Impedance and Wakefields

A. Mostacci, M. Migliorati, L. Palumbo

The CERN Accelerator School

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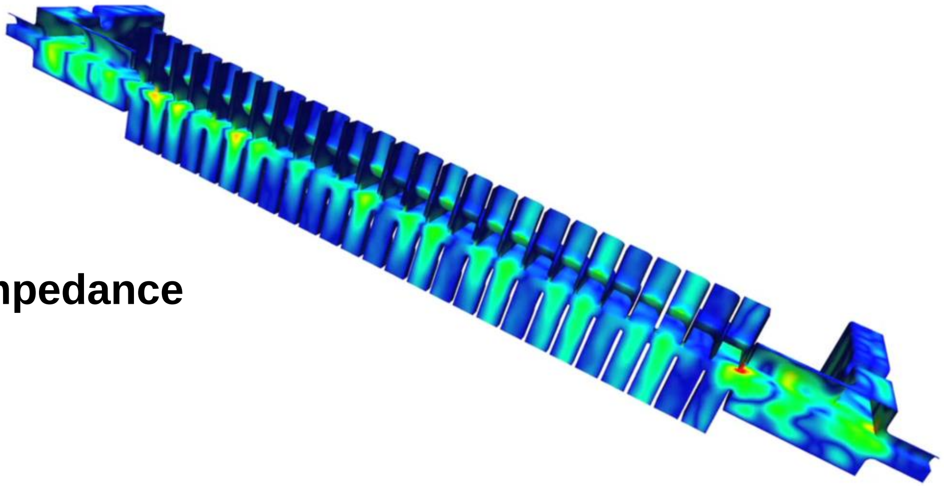
Helmholtz Zentrum Berlin

OUTLINE

Impedance and Wakefields

The wakefields

Perturbative approach and main approximations



Wake functions and wake potentials

Longitudinal and transverse coupling impedance

Examples

Ideas on exploiting the wakefields

Andrea Mostacci, M. Migliorati, L. Palumbo
(Sapienza University, INFN-Roma)

PARTICLE INTERACTION WITH EM FIELDS

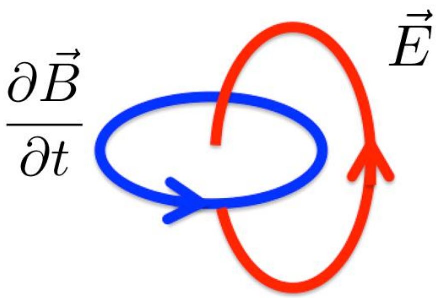
Beam manipulation

Particle acceleration, deflection …

External sources acting on the beam through EM fields.

RF devices (cavities or waveguides)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$



Parasitic effects

Wakefields and coupling impedance

The beam itself is the source of EM fields

Beam Instabilities

Diagnostics

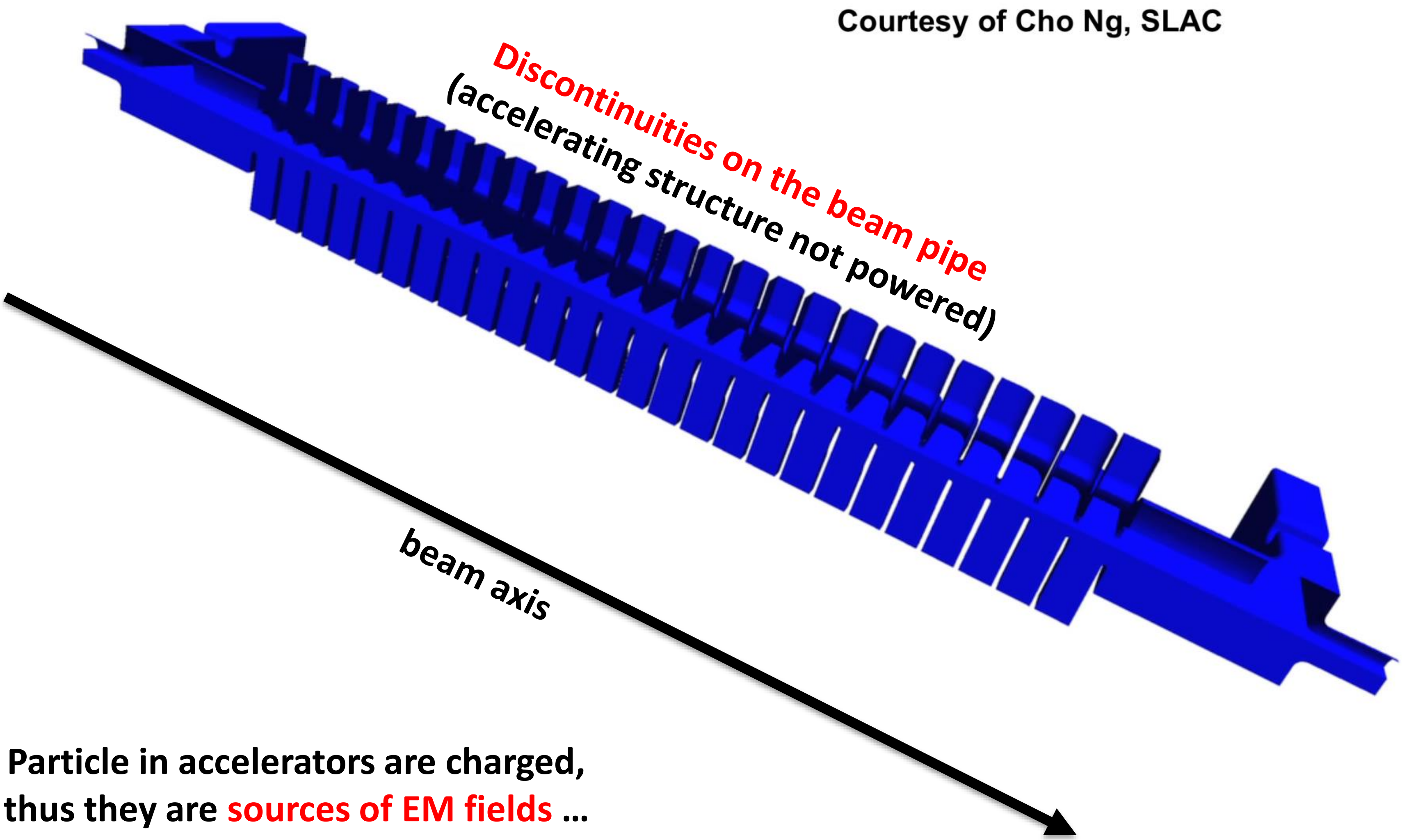
Extraction of beam energy (e.g. klystron)

$$\nabla \cdot \vec{E} = \rho / \epsilon_0$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

Point charge $\vec{J} = \rho \vec{v} = \frac{Q}{2\pi r} \delta(r) \delta(z - vt) \vec{v}$

PARASITIC EFFECTS: THE WAKEFIELDS

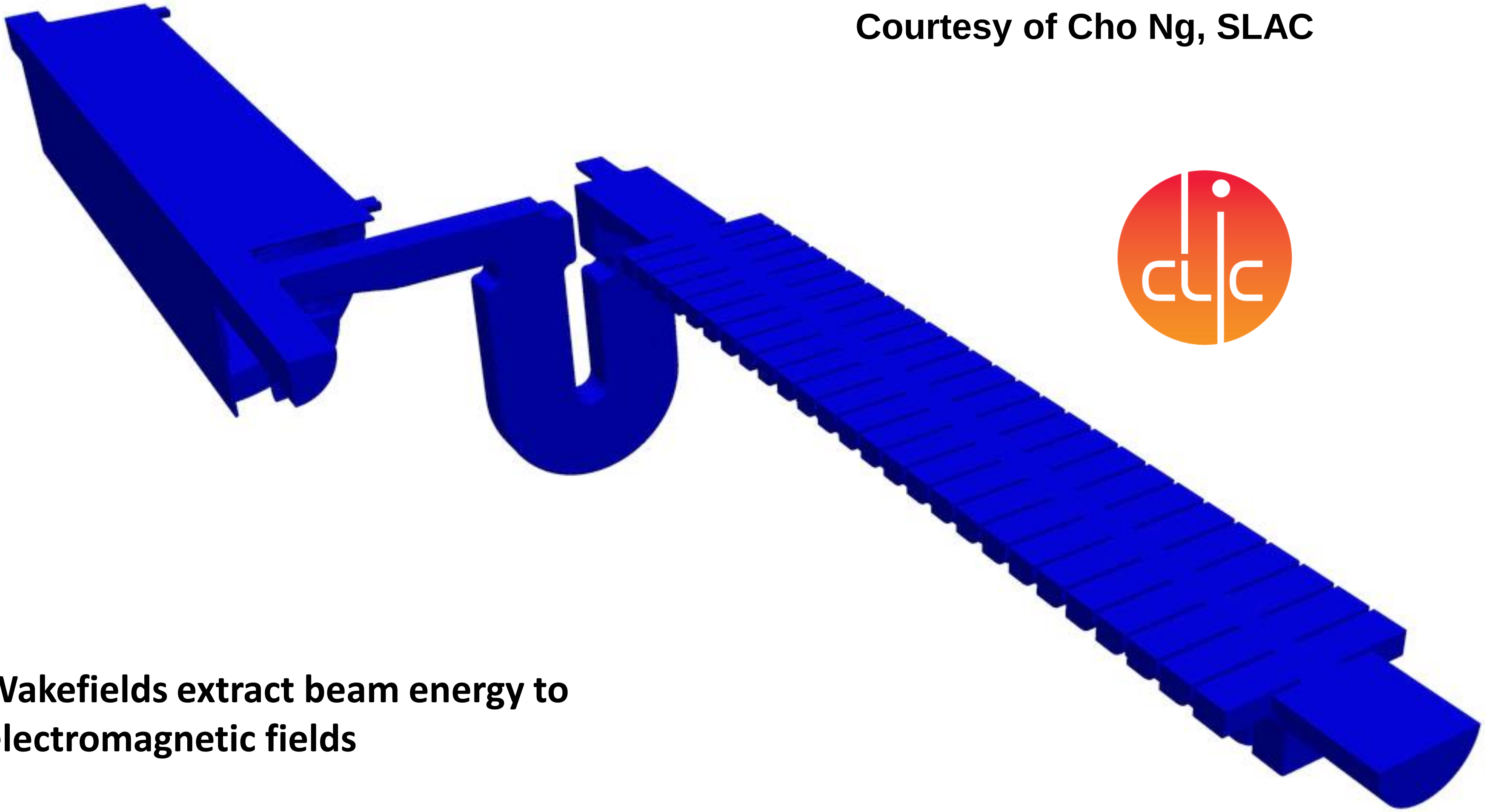


Particle in accelerators are charged, thus they are sources of EM fields …

Andrea Mostacci, M. Migliorati, L. Palumbo
(Sapienza University, INFN-Roma)

WAKEFIELD AND ENERGY CONSERVATION

Courtesy of Cho Ng, SLAC



Wakefields extract beam energy to electromagnetic fields

The principle is used in general purpose RF sources (e.g. klystrons) as well as for direct particle acceleration (e.g. particle wakefield accelerators)

Andrea Mostacci, M. Migliorati, L. Palumbo
(Sapienza University, INFN-Roma)

A PERTURBATIVE APPROACH

Accelerator physics is similar to plasma physics; they use, for instance, Vlasov equation. Both involve nonlinear dynamics (single-particle effects) and collective instabilities (multi-particle effects). However, there is an important difference:

beam self fields > external applied fields (plasma)
beam self fields << external applied fields (accelerators)

This difference means perturbation techniques are applicable to accelerators with

unperturbed motion = external fields (cavities, magnets, …)
perturbed motion = self fields or "wakefields"

In fact, in accelerator physics, a first order perturbation often suffices.

It is important to appreciate the fact that instability analysis in accelerators is based on the validity of this perturbation technique.

In particular, the concept of wakefield is based on the validity of this perturbation technique as applied to high energy accelerators.

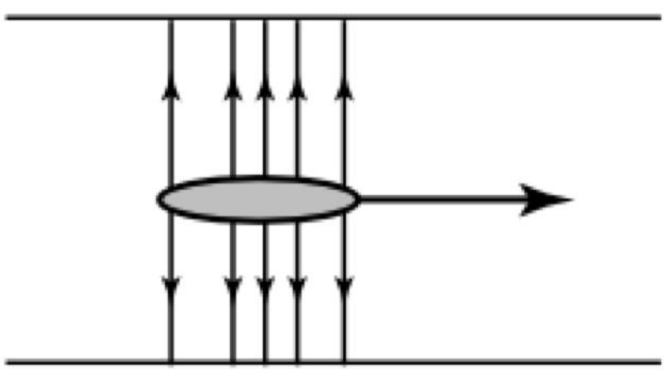
A. Chao, Lecture Notes on Special Topics in Accelerator Physics

BEAM - STRUCTURE INTERACTION

Most wakefields are generated by beam-structure interaction.

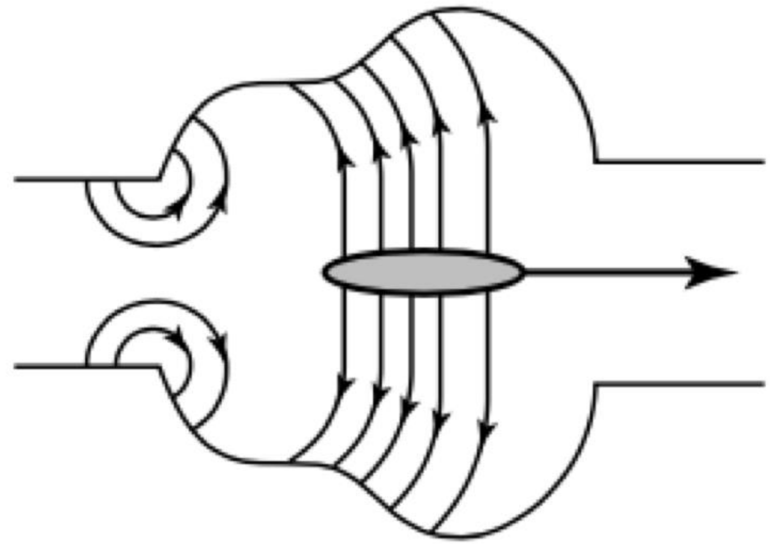
Smooth Pipe

No wakefield



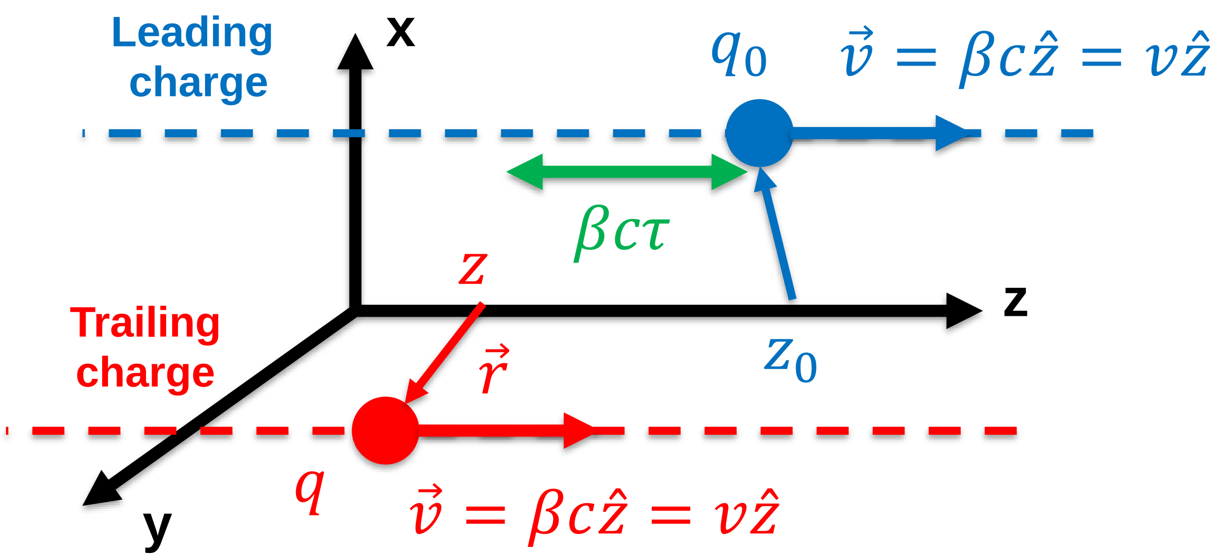
Pipe with structure

Wakefield



Beam-structure interaction is a difficult problem in general. Its solution often involves numerical solution using particle-in-cell (PIC) codes. Applying PIC codes is reasonable for small devices such as electron guns and klystrons, but becomes impractical for large accelerators.

THE PROBLEM



$$z_0(t=0)=0$$

$$z_0(t)=\beta ct$$

$$z(t)=\beta c(t-\tau)$$

q_0 **produces** $\vec{E}, \vec{B}$ **Maxwell equations + Boundary conditions**

Force acting on the trailing charge q

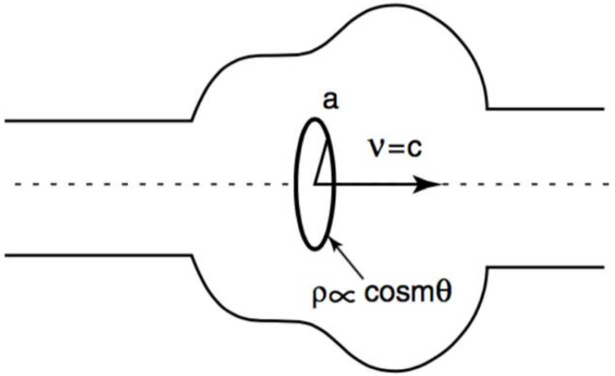
$$\vec{F} = q\left[\vec{E}(\vec{r}, z, \vec{r_0}, z_0; t) + \vec{v} \times \vec{B}(\vec{r}, z, \vec{r_0}, z_0; t)\right]$$

They depend on q_0**, i.e. on beam intensity**

No external fields such as accelerating, focusing, deflecting …

RIGID BEAM APPROXIMATION

At high energies, beam motion is little affected during the passage of a structure. This means one can calculate the wakefields assuming the **beam shape is rigid** and its motion is **ultra-relativistic**.

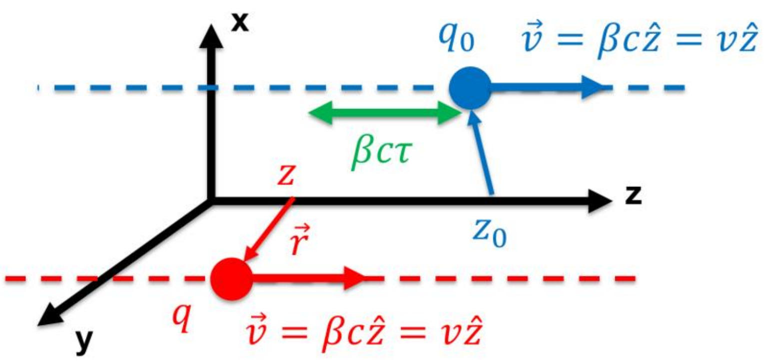


wakefield by a rigid *cos(mθ)* ring beam

m = 0 is monopole moment (net charge)

m = 1 is dipole moment, …

Wake field of a general beam can be obtained by **superposition** of wakefield due to the ring beams with different **m indexes** and different **ring radii**.



Rigid motion of q and q_0.

β doesn't change during the motion

$z_0 - z = \beta c\tau$ constant

$\gamma \gg 1$ High energy

IMPULSE APPROXIMATION

We need to know the impulse of the Lorentz force integrated along the unperturbed trajectory of the trailing charge q.

$\beta c\tau$ **is fixed**

Rigid beam

$$\overrightarrow{\Delta p}(\vec{r},\overrightarrow{r_0};\tau) = \int_{-\infty}^{+\infty} q\left[\vec{E}(\vec{r},z,\overrightarrow{r_0},z_0;t) + \vec{v} \times \vec{B}(\vec{r},z,\overrightarrow{r_0},z_0;t)\right]dt$$

wakefield are much difficult to compute than Δp

The wakefield is localized to the neighbourhood of the finite length structure in the beam pipe.

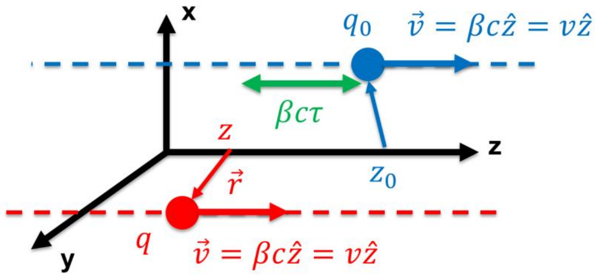
The beam is rigid during the passage, Δp will affect the beam motion after the passage

Kick received by the trailing charge Δp_z $\overrightarrow{\Delta p_\perp}$ **since** F_z $\overrightarrow{F_\perp}$

Andrea Mostacci, M. Migliorati, L. Palumbo
(Sapienza University, INFN-Roma)

PROPERTIES OF LORENTZ FORCE IMPULSE

∇ **applied to trailing charge coordinates** $(\vec{r}, t)$



Rigid beam

$$z(t) = vt - v\tau$$

constant

$$\longrightarrow \quad \frac{\partial}{\partial t}\dots + v\frac{\partial}{\partial z}\dots = \frac{d}{dt}\dots$$

Maxwell Equations

$$\nabla \times \Delta\vec{p} = q\int_{-\infty}^{-\infty} \nabla \times \vec{F}\mathrm{d}t = \cdots = -q\int_{-\infty}^{-\infty} \frac{\mathrm{d}}{\mathrm{d}t}\vec{\mathrm{B}}(\vec{r},\vec{r}_0,\mathrm{z};\mathrm{t})\mathrm{d}t =$$

$$= -q\left[\vec{B}(t=+\infty) - \vec{B}(t=-\infty)\right] = 0$$

wakefield generated in finite structures in finite time

$$\nabla \times \Delta\vec{p} = 0$$

- **Sometimes referred as Panofsky-Wenzel theorem**
- **No particular beam shape/beam pipe assumed**
- **It is valid for any β**

Andrea Mostacci, M. Migliorati, L. Palumbo
(Sapienza University, INFN-Roma)

MOMENTUM KICK AND POTENTIALS

$$\nabla \times \Delta\vec{p} = 0$$

$\Delta\vec{p}$ **is irrotational thus it admits potentials.**

$$\vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t} \qquad \vec{B} = \nabla \times \vec{A}$$

Maxwell Equations

Rigid beam approximation

$$\Delta\vec{p} = -q\nabla \int_{-\infty}^{\infty} (\varphi - vA_z)dt$$

$$\vec{B} = \nabla \times (A_z\hat{z}) \longrightarrow$$

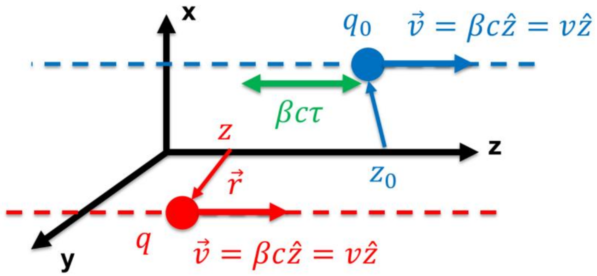
Only Transverse Magnetic (TM) modes can give momentum kicks (longitudinal and/or transverse)

W.K.H. Panofsky and W.A. Wenzel. Some considerations concerning the transverse deflection of charged particles in radio-frequency fields. *Review of Scientific Instruments*, 27, 1956.

WAKE FUNCTION

Rigid beam $z(t) = z_0(t) - s = z_0(t) - \beta c \tau$

constant constant



Charges are moving along the z-axis

$$\beta c \overrightarrow{\Delta p}(\vec{r}, \overrightarrow{r_0}; s) = \int_{-\infty}^{+\infty} \vec{F}\left(\vec{r}, \overrightarrow{r_0}, z, z_0; t = \frac{z+s}{\beta c}\right) dz$$

Definition **Wake function**

$$\vec{w}(\vec{r}, \overrightarrow{r_0}; s) = -\frac{\beta c}{q q_0} \overrightarrow{\Delta p}(\vec{r}, \overrightarrow{r_0}, s) = \hat{z}\, w_z(\vec{r}, \overrightarrow{r_0}; s) + \overrightarrow{w_\perp}(\vec{r}, \overrightarrow{r_0}, s) \qquad \text{(V/C)}$$

Longitudinal wake function — **Beam energy variation**

Transverse wake function — **Beam deflection at constant energy**

WAKE FUNCTION AND WAKE POTENTIAL

$$\vec{w}(\vec{r},\vec{r_0};s) = -\frac{\beta c}{q q_0}\overrightarrow{\Delta p}(\vec{r},\vec{r_0},s) = \hat{z}\, w_z(\vec{r},\vec{r_0};s) + \overrightarrow{w_\perp}(\vec{r},\vec{r_0},s)$$

The wake function describes the effect on a point charge: it is a Green function.

$$\nabla \times \Delta\vec{p} = 0 \longrightarrow \nabla \times \Delta\vec{w} = 0 \longrightarrow \frac{\partial \overrightarrow{w_\perp}}{\partial z}(\vec{r},z) = -\nabla_\perp w_z(\vec{r},z)$$

Panofsky-Wenzel theorem

Definition: **Wake potential**

$$q\vec{w}(\vec{r},\vec{r_0};s) = -\frac{\beta c}{q_0}\overrightarrow{\Delta p}(\vec{r},\vec{r_0},s) \qquad \text{(V)}$$

$$\beta c \Delta p_z(\vec{r},\vec{r_0},s) = -q q_0 w_z(\vec{r},\vec{r_0};s) \qquad \text{(Joule)}$$

Energy gain/loss

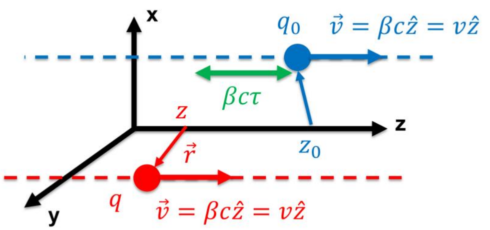
$$\beta c \overrightarrow{\Delta p_\perp}(\vec{r},\vec{r_0},s) = -q q_0 \overrightarrow{w_\perp}(\vec{r},\vec{r_0};s) \qquad \text{(Joule)}$$

Wake-field is not anymore used as a physical quantity

Andrea Mostacci, M. Migliorati, L. Palumbo
(Sapienza University, INFN-Roma)

LONGITUDINAL WAKE FUNCTION AND ENERGY

Trailing charge q



Energy variation is the work done by the longitudinal electric field along the beam trajectory.

$$\beta c\Delta p_z(\vec{r},\vec{r_0},s) = -qq_0 w_z(\vec{r},\vec{r_0};s)$$

deceleration $w_z(\vec{r},\vec{r_0};s)>0$

acceleration $w_z(\vec{r},\vec{r_0};s)<0$

$$w_z(\vec{r},\vec{r_0};s) = -\frac{1}{q_0}\int_{-\infty}^{+\infty} E_z\left(\vec{r},\vec{r_0},z,z_0;t=\frac{z+s}{\beta c}\right)dz$$

No longitudinal contribution of $\vec{v}\times\vec{B}$

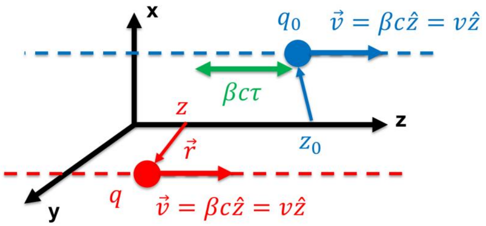
In long uniform or periodic structures

$$\frac{dw_z}{dz}(\vec{r},\vec{r_0};s) = -\frac{1}{q_0}E_z(\vec{r},\vec{r_0},s) \qquad \left(\frac{V}{C\ m}\right)$$

Wake function per unit length

LONGITUDINAL WAKE FUNCTION AND ENERGY

Leading charge q_0 **Energy variation due to its own field.**

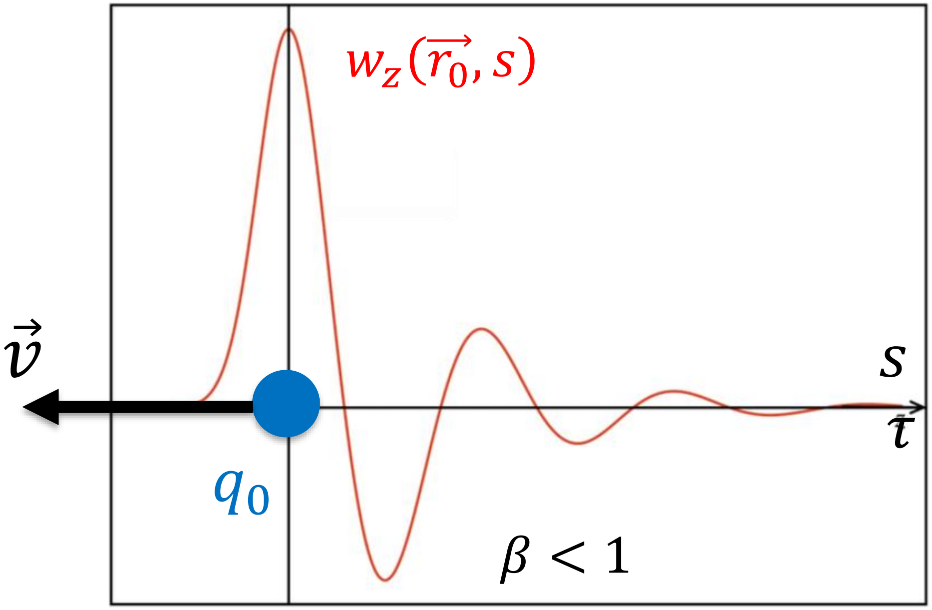


Loss factor

$$k(\vec{r_0}) = w_z(\vec{r_0}; s = 0) = -\frac{1}{q_0}\int_{-\infty}^{+\infty} E_z\left(\vec{r_0}, z; t = \frac{z}{\beta c}\right) dz$$

Energy Loss if k>0

If there is energy lost by the beam, there must be electric field on its path.



$$k(\vec{r_0}) = w_z(\vec{r_0}, 0)$$

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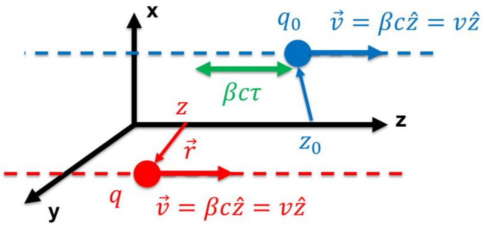
LONGITUDINAL WAKE FUNCTION AND ENERGY

Leading charge q_0 **Energy variation due to its own field.**



Loss factor

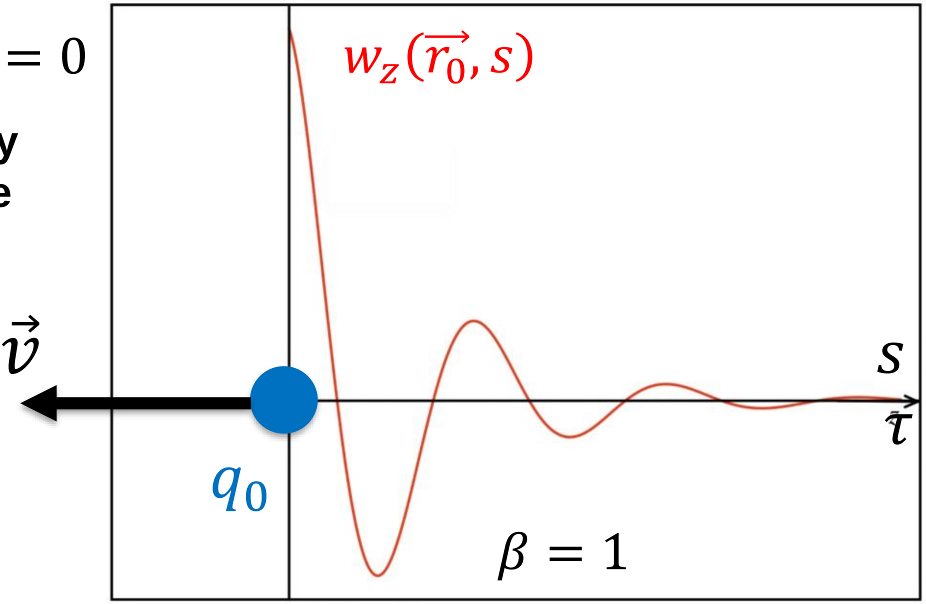
$$k(\overrightarrow{r_0}) = w_z(\overrightarrow{r_0}; s = 0) = -\frac{1}{q_0}\int_{-\infty}^{+\infty} E_z\left(\overrightarrow{r_0}, z; t = \frac{z}{\beta c}\right) dz$$

Energy Loss if k>0

If there is energy lost by the beam, there must be electric field on its path.

$$w_z(s < 0) = 0$$

Causality principle



$$k(\overrightarrow{r_0}) = \frac{w_z(\overrightarrow{r_0}, 0^+)}{2}$$

Beam loading theorem

Andrea Mostacci, M. Migliorati, L. Palumbo
(Sapienza University, INFN-Roma)

WAKE FUNCTION AND SYNCHRONOUS FIELDS

On an infinite path only field components having the phase velocity = β c can exchange energy with the beam.

TM modes

$$E_z(\vec{r},\vec{r_0},z,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}dk\,\widetilde{E_z}(\vec{r},\vec{r_0},\omega,k)e^{j(\omega t-kz)}$$

at any phase velocity ω/k

$$w_z(\dots,\tau)=-\frac{1}{q_0}\int_{-\infty}^{+\infty}E_z\left(\dots,z,t=\frac{z}{\beta c}+\tau\right)dz=$$

$$=-\frac{1}{2\pi q_0}\int_{-\infty}^{\infty}d\omega e^{j\omega\tau}\int_{-\infty}^{\infty}dk\,\widetilde{E_z}(\dots,\omega,k)\int_{-\infty}^{+\infty}e^{-jz(k-\omega/\beta c)}dz$$

$$\delta\left(k-\frac{\omega}{\beta c}\right)$$

$$=-\frac{1}{q_0}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\widetilde{E_z}\left(\omega,k=\frac{\omega}{\beta c}\right)e^{j\omega\tau}d\omega$$

phase velocity = β c **SURFING EFFECT**

Andrea Mostacci, M. Migliorati, L. Palumbo
(Sapienza University, INFN-Roma)

WAKE FUNCTION OF CHARGE DISTRIBUTION (1)

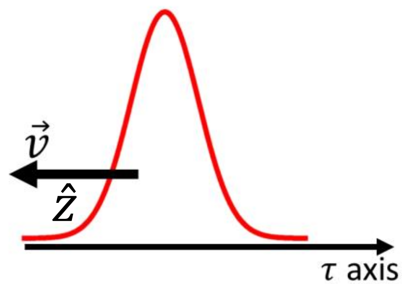
$$w_z(\tau) = -\frac{1}{q_0}\int_{-\infty}^{+\infty} E_z\left(z, t = \frac{z}{\beta c} + \tau\right) dz$$

Point Charge → **Green Function**

Hypothesis: all the particles move on the same trajectory

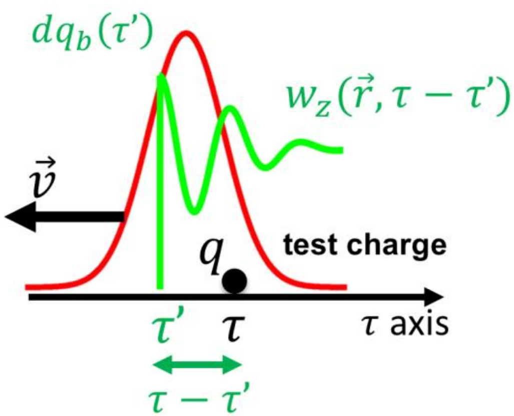
Bunch distribution of total charge

q_b



$$q_b = \int_{-\infty}^{-\infty} i_b(\tau) d\tau$$

GOAL: Compute the wake potential versus the delay τ from the reference particle



Point Charge $dq_b(\tau') = i_b(\tau') d\tau'$ → $w_z(\vec{r}, \tau - \tau')$

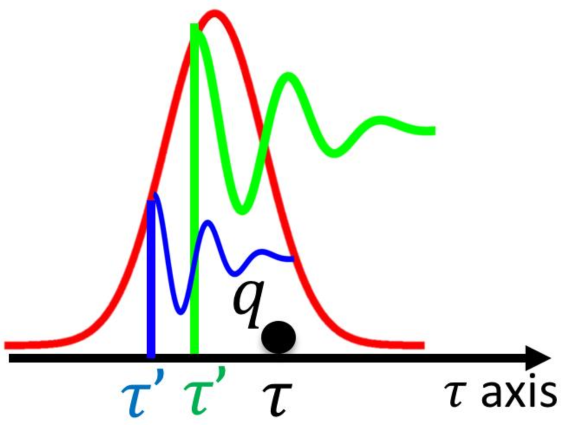
Energy change of the charge q because of the slice at @ τ'

$$dU(\vec{r}, \tau - \tau') = q\, i_b(\tau') w_z(\vec{r}, \tau - \tau') d\tau'$$

Andrea Mostacci, M. Migliorati, L. Palumbo
(Sapienza University, INFN-Roma)

WAKE FUNCTION OF CHARGE DISTRIBUTION (2)

Superposition principle to compute the energy loss/gain by the q due to bunch



Energy loss/gain of the charge q

$$U(\vec{r},\tau) = q \int_{-\infty}^{+\infty} i_b(\tau')\, w_z(\vec{r},\tau-\tau')d\tau'$$

Wake function of a charge distribution

$$W_z(\vec{r},\tau) = \frac{U(\vec{r},\tau)}{q\, q_b} = \frac{1}{q_b}\int_{-\infty}^{+\infty} i_b(\tau')\, w_z(\vec{r},\tau-\tau')d\tau' \quad (\mathrm{V/C})$$

Energy lost per unit charge

$$\frac{U(\vec{r},\tau)}{q} = q_b\, W_z(\vec{r},\tau) \quad (\mathrm{V})$$

Wake potential versus the delay τ of the test charge q

ENERGY LOSS OF CHARGE DISTRIBUTION

Energy loss by the bunch $U_{bunch} = \sum$ **Energy lost per unit charge** $\cdot$ **Charge of the bunch slice** $= \int_{-\infty}^{+\infty} q_b \, W_z(\vec{r},\tau) \, i_b(\tau) d\tau$

Loss Factor $$K(\vec{r}) = \frac{U_{bunch}}{q_b^2} = \frac{U(\vec{r})}{q_b^2} = \frac{1}{q_b} \int_{-\infty}^{+\infty} W_z(\vec{r},\tau) \, i_b(\tau) d\tau$$

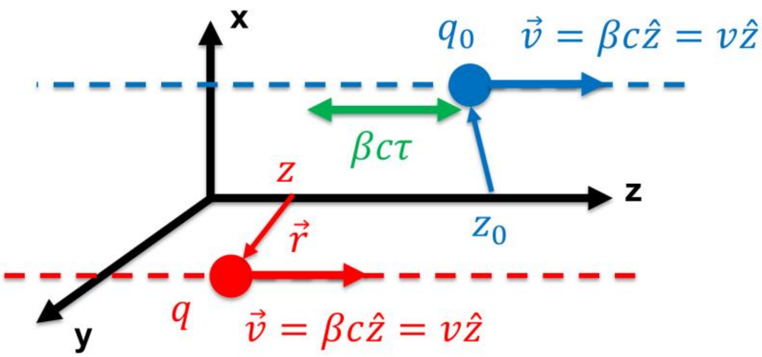
where $$W_z(\vec{r},\tau) = \frac{1}{q_b} \int_{-\infty}^{+\infty} i_b(\tau') \, w_z(\vec{r},\tau-\tau') d\tau'$$

If $\beta = 1$ $\quad w_z(\vec{r},\tau) \neq 0$ only for $\tau > 0 \implies \tau' < \tau$

$$W_z(\vec{r},\tau) = \frac{1}{q_b} \int_{-\infty}^{\tau} i_b(\tau') \, w_z(\vec{r},\tau-\tau') d\tau'$$

Causality principle

TRANSVERSE WAKE FUNCTION



Transvers kick normalised to both charges

$$\overrightarrow{w_\perp}(\vec{r}, \overrightarrow{r_0}, s) = -\frac{\Delta\overrightarrow{p_\perp}(\vec{r}, \overrightarrow{r_0}, s)}{q_0\, q} \qquad (\mathrm{V/C})$$

The dipole transverse kick is dominant and proportional to $\overrightarrow{r_0}$

Transverse dipole wake function

$$\overrightarrow{w_\perp}'(\vec{r}, \overrightarrow{r_0}, s) = \frac{\overrightarrow{w_\perp}(\vec{r}, \overrightarrow{r_0}, s)}{r_0}$$

Transverse wake per unit of transverse displacement

Charge distribution (bunch)

$$\overrightarrow{W_\perp}(\vec{r}, \tau) = \frac{1}{q_b}\int_{-\infty}^{+\infty} \overrightarrow{w_\perp}(\vec{r}, \tau - \tau')\, i_b(\tau')\, d\tau' \qquad (\mathrm{V/C})$$

$$\overrightarrow{W_\perp}'(\vec{r}, \tau) = \frac{\overrightarrow{W_\perp}(\vec{r}, \tau)}{r_0} \qquad \left(\frac{\mathrm{V}}{\mathrm{C\ m}}\right)$$

LONGITUDINAL COUPLING IMPEDANCE

Wake functions → **Beam dynamics in Linear Accelerators**

Circular Accelerators → **Periodic motion** → **Spectrum of the Wake functions**

$$Z_{\parallel}(\vec{r}, \overrightarrow{r_0}, \omega) = \int_{-\infty}^{+\infty} w_z(\vec{r}, \overrightarrow{r_0}, \tau) e^{-j\omega\tau} d\tau \qquad (\Omega)$$

$$w_z(\vec{r}, \overrightarrow{r_0}, \tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} Z_{\parallel}(\vec{r}, \overrightarrow{r_0}, \omega) e^{j\omega\tau} d\omega \qquad (\mathrm{V/C})$$

Andrea Mostacci, M. Migliorati, L. Palumbo
(Sapienza University, INFN-Roma)

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Surfing effect

$$w_z(\vec{r},\vec{r_0},\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} -\frac{\widetilde{E_z}(\omega, k=k_0)}{q_0} e^{j\omega\tau} d\omega$$

$$Z_{\parallel}(\omega) = -\frac{\widetilde{E_z}(\omega, k=k_0)}{q_0}$$

$$Z_{\parallel,real}(\omega) > 0$$

Energy Loss

E_z opposing the motion

Andrea Mostacci, M. Migliorati, L. Palumbo
(Sapienza University, INFN-Roma)

LONGITUDINAL IMPEDANCE AND LOSS FACTOR

$\boldsymbol{\beta \to 1}$

Imaginary part of the impedance is odd

$$k(\overrightarrow{r_0}) = \frac{w_z(\overrightarrow{r_0}, \tau \to 0^+\ )}{2} = \frac{1}{4\pi}\int_{-\infty}^{+\infty} Z_{\parallel}(\overrightarrow{r_0}, \vec{r}, \omega)\, d\omega = \frac{1}{2\pi}\int_{0}^{+\infty} Z_{\parallel,real}(\overrightarrow{r_0}, \vec{r}, \omega)\, d\omega$$

Real part of the longitudinal coupling impedance is the power spectrum of the energy loss of a unit point charge

Charge distribution

Fourier Transform

$i_b(t)$ **(Ampere)** ⟷ $I(\omega)$ **(Coulomb)**

real **symmetric**

$$W_z(\vec{r}, \tau) = \frac{1}{2\pi q_b}\int_{-\infty}^{+\infty} Z_{\parallel,real}(\vec{r}, \omega) I(\omega) e^{j\omega\tau} d\omega$$

$$K(\vec{r}) = \frac{1}{\pi q_b^2}\int_{-\infty}^{+\infty} Z_{\parallel,real}(\vec{r}, \omega) |I(\omega)|^2 d\omega$$

TRANSVERSE COUPLING IMPEDANCE

$$\overrightarrow{Z_\perp}(\vec{r},\overrightarrow{r_0},\omega) = j\int_{-\infty}^{+\infty} \overrightarrow{w_\perp}(\vec{r},\overrightarrow{r_0},\tau)e^{-j\omega\tau}d\tau$$

Panofsky-Wenzel relation

$$\frac{\partial}{\partial\tau}\overrightarrow{w_\perp}(\vec{r},\tau) = -c\nabla_\perp w_z(\vec{r},\tau) \quad \Rightarrow \quad \overrightarrow{Z_\perp}(\vec{r},\overrightarrow{\mathrm{r}_0},\omega) = \frac{c}{\omega}\nabla_\perp Z_\parallel(\vec{r},\overrightarrow{\mathrm{r}_0},\omega)$$

Dipole term in cylindrical symmetry

$$\overrightarrow{Z_\perp}(\vec{r},\overrightarrow{r_0},\omega) = \frac{c}{\omega}\overline{\overline{Z_1}}(\omega)\,\overrightarrow{r_0}$$

Andrea Mostacci, M. Migliorati, L. Palumbo
(Sapienza University, INFN-Roma)

RESISTIVE WALL IMPEDANCE

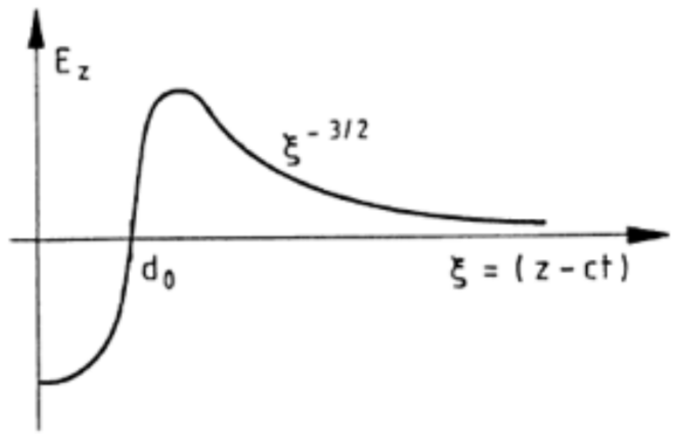


Fig. 7a The longitudinal field behind the source charge for a lossy pipe ($\beta = 1$)

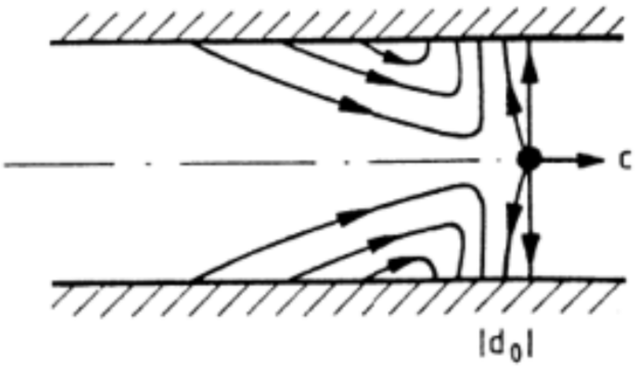


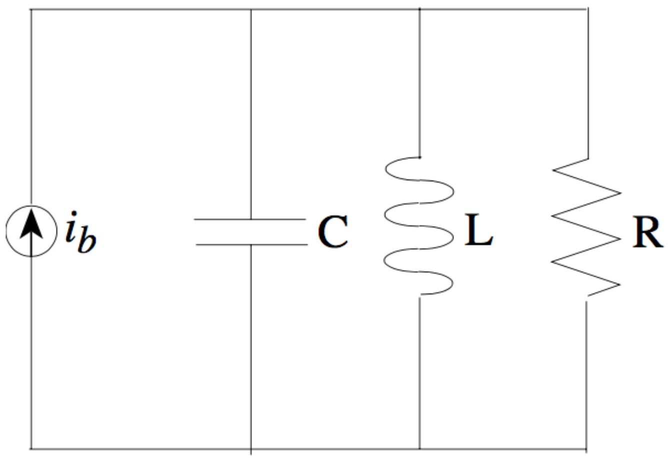
Fig. 7b Circular pipe of finite conductivity σ

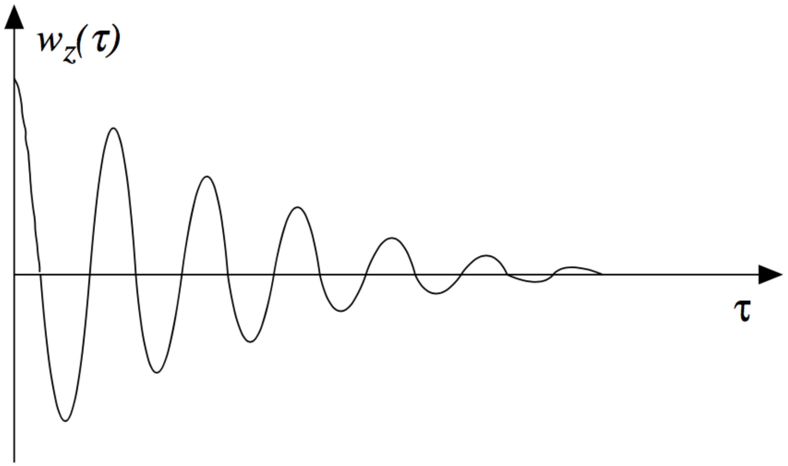
$$\frac{\partial \bar{Z}_{m=0}}{\partial z} = \frac{1+j}{2\pi b}\sqrt{\frac{\omega Z_o}{2c\sigma}}$$

$$\frac{\partial Z_{\perp,1}}{\partial z} = \frac{1+j}{2\pi b^3} Z_o \delta$$

CAVITY HOM: IMPEDANCE AND WAKE FUNCTION

Effect of high order mode (HOM) in cavities





$$w_z(\tau) = \frac{V(\tau)}{q_1} = \frac{e^{-\Gamma\tau}}{C}\left[cos(\overline{\omega}_r\tau) - \frac{\Gamma}{\overline{\omega}_r} sin(\overline{\omega}_r\tau)\right]H(\tau)$$

$$2\Gamma = \frac{1}{RC} \qquad \omega_r^2 = \frac{1}{LC}$$

$$Z(\omega) = \frac{R}{1 + jQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

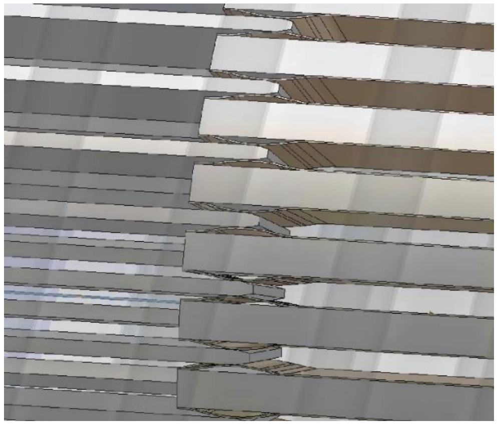
$$\overline{\omega}_r^2 = \omega_r^2 - \Gamma^2 \qquad Q = \frac{\omega_r}{2\Gamma}$$

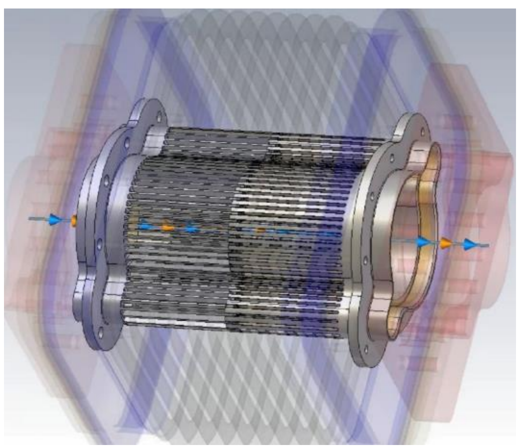
IMPEDANCE DUE TO ACCELERATOR DEVICES

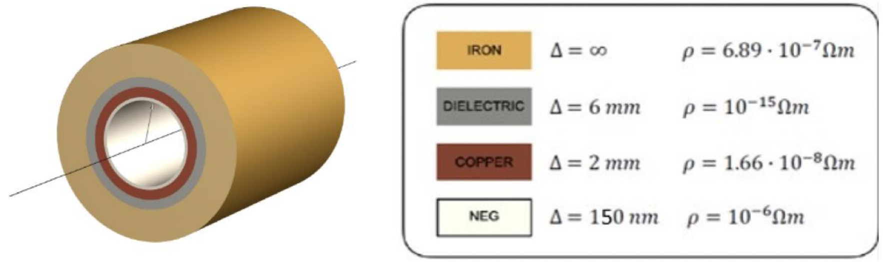
Impedance database for beam dynamics simulation large machines

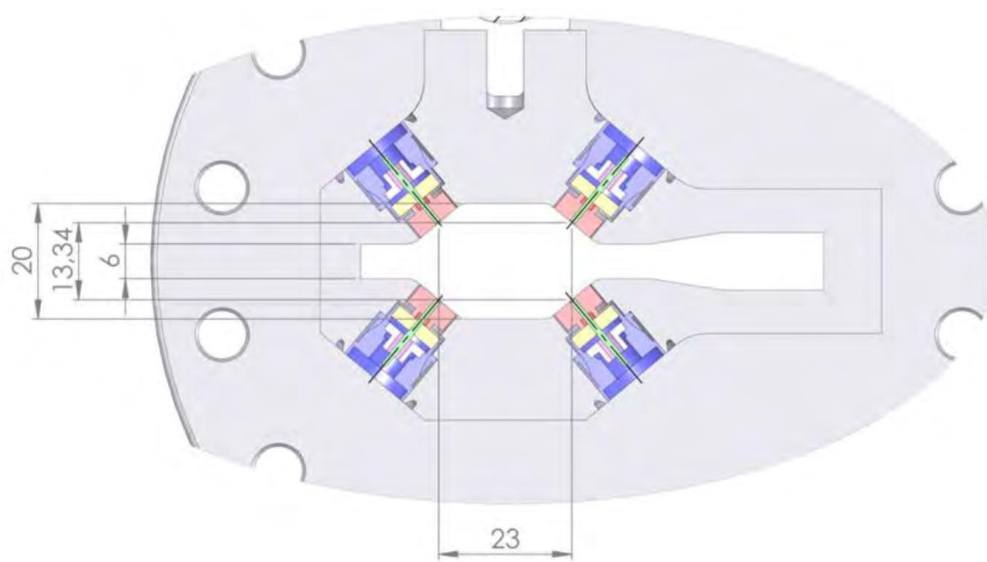
Example

Bellows







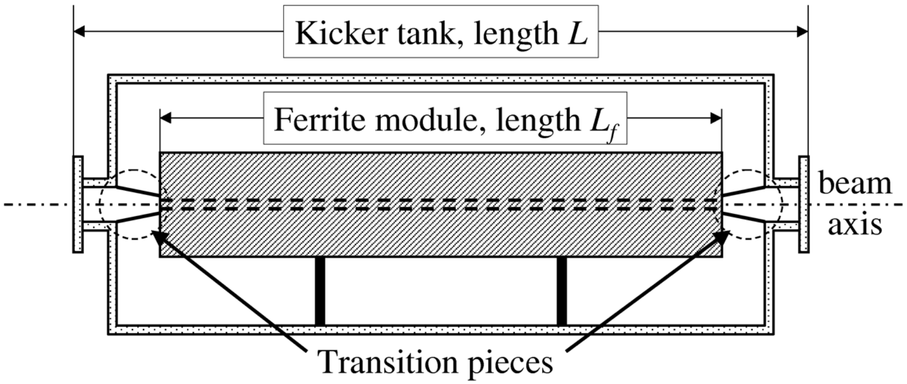


Geometry of a large BPM block (#8 is shown here), button diameter is 8mm.

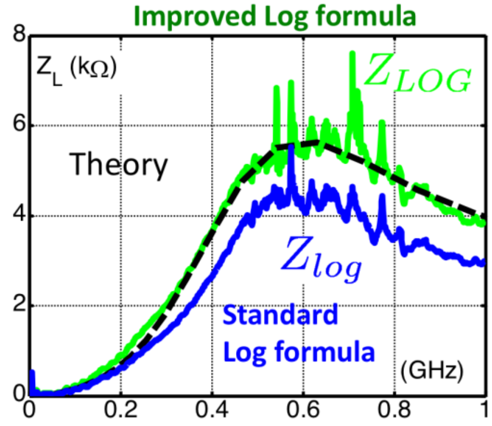
EXISITING MACHINES MAINTENACE @ CERN

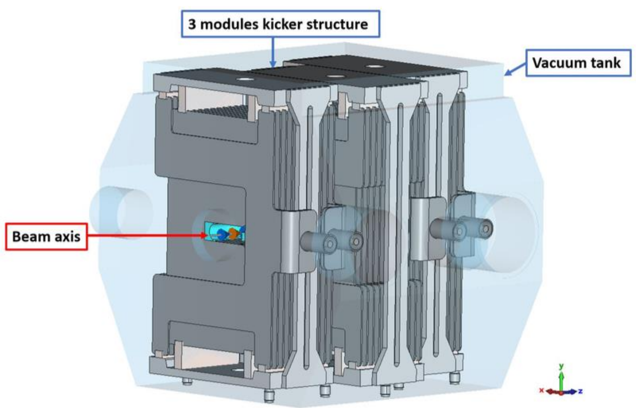
Example

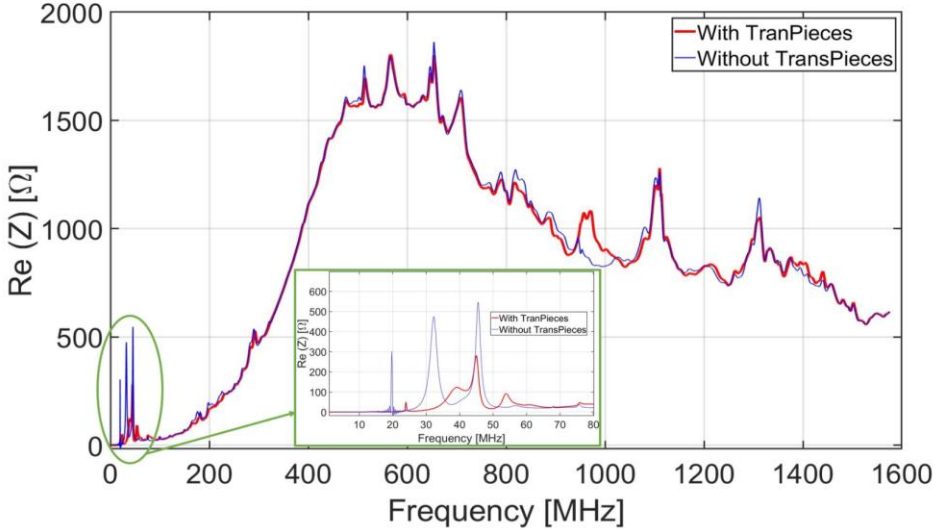
MKE kicker @ SPS: 7 cells module.



A. Mostacci, F. Caspers CERN-SL-2000-071







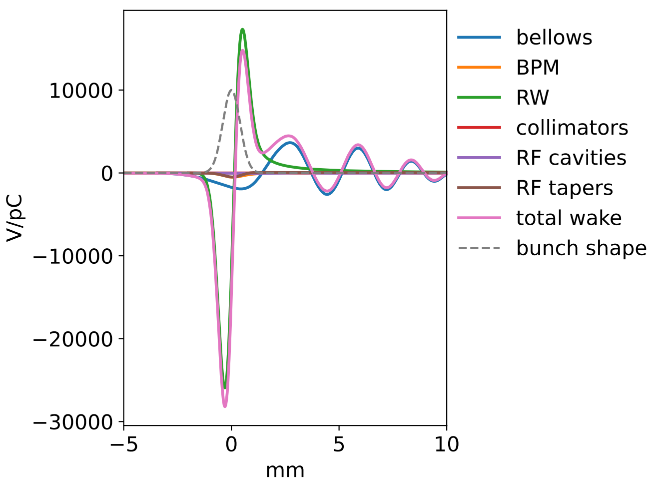
M. Neroni et al, Characterization of the longitudinal beam coupling impedance and mitigation strategy for the fast extraction kicker KFA79 in the cern PS, IPAC2023.

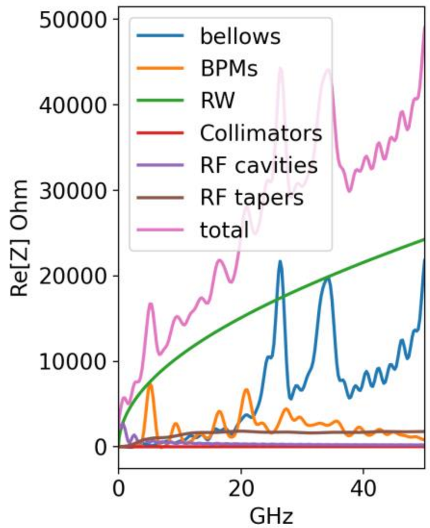
FUTURE CIRCULAR COLLIDER CASE

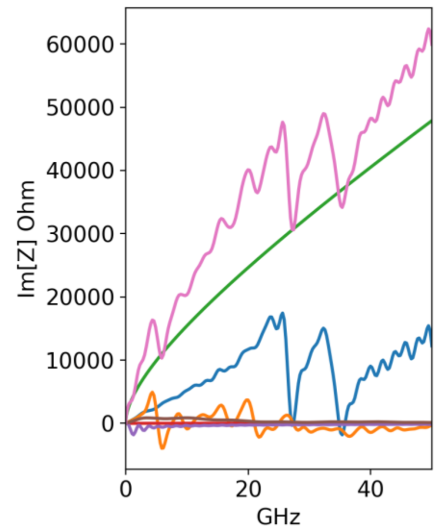
Example

Longitudinal

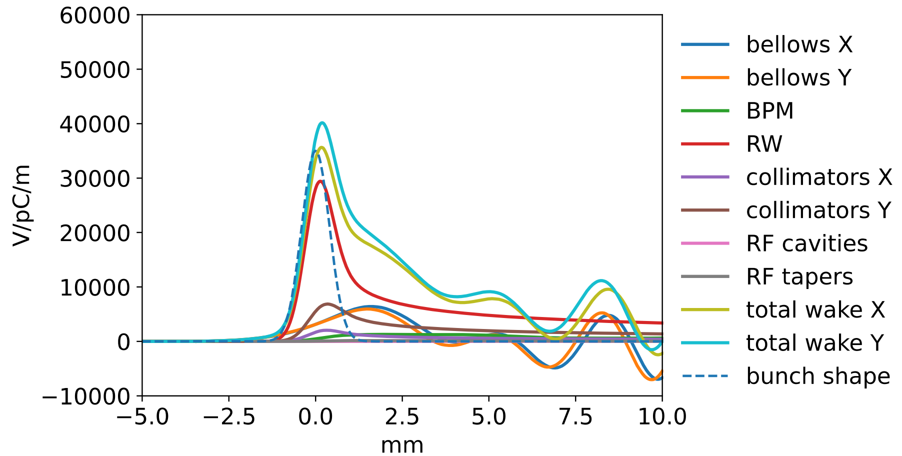
Impedance wise, carefully designed devices, but long machine







Transverse



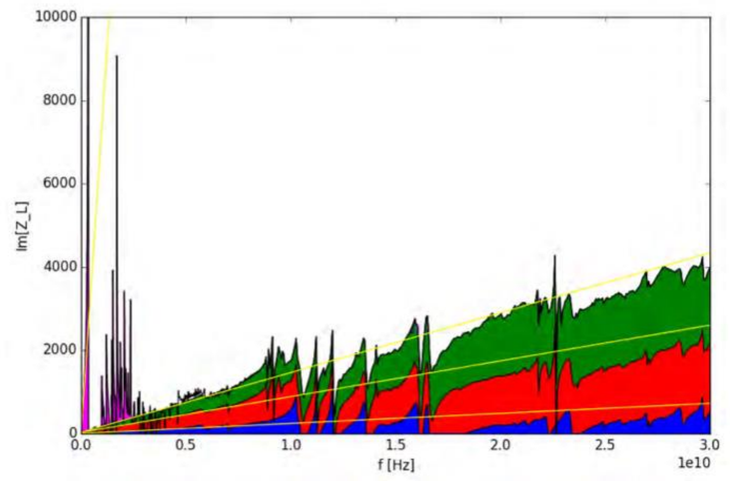
LOW EMITTANCE RING CASE

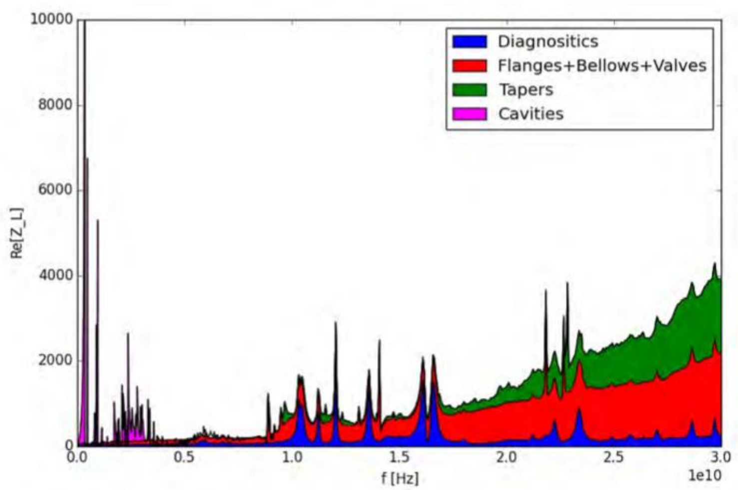
Example

Relevant for novel advanced photon sources with multi-bend achromat lattice

Section	6 x 6	mm
Pipe diameter	3	mm
Copper resistivity	$1.78*10^{-8}$	Ohm m
Insulation	0.5	mm
Conductor area	28.0674	mm^2
Material	1300-100	
Stacking factor	0.98	

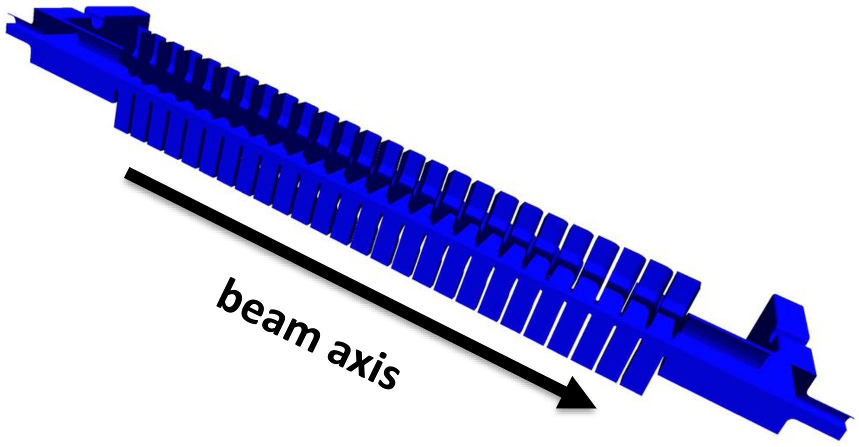
	K_{loss} [V/pC]	Z/n_{eff} [Ω]
Diagnostics	4.02	$8.4\ 10^{-3}$
Flanges+bellows	9.27	$25\ 10^{-3}$
Scrapers	?	?
Tapers	3.2	$20\ 10^{-3}$
Cavities	6.66	$69\ 10^{-3}$
Resistive wall	38.7	0.222
Total	51.85	0.344





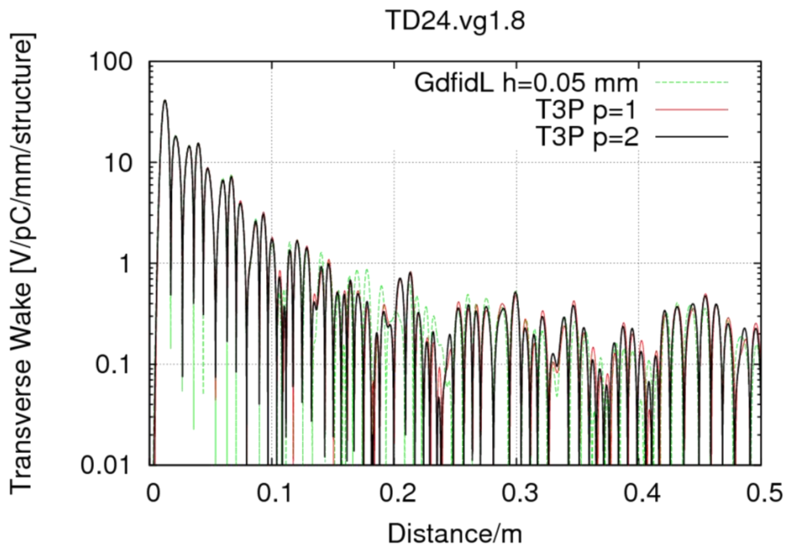
Andrea Mostacci, M. Migliorati, L. Palumbo
(Sapienza University, INFN-Roma)

HIGH ORDER MODES IMPEDANCE

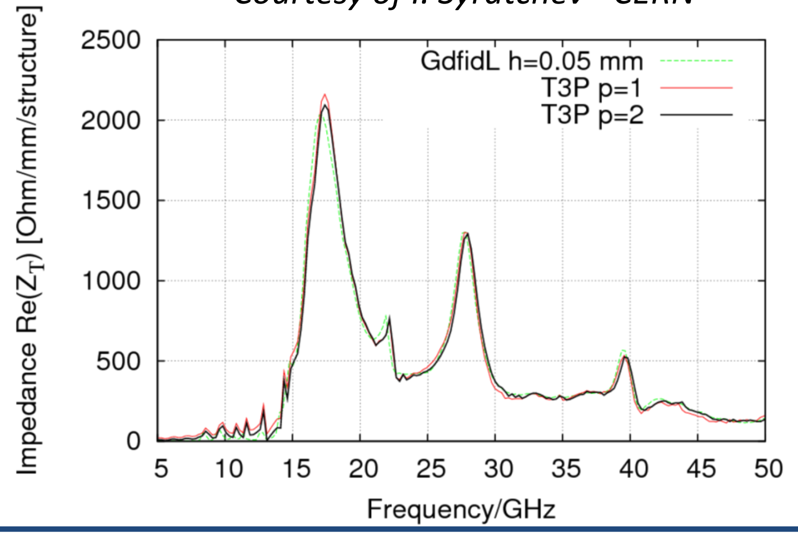


Electric field magnitude shown, one half of the structure
Electric boundary condition in vertical symmetry plane
Gaussian bunch, sigma=2 mm, 1 mm horizontal offset

Cho Ng, XB10 Workshop, Cockcroft Institute, Nov 30 - Dec 3, 2010

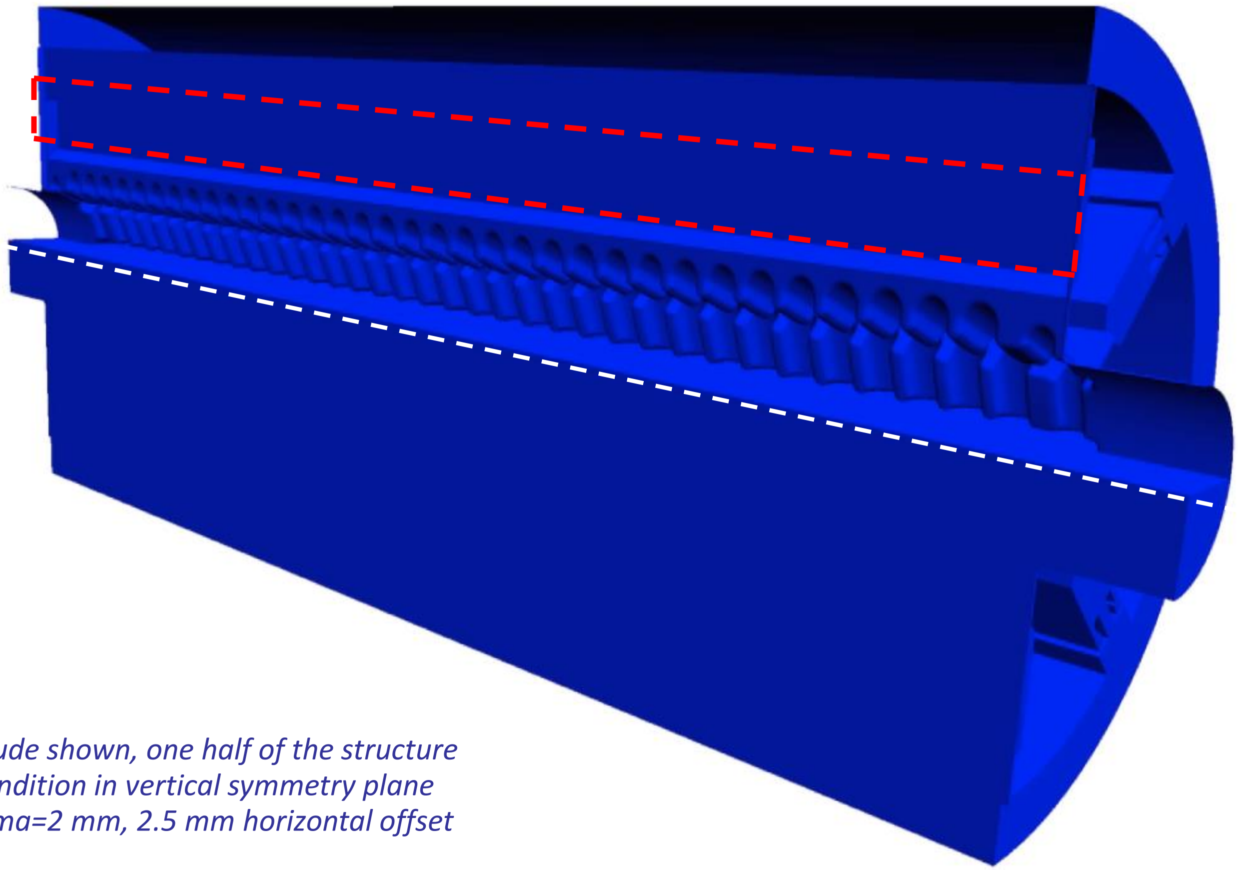


Courtesy of I. Syratchev - CERN



DAMPING OF HIGH ORDER MODES

Dissipation of wakefields in dielectric loads: eps=13, tan(δ)=0.2

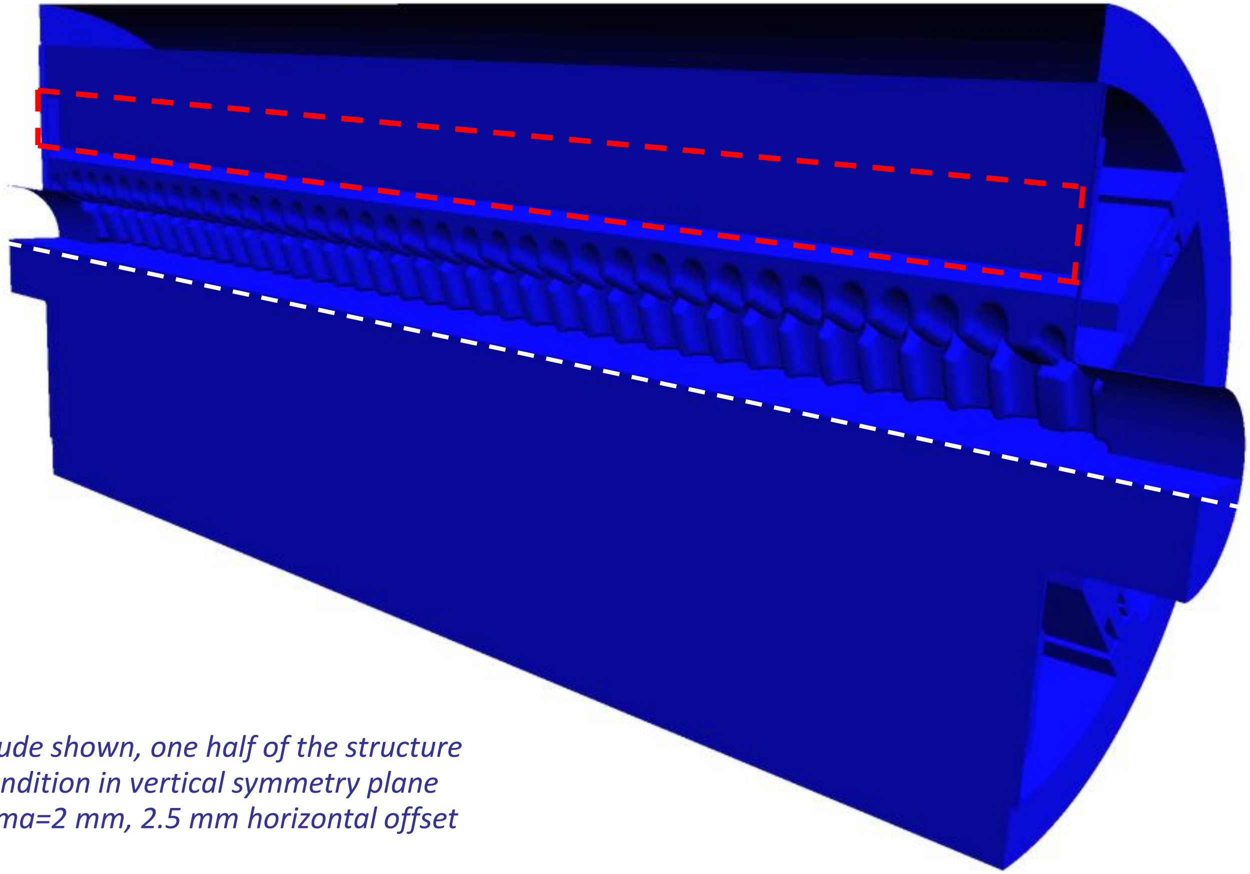


Electric field magnitude shown, one half of the structure
Electric boundary condition in vertical symmetry plane
Gaussian bunch, sigma=2 mm, 2.5 mm horizontal offset

Cho Ng, XB10 Workshop, Cockcroft Institute, Nov 30 - Dec 3, 2010

DAMPING OF HIGH ORDER MODES

Dissipation of wakefields in dielectric loads: eps=13, tan(δ)=0.2



Electric field magnitude shown, one half of the structure
Electric boundary condition in vertical symmetry plane
Gaussian bunch, sigma=2 mm, 2.5 mm horizontal offset

Cho Ng, XB10 Workshop, Cockcroft Institute, Nov 30 - Dec 3, 2010

Andrea Mostacci, M. Migliorati, L. Palumbo
(Sapienza University, INFN-Roma)

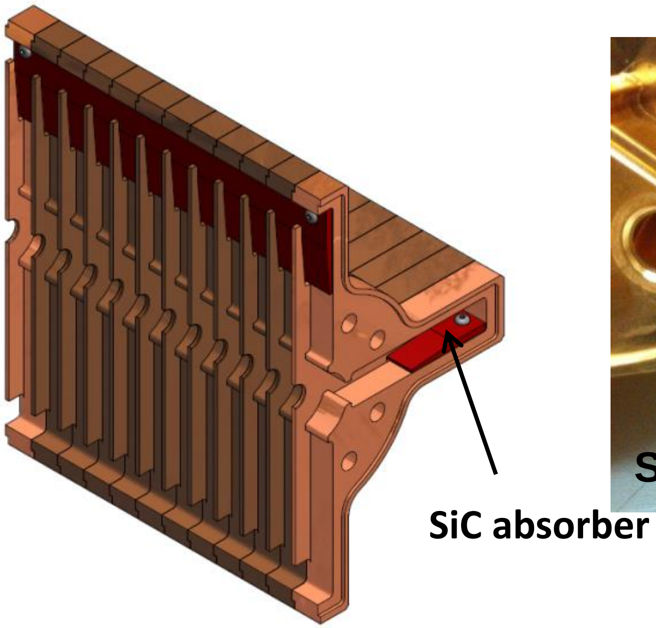
DAMPED C-BAND STRUCTURE

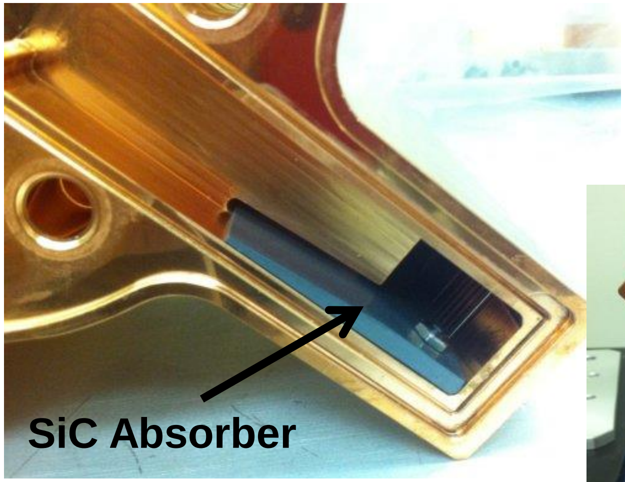
Example

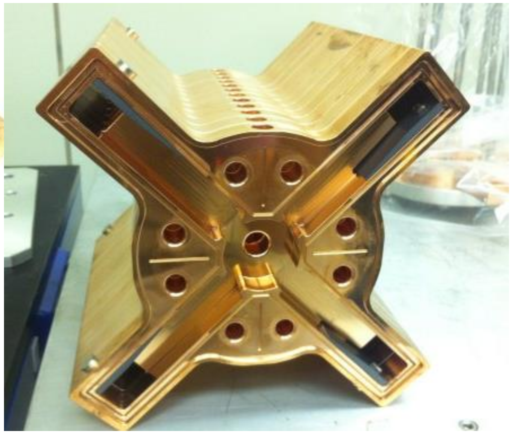
Each cell of the structure has four waveguides that allows the excited HOMs to propagate and dissipate into silicon-carbide (SiC) RF loads.

The SiC tiles have been optimized to avoid reflections and are integrated into the structure.

In each stack of 12 cells there are four SiC long absorbers, each stack is brazed and all stacks are finally assembled and brazed with the input and output couplers.







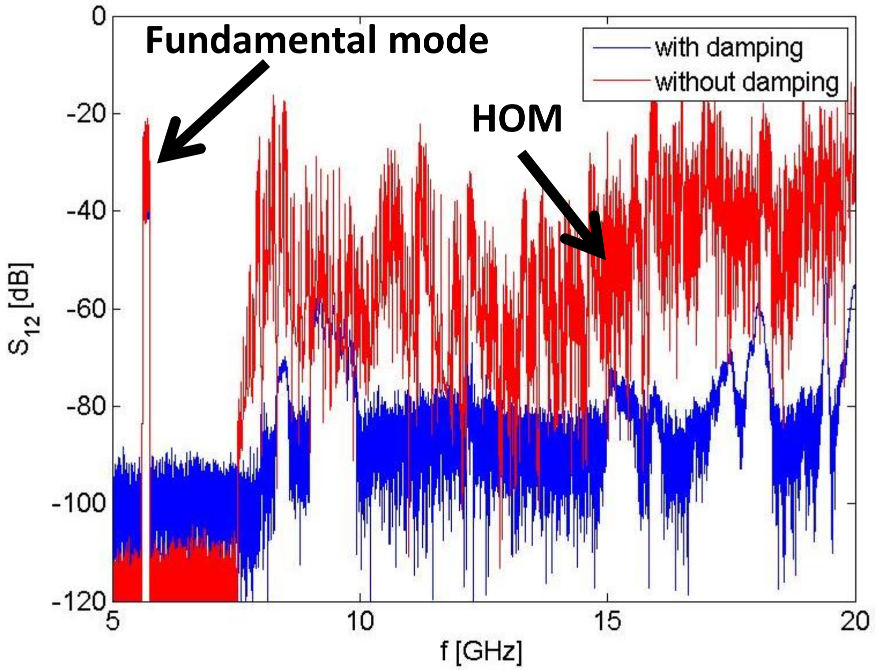
Courtesy of D. Alesini INFN-LNF

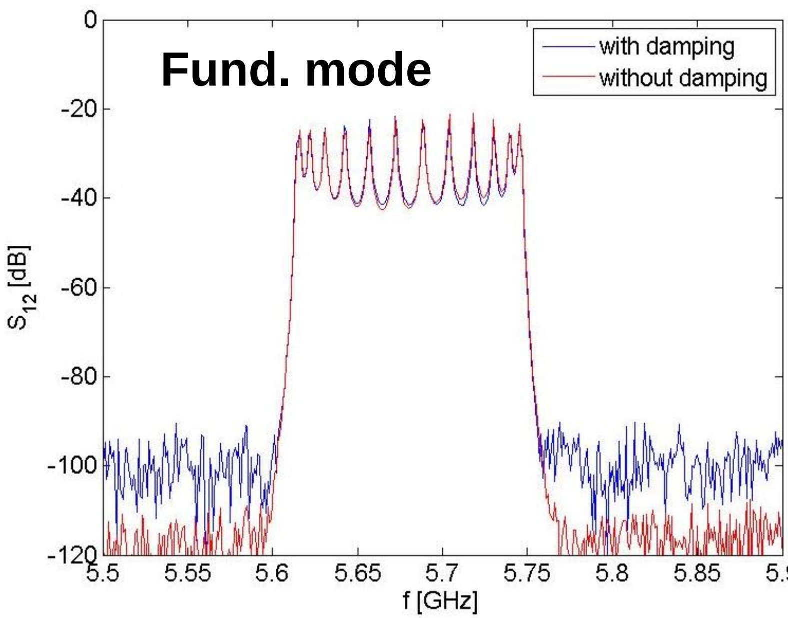
RF TEST OF 12 CELLS PROTOTYPE

Low power RF test in the 12 cells module with and without the SiC absorbers (antenna coupling).

No dumpers: HOM Q factor of few 1000s

With dumpers: HOM hardly measurable.

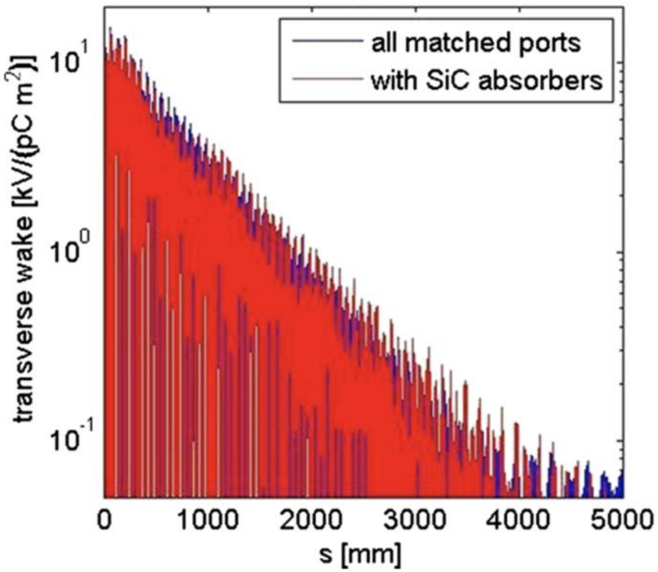


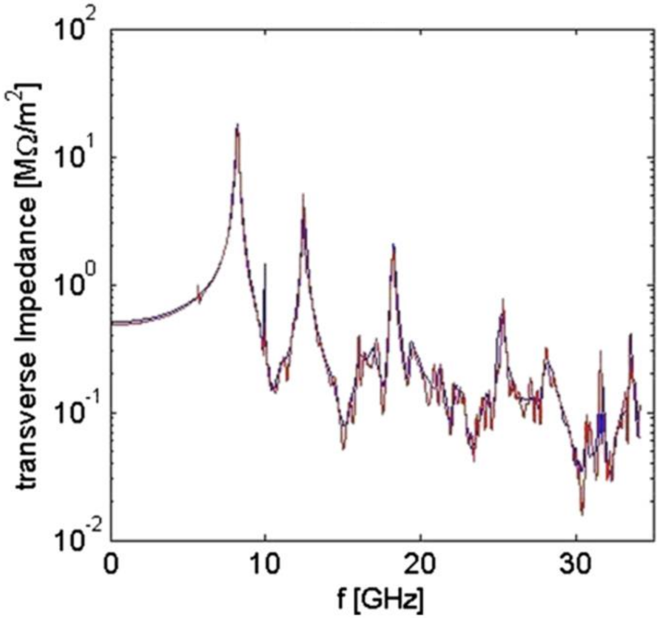


Courtesy of D. Alesini INFN-LNF

DUMPED C-BAND STRUCTURE IMPEDANCE

Example





Attenuation of the transverse wakefield more than 2 orders of magnitude after 16 ns (i.e. 4.8 m). The wakefield has been obtained with a beam sigma of 3 mm, an off axis of the beam of 2 mm and all perfect matched ports.

The quality factor of the first dipole modes are below 100, as can be calculated from the decay time of the wakefield or by the bandwidth of the corresponding impedance.

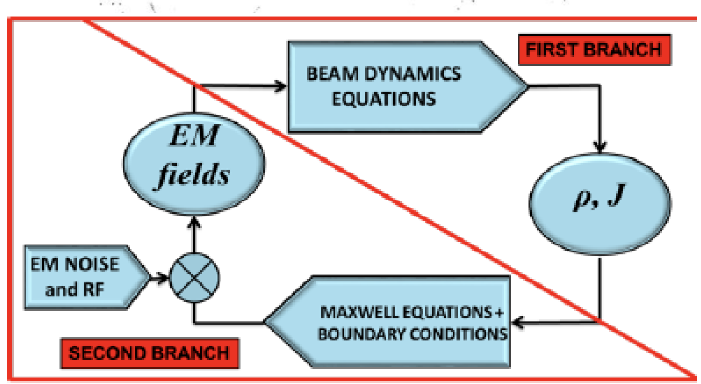
D. Alesini et al., Design of high gradient, high repetition rate damped C-band rf structures, PRAB, 20, 032004 (2017)

HYSTORICAL PERSECTIVE

ISR-RF/66-35
November 18, 1966

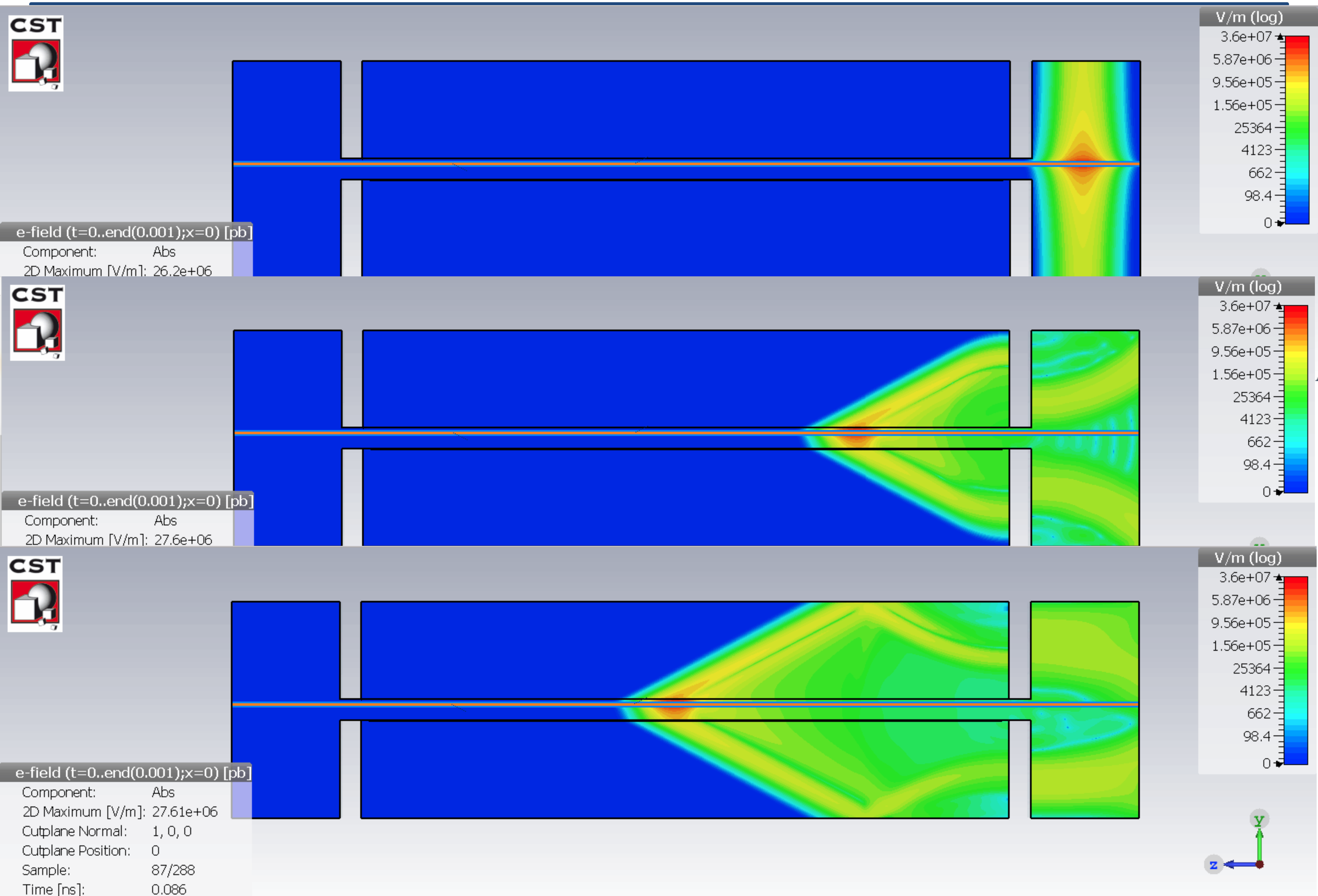
LONGITUDINAL INSTABILITY OF A COASTING BEAM ABOVE TRANSITION, DUE TO THE ACTION OF LUMPED DISCONTINUITIES.

by V.G. Vaccaro

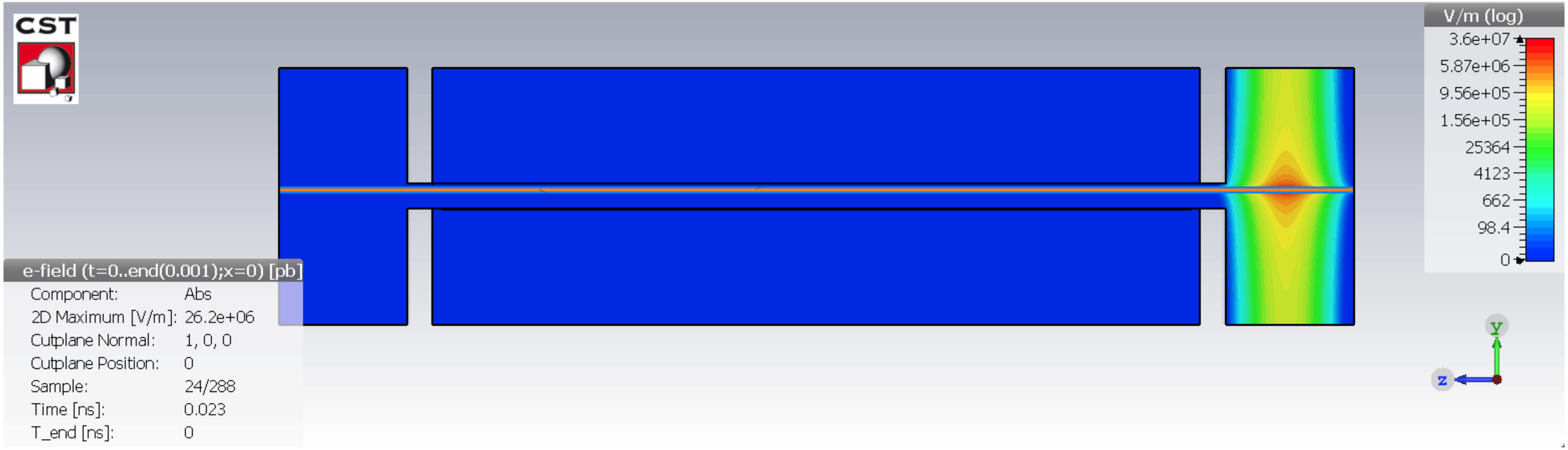


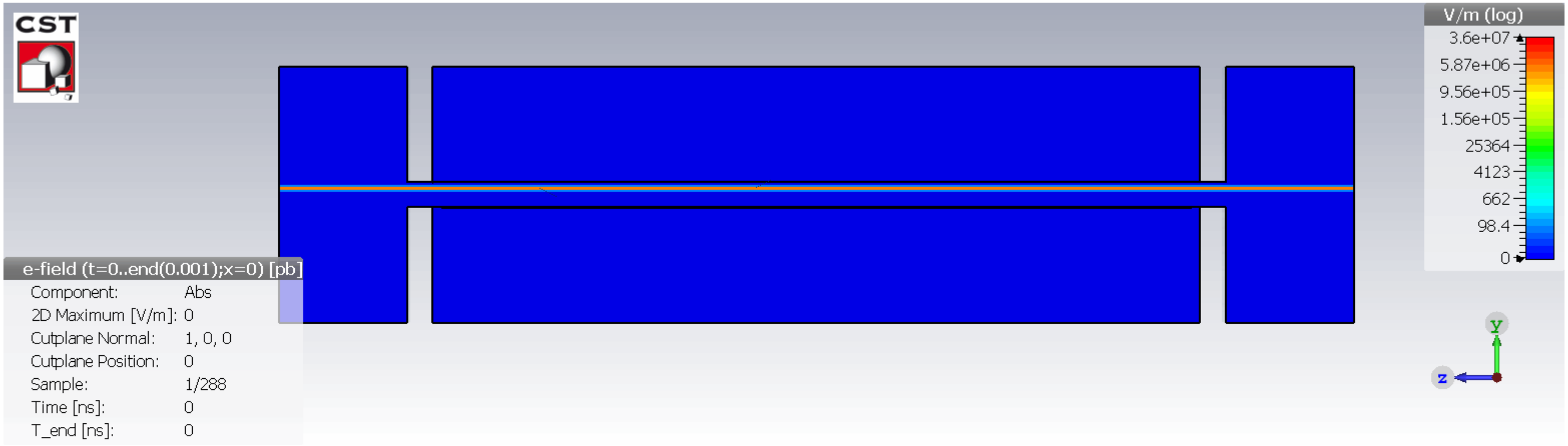
For his pioneering studies on **instabilities in particle beam physics**, the introduction of the **impedance concept** in storage rings and, in the course of his academic career, for **disseminating knowledge in accelerator physics** throughout many generations of young scientists.

WAKEFIELD IN DIELECTRIC STRUCTURES

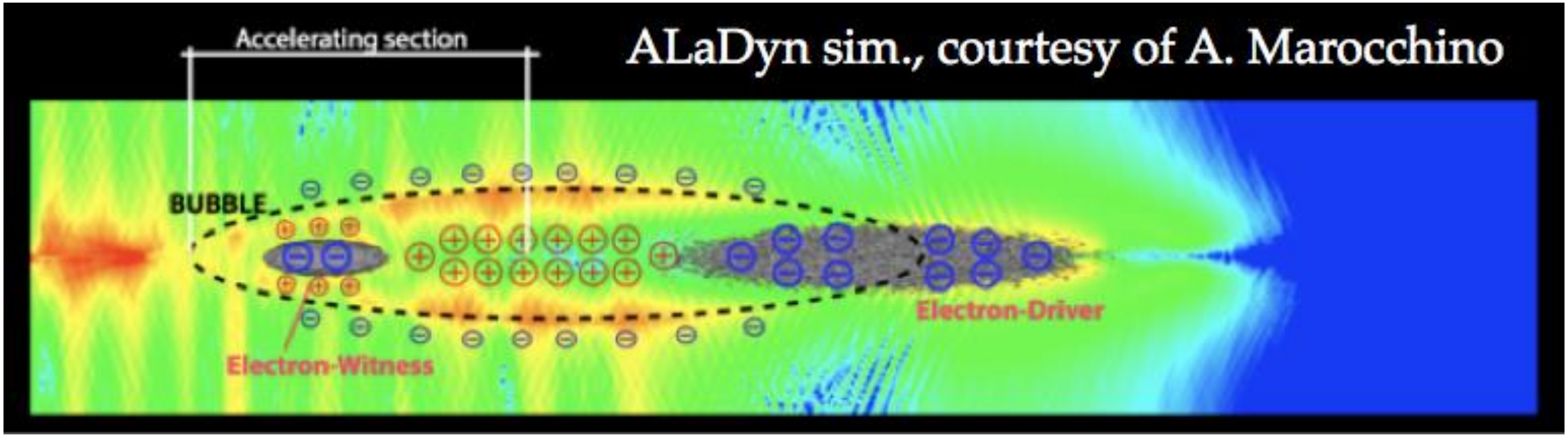


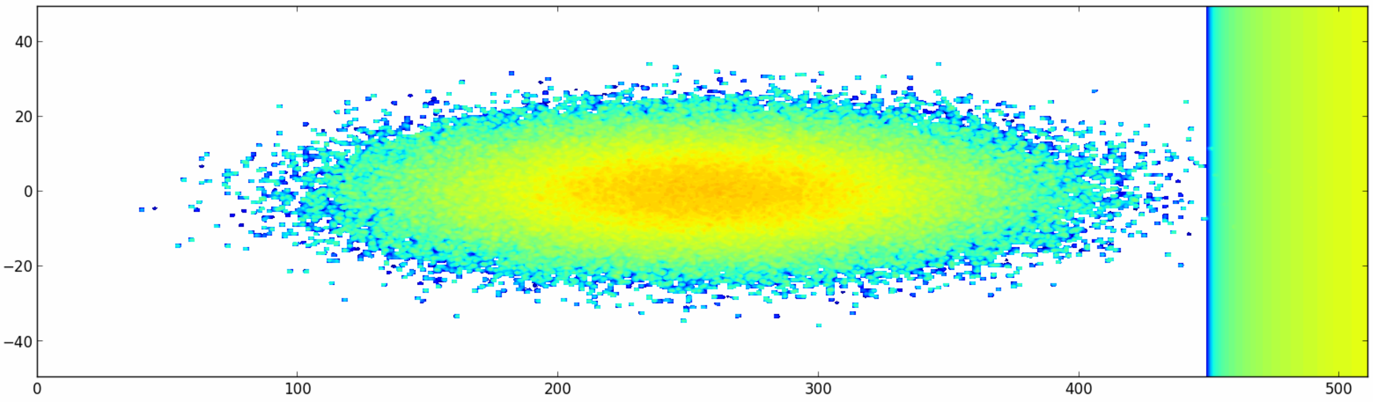
WAKEFIELD IN DIELECTRIC STRUCTURES





WAKEFIELD IN PLASMA





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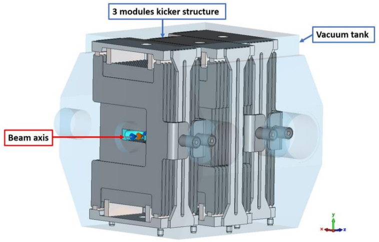
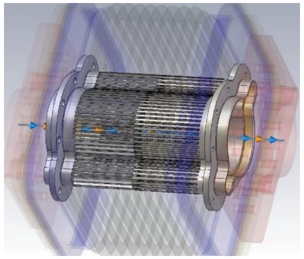
(Sapienza University, INFN-Roma)

CONCLUSIONS



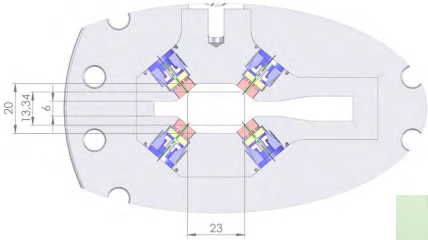
Reminded the basics and the main approximations

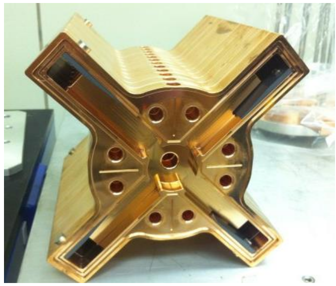
Given the definitions needed for the more advanced applications



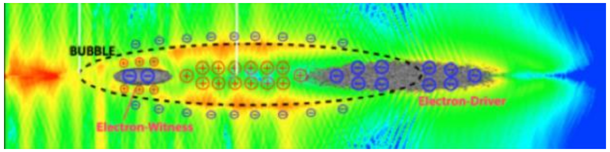
Examples:

- **Maintenance of working machines**
- **Future colliders**
- **Future light sources**
- **HOM damping**





Novel applications



Impedance optimisation is a lively field (even beyond my expectations …)

REFERENCES

SLAC-PUB-9574
November 2002

Lecture Notes on Topics in Accelerator Physics

Alex Chao
Stanford Linear Accelerator Center

These are lecture notes that cover a selection of topics, some of them under current research, in accelerator physics. I try to derive the results from first principles, although the students are assumed to have an introductory knowledge of the basics. The topics covered are:

1. Panofsky-Wenzel and Planar Wake Theorems
2. Echo Effect
3. Crystalline Beam
4. Fast Ion Instability
5. Lawson-Woodward Theorem and Laser Acceleration in Free Space
6. Spin Dynamics and Siberian Snakes
7. Symplectic Approximation of Maps
8. Truncated Power Series Algebra
9. Lie Algebra Technique for nonlinear Dynamics

The purpose of these lectures is not to elaborate, but to prepare the students so that they can do their own research. Each topic can be read independently of the others.

Many useful comments and help at the lecturing from Gennady Stupakov of SLAC and Jeff Holmes of ORNL are greatly appreciated.

*Work supported by Department of Energy Contract DE-AC03-76SF00515.

Physics of Collective Beam Instabilities in High Energy Accelerators

ALEXANDER WU CHAO
Superconducting Super Collider Laboratory
Dallas, Texas

Wiley-Interscience Publication
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New York / Chichester / Brisbane / Toronto / Singapore

Wake Fields and Impedance

L. Palumbo ◊*), V.G. Vaccaro @§) and M. Zobov ◊)

◊) INFN-LNF Frascati
§) INFN, Sezione di Napoli
*) Università di Roma "La Sapienza"
@) Università di Napoli "Federico II"

Abstract

Knowledge of the electromagnetic interaction between a beam and the surrounding vacuum chamber is necessary in order to optimize the accelerator performance in terms of stored current. Many instability phenomena may occur in the machine because of the fields produced by the beam and acting back on itself as in a feedback device. Basically, these fields produce an extra voltage and energy gain, affecting the longitudinal dynamics, and a transverse momentum kick which deflects the beam. In this paper we describe the main features of this interaction with typical machine components.

Lecture given at the
"CAS Advanced School on Accelerator Physics"
Rhodes – Greece 20 Sept. 1 Oct. (1994)

The CERN Accelerator School

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