Basics of RF Electronics

Lecture 2

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Plan

Lecture 1

- Components
- Transmission line theory re-cap
- Connectors
- Reflection
- Co-planar waveguide
- Surface mount components
- Printed structures
- Filters
- Kuroda identities
- Wilkinson splitter

Lecture 2

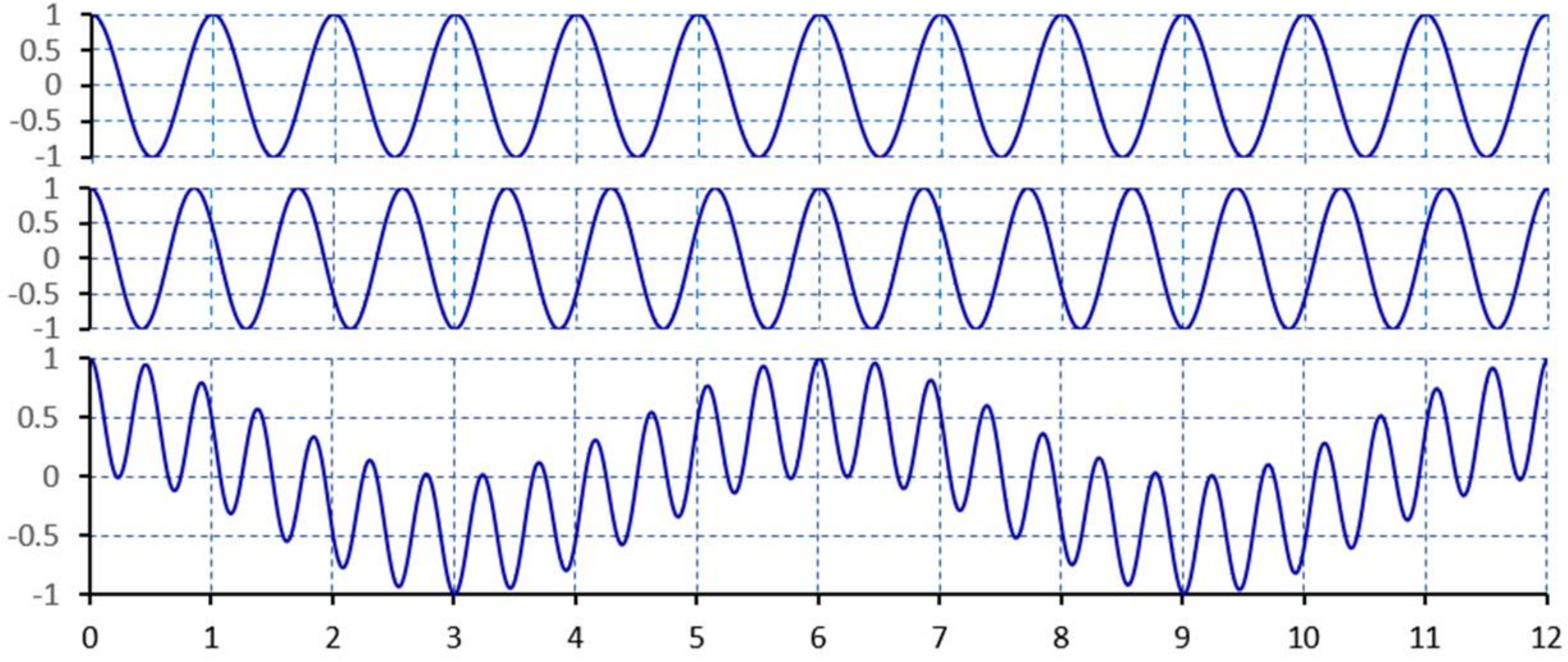
- Mixers
- IQ modulation
- Oscillators
- Phase noise
- Phase locked loops
- Learning resources

Mixing

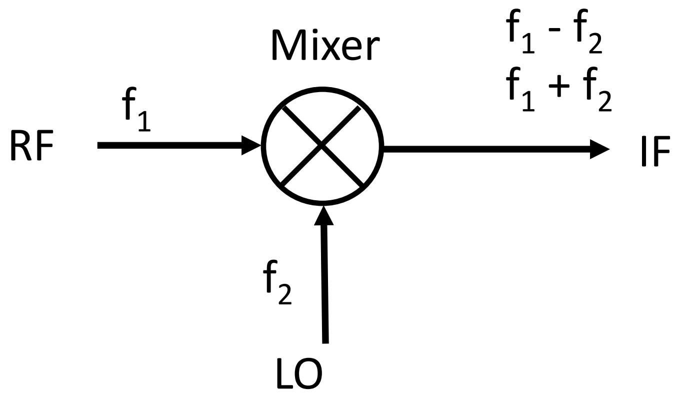
- Mixers perform time domain multiplication of signals.
- For perfect multiplication the output only contains the sum and difference frequencies.

$$2\cos(\omega_1 t+\theta_1)\cos(\omega_2 t+\theta_2)=\cos\{(\omega_1+\omega_2)t+\theta_1+\theta_2\}+\cos\{(\omega_1-\omega_2)t+\theta_1-\theta_2\}$$

- Output phases are also sums and differences of the input phases.
- Output with the summed frequency is known as the up converted signal.
- Output with the difference frequency is known as the down converted signal.



Mixers



For a communication system,

The local oscillator (LO) is sinusoidal

The RF carries information to be transmitted by amplitude and phase modulation of f_1

If f_1 is close to f_2 then the output $f_1 – f_2$ will be at a much lower frequency for convenient processing and is called the intermediate frequency (IF)

Low pass and high pass filter select the output

It is not possible to perform a perfect multiplication with simple analogue devices. For a real mixer only part of the output is a pure multiplication

Types

Operation

Add and square, multiply by square wave

Input and Output Connection

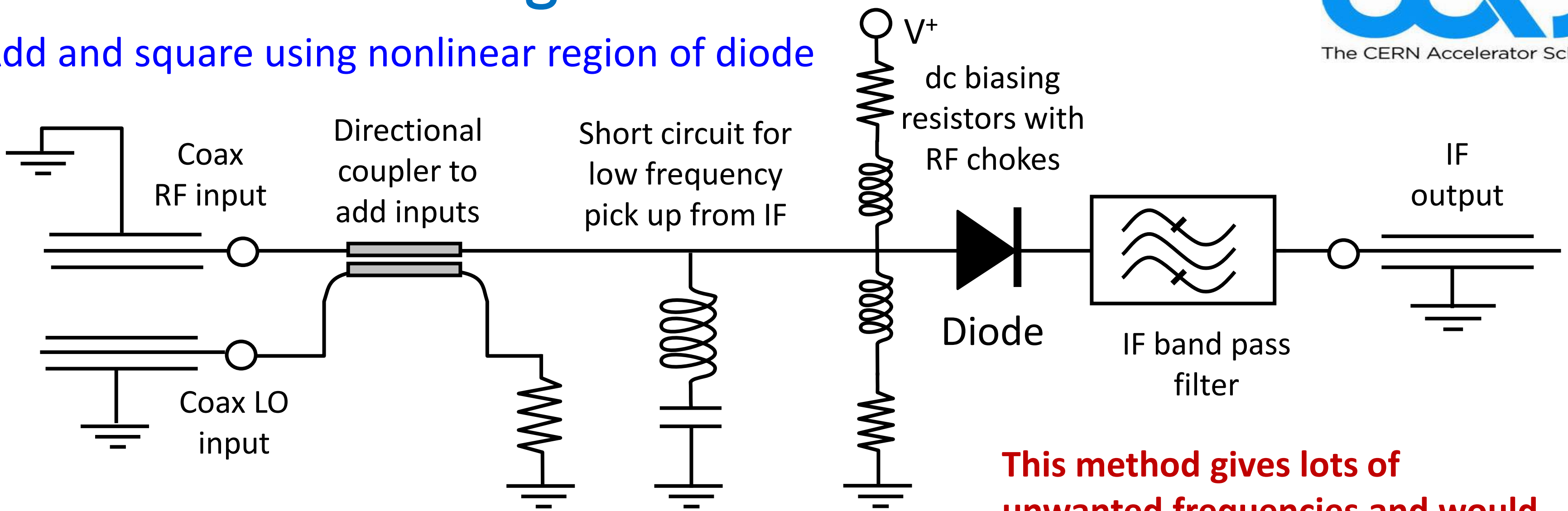
Unbalanced - Single balanced – Double balanced – Triple Balanced, Gilbert Cell

Devices

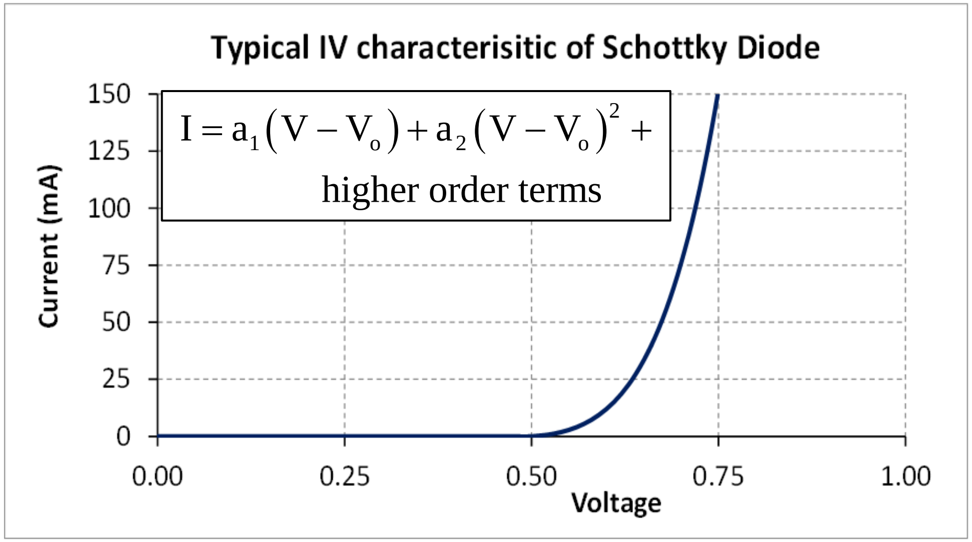
PN junction diodes, Schottky diodes, BJTs, FETs

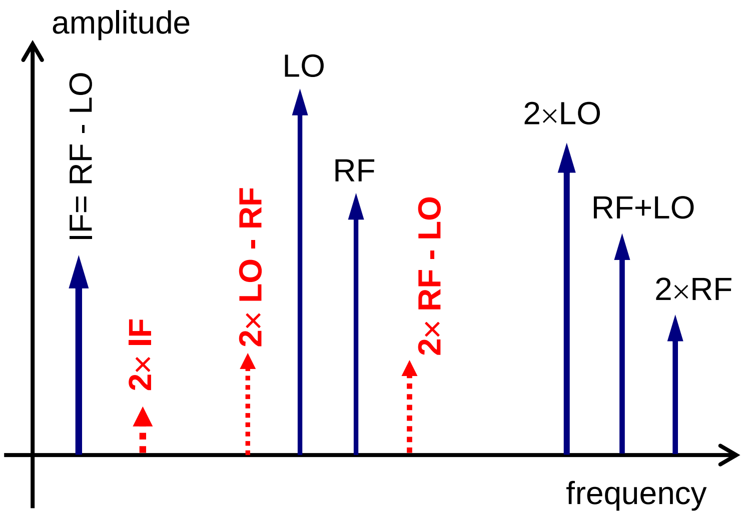
Unbalanced Mixing

Add and square using nonlinear region of diode



This method gives lots of unwanted frequencies and would not be used for accelerators





Higher order terms

For an arbitrary device where V is the input

$$V_{out} = a_o(V_o) + a_1 V + a_2 V^2 + a_3 V^3 + \cdots + a_k V^k + \cdots$$

For input V containing two frequencies

$$V = A_1 \cos 2\pi f_1 t + A_2 \cos 2\pi f_2 t$$

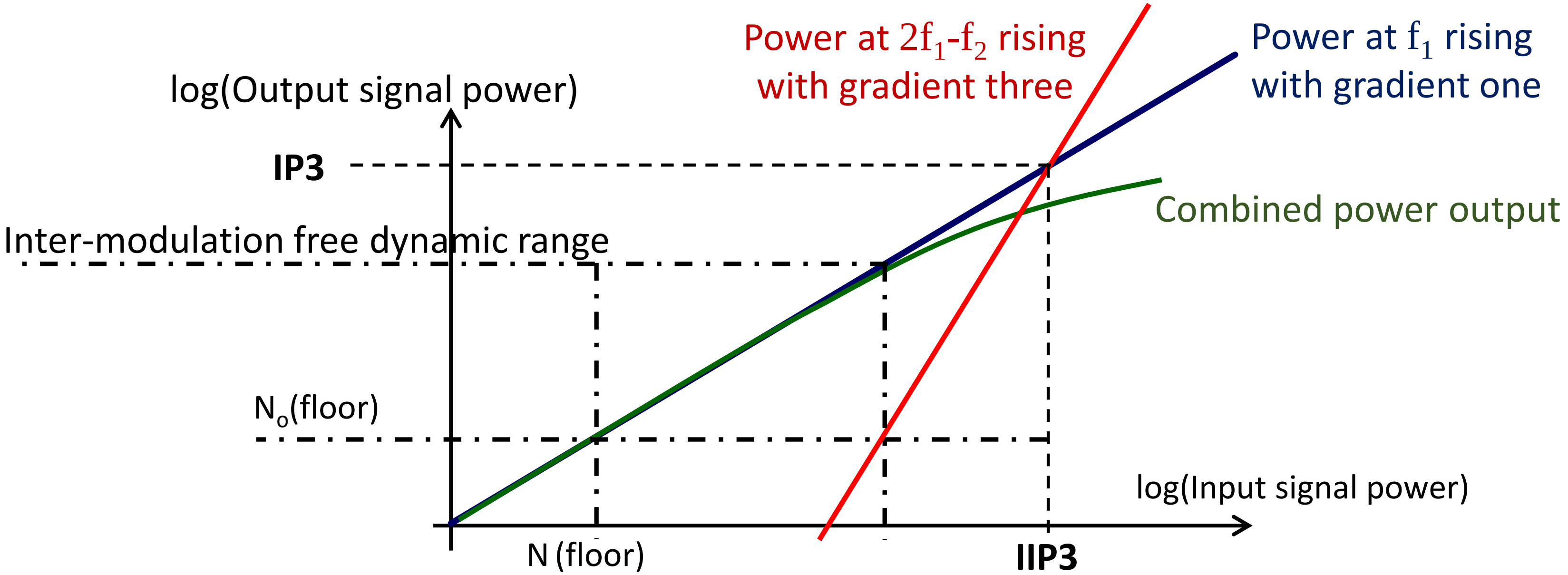
Output frequencies determined as

$$f_{out} = |m f_1 + n f_2|$$

Where f_{out} is positive and m and n are positive or negative integers

Frequencies from k^th harmonic determined with additional constraint $|\mathbf{m}| + |\mathbf{n}| = \mathbf{k}$

Nonlinearities can map odd harmonics back near the fundamental



Square wave multiplication

The double balanced mixer uses the Local Oscillator to turn diodes on and off and for high LO inputs effectively multiplies the RF signal with a square wave.

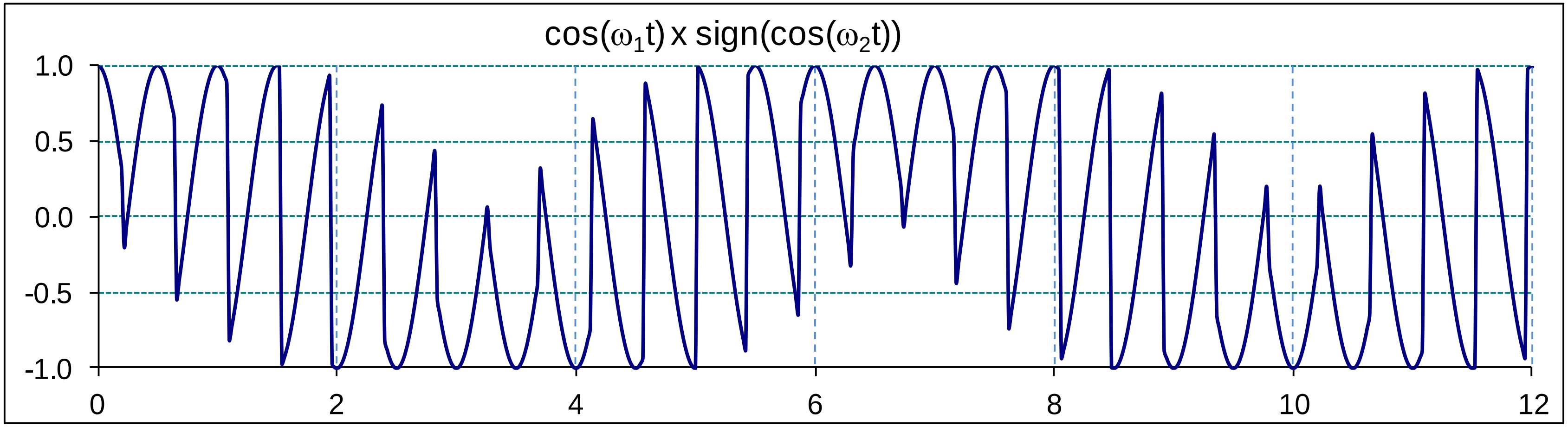
$$V_{IF} = \frac{4}{\pi} V_{RF} \cos(\omega_{RF} t) \underbrace{\left\{ \cos(\omega_{LO} t) - \frac{1}{3}\cos(3\omega_{LO} t) + \frac{1}{5}\cos(5\omega_{LO} t) + \cdots \right\}}_{\text{Square wave}}$$

hence

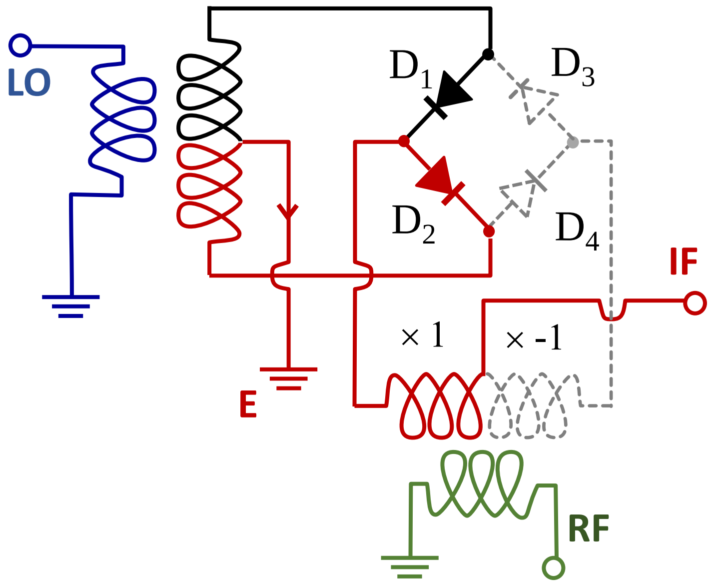
$$V_{IF} = \frac{2}{\pi} V_{RF} \left\{ \cos(\omega_{LO} - \omega_{RF})t + \cos(\omega_{LO} + \omega_{RF})t - \frac{1}{3}\cos(3\omega_{LO} - \omega_{RF})t - \frac{1}{3}\cos(3\omega_{LO} + \omega_{RF})t \right.$$

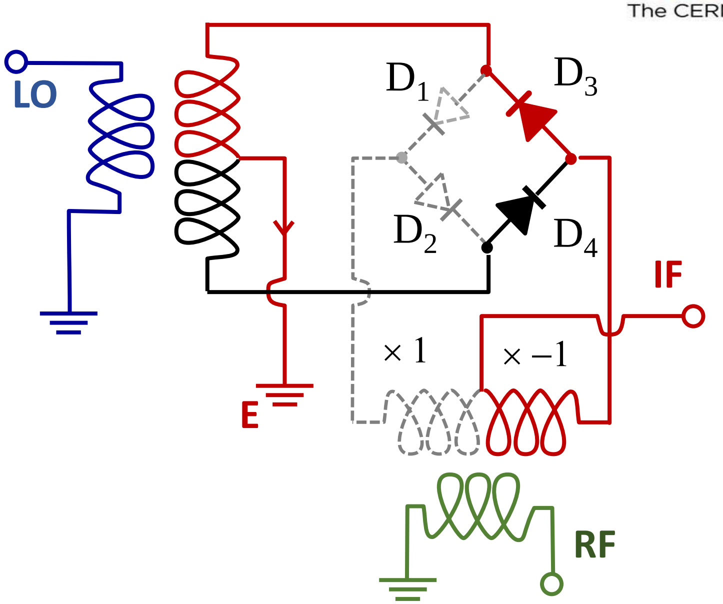
$$\left. \frac{1}{5}\cos(5\omega_{LO} - \omega_{RF})t + \frac{1}{5}\cos(5\omega_{LO} + \omega_{RF})t + \cdots \right\}$$

Mixing gives wave as shown, low frequency w_{LO} - w_{RF} is recovered by low pass filtering



Double balanced mixer operation





First half of LO cycle diodes D_1 and D_2 conduct, second half of LO cycle diodes D_3 and D_4 conduct.

When diodes D_1 and D_2 conduct, the secondary on the RF transformer connects the IF to earth E via diodes D_1 and D_2. It flows with plus sense (x +1) in the RF coil secondary.

When diodes D_3 and D_4 conduct, the secondary on the RF transformer connects the IF to earth E via diodes D_3 and D_4. It flows with minus sense (x -1) in the RF coil secondary.

Multiply by +1 when LO positive and -1 when negative hence multiply by a square wave

If the RF signal is small, equal currents flow in both halves of the LO coil secondary hence pick-up to the LO primary cancels and the mixer is balanced.

Mixer Terminology

Conversion Loss

This is the ratio of the output signal level to the input signal level expressed in decibels (dB).

Isolation

Isolation is the amount of leakage or feed through between the mixer ports.

Noise figure

Signal to noise ratio at input divided by the signal to noise ratio at output expressed in decibels (dB). It does not include the noise figure of any IF amplifier or 1/f noise.

Conversion compression

This is the RF input level above which the RF verses IF output curve deviates from a straight line.

Dynamic Range

This is the amplitude range over which a mixer can operate without degradation of performance and is determined by conversion compression and the noise figure.

Intercept point

This is a figure of merit for intermodulation product suppression.

Level

Nominal power level for LO in dBm

SIM-24MH+ Level 13 mixer example

In figure top mixer mixes RF_1 and RF_2 to ~ DC to measure phase

LO = 11.948 GHz is mixed with RF_1 and RF_2 by lower mixers

RF was 11.994 GHz hence IF = 46 MHz (sampled at 120 MSPS).

Features for the SIM-24MH+ mixers used are:-

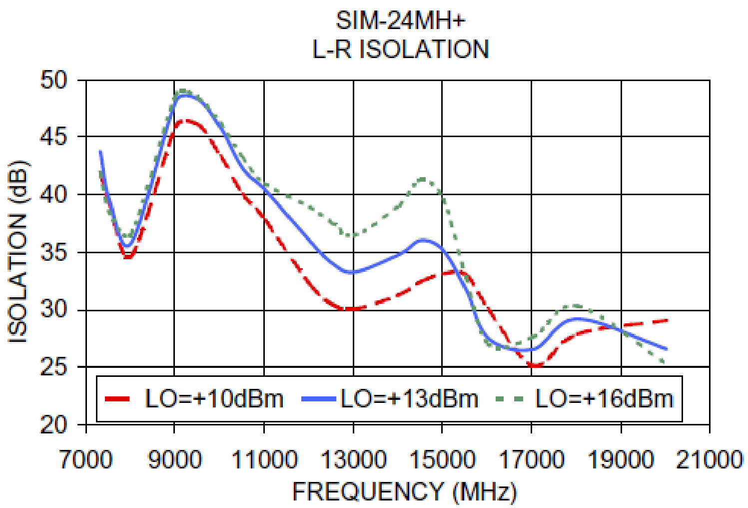
RF/LO frequency range 7.3 GHz to 20 GHz

IF frequency range DC to 7.5 GHz

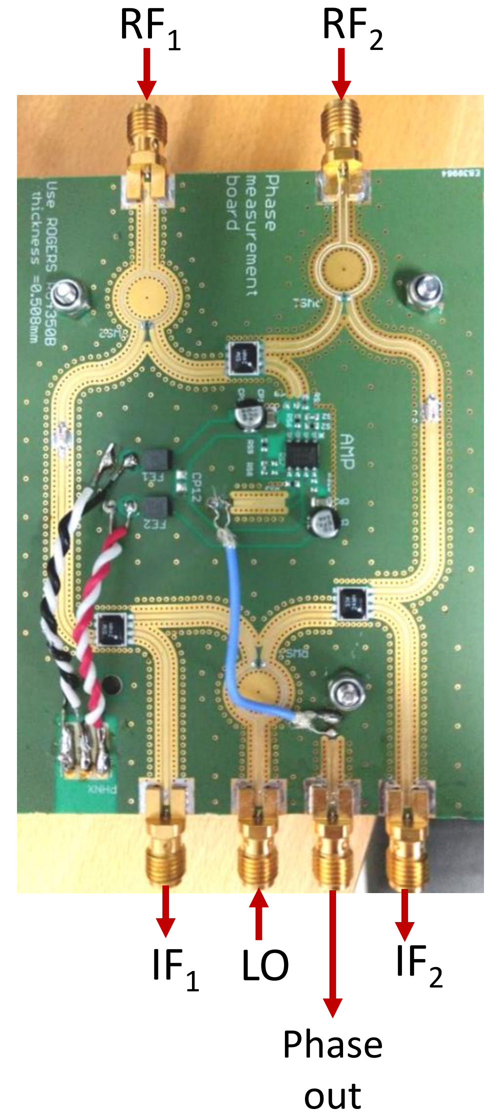
LO Power (+10 dBm to +16dBm) recommended =+13dBm

Conversion loss (5.84 dB at 12 GHz for +13dBm LO)

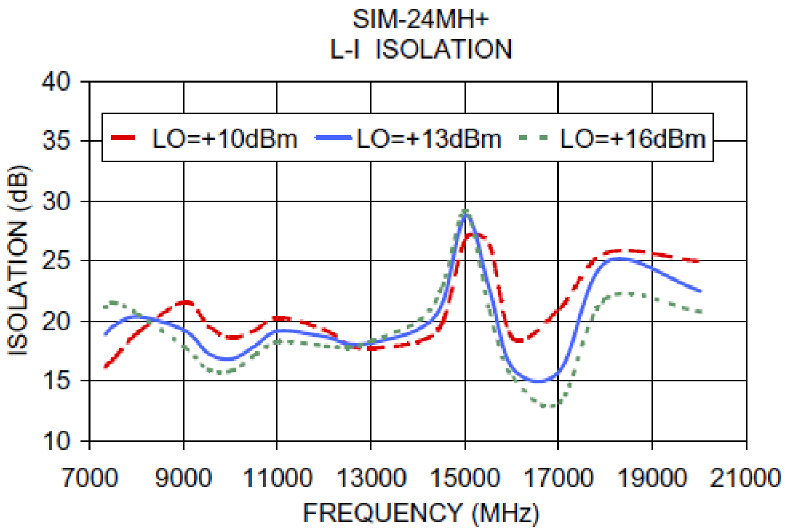
IP3 (16 dBm at 12 GHz)



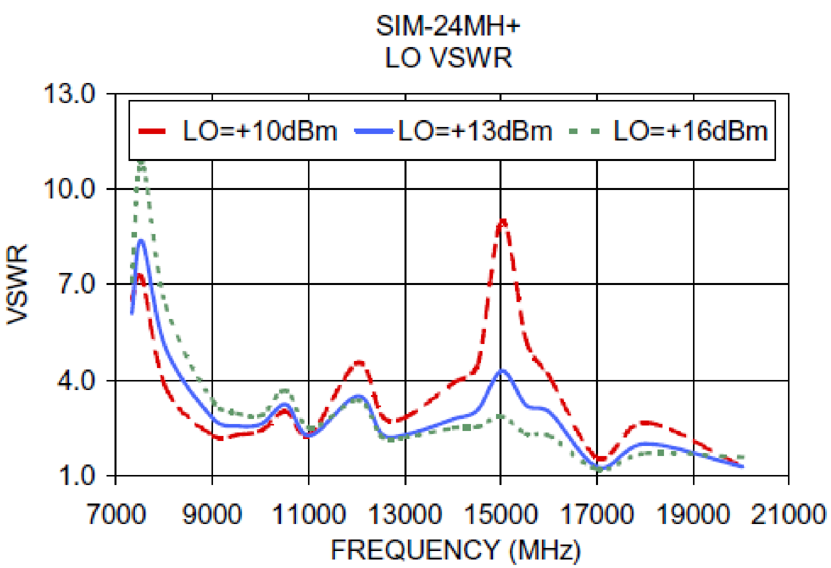
If the Splitter has perfect isolation, LO power leaking as an outgoing wave on the RF input has no detrimental effect if the connectors are reflectionless. This power get completely absorbed in the load of the directional coupler sampling RF from the cavity.



VSWR and LO-IF isolation



Feed through of LO or RF to the IF is easily removed by low pass resistive (RC) filtering when the mixer is down converting. IF on the LO and RF line can be removed by high pass resistive filtering. Reflecting IF back to mixer RF to LO inputs from a high pass filter will up convert to a frequency that can be removed by filtering.



An important observation is that the mixer is never matched. Reflected power is determined from the VSWR as

$$P_R = \left(\frac{VSWR - 1}{VSWR + 1}\right)^2$$

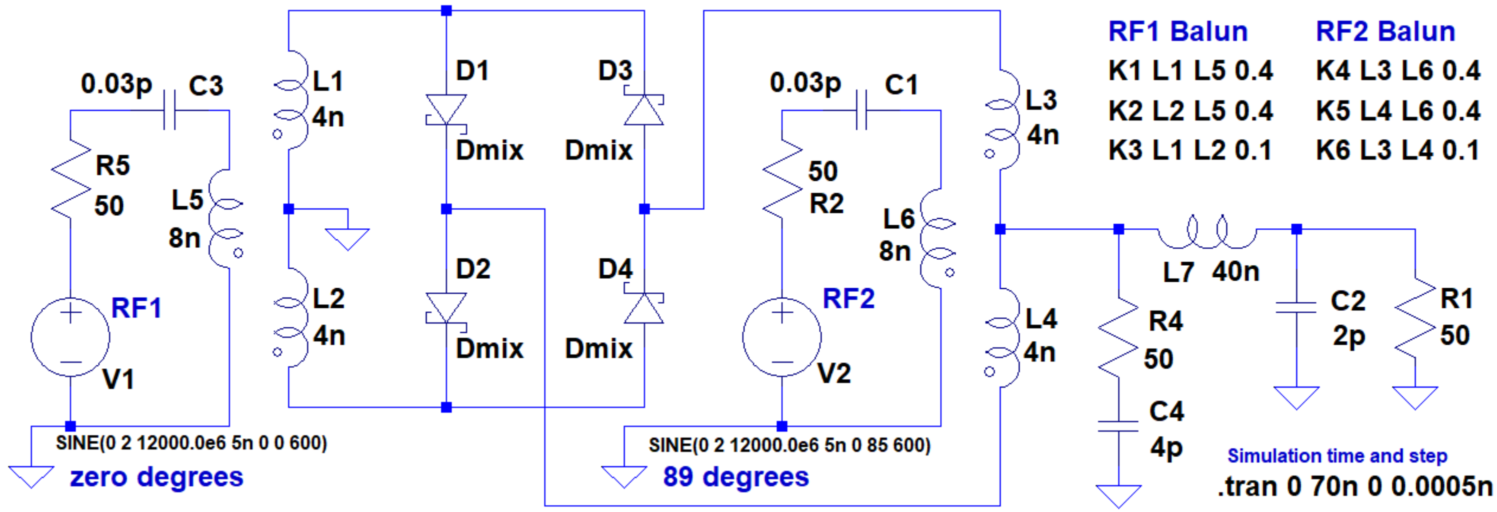
e.g. VSWR = 3 gives 25% reflection. Matching at the frequency of interest will change the phase to be measured hence is probably not useful. Some reflection of the LO may not be an issue if it never changes.

Simulation for phase measurement

$$2\cos(\omega t+\theta_1)\cos(\omega t+\theta_2)=\underbrace{\cos\{2\omega t+\theta_1+\theta_2\}}_{\text{Remove with LP Filter}}+\cos\{\theta_1-\theta_2\}$$

Schottky Diode Model

.model Dmix D(IS=1.6p RS=1.3 BV=5 IBV=10u CJO=0.015p M=0.25 N=1.4 EG=1.43 TT=3p VJ=0.7)



Mixers are specified for operation at large fixed LO power levels giving pseudo square wave mixing with a much smaller RF signals.

How mixers perform as phase detector when RF and LO have the same power level is not answered by the datasheets.

Answering this questions requires a model to be devised that fits data they do give.

This model reveals for identical RF and LO power levels, the DC output varies linearly with phase difference near 90 degrees, but there is an extra DC offset if power levels differ.

IQ modulation

- I stands for that part of the signal that is in phase with the reference oscillator.
- Q stands for that part of the signal that is 90° retarded from the reference signal.
- To correct the phase of a cavity subject to perturbations from the beam and other influences, the phase and amplitude of the RF drive must be varied.
- One method is using electronic phase shifters and variable attenuators on the signal from the master oscillator before it is inputted to the high power amplifier train.
- Operating mode of an accelerating cavity is governed by a 2nd order differential equation.
- Whilst controlling separately on amplitude and phase works in general, it is not optimal in terms of achieving the best possible tracking of the set point.

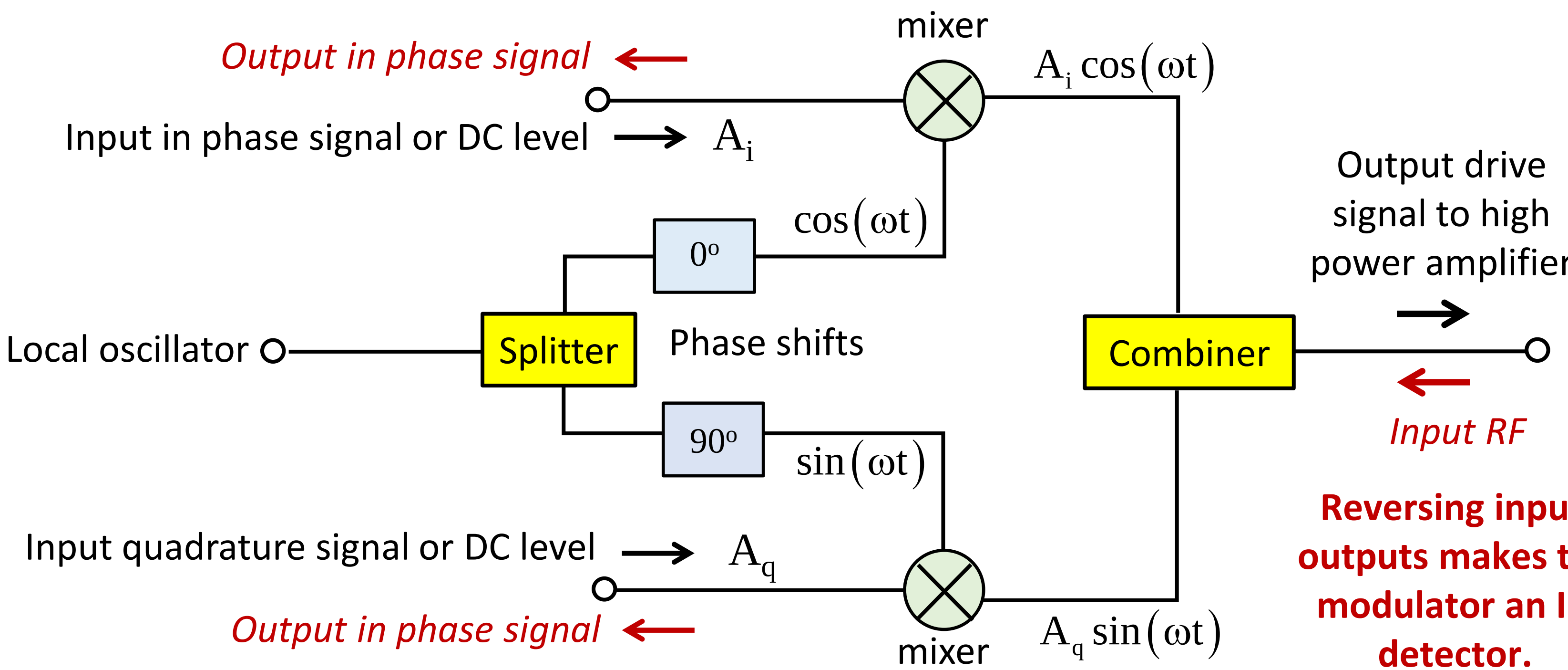
If cavity voltage is decomposed with a steady nominal frequency ω together with in phase and quadrature, slowly varying functions of time $A_i(t)$ and $A_q(t)$ as given by

$$V(t) = \left\{ A_i(t) + jA_q(t) \right\} \exp(-j\omega t)$$

for a high Q cavity, to an excellent approximation, functions $A_i(t)$ and $A_q(t)$ obey simultaneous first order differential equations so the cavity model becomes a dynamical system. The drive for these two equations is simply the in phase and quadrature components of the forward power delivered to the cavity. Proportional-integral (PI) controllers acting separately on the in phase and quadrature components of the drive and dependent on errors in the in phase and quadrature components of the cavity voltage is optimal.

IQ modulator operation

In order to implement this controller I and Q must be measured and then an IQ modulator applied to the drive.



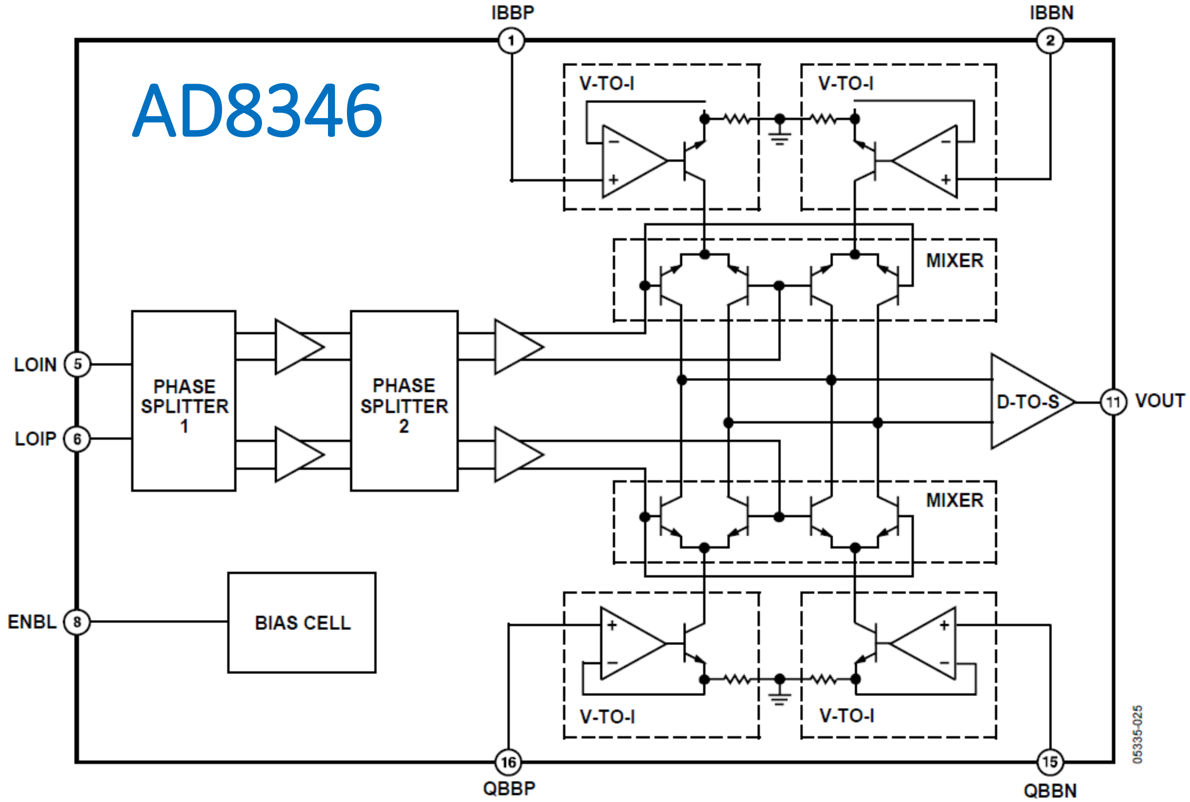
Reversing inputs outputs makes the modulator an IQ detector.

if the mixer IF accepts DC ($\omega_s \to 0$) then mixing multiplies by a constant

$$A_i \cos(\omega_s t)\cos(\omega t) = 0.5 A_i \cos\{(\omega + \omega_s)t\} + \cos\{(\omega - \omega_s)t\} \approx A_i \cos(\omega t)$$

$$A_q \cos(\omega_s t)\sin(\omega t) = 0.5 A_q \sin\{(\omega + \omega_s)t\} + \sin\{(\omega - \omega_s)t\} \approx A_q \sin(\omega t)$$

AD8346



this IQ modulator uses Gilbert Cells to perform the mixing operation

Quadrature phase error ~ 1 degree of immediate concern. Accelerator systems typically need phase control at the level of 5 to 100 milli-degrees rms.

This is the reason to not choose IQ detector chips to determine cavity phase in preference to digitally sampling.

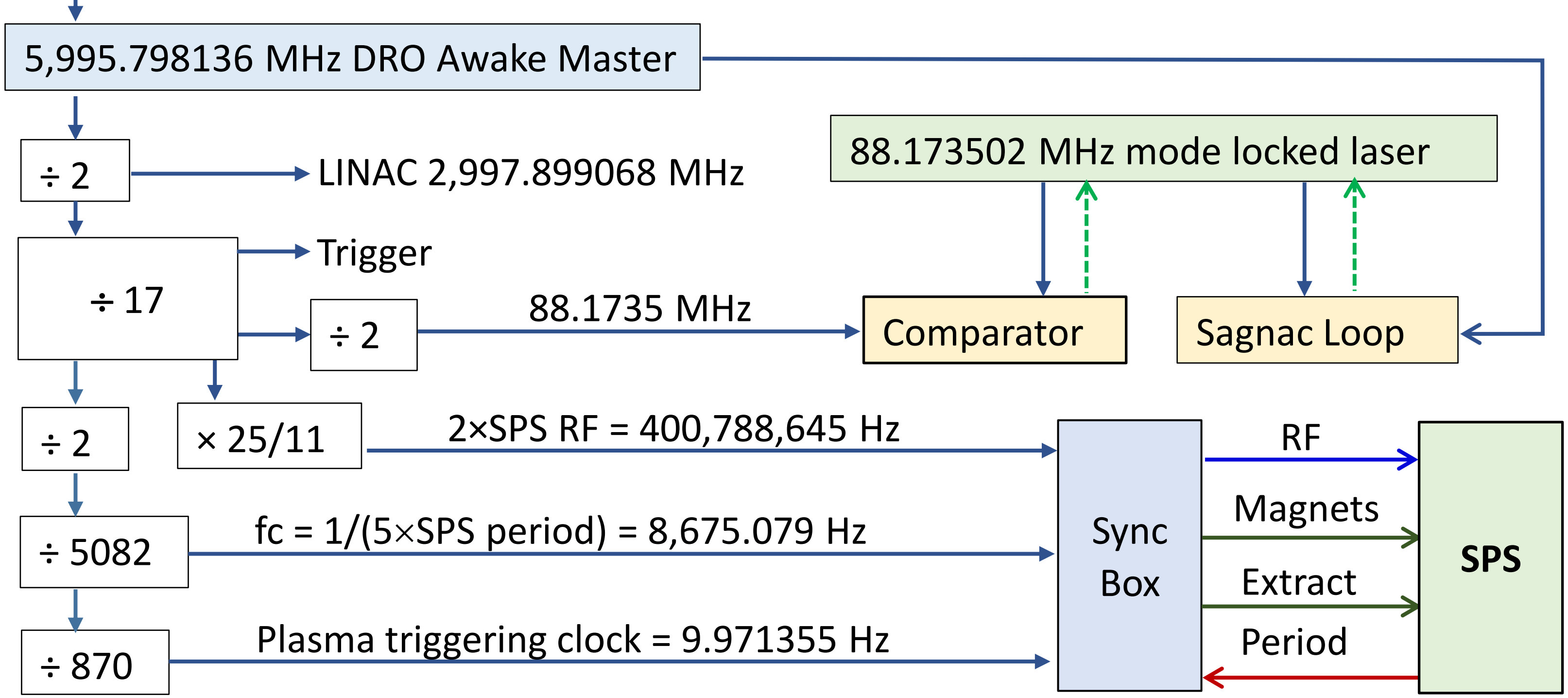
As a modulator this quadrature phase error is far less problematic. Whilst the cavity must be controlled to milli-degrees, because it has a high Q factor, a significantly larger phase shift in the drive is required to start making a milli-degree corrections in the cavity.

RF/LO frequency range	0.8 GHz to 2.5 GHz
IQ modulation bandwidth	DC to 70 MHz
Output Power	-10 dBm (typical)
LO power	-12 dBm to – 6 dBm
IQ Input impedance	12 kΩ
Quadrature phase error	1 degree rms @ 1.9 GHz
I/Q Amplitude balance	0.2 dB @ 1.9 GHz
Side band suppression	-36 dBc @ 1.9 GHz
Noise floor-147 dBm/Hz	

Oscillators at AWAKE

The master oscillator is often an atomic clock with a quartz crystal resonator that has excellent long term stability and can be distributed around the experiment.

There is also a DRO and a mode locked laser synchronised to the mater



A proton beam extracted from the CERN's SPS is injected into a plasma to accelerate bunches of electrons from an electronic LINAC. Experiments like AWAKE need multiple frequencies to be generated all of which must be synchronised at different levels of timing error.

Stability requirements

Frequency stability of commercial, crystal stabilised, oscillators is 10^{-6} - 10^{-7}.

Latter figure is ~ 1 second in 4 months or 1 minute in 20 years.

To time separate events occurring every 20 milliseconds (50 Hz) to an accuracy of 50 femto-seconds a stability of 2.5×10^{-12} is needed.

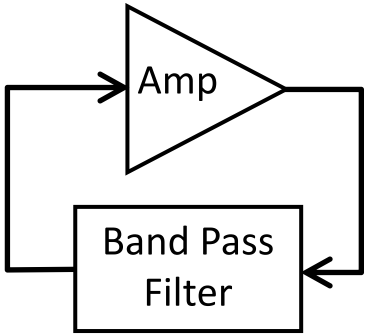
Precision quartz oscillators held at constant temperature and protected from environmental disturbances, have fractional stabilities in the range 10^{-10} to 10^{-12}.

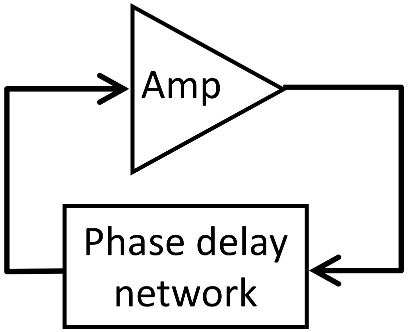
Mode locked lasers can have significantly better fractional stability than 10^{-12}.

At AWAKE the dielectric resonator oscillator (DRO) which is phase locked to the 10 MHz GPS signal, is used to generate all the other necessary frequencies by integer division and multiplication.

The mode locked laser has better stability than the DRO and is used to time the laser pulses. It can also measure the stability of the DRO reporting on short term phase errors.

Oscillator basics





- Reduced to its simplest format, an oscillator consists of an amplifier and filter or phase delay network operating with positive feedback.
- With a band pass filter one amplifies "more of the same" to get oscillation.
- Importantly, for amplification to occur, the phase gain around the whole loop must be 360° otherwise it is not "more of the same". This is the Barkhausen condition – (*right hand image*).
- A band pass filter always gives a phase delay. The oscillator does not oscillate at the lowest impedance of the band pass filter but rather at a frequency nearby, where the loop phase is 360°. If for instance the amplifier is an inverting amplifier with a small delay, then the amplifier might provide 185° in which case, the frequency adjusts itself until the phase delay in the bandpass filter is 175°
- Low pass, high pass and band stop filters all give phase delays so can also be used.

Three terminal amplifiers as oscillators

$$Q_F = \frac{f}{\delta f}$$

δf = 3dB bandwidth

For high stability operation choose a very high Q factor Band Pass Filter, typically a resonant crystal, microwave cavity or ceramic block.

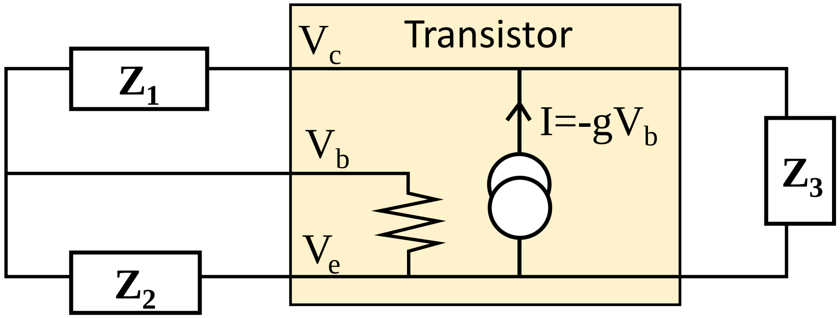
For Q_F very large, bandwidth is very small, hence phase shift across filter changes very rapidly in a small range and oscillator stabilises itself this range.

Crystals and resonators can be modelled with inductors (L), capacitances (C) and resistors.

We consider LC lumped circuit resonators and phase delay networks first.

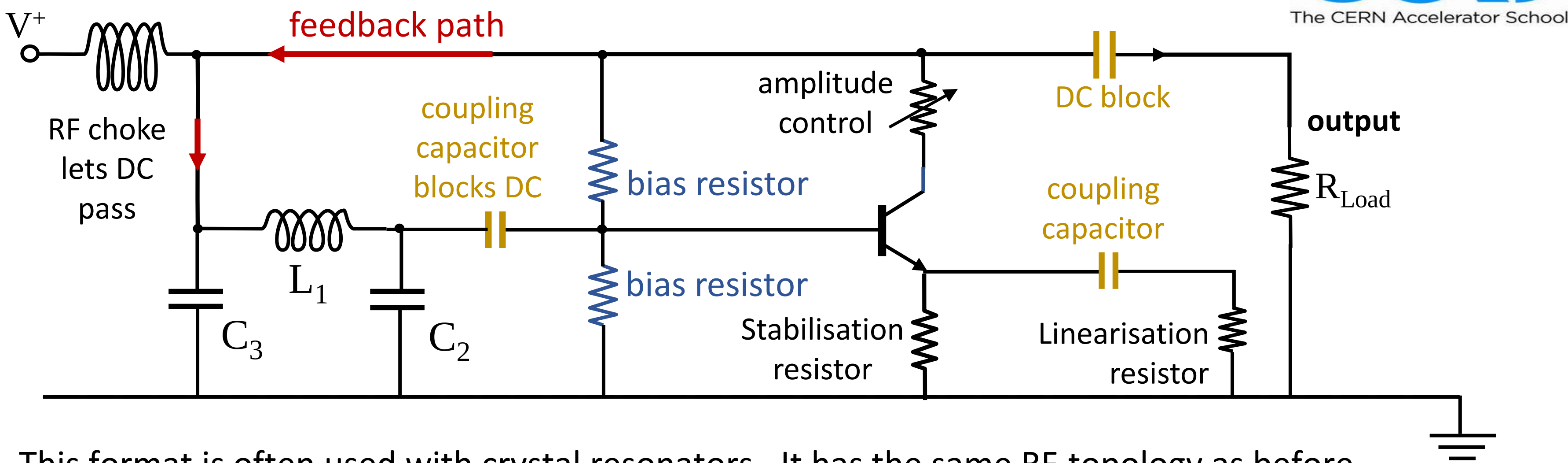
Amplification might be provided by a three terminal active device, e.g. Bipolar Transistor or FET. When driven at a particular frequency, series or parallel combinations of inductors and capacitances can be represented by a single impedance (Z_1 Z_2 and Z_3 in this case).

Impedances can be taken as positive or negative as the oscillator is never perfectly on the resonant frequency of individual groups of components. With just three terminals the RF characteristics can be modelled with just three impedances as shown.



Power supply, output and biasing is not shown as it varies for differing implementations

Pierce Oscillator



This format is often used with crystal resonators. It has the same RF topology as before with L_1 connecting collector and base, C_2 connecting base and emitter and C_3 connecting collector and emitter. When the principal impedances L_1 C_2 and C_3 are drawn as a ladder, then the filter type is apparent - a low pass filter. Very low frequencies are not delayed so arrive at the base 180° degrees out of phase.

The corner frequency of the network is given as $\omega_c = \sqrt{\dfrac{(C_1 + C_2)}{LC_1C_2}}$

At this frequency the phase shift is 180° bringing the circuit close to resonance. For this circuit the frequency of oscillation is slightly below the corner frequency and depends on resistive loading. At microwave frequencies transistor oscillators are designed from its S parameters. The frequency of the oscillation is most easily adjusted by changing values of capacitors C_3 or C_2 .

Voltage Controlled Oscillators

A voltage controller oscillator (VCO) changes its frequency with applied DC voltage.

A reverse biased PN junction diode has 2 charged layers separated by a depletion layer.

In reverse bias the diode becomes a capacitor that varies with voltage.

VCOs are often constructed by replacing a capacitance with a varactor diode.

Mini-Circuits sells over 800 models covering the frequency range from 12.5 MHz to 8 GHz.

For broadband use below 2 GHz, the technology is likely to be LC-Monolithic Microwave Integrated Circuits (MMIC). Their selection table has headings as given below with 3 selected products

Model	Low Freq (MHz)	High Freq (MHz)	Power Output dBm	Tuning voltage range	Phase Noise (dBc/Hz) offset 1kHz	Phase Noise (dBc/Hz) offset 10kHz	Phase Noise (dBc/Hz) offset 100kHz	Phase Noise (dBc/Hz) offset 1MHz
ROS480+	386	480	9.5	3-16	-88	-115	-137	-158
ROS1700W+	770	1700	9	1-24	-72	-99	-120	-141
ROS3050C+	2635	3050	6.5	0.5-16	-77	-104	-125	-145

Model	Pulling (MHz) pk-pk @12dB reflect	Pushing (MHz/V)	Tuning sensitivity (MHz/V)	Harmonics (dBc) typical	Harmonics (dBc) max	3dB control band-width	DC Voltage (V)	DC current (mA)
ROS480+	0.25	0.45	8-12	-20	-13	30	12	31
ROS1700W+	7	0.7	26-65	-25	-9	50	12	35
ROS3050C+	4.5	0.2	35-47	-22	-12	100	8	40

Crystal Oscillators

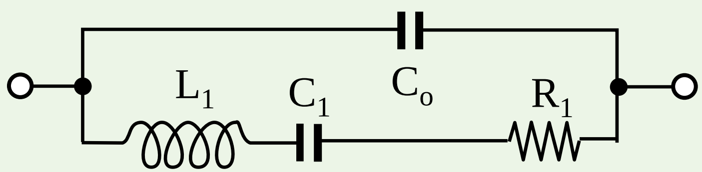
Quartz crystals have piezo electric properties and hence their electrical resonance is coupled to a mechanical resonance.

Their fundamental mode can be accurately modelled with a series LCR circuit and a small parallel capacitance coming from the mounting.

Capacitance C_1 is typically in the range 10^{-3} pF to 10^{-1} pF and is associated with piezoelectric charge movement as a function of voltage.

The inductance depends on crystal mass and can be as large as a few Henry's for a large thick crystal down to a few milli-Henry's for a small crystal.

Values of R_1 vary from 10 Ω for a 20 MHz crystal to 200 kΩ for 1 kHz crystals.



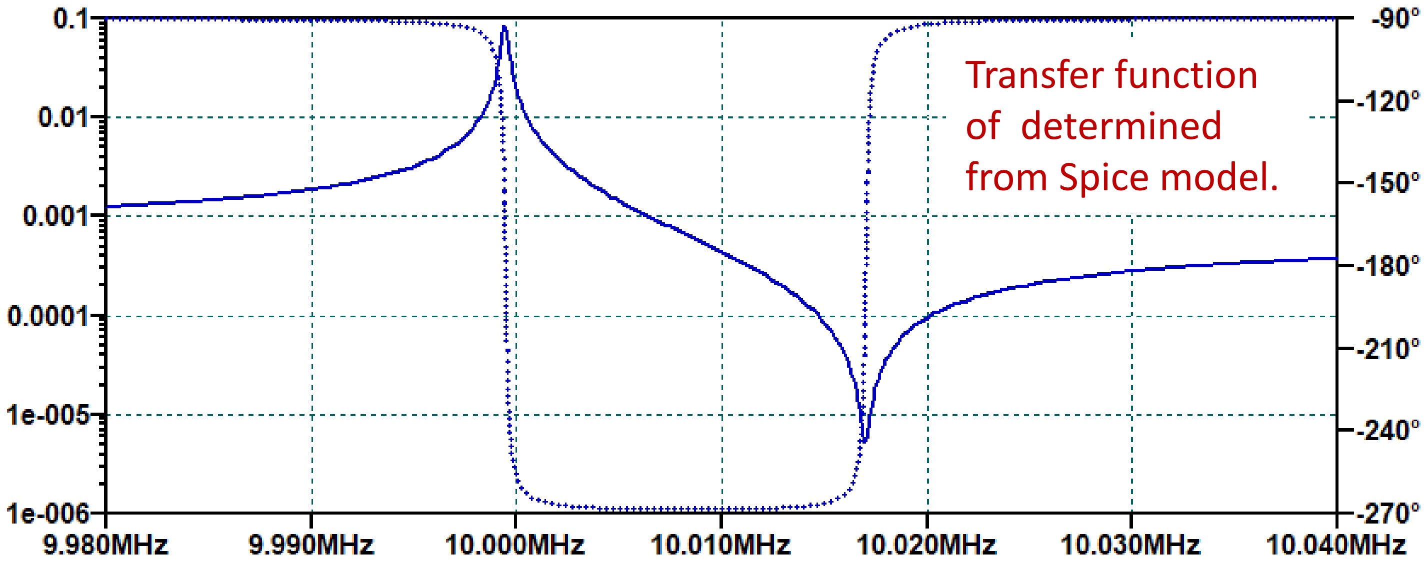
Q factor of series LCR resonator $Q = \frac{1}{R_1}\sqrt{\frac{L_1}{C_1}}$

Resonant frequency $f = \frac{1}{2\pi\sqrt{L_1 C_1}}$

As an example for C_1= 24.6 fF, L_1 =10.298 mH, R_1 = 16 Ω then the frequency is 9.999450 MHz and the Q factor is 40438.

The shunt capacitance C_o is the sum of capacitance dues to electrodes on the crystal (independent of piezo effects) and also stray capacitance due to its mounting. The value is typically in the range 3 pF to 7 pF.

Crystal operational frequency



Using example on previous slide with C_o= 7 pF and for an ac applied peak voltage of 1.5V, current through the circuit is shown.

The solid line gives the current magnitude and the dotted line gives the phase.

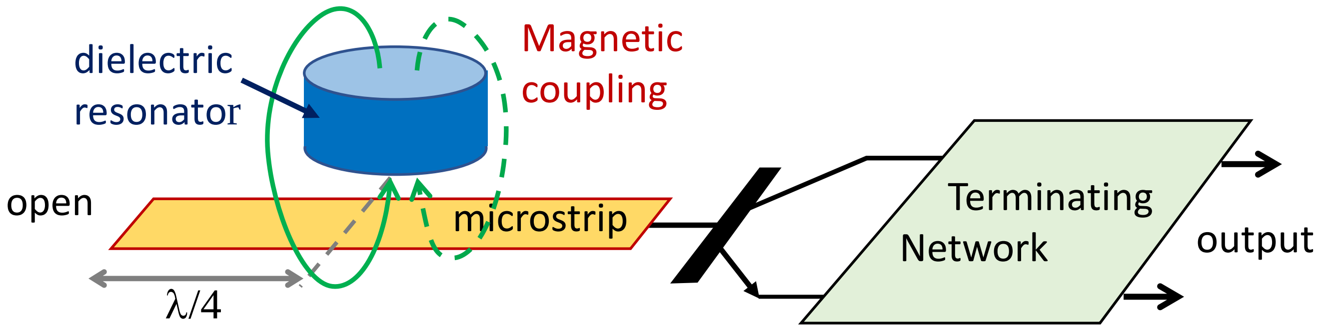
Band pass peak at 9.99945 MHz as expected. For frequencies less than this the crystal is capacitive.

Band stop dip is at 10.01700 MHz. Between the band pass peak and the band stop dip the crystal is inductive.

The oscillation frequency is set in the inductive region and where the phase changes rapidly which in this case would be at 10.000 MHz.

The oscillator can be implemented by replacing the L_1 in slide 22 with the crystal. A small tuning range change be achieved by placing varactor diode in parallel with C_2 and C_3 .

Dielectric Resonance Oscillator



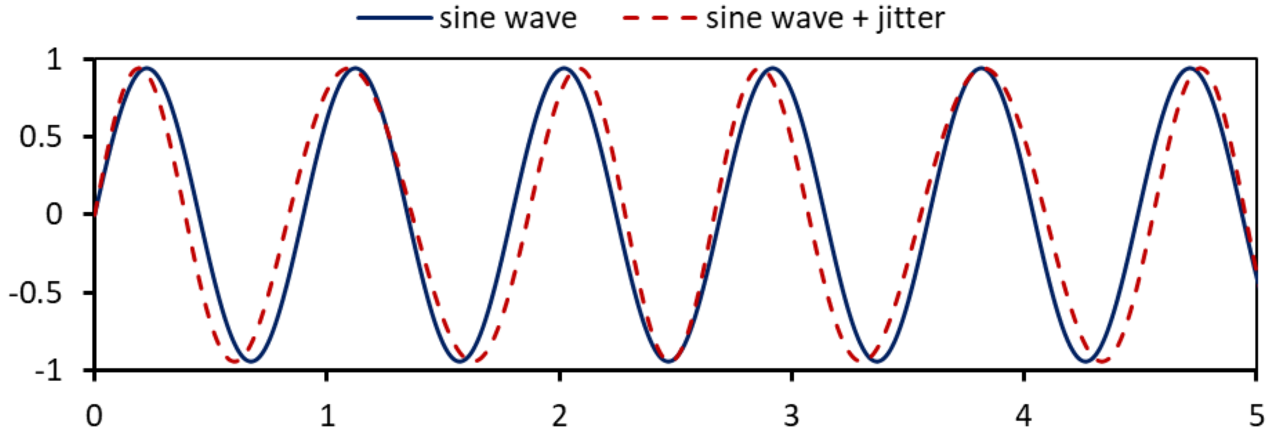
It is important to maximise clock stability at a number of important frequencies employed in an accelerator complex.

Stability is enhanced by high Q oscillators which invariably have a very small tuneable range. One would not choose general wide band VCOs for critical clocks.

Quartz crystal overtones allows high Q oscillators to be made up to about 300 MHz but above that, other resonators are needed. Surface wave acoustic resonators (SAW) are useful in the range of 200 MHz to 2 GHz.

Above this frequency the high Q resonator most widely employed is the DRO. Their fundamental frequencies are usually in the range 6 GHz to 12 GHz. Using the first sub harmonic they might provide frequencies down to 3 GHz and higher harmonic are used up to about 50 GHz. Q factors up to 10^5 are possible.

Jitter



Amplitude noise is usually less pronounced on RF signals as the both the oscillators in the source and the power amplifiers run near saturation.

In principle jitter can be measured on an oscilloscope however the speed and accuracy of sampling is invariably insufficient to get a useful measurements (without down conversion).

Jitter is usually examined in the frequency domain where it is called phase noise.

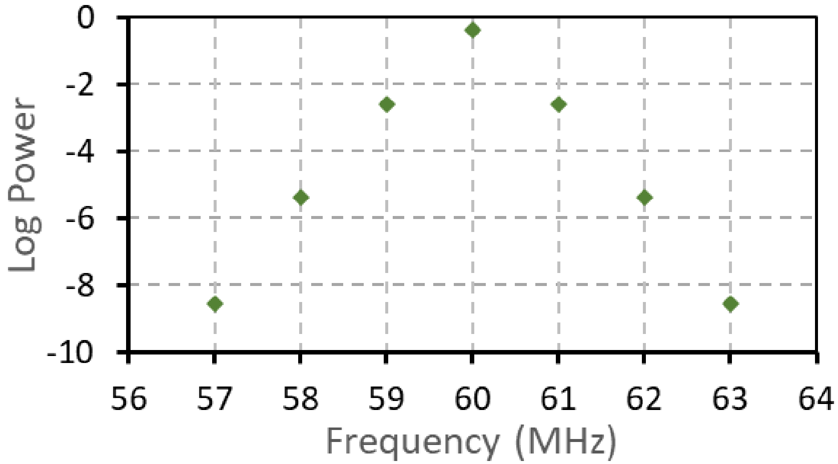
Consider a sine wave with a regular oscillation on the phase representing the unwanted phase jitter and a regular oscillation on the amplitude representing unwanted amplitude jitter then

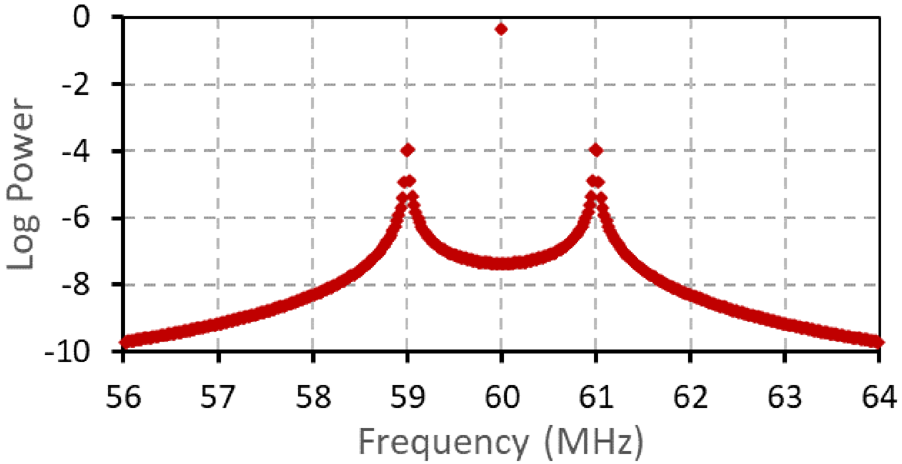
$$V = V_o \left\{1 + a_n \cos\left(\omega_n t\right)\right\} \cos\left\{ \omega t + \varphi_n \cos\left(\omega_n t\right) \right\}$$

a_n is amplitude modulation depth, φ_n is the phase modulation depth, and for the example to be given, ω_n is taken as the angular frequency of both the amplitude and the phase noise.

FFT of amplitude and phase jitter

$$V = V_o\left\{1 + a_n\cos(\omega_n t)\right\}\cos\left\{\omega t + \varphi_n\cos(\omega_n t)\right\}$$





Spectral density derived from the Fast Fourier Transforms for the two cases of a_n= 0 left (phase modulation) and φ_n= 0 right (amplitude modulation).

Noise frequency was taken as 1 MHz in both cases, modulation depth ϕ_n taken as π/20 radians, the amplitude modulation was taken as 5%.

As the maximum modulation for phase is to 180^o out of phase and the maximum useful modulation of amplitude is to one, then there is equivalence in this choice of modulation.

largest noise contribution is for phase modulation at 10^(-2.58) ~ 26.3e-4 whilst for the amplitude modulation it is 10^(-4) ~ 1.0e-4 .

This indicates that the spectral density preferentially picks out the phase noise.

Phase Noise

Expanding the expression for phase jitter gives

$$V = V_o \cos\left\{\omega t + \varphi_n \cos(\omega_n t)\right\} = \cos(\omega t)\cos\left\{\varphi_n \cos(\omega_n t)\right\} - \sin(\omega t)\sin\left\{\varphi_n \cos(\omega_n t)\right\}$$

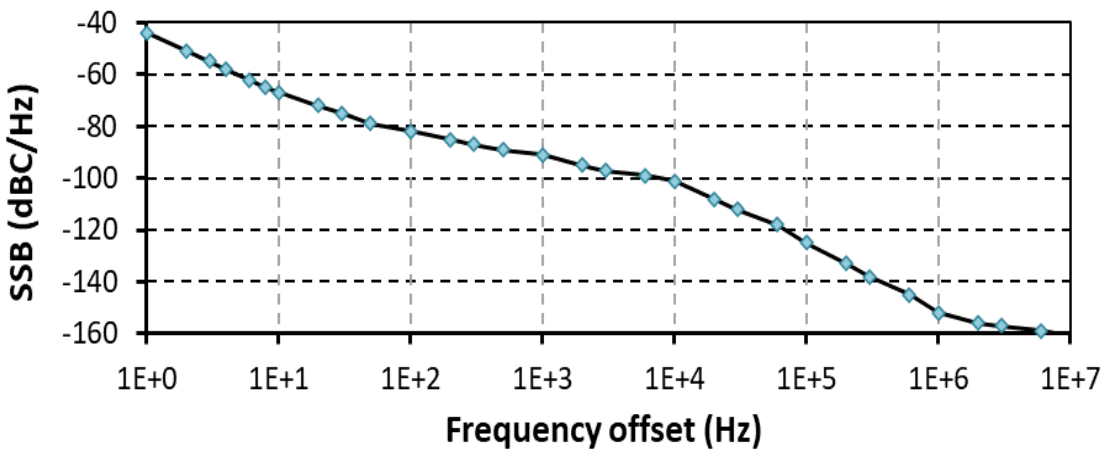
Hence $$V/V_o \approx \cos(\omega t) - \frac{\varphi_n}{2}\sin\left[(\omega+\omega_n)t\right] + \frac{\varphi_n}{2}\sin\left[(\omega-\omega_n)t\right]$$ when φ_n is small

Noise modulating phase at ω_n is mapped to upper and lower side bands at $\omega + \omega_n$ and $\omega - \omega_n$.

Spectrum analysers display output as squares of Fourier coefficients normalised to power/Hz.

Side band power primarily gives a measure of the phase jitter. Phase noise usually is plotted on just one side of the carried in units of decibels below the carrier.

Single side band phase noise (SSB) in dBc per Hz for a DRO locked to a 10 MHz source. Spectrum analysers often have the capability to make this measurement and remove the carrier.



At low frequencies the noise is rising with 3 orders of magnitude per decade hence a $1/f^3$ law applies. This is typical of 1/f flicker noise from a transistor being attenuated by a high Q filter. The $1/f^3$ does not continue to zero as this would imply infinite deviations on very long-time scales.

Phase noise to jitter conversion

Taking values on the plot given in dBC/Hz as SSB, then the magnitude of Fourier coefficient is $\sqrt{10^{-\text{SSB}/10}}$

using $V/V_o \approx \cos(\omega t) - \frac{\varphi_n}{2}\sin\left[(\omega+\omega_n)t\right] + \frac{\varphi_n}{2}\sin\left[(\omega-\omega_n)t\right]$ peak deviation $\varphi_n = 2\sqrt{10^{-\text{SSB}/10}}$

Hence rms phase deviation $\varphi_{rms} = \varphi_n / \sqrt{2}$ so that $\varphi_{rms}^2 = 2\times 10^{-\text{SSB}/10}$

Each uncorrelated frequency present in the phase noise, each add a phase error.

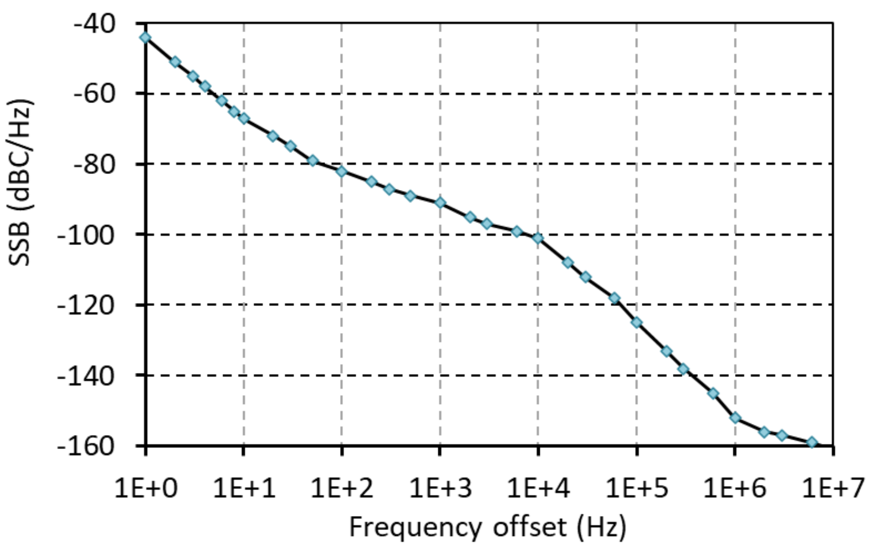
These phase errors come as components of the Fourier transform so are sinusoidal. Random phases add as length vectors in a Brownian random motion i.e.

$$\text{Net distance} = \sum_n \underline{\delta L_i} \approx \sqrt{\sum_n \left|\delta L_i^2\right|}$$

where $\underline{\delta L_i} =$ vectored step length, hence

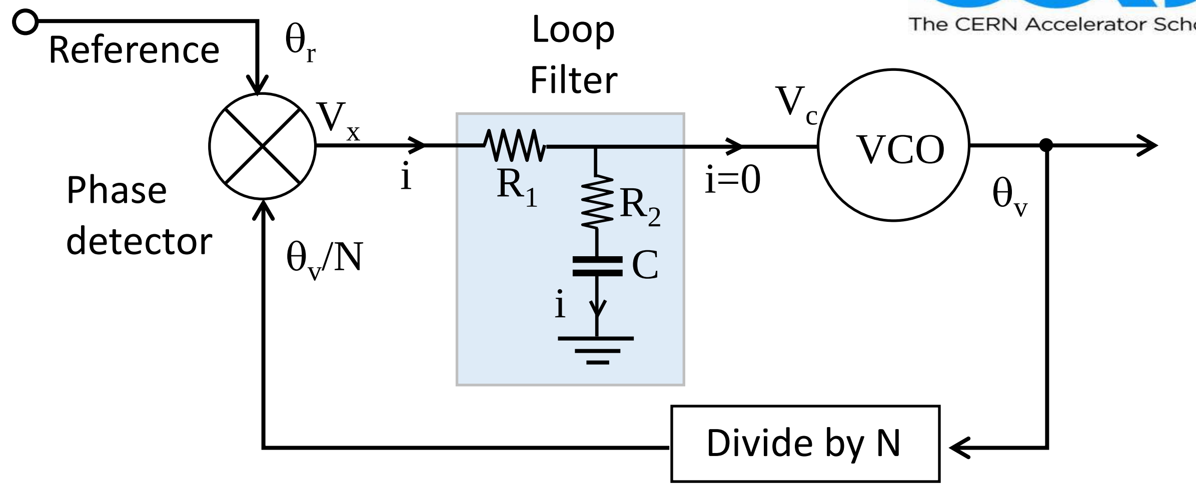
hence $$\varphi_{rms}^2 = \frac{1}{T}\int_0^T \varphi^2(t)\,dt = 2\int_{1/T}^{\infty} 10^{-\text{SSB}/10}\,df$$

where φ(t) is the time domain jitter and f is the frequency. Application gives rms jitters of 0.655º, 0.247º and 0.189º when one integrates from 10 MHz to 1 Hz, 10 Hz and 100 Hz respectively.



Phase locked loops

Configuration where phase comparison is made at the reference frequency. Communication systems typically include further division so that frequencies are compared at a sub harmonic of the reference frequency.



A phase locked loop (PLL) uses a frequency divider in a feedback loop to force a VCO to oscillate at a multiple frequency of an input reference frequency. At low frequency offsets from the centre frequency, phase noise is constrained to follow the performance of the reference. At high frequencies the loop filter ceases to be effective and noise is determined by the performance of the free running VCO.

Let the phase of the reference be given as $\theta_r = \omega_r t$

Let the phase of the VCO be given as $\theta_v = \omega_v t$

Output of mixer $V_x = k_x(\theta_r - \theta_v/N)$

VCO model $\dfrac{d\theta_v}{dt} = k_v V_c$

Loop filter equations

$$V_x = i(R_1 + R_2) + (1/C)\int i\,dt$$

$$V_c = iR_2 + (1/C)\int i\,dt$$

PLL model

Take Laplace Transforms of the previous equations

$$V_x = k_x\left(\theta_r - \theta_v/N\right) \qquad V_x = \tilde{i}\left(R_1 + R_2 + \frac{1}{sC}\right) \qquad V_c = \tilde{i}\left(R_2 + \frac{1}{sC}\right) \qquad s\,\theta_v = k_v V_c$$

Solution gives

$$\theta_v = N\frac{1+sCR_2}{1+s\left(\frac{N}{k_x k_r}+CR_2\right)+s^2\left\{C\left(R_1+R_2\right)\frac{N}{k_x k_r}\right\}}\theta_r$$

Second order system as the denominator has two poles in negative half plane as all coefs. in quadratic are positive.

Steady state response is determined by taking the limit of s $\rightarrow$ 0 so that

$$\theta_v = N\theta_r \quad \text{for} \quad s = 0$$

Time constant for loop filter

$$\tau = 2\pi\sqrt{C\left(R_1+R_2\right)\frac{N}{k_x k_v}}$$

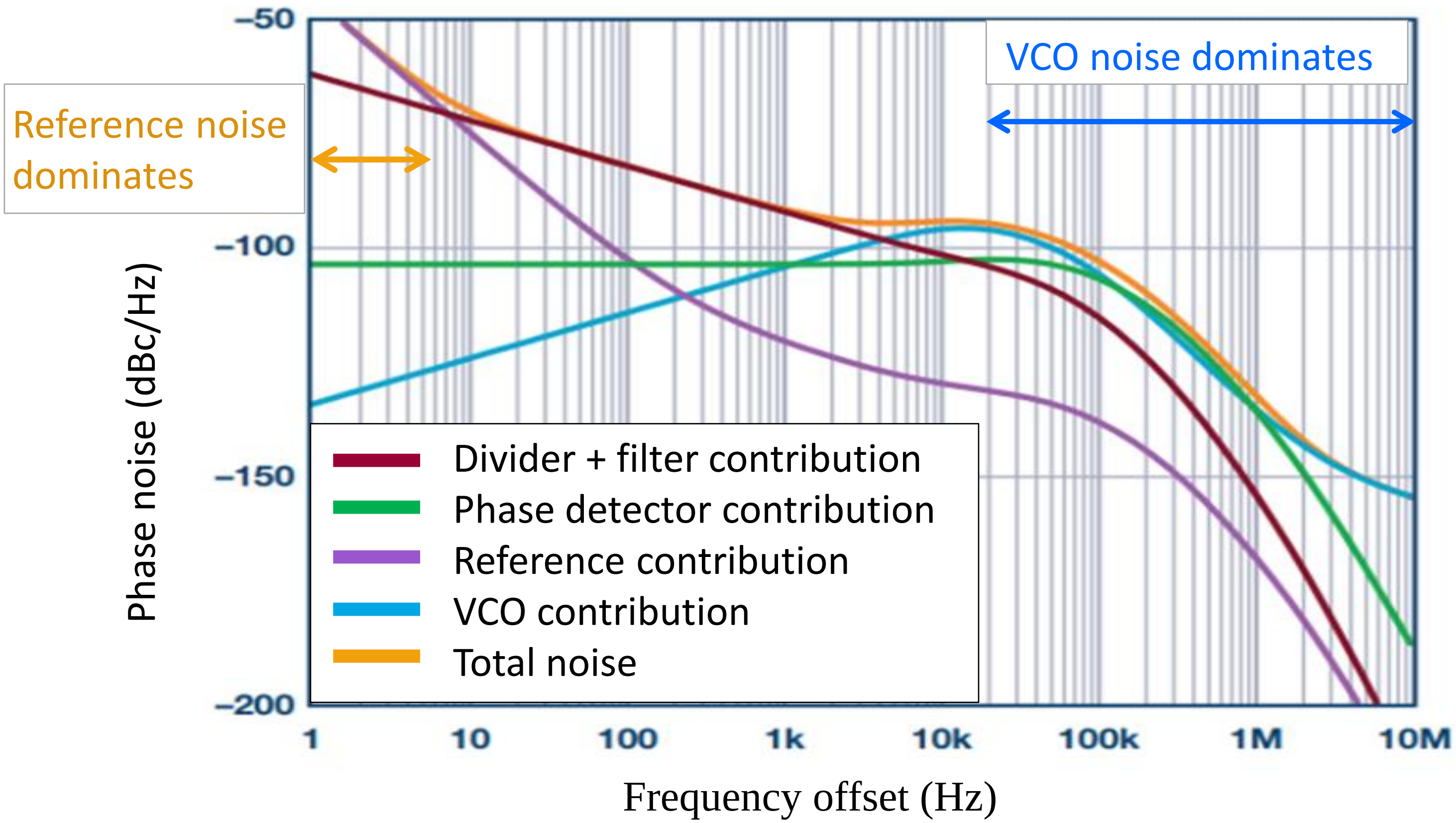
Damping factor

$$\gamma = 0.5\left(R_2 C + \frac{N}{k_x k_v}\right)\sqrt{\frac{k_x k_v}{NC\left(R_1+R_2\right)}}$$

Not included in this model is the time delay for the loop. This limits the gain and hence the minimum time constant τ. Once τ has been made as small as practical, the damping factor can be set. Typically γ is taken as about 0.7 to get an optimally flat frequency transfer characteristic.

PLL simulation

Performing your own PLL analysis is not usually necessary as suppliers like Analog Devices provide software for design using their components. Their specific software is called ADI SimPLL and an example of its output is given. In this example the time constant τ has been set to about 20 μs which is 50 kHz. At higher frequencies the PLL makes no reduction of the free running VCO jitter towards the phase noise level of the reference.



Learning Resources

David M. Pozar, "Microwave Engineering", 4th ed., John Wiley & Sons, Inc.

Reinhold Ludwig and Gene Bogdanov, "RF Circuit Design", 2nd ed. Pearson

https://www.analog.com/en/index.html