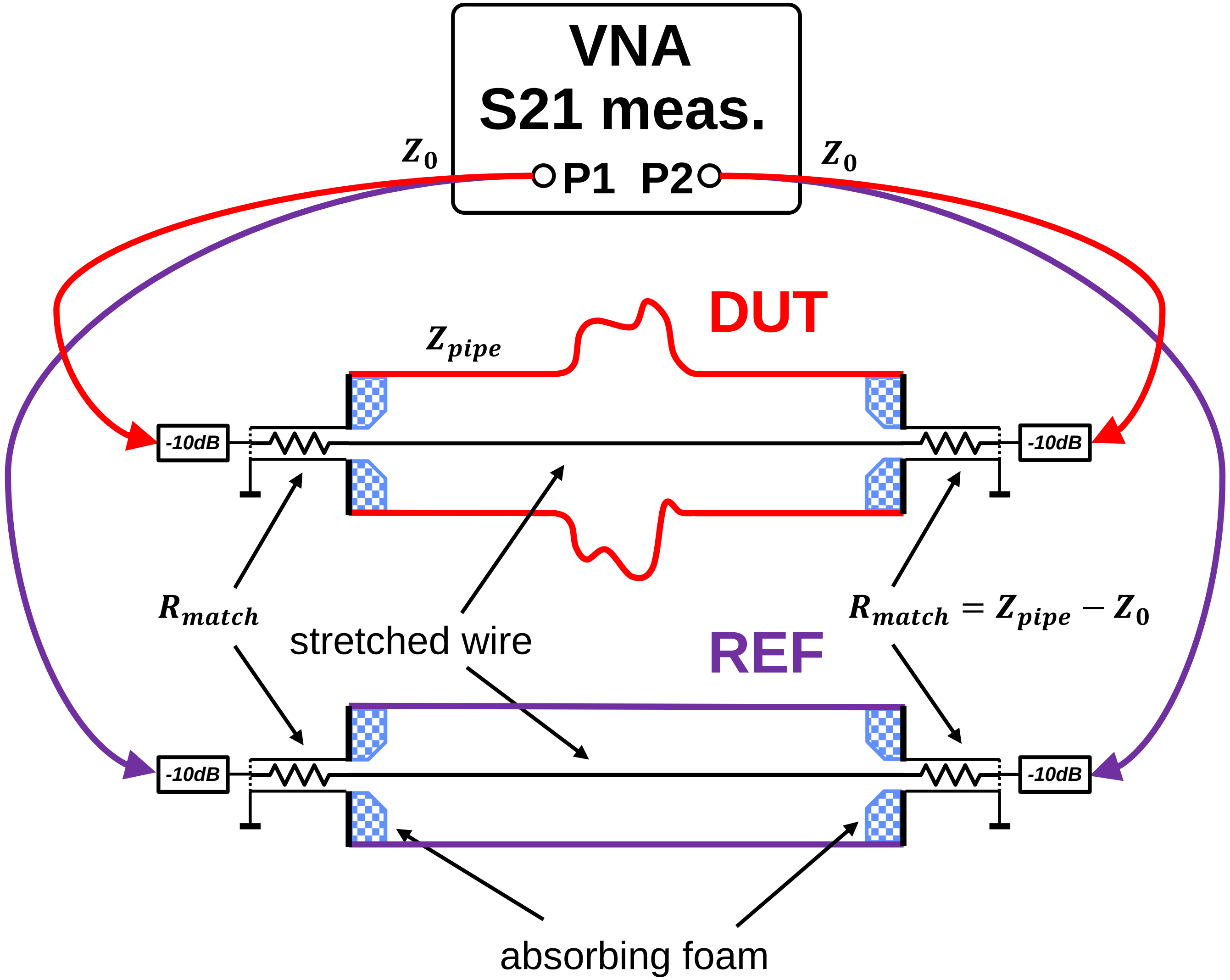


- **Beam coupling impedance**
  - **Frequency domain representation of the wake potential**

$$Z(x_1, y_1, x_2, y_2, \omega) = -\frac{1}{c_0}\int_{-\infty}^{+\infty} ds\,\vec{w}(x_1, y_1, x_2, y_2, s)e^{-j\omega s/c_0}$$

  - **Can be decomposed in longitudinal $Z_{\parallel}$ and transverse $Z_{\perp}$ components (*Panofsky-Wenzel* theorem)**
  - **Resonant structures, i^{th} mode:** $R_{sh,i} = Z_{\parallel,i} = \dfrac{2k_{loss,i}Q_i}{\omega_i}$

Stretched-Wire $Z_{\|}$ Measurement



- **Formulas:**
  - **Normalized electrical length:** $\Theta = 2\pi \frac{L}{\lambda}$
  - **Lumped impedance formula**

$$Z_{\|} = 2Z_{pipe} \frac{1 - S_{21}}{S_{21}} \quad \Theta \leq 1 \quad L < D_{pipe}$$

  - **Log formula**

$$Z_{\|} = -2Z_{pipe} \ln S_{21}$$

  - **Improved log formula**

$$Z_{\|} = -2Z_{pipe} \ln S_{21} \left(1 + j \frac{\ln S_{21}}{2\Theta}\right)$$

  - **Transmission coefficient**

$$S_{21} = \frac{S_{21,DUT}}{S_{21,REF}}$$

  - **Circular beam pipe impedance**

$$Z_{pipe} = \frac{\eta_0}{2\pi\sqrt{\varepsilon_r}} \ln \frac{D}{d} \cong 60\ \Omega \ln \frac{D_{pipe}}{d_{wire}}$$

Summary

- **No summary, just thank you for listening!**
- **Also, a big THANK YOU to all the help from the hands-on instructors!**
- **More THANKS to *Dassault / CST*, *SIMUSERV (Frank)* and *Computer Controls / Keysight***
- **THANK YOU to my colleagues at CERN, in particular *Joel D*.!**

Thank you!

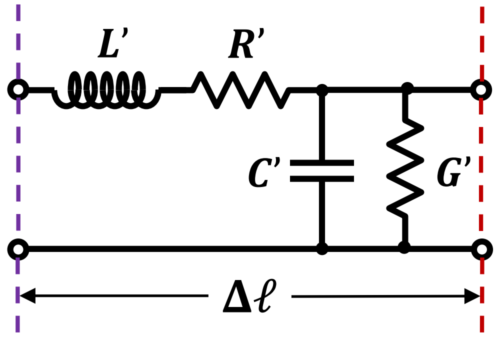
Backup Slides

Refresher: Some TL Equations (1)

- **Characteristic impedance**
  - **for a TEM transmission-line**
    - with losses

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \quad [\Omega]$$

Equivalent circuit TEM TL segment



- lossless, non-magnetic media ($\mu_r = 1$)

$$\boldsymbol{Z_0 \cong \sqrt{\frac{L'}{C'}}} \quad [\Omega]$$

$$Z_0 \cong \frac{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}}{C'} = \frac{\sqrt{\varepsilon_r}}{cC'} = \frac{1}{v_p C'} \quad [\Omega]$$

The characteristic impedance can be calculated from 2D electrostatic equations

$$Z_0 \cong \frac{L'}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}} = \frac{cL'}{\sqrt{\varepsilon_r}} = v_p L' \quad [\Omega]$$

- **Propagation constant**
  - **for a TEM transmission-line**

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

- attenuation constant

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} - 1\right)} \quad [\frac{Np}{m}]$$

- phase constant

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} + 1\right)} \quad [\frac{rad}{m}]$$

$$\boldsymbol{\beta = \frac{2\pi}{\lambda_g}} \quad [\frac{rad}{m}]$$

$$\beta = \omega\sqrt{L'C'}$$

Refresher: Some TL Equations (2)

- **Wave impedance**

in media

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \quad [\Omega]$$

Characteristic impedance of free space

$$\boldsymbol{\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cong 120\pi} \ [\Omega] \cong \mathbf{377} \ [\Omega]$$

- **Speed of light**

$$\boldsymbol{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \cong 2.997925 \cdot 10^8} \quad [\frac{m}{s}]$$

- **Phase velocity**

$$v_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \quad [\frac{m}{s}]$$

  – non-magnetic media ($\mu_r = 1$)

$$\boldsymbol{v_p \cong \frac{c}{\sqrt{\varepsilon_r}}} \quad [\frac{m}{s}]$$

- **Wavelength**

in free space

$$\boldsymbol{\lambda_0 = \lambda = \frac{c}{f}} \quad [m]$$

guide wavelength (in media)

$$\lambda_g = \frac{c}{f\sqrt{\mu_r \varepsilon_r}} = \frac{\lambda_0}{\sqrt{\mu_r \varepsilon_r}} \quad [m]$$

  – non-magnetic media: ($\mu_r = 1$)

$$\boldsymbol{\lambda_g \cong \frac{c}{f\sqrt{\varepsilon_r}}} \quad [m]$$

- **Electrical length**

  – for a TEM line of physical length ℓ

$$\boldsymbol{\theta = \beta\ell = 2\pi\frac{\ell}{\lambda_g}} \quad [rad]$$

$$\theta = \beta\ell = 360\frac{\ell}{\lambda_g} \quad [deg]$$

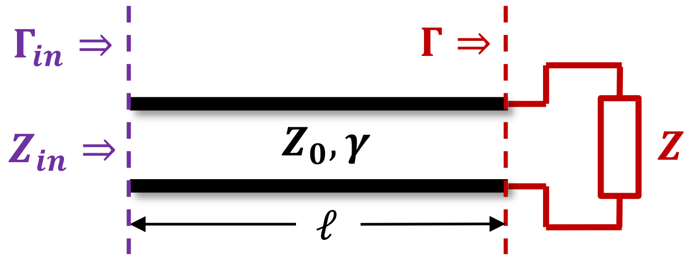
- **Permeability:** $\boldsymbol{\mu = \mu_0 \mu_r}$ $\quad \mu_0 \cong 4\pi \cdot 10^{-7} \quad [H/m]$

- **Permittivity:** $\boldsymbol{\varepsilon = \varepsilon_0 \varepsilon_r}$ $\quad \varepsilon_0 \cong 8.854 \cdot 10^{-12} \quad [F/m]$

Lossless Transmission-lines

- **Transmission-line terminated with an arbitrary impedance Z:**

$$Z_{in} = Z_0 \frac{1+\Gamma_{in}}{1-\Gamma_{in}} \quad \text{with: } \Gamma_{in} = \Gamma e^{-2\gamma\ell} \quad \text{and: } \Gamma = \frac{Z-Z_0}{Z+Z_0}$$



$$\boxed{Z_{in} = Z_0 \left(\frac{Z \cosh\gamma\ell + Z_0 \sinh\gamma\ell}{Z_0 \cosh\gamma\ell + Z \sinh\gamma\ell} \right) = Z_0 \frac{Z + Z_0 \tanh\gamma\ell}{Z_0 + Z \tanh\gamma\ell} \quad [\Omega]}$$

- **Lossless transmission-line:**

$\alpha = 0 \quad \Rightarrow$

$$\boxed{\boldsymbol{Z_{in} = Z_0 \left(\frac{Z \cos\beta\ell + jZ_0 \sin\beta\ell}{Z_0 \cos\beta\ell + jZ \sin\beta\ell} \right) \quad [\Omega]}}$$

– **Popular applications**
  - Quarter-wave line: $\ell = \frac{\lambda}{4} \Rightarrow \theta = \frac{\pi}{2} \Rightarrow \boxed{Z_{in} = \frac{Z_0^2}{Z}}$
  - Terminated (matched) line: $Z = Z_0 \Rightarrow \boxed{Z_{in} = Z_0}$
  - Open line: $Z \to \infty \Rightarrow \boxed{Z_{in} = -j\,Z_0 \cot\beta\ell}$
  - Shorted line: $Z = 0 \Rightarrow \boxed{Z_{in} = j\,Z_0 \tan\beta\ell}$

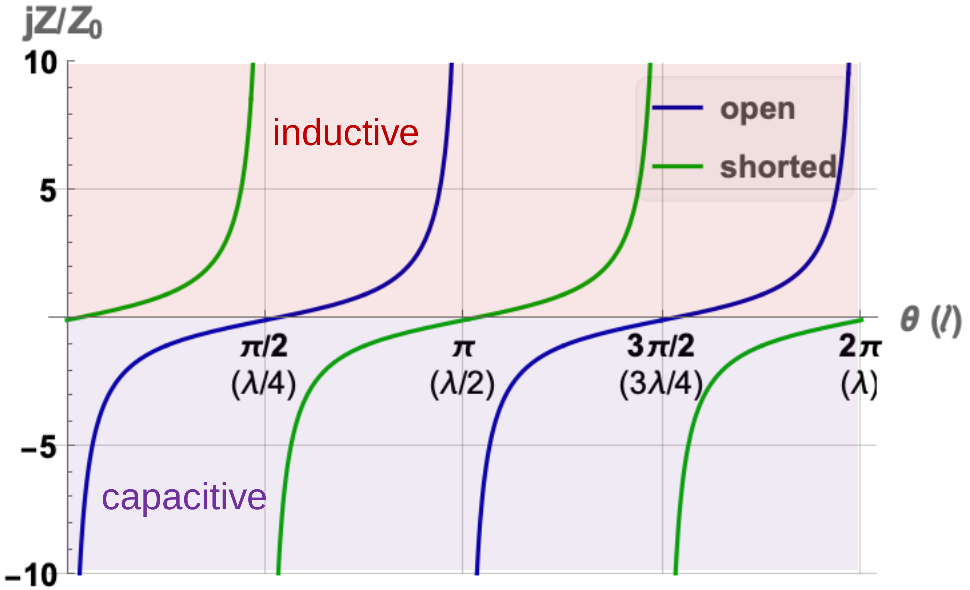
Lossless Transmission-lines

$$Z_{in,open} = -j\,Z_0 \cot\beta\ell$$

$$Z_{in,short} = j\,Z_0 \tan\beta\ell$$

physical length ℓ / electrical length $\theta = \beta\ell$	$0 < \ell < \lambda_g/4$ / $0 < \theta < \pi/2$	$\lambda_g/4 < \ell < \lambda_g/2$ / $\pi/2 < \theta < \pi$	$\lambda_g/2 < \ell < 3\lambda_g/4$ / $\pi < \theta < 3\pi/2$	$3\lambda_g/4 < \ell < \lambda_g$ / $3\pi/2 < \theta < 2\pi$
lossless TL open	"capacitive"	"inductive"	"capacitive"	"inductive"
lossless TL shorted	"inductive"	"capacitive"	"inductive"	"capacitive"

- **A lossless TL with open ($Z = 0$) or shorted ($Z \to \infty$) termination can approximate a lumped reactive element (capacitor or inductor)**
  - **A "capacitive" element has the form:** $Z_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$
  - **An "inductive" element has the form:** $Z_L = j\omega L$
    - A more precise method follows a "T" or "π" LCL or CLC equivalent circuit of the lossless TL.
    - In case of $\theta \ll 1$, we can simplify $\tan\theta \cong \theta$, etc.
    - In practice it is useful to select a low Z_0 for a capacitive, or a high Z_0 for an inductive semi-lumped element approximation

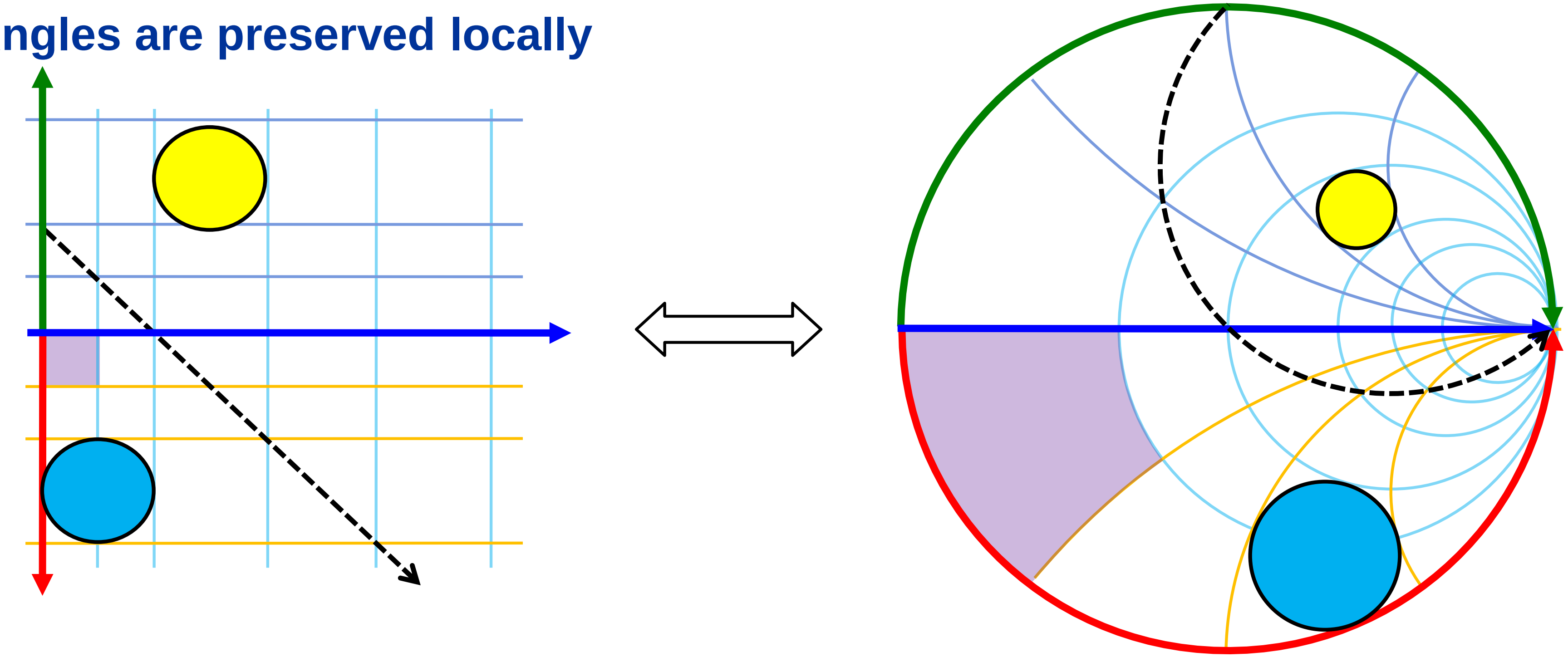


Navigation in the *Smith* Chart (1)

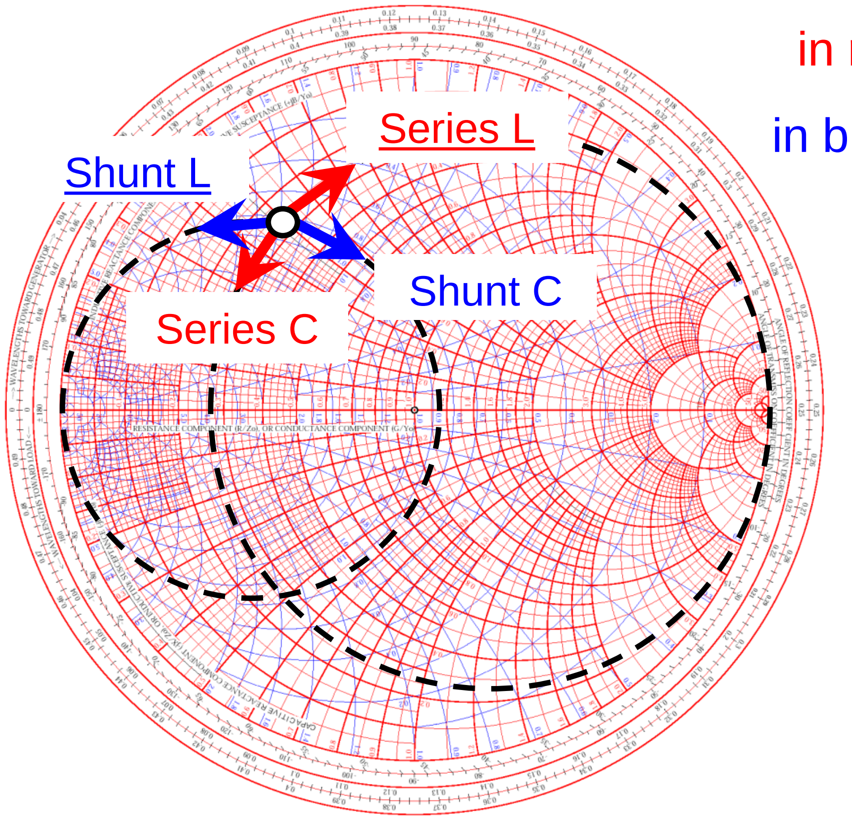
This is a "bilinear" transformation with the following properties:

- **Generalized circles are transformed into generalized circles**
  - **circle → circle**
  - **straight line → circle**
  - **circle → straight line**
  - **straight line → straight line**
- **Angles are preserved locally**

> - a straight line is equivalent to a circle with infinite radius
> - a circle is defined by 3 points
> - a straight line is defined by 2 points



Navigation in the *Smith* Chart (2)

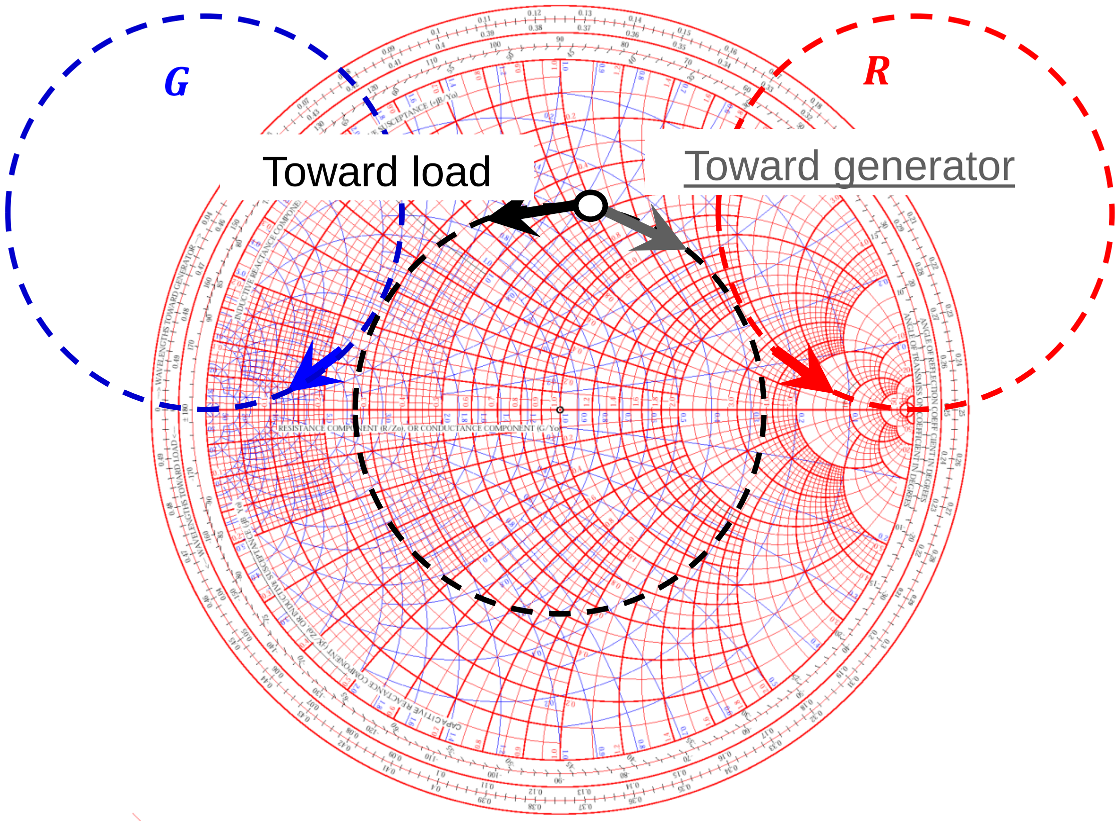


in red: impedance plane $(= z)$

in blue: admittance plane $(= y)$

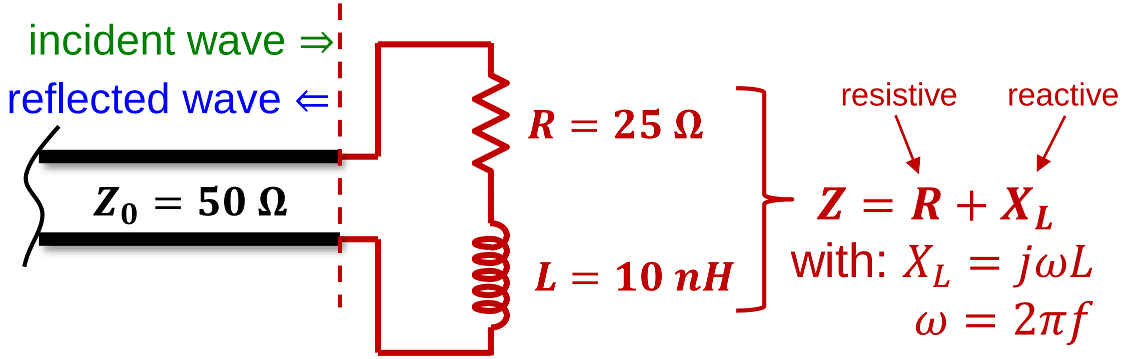
	Up	Down
Red circles	Series L	Series C
Blue circles	Shunt L	Shunt C

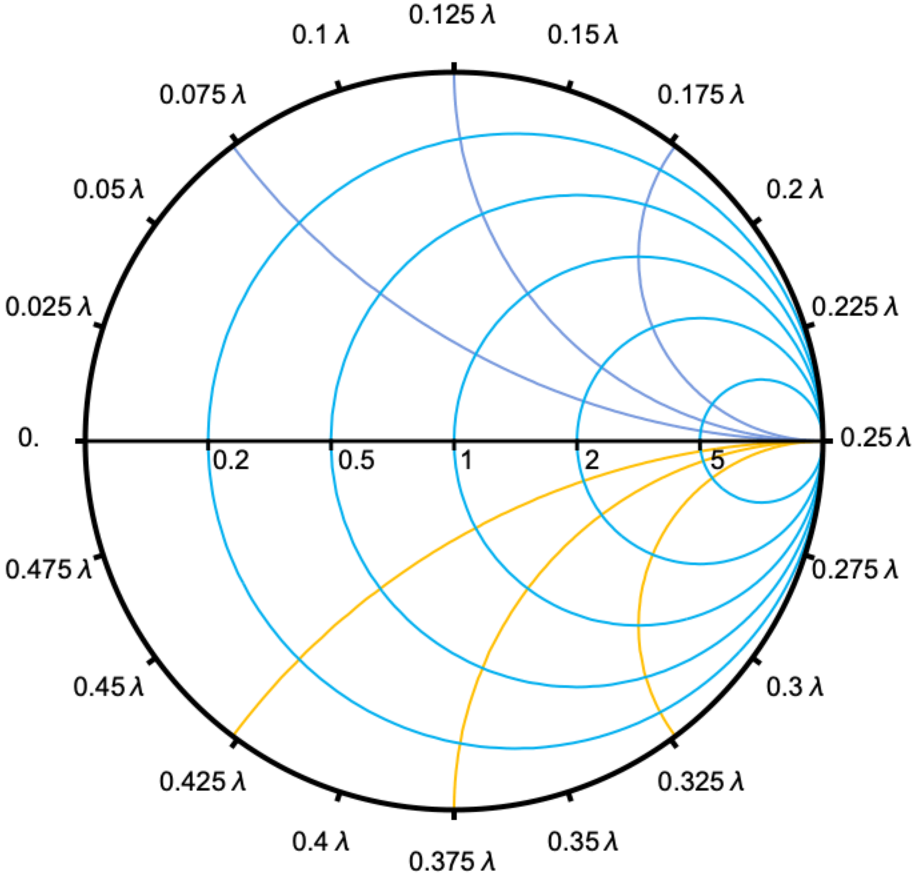
Navigation in the *Smith* Chart (3)



Red arcs	Resistance *R*
Blue arcs	Conductance *G*
Con-centric circle	Transmission line going Toward load Toward generator

The *Smith* Chart – Basic Example (1)

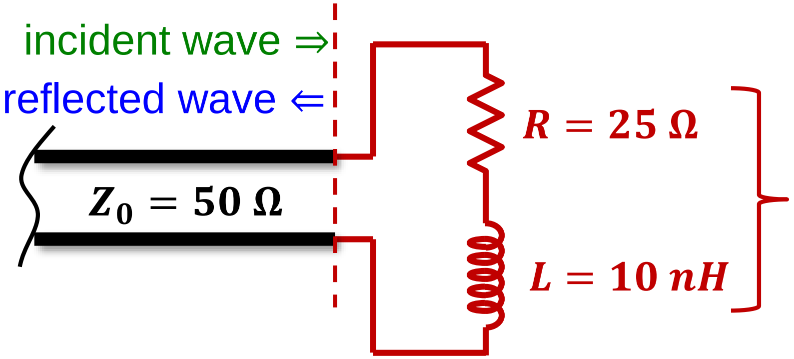




Complex impedance based on lumped element components

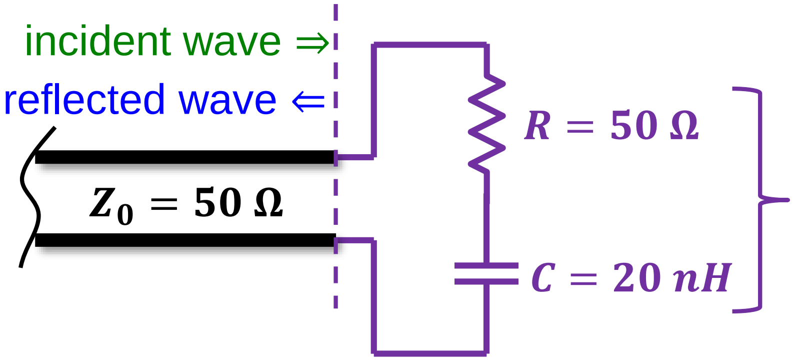
- **Calculate Z for a given frequency, e.g., $f = 50\ MHz$: $Z = (25 + j6.28)\ \Omega$**
- **Calculate the normalized impedance $z = Z/Z_0 = 0.5 + j0.126$**
  - **Locate z in the Smith chart**
  - **Retrieve $\Gamma = 0.34\angle 161° = 0.34 e^{j2.81}$**
- **Repeat for other frequencies…**

The *Smith* Chart – Basic Example (2)

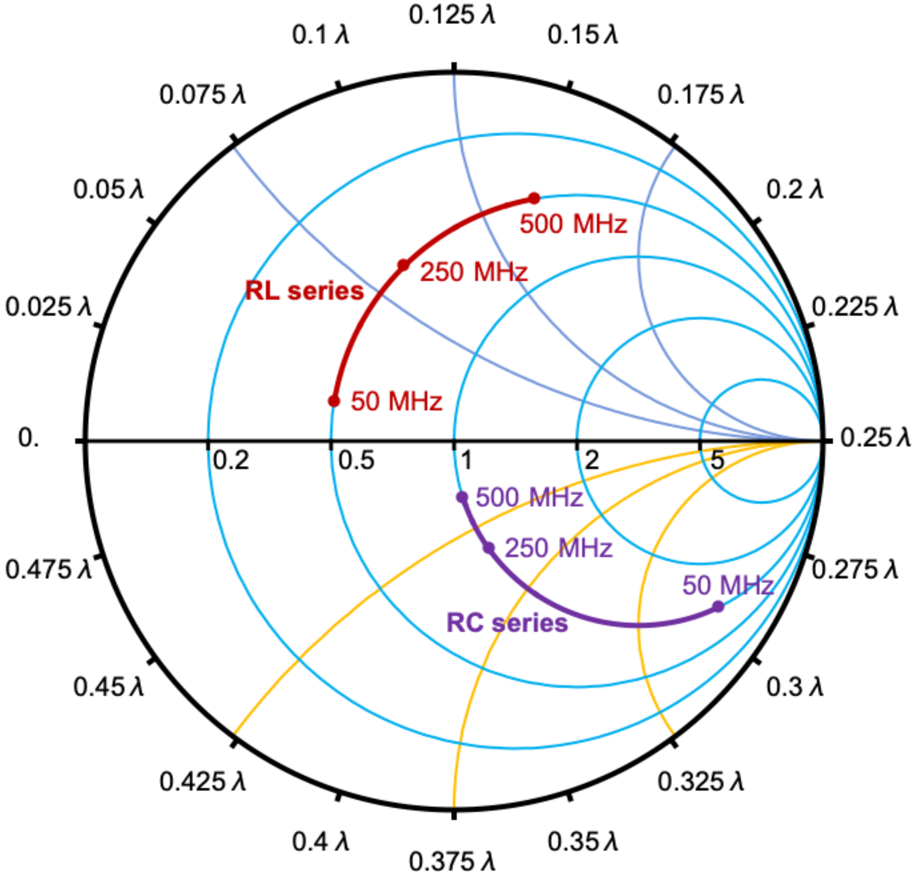


$Z = R + jX_L$
with: $X_L = \omega L$

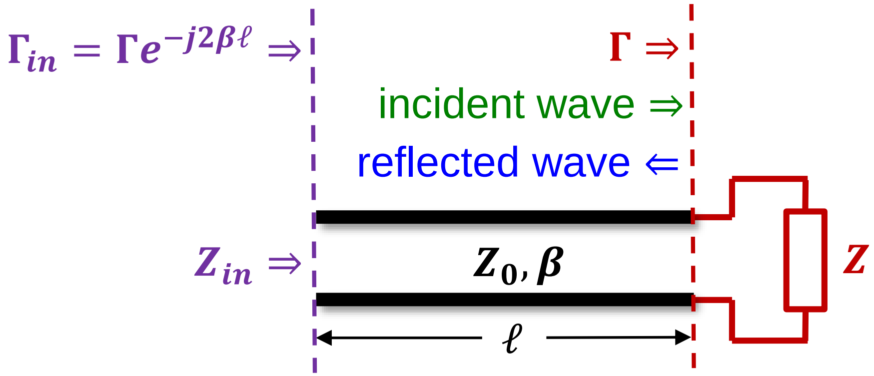
- …and for different component values and circuit combinations



$Z = R + jX_C$
with: $X_C = \frac{-1}{\omega C}$



The *Smith* Chart – TL Transformer (1)



- **S-parameter of a lossless transmission-line:**

$$S = \begin{bmatrix} 0 & e^{-j\beta\ell} \\ e^{-j\beta\ell} & 0 \end{bmatrix}$$

backward transmission coefficient S12

forward transmission coefficient S21

  - **Phase delay (electrical length) of**

$$\theta = \beta\ell$$

with: $\beta = \frac{2\pi}{\lambda_g}$

- **The lossless transmission-line adds a phase delay of $2\beta\ell$, seen at its input, to the reflection coefficient at its output:**

$$\Gamma_{in} = \Gamma e^{-j2\beta\ell}$$

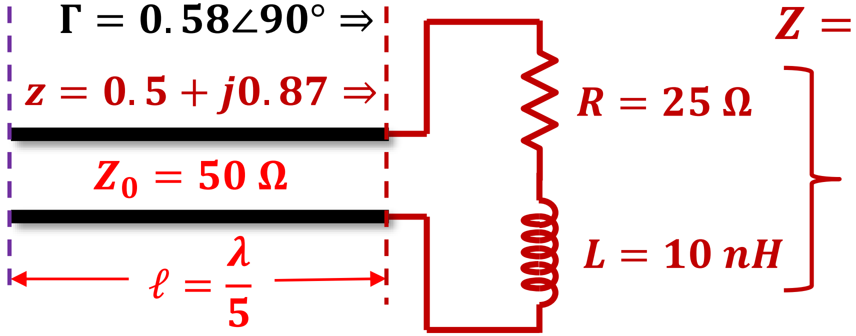
- **This results in a transformation of the impedance Z at the end of the line to a different impedance Z_{in} at the input of the line**
  - **The Smith chart offers an effective, simple graphical way to calculate this transmission-line based impedance transformation**

The *Smith* Chart – TL Transformer (2)

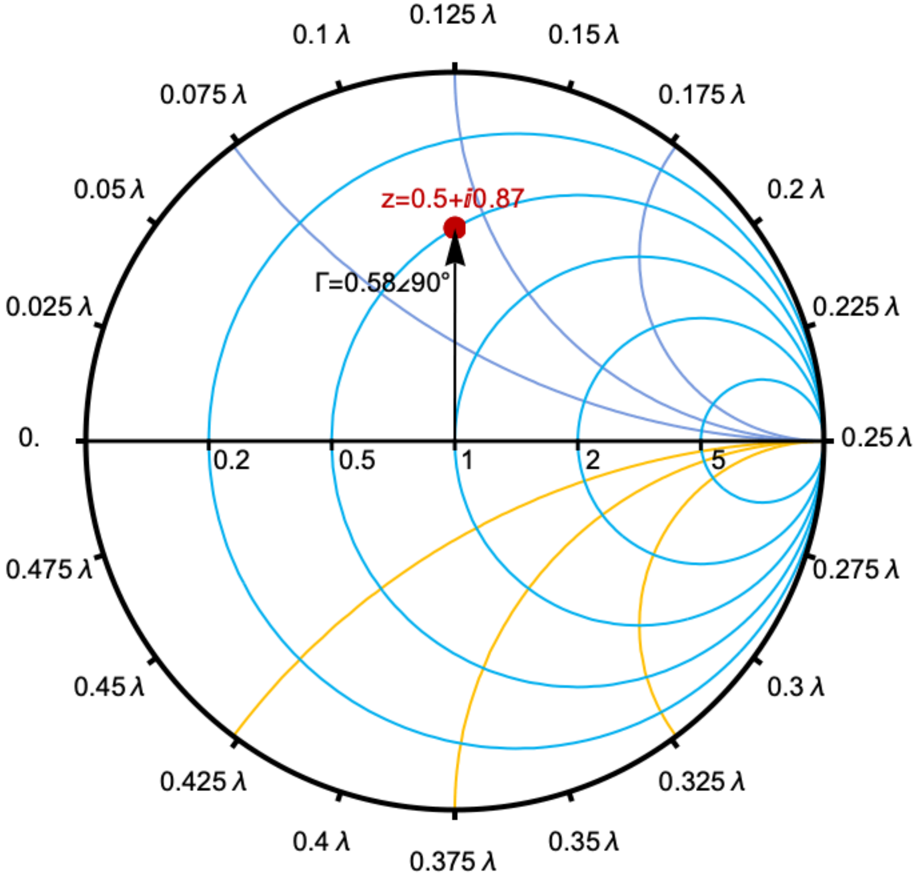
$\Gamma_{in} = 0.58\angle -54° \Rightarrow$ | $\Gamma = 0.58\angle 90° \Rightarrow$

$z_{in} = 1.02 - j1.43 \Rightarrow$ | $z = 0.5 + j0.87 \Rightarrow$

$Z_{in} = (50.8 - j71.3)\ \Omega \Rightarrow$



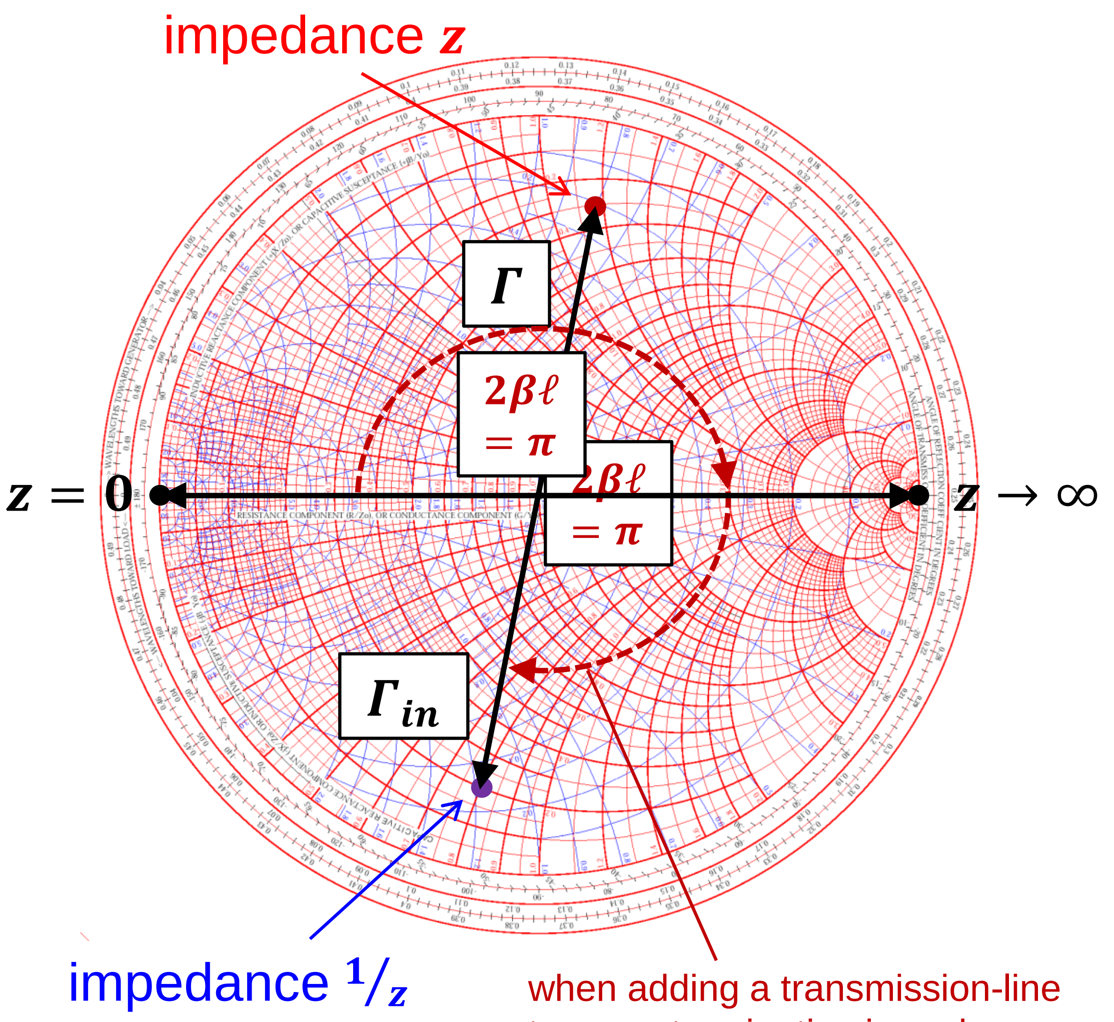
$Z = (25 + j43.4)\ \Omega$ @ $f = 345\ MHz$



Based on the previous example

- **Calculate the normalized impedance z for $f = 345\ MHz$ and locate the point in the Smith chart.**
  - **Notice the corresponding value of Γ, and read the λ-length value on the rim**
- **Add $\ell = \lambda/5$ by rotating Γ by $2\beta\ell = 4\pi/5 \equiv 144°$**
  - **From $0.125\ \lambda$ to $0.325\ \lambda$**
    - The phase is subtracted, therefore clockwise rotation!
- **Notice the value of Γ_{in} and read the corresponding value of the normalized impedance z_{in}**
  - **Calculate the transformed impedance $Z_{in}@f = 345\ MHz$**
  - **What is the equivalent circuit, and what are the compon**
    - …and what is the physical length ℓ of the transmission-line?
      - assuming a coaxial cable as transmission-line with a dielectric constant of $\epsilon_r = 2.1$

The *Smith* Chart – TL Transformer (3)



when adding a transmission-line to some terminating impedance we rotate clockwise through the *Smith*-Chart

- A transmission-line of length

$$\ell = {}^{\lambda}/_{4} \equiv \beta\ell = \frac{\pi}{2}$$

  transforms a reflection Γ at the end of the line to its input as

$$\Gamma_{in} = \Gamma\, e^{-j2\beta\ell} = \Gamma\, e^{-j\pi} = -\Gamma$$

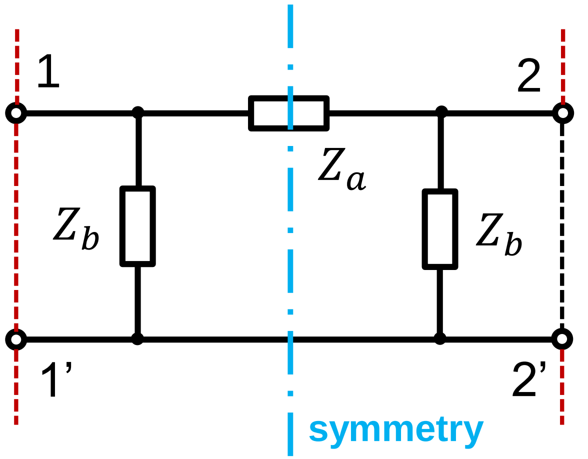
- This results the unitless, normalized impedance z at the end of the line to be transformed into:

$$z_{in} = {}^{1}/_{z}$$

  at the beginning of the line
- A short circuit at the end of the $\lambda/4$-transformer is transformed to an open, and vice versa
  - This is the principle of the $\lambda/4$-resonator.

Examples for Symmetry and Reciprocity

π-network

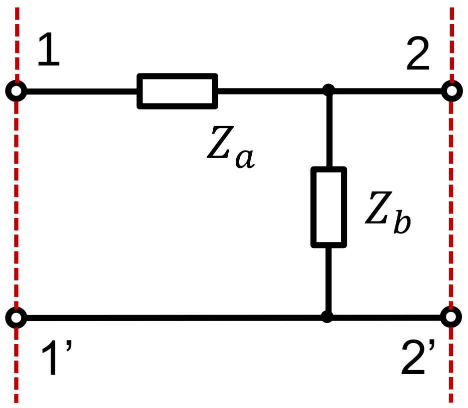


$$(S_\pi) = \frac{1}{\Delta}\begin{pmatrix} Z_aZ_b^2 - Z_0^2(Z_a+2Z_b) & 2Z_0Z_b^2 \\ 2Z_0Z_b^2 & Z_aZ_b^2 - Z_0^2(Z_a+2Z_b) \end{pmatrix}$$

with: $\Delta = (Z_a + Z_b)[Z_aZ_b + Z_0(Z_a + 2Z_b)]$

$S_{12} = S_{21} \wedge S_{11} = S_{22} \Rightarrow$ reciprocal and symmetric

divider-network



$$(S_{div}) = \frac{1}{\Delta}\begin{pmatrix} Z_aZ_b - Z_0(Z_0 - Z_a) & 2Z_0Z_b \\ 2Z_0Z_b & Z_aZ_b - Z_0(Z_0+Z_a) \end{pmatrix}$$

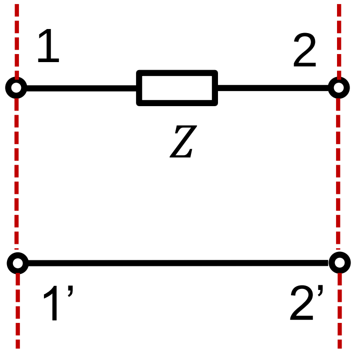
with: $\Delta = Z_0(Z_0 + Z_a) + Z_b(2Z_0 + Z_a)$

$S_{12} = S_{21} \wedge S_{11} \neq S_{22} \Rightarrow$ reciprocal, but not symmetric

- **Without prof: The S-matrix is always symmetric for reciprocal networks.**

Numerical lossless / lossy Examples

2-port series-network



$$(S_{ser}) = \frac{1}{2Z_0 + Z}\begin{pmatrix} Z & 2Z_0 \\ 2Z_0 & Z \end{pmatrix}$$

$$S_{11} = S_{22} \wedge S_{12} = S_{21}$$

$\mathbf{Z = j\omega L = j10}$	$\mathbf{Z = R = 10}$
$\lvert S_{11}\rvert = \sqrt{1 - \lvert S_{12}\rvert^2}$	
$\frac{1}{\sqrt{101}} = \sqrt{1 - \left(\frac{10}{\sqrt{101}}\right)^2}$	$\frac{1}{11} \neq \sqrt{1 - \left(\frac{10}{11}\right)^2}$
$\angle S_{11} - \angle S_{12} = \angle S_{21} - \angle S_{22} - \pi$	
$\tan^{-1}(10) + \tan^{-1}\frac{1}{10} = -\tan\frac{1}{10} - \tan(10) - \pi$	$0 - 0 \neq 0 - 0 - \pi$
⇒ **lossless**	⇒ **lossy**

4-port ideal directional coupler

$$(S_{dc}) = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{3} & j & 0 \\ \sqrt{3} & 0 & 0 & j \\ j & 0 & 0 & \sqrt{3} \\ 0 & j & \sqrt{3} & 0 \end{pmatrix}$$

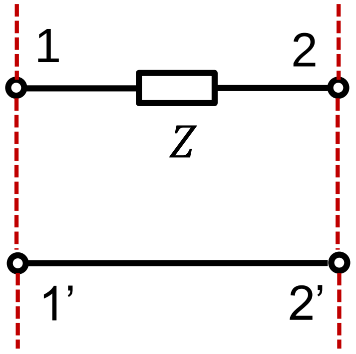
- **It is evident, this ideal 4-port coupler is symmetric and reciprocal**

  $$S_{ij} = S_{ji} \wedge S_{ii} = S_{jj}$$

- **It also is matched:** $S_{ii} = 0$
- **But is it lossless or lossy?**

Numerical lossless / lossy Examples

2-port series-network



$$(S_{ser}) = \frac{1}{2Z_0 + Z}\begin{pmatrix} Z & 2Z_0 \\ 2Z_0 & Z \end{pmatrix}$$

$$S_{11} = S_{22} \wedge S_{12} = S_{21}$$

$\mathbf{Z = j\omega L = j10}$	$\mathbf{Z = R = 10}$
$\lvert S_{11}\rvert = \sqrt{1 - \lvert S_{12}\rvert^2}$	
$\frac{1}{\sqrt{101}} = \sqrt{1 - \left(\frac{10}{\sqrt{101}}\right)^2}$	$\frac{1}{11} \neq \sqrt{1 - \left(\frac{10}{11}\right)^2}$
$\angle S_{11} - \angle S_{12} = \angle S_{21} - \angle S_{22} - \pi$	
$\tan^{-1}(10) + \tan^{-1}\frac{1}{10} = -\tan\frac{1}{10} - \tan(10) - \pi$	$0 - 0 \neq 0 - 0 - \pi$
⇒ **lossless**	⇒ **lossy**

4-port ideal directional coupler

$$(S_{dc}) = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{3} & j & 0 \\ \sqrt{3} & 0 & 0 & j \\ j & 0 & 0 & \sqrt{3} \\ 0 & j & \sqrt{3} & 0 \end{pmatrix}$$

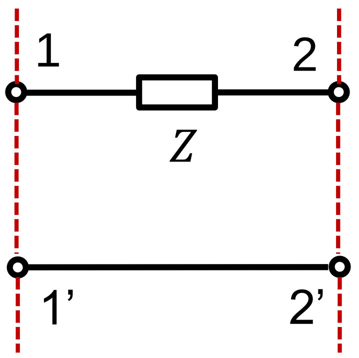
$$(S)^\dagger(S) = (I) \quad \Rightarrow \sum_{k=1}^{N} S_{ki}S_{ki}^* = 1 \;\wedge\; \sum_{k=1}^{N} S_{ki}S_{kj}^* = 0 \;\forall\; i \neq j$$

- **Multiply matrix columns by itself with the conjugate complex**
  - **and test for = 1**

$$S_{11}S_{11}^* + S_{21}S_{21}^* + S_{31}S_{31}^* + S_{41}S_{41}^* = (0\cdot 0 + \sqrt{3}\cdot\sqrt{3} + j\cdot(-j) + 0\cdot 0)/2^2 = 1$$
$$S_{12}S_{12}^* + S_{22}S_{22}^* + S_{32}S_{32}^* + S_{42}S_{42}^* = (\sqrt{3}\cdot\sqrt{3} + 0\cdot 0 + 0\cdot 0 + j\cdot(-j))/2^2 = 1$$
$$S_{13}S_{13}^* + S_{23}S_{23}^* + S_{33}S_{33}^* + S_{43}S_{43}^* = (j\cdot(-j) + 0\cdot 0 + 0\cdot 0 + \sqrt{3}\cdot\sqrt{3})/2^2 = 1$$
$$S_{14}S_{14}^* + S_{24}S_{24}^* + S_{34}S_{34}^* + S_{44}S_{44}^* = (0\cdot 0 + j\cdot(-j) + \sqrt{3}\cdot\sqrt{3} + 0\cdot 0)/2^2 = 1$$

Numerical lossless / lossy Examples

2-port series-network



$$(S_{ser}) = \frac{1}{2Z_0 + Z}\begin{pmatrix} Z & 2Z_0 \\ 2Z_0 & Z \end{pmatrix}$$

$$S_{11} = S_{22} \wedge S_{12} = S_{21}$$

$\mathbf{Z = j\omega L = j10}$	$\mathbf{Z = R = 10}$
$\lvert S_{11}\rvert = \sqrt{1 - \lvert S_{12}\rvert^2}$	
$\frac{1}{\sqrt{101}} = \sqrt{1 - \left(\frac{10}{\sqrt{101}}\right)^2}$	$\frac{1}{11} \neq \sqrt{1 - \left(\frac{10}{11}\right)^2}$
$\angle S_{11} - \angle S_{12} = \angle S_{21} - \angle S_{22} - \pi$	
$\tan^{-1}(10) + \tan^{-1}\frac{1}{10} = -\tan\frac{1}{10} - \tan(10) - \pi$	$0 - 0 \neq 0 - 0 - \pi$
⇒ **lossless**	⇒ **lossy**

4-port ideal directional coupler

$$(\boldsymbol{S_{dc}}) = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{3} & j & 0 \\ \sqrt{3} & 0 & 0 & j \\ j & 0 & 0 & \sqrt{3} \\ 0 & j & \sqrt{3} & 0 \end{pmatrix}$$

$$(S)^{\dagger}(S) = (I) \quad \Rightarrow \sum_{k=1}^{N} S_{ki}S_{ki}^{*} = 1 \;\wedge\; \sum_{k=1}^{N} S_{ki}S_{kj}^{*} = 0 \;\forall\; i \neq j$$

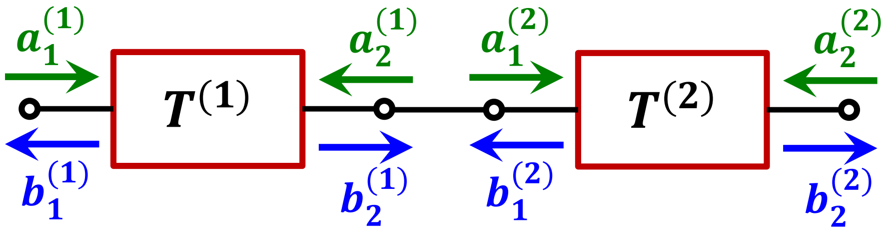
- **Multiply all different matrix columns with the conjugate complex**
  - **and test for = 0**

$$S_{11}S_{12}^{*} + S_{21}S_{22}^{*} + S_{31}S_{32}^{*} + S_{41}S_{42}^{*} = (0\cdot\sqrt{3} + \sqrt{3}\cdot 0 + j\cdot 0 + 0\cdot(-j))/2^2 = 0$$
$$S_{11}S_{13}^{*} + S_{21}S_{23}^{*} + S_{31}S_{33}^{*} + S_{41}S_{43}^{*} = (0\cdot(-j) + \sqrt{3}\cdot 0 + j\cdot 0 + 0\cdot\sqrt{3})/2^2 = 0$$
$$S_{11}S_{14}^{*} + S_{21}S_{24}^{*} + S_{31}S_{34}^{*} + S_{41}S_{44}^{*} = (0\cdot 0 + \sqrt{3}\cdot(-j) + j\cdot\sqrt{3} + 0\cdot 0)/2^2 = 0$$
$$S_{12}S_{13}^{*} + S_{22}S_{23}^{*} + S_{32}S_{33}^{*} + S_{42}S_{43}^{*} = (\sqrt{3}\cdot(-j) + 0\cdot 0 + 0\cdot 0 + j\cdot\sqrt{3})/2^2 = 0$$
$$S_{12}S_{14}^{*} + S_{22}S_{24}^{*} + S_{32}S_{34}^{*} + S_{42}S_{44}^{*} = (\sqrt{3}\cdot 0 + 0\cdot(-j) + 0\cdot\sqrt{3} + j\cdot 0)/2^2 = 0$$
$$S_{13}S_{14}^{*} + S_{23}S_{24}^{*} + S_{33}S_{34}^{*} + S_{43}S_{44}^{*} = (j\cdot 0 + 0\cdot(-j) + 0\cdot\sqrt{3} + \sqrt{3}\cdot 0)/2^2 = 0$$

T-Parameters

- **Cascading e.g., 2-port S-parameter files is important to characterize a larger RF system.**
  - **Solution: Transfer (T) parameters, which directly relates the waves at input and output**

$$(\boldsymbol{T}) = \begin{pmatrix} T_{11} & T_{12} \\ T_{12} & T_{22} \end{pmatrix} \Rightarrow \begin{aligned} b_1 &= T_{11} a_2 + T_{12} b_2 \\ a_1 &= T_{21} a_2 + T_{22} b_2 \end{aligned}$$



  - **T-parameters enable cascaded 2-port networks by simply multiplying their matrices:**

$$(T) = \left(T^{(1)}\right)\left(T^{(2)}\right) \cdots \left(T^{(N)}\right) = \prod_{i=1}^{N} \left(T^{(i)}\right)$$

  - **Relation between 2-port T-parameters and S-parameters:**

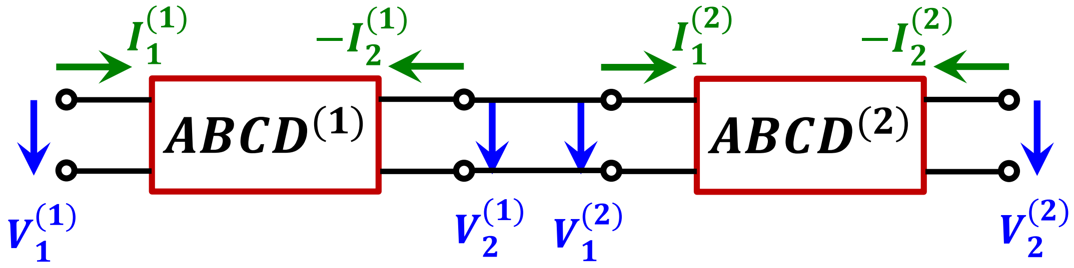
$$(\mathrm{T}) = \frac{1}{S_{21}} \begin{pmatrix} -\det(S) & S_{11} \\ -S_{22} & 1 \end{pmatrix} \qquad (\mathrm{S}) = \frac{1}{T_{22}} \begin{pmatrix} T_{12} & \det(T) \\ 1 & -T_{21} \end{pmatrix}$$

with: $\det(S) = S_{11}S_{22} - S_{12}S_{21}$ with: $\det(T) = T_{11}T_{22} - T_{12}T_{21}$

ABCD-Parameters

- **Also called "chain" parameters, used for cascading networks based on V and I**
  - **Useful for chaining a mix of lumped elements and transmission-lines**

$$(\boldsymbol{ABCD}) = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{pmatrix} \Rightarrow \begin{matrix} \boldsymbol{V_1} = \boldsymbol{A}\boldsymbol{V_2} - \boldsymbol{B}\boldsymbol{I_2} \\ \boldsymbol{I_1} = \boldsymbol{C}\boldsymbol{V_2} - \boldsymbol{D}\boldsymbol{I_2} \end{matrix}$$



  - **Chaining 2-port ABCD-parameter networks:**

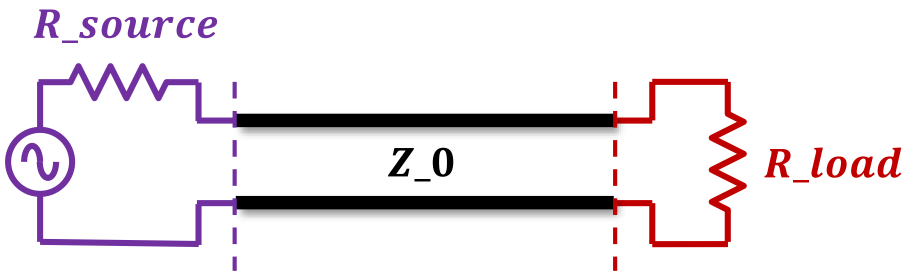
$$(ABCD) = \left(ABCD^{(1)}\right)\left(ABCD^{(2)}\right)\cdots\left(ABCD^{(N)}\right) = \prod_{i=1}^{N}\left(ABCD^{(i)}\right)$$

  - **Relation between 2-port ABCD-parameters and S-parameters:**

$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} \qquad S_{12} = \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D}$$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D} \qquad S_{11} = \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D}$$

Quiz (1)

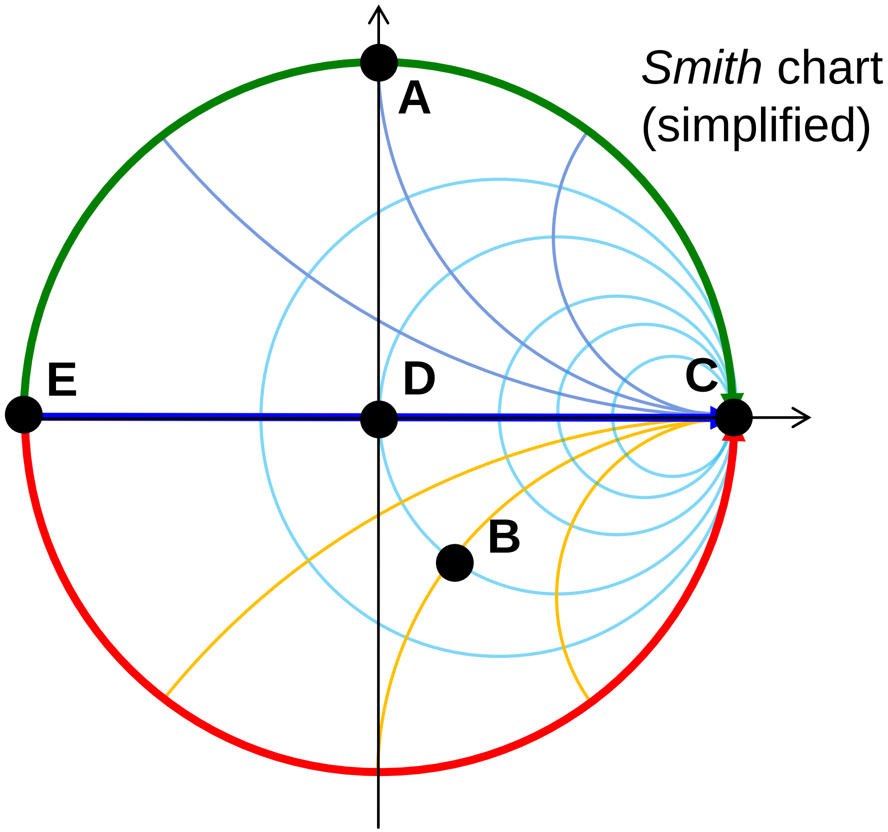


1. **When do no signal reflections occur at the end of a transmission-line?**
   - ○ R_source = R_load
   - ○ R_source = Z_0
   - ✗ Z_0 = R_load
   - ✗ R_source = Z_0 = R_load

2. **The Smith chart transforms the complex impedance plane onto the complex Gamma (reflection coefficient) plane within the unit circle.**

Quiz (2)

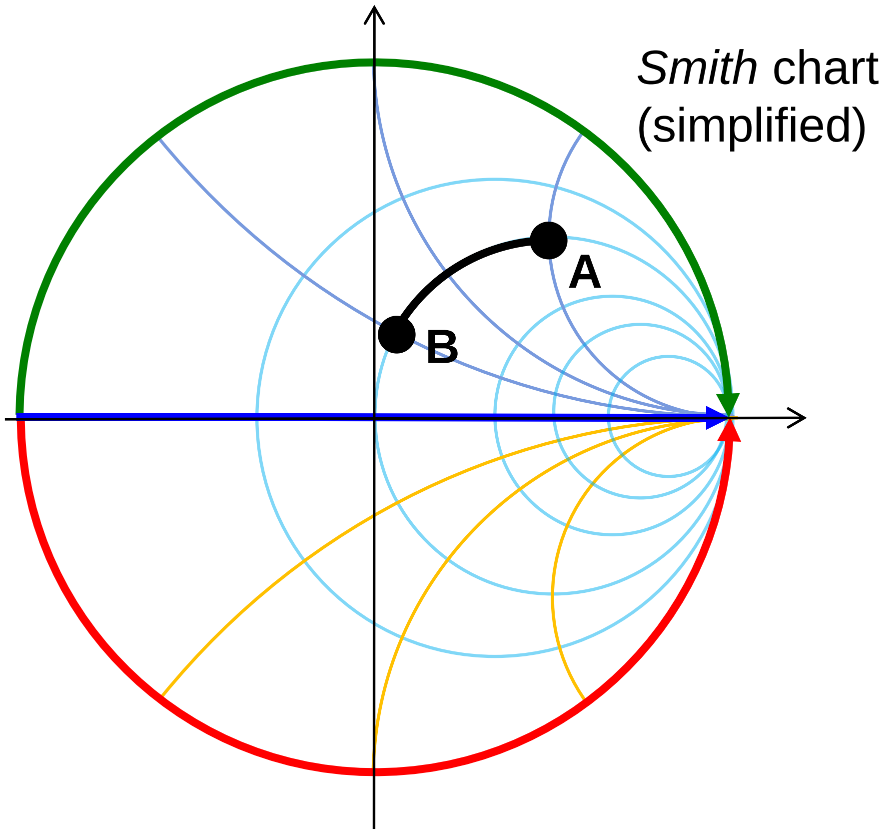
Prompts		Possible Answers
A. Point A	A5	1. Gamma = +1, z -> infinity
B. Point B	B4	2. Gamma = -j
C. Point C	C1	3. Gamma = 0, z = 1, match
D. Point D	D3	4. Point in the capacitive half plane
E. Point E	E6	5. Gamma = +j
		6. Gamma = -1, z = 0
		7. Point in the inductive half plane



Quiz (3)

4. Trace with marker points in the simplified Smith chart for an RL series impedance

- ○ Frequency f at point B > frequency f at point A
- ✗ Frequency f at point B < frequency f at point A
- ○ There is no frequency related to points A and B
- ○ Frequency f at point A = frequency f at point B



Quiz (4)

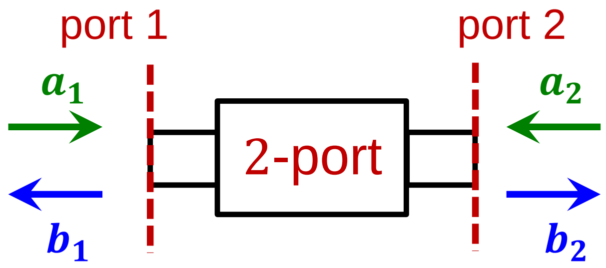
1. **Select all correct answers**

- ○ Y- and Z-parameters of electrical networks require a reference impedance Z_0
- ✗ Scattering parameters of RF networks are based on
  normalize complex voltage waves incident and reflected / transmitted at their ports
- ✗ DUT stands for "Device Under Test", as acronym for the RF network to be characterized
- ○ S-parameters are only defined for a reference impedance of Z_0 = 50 Ohm.
- ✗ Unused ports in a S-parameter measurement setup
  always need to be terminated in their characteristic port impedance

Quiz (5)

Prompts		Possible Answers
A. matched	A4	1. S_ii = S_ij
B. symmetric	B3	2. (S*)^T = (i)
C. reciprocal	C5	3. S_ij = S_ji and S_ii = S_jj
D. passive and lossless	D2	4. S_ii = 0
		5. Gamma = +j
		6. S_ij = S_ji

Quiz (6)



3. **Mark all correct answers for the S-parameters of a 2-port RF network**
- ○ a_1 and b_1 are independent parameters
- ✗ S_11 is the input reflection coefficient
- ✗ a_1 and a_2 are the incident waves at port 1 and port 2, respectively.
- ○ b_1 and b_2 are the transmitted waves between port 1 and port 2, and vice versa.
- ✗ S_21 and S_12 are the forward and reverse transmission gains.
- ✗ To characterize the S-parameters at port 2, port 1 needs to be terminated in its characteristic port impedance.